

$$31535537 \quad 134$$

$$314646472 \quad 10100$$

$\hat{\beta} = (\mathbf{x}^T \mathbf{x})^{-1} \mathbf{x}^T \mathbf{y}$

$$\hat{\beta} = (\mathbf{x}^T \mathbf{x})^{-1} \mathbf{x}^T \mathbf{y} = (\mathbf{x}^T \mathbf{x}^{-1}) \mathbf{x}^T \mathbf{y} = (\mathbf{x}^T \mathbf{x}^{-1}) \mathbf{x}^T \mathbf{y}$$
.1c

$$\theta := \alpha^T \beta = \sum_{j=0}^p \alpha_j \beta_j \quad \alpha = (0, -1, 1, 0, \dots, 0)^T \text{ constant vector}$$

$$\Rightarrow \hat{\theta} = \beta_2 - \beta_1 \quad \Rightarrow \hat{\theta} = \alpha^T \hat{\beta} = \underbrace{\alpha^T (\mathbf{x}^T \mathbf{x})^{-1} \mathbf{x}^T}_{C^T \gamma} \gamma = C^T \gamma \quad .2$$

$$E(\hat{\theta}) = E(\hat{\beta}_2 - \hat{\beta}_1) = E(\hat{\beta}_2) - E(\hat{\beta}_1) = E((\mathbf{x}_2^T \mathbf{x}_2)^{-1} \mathbf{x}_2^T \mathbf{f}_2) - E((\mathbf{x}_1^T \mathbf{x}_1)^{-1} \mathbf{x}_1^T \mathbf{f}_1)$$

$$= A_2 E(\gamma_2) - A_1 E(\gamma_1) = A_2 E(\mathbf{x}_2 \mathbf{f}_2 + \varepsilon_2) - A_1 E(\mathbf{x}_1 \mathbf{f}_1 + \varepsilon_1)$$

$$= A_2 (E(\mathbf{x}_2 \mathbf{f}_2) + E(\varepsilon_2)) - A_1 (E(\mathbf{x}_1 \mathbf{f}_1) + E(\varepsilon_1)) =$$

$$= (\mathbf{x}_2^T \mathbf{x}_2)^{-1} \mathbf{x}_2^T \mathbf{f}_2 - (\mathbf{x}_1^T \mathbf{x}_1)^{-1} \mathbf{x}_1^T \mathbf{f}_1 = \beta_2 - \beta_1$$

$$\text{Cov}(\hat{\theta}) = \text{Cov}(\hat{\beta}_2 - \hat{\beta}_1) = \text{Cov}(\mathbf{x}_2^T \mathbf{x}_2)^{-1} \mathbf{x}_2^T \mathbf{f}_2 - (\mathbf{x}_1^T \mathbf{x}_1)^{-1} \mathbf{x}_1^T \mathbf{f}_1 \quad .3e$$

$$\text{Cov}(\hat{\beta}_2 \hat{\beta}_1) = \text{Cov}(\hat{\theta}) \quad \text{per } \hat{\theta} = \hat{\beta}_2 - \hat{\beta}_1 \quad \text{per } \hat{\theta} = \hat{\beta}_2 \hat{\beta}_1$$

$$A := (\mathbf{x}^T \mathbf{x})^{-1} \mathbf{x}^T \quad \hat{\theta} = A \mathbf{f} \quad \text{per } \hat{\theta} = \hat{\beta}_2 \hat{\beta}_1$$

$$\text{Cov}(\hat{\theta}) = \text{Cov}(\mathbf{f}) \mathbf{A}^T = \sigma^2 \mathbf{I} \mathbf{A}^T = \sigma^2 (\mathbf{x}^T \mathbf{x})^{-1}$$

$$\text{per } \hat{\theta} = \hat{\beta}_2 \hat{\beta}_1 \quad \text{per } \hat{\theta} = \hat{\beta}_2 \hat{\beta}_1 \quad \text{per } \hat{\theta} = \hat{\beta}_2 \hat{\beta}_1$$

$$C = \mathbf{x}^T (\mathbf{x}^T \mathbf{x})^{-1} \quad \hat{\theta} = C^T \gamma$$

der "normalen" Schreibweise ist $\hat{\theta} = C^T \gamma$

def. $\hat{\theta} = C^T \gamma$, $\hat{\theta} = C^T \gamma$ $\hat{\theta} = C^T \gamma$ $\hat{\theta} = C^T \gamma$ $\hat{\theta} = C^T \gamma$ $\hat{\theta} = C^T \gamma$

$$\text{Var}(\hat{\theta}) < \text{Var}(\hat{\theta}) \quad \text{per } \hat{\theta} = C^T \gamma \quad \hat{\theta} = C^T \gamma$$

per $\hat{\theta} = C^T \gamma$ $\hat{\theta} = C^T \gamma$ $\hat{\theta} = C^T \gamma$ $\hat{\theta} = C^T \gamma$

$$\hat{\theta} := \alpha^T \beta = \sum \alpha_j \beta_j \in \mathbb{R}$$

.c

$$\hat{\gamma}_i = x_i \hat{\beta}_i = A(x_i^T x_i)^{-1} x_i^T \gamma_i$$

$$h_i = \text{cov}(A(x_i^T x_i)^{-1} x_i^T \gamma_i, \gamma_i) = A \text{cov}(\gamma_i, \gamma_i) = x_i (x_i^T x_i)^{-1} x_i^T \sigma^2$$

$$\begin{matrix} \gamma \\ \text{cov}(\gamma, \gamma) \\ \text{cov}(x_i (x_i^T x_i)^{-1} x_i^T \gamma, \gamma) \\ \cdot b^2 \end{matrix}$$

2 問題

$$Q(v) = v^T A v \quad \forall v \in \mathbb{R}^n . \quad E(\gamma) = \mu \quad \text{if } \gamma = (\gamma_1, \gamma_2, \dots, \gamma_n) \in \mathbb{R}^n .$$

$$E(Q(\gamma)) = Q(\mu)^T E[Q(\gamma - \mu)] \quad \text{where } A \in \mathbb{R}^{m \times n} \text{ is constant}$$

$E(\varepsilon) = 0$ $\Rightarrow \gamma - \mu + \varepsilon \sim N(\mu, \Sigma)$ $\Rightarrow \mu^T \Sigma^{-1} \mu = \frac{1}{2} \text{tr}(\Sigma)$ $\Rightarrow \text{constant}$

$$\textcircled{2} \quad E[Q(\gamma)] = E(\gamma^T A \gamma) = E((\mu + \varepsilon)^T A (\mu + \varepsilon)) = \mu^T A \mu + E(\varepsilon^T A \varepsilon) = Q(\mu)$$

$$\textcircled{3} \quad E[Q(\gamma - \mu)] = E(Q(\varepsilon)) = E(\varepsilon^T A \varepsilon) : \quad A E(\varepsilon^T \varepsilon) = A E[\varepsilon^T] E(\varepsilon) = 0$$

$$\textcircled{4} \quad E[Q(\gamma - \mu)] + Q(\mu) = Q(\mu) = \mu^T A \mu = E[(\mu + \varepsilon)^T A (\mu + \varepsilon)] = E(\gamma^T A \gamma) = E(Q(\gamma - \mu))$$

... \Rightarrow $\gamma^T A \gamma$ $\sim N(\mu^T A \mu, \text{var}(\gamma^T A \gamma))$ $\Rightarrow \text{constant}$ $\Rightarrow \text{constant}$

also μ is constant when we drop ε from transpose \Rightarrow μ is constant

... \Rightarrow $\gamma^T A \gamma$ transpose \Rightarrow γ

3. 例題

$$y_i = \alpha + \beta x_i + \varepsilon_i \quad \varepsilon_i \sim \text{N}(0, \sigma^2) \quad \text{simple linear regression}$$

simple linear regression $y_i = \alpha + \beta x_i + \varepsilon_i$. Then

$$X = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix} \quad \theta = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix},$$

$\hat{\beta} = (x^T x)^{-1} x^T y$ は、誤差項 ε_i の影響を除いた回帰直線の傾き

$$Q(b) = \|y - xb\|^2 = \|y\|^2 + \|xb\|^2 - 2y^T xb$$

$$= \|y\|^2 + b^T x^T xb - 2(x^T y)^T b \quad / (x^T x) \hat{\beta} = x^T y$$

$$\frac{\partial Q}{\partial b} = 2b^T (x^T x) - 2(x^T y)^T \xrightarrow{\text{等式を解く}} \hat{\beta} = (x^T x)^{-1} x^T y$$

$$Q(\alpha, b) = \sum_{i=1}^n (\alpha + b x_i - y_i)^2 \quad \hat{\beta} = \arg \min_{b} Q(b), \quad \hat{\alpha} = \arg \min_{\alpha} Q(\alpha, \hat{\beta})$$

$$\hat{\beta}_i = \frac{\sum_{j=1}^n (y_j - \bar{y})(x_j - \bar{x})}{\sum_{j=1}^n (x_j - \bar{x})^2} = \frac{s_{ij}}{s_{xx}}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x}$$

B 例題

Quadratic regression: $Y_i = \alpha + \beta x_i + \gamma x_i^2 + \varepsilon_i \quad (i = 1, \dots, n)$.

$$p = 3, \quad X = \begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 \end{pmatrix}, \quad \theta = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}.$$

$$f = x \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$x^T x - \text{誤差項の影響}$$

$$(x^T x)^{-1} x^T f = (x^T x)^{-1} (x^T x) \theta \xrightarrow{\text{等式を解く}} \hat{\alpha} =$$

$$\hat{\beta} = [(x^T x)^{-1} x^T f]_1$$

$$\hat{\gamma} = [(x^T x)^{-1} x^T f]_2 \Rightarrow \hat{\gamma} = [(x^T x)^{-1} x^T f]_3$$

$$\hat{\gamma} = [(x^T x)^{-1} x^T f]_3$$

$$\text{cov}(\hat{\alpha}) = \text{cov}\left[\underbrace{(x^T x)^{-1} x^T}_{A^{-1}} \cdot \beta\right] = A \underbrace{\text{cov}(\beta)}_{\sigma^2 I_n} A^T = \sigma^2 A A^T$$

$$= \sigma^2 (x^T x)^{-1} x^T x^{-1} = [\sigma^2 (x^T x)^{-1}]_{11}$$

$\hat{\beta}_1 = \frac{[\sigma^2 (x^T x)^{-1}]_{11}}{\sigma^2 (x_i - \bar{x})^2} = \frac{\sum_{i=1}^n (x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{\sum_{i=1}^n (x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$

$$\begin{aligned} \text{var}(\hat{\beta}_1) &= \frac{\sum_{i=1}^n (x_i - \bar{x})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{1}{\sum_{i=1}^n (x_i - \bar{x})^2} \text{var}(\sum_{i=1}^n (x_i - \bar{x})) \\ &= \frac{1}{\sum_{i=1}^n (x_i - \bar{x})^2} \sum_{i=1}^n \text{var}(x_i) + \sum_{i \neq j} \text{cov}(x_i - \bar{x}, x_j - \bar{x}) = \\ &= \frac{\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \end{aligned}$$

$$\begin{aligned} \text{var}(\hat{\gamma}) &= \text{var}(\hat{\gamma} - \hat{\beta}_1 \bar{x}) = \text{var}(\hat{\gamma}) + \text{var}(\hat{\beta}_1 \bar{x}) - 2 \text{cov}(\hat{\gamma}, \hat{\beta}_1 \bar{x}) = \\ &= \frac{1}{n^2} \sum_{i=1}^n \text{var}(\gamma_i) + \bar{x}^2 \frac{\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2} - 2 \bar{x} \text{cov}(\hat{\gamma}, \hat{\beta}_1) = \\ &= \sigma^2 \left[\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \right] \end{aligned}$$

↓

VAR \rightarrow $\text{var}(\hat{\gamma}) \approx \text{var}(\hat{\beta}_1 \bar{x})$

$$\text{var}(\hat{\gamma}) = \sigma^2 \left[\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \right]$$

$$\text{Cov}(\bar{\gamma}, \hat{\beta}_1) = 0$$

↓

$$\begin{aligned} &= \text{Cov}(\beta_0 + \beta_1 \bar{x} + \bar{\epsilon}, \hat{\beta}_1) = \text{Cov}(\bar{\epsilon}, \hat{\beta}_1) = \\ &\text{Cov}(\bar{\epsilon}, \frac{\sum_{i=1}^n (x_i - \bar{x})(\beta_0 + \beta_1 x_i + \epsilon_i)}{\sum (x_i - \bar{x})^2}) = \text{Cov}(\bar{\epsilon}, \frac{\sum (x_i - \bar{x})\epsilon_i}{\sum (x_i - \bar{x})^2}) \\ &= \frac{1}{\sum (x_i - \bar{x})^2} \text{Cov}(\bar{\epsilon}, \sum_{i=1}^n (x_i - \bar{x})\epsilon_i) = \frac{1}{\sum (x_i - \bar{x})^2} \cdot \frac{1}{n} \sum \text{Cov}(\epsilon_i, (x_i - \bar{x})\epsilon_i) \\ &= \frac{1}{\sum (x_i - \bar{x})^2} \cdot \frac{1}{n} \sum_{j=1}^n \sum_{i=1}^n (x_i - \bar{x}) \text{Cov}(\epsilon_j, \epsilon_i) = 0 \end{aligned}$$

$$\text{Cov}(\hat{\alpha}, \hat{\beta}) = \text{Cov}(\bar{\gamma} - \hat{\beta}\bar{x}, \hat{\beta}) = \underbrace{\text{Cov}(\bar{\gamma}, \hat{\beta})}_{=0} - \bar{x} \text{Cov}(\hat{\beta}, \hat{\beta}) =$$

$$= -\bar{x} \left[\frac{\sigma^2}{\lambda \alpha' \Gamma(\lambda)} \right] = -\bar{x} [\sigma^2 (\bar{x}' \bar{x})^{-1}]_{B3}$$

$$\begin{aligned} \text{Cov}(\hat{\gamma}) &= \text{Cov}[\underbrace{(\bar{x}' \bar{x})^{-1} \bar{x}'}_{A} \cdot \underbrace{(\bar{\gamma})}_{B}] = A \underbrace{\text{Cov}(\bar{\gamma})}_{\sigma^2 I_n} A' = \sigma^2 A A' \\ &= \sigma^2 (\bar{x}' \bar{x})^{-1} \bar{x}' (\bar{x}' \bar{x})^{-1} = [\sigma^2 (\bar{x}' \bar{x})^{-1}]_{B3} \end{aligned}$$

2.6.2.2 ↗

A spin rpa \Rightarrow δ pva \rightarrow β
 $\hat{\beta} \sim N(\beta, \sigma^2(x^T x))$: sk $\sim N(0, 1)$ rk

: k \Rightarrow pka \approx 0.1

$$\frac{\hat{\beta}_j - \beta_j}{\sqrt{\text{var}(\hat{\beta}_j)}} = \frac{\hat{\beta}_j - \beta_j}{\sqrt{\sigma^2(x^T x)_{jj}}} \cdot \frac{\sqrt{\sigma^2(x^T x)_{jj}}}{\sqrt{\sigma^2(x^T x)_{jj}}} = \frac{(\hat{\beta}_j - \beta_j)/\sqrt{\sigma^2(x^T x)_{jj}}}{\sqrt{\sigma^2(x^T x)_{jj}}} =$$

$$\frac{(\hat{\beta}_j - \beta_j)/\sqrt{\sigma^2(x^T x)_{jj}}}{\sigma/\sqrt{x_{n+j}/(n-p+1)}} \sim \frac{N(0, 1)}{\sqrt{x_{n+j}/(n-p+1)}} \sim t_{n-p+1}$$

$$T_j = \frac{\hat{\beta}_j - \beta_j}{\sqrt{\sigma^2(x^T x)_{jj}}} \sim t_{n-p+1} \quad \text{by corollary 1.2.1}$$

$$1-\alpha = P\left(t_{n-p+1, \frac{\alpha}{2}} \leq T_j \leq t_{n-p+1, 1-\frac{\alpha}{2}}\right) =$$

$$P\left(\hat{\beta}_j - \sqrt{\sigma^2(x^T x)_{jj}} t_{n-p+1, 1-\frac{\alpha}{2}} \leq \beta_j \leq \hat{\beta}_j + \sqrt{\sigma^2(x^T x)_{jj}} t_{n-p+1, \frac{\alpha}{2}}\right)$$

$$\left[\hat{\beta}_j \pm \sqrt{\sigma^2(x^T x)_{jj}} \cdot t_{n-p+1, 1-\frac{\alpha}{2}} \right] \quad \hat{\beta}_j \pm \sigma \cdot t_{n-p+1, \frac{\alpha}{2}}$$

$\hat{\beta}_j = 0$ H_0 \wedge α

$$T_j = \frac{\hat{\beta}_j - 0}{\sqrt{\sigma^2(x^T x)_{jj}}} \sim t_{n-p+1}$$

$H_0: \gamma = 0$ \wedge α

. $\gamma \neq 0$ \wedge α \wedge $\gamma \neq 0$ \wedge α \wedge $\gamma \neq 0$ \wedge α

$H_0: \gamma = 0$ \wedge α

$$\left| \frac{\hat{\beta}_j}{\sqrt{\sigma^2(x^T x)_{jj}}} \right| \geq t_{n-p+1, 1-\frac{\alpha}{2}} \quad \text{rk}$$

Quiz 1

Menachem Sokolik and Idan Keipour

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```
My_lm <- function(X,Y){  
  #' @param X is matrix with n rows and p+1 columns  
  #' @param Y is vector with length n  
  # TODO: Complete the code.  
  #       Don't use any external libraries or functions for computing linear models.  
  X <- as.matrix(X)  
  X <- cbind(1,X) # n*(p+1)  
  Y <- as.matrix(Y)  
  n <- nrow(X)  
  p <- ncol(X) - 1  
  dof <- n-p-1  
  # t(X) is the transpose, %*% is matrix multiplication,  
  # solve takes the inverse  
  A <- solve(t(X) %*% X)  
  # this comes from looking at formula for beta hat and  
  # multiplying the additional terms needed  
  beta_hat <- A %*% t(X) %*% Y  
  Y_hat <- X %*% beta_hat  
  # just filling in formula  
  residuals <- Y - (Y_hat)  
  sigma2_hat <- t(residuals) %*% residuals/dof  
  # I've put as.numeric() at the end to convert it to a single scalar number  
  # instead of a 1 by 1 matrix.  
  sigma2_hat <- as.numeric(sigma2_hat)  
  # covariance matrix of estimated regression coefficients  
  # is just the estimated residual variance times solve(t(X) %*% X)  
  # we calculated earlier and stored as A  
  beta_cov <- sigma2_hat * A  
  # diag takes the diagonal of the matrix, sqrt makes it go  
  # from variance to standard deviation  
  beta_SE <- sqrt(diag(beta_cov))  
  
  T_stat <- beta_hat/beta_SE  
  P_Values <- 2*(1-pt(abs(T_stat),dof))  
  
  # residual sum of squares divided by total sum of squares  
  R_squared <- 1- sum((residuals)^2)/sum((Y - mean(Y))^2)  
  return(list(beta_hat, beta_cov, T_stat, P_Values, sigma2_hat, R_squared, dof, residuals, X,  
            Y, Y_hat))  
}
```

```
df <- read.csv("startups.csv")
```

```
|  
str(df)
```

```
## 'data.frame': 36 obs. of 5 variables:  
## $ R.D.Spend      : num 165349 162598 153442 131877 130298 ...  
## $ Administration : num 136898 151378 101146 99815 145530 ...  
## $ Marketing.Spend: num 471784 443899 407935 362861 323877 ...  
## $ State          : chr "New York" "California" "Florida" "New York" ...  
## $ Profit         : num 192262 191792 191050 156991 155753 ...
```

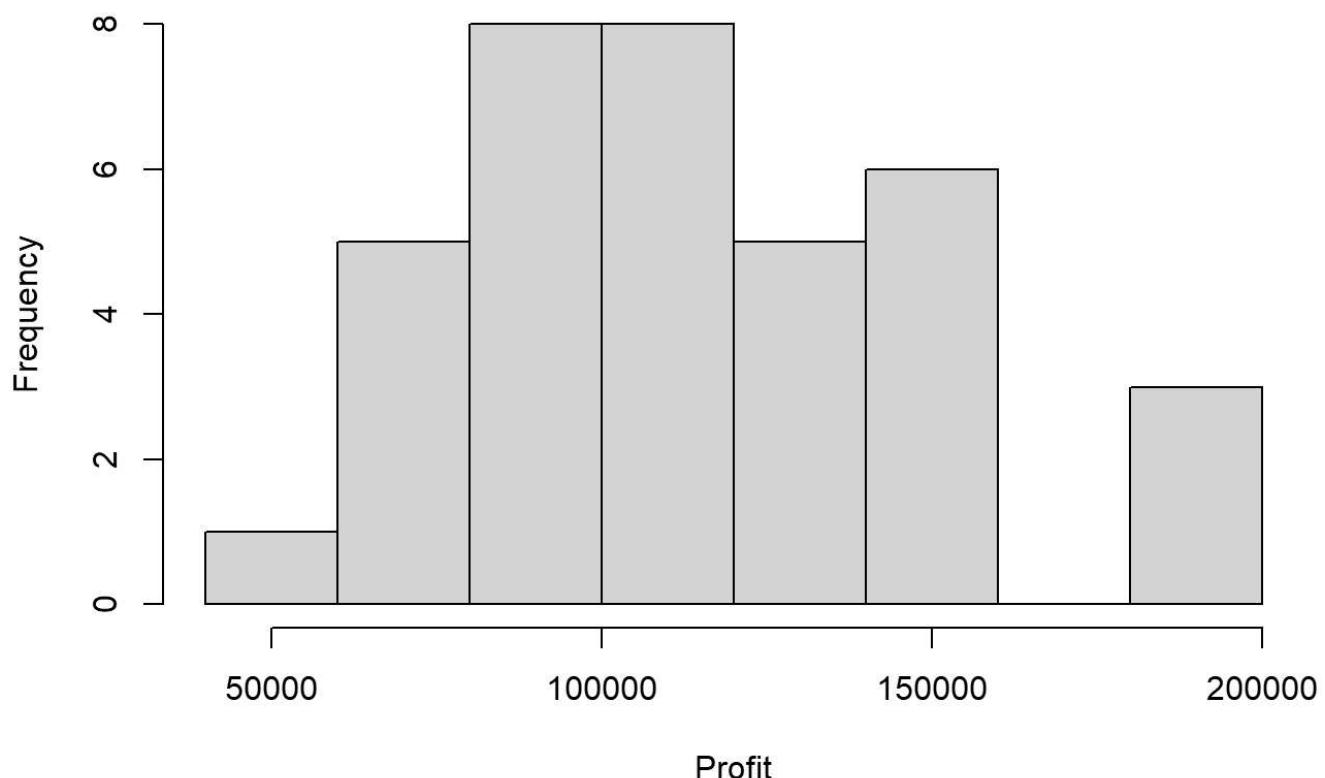
```
summary(df)
```

```
##    R.D.Spend     Administration   Marketing.Spend       State  
## Min.   : 1000   Min.   : 51283   Min.   : 1904   Length:36  
## 1st Qu.: 45837  1st Qu.:105078  1st Qu.:139922  Class :character  
## Median  : 74662  Median :122700   Median :239453  Mode  :character  
## Mean    : 76578  Mean   :123041   Mean   :224241  
## 3rd Qu.:105066 3rd Qu.:144485  3rd Qu.:300633  
## Max.   :165349  Max.   :182646   Max.   :471784  
  
##    Profit  
## Min.   : 49491  
## 1st Qu.: 95212  
## Median :108643  
## Mean   :115300  
## 3rd Qu.:136795  
## Max.   :192262
```

ii

```
hist(df$Profit, xlab = "Profit")
```

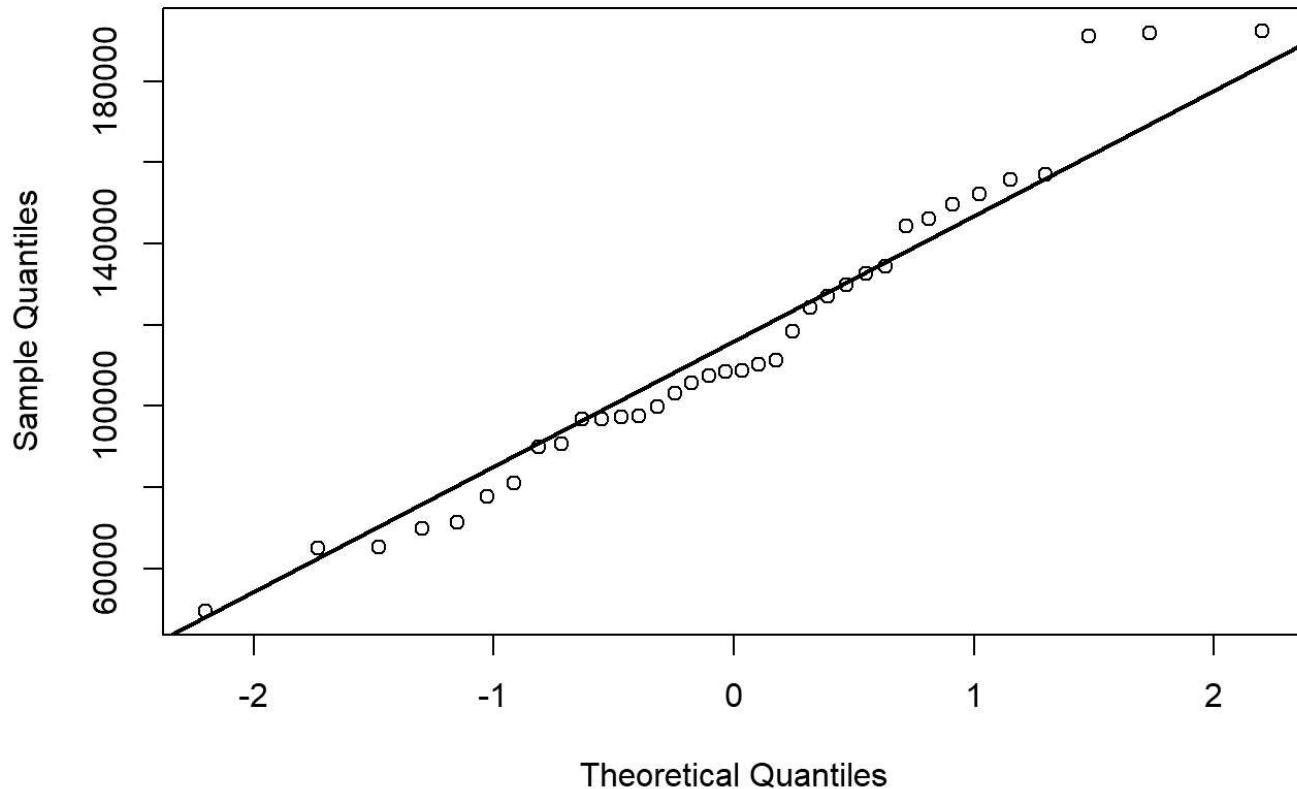
Histogram of df\$Profit



looks like a normal distribution but with long right tail. also can see that with the QQPlot to identify normality.

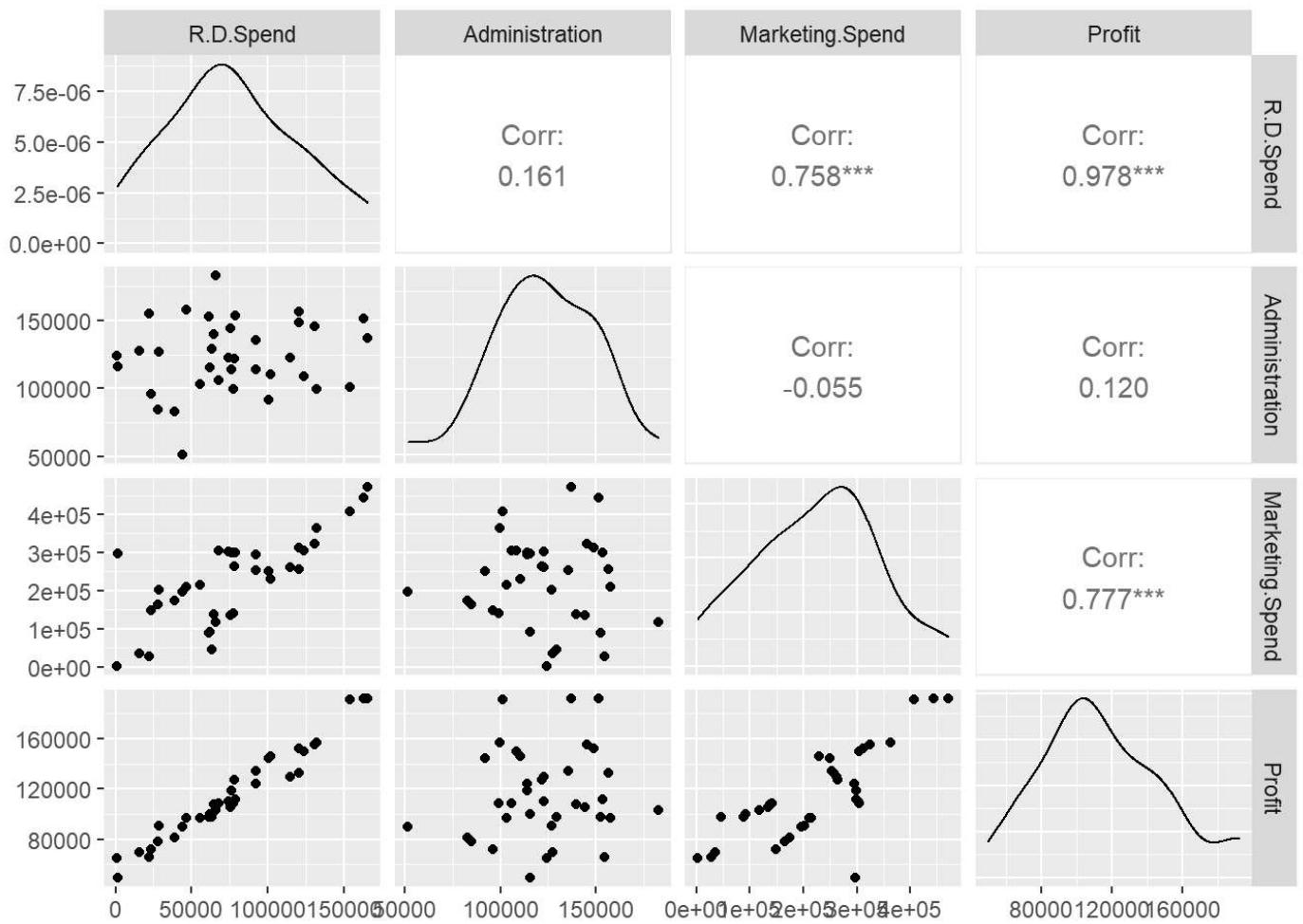
```
qqnorm(df$Profit)
qqline(df$Profit, lwd = 2)
```

Normal Q-Q Plot

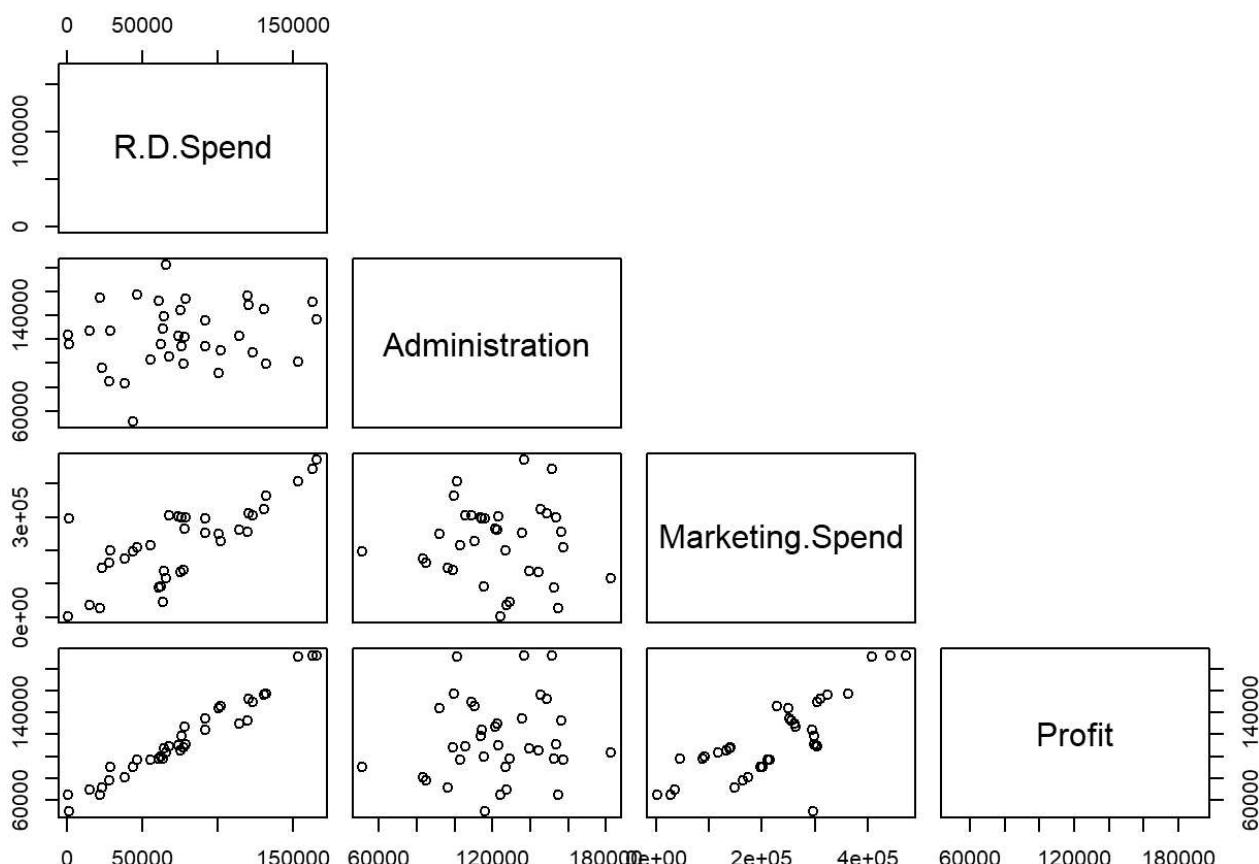


iii

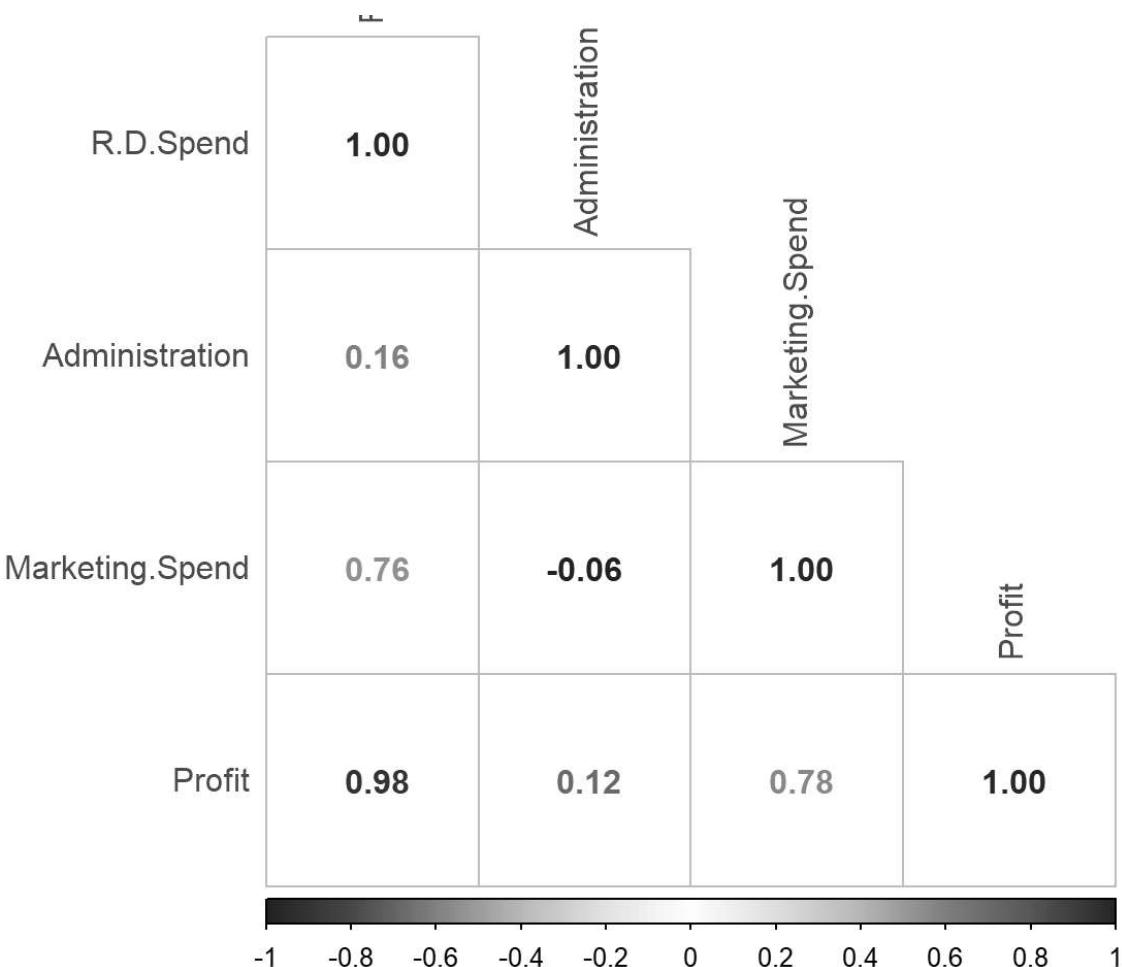
```
# automatic for the correlation coefficients.  
# The diagonal consists of the densities of the three variables and the upper panels consist  
# of the correlation coefficients between the variables.  
ggpairs(df[c(-4)])
```



```
pairs(~ R.D.Spend + Administration + Marketing.Spend + Profit, data = df, upper.panel = NULL)
```



```
# by hand
cor_df <- cor(df[c(-4)])
corrplot(cor_df, method="number", type = "lower" )
```



```
cor_df
```

```
##          R.D.Spend Administration Marketing.Spend      Profit
## R.D.Spend    1.0000000   0.16054446   0.75826916  0.9782919
## Administration 0.1605445   1.00000000  -0.05504912  0.1196753
## Marketing.Spend 0.7582692  -0.05504912   1.00000000  0.7770678
## Profit        0.9782919   0.11967533   0.77706784  1.0000000
```

iv

```
# matrix of profit
# dummy <- for k variables making k-1 categorical.
df$New_York <- ifelse(df$State == "New York", 1,0)
df$Florida <- ifelse(df$State == "Florida", 1,0)
Y <- df["Profit"]
X <- df %>% select(-c("Profit","State"))
My_reg <- My_lm(X,Y)
coeff <- My_reg[[1]]
rownames(coeff)[1] = "Intercept"
coeff
```

```

##                               Profit
## Intercept      5.633027e+04
## R.D.Spend     7.660746e-01
## Administration -2.616449e-02
## Marketing.Spend 2.384049e-02
## New_York       -2.939637e+03
## Florida        -2.792330e+03

```

V

```

t_stat <- My_reg[3]
t_stat

```

```

## [[1]]
##                               Profit
##                         8.1873071
## R.D.Spend      15.8224079
## Administration -0.5077559
## Marketing.Spend 1.3142079
## New_York        -0.8746709
## Florida         -0.9405045

```

VI

```

CI <- function(alpha, coeff, beta_cov, dof){
  CI_betas <- data.frame(beta=NA, lower_beta=NA, upper_beta=NA)
  for(i in 1:length(coeff)){
    beta <- rownames(coeff)[i]
    lower_beta <- coeff[i] - sqrt(beta_cov[i])*qt(1-alpha/2,dof)
    upper_beta <- coeff[i] + sqrt(beta_cov[i])*qt(1-alpha/2,dof)
    CI_betas <- rbind(CI_betas, c(beta, lower_beta, upper_beta))
  }
  return(CI_betas)
}

```

VII

```

alpha= 0.05
beta_cov <- diag(My_reg[[2]])
dof <- My_reg[[7]]
CI(alpha, coeff, beta_cov, dof)

```

	beta	lower_beta	upper_beta
## 1	<NA>	<NA>	<NA>
## 2	Intercept	42279.037676838	70381.5040675557
## 3	R.D.Spend	0.667193797647681	0.864955498705348
## 4	Administration	-0.131402078859794	0.0790731063466653
## 5	Marketing.Spend	-0.0132075109226804	0.060888492906907
## 6	New_York	-9803.4083213011	3924.13344461858
## 7	Florida	-8855.77633934312	3271.11644734578

We will be rejected H_0 when $|T_j|$ in absolute value is greater than $qt(dof)$. So when truth comes out we reject the null hypothesis otherwise we do not reject. Also, can not see by p.value test when p.value greater than alpha does not reject the null hypothesis, otherwise rejected. rejecting null hypothesis of intercept and for R.D spend.

```
abs(t_stat[[1]])>qt(0.975, dof)
```

```
##                  Profit
##                    TRUE
## R.D.Spend      TRUE
## Administration FALSE
## Marketing.Spend FALSE
## New_York        FALSE
## Florida         FALSE
```

```
P.value <- My_reg[[4]]
P.value
```

```
##                  Profit
##            3.866198e-09
## R.D.Spend    4.440892e-16
## Administration 6.153375e-01
## Marketing.Spend 1.987366e-01
## New_York      3.887019e-01
## Florida        3.544707e-01
```

question 4.c

```
new_df <- read.csv("Startups_test.csv")
new_df <- cbind("Intercept"=1,new_df)
new_df$New_York <- ifelse(new_df$State == "New York", 1,0)
new_df$Florida <- ifelse(new_df$State == "Florida", 1,0)
real_profit <- new_df$Profit
new_df<- new_df %>% select(-c("Profit","State"))
new_df <- as.matrix(mutate(new_df,"exp_profit" = as.matrix(new_df)%%coeff))
new_df
```

```

##      Intercept R.D.Spend Administration Marketing.Spend New_York Florida
## [1,] 1 46014.02     85047.44    205517.6 1 0
## [2,] 1 134615.46   147198.87   127716.8 0 0
## [3,] 1 72107.60    127864.55   353183.8 1 0
## [4,] 1 94657.16    145077.58   282574.3 1 0
## [5,] 1 20229.59    65947.93    185265.1 1 0
## [6,] 1 144372.41   118671.85   383199.6 1 0
## [7,] 1 28754.33    118546.05   172795.7 0 0
## [8,] 1 65605.48    153032.06   107138.4 1 0
## [9,] 1 142107.34   91391.77    366168.4 0 1
## [10,] 1 93863.75   127320.38   249839.4 0 1
##      exp_profit
## [1,] 91315.23
## [2,] 158649.21
## [3,] 113705.00
## [4,] 128845.91
## [5,] 71579.33
## [6,] 170021.36
## [7,] 79376.07
## [8,] 102199.55
## [9,] 168741.19
## [10,] 128069.60

```

```

n <- length(real_profit)
RMSE <- sqrt((1/n)*sum((new_df[,7] - real_profit)^2))
RMSE

```

```

## [1] 7561.162

```