Report on Model 4: Caltech Ducted Fan System

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1 Introduction

The goal of this project is to control the Caltech Ducted Fan System (model 4) using reinforcement learning, Vaisbord-Milhstein Algorithm, and linear quadratic optimal control. A scaled replica of a flight vehicle's longitudinal axis is the Caltech Ducted Fan System (1). The goal is to remove a pilot from a flying vehicle so that it can carry out a certain duty. State and control penalty matrices Q and R, respectively, are used to model the system in matrices A and B. The goal is to use reinforcement learning to iteratively optimize control performance and LQ control to obtain optimal stabilization.

2 System Definition and Parameters

System matrices A and B are defined as follows:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\gamma & -\frac{d}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{d}{m} & 0 \\ 0 & 0 & -\frac{m \cdot g \cdot l}{J} & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & \frac{1}{m} \\ \frac{r}{J} & 0 \end{bmatrix}$$

The state penalty matrix Q and control penalty matrix R are:

$$Q = I_{6\times 6}, \quad R = I_{2\times 2}$$

Initial state $x_0 = [1; 1; 1; 1; 1; 1]$ and desired pole locations for the initial stabilizing feedback:

$$\lambda_{\text{desired}} = [-5, -10, -15, -20, -25, -30]$$

3 LQ Optimal Control Solution

Optimal feedback matrix F_{opt} is calculated as:

$$F_{\text{opt}} = \begin{bmatrix} 1.0000 & -0.0000 & 1.0576 & 2.2237 & -0.0000 & 0.9232 \\ -0.0000 & 1.0000 & -0.0000 & -0.0000 & 1.8100 & -0.0000 \end{bmatrix}$$

The optimal cost J_{opt} for the initial state is:

$$J_{\text{opt}} = 15.4630$$

4 Reinforcement Learning Iterative Solution

The initial stabilizing feedback F_0 is:

$$F_0 = \begin{bmatrix} 20.2541 & -0.7376 & 4.7516 & 17.1630 & -0.3533 & -0.2202 \\ -1.2740 & 1.0496 & -0.2191 & -0.9448 & 1.8327 & 0.0749 \end{bmatrix}$$

Eigenvalues of the closed-loop system with F_0:

$$\text{Eigenvalues of } (A - BF_0) = [-30, -5, -10, -25, -20, -15]$$

Performance metric J_i over 10 iterations:

$$\begin{aligned} J_1 &= 1165300 \\ J_2 &= 491340 \\ J_3 &= 194360 \\ J_4 &= 72614 \\ J_5 &= 26200 \\ J_6 &= 9338.8 \\ J_7 &= 3350.1 \\ J_8 &= 1230.8 \\ J_9 &= 472.9988 \\ J_{10} &= 195.2850 \end{aligned}$$

5 Simulation of State Responses

The state response $x(t)$ was simulated over a time period of 0.01 seconds using the optimal feedback F_{opt}. The path for each state variable show that the system converges and stabilizes under the control law.

6 Results and Observations

After ten iterations, the RL algorithm shows rapid convergence in the performance parameter J_i, reaching values that are close to J_{opt}. The system can converge at the 18^{th} iteration, reaching the same value as J_{opt}, by increasing the number of iterations from 10 to 20. The system stabilizes under both LQ and RL-based control techniques, according to the state variable plots. Furthermore, since all real portions are negative, the closed-loop system's eigenvalues with F_0 likewise validate stability.

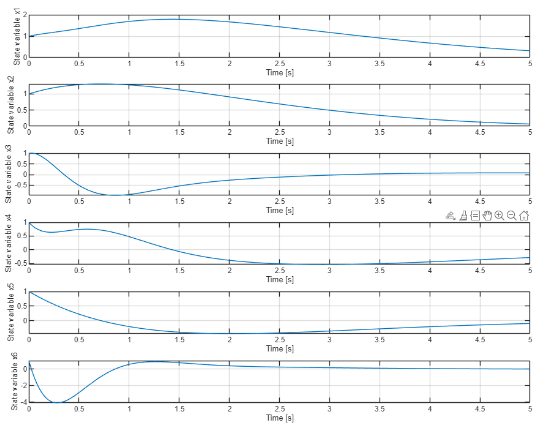


Figure 1: State Variables

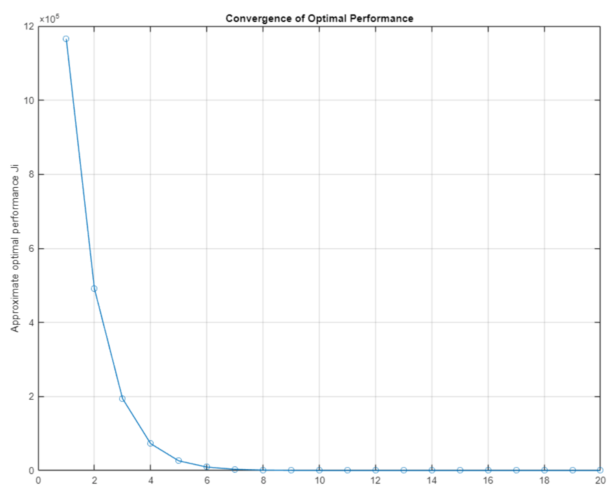


Figure 2: J_i Convergence Graph

7 Conclusion

The project successfully implemented both LQ optimal control and Vaisbord-Milhstein Algorithm for the Caltech Ducted Fan system. The Vaisbord-Milhstein Algorithm demonstrated efficient convergence towards the optimal control policy. The results of this project show how efficient the Vaisbord-Milhstein Algorithm can be for quick convergence to solve the stability control problem for the Caltech Ducted Fan System.

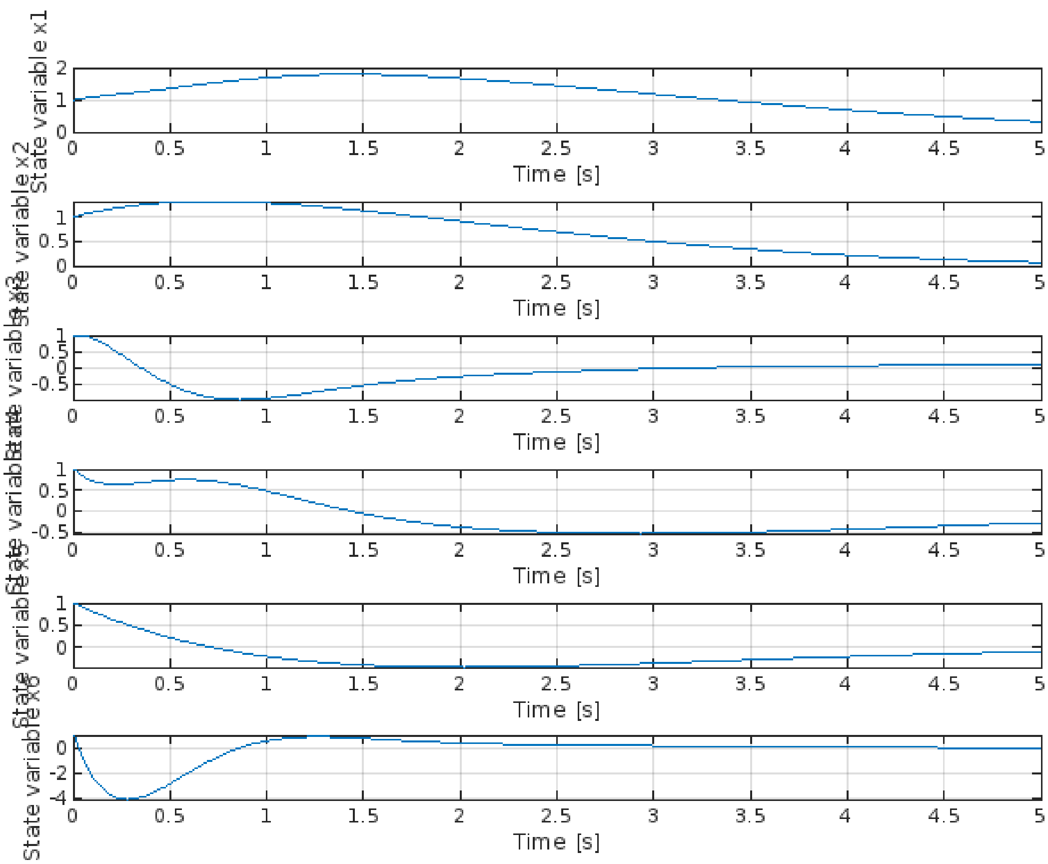
A Appendix: Reference

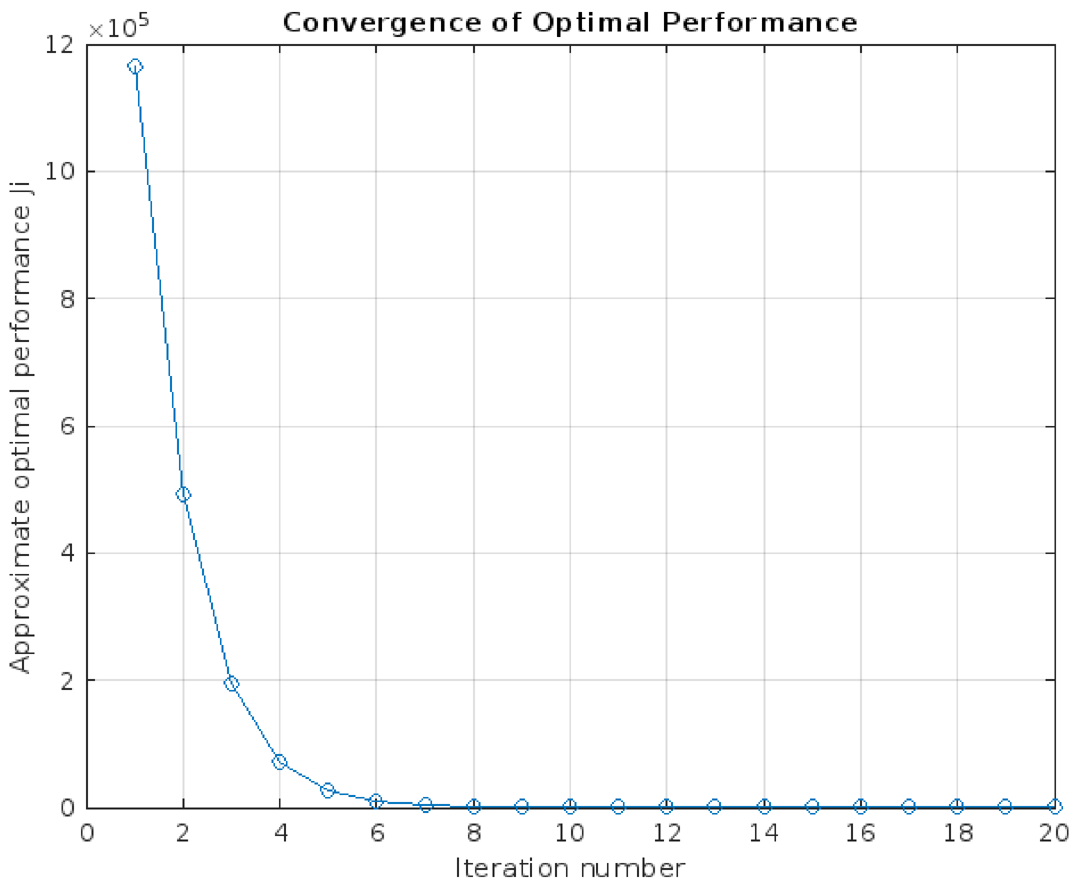
Milam, M., and Murray, R. M. (1999). "A Testbed for Nonlinear Flight Control Techniques: The Caltech Ducted Fan." In *1999 Conference on Control Applications.* (1)

```
clear all;

J = 0.0475;
m = 1.5;
r = 0.25;
g = 10;
gamma = 0.51;
d = 0.2;
l = 0.05;

A = [0 0 0 1 0 0;
     0 0 0 0 1 0;
     0 0 0 0 0 1;
     0 0 -gamma -d/m 0 0;
     0 0 0 0 -d/m 0;
     0 0 -m*g*l/J 0 0 0];

B = [0 0;
     0 0;
     0 0;
     1/m 0;
     0 1/m;
     r/J 0];

x0 = [1; 1; 1; 1; 1; 1];

Q = eye(size(A,1));
R = eye(size(B,2));

CO = ctrb(A, B);
rankCO = rank(CO);

if rankCO < size(A,1)
    error('The system is not fully controllable.');
end

[Fopt, Popt] = lqr(A, B, Q, R);
Jopt = 0.5 * x0' * Popt * x0;

ev = eig(A);
disp('Eigenvalues of A:');
disp(ev);

lambda_desired = [-5; -10; -15; -20; -25; -30];
F0 = place(A, B, lambda_desired);

evF0 = eig(A - B * F0);
disp('Eigenvalues of (A - B * F0):');
disp(evF0);

R_inv = inv(R);
iterations = 20;
```

```
Ji = zeros(1, iterations);

for i = 1:iterations
    Pi = lyap((A - B * F0)', Q + F0' * R * F0);
    Ji(i) = 0.5 * x0' * Pi * x0;
    F0 = R_inv * B' * Pi;
end

time = 0:0.01:5;
x = zeros(size(A,1), length(time));

for j = 1:length(time)
    x(:, j) = expm((A - B * Fopt) * time(j)) * x0;
end

figure;
for k = 1:size(A, 1)
    subplot(size(A,1), 1, k);
    plot(time, x(k, :));
    grid on;
    xlabel('Time [s]');
    ylabel(['State variable x', num2str(k)]);
end

figure;
plot(1:iterations, Ji, 'o-');
grid on;
xlabel('Iteration number');
ylabel('Approximate optimal performance Ji');
title('Convergence of Optimal Performance');
```

```
Eigenvalues of A:
   0.0000 + 0.0000i
  -0.1333 + 0.0000i
   0.0000 + 3.9736i
   0.0000 - 3.9736i
   0.0000 + 0.0000i
  -0.1333 + 0.0000i

Eigenvalues of (A - B * F0):
  -30.0000
   -5.0000
  -10.0000
  -25.0000
  -20.0000
  -15.0000
```


State variable x1
State variable x2
State variable x3
State variable x4
State variable x5
State variable x6
Time [s]
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



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