

**Course: Machine Learning Techniques**  
**Practice Questions**  
**Week 8**

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1. (1 point) (Multiple select) In hard margin SVM, maximizing  $\frac{2}{\|w\|}$  is equivalent to
- A. Maximizing  $\frac{\|w\|}{2}$
  - B. Minimizing  $\|w\|$
  - C. Minimizing  $\|w\|^2$
  - D. Maximizing  $\|w\|^2$

**Answer:** B,C

2. (1 point) (Multiple select) Which of the following are true statements for SVM:
- A. The hyper plane changes if any one of the support vector points is removed from dataset.
  - B. The hyper plane remains same if any one of the support vector points is removed from dataset.
  - C. The hyper plane changes if new data points are added in dataset. These new points are not candidates for support vectors.
  - D. The hyper plane remains the same if new data points are added in dataset. These new points are not candidates for support vectors.

**Answer:** A,D

3. (1 point) Suppose we have eight data points along with their labels as shown in the table. Plot these data points and identify the support vectors.

Index	Data Points	Labels
a	(2,1)	-1
b	(2,-1)	-1
c	(0,2)	-1
d	(4,0)	1
e	(5,1)	1
f	(6,2)	1
g	(-1,0)	-1
h	(5,-1)	1

- A. a,b,d
- B. a,b,e
- C. b,c,d

D. c,d,g

E. c,g,h

F. b,e,h

**Answer:** B

Solution:

By looking at the data, the support vectors are-

$$S_1 = (2, 1), S_2 = (2, -1), S_3 = (4, 0)$$

4. (1 point) Consider the following data set and the labels.

Data Points	Labels
(3,1)	1
(3,-1)	1
(6,1)	1
(6,-1)	1
(1,0)	-1
(0,1)	-1
(0,-1)	-1
(-1,0)	-1

A.  $w=(1,0)$  and  $b=-2$

B.  $w=(1,1)$  and  $b=-4$

C.  $w=(2,2)$  and  $b=2$

D.  $w=(2,1)$  and  $b=1$

**Answer:** A

**Solution:** Using SVM algorithm, find the weight  $w$  and bias  $b$  for the hyper-plane.

Solution:

By looking at the data, the support vectors are-

$$S_1 = (1, 0), S_2 = (3, 1), S_3 = (3, -1)$$

For vector representation we have to add augmented bias

$$\tilde{s}_1 = (1, 0, 1) \quad \tilde{s}_2 = (3, 1, 1) \quad \tilde{s}_3 = (3, -1, 1)$$

Now we have to solve the following system of equation:

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_1 + \alpha_2 \tilde{s}_1 \cdot \tilde{s}_2 + \alpha_3 \tilde{s}_1 \cdot \tilde{s}_3 = -1 \quad (1)$$

$$\alpha_1 \tilde{s}_2 \cdot \tilde{s}_1 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_2 + \alpha_3 \tilde{s}_2 \cdot \tilde{s}_3 = 1 \quad (2)$$

$$\alpha_1 \tilde{s}_3 \cdot \tilde{s}_1 + \alpha_2 \tilde{s}_3 \cdot \tilde{s}_2 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_3 = 1 \quad (3)$$

We will get,

$$\alpha_1 = -3.5\alpha_2 = 0.75\alpha_3 = 0.75$$

$$\text{Hence } W = \sum \alpha_i \tilde{s}_i = (1, 0, -2)w = (1, 0) \text{ and } b = -2$$

(Common data Q4, Q5) Use Lagrange Multipliers to find the global maximum and minimum values of  $f(x, y) = x^2 + 2y^2 - 4y$

subject to the constraint

$$x^2 + y^2 = 9.$$

5. (1 point) What is the global maximum value?

**Answer:** 30

6. (1 point) What is the global minimum value?

**Answer:** 5

**Solution**

$$g(x, y) = x^2 + y^2$$

Then solving the equation

$\Delta f(x, y) = \lambda \Delta g(x, y)$  along with the constraint, we get the following system of equation-

$$2x = \lambda 2y$$

$$4y - 4 = \lambda 2y$$

$$x^2 + y^2 = 9$$

Solving (1) we get 2 solution  $x=0$  or  $\lambda = 1$

For  $x=0$ ,  $y = \pm 3$

For  $\lambda = 1$

$$y=2 \text{ and } x=\pm\sqrt{5}$$

Now evaluating the function at these 4 points we find that,

the global maximum is 30, which occurs at  $(0, 3)$ , and the global minimum is 5, which occurs at the two points  $(\pm\sqrt{5}, 2)$ .

7. (1 point) Let  $K(x, y)$  and  $K'(x, y)$  be kernels then  $K+K'$  is also a kernel.

A. True

B. False

**Answer:** A

8. (1 point) Let  $f : \mathbb{R} \rightarrow \mathbb{R}^3$  be defined by

$$f(x) = (1, \sqrt{2}x, x^2)$$

Which of the following will be the most suitable kernel to find  $\langle f(x), f(y) \rangle$ .

- A.  $K(x, y) = (\langle x, y \rangle)^2$
- B.  $K(x, y) = (1 + \langle x, y \rangle)^2$
- C.  $K(x, y) = (\langle x, y \rangle - 1)^2$
- D.  $K(x, y) = (\langle x, y \rangle + \langle y, x \rangle)$

**Answer:** B

9. (1 point) Let  $K(x, y) = fK'(x, y)$  where  $f > 0$ . Let  $\phi$  and  $\phi'$  be the feature composition corresponding to  $K$  and  $K'$  respectively such that  $\phi(x) = g\phi'(x)$  then  $g =$

- A.  $f$
- B.  $\sqrt{f}$
- C.  $f^2$

**Answer:** B

10. (1 point) Let  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^9$  be defined by

$$f(x_1, x_2, x_3) = (x_1x_1, x_1x_2, x_1x_3, x_2x_1, x_2x_2, x_2x_3, x_3x_1, x_3x_2, x_3x_3)$$

Which of the following will be the most suitable kernel to find  $\langle f(x), f(y) \rangle$ .

- A.  $K(x, y) = (\langle x, y \rangle)^2$
- B.  $K(x, y) = (1 + \langle x, y \rangle)^2$
- C.  $K(x, y) = (\langle x, y \rangle - 1)^2$
- D.  $K(x, y) = (\langle x, y \rangle + \langle y, x \rangle)$

**Answer:** A

11. (1 point) (Multiple select) Consider the following data set and the labels.

Data Points	Labels
(2,0)	1
(0,2)	1
(-2,0)	1
(0,-2)	1
(1,1)	-1
(1,-1)	-1
(-1,1)	-1
(-1,-1)	-1

Since the data-points are not linearly separable, use the following transformation to identify which of the following are support vectors in the new space.

$$f(x_1, x_2) = \begin{cases} (6 - x_1 + (x_1 - x_2)^2, 6 - x_2 + (x_1 - x_2)^2), & \sqrt{x_1^2 + x_2^2} \geq 2 \\ (x_1, x_2), & \text{otherwise} \end{cases}$$

- A. (8,10)
- B. (10,8)
- C. (12,10)
- D. (1,1)

**Answer:** A,B,D

**Solution:**

After transformation, the new points will be

Data Points	Labels
(8,10)	1
(10,8)	1
(12,10)	1
(10,12)	1
(1,1)	-1
(1,-1)	-1
(-1,1)	-1
(-1,-1)	-1

Plotting the points will reveal that A, B and D are the support vectors.

12. (1 point) ----- are sometimes used when classifying text documents or DNA sequences.
- A. Exponential Trick
  - B. RBF kernel
  - C. String kernel
  - D. Linear kernel

**Answer:** C