**Course: Machine Learning Techniques**
**Graded Questions**
**Week 8**

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1. (1 point) Which of the following is true about support vectors-
   a. Support vectors are the data points that lie closest to the decision surface.
   b. Support vectors have direct bearing on the optimum location of the decision surface.

   A. Only a is True.

   B. Only b is True.

   C. Both are True.

   D. Both are False.

   **Answer:** C

   (Common data for Q2, Q3) Consider the following data set and the labels.

| Data Points | Labels |
|---|---|
| (2,1) | -1 |
| (2,-1) | -1 |
| (0,2) | -1 |
| (4,0) | 1 |
| (5,1) | 1 |
| (6,2) | 1 |
| (-1,0) | -1 |
| (5,-1) | 1 |

   Using SVM algorithm, find the weight w and bias b for the hyper-plane.

2. (1 point) Select the correct weight vector from the following.

   A. w=(1,0)

   B. w=(2,1)

   C. w=(2,4)

   D. w=(2,0)

   **Answer:** A

3. (1 point) Select the correct bias value from the following.

   A. b=-2

   B. b=-3

   C. b=-4

   D. b=-5

**Answer:** B

**Solution:**

By looking at the data, the support vectors are-

$s_1 = (2, 1), s_2 = (2, -1), s_3 = (4, 0)$

For vector representation we have to add augmented bias

$\tilde{s}_1 = (2, 1, 1)$ $\tilde{s}_2 = (2, -1, 1)$ $\tilde{s}_3 = (4, 0, 1)$

Now we have to solve the following system of equation:

$$\alpha_1 \tilde{s}_1.\tilde{s}_1 + \alpha_2 \tilde{s}_1.\tilde{s}_2 + \alpha_3 \tilde{s}_1.\tilde{s}_3 = -1 \tag{1}$$

$$\alpha_1 \tilde{s}_2.\tilde{s}_1 + \alpha_2 \tilde{s}_2.\tilde{s}_2 + \alpha_3 \tilde{s}_2.\tilde{s}_3 = -1 \tag{2}$$

$$\alpha_1 \tilde{s}_3.\tilde{s}_1 + \alpha_2 \tilde{s}_3.\tilde{s}_2 + \alpha_3 \tilde{s}_3.\tilde{s}_3 = 1 \tag{3}$$

We will get,

$\alpha_1 = -3.25\alpha_2 = -3..25\alpha_3 = 3.5$

Hence W=$\sum \alpha_i \tilde{s}_i = (1, 0, -3)$

w=(1,0) and b=-3

(Common data for Q4, Q5) Consider the following data set and the labels.

| Data Points | Labels |
|---|---|
| (2,2) | 1 |
| (2,-2) | 1 |
| (-2,2) | 1 |
| (-2,-2) | 1 |
| (1,1) | -1 |
| (1,-1) | -1 |
| (-1,1) | -1 |
| (-1,-1) | -1 |

Since the data-points are not linearly seperable, use the following transformation and then using SVM algorithm, find the weight w and bias b for the hyper-plane.

$$f(x_1, x_2) = \begin{cases} (4 - x_2 + |x_1 - x_2|, 4 - x_1 + |x_1 - x_2|), & \sqrt{{x_1}^2 + {x_2}^2} > 2 \\ (x_1, x_2), & \text{otherwise} \end{cases}$$

4. (1 point) Select the correct weight vector from the following.

   A. w=(2,2)

B. w=(3,1)

C. w=(1,1)

D. w=(2,3)

**Answer:** C

5. (1 point) Select the correct bias value from the following.

A. b=-2

B. b=-3

C. b=-4

D. b=-5

**Answer:** B

**Solution:**

After transformation, the new points will be

| Data Points | Labels |
|---|---|
| (2,2) | 1 |
| (10,6) | 1 |
| (6,10) | 1 |
| (6,6) | 1 |
| (1,1) | -1 |
| (1,-1) | -1 |
| (-1,1) | -1 |
| (-1,-1) | -1 |

By looking at the data, the support vectors are- $s_1 = (2, 2), s_2 = (1, 1)$

For vector representation we have to add augmented bias

$\tilde{s}_1 = (2, 2, 1)$ $\tilde{s}_2 = (1, 1, 1)$

Now we have to solve the following system of equation:

$$\alpha_1 \tilde{s}_1.\tilde{s}_1 + \alpha_2 \tilde{s}_1.\tilde{s}_2 = 1 \tag{4}$$

$$\alpha_1 \tilde{s}_2.\tilde{s}_1 + \alpha_2 \tilde{s}_2.\tilde{s}_2 = -1 \tag{5}$$

We will get,

$\alpha_1 = -7\alpha_2 = 4$

Hence W=$\sum \alpha_i \tilde{s}_i = (1, 1, -3)$

w=(1,1) and b=-3

6. (1 point) Suppose we have eight data points along with their labels as shown in the table. Plot these data points and identify the support vectors.

| Index | Data Points | Labels |
|---|---|---|
| A | (3,1) | 1 |
| B | (3,-1) | 1 |
| C | (6,1) | 1 |
| D | (6,-1) | 1 |
| E | (1,0) | -1 |
| F | (0,1) | -1 |
| G | (0,-1) | -1 |
| H | (-1,0) | -1 |

A. A,B,D

B. A,B,E

C. B,C,D

D. A,B,E

E. C,G,H

F. B,E,H

**Answer:** D

**Solution**

After plotting these points, we find that there are three support vectors namely,

$S_1 = (1, 0), S_2 = (3, 1), S_3 = (3, -1)$

(Common data Q7,Q8) Use Lagrange Multipliers to find the global maximum and minimum values of $f(x, y) = x^2 + 2y^2 - 4y$ on the unit circle.

7. (1 point) What is the global maximum value?

**Answer:** 2.25

range 2,3

8. (1 point) What is the global minimum value?

**Answer:** 0

Solution

$g(x, y) = x^2 + y^2$

Then solving the equation

$\Delta f(x, y) = \lambda \Delta g(x, y)$ along with the constraint, we get the following system of equation-

$2x + 1 = \lambda 2x$

4y=$\lambda 2y$

x$^2 + y^2 = 1$

Solving (2) we get 2 solution y=0 or $\lambda = 2$

For y=0, x= $\pm 1$

For $\lambda = 2$

$x = \frac{1}{2}$ and $y = \pm\frac{\sqrt{3}}{2}$

Now evaluating the function at these 4 points we find that, the global maximum is $\frac{9}{4}$, which occurs at two points $(\frac{1}{2}, \pm\frac{\sqrt{3}}{2})$, and the global minimum is 0, which occurs at (-1, 0).

9. (1 point) Let K(x,y) anf K'(x,y) be kernels then which of the following will also be a kernel.

   A. K+K'
   B. $\alpha K : \alpha \in \mathbb{R}$
   C. $\alpha$K : $\alpha > 0$
   D. $\alpha_1 K + \alpha_2 K' : \alpha_i > 0$

   **Answer:** A,C,D

10. (1 point) Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by

    $f(x_1, x_2) = ({x_1}^2, {x_2}^2, \sqrt{2}x_1x_2)$

    Let $x = (1, 2)$ and $y = (-2, 3)$

    Use a suitable kernel to find $< f(x), f(y) >$ .

    Which of the following is the correct answer?

    A. 10
    B. 6
    C. 16
    D. 3

    **Answer:** C

    **Solution** 16

    Kernel used : $(< x, y >)^2$

11. (1 point) Let $f : \mathbb{R}^3 \to \mathbb{R}^9$ be defined by

    $f(x_1, x_2, x_3) = (x_1x_1, x_1x_2, x_1x_3, x_2x_1, x_2x_2, x_2x_3, x_3x_1, x_3x_2, x_3x_3)$

    Let $x = (5, 6, 7)$ and $y = (2, 3, 4)$

    Use a suitable kernel to find $< f(x), f(y) >$. Enter the correct answer.

    **Answer:** 3136

    **solution** 3136