

# 1 Logic

A few interesting formal relations:

1.  $\neg(\neg p) \Leftrightarrow p$
2.  $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$
3.  $\neg(\forall x p(x)) \Leftrightarrow \exists x \neg p(x)$

# 2 Sets

1. Equality of sets:  $x = y \Leftrightarrow \forall z (z \in x \Leftrightarrow z \in y)$ .
2. De Morgan's laws:

$$\left[ \bigcap_{i \in I} A_i \right]^c = \bigcup_{i \in I} A_i^c,$$
$$\left[ \bigcup_{i \in I} A_i \right]^c = \bigcap_{i \in I} A_i^c.$$

3. Distributive property:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C),$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

# 3 Functions

1. Suppose  $f: A \rightarrow B$ , and  $C \subseteq A$ . Definition:  $f(C) = \{f(c) \mid c \in C\}$ .
2. Suppose  $D \subseteq B$ . Definition:  $f^{-1}(D) = \{a \in A \mid f(a) \in D\}$ . This is overloaded by the inverse function, when the inverse function exists.
3. Definition:  $f$  is *injective* or *one-to-one* if, for all  $x, y \in A$ ,

$$f(x) = f(y) \Rightarrow x = y.$$

4. Definition:  $f$  is *surjective* or *onto* if  $f(A) = B$ .

# 4 Groups