Dimensionality Reduction and Reduced Order Modeling for Traveling Wave Physics

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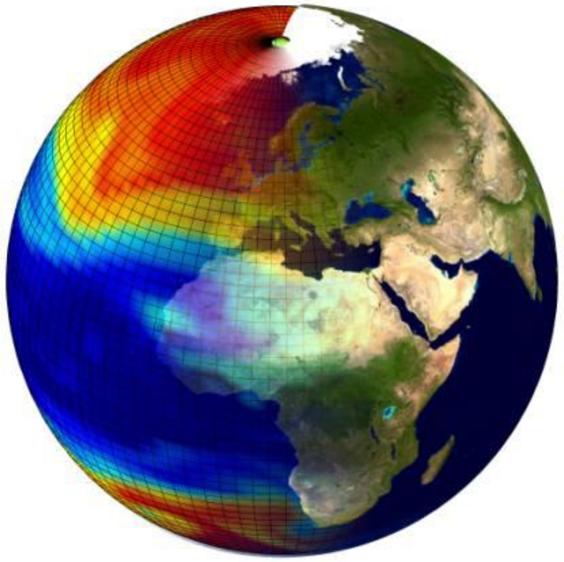
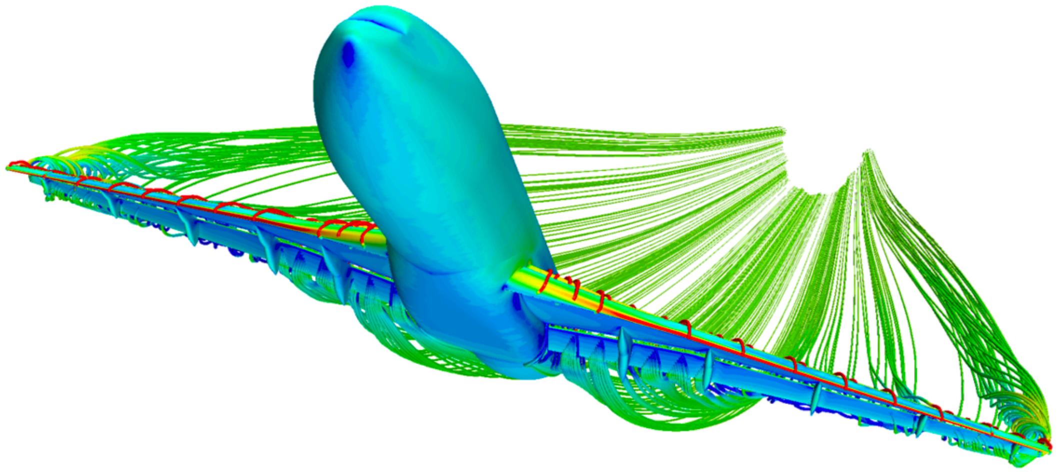
UNIVERSITY of WASHINGTON

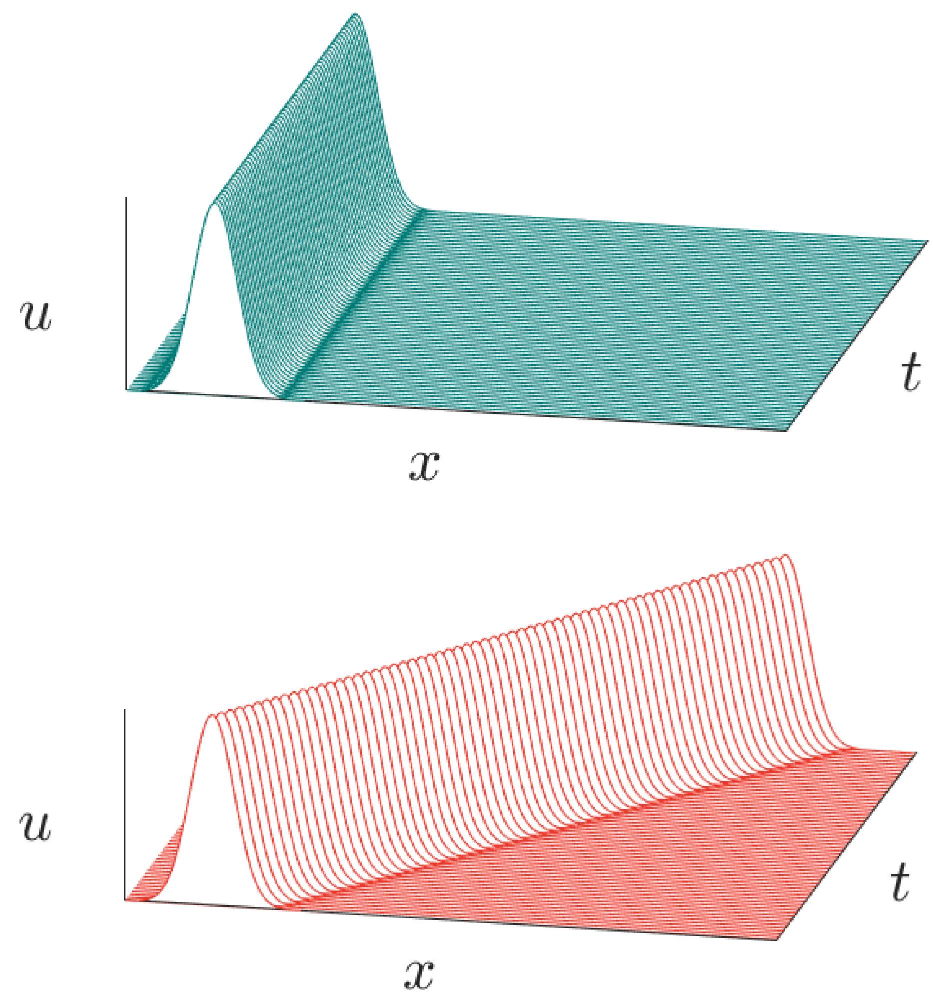



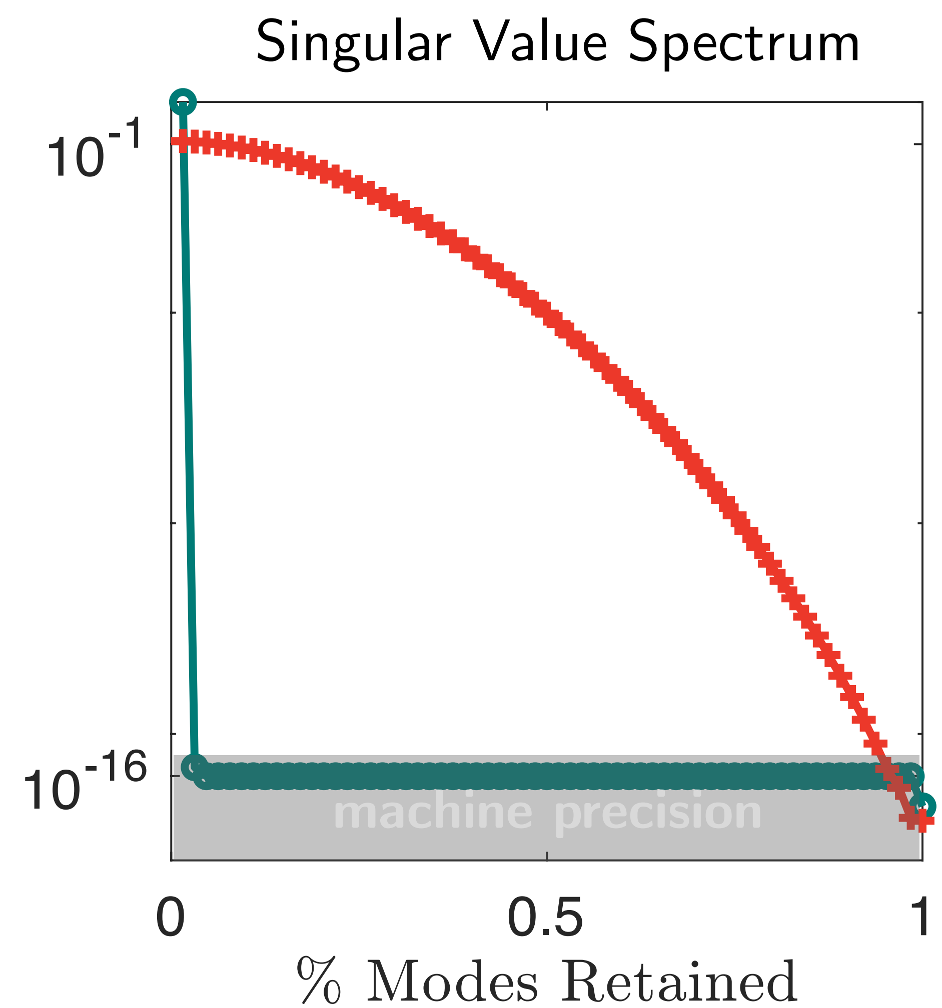
We want to understand, predict, and control multi-scale, chaotic, and nonlinear fluids systems at large time and space scales




SVD/POD Inappropriate for Traveling Waves

$$u(x,t) = \sum \alpha(t)\phi(x)$$



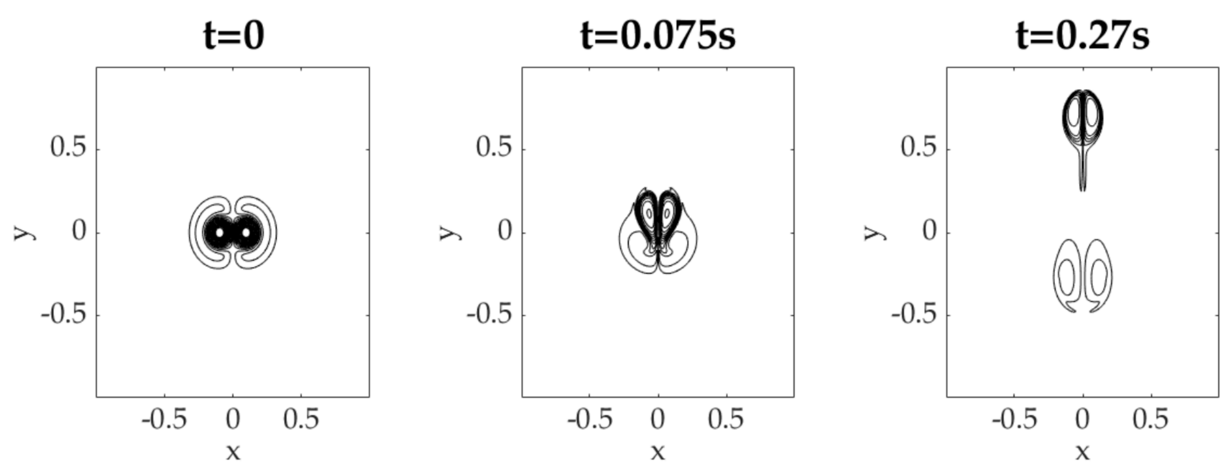


Background

Main concept: move waves into traveling frame where they appear stationary

Current methods:

- Utilize underlying equations or templates
- Highly supervised
- Not adapted to non-constant wave speeds
- Not adapted to rapidly-changing wave shapes

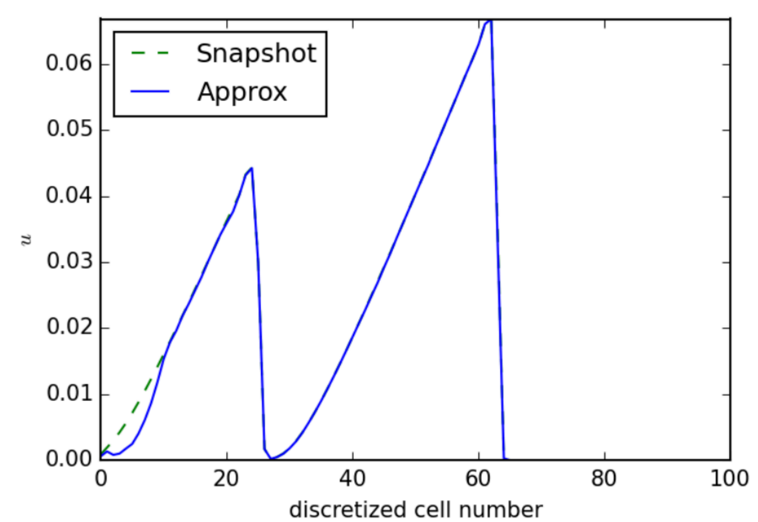


Reiss et al, 2018

Kirby and Armbruster, 1992
Rowley and Marsden, 2000
Lucia et al., 2014
Mojgani and Balajewicz, 2017
Cagniart, Maday, Stamm, 2019

Desired methods:

+ Equation-free
+ Unsupervised
+ Handle non-constant wave speeds
+ Handle rapidly-changing wave shapes
+ Interpretable



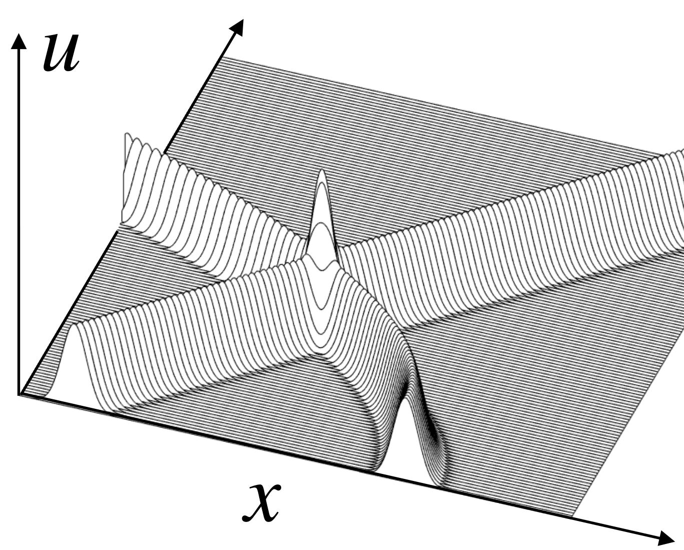
Rim, Moe, LeVeque, 2018

Our Approach: UnTWIST

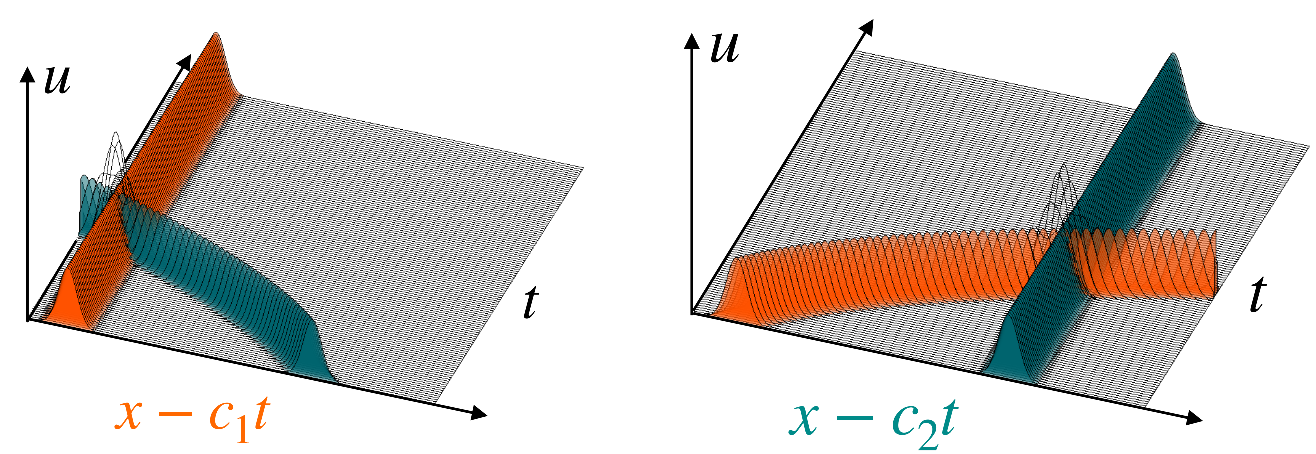
Unsupervised Traveling Wave Identification with Shifting and Truncation

Leverage machine learning to detect interpretable wave speeds

Simultaneous separation and model discovery



Traveling wave data



$$u = u_1(x - c_1 t) + u_2(x - c_2 t)$$

Low-dimensional models

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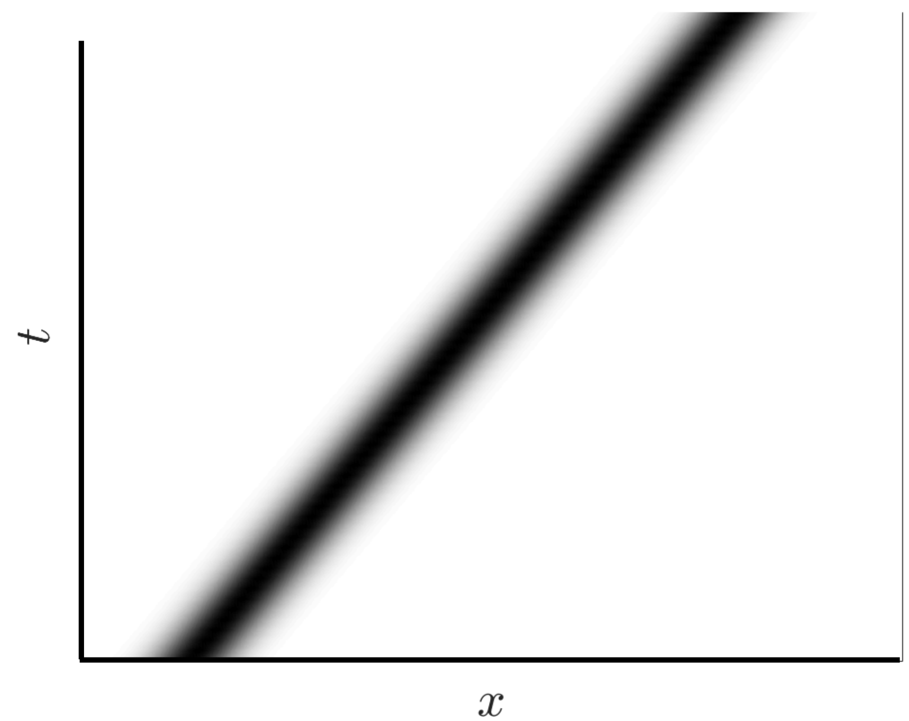
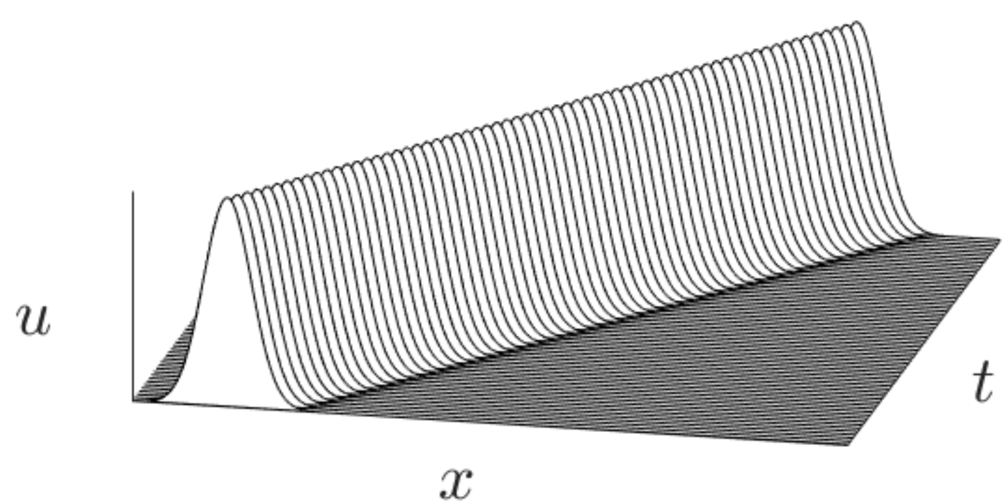
Single Wave Example





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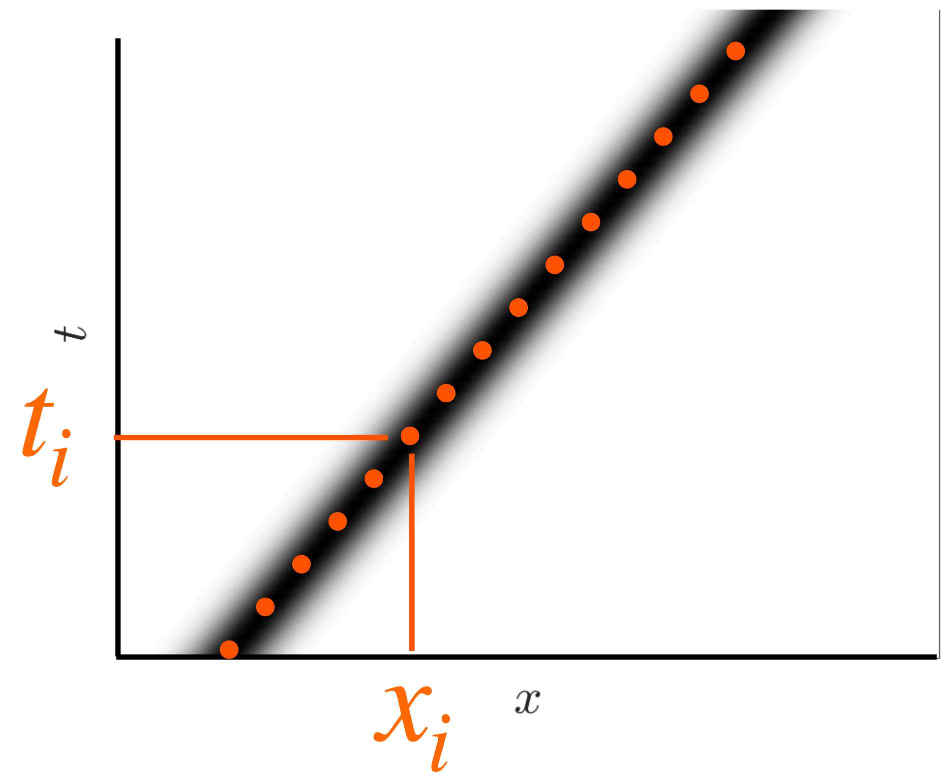
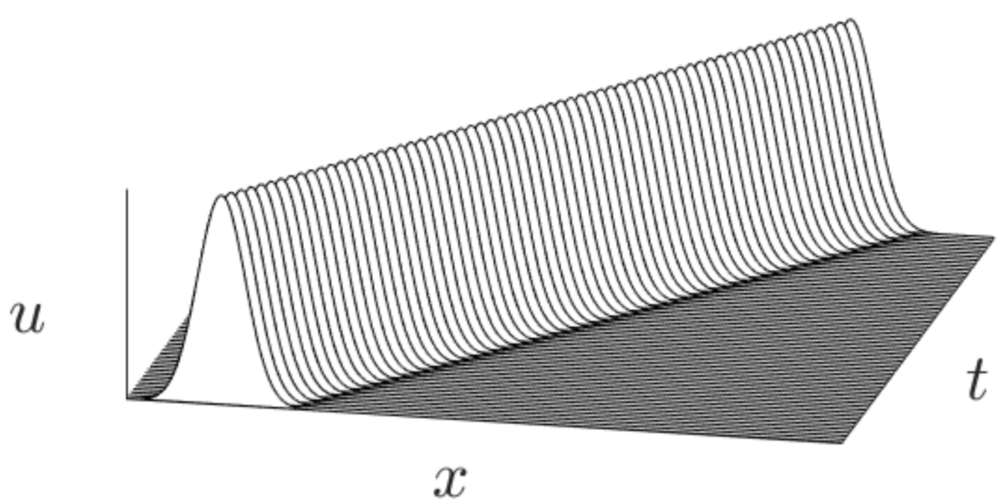
Single Wave Example



$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \mathbf{t} = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix}$$



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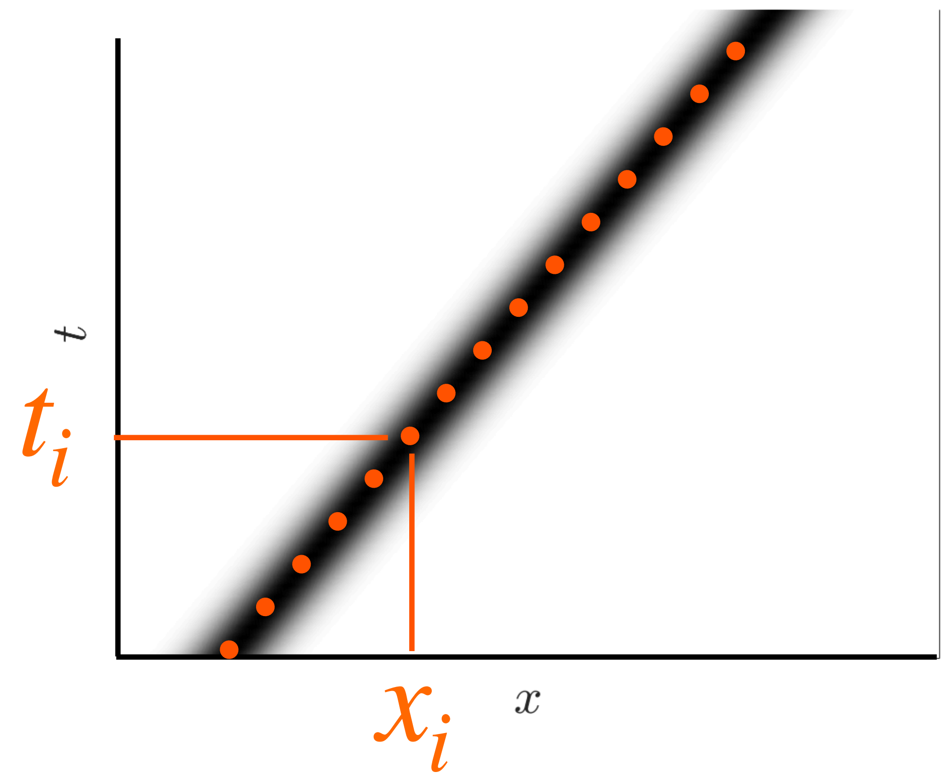
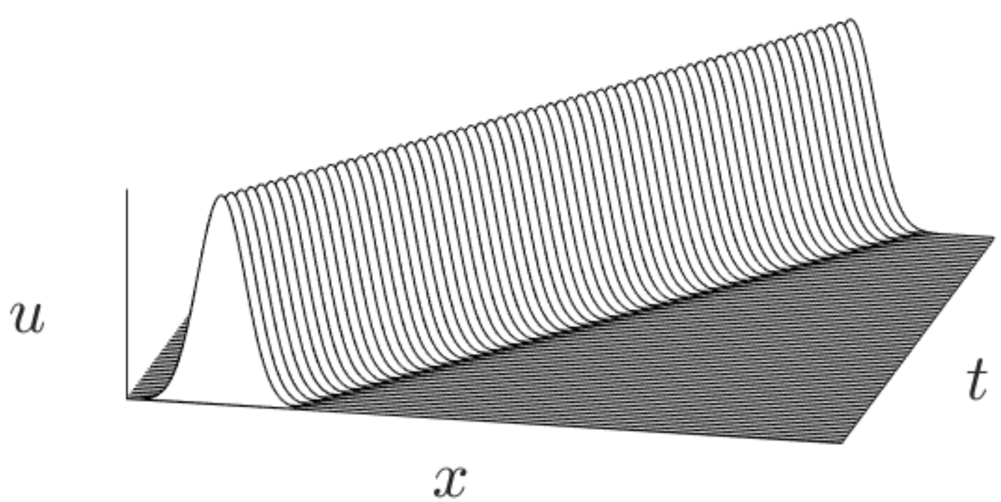
Single Wave Example



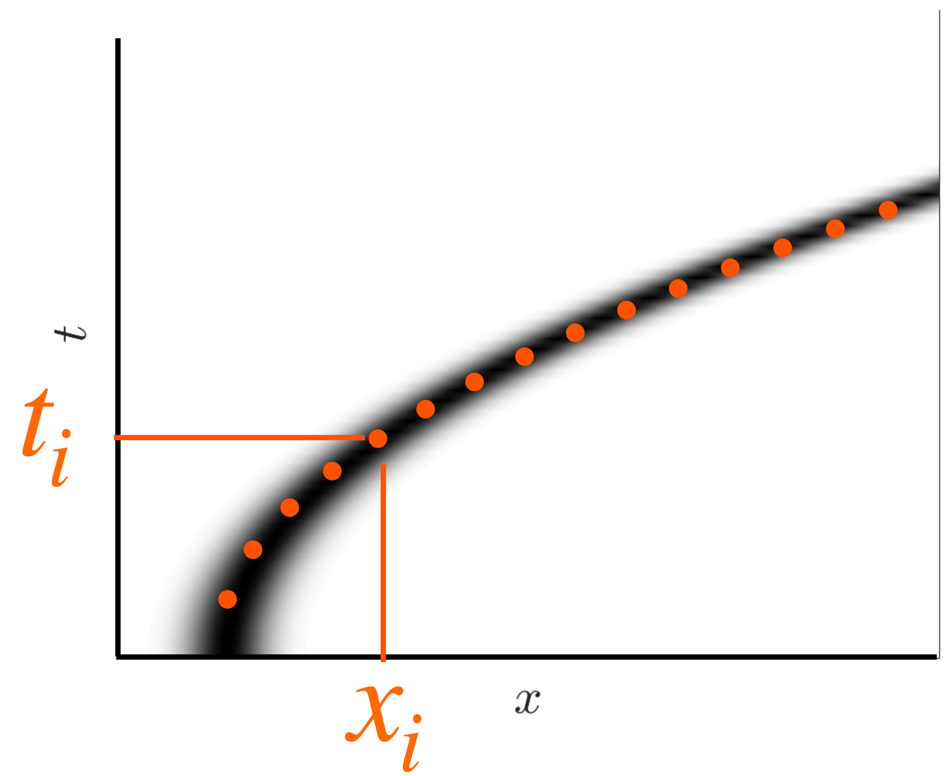


$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \mathbf{t} = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix}$$

Solve: $\mathbf{x} - c\mathbf{t} = 0$

UnTWIST

Single Wave Example



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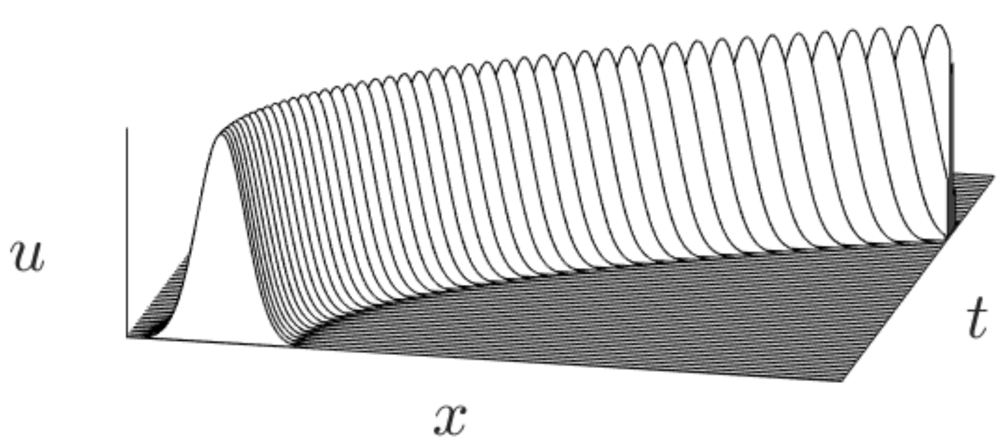

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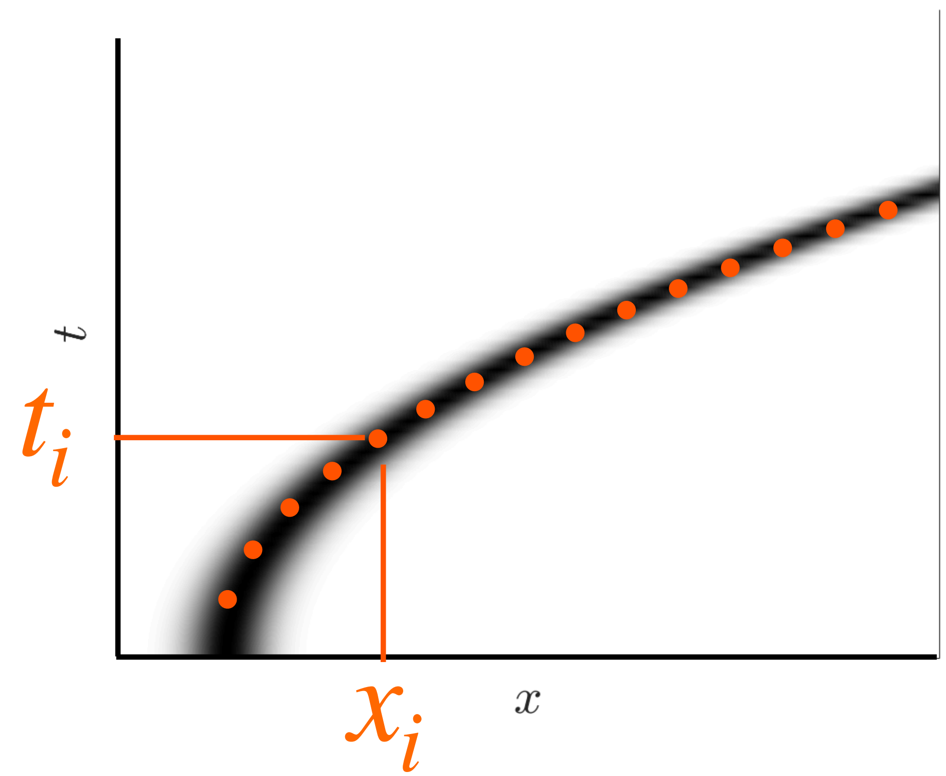
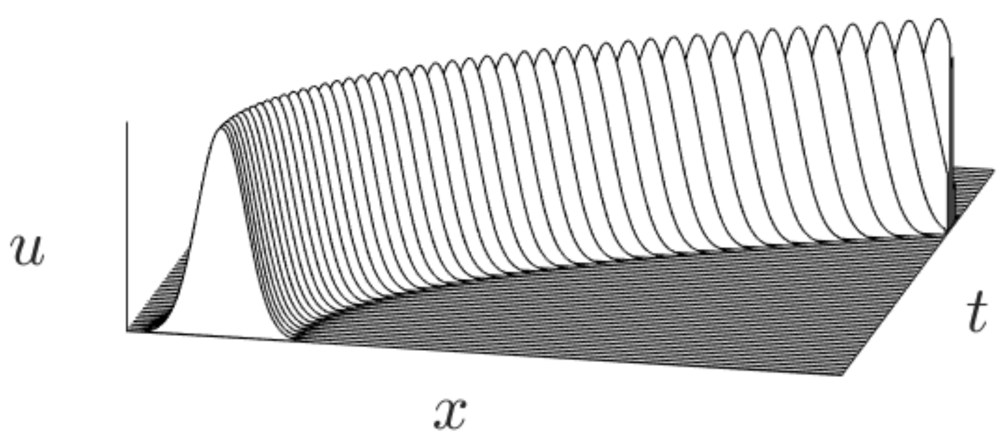
Single Wave Example





$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \mathbf{t} = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix}$$

$$\mathbf{c} = [c_1, c_2, c_3, \ldots] \qquad \mathbf{T} = \begin{bmatrix} -1- \\ -t- \\ -t^2- \\ \vdots \end{bmatrix}$$

Solve: $\mathbf{x} - \mathbf{CT} = 0$

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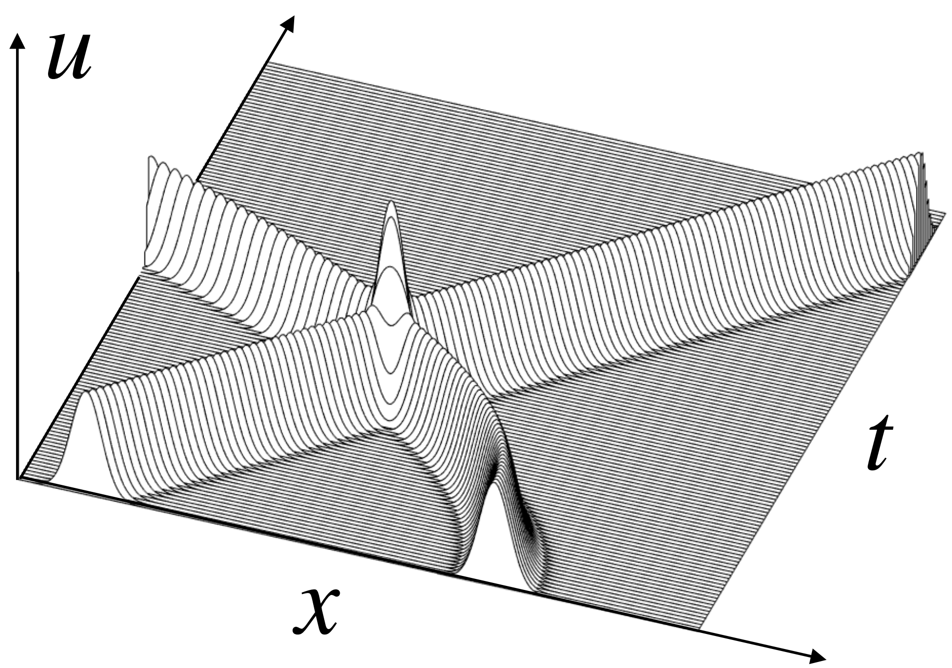
Step 1: Input

Original Data



Model Library

$$\theta_1(\tau) = 1$$

$$\theta_2(\tau) = \tau$$

$$\theta_3(\tau) = \sqrt{\tau} \qquad n = 2$$

$$\vdots$$

$$\theta_k(\tau) = \tau^2$$

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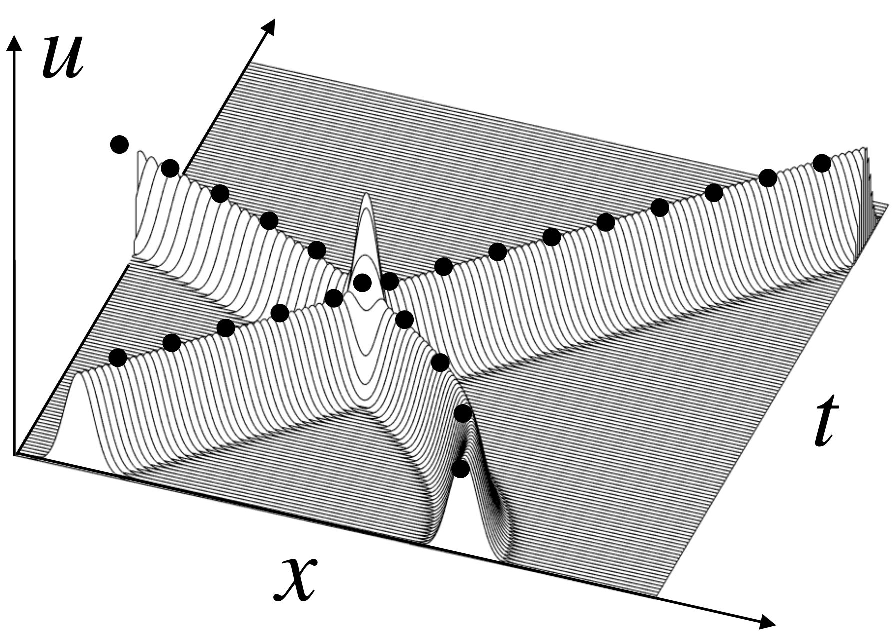
Step 2: Initialization

Ridge detection[1]



[1] Rasche, 2017, *IEEE Trans. Image Process.*

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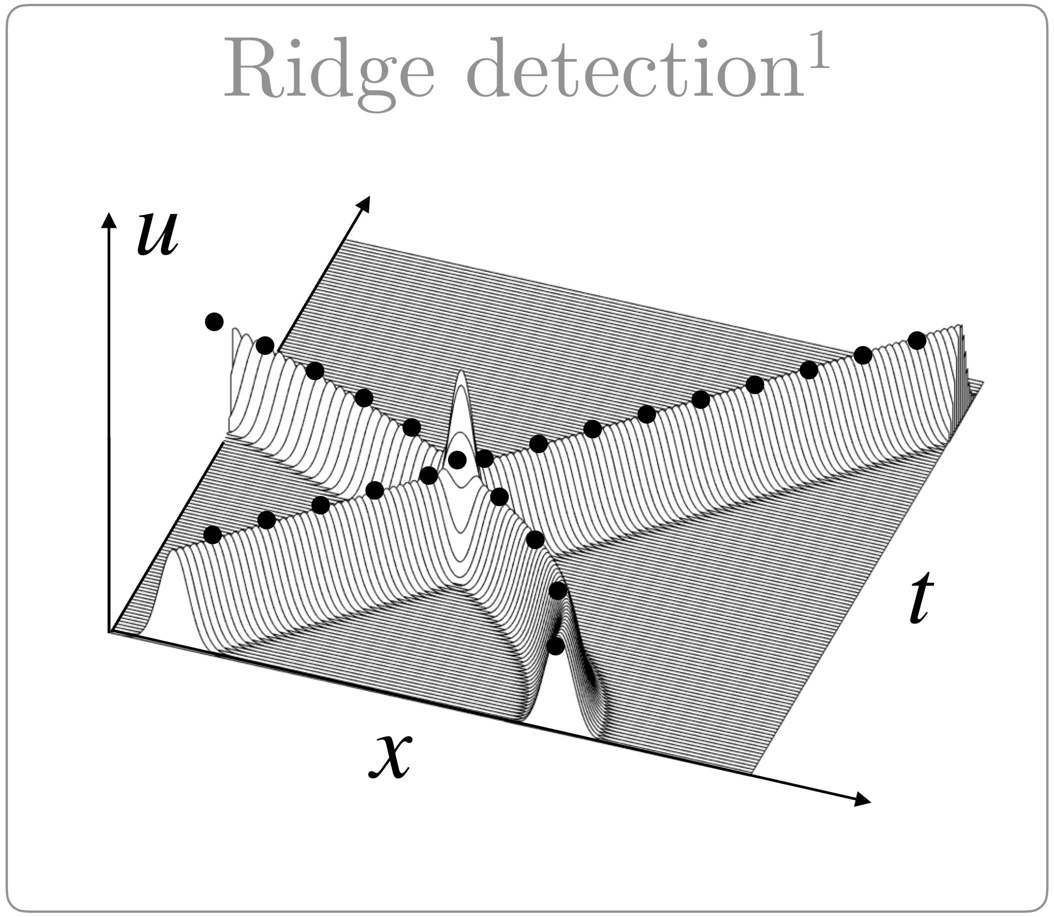
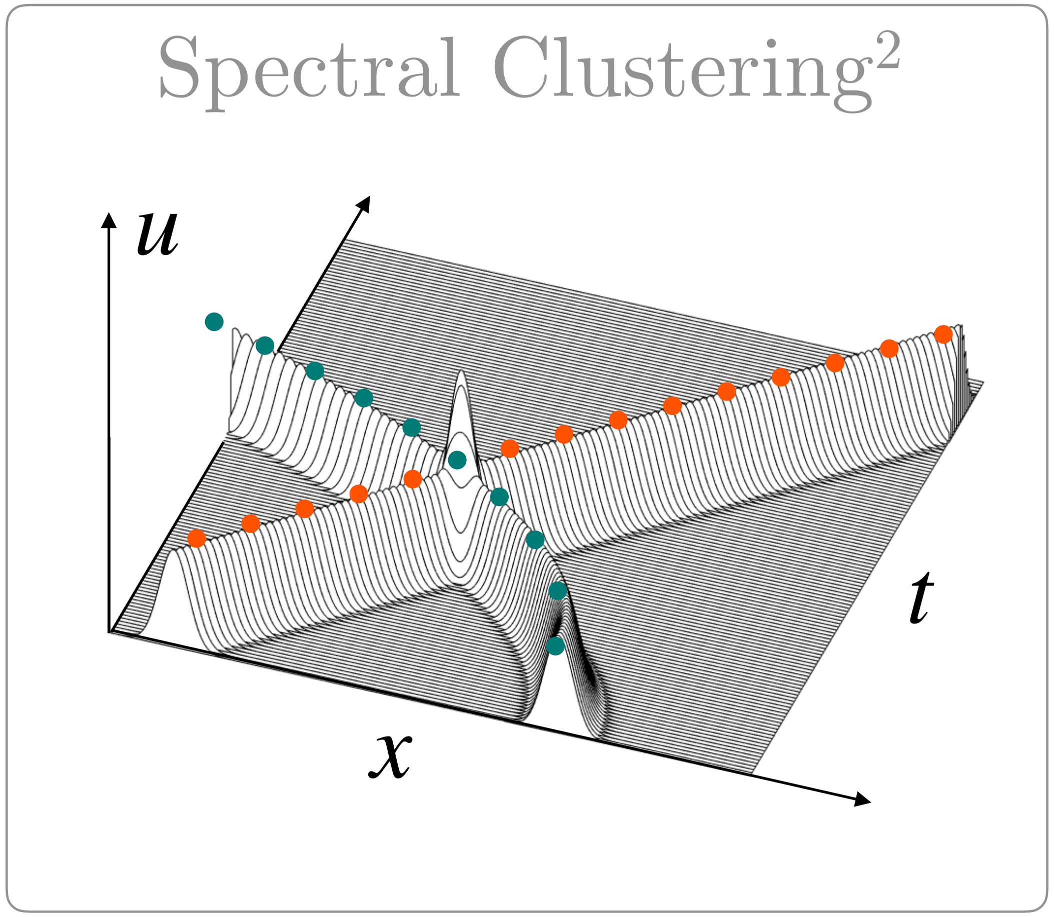
Step 2: Initialization





[1] Rasche, 2017, *IEEE Trans. Image Process.*

[2] Ng et al., 2002, *Adv Neural Inf Process Syst*

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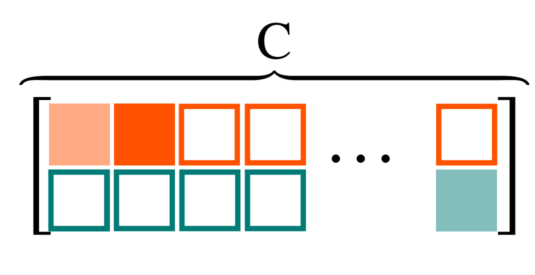
Step 3: Optimization

$$\min_{\mathbf{C},\mathbf{W}\in\Omega} \frac{1}{2}\mathbf{W} \odot \|\mathbf{X} - \mathbf{TC}\|^2$$

$$\overbrace{\begin{bmatrix} | & | \\ x & x \\ | & | \end{bmatrix}}^{\mathrm{X}(\mathrm{x})} - \overbrace{\begin{bmatrix} - & 1 & - \\ - & t & - \\ - & \sqrt{t} & - \\ & \vdots & \\ - & t^2 & - \end{bmatrix}}^{\mathrm{T}(t)}$$



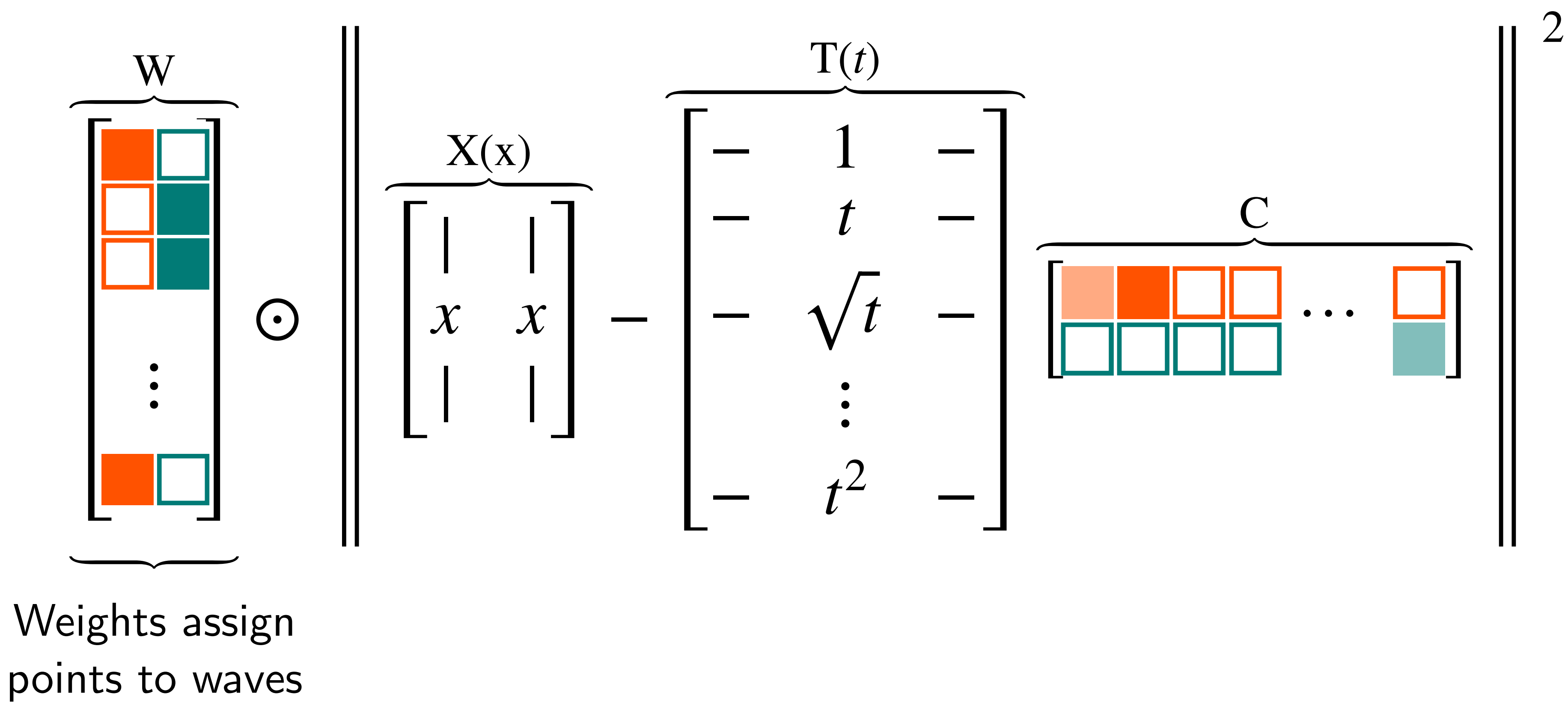
Sparse model coefficients

Library of models applied to our data

UnTWIST

Step 3: Optimization

$$\min_{\mathbf{C},\mathbf{W}\in\Omega} \frac{1}{2}\mathbf{W} \odot \|\mathbf{X} - \mathbf{T}\mathbf{C}\|^2$$



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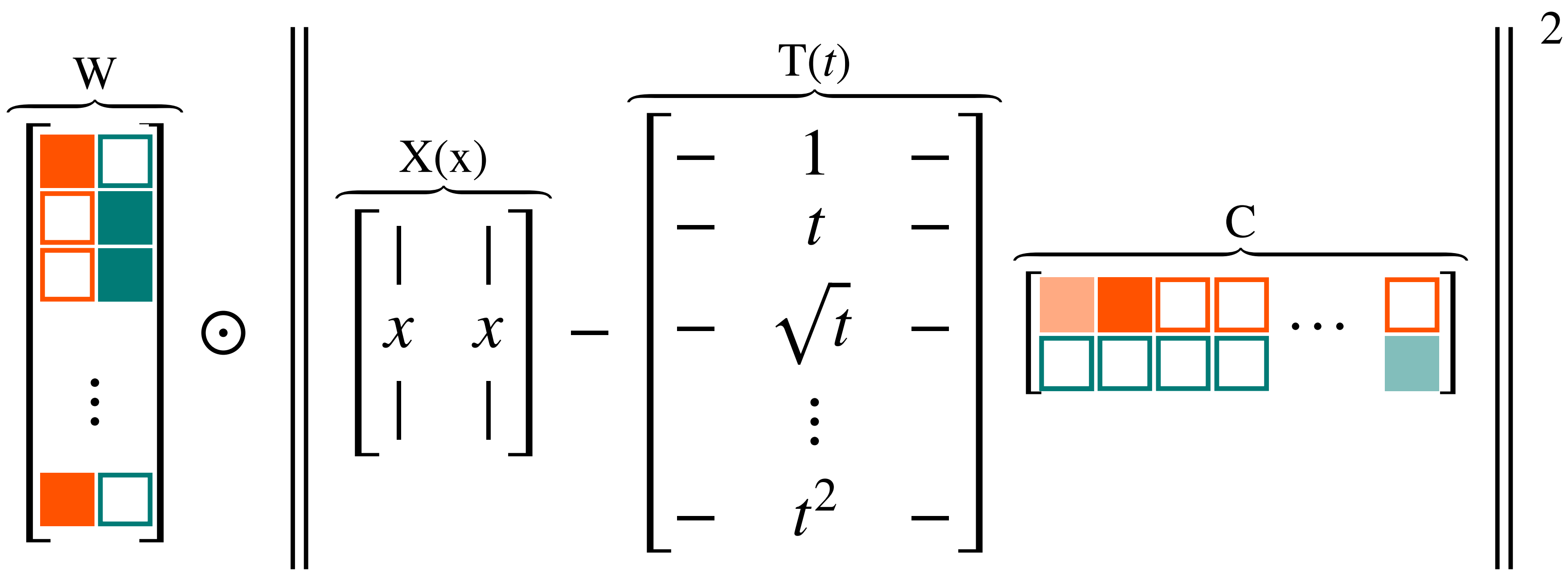
Step 3: Optimization

$$\min_{\mathbf{C},\mathbf{B},\mathbf{W}\in\Omega} \frac{1}{2}\mathbf{W} \odot \|\mathbf{X} - \mathbf{TC}\|^2 + \lambda R(\mathbf{B}) + \frac{1}{2\zeta}\|\mathbf{C} - \mathbf{B}\|^2$$



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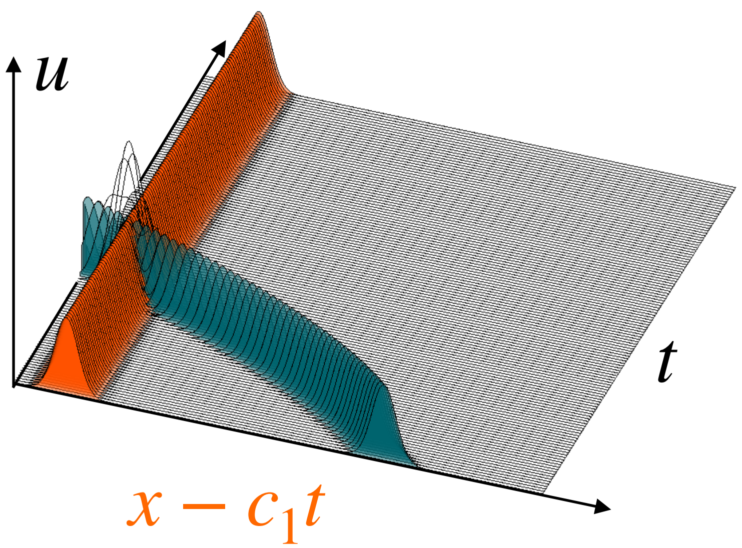
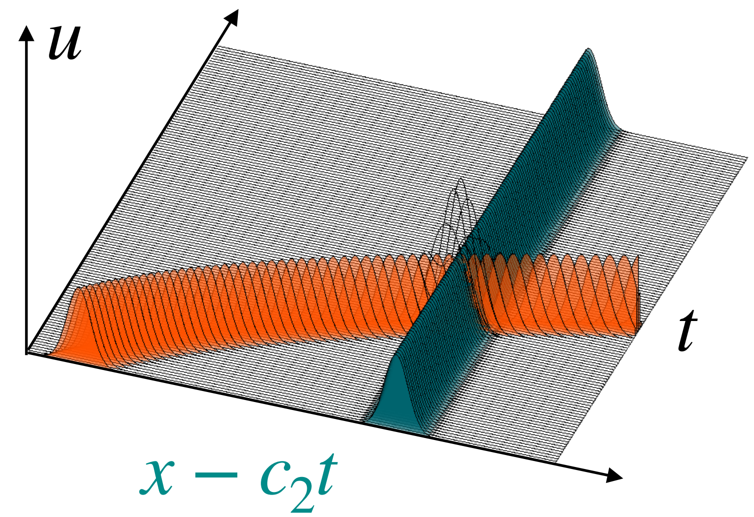
Step 4: ROMs



Shift in one coordinate frame for each wave



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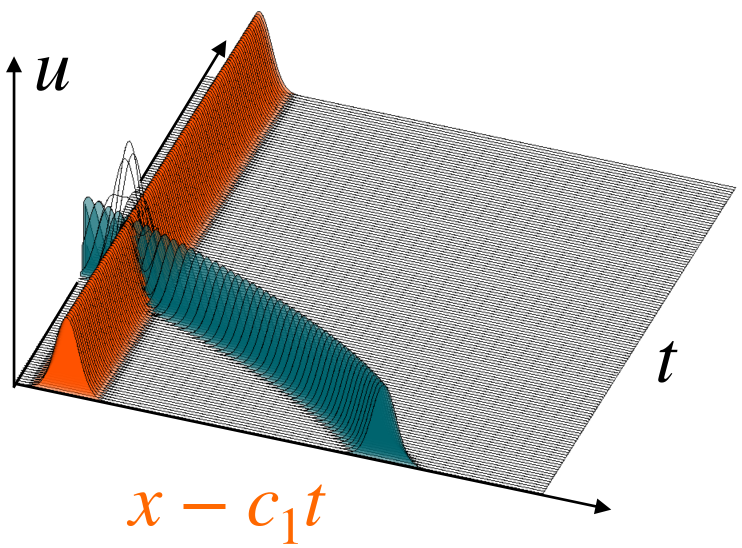
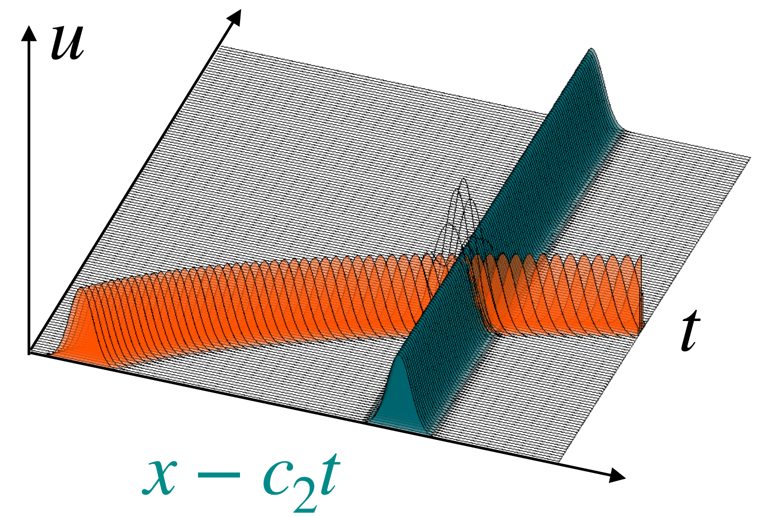
Step 4: ROMs



Shift in one coordinate frame for each wave



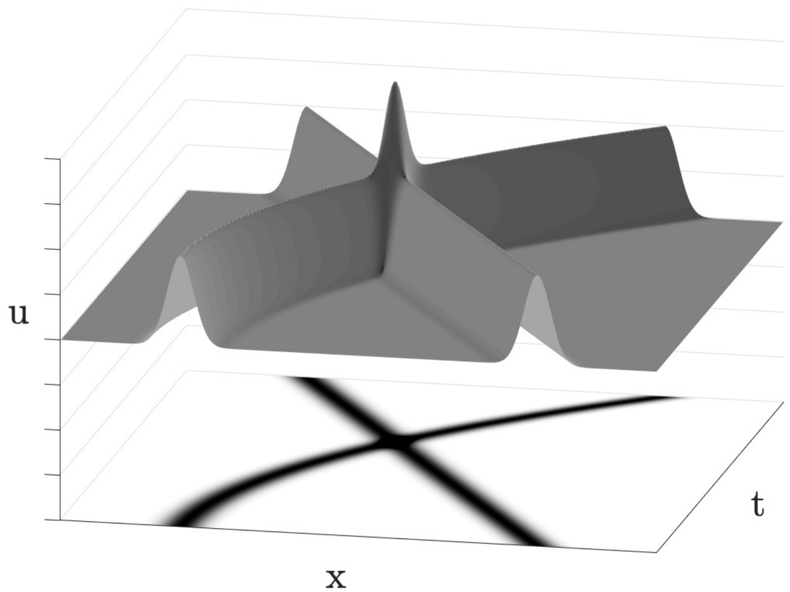
Traditional method (SVD)

$$u = \sum_{r_1} \alpha_1(t)\phi_1(x - c_1 t) + \sum_{r_2} \alpha_2(t)\phi_2(x - c_2 t)$$

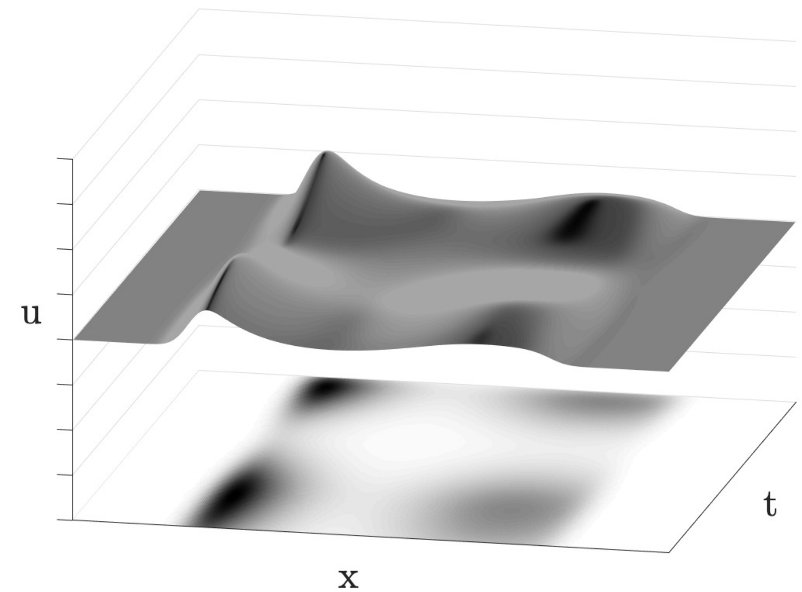
Low-rank models in shifted frames

Results

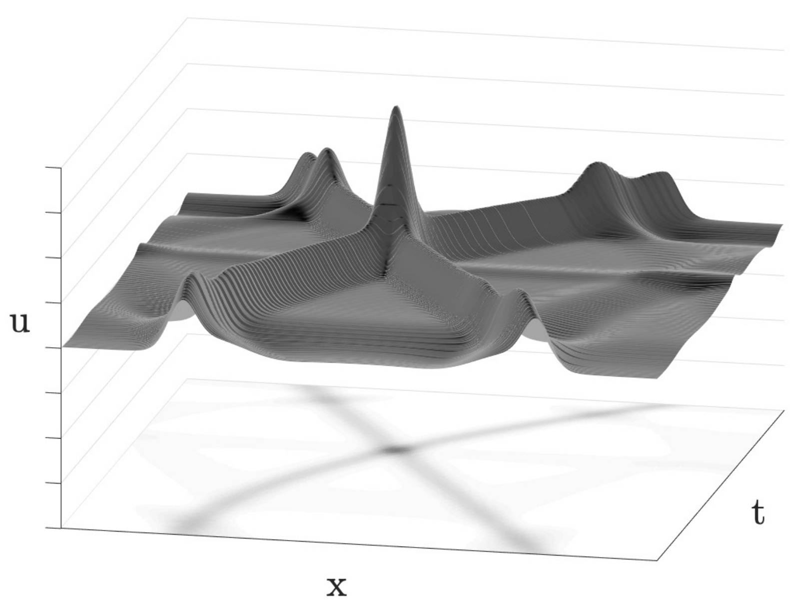
Rank-2 Reconstructions



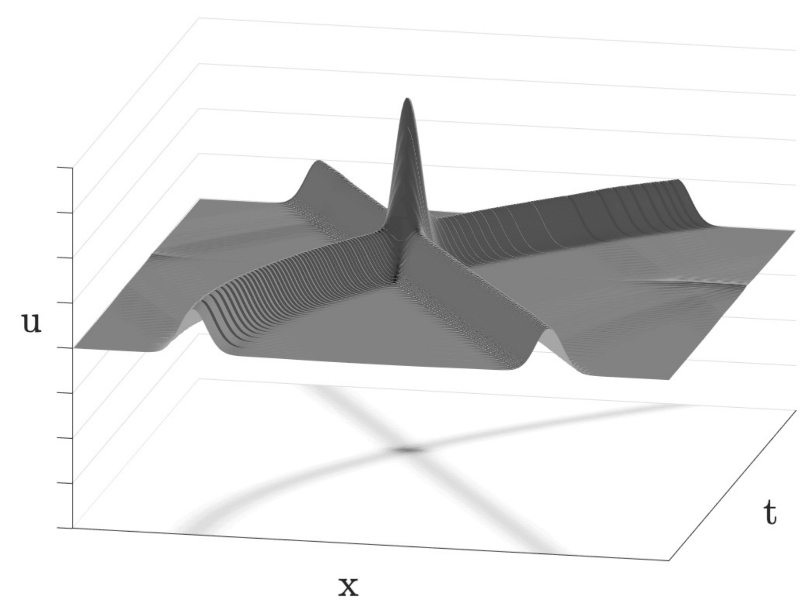
Original Data,
full rank



POD



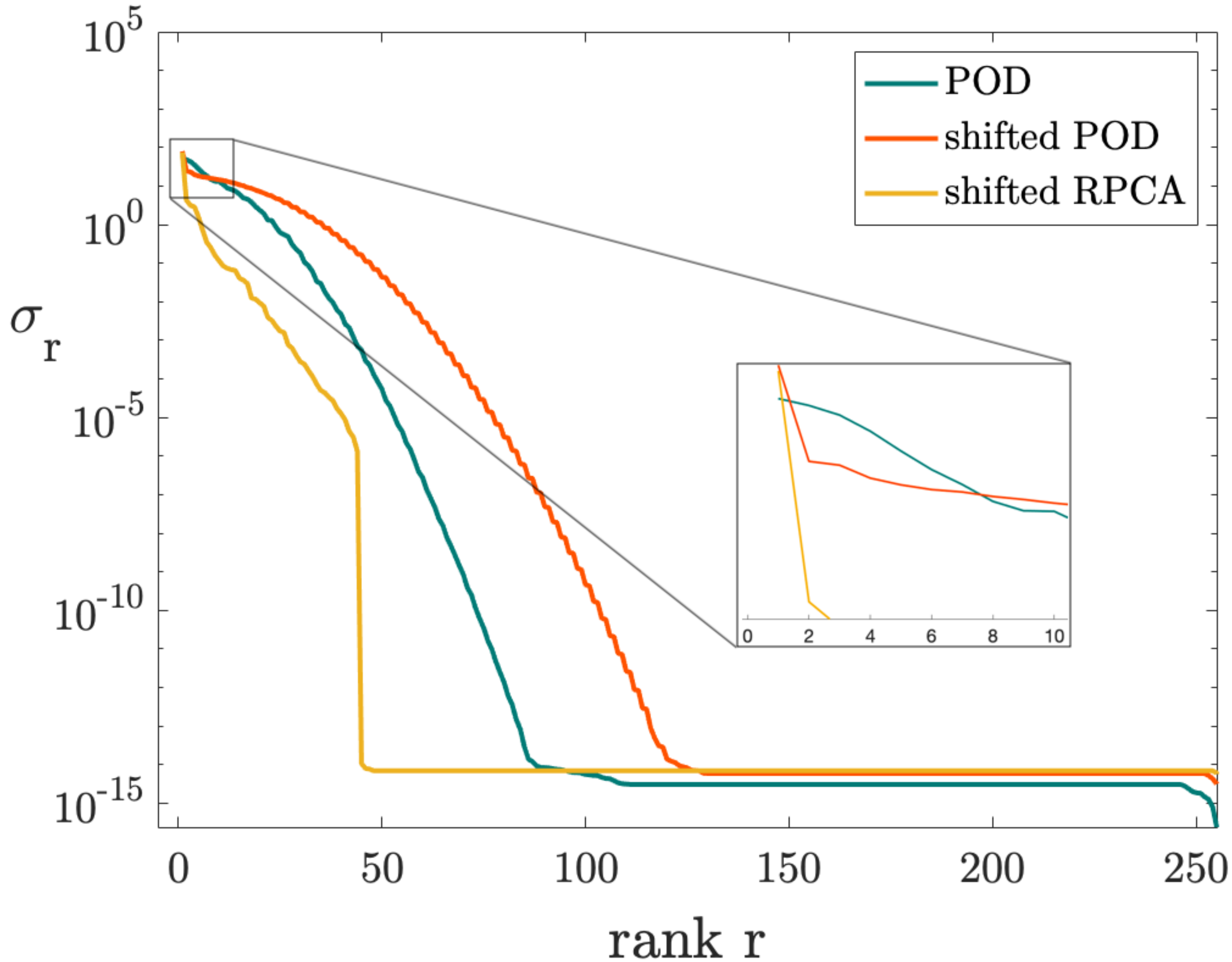
UnTWIST+POD



UnTWIST+sRPCA

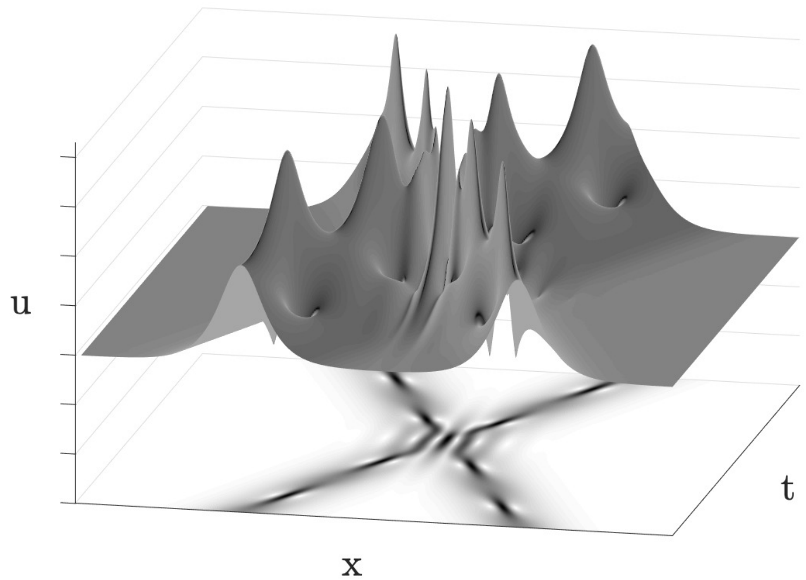
Results

Singular Value Decay

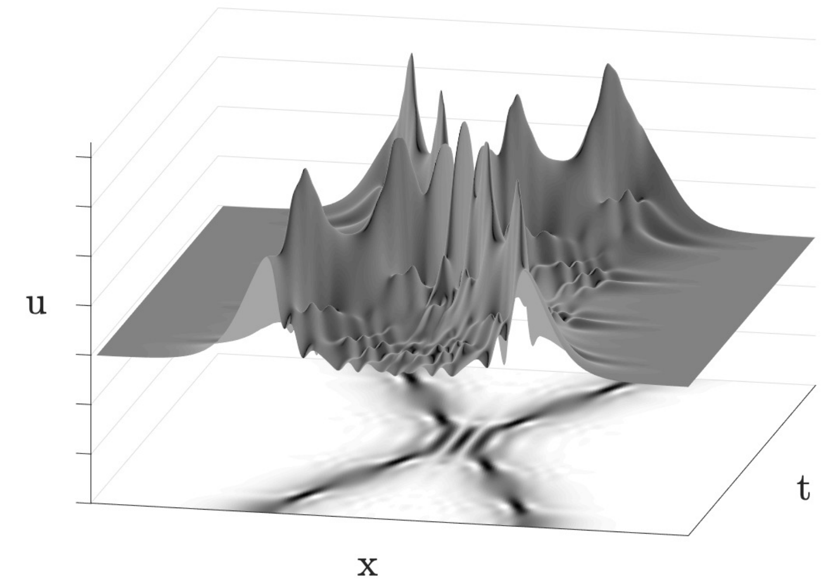


Results

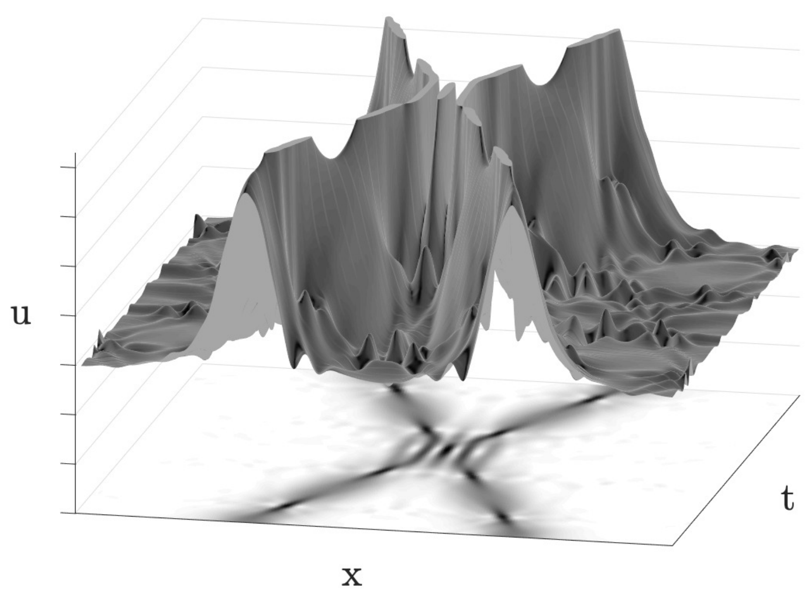
Rank-2 Reconstructions



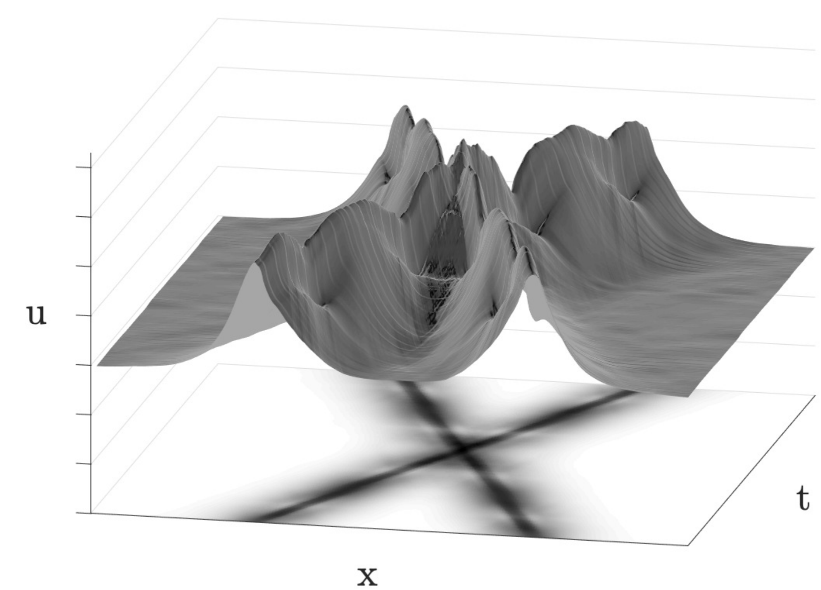
Original Data,
Full rank



POD



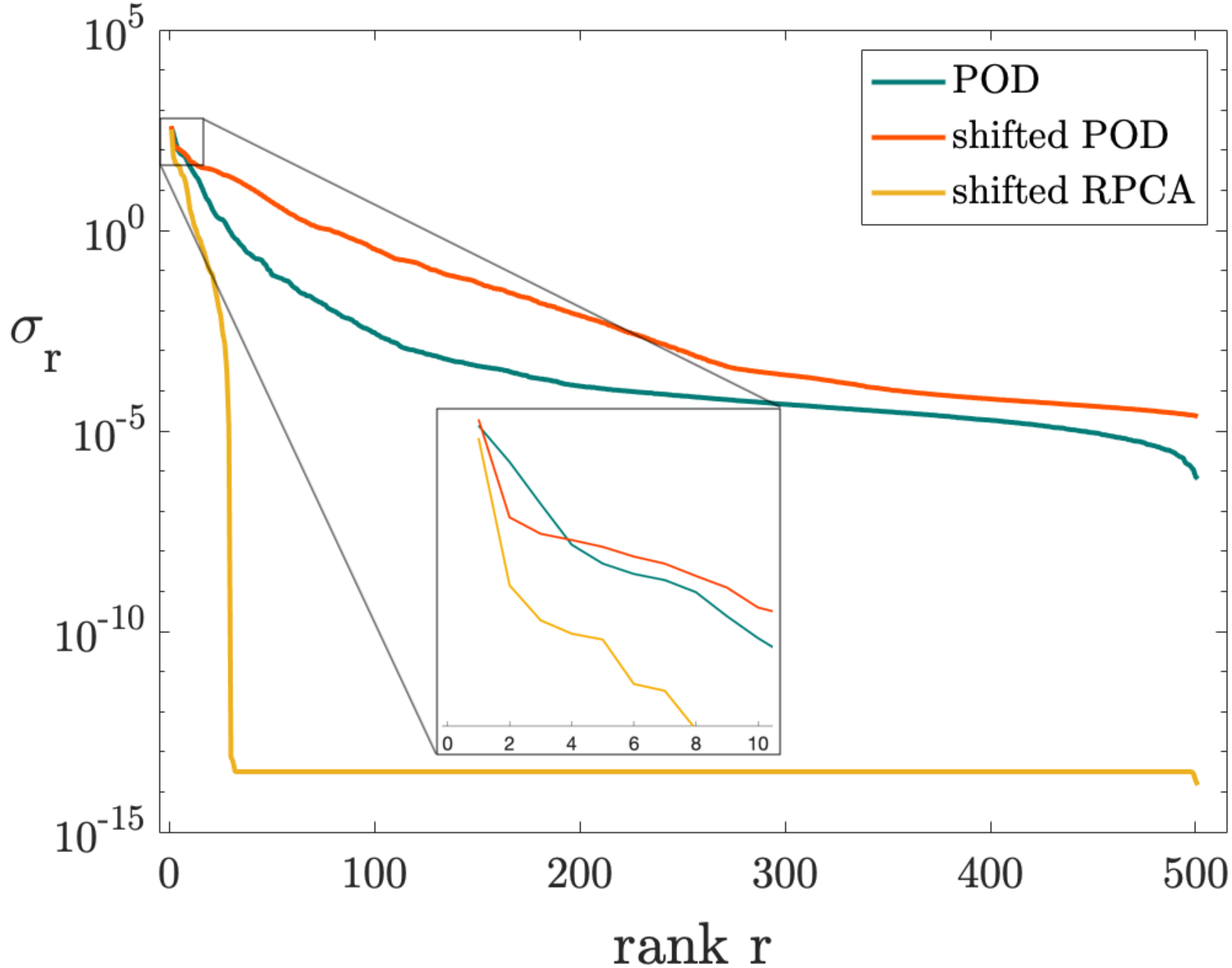
UnTWIST+POD



UnTWIST+sRPCA

Results

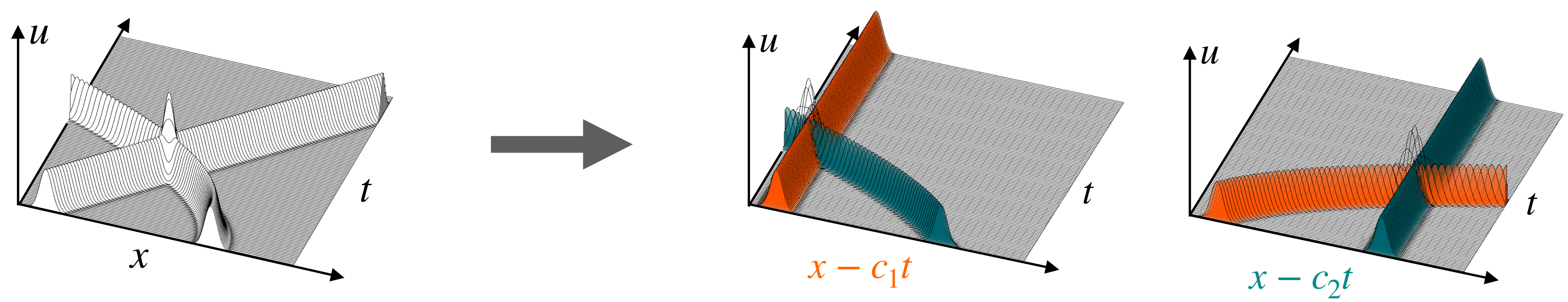
Singular Value Decay



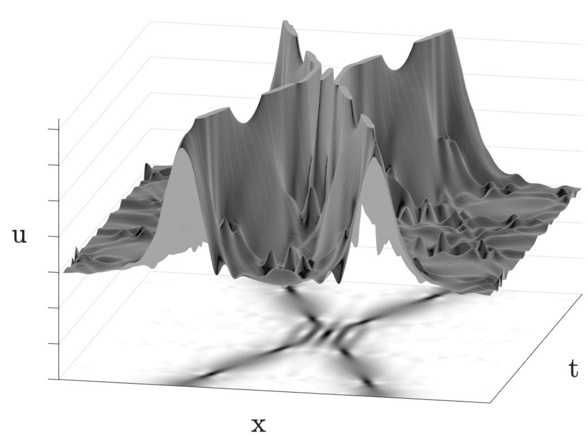
Conclusions and Future Work

Pre-print available arXiv:1911.00565v1

Created unsupervised machine learning method for automatically identifying interpretable models for translationally symmetric systems



ROMs are promising, though more work must be done to ensure energy is captured properly



Use machine learning to discover physics-based symmetries:

$$u(x,t) = \sum_r \phi_r(x; c(t))$$

$$\dot{c} = f(c)$$

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