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Abstract

1 Original Data

This data comes from a Rotating Detonation Engine (RDE) with increasing fuel input. The data is visualized as an x-t diagram, where the x dimension is an unwrapped representation of the periodic annulus. As the mass flux ratio of the fuel decreases the two co-rotating shock waves converge into one shock wave with a higher peak.

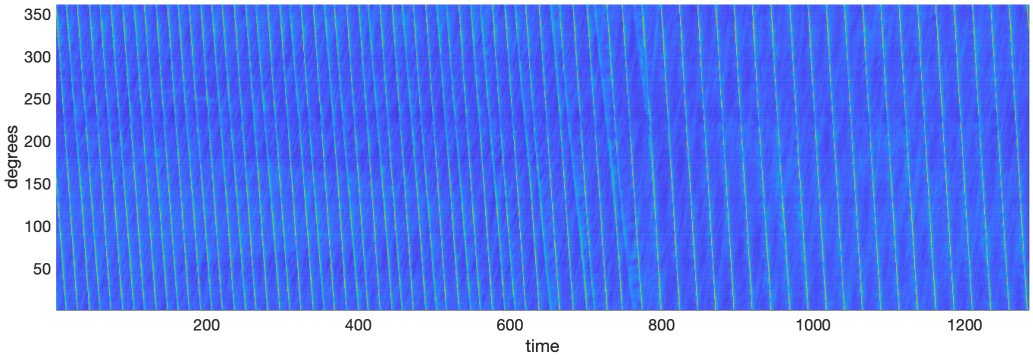


Figure 1: Original RDE data of two co-rotating shock waves coalescing into one shock wave around time $t = 700$. The waves appear to be traveling with a linear wave speed.

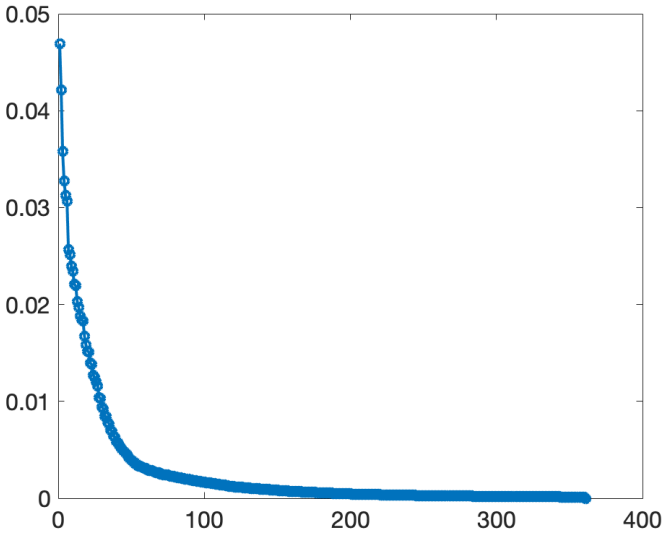


Figure 2: Log plot of the singular value spectrum of the original data after mean centering.

2 First Shift

In principle, the knowledge that there are two waves in one regime and one wave in another regime leads us to believe that the the system should be very low-rank in some coordinate frame. When we apply the UnTWIST method on the first 10 time steps of the data, which appears approximately linear, and shift in to this moving coordinate frame, we obtain a clearer picture of what is happening in the underlying dynamics.

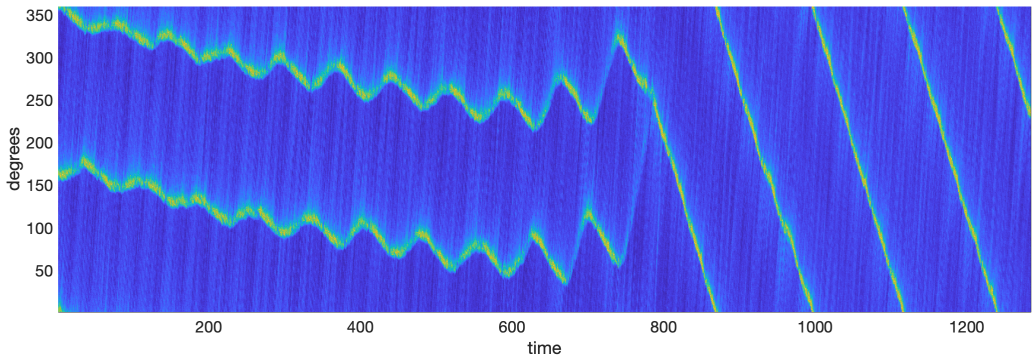


Figure 3: The full RDE data after being shifted into a linear speed moving coordinate frame. It can be seen more clearly that the two shock waves are oscillating in their speeds through time, and eventually converge into one single shock wave.

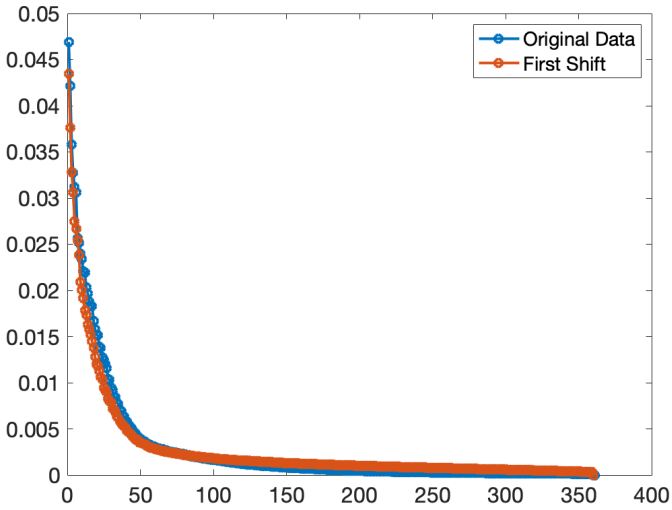


Figure 4: Singular value spectrum of the data in Figure 3 compared to that of the original data in Figure 1.

The spectrum of the original data decays more quickly than the shifted data, meaning we have gained little from this first shift. If we shift again into a more constant wave frame, will this give us an advantage?

3 Second Shift

From here, it is clear that another shift is necessary to redistribute energy to the early modes of the spectrum.Taking the regime to be only the first 700 time steps of the data, since this is approximately where the two waves converge to one, we can apply a second UnTWIST shifting to see how this affects the singular value spectrum.

3.1 Centering

When we shift the waves into one single, x-centered coordinate frame, we can even more clearly see the oscillatory pattern between the two wave fronts.

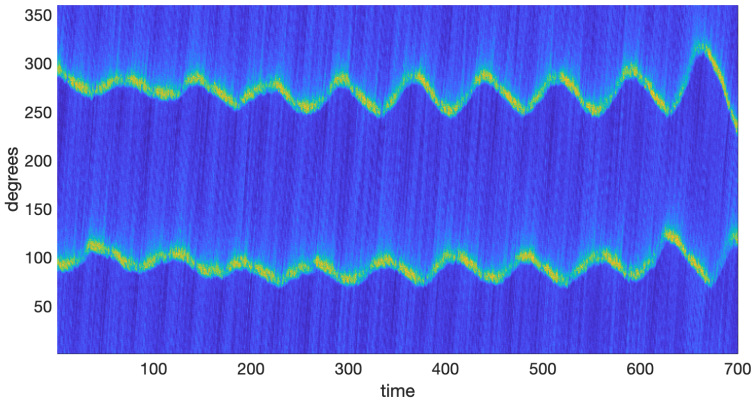


Figure 5: The second shift of the data which prioritizes centering along the x-dimension for both waves simultaneously.

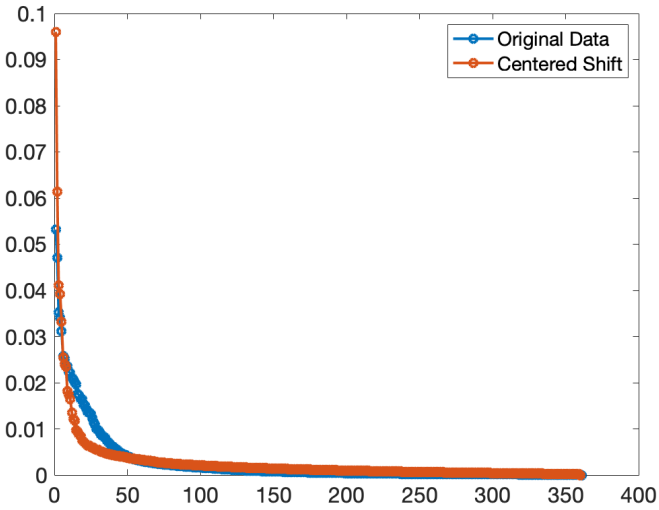
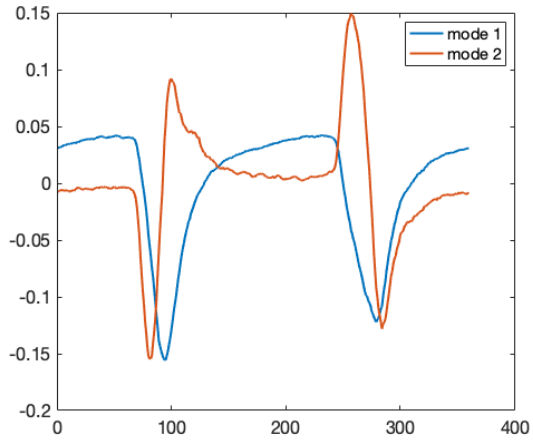


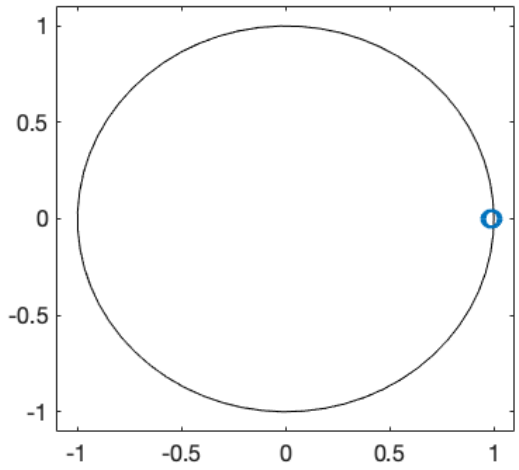
Figure 6: Spectrum comparisons of the data from Figure 1 (time steps 1-700) to Figure 5, showing an improvement in the energy allocated to the earlier modes.

This shifted coordinate frame is closer to what we need to have extremely low rank modes, but is not yet there. We can explore what we can be done by applying DMD to this data.

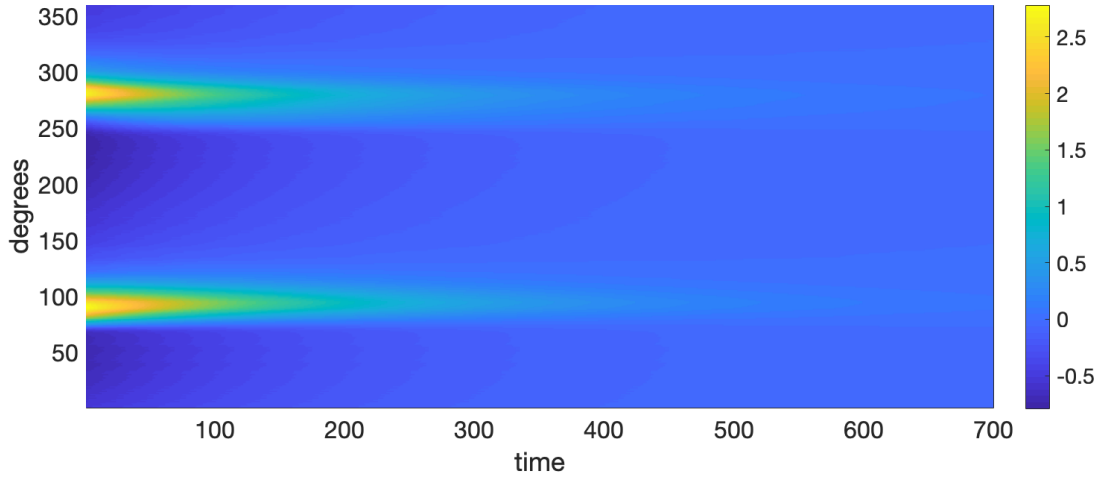
3.1.1 Exact DMD



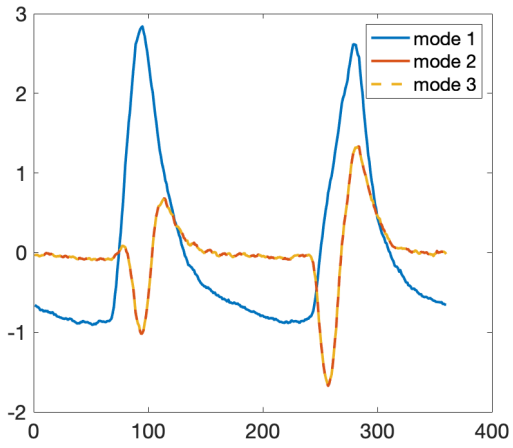
(a) Two modes showing similarities to the mode shapes of a coupled oscillator system, which has eigenvectors of $[1, 1]$ and $[1, -1]$



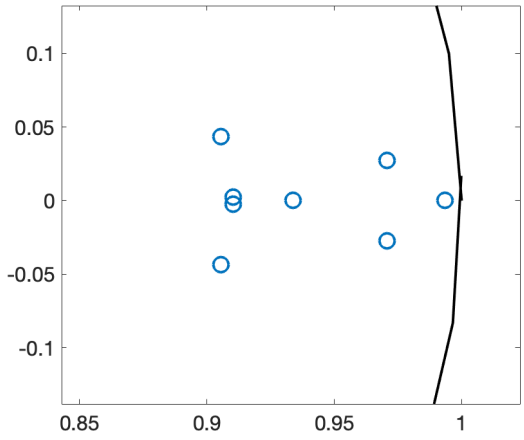
(b) Two real, unstable, eigenvalues of $\lambda_1 = 0.9945$ and $\lambda_2 = 0.9811$



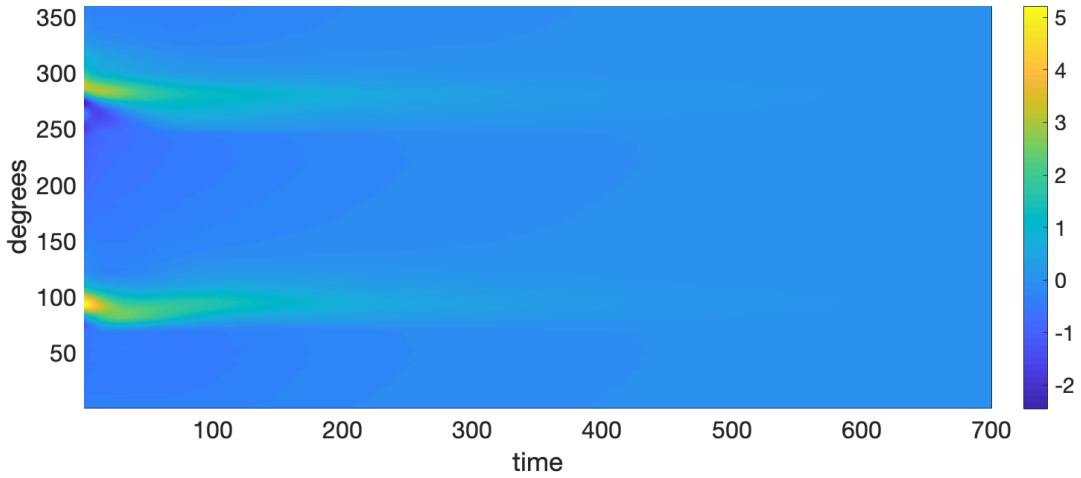
(c) 2-mode reconstruction.



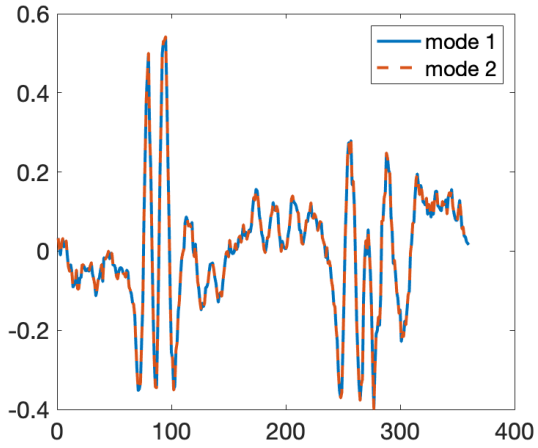
(a) First three modes of an 8-mode DMD also showing similarities to the mode shapes of a coupled oscillator system.



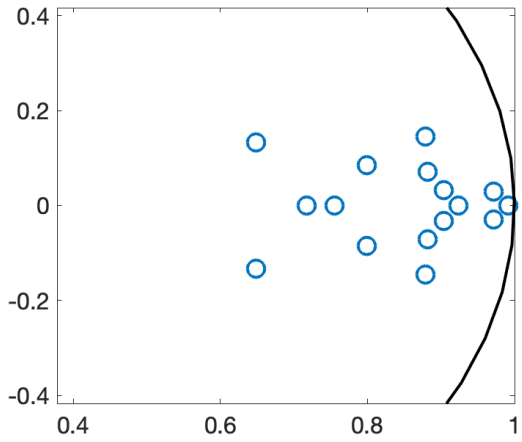
(b) The eigenvalues in an 8-mode DMD now show that the modes oscillate, but are still unstable.



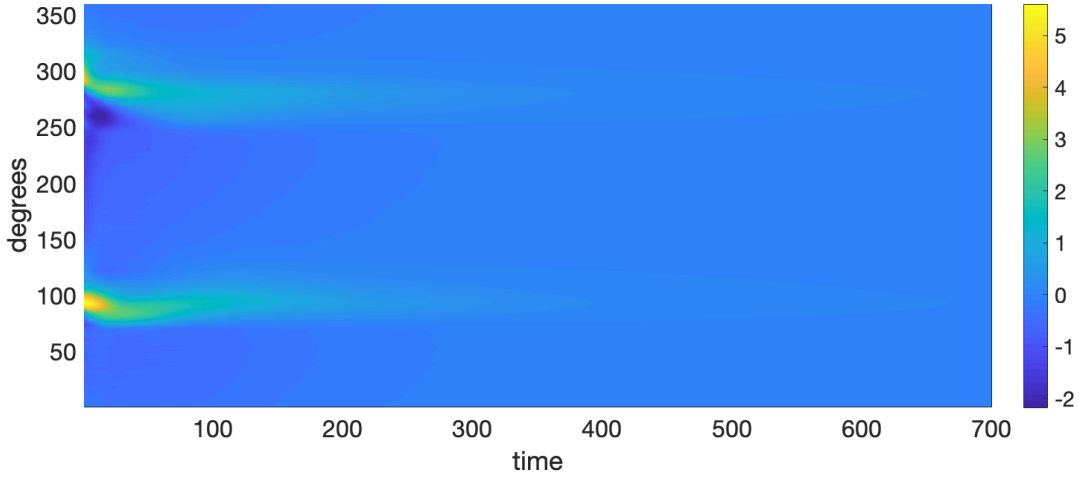
(c) 8-mode reconstruction.



(a) First two modes in a 16-mode DMD now are identical and show little resemblance to the normal modes of the couple oscillator.



(b) Eigenvalues are also unstable in the 16-mode DMD system.



(c) 16-mode reconstruction.

DMD is sensitive to noise, so we move to optimized DMD to alleviate the noise issues.

3.2 Straightening in Two Coordinate Frames

If we can isolate one wave into a constant coordinate frame we can capture its single mode to describe that shock front.