

## Trabajo Práctico N° 2: Límites.

### Ejercicio 1.

Calcular los límites de las siguientes funciones, si existen.

(a)  $f(x) = \begin{cases} x^2, & \text{si } x > 2 \\ 1, & \text{si } x \leq 2 \end{cases}$ . Calcular  $\lim_{x \rightarrow 2^+} f(x)$  y  $\lim_{x \rightarrow 2^-} f(x)$ .

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} 1 = 1.$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} x^2 = 2^2 = 4.$$

Por lo tanto, ya que  $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$ ,  $\nexists \lim_{x \rightarrow 2} f(x)$ .

(b) Sea  $g$  la función valor absoluto,  $g(x) = |x| = \begin{cases} x, & \text{si } x \geq 0 \\ -x, & \text{si } x < 0 \end{cases}$ . Calcular  $\lim_{x \rightarrow 0^+} g(x)$  y  $\lim_{x \rightarrow 0^-} g(x)$ . ¿Qué se puede decir respecto al  $\lim_{x \rightarrow 0} g(x)$ ?

$$\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} -x = 0.$$

$$\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} x = 0.$$

Por lo tanto, ya que  $\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^+} g(x)$ ,  $\exists \lim_{x \rightarrow 0} g(x)$ .

**Ejercicio 2.**

Dada la función  $f(x) = \begin{cases} \frac{x-1}{x^2-1}, & \text{si } x \neq 1 \\ 3, & \text{si } x = 1 \end{cases}$ , calcular  $\lim_{x \rightarrow 1^+} f(x)$  y  $\lim_{x \rightarrow 1^-} f(x)$ .

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} \frac{x-1}{x^2-1} = \frac{1-1}{1^2-1} = \frac{0}{1-1} = \left(\frac{0}{0}\right).$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} \frac{x-1}{(x+1)(x-1)}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}.$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{x-1}{x^2-1} = \frac{1-1}{1^2-1} = \frac{0}{1-1} = \left(\frac{0}{0}\right).$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{x-1}{(x+1)(x-1)}$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}.$$

**Ejercicio 3.**

Calcular, si existen, los siguientes límites:

(a)  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+9}-3}{x^2}.$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+9}-3}{x^2} = \frac{\sqrt{0^2+9}-3}{0^2} = \frac{\sqrt{0+9}-3}{0} = \frac{\sqrt{9}-3}{0} = \frac{3-3}{0} = \frac{0}{0}.$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \rightarrow 0} \frac{\sqrt{x^2+9}-3}{x^2} \cdot \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \rightarrow 0} \frac{x^2+9-3\sqrt{x^2+9}+3\sqrt{x^2+9}-9}{x^2(\sqrt{x^2+9}+3)}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \rightarrow 0} \frac{x^2}{x^2(\sqrt{x^2+9}+3)}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{x^2+9}+3} = \frac{1}{\sqrt{0^2+9}+3} = \frac{1}{\sqrt{0+9}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}.$$

(b)  $\lim_{x \rightarrow 1} \frac{x^2-2x+1}{x^2-3x+2}.$

$$\lim_{x \rightarrow 1} \frac{x^2-2x+1}{x^2-3x+2} = \frac{1^2-2*1+1}{1^2-3*1+2} = \frac{1-2+1}{1-3+2} = \frac{0}{0}.$$

$$\lim_{x \rightarrow 1} \frac{x^2-2x+1}{x^2-3x+2} = \lim_{x \rightarrow 1} \frac{(x-1)^2}{(x-2)(x-1)}$$

(\*) y (\*\*)

$$\lim_{x \rightarrow 1} \frac{x^2-2x+1}{x^2-3x+2} = \lim_{x \rightarrow 1} \frac{x-1}{x-2} = \frac{1-1}{1-2} = \frac{0}{-1} = 0.$$

$$(*) \quad x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4*1*1}}{2*1}$$

$$x_1, x_2 = \frac{2 \pm \sqrt{4-4}}{2}$$

$$x_1, x_2 = \frac{2 \pm \sqrt{0}}{2}$$

$$x_1, x_2 = \frac{2 \pm 0}{2}$$

$$x_1 = \frac{2+0}{2} = \frac{2}{2} = 1.$$

$$x_2 = \frac{2-0}{2} = \frac{2}{2} = 1.$$

$$(**) \quad x_1, x_2 = \frac{-(-3) \pm \sqrt{(-3)^2 - 4*1*2}}{2*1}$$

$$x_1, x_2 = \frac{3 \pm \sqrt{9-8}}{2}$$

$$x_1, x_2 = \frac{3 \pm \sqrt{1}}{2}$$

$$x_1, x_2 = \frac{3 \pm 1}{2}$$

$$x_1 = \frac{3+1}{2} = \frac{4}{2} = 2.$$

$$x_2 = \frac{3-1}{2} = \frac{2}{2} = 1.$$

**Ejercicio 4.***Calcular los siguientes límites:*

(a)  $\lim_{x \rightarrow 3} \frac{x+2}{x+3}.$

$$\lim_{x \rightarrow 3} \frac{x+2}{x+3} = \frac{3+2}{3+3} = \frac{5}{6}.$$

(b)  $\lim_{x \rightarrow 1} x^3 + 5x^2 + 10.$

$$\lim_{x \rightarrow 1} x^3 + 5x^2 + 10 = 1^3 + 5 * 1^2 + 10 = 1 + 5 * 1 + 10 = 1 + 5 + 10 = 16.$$

(c)  $\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}.$

$$\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - 1}{5 - 3x} = \frac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)} = \frac{-8 + 2*4 - 1}{5 + 6} = \frac{-8 + 8 - 1}{11} = \frac{-1}{11}.$$

(d)  $\lim_{x \rightarrow \pi} \text{sen}(x - \pi).$

$$\lim_{x \rightarrow \pi} \text{sen}(x - \pi) = \text{sen}(\pi - \pi) = \text{sen } 0 = 0.$$

(e)  $\lim_{x \rightarrow 2} \frac{\sqrt[3]{x^2 + 2}}{x}.$

$$\lim_{x \rightarrow 2} \frac{\sqrt[3]{x^2 + 2}}{x} = \frac{\sqrt[3]{2^2 + 2}}{2} = \frac{\sqrt[3]{4 + 2}}{2} = \frac{\sqrt[3]{6}}{2}.$$

(f)  $\lim_{x \rightarrow 1} \ln \frac{2}{x^3 + 1}.$

$$\lim_{x \rightarrow 1} \ln \frac{2}{x^3 + 1} = \ln \frac{2}{1^3 + 1} = \ln \frac{2}{1 + 1} = \ln \frac{2}{2} = \ln 1 = 0.$$

(g)  $\lim_{x \rightarrow 0} \frac{\ln(x+1)}{e^x}.$

$$\lim_{x \rightarrow 0} \frac{\ln(x+1)}{e^x} = \frac{\ln(0+1)}{e^0} = \frac{\ln 1}{1} = \frac{0}{1} = 0.$$

(h)  $\lim_{x \rightarrow 0} 3 \cos x^2 (1+x)^4.$

$$\lim_{x \rightarrow 0} 3 \cos x^2 (1+x)^4 = 3 \cos 0^2 (1+0)^4 = 3 \cos 0 * 1^4 = 3 * 1 * 1 = 3.$$

(i)  $\lim_{x \rightarrow 3} |x - 3|.$

$$\lim_{x \rightarrow 3} |x - 3| = |3 - 3| = |0| = 0.$$

(j)  $\lim_{x \rightarrow 3} \frac{|x-3|}{x}.$

$$\lim_{x \rightarrow 3} \frac{|x-3|}{x} = \frac{|3-3|}{3} = \frac{|0|}{3} = \frac{0}{3} = 0.$$

(k)  $\lim_{x \rightarrow 0} \frac{x}{x^2+x}.$

$$\lim_{x \rightarrow 0} \frac{x}{x^2+x} = \frac{0}{0^2+0} = \frac{0}{0+0} = \left(\frac{0}{0}\right).$$

$$\lim_{x \rightarrow 0} \frac{x}{x^2+x} = \lim_{x \rightarrow 0} \frac{x}{x(x+1)}$$

$$\lim_{x \rightarrow 0} \frac{x}{x^2+x} = \lim_{x \rightarrow 0} \frac{1}{x+1} = \frac{1}{0+1} = \frac{1}{1} = 1.$$

(l)  $\lim_{x \rightarrow 2} \frac{x^2-x-2}{x^2-4}.$

$$\lim_{x \rightarrow 2} \frac{x^2-x-2}{x^2-4} = \frac{2^2-2-2}{2^2-4} = \frac{4-2-2}{4-4} = \left(\frac{0}{0}\right).$$

$$\lim_{x \rightarrow 2} \frac{x^2-x-2}{x^2-4} = \lim_{x \rightarrow 2} \frac{(x-2)(x+1)}{(x+2)(x-2)}$$

$$\lim_{x \rightarrow 2} \frac{x^2-x-2}{x^2-4} = \lim_{x \rightarrow 2} \frac{x+1}{x+2} = \frac{2+1}{2+2} = \frac{3}{4}.$$

$$(*) x_1, x_2 = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 * 1 * (-2)}}{2 * 1}$$

$$x_1, x_2 = \frac{1 \pm \sqrt{1+8}}{2}$$

$$x_1, x_2 = \frac{1 \pm \sqrt{9}}{2}$$

(\*)

$$x_1, x_2 = \frac{1 \pm 3}{2}$$

$$x_1 = \frac{1+3}{2} = \frac{4}{2} = 2.$$

$$x_2 = \frac{1-3}{2} = \frac{-2}{2} = -1.$$

$$(m) \lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + 3x + 2}.$$

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + 3x + 2} = \frac{(-1)^2 - 1}{(-1)^2 + 3(-1) + 2} = \frac{1 - 1}{1 - 3 + 2} = \left(\frac{0}{0}\right).$$

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + 3x + 2} = \lim_{x \rightarrow -1} \frac{(x+1)(x-1)}{(x-2)(x-1)} \quad (*)$$

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + 3x + 2} = \lim_{x \rightarrow -1} \frac{x+1}{x-2} = \frac{-1+1}{-1-2} = \frac{0}{-3} = 0.$$

$$(*) x_1, x_2 = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$

$$x_1, x_2 = \frac{3 \pm \sqrt{9-8}}{2}$$

$$x_1, x_2 = \frac{3 \pm \sqrt{1}}{2}$$

$$x_1, x_2 = \frac{3 \pm 1}{2}$$

$$x_1 = \frac{3+1}{2} = \frac{4}{2} = 2.$$

$$x_2 = \frac{3-1}{2} = \frac{2}{2} = 1.$$

$$(n) \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}.$$

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = \frac{1-1}{\sqrt{1}-1} = \frac{0}{1-1} = \left(\frac{0}{0}\right).$$

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x}+1)}{x+\sqrt{x}-\sqrt{x}-1}$$

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x}+1)}{x-1}$$

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \rightarrow 1} \sqrt{x} + 1 = \sqrt{1} + 1 = 1 + 1 = 2.$$

$$(\tilde{n}) \lim_{x \rightarrow 0} \frac{x^3}{x^2+x}.$$

$$\lim_{x \rightarrow 0} \frac{x^3}{x^2+x} = \frac{0^3}{0^2+0} = \frac{0}{0+0} = \left(\frac{0}{0}\right).$$

$$\lim_{x \rightarrow 0} \frac{x^3}{x^2+x} = \lim_{x \rightarrow 0} \frac{x^3}{x(x+1)}$$

$$\lim_{x \rightarrow 0} \frac{x^3}{x^2+x} = \lim_{x \rightarrow 0} \frac{x^2}{x+1} = \frac{0^2}{0+1} = \frac{0}{1} = 0.$$

(o)  $\lim_{x \rightarrow 2} \frac{x+2}{2-x}$ .

$$\lim_{x \rightarrow 2} \frac{x+1}{2-x} = \frac{2+1}{2-2} = \frac{3}{0} = +\infty.$$

(p)  $\lim_{x \rightarrow 3} \frac{x}{x-3}$ .

$$\lim_{x \rightarrow 3} \frac{x}{x-3} = \frac{3}{3-3} = \frac{3}{0} = +\infty.$$

(q)  $\lim_{x \rightarrow -2} \frac{x}{|x+2|}$ .

$$\lim_{x \rightarrow -2} \frac{x}{|x+2|} = \frac{-2}{|-2+2|} = \frac{-2}{|0|} = \frac{-2}{0} = -\infty.$$

**Ejercicio 5.**

Calcular los siguientes límites al infinito:

(a)  $\lim_{x \rightarrow -\infty} x - 3x^4.$

$$\lim_{x \rightarrow -\infty} x - 3x^4 = -\infty.$$

(b)  $\lim_{x \rightarrow -\infty} \frac{5}{x} - \frac{3}{x}.$

$$\lim_{x \rightarrow -\infty} \frac{5}{x} - \frac{3}{x} = 0 - 0 = 0.$$

(c)  $\lim_{x \rightarrow +\infty} \frac{1}{x^2 + 3x - 1}.$

$$\lim_{x \rightarrow +\infty} \frac{1}{x^2 + 3x - 1} = 0.$$

(d)  $\lim_{x \rightarrow +\infty} \frac{3x^2 + 2x - 16}{x^2 - x - 2}.$

$$\lim_{x \rightarrow +\infty} \frac{3x^2 + 2x - 16}{x^2 - x - 2} = \left(\frac{\infty}{\infty}\right).$$

$$\lim_{x \rightarrow +\infty} \frac{3x^2 + 2x - 16}{x^2 - x - 2} = \lim_{x \rightarrow +\infty} \frac{x^2(3 + \frac{2}{x} - \frac{16}{x^2})}{x^2(1 - \frac{1}{x} - \frac{2}{x^2})}$$

$$\lim_{x \rightarrow +\infty} \frac{3x^2 + 2x - 16}{x^2 - x - 2} = \lim_{x \rightarrow +\infty} \frac{3 + \frac{2}{x} - \frac{16}{x^2}}{1 - \frac{1}{x} - \frac{2}{x^2}} = \frac{3 + 0 - 0}{1 - 0 - 0} = \frac{3}{1} = 3.$$

(e)  $\lim_{x \rightarrow -\infty} \frac{x^2 - 2x - 3}{x^3 - x^2 - 6x - 2}.$

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 2x - 3}{x^3 - x^2 - 6x - 2} = 0.$$

(f)  $\lim_{x \rightarrow -\infty} \frac{x^4 - 2x^2}{\sqrt{x^4 - 6x}}.$



$$\lim_{x \rightarrow -\infty} \frac{x^4 - 2x^2}{\sqrt{x^4 - 6x}} = \frac{x^4(1 - \frac{2}{x^2})}{\sqrt{x^4(1 - \frac{6}{x^3})}}$$

$$\lim_{x \rightarrow -\infty} \frac{x^4 - 2x^2}{\sqrt{x^4 - 6x}} = \frac{x^4(1 - \frac{2}{x^2})}{\sqrt{x^4} \sqrt{1 - \frac{6}{x^3}}}$$

$$\lim_{x \rightarrow -\infty} \frac{x^4 - 2x^2}{\sqrt{x^4 - 6x}} = \frac{x^4(1 - \frac{2}{x^2})}{x^2 \sqrt{1 - \frac{6}{x^3}}}$$

$$\lim_{x \rightarrow -\infty} \frac{x^4 - 2x^2}{\sqrt{x^4 - 6x}} = \frac{x^2(1 - \frac{2}{x^2})}{\sqrt{1 - \frac{6}{x^3}}} = +\infty.$$

**Ejercicio 6.**

Calcular los siguientes límites, si es que existen:

(a) Dada  $h(x) = \begin{cases} \sqrt{x-4}, & \text{si } x > 4 \\ 8-2x, & \text{si } x < 4 \end{cases}$ . Calcular  $\lim_{x \rightarrow 4} h(x)$ .

$$\lim_{x \rightarrow 4^-} h(x) = \lim_{x \rightarrow 4^-} 8 - 2 * 4 = 8 - 8 = 0.$$

$$\lim_{x \rightarrow 4^+} h(x) = \lim_{x \rightarrow 4^+} \sqrt{x-4} = \sqrt{4-4} = \sqrt{0} = 0.$$

Por lo tanto, ya que  $\lim_{x \rightarrow 4^-} h(x) = \lim_{x \rightarrow 4^+} h(x) = 0$ , entonces,  $\lim_{x \rightarrow 4} h(x) = 0$ .

(b) Sea  $f(x) = \begin{cases} \frac{x}{e^x}, & \text{si } x \leq 0 \\ x^2, & \text{si } x > 0 \end{cases}$ . Calcular  $\lim_{x \rightarrow 0} f(x)$  y  $\lim_{x \rightarrow -1} f(x)$ .

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x}{e^x} = \frac{0}{e^0} = \frac{0}{1} = 0.$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2 = 0^2 = 0.$$

Por lo tanto, ya que  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = 0$ , entonces,  $\lim_{x \rightarrow 0} f(x) = 0$ .

$$\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} x^2 = (-1)^2 = 1.$$

Por lo tanto,  $\lim_{x \rightarrow -1} f(x) = 1$ .

(c)  $\lim_{x \rightarrow +\infty} \frac{(\ln x)^2}{x}$ .

$$\lim_{x \rightarrow +\infty} \frac{(\ln x)^2}{x} = 0.$$

(d)  $\lim_{x \rightarrow +\infty} \frac{3x^2+2x+1}{e^x}$ .

$$\lim_{x \rightarrow +\infty} \frac{3x^2+2x+1}{e^x} = 0.$$

(e)  $\lim_{x \rightarrow 0^+} x \ln x$ .

$$\begin{aligned}\lim_{x \rightarrow 0^+} x \ln x &= \lim_{y \rightarrow +\infty} \frac{1}{y} \ln \frac{1}{y} \\ \lim_{x \rightarrow 0^+} x \ln x &= \lim_{y \rightarrow +\infty} \frac{\ln \frac{1}{y}}{y} \\ \lim_{x \rightarrow 0^+} x \ln x &= \lim_{y \rightarrow +\infty} \frac{\ln 1 - \ln y}{y} \\ \lim_{x \rightarrow 0^+} x \ln x &= \lim_{y \rightarrow +\infty} \frac{0 - \ln y}{y} \\ \lim_{x \rightarrow 0^+} x \ln x &= \lim_{y \rightarrow +\infty} \frac{-\ln y}{y} = 0.\end{aligned}$$

**Ejercicio 7.**

Calcular las asíntotas verticales y horizontales, si existen, de las funciones dadas a continuación:

(a)  $f(x) = \frac{x^2}{x+1}$ .

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} \frac{x^2}{x+1} = \frac{(-1)^2}{-1+1} = \frac{1}{0} = -\infty.$$

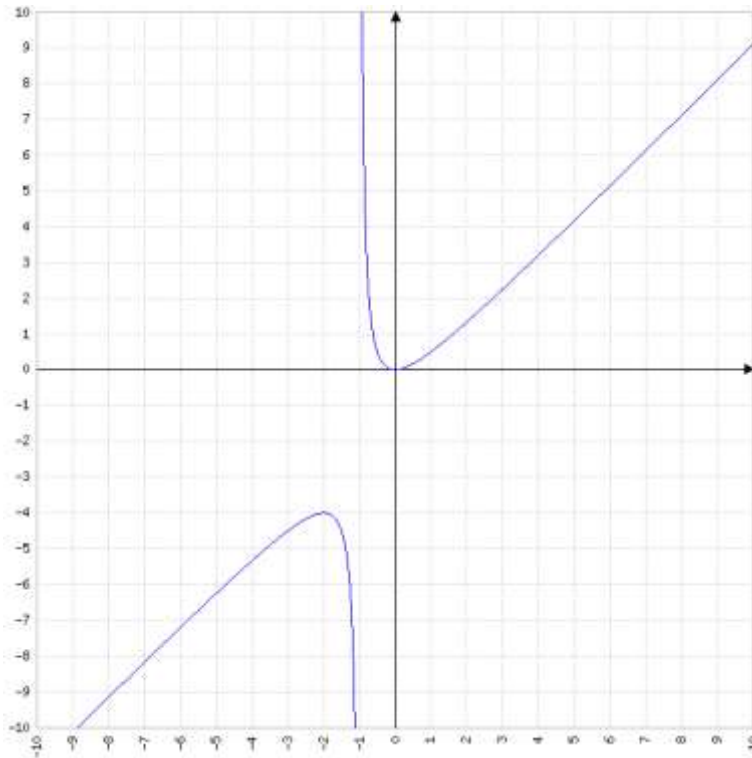
$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} \frac{x^2}{x+1} = \frac{(-1)^2}{-1+1} = \frac{1}{0} = +\infty.$$

Por lo tanto,  $f(x)$  tiene una asíntota vertical en  $x = -1$ .

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x^2}{x+1} = -\infty.$$

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{x^2}{x+1} = +\infty.$$

Por lo tanto,  $f(x)$  no tiene asíntotas horizontales.



(b)  $g(x) = \frac{4x^2 + 2x - 2}{3x - 1}$ .

$$\lim_{x \rightarrow \frac{1}{3}^-} g(x) = \lim_{x \rightarrow \frac{1}{3}^-} \frac{4x^2 + 2x - 2}{3x - 1} = \frac{4(\frac{1}{3})^2 + 2\frac{1}{3} - 2}{3(\frac{1}{3}) - 1} = \frac{4\frac{1}{9} + \frac{2}{3} - 2}{1 - 1} = \frac{\frac{4}{9} + \frac{2}{3} - 2}{0} = \frac{-8}{0} = +\infty.$$

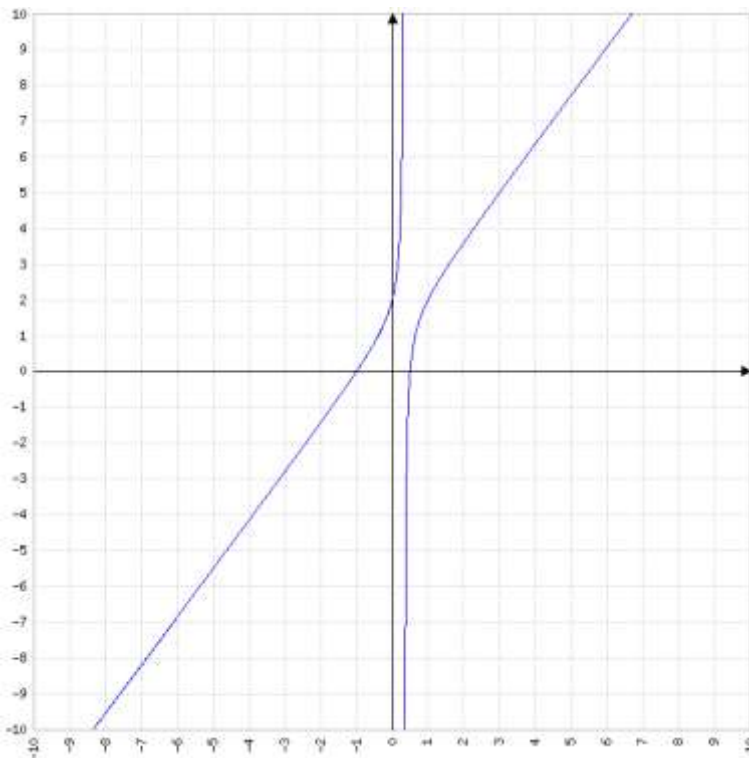
$$\lim_{x \rightarrow \frac{1}{3}^+} g(x) = \lim_{x \rightarrow \frac{1}{3}^+} \frac{4x^2 + 2x - 2}{3x - 1} = \frac{4(\frac{1}{3})^2 + 2\frac{1}{3} - 2}{3(\frac{1}{3}) - 1} = \frac{4\frac{1}{9} + \frac{2}{3} - 2}{1 - 1} = \frac{\frac{4}{9} + \frac{2}{3} - 2}{0} = \frac{-8}{0} = -\infty.$$

Por lo tanto,  $g(x)$  tiene un asíntota vertical en  $x = \frac{1}{3}$ .

$$\lim_{x \rightarrow -\infty} g(x) = \lim_{x \rightarrow -\infty} \frac{4x^2 + 2x - 2}{3x - 1} = -\infty.$$

$$\lim_{x \rightarrow +\infty} g(x) = \lim_{x \rightarrow +\infty} \frac{4x^2 + 2x - 2}{3x - 1} = +\infty.$$

Por lo tanto,  $g(x)$  no tiene asíntotas horizontales.



(c)  $h(x) = \frac{x-2}{x^2-4x+4}$ .

$$x_1, x_2 = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$

$$x_1, x_2 = \frac{4 \pm \sqrt{16-16}}{2}$$

$$x_1, x_2 = \frac{4 \pm \sqrt{0}}{2}$$

$$x_1, x_2 = \frac{4 \pm 0}{2}$$

$$x_1 = \frac{4+0}{2} = \frac{4}{2} = 2.$$

$$x_2 = \frac{4-0}{2} = \frac{4}{2} = 2.$$

$$h(x) = \frac{x-2}{(x-2)^2}$$

$$h(x) = \frac{1}{x-2}$$

$$\lim_{x \rightarrow 2^-} h(x) = \lim_{x \rightarrow 2^-} \frac{1}{x-2} = \frac{1}{2-2} = \frac{1}{0} = -\infty.$$

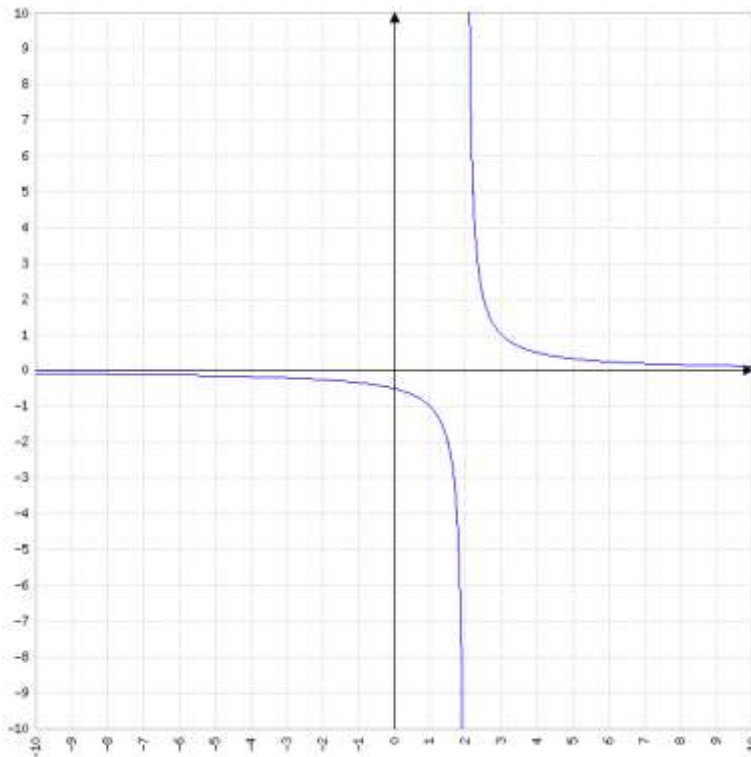
$$\lim_{x \rightarrow 2^+} h(x) = \lim_{x \rightarrow 2^+} \frac{1}{x-2} = \frac{1}{2-2} = \frac{1}{0} = +\infty.$$

Por lo tanto,  $h(x)$  tiene un asíntota vertical en  $x=2$ .

$$\lim_{x \rightarrow -\infty} h(x) = \lim_{x \rightarrow -\infty} \frac{1}{x-2} = 0.$$

$$\lim_{x \rightarrow +\infty} h(x) = \lim_{x \rightarrow +\infty} \frac{1}{x-2} = 0.$$

Por lo tanto,  $h(x)$  tiene un asíntota horizontal en  $y=0$ .



$$(d) \ k(x) = \begin{cases} \frac{x}{x^2-1}, & \text{si } x < 2 \\ 3x^3 - 2x, & \text{si } x \geq 2 \end{cases}.$$

$$\lim_{x \rightarrow -1^-} k(x) = \lim_{x \rightarrow -1^-} \frac{x}{x^2-1} = \frac{1}{(-1)^2-1} = \frac{1}{1-1} = \frac{1}{0} = -\infty.$$

$$\lim_{x \rightarrow -1^+} k(x) = \lim_{x \rightarrow -1^+} \frac{x}{x^2-1} = \frac{1}{(-1)^2-1} = \frac{1}{1-1} = \frac{1}{0} = +\infty.$$

$$\lim_{x \rightarrow 1^-} k(x) = \lim_{x \rightarrow 1^-} \frac{x}{x^2-1} = \frac{1}{1^2-1} = \frac{1}{1-1} = \frac{1}{0} = -\infty.$$

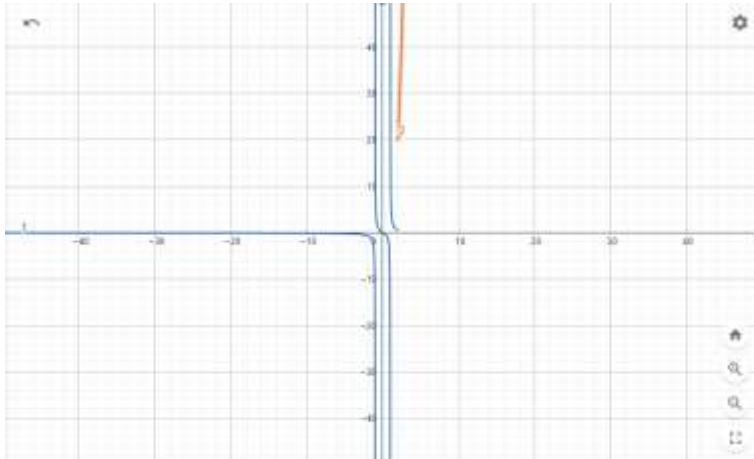
$$\lim_{x \rightarrow 1^+} k(x) = \lim_{x \rightarrow 1^+} \frac{x}{x^2-1} = \frac{1}{1^2-1} = \frac{1}{1-1} = \frac{1}{0} = +\infty.$$

Por lo tanto,  $k(x)$  tiene asíntotas verticales en  $x = -1$  y  $x = 1$ .

$$\lim_{x \rightarrow -\infty} k(x) = \lim_{x \rightarrow -\infty} \frac{x}{x^2-1} = 0.$$

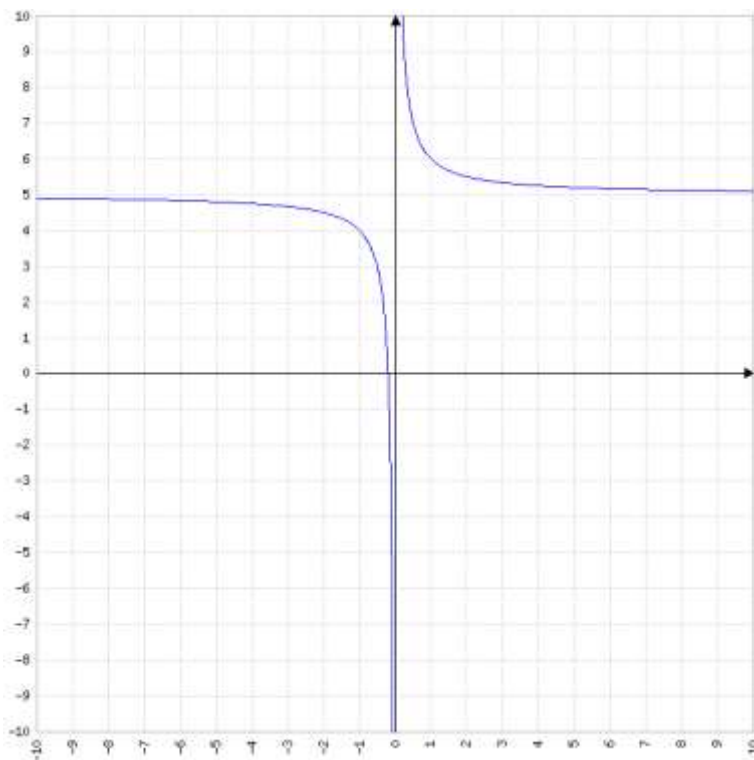
$$\lim_{x \rightarrow +\infty} k(x) = \lim_{x \rightarrow +\infty} \frac{x}{x^2-1} = 0.$$

Por lo tanto,  $k(x)$  tiene una asíntota horizontal en  $y = 0$ .



**Ejercicio 8.**

Utilizar GeoGebra para verificar, gráficamente, el comportamiento de las funciones de los Ejemplos (11) y (15).

**Ejemplo 11:****Ejemplo 15:**



