

Trabajo Práctico N° 6: Integrales.

Ejercicio 1.

(a) Si $\int_0^9 f(x) dx = 37$ y $\int_0^9 g(x) dx = 16$, encontrar el valor de $\int_0^9 2f(x) - \frac{1}{4}g(x) dx$.

$$\begin{aligned}\int_0^9 2f(x) - \frac{1}{4}g(x) dx &= \int_0^9 2f(x) dx + \int_0^9 \frac{-1}{4}g(x) dx \\ \int_0^9 2f(x) - \frac{1}{4}g(x) dx &= 2 \int_0^9 f(x) dx - \frac{1}{4} \int_0^9 g(x) dx \\ \int_0^9 2f(x) - \frac{1}{4}g(x) dx &= 2 * 37 - \frac{1}{4} * 16 \\ \int_0^9 2f(x) - \frac{1}{4}g(x) dx &= 74 - 4 \\ \int_0^9 2f(x) - \frac{1}{4}g(x) dx &= 70.\end{aligned}$$

(b) Si $\int_{-2}^3 h(x) dx = 12$ y $\int_0^3 h(x) dx = 3$, hallar el valor de $\int_{-2}^0 h(x) dx$.

$$\begin{aligned}\int_{-2}^0 h(x) dx &= \int_{-2}^3 h(x) dx - \int_0^3 h(x) dx \\ \int_{-2}^0 h(x) dx &= 12 - 3 \\ \int_{-2}^0 h(x) dx &= 9.\end{aligned}$$

(c) Si $\int_{-1}^3 f(t) dt = 3$ y $\int_{-1}^4 f(t) dt = 7$, determinar el valor de $\int_3^4 f(t) dt$.

$$\begin{aligned}\int_3^4 f(t) dt &= \int_{-1}^4 f(t) dt - \int_{-1}^3 f(t) dt \\ \int_3^4 f(t) dt &= 7 - 3 \\ \int_3^4 f(t) dt &= 4.\end{aligned}$$

Ejercicio 2.

Calcular las siguientes integrales utilizando las propiedades y, en caso de ser posible, usando la regla de Barrow.

(a) $\int_{-2}^3 2x - 1 \, dx.$

$$\int_{-2}^3 2x - 1 \, dx = \int_{-2}^3 2x \, dx + \int_{-2}^3 -1 \, dx$$

$$\int_{-2}^3 2x - 1 \, dx = 2 \int_{-2}^3 x \, dx - \int_{-2}^3 1 \, dx$$

$$\int_{-2}^3 2x - 1 \, dx = 2 \frac{x^2}{2} \Big|_{-2}^3 - x \Big|_{-2}^3$$

$$\int_{-2}^3 2x - 1 \, dx = x^2 \Big|_{-2}^3 - [3 - (-2)]$$

$$\int_{-2}^3 2x - 1 \, dx = [3^2 - (-2)^2] - (3 + 2)$$

$$\int_{-2}^3 2x - 1 \, dx = (9 - 4) - 5$$

$$\int_{-2}^3 2x - 1 \, dx = 5 - 5$$

$$\int_{-2}^3 2x - 1 \, dx = 0.$$

(b) $\int x^2 + 2x + 8 \, dx.$

$$\int x^2 + 2x + 8 \, dx = \int x^2 \, dx + \int 2x \, dx + \int 8 \, dx$$

$$\int x^2 + 2x + 8 \, dx = \frac{x^3}{3} + 2 \int x \, dx + 8 \int 1 \, dx$$

$$\int x^2 + 2x + 8 \, dx = \frac{x^3}{3} + 2 \frac{x^2}{2} + 8x$$

$$\int x^2 + 2x + 8 \, dx = \frac{x^3}{3} + x^2 + 8x + C.$$

(c) $\int_0^{2\pi} \sin x + x \, dx.$

$$\int_0^{2\pi} \sin x + x \, dx = \int_0^{2\pi} \sin x \, dx + \int_0^{2\pi} x \, dx$$

$$\int_0^{2\pi} \sin x + x \, dx = -\cos x \Big|_0^{2\pi} + \frac{x^2}{2} \Big|_0^{2\pi}$$

$$\int_0^{2\pi} \sin x + x \, dx = -(\cos 2\pi - \cos 0) + \frac{1}{2} [(2\pi)^2 - 0^2]$$

$$\int_0^{2\pi} \sin x + x \, dx = -(1 - 1) + \frac{1}{2} (4\pi^2 - 0)$$

$$\int_0^{2\pi} \sin x + x \, dx = -0 + \frac{1}{2} 4\pi^2$$

$$\int_0^{2\pi} \sin x + x \, dx = 2\pi^2.$$

(d) $\int_0^4 2e^x + 3x^4 \, dx.$

$$\begin{aligned}
 \int_0^4 2e^x + 3x^4 \, dx &= \int_0^4 2e^x \, dx + \int_0^4 3x^4 \, dx \\
 \int_0^4 2e^x + 3x^4 \, dx &= 2 \int_0^4 e^x \, dx + 3 \int_0^4 x^4 \, dx \\
 \int_0^4 2e^x + 3x^4 \, dx &= 2e^x \Big|_0^4 + 3 \frac{x^5}{5} \Big|_0^4 \\
 \int_0^4 2e^x + 3x^4 \, dx &= 2(e^4 - e^0) + \frac{3}{5}(4^5 - 0^5) \\
 \int_0^4 2e^x + 3x^4 \, dx &= 2(e^4 - 1) + \frac{3}{5}(1024 - 0) \\
 \int_0^4 2e^x + 3x^4 \, dx &= 2e^4 - 2 + \frac{3}{5} * 1024 \\
 \int_0^4 2e^x + 3x^4 \, dx &= 2e^4 - 2 + \frac{3072}{5} \\
 \int_0^4 2e^x + 3x^4 \, dx &= \frac{10e^4 + 3062}{5}.
 \end{aligned}$$

(e) $\int 3\frac{1}{x} + 2e^x \, dx.$

$$\begin{aligned}
 \int 3\frac{1}{x} + 2e^x \, dx &= \int 3\frac{1}{x} \, dx + \int 2e^x \, dx \\
 \int 3\frac{1}{x} + 2e^x \, dx &= 3 \int \frac{1}{x} \, dx + 2 \int e^x \, dx \\
 \int 3\frac{1}{x} + 2e^x \, dx &= 3 \ln|x| + 2e^x + C.
 \end{aligned}$$

(f) $\int \cos x + \operatorname{sen} x + 2x^{\frac{3}{5}} \, dx.$

$$\begin{aligned}
 \int \cos x + \operatorname{sen} x + 2x^{\frac{3}{5}} \, dx &= \int \cos x \, dx + \int \operatorname{sen} x \, dx + \int 2x^{\frac{3}{5}} \, dx \\
 \int \cos x + \operatorname{sen} x + 2x^{\frac{3}{5}} \, dx &= \operatorname{sen} x - \cos x + 2 \int x^{\frac{3}{5}} \, dx \\
 \int \cos x + \operatorname{sen} x + 2x^{\frac{3}{5}} \, dx &= \operatorname{sen} x - \cos x + 2 \frac{x^{\frac{8}{5}}}{\frac{8}{5}} \\
 \int \cos x + \operatorname{sen} x + 2x^{\frac{3}{5}} \, dx &= \operatorname{sen} x - \cos x + \frac{5}{4}x^{\frac{8}{5}} + C.
 \end{aligned}$$

(g) $\int_{-5}^1 x^2 + 2x + 8 \, dx.$

$$\begin{aligned}
 \int_{-5}^1 x^2 + 2x + 8 \, dx &= \int_{-5}^1 x^2 \, dx + \int_{-5}^1 2x \, dx + \int_{-5}^1 8 \, dx \\
 \int_{-5}^1 x^2 + 2x + 8 \, dx &= \frac{x^3}{3} \Big|_{-5}^1 + 2 \int_{-5}^1 x \, dx + 8 \int_{-5}^1 1 \, dx \\
 \int_{-5}^1 x^2 + 2x + 8 \, dx &= \frac{1}{3}[1^3 - (-5)^3] + 2 \frac{x^2}{2} \Big|_{-5}^1 + 8x \Big|_{-5}^1 \\
 \int_{-5}^1 x^2 + 2x + 8 \, dx &= \frac{1}{3}[1 - (-125)] + [1^2 - (-5)^2] + 8[1 - (-5)] \\
 \int_{-5}^1 x^2 + 2x + 8 \, dx &= \frac{1}{3}(1 + 125) + (1 - 25) + 8(1 + 5) \\
 \int_{-5}^1 x^2 + 2x + 8 \, dx &= \frac{1}{3} * 126 - 24 + 8 * 6 \\
 \int_{-5}^1 x^2 + 2x + 8 \, dx &= \frac{126}{3} - 24 + 48
 \end{aligned}$$

$$\int_{-5}^1 x^2 + 2x + 8 \, dx = 66.$$

(h) $\int x - x^{\frac{2}{5}} + 3e^x - \cos x \, dx.$

$$\begin{aligned}\int x - x^{\frac{2}{5}} + 3e^x - \cos x \, dx &= \int x \, dx + \int -x^{\frac{2}{5}} \, dx + \int 3e^x \, dx + \int -\cos x \, dx \\ \int x - x^{\frac{2}{5}} + 3e^x - \cos x \, dx &= \frac{x^2}{2} - \int x^{\frac{2}{5}} \, dx + 3 \int e^x \, dx - \int \cos x \, dx \\ \int x - x^{\frac{2}{5}} + 3e^x - \cos x \, dx &= \frac{x^2}{2} - \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + 3e^x - \sin x \\ \int x - x^{\frac{2}{5}} + 3e^x - \cos x \, dx &= \frac{1}{2}x^2 - \frac{5}{7}x^{\frac{7}{5}} + 3e^x - \sin x + C.\end{aligned}$$

Ejercicio 3.

Calcular las siguientes integrales utilizando los métodos vistos.

(a) $\int (3x^4 + 5x^2 + 8)^4(12x^3 + 10x) dx.$

$$\int (3x^4 + 5x^2 + 8)^4(12x^3 + 10x) dx = \int u^4 du \quad (*)$$

$$\int (3x^4 + 5x^2 + 8)^4(12x^3 + 10x) dx = \frac{u^5}{5}$$

$$\int (3x^4 + 5x^2 + 8)^4(12x^3 + 10x) dx = \frac{(3x^4 + 5x^2 + 8)^5}{5} + C.$$

(*) $u = 3x^4 + 5x^2 + 8; du = (12x^3 + 10x) dx.$

(b) $\int x \cos x dx.$

$$\int x \cos x dx = x \sin x - \int \sin x dx \quad (*)$$

$$\int x \cos x dx = x \sin x - (-\cos x)$$

$$\int x \cos x dx = x \sin x + \cos x + C.$$

(*) $u = x; du = dx; dv = \cos x dx; v = \sin x.$

(c) $\int x^3 \ln x dx.$

$$\int x^3 \ln x dx = \ln x \frac{x^4}{4} - \int \frac{x^4}{4} \frac{1}{x} dx \quad (*)$$

$$\int x^3 \ln x dx = \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 dx$$

$$\int x^3 \ln x dx = \frac{x^4}{4} \ln x - \frac{1}{4} \frac{x^4}{4}$$

$$\int x^3 \ln x dx = \frac{x^4}{4} \ln x - \frac{x^4}{16}$$

$$\int x^3 \ln x dx = \frac{x^4}{4} \left(\ln x - \frac{1}{4} \right) + C.$$

(*) $u = \ln x; du = \frac{1}{x} dx; dv = x^3 dx; v = \frac{x^4}{4}.$

(d) $\int \cos 5x * 5 dx.$

$$\int \cos 5x * 5 dx = \int \cos u du \quad (*)$$

$$\int \cos 5x * 5 dx = \sin u$$

$$\int \cos 5x * 5 dx = \sin 5x + C.$$

(*) $u = 5x; du = 5 dx.$

(e) $\int \frac{2+e^x}{e^x+2x} dx.$

$$\begin{aligned}\int \frac{2+e^x}{e^x+2x} dx &= \int \frac{1}{u} du && (*) \\ \int \frac{2+e^x}{e^x+2x} dx &= \ln |u| \\ \int \frac{2+e^x}{e^x+2x} dx &= \ln |e^x + 2x| + C.\end{aligned}$$

(*) $u = e^x + 2x$; $du = (e^x + 2) dx.$

(f) $\int x\sqrt{x-1} dx.$

(*) $u = \sqrt{x-1}$; $du = \frac{1}{2\sqrt{x-1}} dx.$

$$\begin{aligned}\int x\sqrt{x-1} dx &= \int (u+1)\sqrt{u} du && (*) \\ \int x\sqrt{x-1} dx &= \int u^{\frac{3}{2}} + u^{\frac{1}{2}} du \\ \int x\sqrt{x-1} dx &= \int u^{\frac{3}{2}} du + \int u^{\frac{1}{2}} du \\ \int x\sqrt{x-1} dx &= \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \\ \int x\sqrt{x-1} dx &= \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} \\ \int x\sqrt{x-1} dx &= 2 \left[\frac{(x-1)^{\frac{5}{2}}}{5} + \frac{(x-1)^{\frac{3}{2}}}{3} \right] + C.\end{aligned}$$

(*) $u = x - 1$; $du = dx.$

(g) $\int_0^8 \frac{1}{\sqrt{x+1}} dx.$

$$\begin{aligned}\int_0^8 \frac{1}{\sqrt{x+1}} dx &= \int_0^8 \frac{1}{\sqrt{u}} du && (*) \\ \int_0^8 \frac{1}{\sqrt{x+1}} dx &= \int_{0+1}^{8+1} u^{-\frac{1}{2}} du \\ \int_0^8 \frac{1}{\sqrt{x+1}} dx &= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \Big|_{0+1}^{8+1} \\ \int_0^8 \frac{1}{\sqrt{x+1}} dx &= 2(x+1)^{\frac{1}{2}} \Big|_0^8 \\ \int_0^8 \frac{1}{\sqrt{x+1}} dx &= 2\sqrt{x+1} \Big|_0^8 \\ \int_0^8 \frac{1}{\sqrt{x+1}} dx &= 2(\sqrt{8+1} - \sqrt{0+1}) \\ \int_0^8 \frac{1}{\sqrt{x+1}} dx &= 2(\sqrt{9} - \sqrt{1})\end{aligned}$$

$$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 2 (3 - 1)$$

$$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 2 * 2$$

$$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 4.$$

(*) $u = x + 1$; $du = dx$.

(h) $\int_0^{2\pi} x \sin x dx$.

$$\int_0^{2\pi} x \sin x dx = x (-\cos x) \Big|_0^{2\pi} - \int_0^{2\pi} -\cos x dx \quad (*)$$

$$\int_0^{2\pi} x \sin x dx = -x \cos x \Big|_0^{2\pi} + \int_0^{2\pi} \cos x dx$$

$$\int_0^{2\pi} x \sin x dx = -(2\pi \cos 2\pi - 0 \cos 0) + \sin x \Big|_0^{2\pi}$$

$$\int_0^{2\pi} x \sin x dx = -(2\pi * 1 - 0 * 1) + (\sin 2\pi - \sin 0)$$

$$\int_0^{2\pi} x \sin x dx = -(2\pi - 0) + (0 - 0)$$

$$\int_0^{2\pi} x \sin x dx = -2\pi + 0$$

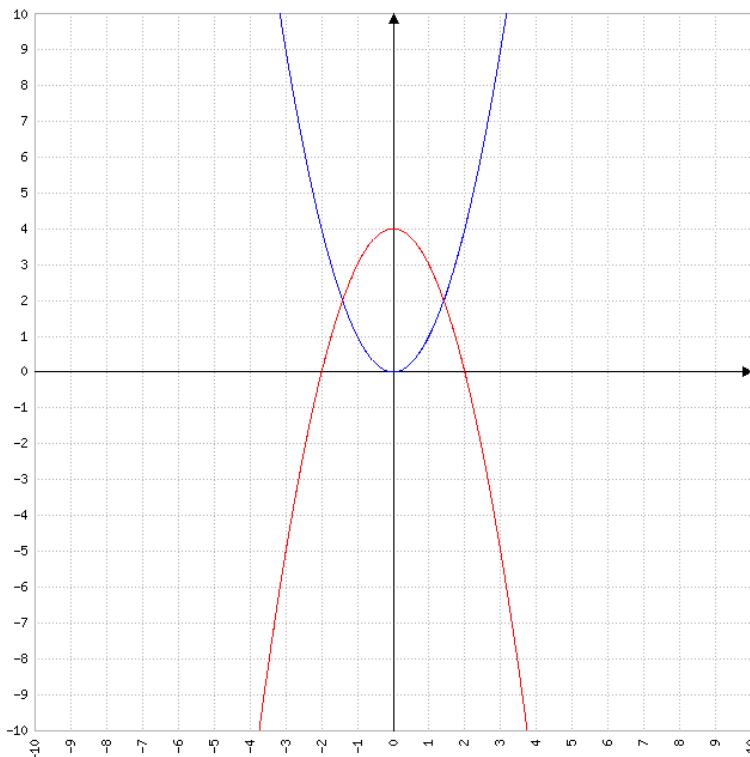
$$\int_0^{2\pi} x \sin x dx = -2\pi.$$

(*) $u = x$; $du = dx$; $dv = \sin x dx$; $v = -\cos x$.

Ejercicio 4.

Hallar el área comprendida entre las gráficas de las siguientes pares de funciones:

(a) $f(x) = x^2$ y $g(x) = -x^2 + 4$.



$$f(x) = g(x)$$

$$x^2 = -x^2 + 4$$

$$x^2 + x^2 = 4$$

$$2x^2 = 4$$

$$x^2 = \frac{4}{2}$$

$$x^2 = 2$$

$$\sqrt{x^2} = \sqrt{2}$$

$$|x| = \sqrt{2}$$

$$x = \pm \sqrt{2}.$$

Intervalo	$(-\sqrt{2}, \sqrt{2})$
VP	0
$f(x)$	0
$g(x)$	4

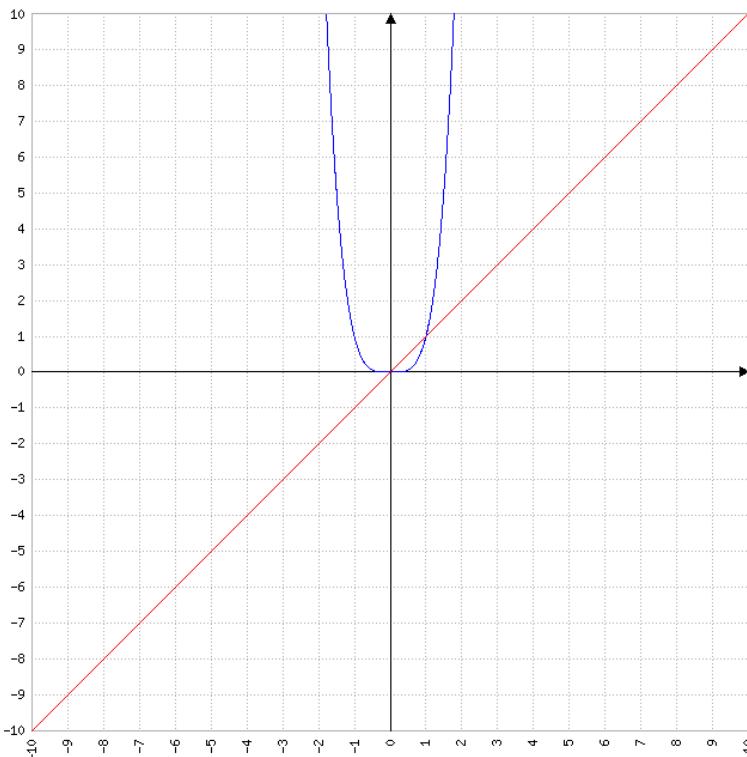
$$A = \int_{-\sqrt{2}}^{\sqrt{2}} g(x) - f(x) dx$$

$$A = \int_{-\sqrt{2}}^{\sqrt{2}} (-x^2 + 4) - x^2 dx$$

$$A = \int_{-\sqrt{2}}^{\sqrt{2}} -x^2 + 4 - x^2 dx$$

$$\begin{aligned}
 A &= \int_{-\sqrt{2}}^{\sqrt{2}} -2x^2 + 4 \, dx \\
 A &= \int_{-\sqrt{2}}^{\sqrt{2}} -2x^2 \, dx + \int_{-\sqrt{2}}^{\sqrt{2}} 4 \, dx \\
 A &= -2 \int_{-\sqrt{2}}^{\sqrt{2}} x^2 \, dx + 4 \int_{-\sqrt{2}}^{\sqrt{2}} 1 \, dx \\
 A &= -2 \frac{x^3}{3} \Big|_{-\sqrt{2}}^{\sqrt{2}} + 4x \Big|_{-\sqrt{2}}^{\sqrt{2}} \\
 A &= \frac{-2}{3} [(\sqrt{2})^3 - (-\sqrt{2})^3] + 4 [\sqrt{2} - (-\sqrt{2})] \\
 A &= \frac{-2}{3} [(\sqrt{2})^3 + (\sqrt{2})^3] + 4 (\sqrt{2} + \sqrt{2}) \\
 A &= \frac{-2}{3} * 2 (\sqrt{2})^3 + 4 * 2 \sqrt{2} \\
 A &= \frac{-4}{3} (\sqrt{2})^2 \sqrt{2} + 8 \sqrt{2} \\
 A &= \frac{-4}{3} * 2 \sqrt{2} + 8 \sqrt{2} \\
 A &= \frac{-8}{3} \sqrt{2} + 8 \sqrt{2} \\
 A &= \frac{16}{3} \sqrt{2}.
 \end{aligned}$$

(b) $f(x) = x^4$ y $g(x) = x$.



$$f(x) = g(x)$$

$$x^4 = x$$

$$x^4 - x = 0$$

$$x^3 (x - 1) = 0.$$

$$x_1 = 0; x_2 = 1.$$

Intervalo	(0, 1)
VP	$\frac{1}{2}$
f(x)	$\frac{1}{16}$
g(x)	$\frac{1}{2}$

$$A = \int_0^1 g(x) - f(x) dx$$

$$A = \int_0^1 x - x^4 dx$$

$$A = \int_0^1 x dx + \int_0^1 -x^4 dx$$

$$A = \frac{x^2}{2} \Big|_0^1 - \int_0^1 x^4 dx$$

$$A = \frac{1}{2}(1^2 - 0^2) - \frac{x^5}{5} \Big|_0^1$$

$$A = \frac{1}{2}(1 - 0) - \frac{1}{5}(1^5 - 0^5)$$

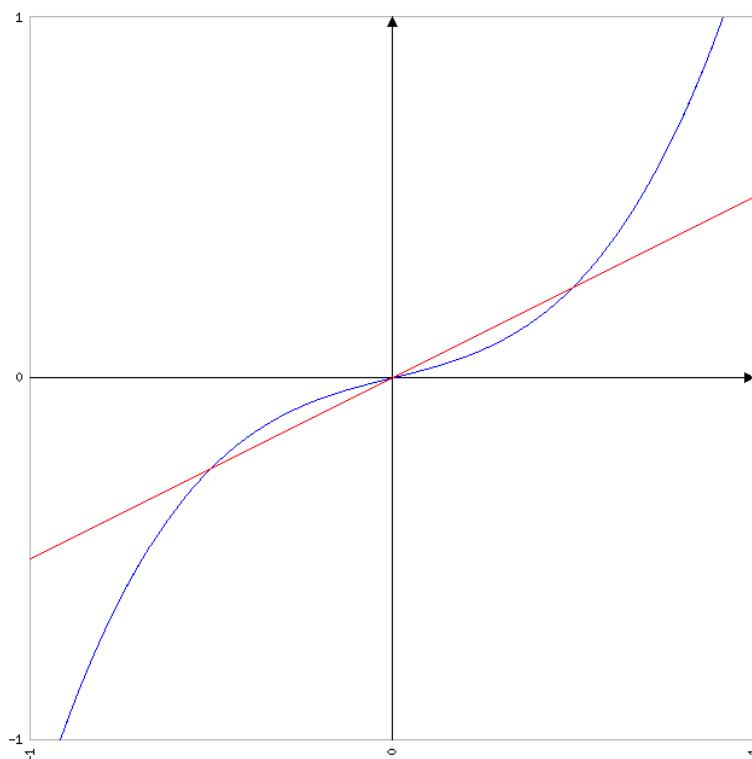
$$A = \frac{1}{2} * 1 - \frac{1}{5}(1 - 0)$$

$$A = \frac{1}{2} - \frac{1}{5} * 1$$

$$A = \frac{1}{2} - \frac{1}{5}$$

$$A = \frac{3}{10}.$$

(c) $f(x) = x^3 + \frac{1}{4}x$ y $g(x) = \frac{1}{2}x$.



$$\begin{aligned}
 f(x) &= g(x) \\
 x^3 + \frac{1}{4}x &= \frac{1}{2}x \\
 x^3 + \frac{1}{4}x - \frac{1}{2}x &= 0 \\
 x^3 - \frac{1}{4}x &= 0 \\
 x(x^2 - \frac{1}{4}) &= 0.
 \end{aligned}$$

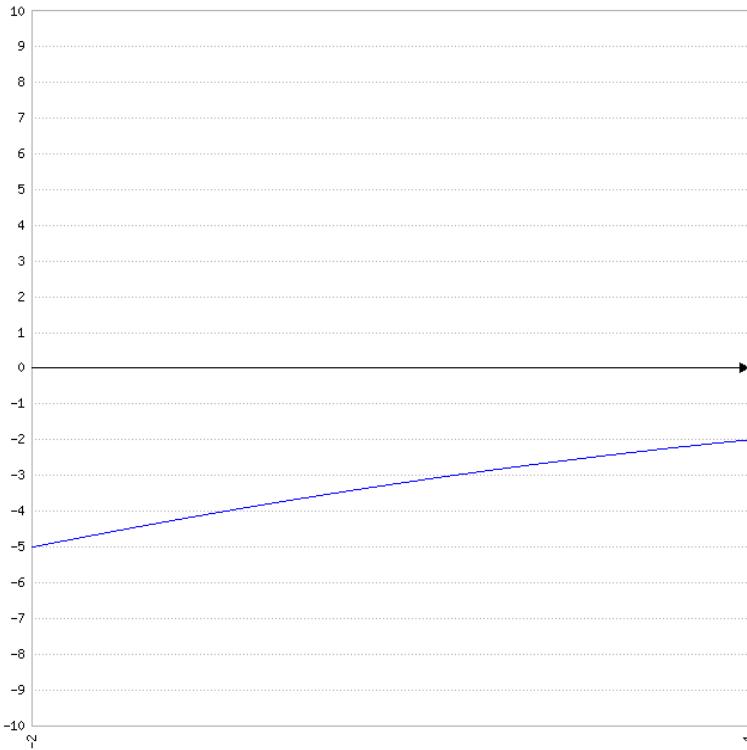
$$x_1 = 0; x_2 = \frac{-1}{2}; x_3 = \frac{1}{2}.$$

Intervalo	$(-\frac{1}{2}, 0)$	$(0, \frac{1}{2})$
VP	$\frac{-1}{4}$	$\frac{1}{4}$
$f(x)$	$\frac{-5}{64}$	$\frac{5}{64}$
$g(x)$	$\frac{-1}{8}$	$\frac{1}{8}$

$$\begin{aligned}
 A &= \int_{-\frac{1}{2}}^0 f(x) - g(x) dx + \int_0^{\frac{1}{2}} g(x) - f(x) dx \\
 A &= \int_{-\frac{1}{2}}^0 (x^3 + \frac{1}{4}x) - \frac{1}{2}x dx + \int_0^{\frac{1}{2}} \frac{1}{2}x - (x^3 + \frac{1}{4}x) dx \\
 A &= \int_{-\frac{1}{2}}^0 x^3 + \frac{1}{4}x - \frac{1}{2}x dx + \int_0^{\frac{1}{2}} \frac{1}{2}x - x^3 - \frac{1}{4}x dx \\
 A &= \int_{-\frac{1}{2}}^0 x^3 - \frac{1}{4}x dx + \int_0^{\frac{1}{2}} -x^3 + \frac{1}{4}x dx \\
 A &= \int_{-\frac{1}{2}}^0 x^3 dx + \int_{-\frac{1}{2}}^0 \frac{-1}{4}x dx + \int_0^{\frac{1}{2}} -x^3 dx + \int_0^{\frac{1}{2}} \frac{1}{4}x dx \\
 A &= \int_{-\frac{1}{2}}^0 x^3 dx - \frac{1}{4} \int_{-\frac{1}{2}}^0 x dx - \int_0^{\frac{1}{2}} x^3 dx + \frac{1}{4} \int_0^{\frac{1}{2}} x dx \\
 A &= \frac{x^4}{4} \Big|_{-\frac{1}{2}}^0 - \frac{1}{4} \frac{x^2}{2} \Big|_{-\frac{1}{2}}^0 - \frac{x^4}{4} \Big|_0^{\frac{1}{2}} + \frac{1}{4} \frac{x^2}{2} \Big|_0^{\frac{1}{2}} \\
 A &= \frac{1}{4} [0^4 - (\frac{-1}{2})^4] - \frac{1}{8} [0^2 - (\frac{-1}{2})^2] - \frac{1}{4} [(\frac{1}{2})^4 - 0^4] + \frac{1}{8} [(\frac{1}{2})^2 - 0^2] \\
 A &= \frac{1}{4} (0 - \frac{1}{16}) - \frac{1}{8} (0 - \frac{1}{4}) - \frac{1}{4} (\frac{1}{16} - 0) + \frac{1}{8} (\frac{1}{4} - 0) \\
 A &= \frac{1}{4} \frac{(-1)}{16} - \frac{1}{8} \frac{(-1)}{4} - \frac{1}{4} \frac{1}{16} + \frac{1}{8} \frac{1}{4} \\
 A &= \frac{-1}{64} + \frac{1}{32} - \frac{1}{64} + \frac{1}{32} \\
 A &= \frac{1}{32}.
 \end{aligned}$$

Ejercicio 5.

Calcular el área de la región comprendida entre el eje x y el gráfico de la función $f(x) = -x^2 - 1$ entre $-2 \leq x \leq -1$.



$$\begin{aligned}f(x) &= 0 \\-x^2 - 1 &= 0 \\x^2 &\neq -1.\end{aligned}$$

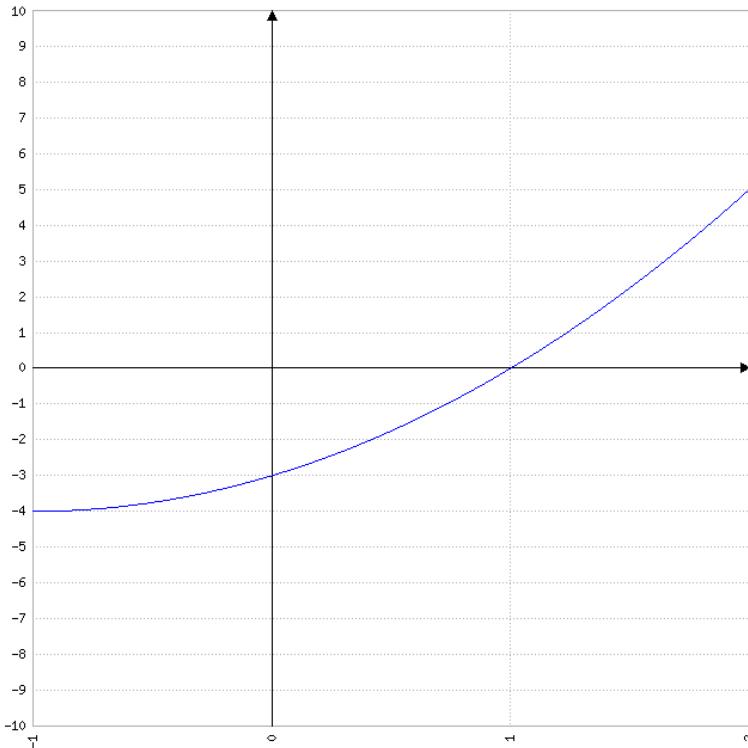
Intervalo	(-2, -1)
VP	$\frac{-3}{2}$
$f(x)$	< 0

$$\begin{aligned}A &= -\int_{-2}^{-1} -x^2 - 1 \, dx \\A &= -(\int_{-2}^{-1} -x^2 \, dx + \int_{-2}^{-1} -1 \, dx) \\A &= -(-\int_{-2}^{-1} x^2 \, dx - \int_{-2}^{-1} 1 \, dx) \\A &= -\left(\frac{-x^3}{3}\right) \Big|_{-2}^{-1} - \left(x\right) \Big|_{-2}^{-1} \\A &= -\left\{\frac{-1}{3} [(-1)^3 - (-2)^3] - [-1 - (-2)]\right\} \\A &= -\left\{\frac{-1}{3} [-1 - (-8)] - (-1 + 2)\right\} \\A &= -\left[\frac{-1}{3} (-1 + 8) - 1\right] \\A &= -\left(\frac{-1}{3} * 7 - 1\right) \\A &= -\left(\frac{-7}{3} - 1\right)\end{aligned}$$

$$A = -\left(\frac{-10}{3}\right)$$
$$A = \frac{10}{3}.$$

Ejercicio 6.

Calcular el área de la región comprendida entre el eje x y el gráfico de la función $f(x) = x^2 + 2x - 3$ entre $-1 \leq x \leq 2$.



$$f(x) = 0 \\ x^2 + 2x - 3 = 0.$$

$$x_1, x_2 = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} \\ x_1, x_2 = \frac{-2 \pm \sqrt{4 + 12}}{2} \\ x_1, x_2 = \frac{-2 \pm \sqrt{16}}{2} \\ x_1, x_2 = \frac{-2 \pm 4}{2} \\ x_1 = \frac{-2 + 4}{2} = \frac{2}{2} = 1. \\ x_2 = \frac{-2 - 4}{2} = \frac{-6}{2} = -3.$$

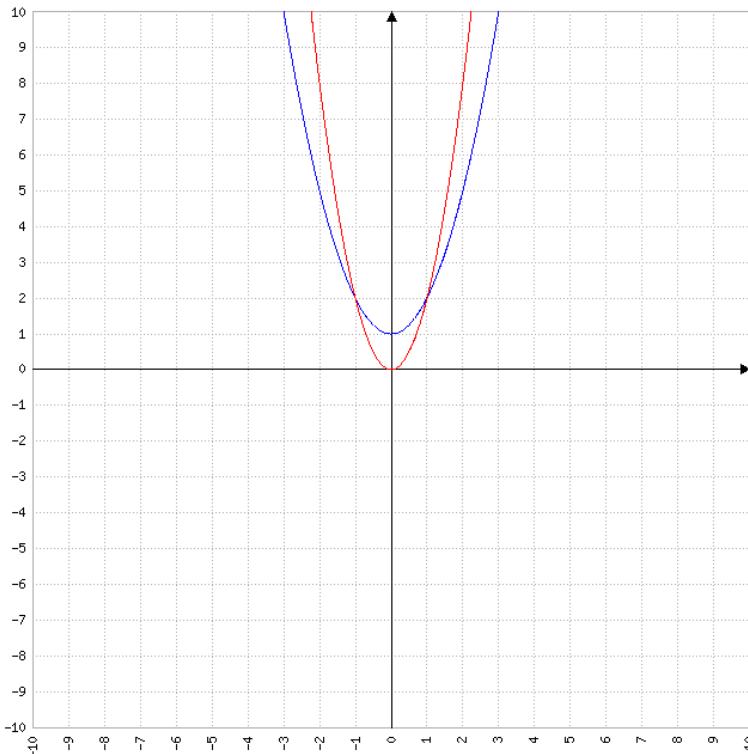
Intervalo	(-1, 1)	(1, 2)
VP	0	$\frac{3}{2}$
$f(x)$	< 0	> 0

$$A = -\int_{-1}^1 x^2 + 2x - 3 \, dx + \int_1^2 x^2 + 2x - 3 \, dx \\ A = -(\int_{-1}^1 x^2 \, dx + \int_{-1}^1 2x \, dx + \int_{-1}^1 -3 \, dx) + (\int_1^2 x^2 \, dx + \int_1^2 2x \, dx + \int_1^2 -3 \, dx) \\ A = -\left(\frac{x^3}{3}\Big|_{-1}^1 + 2 \int_{-1}^1 x \, dx - 3 \int_{-1}^1 1 \, dx\right) + \left(\frac{x^3}{3}\Big|_1^2 + 2 \int_1^2 x \, dx - 3 \int_1^2 1 \, dx\right)$$

$$\begin{aligned} A &= -\left\{\frac{1}{3}[1^3 - (-1)^3] + 2 \frac{x^2}{2} |-1 - 3x|_{-1}^1 + \left[\frac{1}{3}(2^3 - 1^3) + 2 \frac{x^2}{2} |_1^2 - 3x|_1^2\right]\right. \\ A &= -\left\{\frac{1}{3}[1 - (-1)] + [1^2 - (-1)^2] - 3[1 - (-1)]\right\} + \left[\frac{1}{3}(8 - 1) + (2^2 - 1^2) - 3(2 - 1)\right] \\ A &= -\left[\frac{1}{3}(1 + 1) + (1 - 1) - 3(1 + 1)\right] + \left[\frac{1}{3} * 7 + (4 - 1) - 3 * 1\right] \\ A &= -\left(\frac{1}{3} * 2 + 0 - 3 * 2\right) + \left(\frac{7}{3} + 3 - 3\right) \\ A &= -\left(\frac{2}{3} + 0 - 6\right) + \frac{7}{3} \\ A &= -\left(\frac{-16}{3}\right) + \frac{7}{3} \\ A &= \frac{16}{3} + \frac{7}{3} \\ A &= \frac{23}{3}. \end{aligned}$$

Ejercicio 7.

Calcular el área de la región comprendida entre los gráficos de $f(x) = x^2 + 1$ y $g(x) = 2x^2$ para $0 \leq x \leq 2$.



$$f(x) = g(x)$$

$$x^2 + 1 = 2x^2$$

$$2x^2 - x^2 = 1$$

$$x^2 = 1$$

$$\sqrt{x^2} = \sqrt{1}$$

$$|x| = 1$$

$$x = \pm 1.$$

Intervalo	(-1, 1)
VP	0
f(x)	1
g(x)	0

$$A = \int_{-1}^1 (x^2 + 1) - 2x^2 dx$$

$$A = \int_{-1}^1 x^2 + 1 - 2x^2 dx$$

$$A = \int_{-1}^1 -x^2 + 1 dx$$

$$A = \int_{-1}^1 -x^2 dx + \int_{-1}^1 1 dx$$

$$A = -\int_{-1}^1 x^2 dx + x \Big|_{-1}^1$$

$$A = \frac{-x^3}{3} \Big|_{-1}^1 + [1 - (-1)]$$

$$A = \frac{-1}{3} [1^3 - (-1)^3] + (1 + 1)$$

$$A = \frac{-1}{3} [1 - (-1)] + 2$$

$$A = \frac{-1}{3} (1 + 1) + 2$$

$$A = \frac{-1}{3} * 2 + 2$$

$$A = \frac{-2}{3} + 2$$

$$A = \frac{4}{3}$$

Ejercicio 8.

Hallar $f(x)$ sabiendo que $f'(x) = x + \frac{1}{x^2}$ y $f(1) = 1$.

$$\begin{aligned}f(x) &= \int f'(x) dx \\f(x) &= \int x + \frac{1}{x^2} dx \\f(x) &= \int x dx + \int \frac{1}{x^2} dx \\f(x) &= \frac{x^2}{2} + \int x^{-2} dx \\f(x) &= \frac{x^2}{2} + \frac{x^{-1}}{-1} \\f(x) &= \frac{1}{2}x^2 - \frac{1}{x} + C.\end{aligned}$$

$$\begin{aligned}f(1) &= 1 \\ \frac{1}{2} * 1^2 - \frac{1}{1} + C &= 1 \\ \frac{1}{2} * 1 - 1 + C &= 1 \\ \frac{1}{2} - 1 + C &= 1 \\ \frac{-1}{2} + C &= 1 \\ C &= 1 + \frac{1}{2} \\ C &= \frac{3}{2}.\end{aligned}$$

$$f(x) = \frac{1}{2}x^2 - \frac{1}{x} + \frac{3}{2}.$$

Ejercicio 9.

Sabiendo que $f'(x) = 3x^2 - 8x + 2$ y, además, que $f(3) = -4$, hallar la función $f(x)$.

$$\begin{aligned}f(x) &= \int f'(x) dx \\f(x) &= \int 3x^2 - 8x + 2 dx \\f(x) &= \int 3x^2 dx + \int -8x dx + \int 2 dx \\f(x) &= 3 \int x^2 dx - 8 \int x dx + 2 \int 1 dx \\f(x) &= 3 \frac{x^3}{3} - 8 \frac{x^2}{2} + 2x \\f(x) &= x^3 - 4x^2 + 2x + C.\end{aligned}$$

$$\begin{aligned}f(3) &= -4 \\3^3 - 4 * 3^2 + 2 * 3 + C &= -4 \\27 - 4 * 9 + 6 + C &= -4 \\27 - 36 + 6 + C &= -4 \\-3 + C &= -4 \\C &= -4 + 3 \\C &= -1.\end{aligned}$$

$$f(x) = x^3 - 4x^2 + 2x - 1.$$

Ejercicio 10.

Hallar todas las funciones cuya derivada es $g'(x) = x^2 \cos x$.

$$\begin{aligned} g(x) &= \int g'(x) dx \\ g(x) &= \int x^2 \cos x dx \\ g(x) &= x^2 \operatorname{sen} x - \int \operatorname{sen} x * 2x dx && (*) \\ g(x) &= x^2 \operatorname{sen} x - 2 \int x \operatorname{sen} x dx \\ g(x) &= x^2 \operatorname{sen} x - 2 [x(-\cos x) - \int -\cos x dx] && (**) \\ g(x) &= x^2 \operatorname{sen} x - 2 (-x \cos x + \int \cos x dx) \\ g(x) &= x^2 \operatorname{sen} x - 2 (-x \cos x + \operatorname{sen} x) \\ g(x) &= x^2 \operatorname{sen} x - 2x \cos x - 2 \operatorname{sen} x \\ g(x) &= (x^2 - 2) \operatorname{sen} x - 2x \cos x + C. \end{aligned}$$

(*) $u = x^2$; $du = 2x dx$; $dv = \cos x dx$; $v = \operatorname{sen} x$.

(**) $u = x$; $du = dx$; $dv = \operatorname{sen} x$; $v = -\cos x$.

Ejercicio 11.

Sea $g''(x) = 2x^3 - 4x^7$, $g'(1) = -2$ y $g(0) = 0$, hallar la función $g(x)$.

$$\begin{aligned} g'(x) &= \int g''(x) dx \\ g'(x) &= \int 2x^3 - 4x^7 dx \\ g'(x) &= \int 2x^3 dx + \int -4x^7 dx \\ g'(x) &= 2 \int x^3 dx - 4 \int x^7 dx \\ g'(x) &= 2 \frac{x^4}{4} - 4 \frac{x^8}{8} \\ g'(x) &= \frac{1}{2}x^4 - \frac{1}{2}x^8 \\ g'(x) &= \frac{1}{2}(x^4 - x^8) + C. \end{aligned}$$

$$\begin{aligned} g'(1) &= -2 \\ \frac{1}{2}(1^4 - 1^8) + C &= -2 \\ \frac{1}{2}(1 - 1) + C &= -2 \\ \frac{1}{2} * 0 + C &= -2 \\ 0 + C &= -2 \\ C &= -2. \end{aligned}$$

$$g'(x) = \frac{1}{2}(x^4 - x^8) - 2.$$

$$\begin{aligned} g(x) &= \int g'(x) dx \\ g(x) &= \int \frac{1}{2}(x^4 - x^8) - 2 dx \\ g(x) &= \int \frac{1}{2}x^4 - \frac{1}{2}x^8 - 2 dx \\ g(x) &= \int \frac{1}{2}x^4 dx + \int \frac{-1}{2}x^8 dx + \int -2 dx \\ g(x) &= \frac{1}{2} \int x^4 dx - \frac{1}{2} \int x^8 dx - 2 \int 1 dx \\ g(x) &= \frac{1}{2} \frac{x^5}{5} - \frac{1}{2} \frac{x^9}{9} - 2x \\ g(x) &= \frac{1}{10}x^5 - \frac{1}{18}x^9 - 2x + C. \end{aligned}$$

$$\begin{aligned} g(0) &= 0 \\ \frac{1}{10}0^5 - \frac{1}{18}0^9 - 2 * 0 + C &= 0 \\ \frac{1}{10} * 0 - \frac{1}{18} * 0 - 0 + C &= 0 \\ 0 - 0 - 0 + C &= 0 \\ C &= 0. \end{aligned}$$

$$g(x) = \frac{1}{10}x^5 - \frac{1}{18}x^9 - 2x.$$