Trabajo Práctico N° 6:
Integrales.

Ejercicio 1.

(a) *Si $\int_0^9 f(x)\,dx = 37$ y $\int_0^9 g(x)\,dx = 16$, encontrar el valor de $\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx$.*

$\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx = \int_0^9 2f(x)\,dx + \int_0^9 \frac{-1}{4}g(x)\,dx$

$\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx = 2\int_0^9 f(x)\,dx - \frac{1}{4}\int_0^9 g(x)\,dx$

$\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx = 2*37 - \frac{1}{4}*16$

$\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx = 74 - 4$

$\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx = 70.$

(b) *Si $\int_{-2}^3 h(x)\,dx = 12$ y $\int_0^3 h(x)\,dx = 3$, hallar el valor de $\int_{-2}^0 h(x)\,dx$.*

$\int_{-2}^0 h(x)\,dx = \int_{-2}^3 h(x)\,dx - \int_0^3 h(x)\,dx$

$\int_{-2}^0 h(x)\,dx = 12 - 3$

$\int_{-2}^0 h(x)\,dx = 9.$

(c) *Si $\int_{-1}^3 f(t)\,dt = 3$ y $\int_{-1}^4 f(t)\,dt = 7$, determinar el valor de $\int_3^4 f(t)\,dt$.*

$\int_3^4 f(t)\,dt = \int_{-1}^4 f(t)\,dt - \int_{-1}^3 f(t)\,dt$

$\int_3^4 f(t)\,dt = 7 - 3$

$\int_3^4 f(t)\,dt = 4.$

Ejercicio 2.

Calcular las siguientes integrales utilizando las propiedades y, en caso de ser posible, usando la regla de Barrow.

(a) $\int_{-2}^{3} 2x - 1 \, dx$.

$\int_{-2}^{3} 2x - 1 \, dx = \int_{-2}^{3} 2x \, dx + \int_{-2}^{3} -1 \, dx$

$\int_{-2}^{3} 2x - 1 \, dx = 2 \int_{-2}^{3} x \, dx - \int_{-2}^{3} 1 \, dx$

$\int_{-2}^{3} 2x - 1 \, dx = 2 \frac{x^2}{2} |_{-2}^{3} - x |_{-2}^{3}$

$\int_{-2}^{3} 2x - 1 \, dx = x^2 |_{-2}^{3} - [3 - (-2)]$

$\int_{-2}^{3} 2x - 1 \, dx = [3^2 - (-2)^2] - (3 + 2)$

$\int_{-2}^{3} 2x - 1 \, dx = (9 - 4) - 5$

$\int_{-2}^{3} 2x - 1 \, dx = 5 - 5$

$\int_{-2}^{3} 2x - 1 \, dx = 0$.

(b) $\int x^2 + 2x + 8 \, dx$.

$\int x^2 + 2x + 8 \, dx = \int x^2 \, dx + \int 2x \, dx + \int 8 \, dx$

$\int x^2 + 2x + 8 \, dx = \frac{x^3}{3} + 2 \int x \, dx + 8 \int 1 \, dx$

$\int x^2 + 2x + 8 \, dx = \frac{x^3}{3} + 2 \frac{x^2}{2} + 8x$

$\int x^2 + 2x + 8 \, dx = \frac{x^3}{3} + x^2 + 8x + C$.

(c) $\int_{0}^{2\pi} sen \, x + x \, dx$.

$\int_{0}^{2\pi} sen \, x + x \, dx = \int_{0}^{2\pi} sen \, x \, dx + \int_{0}^{2\pi} x \, dx$

$\int_{0}^{2\pi} sen \, x + x \, dx = -\cos x |_{0}^{2\pi} + \frac{x^2}{2} |_{0}^{2\pi}$

$\int_{0}^{2\pi} sen \, x + x \, dx = -(\cos 2\pi - \cos 0) + \frac{1}{2} [(2\pi)^2 - 0^2]$

$\int_{0}^{2\pi} sen \, x + x \, dx = -(1 - 1) + \frac{1}{2} (4\pi^2 - 0)$

$\int_{0}^{2\pi} sen \, x + x \, dx = -0 + \frac{1}{2} 4\pi^2$

$\int_{0}^{2\pi} sen \, x + x \, dx = 2\pi^2$.

(d) $\int_{0}^{4} 2e^x + 3x^4 \, dx$.

$\int_0^4 2e^x + 3x^4\ dx = \int_0^4 2e^x\ dx + \int_0^4 3x^4\ dx$

$\int_0^4 2e^x + 3x^4\ dx = 2\int_0^4 e^x\ dx + 3\int_0^4 x^4\ dx$

$\int_0^4 2e^x + 3x^4\ dx = 2e^x\ |_0^4 + 3\frac{x^5}{5}\ |_0^4$

$\int_0^4 2e^x + 3x^4\ dx = 2\ (e^4 - e^0) + \frac{3}{5}(4^5 - 0^5)$

$\int_0^4 2e^x + 3x^4\ dx = 2\ (e^4 - 1) + \frac{3}{5}(1024 - 0)$

$\int_0^4 2e^x + 3x^4\ dx = 2e^4 - 2 + \frac{3}{5} * 1024$

$\int_0^4 2e^x + 3x^4\ dx = 2e^4 - 2 + \frac{3072}{5}$

$\int_0^4 2e^x + 3x^4\ dx = \frac{10e^4 + 3062}{5}$.

(e) $\int 3\frac{1}{x} + 2e^x\ dx.$

$\int 3\frac{1}{x} + 2e^x\ dx = \int 3\frac{1}{x}\ dx + \int 2e^x\ dx$

$\int 3\frac{1}{x} + 2e^x\ dx = 3\int \frac{1}{x}\ dx + 2\int e^x\ dx$

$\int 3\frac{1}{x} + 2e^x\ dx = 3\ln |x| + 2e^x + C.$

(f) $\int \cos x + sen\ x + 2x^{\frac{3}{5}}\ dx.$

$\int \cos x + sen\ x + 2x^{\frac{3}{5}}\ dx = \int \cos x\ dx + \int sen\ x\ dx + \int 2x^{\frac{3}{5}}\ dx$

$\int \cos x + sen\ x + 2x^{\frac{3}{5}}\ dx = sen\ x - \cos x + 2\int x^{\frac{3}{5}}\ dx$

$\int \cos x + sen\ x + 2x^{\frac{3}{5}}\ dx = sen\ x - \cos x + 2\frac{x^{\frac{8}{5}}}{\frac{8}{5}}$

$\int \cos x + sen\ x + 2x^{\frac{3}{5}}\ dx = sen\ x - \cos x + \frac{5}{4}x^{\frac{8}{5}} + C.$

(g) $\int_{-5}^1 x^2 + 2x + 8\ dx.$

$\int_{-5}^1 x^2 + 2x + 8\ dx = \int_{-5}^1 x^2\ dx + \int_{-5}^1 2x\ dx + \int_{-5}^1 8\ dx$

$\int_{-5}^1 x^2 + 2x + 8\ dx = \frac{x^3}{3}\ |_{-5}^1 + 2\int_{-5}^1 x\ dx + 8\int_{-5}^1 1\ dx$

$\int_{-5}^1 x^2 + 2x + 8\ dx = \frac{1}{3}[1^3 - (-5)^3] + 2\frac{x^2}{2}\ |_{-5}^1 + 8\ x\ |_{-5}^1$

$\int_{-5}^1 x^2 + 2x + 8\ dx = \frac{1}{3}[1 - (-125)] + [1^2 - (-5)^2] + 8\ [1 - (-5)]$

$\int_{-5}^1 x^2 + 2x + 8\ dx = \frac{1}{3}(1 + 125) + (1 - 25) + 8\ (1 + 5)$

$\int_{-5}^1 x^2 + 2x + 8\ dx = \frac{1}{3} * 126 - 24 + 8 * 6$

$\int_{-5}^1 x^2 + 2x + 8\ dx = \frac{126}{3} - 24 + 48$

$\int_{-5}^{1} x^2 + 2x + 8 \ dx = 66.$

(h) $\int x - x^{\frac{2}{5}} + 3e^x - \cos x \ dx.$

$\int x - x^{\frac{2}{5}} + 3e^x - \cos x \ dx = \int x \ dx + \int -x^{\frac{2}{5}} \ dx + \int 3e^x \ dx + \int - \cos x \ dx$

$\int x - x^{\frac{2}{5}} + 3e^x - \cos x \ dx = \frac{x^2}{2} - \int x^{\frac{2}{5}} \ dx + 3 \int e^x \ dx - \int \cos x \ dx$

$\int x - x^{\frac{2}{5}} + 3e^x - \cos x \ dx = \frac{x^2}{2} - \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + 3e^x - \text{sen x}$

$\int x - x^{\frac{2}{5}} + 3e^x - \cos x \ dx = \frac{1}{2}x^2 - \frac{5}{7}x^{\frac{7}{5}} + 3e^x - \text{sen x} + \text{C}.$

Ejercicio 3.

Calcular las siguientes integrales utilizando los métodos vistos.

(a) $\int (3x^4 + 5x^2 + 8)^4(12x^3 + 10x)\, dx$.

$\int (3x^4 + 5x^2 + 8)^4(12x^3 + 10x)\, dx = \int u^4\, du$ (*)

$\int (3x^4 + 5x^2 + 8)^4(12x^3 + 10x)\, dx = \dfrac{u^5}{5}$

$\int (3x^4 + 5x^2 + 8)^4(12x^3 + 10x)\, dx = \dfrac{(3x^4+5x^2+8)^5}{5} + \text{C}$.

(*) u= $3x^4 + 5x^2 + 8$; du= $(12x^3 + 10x)$ dx.

(b) $\int x \cos x\ dx$.

$\int x \cos x\ dx =$ x sen x - $\int sen\ x\ dx$ (*)

$\int x \cos x\ dx =$ x sen x - (-cos x)

$\int x \cos x\ dx =$ x sen x + cos x + C.

(*) u= x; du= dx; dv= cos x dx; v= sen x.

(c) $\int x^3 \ln x\ dx$.

$\int x^3 \ln x\ dx =$ ln x $\dfrac{x^4}{4}$ - $\int \dfrac{x^4}{4}\dfrac{1}{x}\ dx$ (*)

$\int x^3 \ln x\ dx = \dfrac{x^4}{4}$ ln x - $\dfrac{1}{4}\int x^3\ dx$

$\int x^3 \ln x\ dx = \dfrac{x^4}{4}$ ln x - $\dfrac{1}{4}\dfrac{x^4}{4}$

$\int x^3 \ln x\ dx = \dfrac{x^4}{4}$ ln x - $\dfrac{x^4}{16}$

$\int x^3 \ln x\ dx = \dfrac{x^4}{4}$ (ln x - $\dfrac{1}{4}$) + C.

(*) u= ln x; du= $\dfrac{1}{x}$ dx; dv= x^3 dx; v= $\dfrac{x^4}{4}$.

(d) $\int \cos 5x * 5\ dx$.

$\int \cos 5x * 5\ dx = \int \cos u\ du$ (*)

$\int \cos 5x * 5\ dx =$ sen u

$\int \cos 5x * 5\ dx =$ sen 5x + C.

(*) u= 5x; du= 5 dx.

(e) $\int \frac{2+e^x}{e^x+2x} dx$.

$\int \frac{2+e^x}{e^x+2x} dx = \int \frac{1}{u} du$ (*)

$\int \frac{2+e^x}{e^x+2x} dx = \ln |u|$

$\int \frac{2+e^x}{e^x+2x} dx = \ln |e^x + 2x| + C$.

(*) u= $e^x + 2x$; du= $(e^x + 2)$ dx.

(f) $\int x\sqrt{x-1} \, dx$.

(*) u= $\sqrt{x-1}$; du= $\frac{1}{2\sqrt{x-1}}$ dx.

$\int x\sqrt{x-1} \, dx = \int (u+1)\sqrt{u} \, du$ (*)

$\int x\sqrt{x-1} \, dx = \int u^{\frac{3}{2}} + u^{\frac{1}{2}} \, du$

$\int x\sqrt{x-1} \, dx = \int u^{\frac{3}{2}} \, du + \int u^{\frac{1}{2}} \, du$

$\int x\sqrt{x-1} \, dx = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}}$

$\int x\sqrt{x-1} \, dx = \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}}$

$\int x\sqrt{x-1} \, dx = 2 \left[\frac{(x-1)^{\frac{5}{2}}}{5} + \frac{(x-1)^{\frac{3}{2}}}{3} \right] + C$.

(*) u= x - 1; du= dx.

(g) $\int_0^8 \frac{1}{\sqrt{x+1}} dx$.

$\int_0^8 \frac{1}{\sqrt{x+1}} dx = \int_0^8 \frac{1}{\sqrt{u}} dx$ (*)

$\int_0^8 \frac{1}{\sqrt{x+1}} dx = \int_{0+1}^{8+1} u^{\frac{-1}{2}} \, du$

$\int_0^8 \frac{1}{\sqrt{x+1}} dx = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \Big|_{0+1}^{8+1}$

$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 2(x+1)^{\frac{1}{2}} \Big|_0^8$

$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 2\sqrt{x+1} \Big|_0^8$

$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 2(\sqrt{8+1} - \sqrt{0+1})$

$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 2(\sqrt{9} - \sqrt{1})$

$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 2\ (3 - 1)$

$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 2 * 2$

$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 4.$

(*) u= x + 1; du= dx.

(h) $\int_0^{2\pi} x\ sen\ x\ dx.$

$\int_0^{2\pi} x\ sen\ x\ dx = $ x (-cos x) $|_0^{2\pi}$ - $\int_0^{2\pi} -\cos x\ dx$ (*)

$\int_0^{2\pi} x\ sen\ x\ dx = $ -x cos x $|_0^{2\pi}$ + $\int_0^{2\pi} \cos x\ dx$

$\int_0^{2\pi} x\ sen\ x\ dx = $ -(2π cos 2π - 0 cos 0) + sen x $|_0^{2\pi}$

$\int_0^{2\pi} x\ sen\ x\ dx = $ -(2π * 1 - 0 * 1) + (sen 2π - sen 0)

$\int_0^{2\pi} x\ sen\ x\ dx = $ -(2π - 0) + (0 - 0)

$\int_0^{2\pi} x\ sen\ x\ dx = $ -2π + 0

$\int_0^{2\pi} x\ sen\ x\ dx = $ -2π.

(*) u= x; du= dx; dv= sen x dx; v= -cos x.

Ejercicio 4.

Hallar el área comprendida entre las gráficas de las siguientes pares de funciones:

(a) $f(x) = x^2$ y $g(x) = -x^2 + 4$.
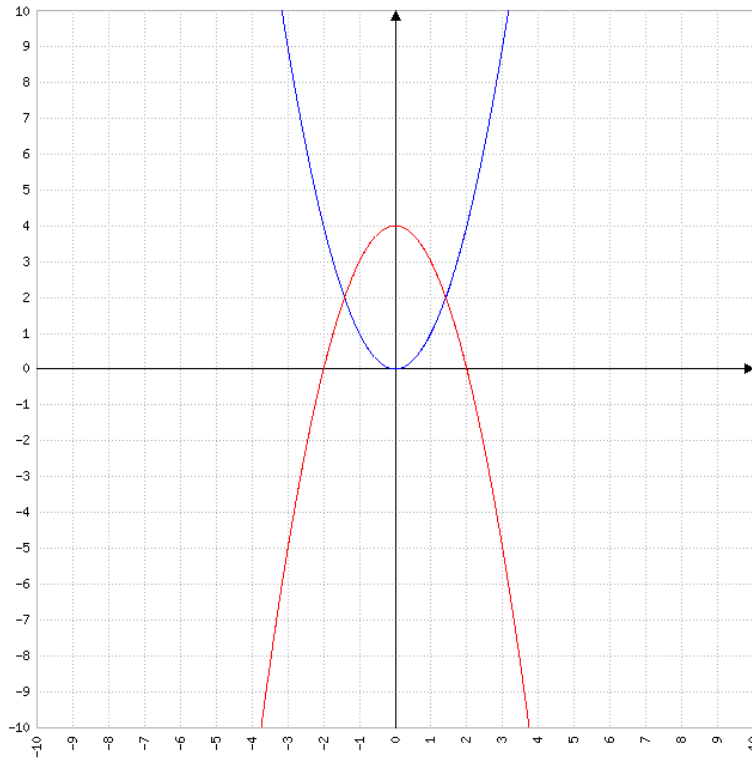
$f(x) = g(x)$
$x^2 = -x^2 + 4$
$x^2 + x^2 = 4$
$2x^2 = 4$
$x^2 = \frac{4}{2}$
$x^2 = 2$
$\sqrt{x^2} = \sqrt{2}$
$|x| = \sqrt{2}$
$x = \pm \sqrt{2}$.

Intervalo	$(-\sqrt{2}, \sqrt{2})$
VP	0
f (x)	0
g (x)	4

$A = \int_{-\sqrt{2}}^{\sqrt{2}} g(x) - f(x)\, dx$

$A = \int_{-\sqrt{2}}^{\sqrt{2}} (-x^2 + 4) - x^2\, dx$

$A = \int_{-\sqrt{2}}^{\sqrt{2}} -x^2 + 4 - x^2\, dx$

A= $\int_{-\sqrt{2}}^{\sqrt{2}} -2x^2 + 4 \, dx$

A= $\int_{-\sqrt{2}}^{\sqrt{2}} -2x^2 \, dx + \int_{-\sqrt{2}}^{\sqrt{2}} 4 \, dx$

A= -2 $\int_{-\sqrt{2}}^{\sqrt{2}} x^2 \, dx + 4 \int_{-\sqrt{2}}^{\sqrt{2}} 1 \, dx$

A= -2 $\frac{x^3}{3} \Big|_{-\sqrt{2}}^{\sqrt{2}} + 4x \Big|_{-\sqrt{2}}^{\sqrt{2}}$

A= $\frac{-2}{3} [(\sqrt{2})^3 - (-\sqrt{2})^3] + 4 [\sqrt{2} - (-\sqrt{2})]$

A= $\frac{-2}{3} [(\sqrt{2})^3 + (\sqrt{2})^3] + 4 (\sqrt{2} + \sqrt{2})$
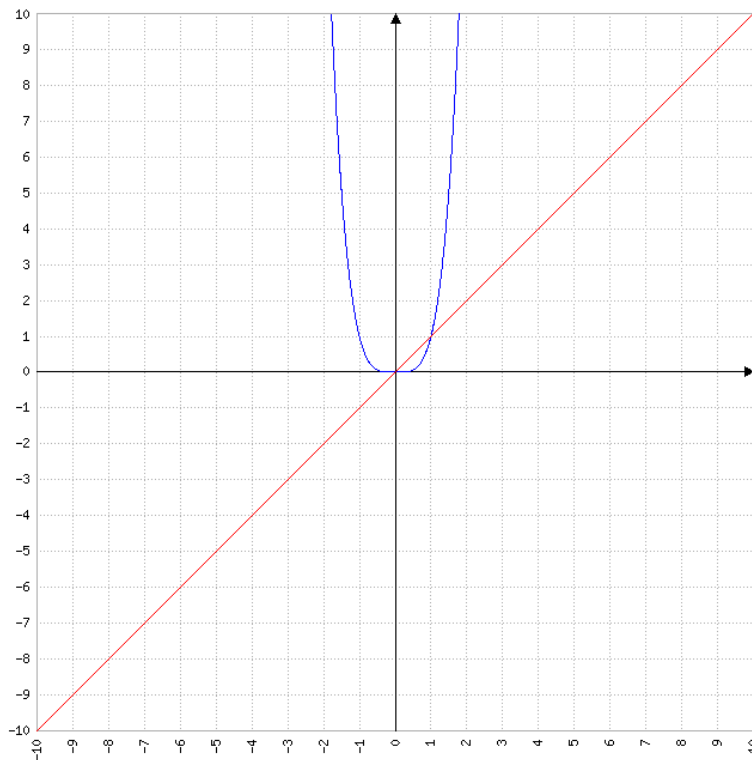
A= $\frac{-2}{3} * 2 (\sqrt{2})^3 + 4 * 2 \sqrt{2}$

A= $\frac{-4}{3} (\sqrt{2})^2 \sqrt{2} + 8 \sqrt{2}$

A= $\frac{-4}{3} * 2 \sqrt{2} + 8 \sqrt{2}$

A= $\frac{-8}{3} \sqrt{2} + 8 \sqrt{2}$

A= $\frac{16}{3} \sqrt{2}.$

(b) $f(x) = x^4$ y $g(x) = x.$

f (x)= g (x)

$x^4 = x$

$x^4 - x = 0$

$x^3 (x - 1) = 0.$

$x_1 = 0; x_2 = 1.$

Intervalo	(0, 1)
VP	$\frac{1}{2}$
f (x)	$\frac{1}{16}$
g (x)	$\frac{1}{2}$

$A= \int_0^1 g\,(x) - f\,(x)\,dx$

$A= \int_0^1 x - x^4\,dx$

$A= \int_0^1 x\,dx + \int_0^1 -x^4\,dx$

$A= \frac{x^2}{2}\,|_0^1 - \int_0^1 x^4\,dx$

$A= \frac{1}{2}\,(1^2 - 0^2) - \frac{x^5}{5}\,|_0^1$
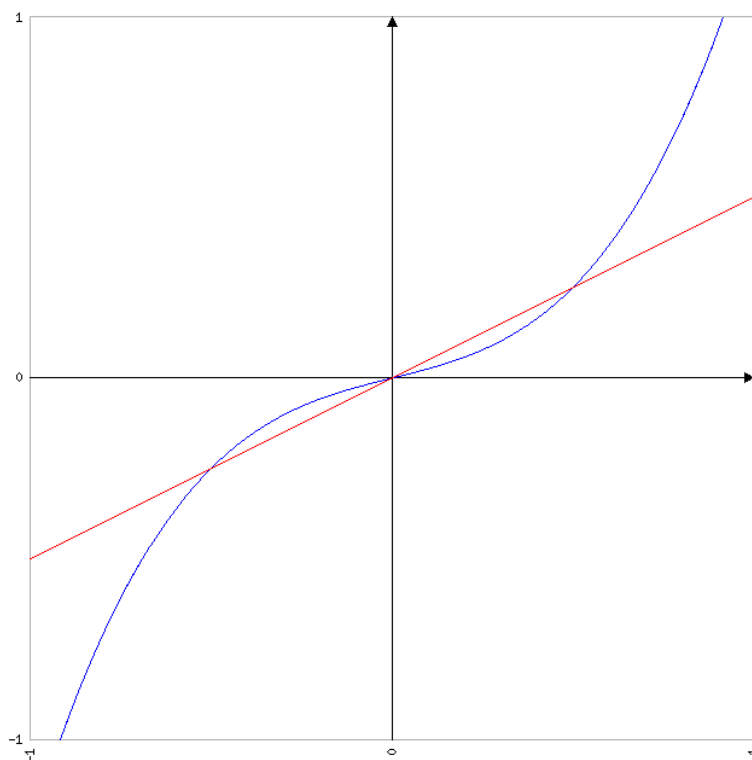
$A= \frac{1}{2}\,(1 - 0) - \frac{1}{5}\,(1^5 - 0^5)$

$A= \frac{1}{2} * 1 - \frac{1}{5}\,(1 - 0)$

$A= \frac{1}{2} - \frac{1}{5} * 1$

$A= \frac{1}{2} - \frac{1}{5}$

$A= \frac{3}{10}.$

(c) $f\,(x)= x^3 + \frac{1}{4}\,x$ y $g\,(x)= \frac{1}{2}\,x.$

f (x)= g (x)

$x^3 + \frac{1}{4}$ x= $\frac{1}{2}$ x

$x^3 + \frac{1}{4}$ x - $\frac{1}{2}$ x= 0

$x^3 - \frac{1}{4}$ x= 0

x $(x^2 - \frac{1}{4})$= 0.

x_1= 0; x_2= $\frac{-1}{2}$; x_3= $\frac{1}{2}$.

Intervalo	$(\frac{-1}{2}, 0)$	$(0, \frac{1}{2})$
VP	$\frac{-1}{4}$	$\frac{1}{4}$
f (x)	$\frac{-5}{64}$	$\frac{5}{64}$
g (x)	$\frac{-1}{8}$	$\frac{1}{8}$

A= $\int_{\frac{-1}{2}}^{0} f (x) - g (x)\, dx + \int_{0}^{\frac{1}{2}} g (x) - f (x)\, dx$

A= $\int_{\frac{-1}{2}}^{0} (x^3 + \frac{1}{4}x) - \frac{1}{2}x\, dx + \int_{0}^{\frac{1}{2}} \frac{1}{2}x - (x^3 + \frac{1}{4}x)\, dx$

A= $\int_{\frac{-1}{2}}^{0} x^3 + \frac{1}{4}x - \frac{1}{2}x\, dx + \int_{0}^{\frac{1}{2}} \frac{1}{2}x - x^3 - \frac{1}{4}x\, dx$

A= $\int_{\frac{-1}{2}}^{0} x^3 - \frac{1}{4}x\, dx + \int_{0}^{\frac{1}{2}} -x^3 + \frac{1}{4}x\, dx$

A= $\int_{\frac{-1}{2}}^{0} x^3\, dx + \int_{\frac{-1}{2}}^{0} \frac{-1}{4}x\, dx + \int_{0}^{\frac{1}{2}} -x^3\, dx + \int_{0}^{\frac{1}{2}} \frac{1}{4}x\, dx$

A= $\int_{\frac{-1}{2}}^{0} x^3\, dx - \frac{1}{4}\int_{\frac{-1}{2}}^{0} x\, dx - \int_{0}^{\frac{1}{2}} x^3\, dx + \frac{1}{4}\int_{0}^{\frac{1}{2}} x\, dx$

A= $\frac{x^4}{4}\,|_{\frac{-1}{2}}^{0} - \frac{1}{4}\frac{x^2}{2}\,|_{\frac{-1}{2}}^{0} - \frac{x^4}{4}\,|_{0}^{\frac{1}{2}} + \frac{1}{4}\frac{x^2}{2}\,|_{0}^{\frac{1}{2}}$

A= $\frac{1}{4}[0^4 - (\frac{-1}{2})^4] - \frac{1}{8}[0^2 - (\frac{-1}{2})^2] - \frac{1}{4}[(\frac{1}{2})^4 - 0^4] + \frac{1}{8}[(\frac{1}{2})^2 - 0^2]$

A= $\frac{1}{4}(0 - \frac{1}{16}) - \frac{1}{8}(0 - \frac{1}{4}) - \frac{1}{4}(\frac{1}{16} - 0) + \frac{1}{8}(\frac{1}{4} - 0)$

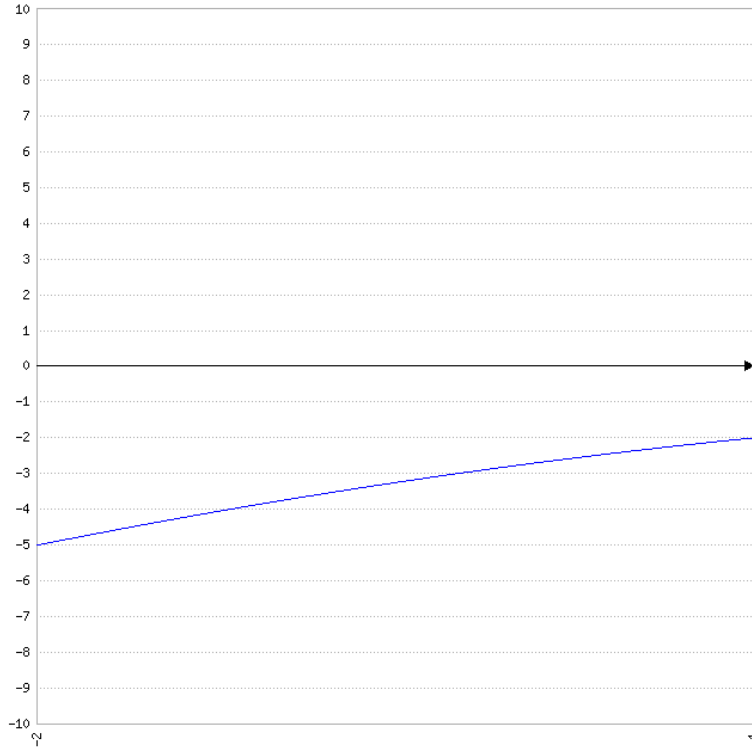A= $\frac{1}{4}(\frac{-1}{16}) - \frac{1}{8}(\frac{-1}{4}) - \frac{1}{4}\frac{1}{16} + \frac{1}{8}\frac{1}{4}$

A= $\frac{-1}{64} + \frac{1}{32} - \frac{1}{64} + \frac{1}{32}$

A= $\frac{1}{32}$.

Ejercicio 5.

Calcular el área de la región comprendida entre el eje x y el gráfico de la función f (x)= -x² - 1 entre -2 ≤ x ≤ -1.

f (x)= 0
$-x^2 - 1 = 0$
$x^2 \neq -1.$

Intervalo	(-2, -1)
VP	$\dfrac{-3}{2}$
f (x)	< 0

A= $-\int_{-2}^{-1} -x^2 - 1 \, dx$

A= $-(\int_{-2}^{-1} -x^2 \, dx + \int_{-2}^{-1} -1 \, dx)$

A= $-(-\int_{-2}^{-1} x^2 \, dx - \int_{-2}^{-1} 1 \, dx)$

A= $-(\frac{-x^3}{3} \mid_{-2}^{-1} - x \mid_{-2}^{-1})$

A= $-\{\frac{-1}{3} [(-1)^3 - (-2)^3] - [-1 - (-2)]\}$

A= $-\{\frac{-1}{3} [-1 - (-8)] - (-1 + 2)\}$

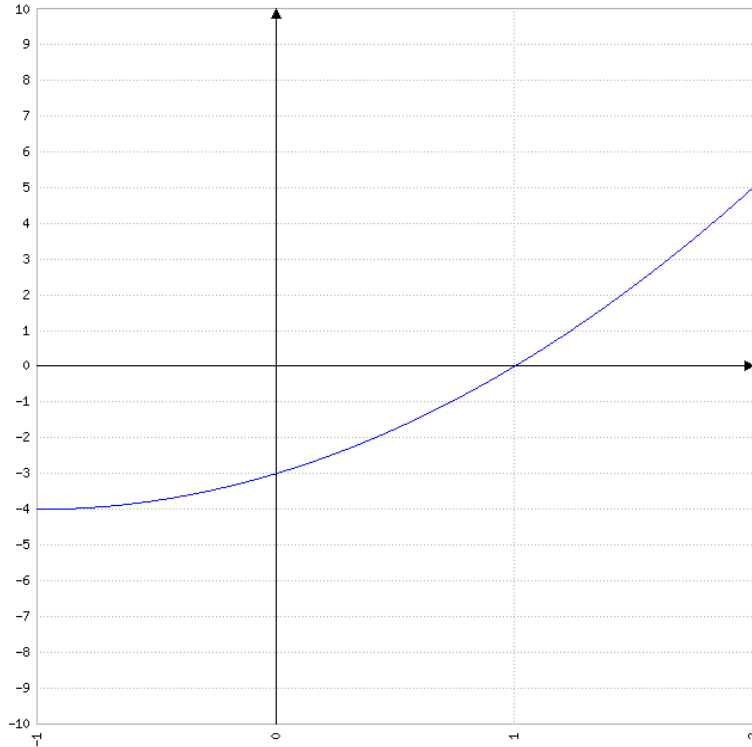A= $-[\frac{-1}{3} (-1 + 8) - 1]$

A= $-(\frac{-1}{3} * 7 - 1)$

A= $-(\frac{-7}{3} - 1)$

$A = -\left(\dfrac{-10}{3}\right)$

$A = \dfrac{10}{3}.$

Ejercicio 6.

Calcular el área de la región comprendida entre el eje x y el gráfico de la función f (x)=
$x^2 + 2x - 3$ *entre -1 ≤ x ≤ 2.*

f (x)= 0
$x^2 + 2x - 3 = 0.$

$x_1, x_2 = \frac{-2 \pm \sqrt{2^2 - 4*1(-3)}}{2*1}$

$x_1, x_2 = \frac{-2 \pm \sqrt{4+12}}{2}$

$x_1, x_2 = \frac{-2 \pm \sqrt{16}}{2}$

$x_1, x_2 = \frac{-2 \pm 4}{2}$

$x_1 = \frac{-2+4}{2} = \frac{2}{2} = 1.$

$x_2 = \frac{-2-4}{2} = \frac{-6}{2} = -3.$

Intervalo	(-1, 1)	(1, 2)
VP	0	$\frac{3}{2}$
f (x)	< 0	> 0

A= $-\int_{-1}^{1} x^2 + 2x - 3 \, dx + \int_{1}^{2} x^2 + 2x - 3 \, dx$

A= $-(\int_{-1}^{1} x^2 \, dx + \int_{-1}^{1} 2x \, dx + \int_{-1}^{1} -3 \, dx) + (\int_{1}^{2} x^2 \, dx + \int_{1}^{2} 2x \, dx + \int_{1}^{2} -3 \, dx)$

A= $-(\frac{x^3}{3} |_{-1}^{1} + 2 \int_{-1}^{1} x \, dx - 3 \int_{-1}^{1} 1 \, dx) + (\frac{x^3}{3} |_{1}^{2} + 2 \int_{1}^{2} x \, dx - 3 \int_{1}^{2} 1 \, dx)$

$A = -\{\frac{1}{3}[1^3 - (-1)^3] + 2\frac{x^2}{2}\big|_{-1}^{1} - 3x\big|_{-1}^{1}\} + [\frac{1}{3}(2^3 - 1^3) + 2\frac{x^2}{2}\big|_{1}^{2} - 3x\big|_{1}^{2}]$

$A = -\{\frac{1}{3}[1 - (-1)] + [1^2 - (-1)^2] - 3[1 - (-1)]\} + [\frac{1}{3}(8 - 1) + (2^2 - 1^2) - 3(2 - 1)]$

$A = -[\frac{1}{3}(1 + 1) + (1 - 1) - 3(1 + 1)] + [\frac{1}{3} * 7 + (4 - 1) - 3 * 1]$

$A = -(\frac{1}{3} * 2 + 0 - 3 * 2) + (\frac{7}{3} + 3 - 3)$

$A = -(\frac{2}{3} + 0 - 6) + \frac{7}{3}$
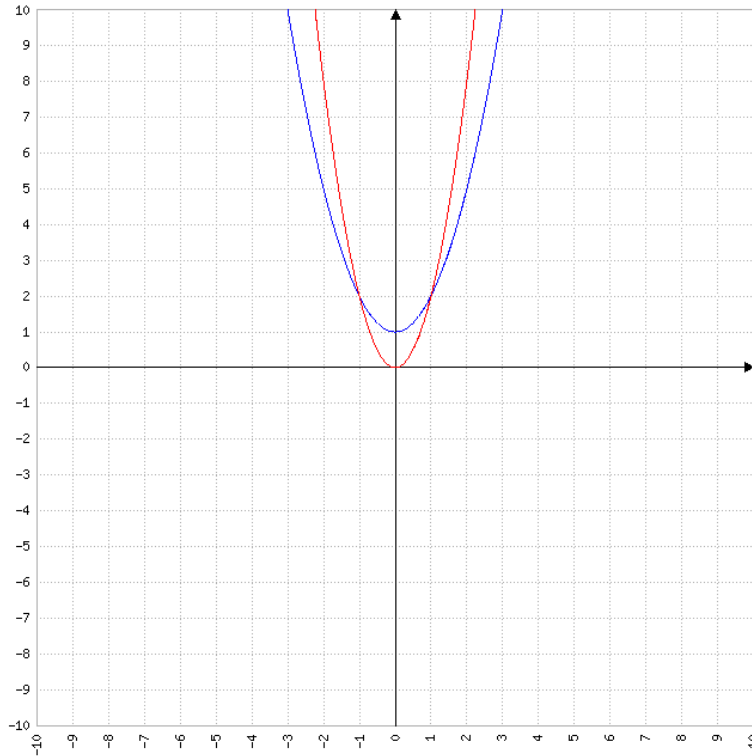
$A = -(\frac{-16}{3}) + \frac{7}{3}$

$A = \frac{16}{3} + \frac{7}{3}$

$A = \frac{23}{3}.$

Ejercicio 7.

Calcular el área de la región comprendida entre los gráficos de f (x)= x^2 + 1 y g (x)= $2x^2$ para 0 ≤ x ≤ 2.

f (x)= g (x)
x^2 + 1= $2x^2$
$2x^2$ - x^2= 1
x^2= 1
$\sqrt{x^2}$= $\sqrt{1}$
$|x|$= 1
x= ±1.

Intervalo	(-1, 1)
VP	0
f (x)	1
g (x)	0

A= $\int_{-1}^{1}(x^2 + 1) - 2x^2\, dx$
A= $\int_{-1}^{1} x^2 + 1 - 2x^2\, dx$
A= $\int_{-1}^{1} -x^2 + 1\, dx$
A= $\int_{-1}^{1} -x^2\, dx + \int_{-1}^{1} 1\, dx$
A= $-\int_{-1}^{1} x^2\, dx + $ x $|_{-1}^{1}$
A= $\frac{-x^3}{3} |_{-1}^{1} + [1 - (-1)]$

A$= \frac{-1}{3}$ $[1^3 - (-1)^3] + (1 + 1)$

A$= \frac{-1}{3}$ $[1 - (-1)] + 2$

A$= \frac{-1}{3}$ $(1 + 1) + 2$

A$= \frac{-1}{3}$ $* 2 + 2$

A$= \frac{-2}{3}$ $+ 2$

A$= \frac{4}{3}$.

Ejercicio 8.

Hallar f (x) sabiendo que f´ (x)= $x + \frac{1}{x^2}$ y f (1)= 1.

f (x)= $\int f´(x)\, dx$

f (x)= $\int x + \frac{1}{x^2}\, dx$

f (x)= $\int x\, dx + \int \frac{1}{x^2}\, dx$

f (x)= $\frac{x^2}{2} + \int x^{-2}\, dx$

f (x)= $\frac{x^2}{2} + \frac{x^{-1}}{-1}$

f (x)= $\frac{1}{2}x^2 - \frac{1}{x} + C.$

f (1)= 1

$\frac{1}{2} * 1^2 - \frac{1}{1} + C = 1$

$\frac{1}{2} * 1 - 1 + C = 1$

$\frac{1}{2} - 1 + C = 1$

$\frac{-1}{2} + C = 1$

C= $1 + \frac{1}{2}$

C= $\frac{3}{2}.$

f (x)= $\frac{1}{2}x^2 - \frac{1}{x} + \frac{3}{2}.$

Ejercicio 9.

Sabiendo que f´ (x)= $3x^2$ - 8x + 2 y, además, que f (3)= -4, hallar la función f (x).

f (x)= $\int f´(x)\,dx$
f (x)= $\int 3x^2 - 8x + 2\,dx$
f (x)= $\int 3x^2\,dx + \int -8x\,dx + \int 2\,dx$
f (x)= 3 $\int x^2\,dx$ - 8 $\int x\,dx$ + 2 $\int 1\,dx$
f (x)= 3 $\frac{x^3}{3}$ - 8 $\frac{x^2}{2}$ + 2x
f (x)= x^3 - $4x^2$ + 2x + C.

f (3)= -4
3^3 - 4 * 3^2 + 2 * 3 + C= -4
27 - 4 * 9 + 6 + C= -4
27 - 36 + 6 + C= -4
-3 + C= -4
C= -4 + 3
C= -1.

f (x)= x^3 - $4x^2$ + 2x - 1.

Ejercicio 10.

Hallar todas las funciones cuya derivada es $g'(x) = x^2 \cos x$.

$g(x) = \int g'(x)\, dx$

$g(x) = \int x^2 \cos x\, dx$

$g(x) = x^2 \operatorname{sen} x - \int \operatorname{sen} x * 2x\, dx$ (*)

$g(x) = x^2 \operatorname{sen} x - 2 \int x \operatorname{sen} x\, dx$

$g(x) = x^2 \operatorname{sen} x - 2\,[x\,(-\cos x) - \int -\cos x\, dx]$ (**)

$g(x) = x^2 \operatorname{sen} x - 2\,(-x \cos x + \int \cos x\, dx)$

$g(x) = x^2 \operatorname{sen} x - 2\,(-x \cos x + \operatorname{sen} x)$

$g(x) = x^2 \operatorname{sen} x - 2x \cos x - 2 \operatorname{sen} x$

$g(x) = (x^2 - 2) \operatorname{sen} x - 2x \cos x + C$.

(*) $u = x^2$; $du = 2x\, dx$; $dv = \cos x\, dx$; $v = \operatorname{sen} x$.

(**) $u = x$; $du = dx$; $dv = \operatorname{sen} x$; $v = -\cos x$.

Ejercicio 11.

Sea g´´(x)= 2x³ - 4x⁷, g´(1)= -2 y g(0)= 0, hallar la función g(x).

$g´(x)= \int g´´(x)\,dx$
$g´(x)= \int 2x^3 - 4x^7\,dx$
$g´(x)= \int 2x^3\,dx + \int -4x^7\,dx$
$g´(x)= 2\int x^3\,dx - 4\int x^7\,dx$
$g´(x)= 2\frac{x^4}{4} - 4\frac{x^8}{8}$
$g´(x)= \frac{1}{2}x^4 - \frac{1}{2}x^8$
$g´(x)= \frac{1}{2}(x^4 - x^8) + C.$

$g´(1)= -2$
$\frac{1}{2}(1^4 - 1^8) + C= -2$
$\frac{1}{2}(1 - 1) + C= -2$
$\frac{1}{2} * 0 + C= -2$
$0 + C= -2$
$C= -2.$

$g´(x)= \frac{1}{2}(x^4 - x^8) - 2.$

$g(x)= \int g´(x)\,dx$
$g(x)= \int \frac{1}{2}(x^4 - x^8) - 2\,dx$
$g(x)= \int \frac{1}{2}x^4 - \frac{1}{2}x^8 - 2\,dx$
$g(x)= \int \frac{1}{2}x^4\,dx + \int \frac{-1}{2}x^8\,dx + \int -2\,dx$
$g(x)= \frac{1}{2}\int x^4\,dx - \frac{1}{2}\int x^8\,dx - 2\int 1\,dx$
$g(x)= \frac{1}{2}\frac{x^5}{5} - \frac{1}{2}\frac{x^9}{9} - 2x$
$g(x)= \frac{1}{10}x^5 - \frac{1}{18}x^9 - 2x + C.$

$g(0)= 0$
$\frac{1}{10}0^5 - \frac{1}{18}0^9 - 2 * 0 + C= 0$
$\frac{1}{10} * 0 - \frac{1}{18} * 0 - 0 + C= 0$
$0 - 0 - 0 + C= 0$
$C= 0.$

$g(x)= \frac{1}{10}x^5 - \frac{1}{18}x^9 - 2x.$