

Trabajo Práctico N° 3: **Números.**

Ejercicio 1.

Probar que no hay enteros, simultáneamente, pares e impares.

Número par: Un número entero n es par si existe un número entero k tal que $n = 2k$, es decir, n es divisible por 2.

Número impar: Un número entero n es impar si existe un número entero k tal que $n = 2k + 1$, es decir, n no es divisible por 2.

Ahora, para que un número sea, simultáneamente, par e impar, tendría que cumplir ambas definiciones al mismo tiempo. Es decir, existirían enteros tales que:

$$\begin{aligned}n &= 2k_1, \text{ con } k_1 \in \mathbb{Z}, \\n &= 2k_2 + 1, \text{ con } k_2 \in \mathbb{Z}.\end{aligned}$$

Esto implica que:

$$\begin{aligned}2k_1 &= 2k_2 + 1 \\2k_1 - 2k_2 &= 1 \\2(k_1 - k_2) &= 1.\end{aligned}$$

Aquí, se llega a una contradicción, ya que el lado izquierdo de la ecuación es, claramente, un número par (ya que es múltiplo de 2), mientras que el lado derecho es 1, que es impar. No hay ningún par de enteros k_1 y k_2 que satisfagan esta ecuación.

Por lo tanto, no hay enteros, simultáneamente, pares e impares.

Ejercicio 2.

Analizar si las siguientes afirmaciones son verdaderas o falsas:

(a) Si $a|1$, entonces, $a = 1$ o $a = -1$.

$$1 = ak, \text{ con } k \in \mathbb{Z}$$

$$k = \frac{1}{a}.$$

Para que esta ecuación sea verdadera, k debe ser igual a $\frac{1}{a}$. Pero, para que k sea un entero, a debe ser un divisor de 1. Los divisores de 1 son aquellos números enteros que, multiplicados por otro entero, dan como resultado 1. Estos divisores son, únicamente, $a = 1$ y $a = -1$, ya que $1 = 1 * 1$ y $1 = (-1) (-1)$.

La afirmación es VERDADERA.

(b) $a|b$ y $b|c$, entonces, $a|c$.

$$b = ak_1, \text{ con } k_1 \in \mathbb{Z}.$$

$$c = bk_2, \text{ con } k_2 \in \mathbb{Z}.$$

$$c = ak_1k_2$$

$$c = ak_3, \text{ con } k_3 \equiv k_1k_2 \in \mathbb{Z}.$$

La afirmación es VERDADERA.

(c) $a(a - 1)$ es par.

Si a es par, $(a - 1)$ es impar y, entonces, el producto es par.

Si a es impar, $(a - 1)$ es par y, entonces, el producto es par.

La afirmación es VERDADERA.

(d) $x|y$ y $y|z$, entonces, $x|yz$.

$$y = xk_1, \text{ con } k_1 \in \mathbb{Z}.$$

$$z = yk_2, \text{ con } k_2 \in \mathbb{Z}.$$

$$yz = xk_1z$$

$$yz = xk_3, \text{ con } k_3 \equiv k_1z \in \mathbb{Z}.$$

La afirmación es VERDADERA.

Ejercicio 3.

Si a un número se lo divide por 5, el resto es 3 y, si se lo divide por 7, el resto es 4. ¿Cuál es el resto si se lo divide por 35?

$$x = 5k_1 + 3, \text{ con } k_1 \in \mathbb{Z}.$$

$$x = 7k_2 + 4, \text{ con } k_2 \in \mathbb{Z}.$$

$$5k_1 + 3 = 7k_2 + 4$$

$$5k_1 = 7k_2 + 4 - 3$$

$$5k_1 = 7k_2 + 1.$$

$$k_1 = 3; k_2 = 2.$$

$$x = 5 * 3 + 3$$

$$x = 15 + 3$$

$$x = 18.$$

$$x = 7 * 2 + 4$$

$$x = 14 + 4$$

$$x = 18.$$

$$x = 35k_3 + 18, \text{ con } k_3 \in \mathbb{Z}.$$

Por lo tanto, si se lo divide por 35, el resto es 18.

Ejercicio 4.

Sean a y b dos números enteros que tienen restos 4 y 7, respectivamente, en la división por 11. Hallar los restos de la división por 11 de $(a + b)^2$.

$$a = 11k_1 + 4, \text{ con } k_1 \in \mathbb{Z}.$$

$$b = 11k_2 + 7, \text{ con } k_2 \in \mathbb{Z}.$$

$$a + b = 11k_1 + 4 + 11k_2 + 7$$

$$a + b = 11(k_1 + k_2) + 11$$

$$a + b = 11(k_1 + k_2 + 1)$$

$$a + b = 11k_3, \text{ con } k_3 \equiv (k_1 + k_2 + 1) \in \mathbb{Z}.$$

$$(a + b)^2 = (11k_3)^2$$

$$(a + b)^2 = 11 * 11k_3^2$$

$$(a + b)^2 = 11k_4, \text{ con } k_4 \equiv 11k_3^2 \in \mathbb{Z}.$$

Por lo tanto, el resto de la división por 11 de $(a + b)^2$ es 0.

Ejercicio 5.

Convertir los siguientes números de base 10 a base 8:

(a) 98.

$$98 = 8 * 12 + 2.$$

$$12 = 8 * 1 + 4.$$

$$1 = 8 * 0 + 1.$$

$$98 = (142)_8.$$

(b) 44.

$$44 = 8 * 5 + 4.$$

$$5 = 8 * 0 + 5.$$

$$44 = (54)_8.$$

(c) 20.

$$20 = 8 * 2 + 4.$$

$$2 = 8 * 0 + 2.$$

$$20 = (24)_8.$$

Ejercicio 6.

Calcular el máximo común divisor entre:

(a) $(16, 24)$.

$$(24, 16) = (16, 8) = (8, 0) = 8.$$

$$\text{mcd}(16, 24) = 8.$$

(b) $(70, 50)$.

$$(70, 50) = (50, 20) = (20, 10) = (10, 0) = 10.$$

$$\text{mcd}(70, 50) = 10.$$

(c) $(121, 88)$.

$$(121, 88) = (88, 33) = (33, 22) = (22, 11) = (11, 0) = 11.$$

$$\text{mcd}(121, 88) = 11.$$

(d) $(-90, 90)$.

$$(-90, 90) = (90, 0).$$

$$\text{mcd}(-90, 90) = 90.$$

(e) $(980, 224)$.

$$(980, 224) = (224, 84) = (84, 56) = (56, 28) = (28, 0) = 28.$$

$$\text{mcd}(980, 224).$$

Ejercicio 7.

Probar que, si a y b son enteros:

(a) $a + b$ es coprimo con a .

$$a = dk_1, \text{ con } d, k_1 \in \mathbb{Z}.$$

$$a + b = dk_2, \text{ con } d, k_2 \in \mathbb{Z}.$$

$$dk_1 + b = dk_2$$

$$b = dk_2 - dk_1$$

$$b = d(k_2 - k_1)$$

$$b = dk_3, k_3 \equiv (k_2 - k_1) \in \mathbb{Z}.$$

No hay razón para que a y b tengan un divisor común que no sea $d = 1$, entonces, $(a + b, a) = 1$.

(b) si a es no nulo, $(a, 0) = |a|$.

$$a = |a|k_1, \text{ con } k_1 \in \mathbb{Z}.$$

$$0 = |a|k_2, \text{ con } k_2 \in \mathbb{Z}.$$

No hay divisores comunes entre a y 0 que sean mayores a $|a|$, entonces, $(a, 0) = |a|$.

(c) $(a, b) = 1$, entonces, $ma + nb = k$, con m, n y k enteros.

$$ma + nb = 1.$$

Ejercicio 8.

Hallar $\text{mcd}(5k + 3, 3k + 2)$ para cualquier k entero.

$$(5k + 3, 3k + 2) = (3k + 2, 2k + 1) = (2k + 1, k + 1) = (k + 1, k) = (k, 1) = (1, 0).$$

$$\text{mcd}(5k + 3, 3k + 2) = 1.$$

Ejercicio 9.

Sean $a, b \in \mathbb{Z}$ y sea p primo. Demostrar que, si $p|ab$, entonces, $p|a$ o $p|b$. Mostrar que esto no se cumple si p no es primo.

$$a = p_1^{e_1} p_2^{e_2} \dots p_m^{e_m}, \text{ con } p_i \in \mathbb{Z}, i = 1, 2, \dots, m.$$

$$b = q_1^{f_1} q_2^{f_2} \dots q_n^{f_n}, \text{ con } q_i \in \mathbb{Z}, i = 1, 2, \dots, n.$$

$$ab = p^s k, \text{ con } k \in \mathbb{Z},$$

donde k es el producto de los factores primos que no son p y s es el número de veces que p divide el producto ab .

Dado que p es primo y que ab se compone de factores primos, p debe aparecer en la factorización de, al menos, uno de los factores a o b .

Esto no se cumple si p no es primo. Considerar un contraejemplo:

$$p = 6, a = 2 \text{ y } b = 3.$$

$$p|ab = 6|6, \text{ pero } p \nmid a = 6 \nmid 2 \text{ y } p \nmid b = 6 \nmid 3.$$

Ejercicio 10.

Hallar, si existe, un número entero q tal que $7290q$ es el cubo de un entero.

$$7290 = 3645 * 2$$

$$7290 = 1215 * 3 * 2$$

$$7290 = 405 * 3^2 * 2$$

$$7290 = 135 * 3^3 * 2$$

$$7290 = 45 * 3^4 * 2$$

$$7290 = 15 * 3^5 * 2$$

$$7290 = 5 * 3^6 * 2.$$

$$q = 5^2 2^2$$

$$q = 25 * 4$$

$$q = 100.$$

$$n^3 = 7290 * 100$$

$$n^3 = 5 * 3^6 * 2 * 5^2 * 2^2$$

$$n^3 = 5^3 3^6 2^3$$

$$n = (5^3 3^6 2^3)^{\frac{1}{3}}$$

$$n = 5 * 3^2 * 2$$

$$n = 5 * 9 * 2$$

$$n = 90.$$

Ejercicio 11.

Demostrar que, dados a y b en \mathbb{Q} tales que $a < b$, existe otro número racional x tal que $a < x < b$.

$$a + a < a + b$$

$$2a < a + b$$

$$a < \frac{a+b}{2}.$$

$$a + b < b + b$$

$$a + b < 2b$$

$$\frac{a+b}{2} < b.$$

$$a < \frac{a+b}{2} < b.$$

Ejercicio 12.

Probar que no existe un número racional cuyo cubo sea igual a 2.

Se supone que existe un número racional $\frac{p}{q}$, donde p y q son enteros y $\frac{p}{q}$ está en su forma más simple, $q \neq 0$, tal que:

$$\left(\frac{p}{q}\right)^3 = 2.$$

$$\frac{p^3}{q^3} = 2$$
$$p^3 = 2q^3.$$

Esto implica que p^3 es un número par y esto, a su vez, implica que p también debe ser par. Entonces, se tiene que $p = 2k$, donde k es un número entero.

$$(2k)^3 = 2q^3$$
$$8k^3 = 2q^3$$
$$q^3 = \frac{8}{2} k^3$$
$$q^3 = 4k^3$$
$$q^3 = 2 * 2k^3.$$

Esto implica que q^3 es un número par y esto, a su vez, implica que q también debe ser par.

Sin embargo, esto contradice la suposición inicial de que $\frac{p}{q}$ está en su forma más simple (es decir, p y q son coprimos).

Por lo tanto, queda demostrado que no existe un número racional cuyo cubo sea igual a 2.

Ejercicio 13.

Indicar la parte real $Re(z)$ y la parte imaginaria $Im(z)$ de los siguientes complejos:

(a) $z = \sqrt{-49}$.

$$z = \sqrt{-1 * 49}$$

$$z = \sqrt{49} \sqrt{-1}$$

$$z = 7 \sqrt{-1}$$

$$z = 7i.$$

$$Re(z) = 0.$$

$$Im(z) = 7.$$

(b) $z = \sqrt{-20}$.

$$z = \sqrt{-1 * 20}$$

$$z = \sqrt{20} \sqrt{-1}$$

$$z = \sqrt{20}i.$$

$$Re(z) = 0.$$

$$Im(z) = \sqrt{20}.$$

(c) $z = \sqrt{\frac{-9}{16}}$.

$$z = \frac{\sqrt{-9}}{\sqrt{16}}$$

$$z = \frac{\sqrt{-1 * 9}}{4}$$

$$z = \frac{\sqrt{9} \sqrt{-1}}{4}$$

$$z = \frac{3 \sqrt{-1}}{4}$$

$$z = \frac{3}{4} i.$$

$$Re(z) = 0.$$

$$Im(z) = \frac{3}{4}.$$

(d) $z = -8$.

$$Re(z) = -8.$$

$$\text{Im}(z) = 0.$$

$$(e) z = 7i.$$

$$\text{Re}(z) = 0.$$

$$\text{Im}(z) = 7.$$

$$(f) z = (3 + i) + (5 - 4i).$$

$$z = 3 + i + 5 - 4i$$

$$z = 8 - 3i.$$

$$\text{Re}(z) = 8.$$

$$\text{Im}(z) = -3.$$

$$(g) z = 3i - (5 - 2i).$$

$$z = 3i - 5 + 2i$$

$$z = -5 + 5i.$$

$$\text{Re}(z) = -5.$$

$$\text{Im}(z) = 5.$$

$$(h) z = \frac{1+3i}{3-i}.$$

$$z = \frac{1+3i}{3-i} \cdot \frac{3+i}{3+i}$$

$$z = \frac{3+i+9i+3i^2}{9+3i-3i-i^2}$$

$$z = \frac{3+10i+3(-1)}{9-(-1)}$$

$$z = \frac{3+10i-3}{9+1}$$

$$z = \frac{10}{10} i$$

$$z = i.$$

$$\text{Re}(z) = 0.$$

$$\text{Im}(z) = 1.$$

$$(i) z = \frac{1-i}{(1+i)^2}.$$

$$\begin{aligned}Z &= \frac{1-i}{(1+i)^2} \\Z &= \frac{1-i}{(1+i)(1+i)} \\Z &= \frac{1-i}{1+i+i^2} \\Z &= \frac{1-i}{1+2i+(-1)} \\Z &= \frac{1-i}{1+2i-1} \\Z &= \frac{1-i}{2i} \\Z &= \frac{1-i}{2i} \left(\frac{-2i}{-2i} \right) \\Z &= \frac{-2i+2i^2}{-4i^2} \\Z &= \frac{-2i+2(-1)}{-4(-1)} \\Z &= \frac{-2i-2}{4} \\Z &= \frac{-1}{2} + \frac{-1}{2} i.\end{aligned}$$

$$\begin{aligned}\operatorname{Re}(z) &= \frac{-1}{2}. \\ \operatorname{Im}(z) &= \frac{-1}{2}.\end{aligned}$$

Ejercicio 14.

La suma de un número complejo y su conjugado es -8 y la suma de sus módulos es 10.
¿De qué números complejos se trata?

$$z = a + ib.$$

$$\bar{z} = a - ib.$$

$$z + \bar{z} = -8$$

$$a + ib + a - ib = -8$$

$$2a = -8$$

$$a = \frac{-8}{2}$$

$$a = -4.$$

$$|z| + |\bar{z}| = 10$$

$$|z| + |z| = 10$$

$$2|z| = 10$$

$$|z| = \frac{10}{2}$$

$$|z| = 5$$

$$\sqrt{a^2 + b^2} = 5$$

$$\sqrt{(-4)^2 + b^2} = 5$$

$$\sqrt{16 + b^2} = 5$$

$$16 + b^2 = 5^2$$

$$16 + b^2 = 25$$

$$b^2 = 25 - 16$$

$$b^2 = 9$$

$$\sqrt{b^2} = \sqrt{9}$$

$$|b| = 3$$

$$b = \pm 3.$$

$$z_1 = -4 + 3i.$$

$$z_2 = -4 - 3i.$$

Ejercicio 15.

Hallar, si existe, x real tal que $\operatorname{Re}(z) = \operatorname{Im}(z)$, siendo $z = \frac{x+2i}{4-3i}$.

$$z = \frac{x+2i}{4-3i}$$

$$z = \frac{x+2i}{4-3i} \cdot \frac{4+3i}{4+3i}$$

$$z = \frac{4x+3xi+8i+6i^2}{16+12i-12i-9i^2}$$

$$z = \frac{4x+3xi+8i+6(-1)}{16-9(-1)}$$

$$z = \frac{4x+3xi+8i-6}{16+9}$$

$$z = \frac{(4x-6)+(3x+8)i}{25}$$

$$z = \frac{4x-6}{25} + \frac{3x+8}{25}i.$$

$$\frac{4x-6}{25} = \frac{3x+8}{25}$$

$$4x - 6 = 3x + 8$$

$$4x - 3x = 8 + 6$$

$$x = 14.$$

$$z = \frac{4 \cdot 14 - 6}{25} + \frac{3 \cdot 14 + 8}{25}i$$

$$z = \frac{56-6}{25} + \frac{42+8}{25}i$$

$$z = \frac{50}{25} + \frac{50}{25}i$$

$$z = 2 + 2i$$

Ejercicio 16.

Encontrar, si existe, un valor de k real para que el complejo $z = \frac{2-(1+k)i}{1-ki}$ sea un número real.

$$\begin{aligned}
 z &= \frac{2-(1+k)i}{1-ki} \\
 z &= \frac{2-(1+k)i}{1-ki} \cdot \frac{1+ki}{1+ki} \\
 z &= \frac{2+2ki-(1+k)i-(1+k)ki^2}{1-ki-ki-k^2i^2} \\
 z &= \frac{2+2ki-(1+k)i-(1+k)k(-1)}{1-k^2(-1)} \\
 z &= \frac{2+2ki-(1+k)i+(1+k)k}{1+k^2} \\
 z &= \frac{2+k+k^2+[2k-(1+k)]i}{1+k^2} \\
 z &= \frac{2+k+k^2+(2k-1-k)i}{1+k^2} \\
 z &= \frac{2+k+k^2+(k-1)i}{1+k^2} \\
 z &= \frac{2+k+k^2}{1+k^2} + \frac{k-1}{1+k^2} i.
 \end{aligned}$$

$$\begin{aligned}
 \frac{k-1}{1+k^2} &= 0 \\
 k-1 &= 0 \quad (1+k^2) \\
 k-1 &= 0 \\
 k &= 1.
 \end{aligned}$$

$$\begin{aligned}
 z &= \frac{2+1+1^2}{1+1^2} + \frac{1-1}{1+1^2} i \\
 z &= \frac{2+1+1}{1+1} + \frac{0}{1+1} i \\
 z &= \frac{4}{2} + \frac{0}{2} i \\
 z &= 2 + 0i \\
 z &= 2 + 0 \\
 z &= 2.
 \end{aligned}$$

Ejercicio 17.*Calcular las siguientes potencias:*

(a) i^{489} .

$$i^{489} = i^{4 \cdot 122 + 1}$$

$$i^{489} = i^{4 \cdot 122} i^1$$

$$i^{489} = (i^4)^{122} i$$

$$i^{489} = 1^{122} i$$

$$i^{489} = 1i$$

$$i^{489} = i.$$

(b) $-i^{1026}$.

$$-i^{1026} = -i^{4 \cdot 256 + 2}$$

$$-i^{1026} = -i^{4 \cdot 256} i^2$$

$$-i^{1026} = -(i^4)^{256} (-1)$$

$$-i^{1026} = -1^{256} (-1)$$

$$-i^{1026} = -1 (-1)$$

$$-i^{1026} = 1.$$

(c) $(3i)^{168}$.

$$(3i)^{168} = 3^{168} i^{168}$$

$$(3i)^{168} = 3^{168} i^{4 \cdot 42 + 0}$$

$$(3i)^{168} = 3^{168} i^{4 \cdot 42} i^0$$

$$(3i)^{168} = 3^{168} (i^4)^{42} * 1$$

$$(3i)^{168} = 3^{168} 1^{42} * 1$$

$$(3i)^{168} = 3^{168} * 1 * 1$$

$$(3i)^{168} = 3^{168}.$$

Ejercicio 18.

Dados los siguientes números complejos, encontrar la forma más adecuada para realizar las operaciones pedidas:

$$z_1 = 3 + 3i;$$

$$z_2 = -1 + i;$$

$$z_3 = 5 + 4i;$$

$$z_4 = 9;$$

$$z_5 = 5i;$$

$$z_6 = -7;$$

$$z_7 = -4 - 4i;$$

$$z_8 = -8i;$$

$$z_9 = 2 - 2i;$$

$$z_{10} = 3 - 4i.$$

(a) $z_1 + z_7.$

$$z_1 + z_7 = (3 + 3i) + (-4 - 4i)$$

$$z_1 + z_7 = 3 + 3i - 4 - 4i$$

$$z_1 + z_7 = -1 - i.$$

(b) $z_5 - z_3.$

$$z_5 - z_3 = 5i - (5 + 4i)$$

$$z_5 - z_3 = 5i - 5 - 4i$$

$$z_5 - z_3 = -5 + i.$$

(c) $z_9 z_6.$

$$z_9 z_6 = (2 - 2i)(-7)$$

$$z_9 z_6 = -14 + 14i.$$

(d) $\frac{z_8}{z_{10}}.$

$$\frac{z_8}{z_{10}} = \frac{-8i}{3-4i}$$

$$\frac{z_8}{z_{10}} = \frac{-8i}{3-4i} \cdot \frac{3+4i}{3+4i}$$

$$\frac{z_8}{z_{10}} = \frac{-8i(3+4i)}{(3-4i)(3+4i)}$$

$$\frac{z_8}{z_{10}} = \frac{-24i-32i^2}{9+4i-4i-16i^2}$$

$$\frac{z_8}{z_{10}} = \frac{-24i-32(-1)}{9-16(-1)}$$

$$\frac{z_8}{z_{10}} = \frac{-24i+32}{9+16}$$

$$\begin{aligned} \frac{z_8}{z_{10}} &= \frac{-24i+32}{9+16} \\ \frac{z_8}{z_{10}} &= \frac{32-24i}{25} \\ \frac{z_8}{z_{10}} &= \frac{32}{25} - \frac{24}{25}i. \end{aligned}$$

(e) $z_3 + z_6$.

$$\begin{aligned} z_3 + z_6 &= (5 + 4i) + (-7) \\ z_3 + z_6 &= 5 + 4i - 7 \\ z_3 + z_6 &= -2 + 4i. \end{aligned}$$

(f) $z_2 - z_6$.

$$\begin{aligned} z_2 - z_6 &= (-1 + i) - (-7) \\ z_2 - z_6 &= -1 + i + 7 \\ z_2 - z_6 &= 6 + i. \end{aligned}$$

(g) $z_3 z_{10}$.

$$\begin{aligned} z_3 z_{10} &= (5 + 4i)(3 - 4i) \\ z_3 z_{10} &= 15 - 20i + 12i - 16i^2 \\ z_3 z_{10} &= 15 - 8i - 16(-1) \\ z_3 z_{10} &= 15 - 8i + 16 \\ z_3 z_{10} &= 31 - 8i. \end{aligned}$$

(h) z_1^3 .

$$\begin{aligned} z_1^3 &= (3 + 3i)^3 \\ z_1^3 &= 3^3 + 3 * 3^2 * 3i + 3 * 3 (3i)^2 + (3i)^3 \\ z_1^3 &= 27 + 81i + 9 * 9i^2 + 27i^3 \\ z_1^3 &= 27 + 81i + 81(-1) + 27(-i) \\ z_1^3 &= 27 + 81i - 81 - 27i \\ z_1^3 &= -54 + 54i. \end{aligned}$$

$$\begin{aligned} |z_1| &= \sqrt{3^2 + 3^2} \\ |z_1| &= \sqrt{9 + 9} \\ |z_1| &= \sqrt{9 * 2} \\ |z_1| &= \sqrt{9} \sqrt{2} \\ |z_1| &= 3 \sqrt{2}. \end{aligned}$$

$$\alpha = \tan^{-1} \left(\frac{3}{1} \right)$$

$$\alpha = \tan^{-1} (1)$$

$$\alpha = \frac{\pi}{4}.$$

$$z_1^3 = |z_1|^3 e^{i3\alpha}$$

$$z_1^3 = (3\sqrt{2})^3 e^{i3\frac{\pi}{4}}$$

$$z_1^3 = 27 * 2\sqrt{2} e^{i\frac{3}{4}\pi}$$

$$z_1^3 = 54\sqrt{2} e^{i\frac{3}{4}\pi}$$

$$z_1^3 = 54\sqrt{2} \left(\cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi \right)$$

$$z_1^3 = 54\sqrt{2} \left(\frac{-3}{3\sqrt{2}} + i \frac{3}{3\sqrt{2}} \right)$$

$$z_1^3 = 54\sqrt{2} \left(\frac{-1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right)$$

$$z_1^3 = -54 + 54i.$$

$$z_1^3 = (|z_1|)^3 (\cos 3\alpha + i \sin 3\alpha)$$

$$z_1^3 = (3\sqrt{2})^3 \left[\cos \left(3\frac{\pi}{4} \right) + i \sin \left(3\frac{\pi}{4} \right) \right]$$

$$z_1^3 = 27 * 2\sqrt{2} \left(\cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi \right)$$

$$z_1^3 = 54\sqrt{2} \left(\frac{-3}{3\sqrt{2}} + i \frac{3}{3\sqrt{2}} \right)$$

$$z_1^3 = 54\sqrt{2} \left(\frac{-1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right)$$

$$z_1^3 = -54 + 54i.$$

(i) z_9 .

$$z_9 = (2 - 2i)^9.$$

$$|z_9| = \sqrt{2^2 + (-2)^2}$$

$$|z_9| = \sqrt{4 + 4}$$

$$|z_9| = \sqrt{8}$$

$$|z_9| = \sqrt{4 * 2}$$

$$|z_9| = \sqrt{4} \sqrt{2}$$

$$|z_9| = 2\sqrt{2}.$$

$$\alpha = \tan^{-1} \left(\frac{-2}{2} \right)$$

$$\alpha = \tan^{-1} (-1)$$

$$\alpha = \frac{-\pi}{4}.$$

$$z_9^9 = |z_9|^9 e^{i9\alpha}$$

$$z_9^9 = (2\sqrt{2})^9 e^{i9\left(\frac{-\pi}{4}\right)}$$

$$z_9^9 = 2^9 (\sqrt{2})^9 e^{\frac{-\pi}{4}i}$$

$$z_9^9 = 2^9 2^4 \sqrt{2} e^{\frac{-\pi}{4}i}$$

$$\begin{aligned}
z_9^9 &= 2^{13} \sqrt{2} e^{\frac{-\pi}{4}i} \\
z_9^9 &= 2^{13} \sqrt{2} [\cos(\frac{-\pi}{4}) + i \sin(\frac{-\pi}{4})] \\
z_9^9 &= 2^{13} \sqrt{2} [\frac{2}{2\sqrt{2}} + i(\frac{-2}{2\sqrt{2}})] \\
z_9^9 &= 2^{13} \sqrt{2} [\frac{1}{\sqrt{2}} + i(\frac{-1}{\sqrt{2}})] \\
z_9^9 &= 2^{13} \sqrt{2} (\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}}) \\
z_9^9 &= 2^{13} - 2^{13}i.
\end{aligned}$$

$$\begin{aligned}
z_9^9 &= (|z_9|)^9 (\cos 9\alpha + i \sin 9\alpha) \\
z_9^9 &= (2\sqrt{2})^9 \{ \cos [9(\frac{-\pi}{4})] + i \sin [9(\frac{-\pi}{4})] \} \\
z_9^9 &= 2^9 (\sqrt{2})^9 [\cos(\frac{-\pi}{4}) + i \sin(\frac{-\pi}{4})] \\
z_9^9 &= 2^9 2^4 \sqrt{2} [\frac{2}{2\sqrt{2}} + i(\frac{-2}{2\sqrt{2}})] \\
z_9^9 &= 2^{13} \sqrt{2} [\frac{1}{\sqrt{2}} + i(\frac{-1}{\sqrt{2}})] \\
z_9^9 &= 2^{13} \sqrt{2} (\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}}) \\
z_9^9 &= 2^{13} - 2^{13}i.
\end{aligned}$$

(j) z_5^{15} .

$$\begin{aligned}
z_5^{15} &= (5i)^{15} \\
z_5^{15} &= 5^{15} i^{15} \\
z_5^{15} &= 5^{15} i^{4*3+3} \\
z_5^{15} &= 5^{15} (i^4)^3 i^3 \\
z_5^{15} &= 5^{15} 1^3 (-i) \\
z_5^{15} &= 5^{15} * 1 (-i) \\
z_5^{15} &= -5^{15}i.
\end{aligned}$$

(k) z_{10}^3 .

$$\begin{aligned}
z_{10}^3 &= (3 - 4i)^3 \\
z_{10}^3 &= 3^3 - 3 * 3^2 * 4i + 3 * 3 * 4i^2 - (4i)^3 \\
z_{10}^3 &= 27 - 3 * 9 * 4i + 3 * 3 * 16i^2 - 64i^3 \\
z_{10}^3 &= 27 - 108i + 144(-1) - 64(-i) \\
z_{10}^3 &= 27 - 108i - 144 + 64i \\
z_{10}^3 &= -117 - 44i.
\end{aligned}$$

$$\begin{aligned}
|z_{10}| &= \sqrt{3^2 + (-4)^2} \\
|z_{10}| &= \sqrt{9 + 16} \\
|z_{10}| &= \sqrt{25} \\
|z_{10}| &= 5.
\end{aligned}$$

$$\alpha = \tan^{-1} \left(\frac{-4}{3} \right)$$

$$\alpha = -0,927.$$

$$z_{10}^3 = |z_{10}|^3 e^{i3\alpha}$$

$$z_{10}^3 = 5^3 e^{i3(-0,927)}$$

$$z_{10}^3 = 125 e^{-2,782i}$$

$$z_{10}^3 = 125 [\cos(-2,782) + i \sin(-2,782)]$$

$$z_{10}^3 = 125 [-0,936 + i(-0,352)]$$

$$z_{10}^3 = 125 (-0,936 - 0,352i)$$

$$z_{10}^3 = -117 - 44i.$$

$$z_{10}^3 = (|z_{10}|)^3 (\cos 3\alpha + i \sin 3\alpha)$$

$$z_{10}^3 = 5^3 \{ \cos [3(-0,927)] + i \sin [3(-0,927)] \}$$

$$z_{10}^3 = 125 [\cos(-2,782) + i \sin(-2,782)]$$

$$z_{10}^3 = 125 [-0,936 + i(-0,352)]$$

$$z_{10}^3 = 125 (-0,936 - 0,352i)$$

$$z_{10}^3 = -117 - 44i.$$

(I) Hallar las raíces cuartas de z_2 .

$$z_2 = -1 + i.$$

$$|z_2| = \sqrt{(-1)^2 + 1^2}$$

$$|z_2| = \sqrt{1 + 1}$$

$$|z_2| = \sqrt{2}.$$

$$\alpha = \tan^{-1} \left(\frac{1}{-1} \right)$$

$$\alpha = \tan^{-1} (-1)$$

$$\alpha = \frac{-\pi}{4}.$$

$$z_2 = \sqrt{2} e^{i(\frac{-\pi}{4})}.$$

$$|w| = \sqrt[4]{|z_2|}$$

$$|w| = \sqrt[4]{\sqrt{2}}$$

$$|w| = \sqrt[8]{2}.$$

$$\varphi_k = \frac{\frac{-\pi}{4} + 2k\pi}{4}.$$

$$\varphi_0 = \frac{\frac{-\pi}{4} + 2 \cdot 0 \pi}{4}$$

$$\varphi_0 = \frac{\frac{-\pi}{4} + 0}{4}$$

$$\varphi_0 = \frac{\frac{-\pi}{4}}{4}$$

$$\varphi_0 = \frac{-\pi}{16}.$$

$$\varphi_1 = \frac{\frac{-\pi}{4} + 2 \cdot 1 \pi}{4}$$

$$\varphi_1 = \frac{\frac{-\pi}{4} + 2\pi}{4}$$

$$\varphi_1 = \frac{\frac{7}{4}\pi}{4}$$

$$\varphi_1 = \frac{7}{16} \pi.$$

$$\varphi_2 = \frac{\frac{-\pi}{4} + 2 \cdot 2 \pi}{4}$$

$$\varphi_2 = \frac{\frac{-\pi}{4} + 4\pi}{4}$$

$$\varphi_2 = \frac{\frac{15}{4}\pi}{4}$$

$$\varphi_2 = \frac{15}{16} \pi.$$

$$\varphi_3 = \frac{\frac{-\pi}{4} + 2 \cdot 3 \pi}{4}$$

$$\varphi_3 = \frac{\frac{-\pi}{4} + 6\pi}{4}$$

$$\varphi_3 = \frac{\frac{23}{4}\pi}{4}$$

$$\varphi_3 = \frac{23}{16} \pi.$$

$$w_k = |w| e^{i\varphi_k}$$

$$w_k = \sqrt[8]{2} e^{i(\frac{\frac{-\pi}{4} + 2k\pi}{4})}.$$

$$w_0 = \sqrt[8]{2} e^{i(\frac{-\pi}{16})}.$$

$$w_1 = \sqrt[8]{2} e^{i(\frac{7}{16}\pi)}.$$

$$w_2 = \sqrt[8]{2} e^{i(\frac{15}{16}\pi)}.$$

$$w_3 = \sqrt[8]{2} e^{i(\frac{23}{16}\pi)}.$$

(m) Hallar las raíces cúbicas de z_4 .

$$z_4 = 9.$$

$$|z_4| = \sqrt{9^2}$$

$$|z_4| = 9.$$

$$\alpha = \tan^{-1} \left(\frac{0}{9} \right)$$

$$\alpha = \tan^{-1} (0)$$

$$\alpha = 0.$$

$$z_4 = 9e^{i \cdot 0}$$

$$z_4 = 9e^0$$

$$z_4 = 9 * 1$$

$$z_4 = 9.$$

$$|w| = \sqrt[3]{|z_4|}$$

$$|w| = \sqrt[3]{9}.$$

$$\varphi_k = \frac{0 + 2k\pi}{3}$$

$$\varphi_k = \frac{2k\pi}{3}.$$

$$\varphi_0 = \frac{2 \cdot 0 \pi}{3}$$

$$\varphi_0 = \frac{0}{3}$$

$$\varphi_0 = 0.$$

$$\varphi_1 = \frac{2 \cdot 1 \pi}{3}$$

$$\varphi_1 = \frac{2}{3} \pi.$$

$$\varphi_2 = \frac{2 \cdot 2 \pi}{3}$$

$$\varphi_2 = \frac{4}{3} \pi.$$

$$w_k = |w| e^{i\varphi_k}$$

$$w_k = \sqrt[3]{9} e^{i \frac{2k\pi}{3}}.$$

$$w_0 = \sqrt[3]{9} e^{i \cdot 0} = \sqrt[3]{9} e^0 = \sqrt[3]{9} * 1 = \sqrt[3]{9}.$$

$$w_1 = \sqrt[3]{9} e^{i(\frac{2}{3}\pi)}.$$

$$w_2 = \sqrt[3]{9} e^{i(\frac{4}{3}\pi)}.$$

(n) Hallar las raíces séptimas de i .

$$z = i.$$

$$|z| = \sqrt{1^2}$$

$$|z| = 1.$$

$$\alpha = \frac{\pi}{2}.$$

$$z = 1 e^{i \frac{\pi}{2}}$$

$$z = e^{i \frac{\pi}{2}}.$$

$$|w| = \sqrt[7]{|z|}$$

$$|w| = \sqrt[7]{1}$$

$$|w| = 1.$$

$$\varphi_k = \frac{\frac{\pi}{2} + 2k\pi}{7}.$$

$$\varphi_0 = \frac{\frac{\pi}{2} + 2 \cdot 0 \pi}{7}$$

$$\varphi_0 = \frac{\frac{\pi}{2} + 0}{7}$$

$$\varphi_0 = \frac{\frac{\pi}{2}}{7}$$

$$\varphi_0 = \frac{\pi}{14}.$$

$$\varphi_1 = \frac{\frac{\pi}{2} + 2 \cdot 1 \pi}{7}$$

$$\varphi_1 = \frac{\frac{\pi}{2} + 2\pi}{7}$$

$$\varphi_1 = \frac{\frac{5}{2}\pi}{7}$$

$$\varphi_1 = \frac{5}{14} \pi.$$

$$\varphi_2 = \frac{\frac{\pi}{2} + 2 \cdot 2 \pi}{7}$$

$$\varphi_2 = \frac{\frac{\pi}{2} + 4\pi}{7}$$

$$\varphi_2 = \frac{\frac{9}{2}\pi}{7}$$

$$\varphi_2 = \frac{9}{14} \pi.$$

$$\varphi_3 = \frac{\frac{\pi}{2} + 2 \cdot 3 \pi}{7}$$

$$\varphi_3 = \frac{\frac{\pi}{2} + 6\pi}{7}$$

$$\varphi_3 = \frac{\frac{13}{2}\pi}{7}$$

$$\varphi_3 = \frac{13}{14} \pi.$$

$$\varphi_4 = \frac{\frac{\pi}{2} + 2 \cdot 4 \pi}{7}$$

$$\varphi_4 = \frac{\frac{\pi}{2} + 8\pi}{7}$$

$$\varphi_4 = \frac{\frac{17}{2}\pi}{7}$$

$$\varphi_4 = \frac{17}{14} \pi.$$

$$\varphi_5 = \frac{\frac{\pi}{2} + 2 \cdot 5 \pi}{7}$$

$$\varphi_5 = \frac{\frac{\pi}{2} + 10\pi}{7}$$

$$\varphi_5 = \frac{\frac{21}{2}\pi}{7}$$

$$\varphi_5 = \frac{3}{2}\pi.$$

$$\varphi_6 = \frac{\frac{\pi}{2} + 2 \cdot 6\pi}{7}$$

$$\varphi_6 = \frac{\frac{\pi}{2} + 12\pi}{7}$$

$$\varphi_6 = \frac{\frac{25}{2}\pi}{7}$$

$$\varphi_6 = \frac{25}{14}\pi.$$

$$w_k = |w| e^{i\varphi_k}$$

$$w_k = \sqrt[6]{1} e^{i\frac{\frac{\pi}{2} + 2k\pi}{7}}$$

$$w_k = 1 e^{i\frac{\frac{\pi}{2} + 2k\pi}{7}}$$

$$w_k = e^{i\frac{\frac{\pi}{2} + 2k\pi}{7}}.$$

$$w_0 = e^{i\frac{\pi}{14}}.$$

$$w_1 = e^{i\frac{5}{14}\pi}.$$

$$w_2 = e^{i\frac{8}{14}\pi}.$$

$$w_3 = e^{i\frac{13}{14}\pi}.$$

$$w_4 = e^{i\frac{17}{14}\pi}.$$

$$w_5 = e^{i\frac{3}{2}\pi}.$$

$$w_6 = e^{i\frac{25}{14}\pi}.$$