Juan Menduiña

Trabajo Práctico N° 2:
Límites.

Ejercicio 1.

Calcular los límites de las siguientes funciones, si existen.

(a) $f(x) = \begin{cases} x^2, si\ x > 2 \\ 1, si\ x \leq 2 \end{cases}$. *Calcular* $\lim\limits_{x \to 2^+} f(x)$ *y* $\lim\limits_{x \to 2^-} f(x)$.

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} 1 = 1.$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} x^2 = 2^2 = 4.$

Por lo tanto, ya que $\lim\limits_{x \to 2^-} f(x) \neq \lim\limits_{x \to 2^+} f(x), \nexists \lim\limits_{x \to 2} f(x).$

(b) *Sea g la función valor absoluto, $g(x) = |x| = \begin{cases} x, si\ x \geq 0 \\ -x, si\ x < 0 \end{cases}$. Calcular* $\lim\limits_{x \to 0^+} g(x)$ *y* $\lim\limits_{x \to 0^-} g(x)$. *¿Qué se puede decir respecto al* $\lim\limits_{x \to 0} g(x)$?

$\lim\limits_{x \to 0^-} g(x) = \lim\limits_{x \to 0^-} -x = 0.$

$\lim\limits_{x \to 0^+} g(x) = \lim\limits_{x \to 0^+} x = 0.$

Por lo tanto, ya que $\lim\limits_{x \to 0^-} g(x) = \lim\limits_{x \to 0^+} g(x), \exists \lim\limits_{x \to 0} g(x).$

Ejercicio 2.

Dada la función f (x)= $\begin{cases} \frac{x-1}{x^2-1}, si\ x \neq 1 \\ 3, si\ x = 1 \end{cases}$, *calcular* $\lim\limits_{x\to 1^+} f(x)$ *y* $\lim\limits_{x\to 1^-} f(x)$.

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} \frac{x-1}{x^2-1} = \frac{1-1}{1^2-1} = \frac{0}{1-1} = \left(\frac{0}{0}\right).$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} \frac{x-1}{(x+1)(x-1)}$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}.$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \frac{x-1}{x^2-1} = \frac{1-1}{1^2-1} = \frac{0}{1-1} = \left(\frac{0}{0}\right).$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \frac{x-1}{(x+1)(x-1)}$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}.$

Ejercicio 3.

Calcular, si existen, los siguientes límites:

(a) $\lim\limits_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}$.

$\lim\limits_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \frac{\sqrt{0^2+9}-3}{0^2} = \frac{\sqrt{0+9}-3}{0} = \frac{\sqrt{9}-3}{0} = \frac{3-3}{0} = \left(\frac{0}{0}\right)$.

$\lim\limits_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim\limits_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} \cdot \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3}$

$\lim\limits_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim\limits_{x \to 0} \frac{x^2+9+3\sqrt{x^2+9}-3\sqrt{x^2+9}-9}{x^2(\sqrt{x^2+9}+3)}$

$\lim\limits_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim\limits_{x \to 0} \frac{x^2}{x^2(\sqrt{x^2+9}+3)}$

$\lim\limits_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim\limits_{x \to 0} \frac{1}{\sqrt{x^2+9}+3} = \frac{1}{\sqrt{0^2+9}+3} = \frac{1}{\sqrt{0+9}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$.

(b) $\lim\limits_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2}$.

$\lim\limits_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2} = \frac{1^2-2*1+1}{1^2-3*1+2} = \frac{1-2+1}{1-3+2} = \left(\frac{0}{0}\right)$.

$\lim\limits_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2} = \lim\limits_{x \to 1} \frac{(x-1)^2}{(x-2)(x-1)}$ (*) y (**)

$\lim\limits_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2} = \lim\limits_{x \to 1} \frac{x-1}{x-2} = \frac{1-1}{1-2} = \frac{0}{-1} = 0$.

(*) $x_1, x_2 = \frac{-(-2)\pm\sqrt{(-2)^2-4*1*1}}{2*1}$

$\quad x_1, x_2 = \frac{2\pm\sqrt{4-4}}{2}$

$\quad x_1, x_2 = \frac{2\pm\sqrt{0}}{2}$

$\quad x_1, x_2 = \frac{2\pm 0}{2}$

$\quad x_1 = \frac{2+0}{2} = \frac{2}{2} = 1$.

$\quad x_2 = \frac{2-0}{2} = \frac{2}{2} = 1$.

(**) $x_1, x_2 = \frac{-(-3)\pm\sqrt{(-3)^2-4*1*2}}{2*1}$

$\quad x_1, x_2 = \frac{3\pm\sqrt{9-8}}{2}$

$\quad x_1, x_2 = \frac{3\pm\sqrt{1}}{2}$

$\quad x_1, x_2 = \frac{3\pm 1}{2}$

$\quad x_1 = \frac{3+1}{2} = \frac{4}{2} = 2$.

$\quad x_2 = \frac{3-1}{2} = \frac{2}{2} = 1$.

Ejercicio 4.

Calcular los siguientes límites:

(a) $\lim\limits_{x \to 3} \dfrac{x+2}{x+3}$.

$\lim\limits_{x \to 3} \dfrac{x+2}{x+3} = \dfrac{3+2}{3+3} = \dfrac{5}{6}$.

(b) $\lim\limits_{x \to 1} x^3 + 5x^2 + 10$.

$\lim\limits_{x \to 1} x^3 + 5x^2 + 10 = 1^3 + 5 * 1^2 + 10 = 1 + 5 * 1 + 10 = 1 + 5 + 10 = 16$.

(c) $\lim\limits_{x \to -2} \dfrac{x^3 + 2x^2 - 1}{5 - 3x}$.

$\lim\limits_{x \to -2} \dfrac{x^3 + 2x^2 - 1}{5 - 3x} = \dfrac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)} = \dfrac{-8 + 2*4 - 1}{5 + 6} = \dfrac{-8 + 8 - 1}{11} = \dfrac{-1}{11}$.

(d) $\lim\limits_{x \to \pi} sen \, (x - \pi)$.

$\lim\limits_{x \to \pi} sen \, (x - \pi) = sen \, (\pi - \pi) = sen \, 0 = 0$.

(e) $\lim\limits_{x \to 2} \dfrac{\sqrt[3]{x^2 + 2}}{x}$.

$\lim\limits_{x \to 2} \dfrac{\sqrt[3]{x^2 + 2}}{x} = \dfrac{\sqrt[3]{2^2 + 2}}{2} = \dfrac{\sqrt[3]{4 + 2}}{2} = \dfrac{\sqrt[3]{6}}{2}$.

(f) $\lim\limits_{x \to 1} ln \dfrac{2}{x^3 + 1}$.

$\lim\limits_{x \to 1} ln \dfrac{2}{x^3 + 1} = ln \dfrac{2}{1^3 + 1} = ln \dfrac{2}{1 + 1} = ln \dfrac{2}{2} = ln \, 1 = 0$.

(g) $\lim\limits_{x \to 0} \dfrac{ln(x+1)}{e^x}$.

$$\lim_{x \to 0} \frac{\ln(x+1)}{e^x} = \frac{\ln(0+1)}{e^0} = \frac{\ln 1}{1} = \frac{0}{1} = 0.$$

(h) $\lim_{x \to 0} 3 \cos x^2 (1 + x)^4.$

$$\lim_{x \to 0} 3 \cos x^2 (1 + x)^4 = 3 \cos 0^2 (1 + 0)^4 = 3 \cos 0 * 1^4 = 3 * 1 * 1 = 3.$$

(i) $\lim_{x \to 3} |x - 3|.$

$$\lim_{x \to 3} |x - 3| = |3 - 3| = |0| = 0.$$

(j) $\lim_{x \to 3} \frac{|x-3|}{x}.$

$$\lim_{x \to 3} \frac{|x-3|}{x} = \frac{|3-3|}{3} = \frac{|0|}{3} = \frac{0}{3} = 0.$$

(k) $\lim_{x \to 0} \frac{x}{x^2+x}.$

$$\lim_{x \to 0} \frac{x}{x^2+x} = \frac{0}{0^2+0} = \frac{0}{0+0} = \left(\frac{0}{0}\right).$$
$$\lim_{x \to 0} \frac{x}{x^2+x} = \lim_{x \to 0} \frac{x}{x(x+1)}$$
$$\lim_{x \to 0} \frac{x}{x^2+x} = \lim_{x \to 0} \frac{1}{x+1} = \frac{1}{0+1} = \frac{1}{1} = 1.$$

(l) $\lim_{x \to 2} \frac{x^2-x-2}{x^2-4}.$

$$\lim_{x \to 2} \frac{x^2-x-2}{x^2-4} = \frac{2^2-2-2}{2^2-4} = \frac{4-2-2}{4-4} = \left(\frac{0}{0}\right).$$
$$\lim_{x \to 2} \frac{x^2-x-2}{x^2-4} = \lim_{x \to 2} \frac{(x-2)(x+1)}{(x+2)(x-2)} \qquad (*)$$
$$\lim_{x \to 2} \frac{x^2-x-2}{x^2-4} = \lim_{x \to 2} \frac{x+1}{x+2} = \frac{2+1}{2+2} = \frac{3}{4}.$$

$$(*) \; x_1, x_2 = \frac{-(-1) \pm \sqrt{(-1)^2 - 4*1*(-2)}}{2*1}$$
$$x_1, x_2 = \frac{1 \pm \sqrt{1+8}}{2}$$
$$x_1, x_2 = \frac{1 \pm \sqrt{9}}{2}$$

$$x_1, x_2 = \frac{1 \pm 3}{2}$$
$$x_1 = \frac{1+3}{2} = \frac{4}{2} = 2.$$
$$x_2 = \frac{1-3}{2} = \frac{-2}{2} = -1.$$

(m) $\lim\limits_{x \to -1} \frac{x^2-1}{x^2+3x+2}$.

$$\lim\limits_{x \to -1} \frac{x^2-1}{x^2+3x+2} = \frac{(-1)^2-1}{(-1)^2+3(-1)+2} = \frac{1-1}{1-3+2} = \left(\frac{0}{0}\right).$$
$$\lim\limits_{x \to -1} \frac{x^2-1}{x^2+3x+2} = \lim\limits_{x \to -1} \frac{(x+1)(x-1)}{(x-2)(x-1)} \qquad\qquad (*)$$
$$\lim\limits_{x \to -1} \frac{x^2-1}{x^2+3x+2} = \lim\limits_{x \to -1} \frac{x+1}{x-2} = \frac{-1+1}{-1-2} = \frac{0}{-3} = 0.$$

$(*)$ $x_1, x_2 = \frac{-(-3) \pm \sqrt{(-3)^2 - 4*1*2}}{2*1}$

$\qquad x_1, x_2 = \frac{3 \pm \sqrt{9-8}}{2}$

$\qquad x_1, x_2 = \frac{3 \pm \sqrt{1}}{2}$

$\qquad x_1, x_2 = \frac{3 \pm 1}{2}$

$\qquad x_1 = \frac{3+1}{2} = \frac{4}{2} = 2.$

$\qquad x_2 = \frac{3-1}{2} = \frac{2}{2} = 1.$

(n) $\lim\limits_{x \to 1} \frac{x-1}{\sqrt{x}-1}$.

$$\lim\limits_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \frac{1-1}{\sqrt{1}-1} = \frac{0}{1-1} = \left(\frac{0}{0}\right).$$
$$\lim\limits_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim\limits_{x \to 1} \frac{x-1}{\sqrt{x}-1} \frac{\sqrt{x}+1}{\sqrt{x}+1}$$
$$\lim\limits_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim\limits_{x \to 1} \frac{(x-1)(\sqrt{x}+1)}{x+\sqrt{x}-\sqrt{x}-1}$$
$$\lim\limits_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim\limits_{x \to 1} \frac{(x-1)(\sqrt{x}+1)}{x-1}$$
$$\lim\limits_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim\limits_{x \to 1} \sqrt{x} + 1 = \sqrt{1} + 1 = 1 + 1 = 2.$$

(ñ) $\lim\limits_{x \to 0} \frac{x^3}{x^2+x}$.

$$\lim\limits_{x \to 0} \frac{x^3}{x^2+x} = \frac{0^3}{0^2+0} = \frac{0}{0+0} = \left(\frac{0}{0}\right).$$
$$\lim\limits_{x \to 0} \frac{x^3}{x^2+x} = \lim\limits_{x \to 0} \frac{x^3}{x(x+1)}$$
$$\lim\limits_{x \to 0} \frac{x^3}{x^2+x} = \lim\limits_{x \to 0} \frac{x^2}{x+1} = \frac{0^2}{0+1} = \frac{0}{1} = 0.$$

(o) $\lim\limits_{x\to 2} \dfrac{x+2}{2-x}.$

$$\lim\limits_{x\to 2} \frac{x+1}{2-x} = \frac{2+1}{2-2} = \frac{3}{0} = +\infty.$$

(p) $\lim\limits_{x\to 3} \dfrac{x}{x-3}.$

$$\lim\limits_{x\to 3} \frac{x}{x-3} = \frac{3}{3-3} = \frac{3}{0} = +\infty.$$

(q) $\lim\limits_{x\to -2} \dfrac{x}{|x+2|}.$

$$\lim\limits_{x\to -2} \frac{x}{|x+2|} = \frac{-2}{|-2+2|} = \frac{-2}{|0|} = \frac{-2}{0} = -\infty.$$

Ejercicio 5.

Calcular los siguientes límites al infinito:

(a) $\lim\limits_{x \to -\infty} x - 3x^4$.

$\lim\limits_{x \to -\infty} x - 3x^4 = -\infty$.

(b) $\lim\limits_{x \to -\infty} \frac{5}{x} - \frac{3}{x}$.

$\lim\limits_{x \to -\infty} \frac{5}{x} - \frac{3}{x} = 0 - 0 = 0$.

(c) $\lim\limits_{x \to +\infty} \frac{1}{x^2 + 3x - 1}$.

$\lim\limits_{x \to +\infty} \frac{1}{x^2 + 3x - 1} = 0$.

(d) $\lim\limits_{x \to +\infty} \frac{3x^2 + 2x - 16}{x^2 - x - 2}$.

$\lim\limits_{x \to +\infty} \frac{3x^2 + 2x - 16}{x^2 - x - 2} = \left(\frac{\infty}{\infty}\right)$.

$\lim\limits_{x \to +\infty} \frac{3x^2 + 2x - 16}{x^2 - x - 2} = \lim\limits_{x \to +\infty} \frac{x^2\left(3 + \frac{2}{x} - \frac{16}{x^2}\right)}{x^2\left(1 - \frac{1}{x} - \frac{2}{x^2}\right)}$

$\lim\limits_{x \to +\infty} \frac{3x^2 + 2x - 16}{x^2 - x - 2} = \lim\limits_{x \to +\infty} \frac{3 + \frac{2}{x} - \frac{16}{x^2}}{1 - \frac{1}{x} - \frac{2}{x^2}} = \frac{3 + 0 - 0}{1 - 0 - 0} = \frac{3}{1} = 3$.

(e) $\lim\limits_{x \to -\infty} \frac{x^2 - 2x - 3}{x^3 - x^2 - 6x - 2}$.

$\lim\limits_{x \to -\infty} \frac{x^2 - 2x - 3}{x^3 - x^2 - 6x - 2} = 0$.

(f) $\lim\limits_{x \to -\infty} \frac{x^4 - 2x^2}{\sqrt{x^4 - 6x}}$.

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^4(1-\frac{2}{x^2})}{\sqrt{x^4(1-\frac{6}{x^3})}}$$

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^4(1-\frac{2}{x^2})}{\sqrt{x^4}\sqrt{1-\frac{6}{x^3}}}$$

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^4(1-\frac{2}{x^2})}{x^2\sqrt{1-\frac{6}{x^3}}}$$

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^2(1-\frac{2}{x^2})}{\sqrt{1-\frac{6}{x^3}}} = +\infty.$$

Ejercicio 6.

Calcular los siguientes límites, si es que existen:

(a) *Dada* $h\ (x)=\begin{cases}\sqrt{x-4},\ si\ x>4\\8-2x,\ si\ x<4\end{cases}$. *Calcular* $\lim\limits_{x\to4} h\ (x)$.

$\lim\limits_{x\to4^-} h\ (x)=\lim\limits_{x\to4^-} 8-2*4=$ 8 - 8= 0.

$\lim\limits_{x\to4^+} h\ (x)=\lim\limits_{x\to4^+} \sqrt{x-4}=\sqrt{4-4}=\sqrt{0}=$ 0.

Por lo tanto, ya que $\lim\limits_{x\to4^-} h\ (x)=\lim\limits_{x\to4^+} h\ (x)=$ 0, entonces, $\lim\limits_{x\to4} h\ (x)=$ 0.

(b) *Sea* $f\ (x)=\begin{cases}\frac{x}{e^x},\ si\ x\leq0\\x^2,\ si\ x>0\end{cases}$. *Calcular* $\lim\limits_{x\to0} f\ (x)$ *y* $\lim\limits_{x\to-1} f\ (x)$.

$\lim\limits_{x\to0^-} f\ (x)=\lim\limits_{x\to0^-} \frac{x}{e^x}=\frac{0}{e^0}=\frac{0}{1}=$ 0.

$\lim\limits_{x\to0^+} f\ (x)=\lim\limits_{x\to0^+} x^2=0^2=$ 0.

Por lo tanto, ya que $\lim\limits_{x\to0^-} f\ (x)=\lim\limits_{x\to0^+} f\ (x)=$ 0, entonces, $\lim\limits_{x\to0} f\ (x)=$ 0.

$\lim\limits_{x\to-1} f\ (x)=\lim\limits_{x\to-1} x^2=(-1)^2=$ 1.

Por lo tanto, $\lim\limits_{x\to-1} f\ (x)=$ 1.

(c) $\lim\limits_{x\to+\infty} \frac{(\ln x)^2}{x}$.

$\lim\limits_{x\to+\infty} \frac{(\ln x)^2}{x}=$ 0.

(d) $\lim\limits_{x\to+\infty} \frac{3x^2+2x+1}{e^x}$.

$\lim\limits_{x\to+\infty} \frac{3x^2+2x+1}{e^x}=$ 0.

(e) $\lim\limits_{x\to0^+} x\ \ln x$.

$$\lim_{x \to 0^+} x \ln x = \lim_{y \to +\infty} \frac{1}{y} \ln \frac{1}{y}$$

$$\lim_{x \to 0^+} x \ln x = \lim_{y \to +\infty} \frac{\ln \frac{1}{y}}{y}$$

$$\lim_{x \to 0^+} x \ln x = \lim_{y \to +\infty} \frac{\ln 1 - \ln y}{y}$$

$$\lim_{x \to 0^+} x \ln x = \lim_{y \to +\infty} \frac{0 - \ln y}{y}$$

$$\lim_{x \to 0^+} x \ln x = \lim_{y \to +\infty} \frac{-\ln y}{y} = 0.$$

Ejercicio 7.

Calcular las asíntotas verticales y horizontales, si existen, de las funciones dadas a continuación:

(a) $f(x) = \frac{x^2}{x+1}$.

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{x^2}{x+1} = \frac{(-1)^2}{-1+1} = \frac{1}{0} = -\infty.$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \frac{x^2}{x+1} = \frac{(-1)^2}{-1+1} = \frac{1}{0} = +\infty.$$

Por lo tanto, $f(x)$ tiene un asíntota vertical en x= -1.

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x^2}{x+1} = -\infty.$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{x^2}{x+1} = +\infty.$$

Por lo tanto, $f(x)$ no tiene asíntotas horizontales.
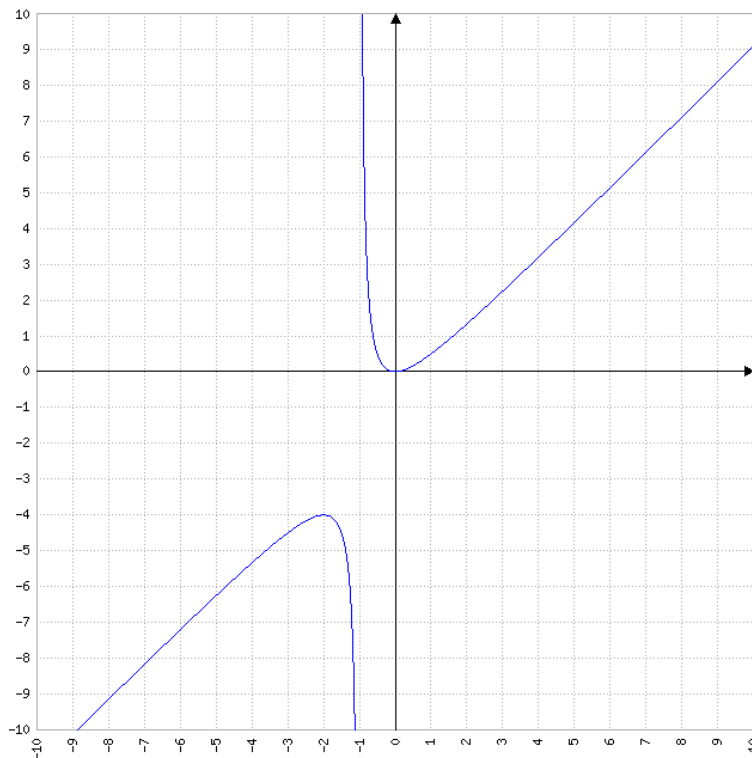
(b) $g(x) = \frac{4x^2 + 2x - 2}{3x - 1}$.

$$\lim_{x \to \frac{1}{3}^-} g\,(x) = \lim_{x \to \frac{1}{3}^-} \frac{4x^2 + 2x - 2}{3x - 1} = \frac{4(\frac{1}{3})^2 + 2\frac{1}{3} - 2}{3(\frac{1}{3}) - 1} = \frac{4\frac{1}{9} + \frac{2}{3} - 2}{1 - 1} = \frac{\frac{4}{9} + \frac{2}{3} - 2}{0} = \frac{\frac{-8}{9}}{0} = +\infty.$$
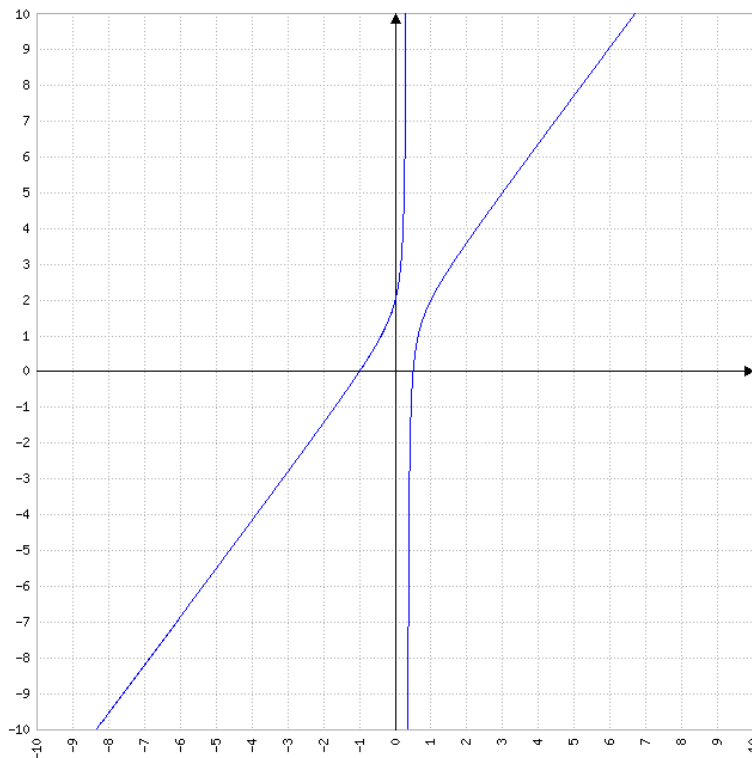
$$\lim_{x \to \frac{1}{3}^+} g\,(x) = \lim_{x \to \frac{1}{3}^+} \frac{4x^2 + 2x - 2}{3x - 1} = \frac{4(\frac{1}{3})^2 + 2\frac{1}{3} - 2}{3(\frac{1}{3}) - 1} = \frac{4\frac{1}{9} + \frac{2}{3} - 2}{1 - 1} = \frac{\frac{4}{9} + \frac{2}{3} - 2}{0} = \frac{\frac{-8}{9}}{0} = -\infty.$$

Por lo tanto, g (x) tiene un asíntota vertical en x$=\frac{1}{3}$.

$$\lim_{x \to -\infty} g\,(x) = \lim_{x \to -\infty} \frac{4x^2 + 2x - 2}{3x - 1} = -\infty.$$

$$\lim_{x \to +\infty} g\,(x) = \lim_{x \to +\infty} \frac{4x^2 + 2x - 2}{3x - 1} = +\infty.$$

Por lo tanto, g (x) no tiene asíntotas horizontales.

(c) $h\,(x) = \frac{x-2}{x^2 - 4x + 4}$.

$$x_1, x_2 = \frac{-(-4) \pm \sqrt{(-4)^2 - 4*1*4}}{2*1}$$
$$x_1, x_2 = \frac{4 \pm \sqrt{16 - 16}}{2}$$
$$x_1, x_2 = \frac{4 \pm \sqrt{0}}{2}$$
$$x_1, x_2 = \frac{4 \pm 0}{2}$$

$x_1 = \dfrac{4+0}{2} = \dfrac{4}{2} = 2.$

$x_2 = \dfrac{4-0}{2} = \dfrac{4}{2} = 2.$

$h(x) = \dfrac{x-2}{(x-2)^2}$

$h(x) = \dfrac{1}{x-2}$

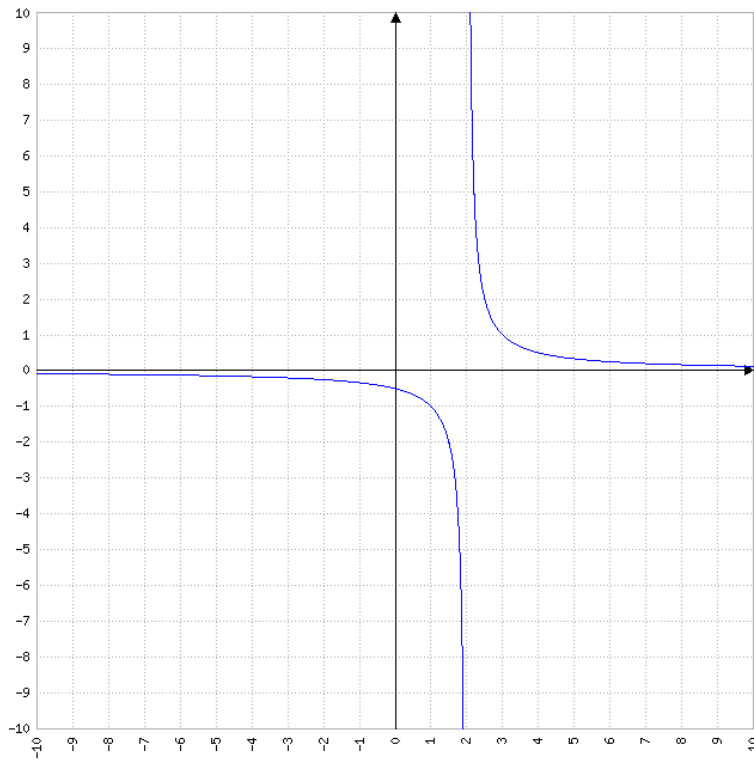$\displaystyle\lim_{x\to 2^-} h(x) = \lim_{x\to 2^-} \dfrac{1}{x-2} = \dfrac{1}{2-2} = \dfrac{1}{0} = -\infty.$

$\displaystyle\lim_{x\to 2^+} h(x) = \lim_{x\to 2^+} \dfrac{1}{x-2} = \dfrac{1}{2-2} = \dfrac{1}{0} = +\infty.$

Por lo tanto, h (x) tiene un asíntota vertical en x= 2.

$\displaystyle\lim_{x\to -\infty} h(x) = \lim_{x\to -\infty} \dfrac{1}{x-2} = 0.$

$\displaystyle\lim_{x\to +\infty} h(x) = \lim_{x\to +\infty} \dfrac{1}{x-2} = 0.$

Por lo tanto, h (x) tiene un asíntota horizontal en y= 0.

(d) $k(x) = \begin{cases} \dfrac{x}{x^2-1}, & si\ x < 2 \\ 3x^3 - 2x, & si\ x \geq 2 \end{cases}.$

$$\lim_{x\to-1^-} k\,(x) = \lim_{x\to-1^-} \frac{x}{x^2-1} = \frac{1}{(-1)^2-1} = \frac{1}{1-1} = \frac{1}{0} = -\infty.$$

$$\lim_{x\to-1^+} k\,(x) = \lim_{x\to-1^+} \frac{x}{x^2-1} = \frac{1}{(-1)^2-1} = \frac{1}{1-1} = \frac{1}{0} = +\infty.$$

$$\lim_{x\to1^-} k\,(x) = \lim_{x\to1^-} \frac{x}{x^2-1} = \frac{1}{1^2-1} = \frac{1}{1-1} = \frac{1}{0} = -\infty.$$

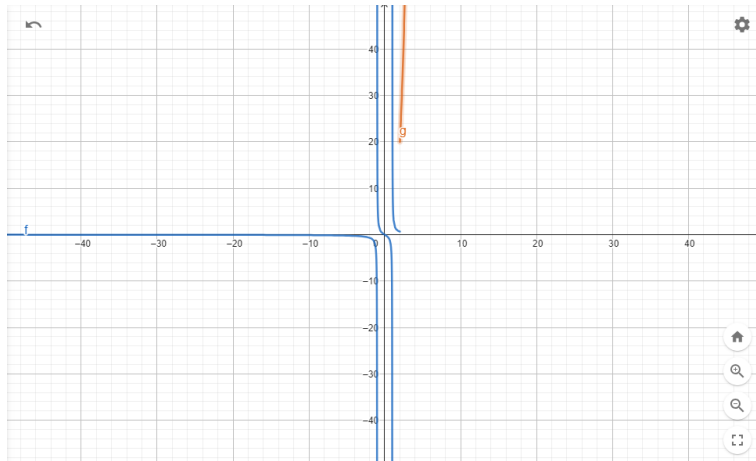$$\lim_{x\to1^+} k\,(x) = \lim_{x\to1^+} \frac{x}{x^2-1} = \frac{1}{1^2-1} = \frac{1}{1-1} = \frac{1}{0} = +\infty.$$

Por lo tanto, k (x) tiene asíntotas verticales en x= -1 y x= 1.

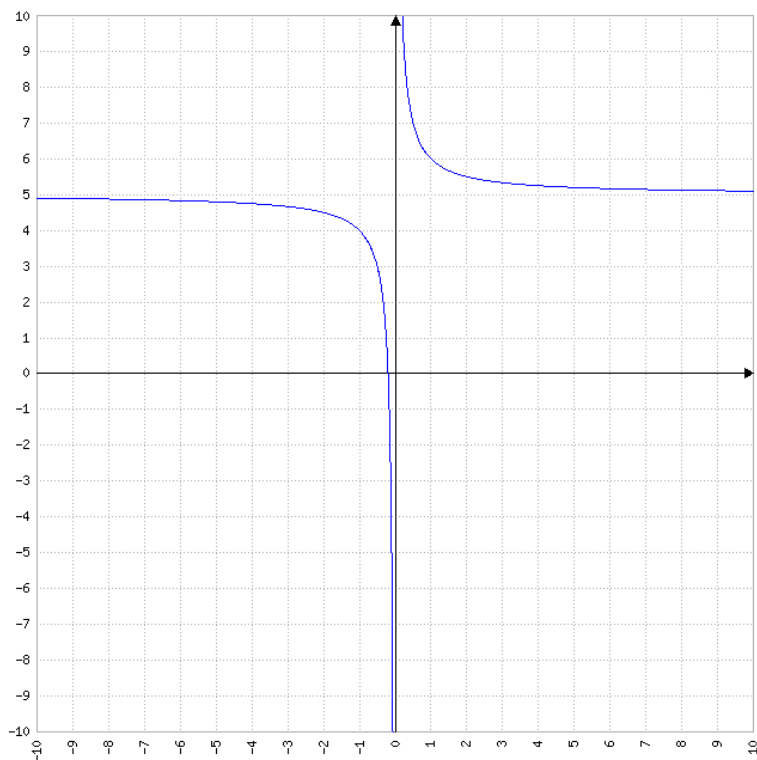$$\lim_{x\to-\infty} k\,(x) = \lim_{x\to-\infty} \frac{x}{x^2-1} = 0.$$

$$\lim_{x\to+\infty} k\,(x) = \lim_{x\to+\infty} 3x^3 - 2x = +\infty.$$

Por lo tanto, k (x) tiene un asíntota horizontal en y= 0.

Ejercicio 8.

Utilizar GeoGebra para verificar, gráficamente, el comportamiento de las funciones de los Ejemplos (11) y (15).

Ejemplo 11:

Ejemplo 15:

Juan Menduiña