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Trabajo Práctico N° 6: Integrales.

Ejercicio 1.

(a) *Si* $\int_0^9 f(x)\,dx$ *= 37 y* $\int_0^9 g(x)\,dx$ *= 16, encontrar el valor de* $\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx$.

$\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx = \int_0^9 2f(x)\,dx + \int_0^9 \frac{-1}{4}g(x)\,dx$

$\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx = 2\int_0^9 f(x)\,dx - \frac{1}{4}\int_0^9 g(x)\,dx$

$\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx = 2 * 37 - \frac{1}{4} * 16$

$\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx = 74 - 4$

$\int_0^9 2f(x) - \frac{1}{4}g(x)\,dx = 70.$

(b) *Si* $\int_{-2}^3 h(x)\,dx$ *= 12 y* $\int_0^3 h(x)\,dx$ *= 3, hallar el valor de* $\int_{-2}^0 h(x)\,dx$.

$\int_{-2}^0 h(x)\,dx = \int_{-2}^3 h(x)\,dx - \int_0^3 h(x)\,dx$

$\int_{-2}^0 h(x)\,dx = 12 - 3$

$\int_{-2}^0 h(x)\,dx = 9.$

(c) *Si* $\int_{-1}^3 f(t)\,dt$ *= 3 y* $\int_{-1}^4 f(t)\,dt$ *= 7, determinar el valor de* $\int_3^4 f(t)\,dt$.

$\int_3^4 f(t)\,dt = \int_{-1}^4 f(t)\,dt - \int_{-1}^3 f(t)\,dt$

$\int_3^4 f(t)\,dt = 7 - 3$

$\int_3^4 f(t)\,dt = 4.$

Ejercicio 2.

Calcular las siguientes integrales utilizando las propiedades y, en caso de ser posible, usando la regla de Barrow.

(a) $\int_{-2}^{3} 2x - 1\, dx$.

$\int_{-2}^{3} 2x - 1\, dx = \int_{-2}^{3} 2x\, dx + \int_{-2}^{3} -1\, dx$
$\int_{-2}^{3} 2x - 1\, dx = 2 \int_{-2}^{3} x\, dx - \int_{-2}^{3} 1\, dx$
$\int_{-2}^{3} 2x - 1\, dx = 2 \frac{x^2}{2} |_{-2}^{3} - x |_{-2}^{3}$
$\int_{-2}^{3} 2x - 1\, dx = x^2 |_{-2}^{3} - [3 - (-2)]$
$\int_{-2}^{3} 2x - 1\, dx = [3^2 - (-2)^2] - (3 + 2)$
$\int_{-2}^{3} 2x - 1\, dx = (9 - 4) - 5$
$\int_{-2}^{3} 2x - 1\, dx = 5 - 5$
$\int_{-2}^{3} 2x - 1\, dx = 0$.

(b) $\int x^2 + 2x + 8\, dx$.

$\int x^2 + 2x + 8\, dx = \int x^2\, dx + \int 2x\, dx + \int 8\, dx$
$\int x^2 + 2x + 8\, dx = \frac{x^3}{3} + 2 \int x\, dx + 8 \int 1\, dx$
$\int x^2 + 2x + 8\, dx = \frac{x^3}{3} + 2 \frac{x^2}{2} + 8x$
$\int x^2 + 2x + 8\, dx = \frac{x^3}{3} + x^2 + 8x + C$.

(c) $\int_{0}^{2\pi} sen\, x + x\, dx$.

$\int_{0}^{2\pi} sen\, x + x\, dx = \int_{0}^{2\pi} sen\, x\, dx + \int_{0}^{2\pi} x\, dx$
$\int_{0}^{2\pi} sen\, x + x\, dx = -\cos x |_{0}^{2\pi} + \frac{x^2}{2} |_{0}^{2\pi}$
$\int_{0}^{2\pi} sen\, x + x\, dx = -(\cos 2\pi - \cos 0) + \frac{1}{2} [(2\pi)^2 - 0^2]$
$\int_{0}^{2\pi} sen\, x + x\, dx = -(1 - 1) + \frac{1}{2} (4\pi^2 - 0)$
$\int_{0}^{2\pi} sen\, x + x\, dx = -0 + \frac{1}{2} 4\pi^2$
$\int_{0}^{2\pi} sen\, x + x\, dx = 2\pi^2$.

(d) $\int_{0}^{4} 2e^x + 3x^4\, dx$.

$\int_0^4 2e^x + 3x^4\,dx$= $\int_0^4 2e^x\,dx$ + $\int_0^4 3x^4\,dx$
$\int_0^4 2e^x + 3x^4\,dx$= 2 $\int_0^4 e^x\,dx$ + 3 $\int_0^4 x^4\,dx$
$\int_0^4 2e^x + 3x^4\,dx$= $2e^x$ $|_0^4$ + 3 $\frac{x^5}{5}$ $|_0^4$
$\int_0^4 2e^x + 3x^4\,dx$= 2 (e^4 - e^0) + $\frac{3}{5}$ (4^5 - 0^5)
$\int_0^4 2e^x + 3x^4\,dx$= 2 (e^4 - 1) + $\frac{3}{5}$ (1024 - 0)
$\int_0^4 2e^x + 3x^4\,dx$= $2e^4$ - 2 + $\frac{3}{5}$ * 1024
$\int_0^4 2e^x + 3x^4\,dx$= $2e^4$ - 2 + $\frac{3072}{5}$
$\int_0^4 2e^x + 3x^4\,dx$= $\frac{10e^4+3062}{5}$.

(e) $\int 3\frac{1}{x} + 2e^x\,dx$.

$\int 3\frac{1}{x} + 2e^x\,dx$= $\int 3\frac{1}{x}\,dx$ + $\int 2e^x\,dx$
$\int 3\frac{1}{x} + 2e^x\,dx$= 3 $\int \frac{1}{x}\,dx$ + 2 $\int e^x\,dx$
$\int 3\frac{1}{x} + 2e^x\,dx$= 3 ln $|x|$ + $2e^x$ + C.

(f) $\int \cos x + sen\ x + 2x^{\frac{3}{5}}\,dx$.

$\int \cos x + sen\ x + 2x^{\frac{3}{5}}\,dx$= $\int \cos x\ \,dx$ + $\int sen\ x\,dx$ + $\int 2x^{\frac{3}{5}}\,dx$
$\int \cos x + sen\ x + 2x^{\frac{3}{5}}\,dx$= sen x - cos x + 2 $\int x^{\frac{3}{5}}\,dx$
$\int \cos x + sen\ x + 2x^{\frac{3}{5}}\,dx$= sen x - cos x + 2 $\frac{x^{\frac{8}{5}}}{\frac{8}{5}}$
$\int \cos x + sen\ x + 2x^{\frac{3}{5}}\,dx$= sen x - cos x + $\frac{5}{4}$ $x^{\frac{8}{5}}$ + C.

(g) $\int_{-5}^1 x^2 + 2x + 8\,dx$.

$\int_{-5}^1 x^2 + 2x + 8\,dx$= $\int_{-5}^1 x^2\,dx$ + $\int_{-5}^1 2x\,dx$ + $\int_{-5}^1 8\,dx$
$\int_{-5}^1 x^2 + 2x + 8\,dx$= $\frac{x^3}{3}$ $|_{-5}^1$ + 2 $\int_{-5}^1 x\,dx$ + 8 $\int_{-5}^1 1\,dx$
$\int_{-5}^1 x^2 + 2x + 8\,dx$= $\frac{1}{3}$ [1^3 - $(-5)^3$] + 2 $\frac{x^2}{2}$ $|_{-5}^1$ + 8 x $|_{-5}^1$
$\int_{-5}^1 x^2 + 2x + 8\,dx$= $\frac{1}{3}$ [1 - (-125)] + [1^2 - $(-5)^2$] + 8 [1 - (-5)]
$\int_{-5}^1 x^2 + 2x + 8\,dx$= $\frac{1}{3}$ (1 + 125) + (1 - 25) + 8 (1 + 5)
$\int_{-5}^1 x^2 + 2x + 8\,dx$= $\frac{1}{3}$ * 126 - 24 + 8 * 6
$\int_{-5}^1 x^2 + 2x + 8\,dx$= $\frac{126}{3}$ - 24 + 48

$\int_{-5}^{1} x^2 + 2x + 8\ dx$= 66.

(h) $\int x - x^{\frac{2}{5}} + 3e^x - cos\, x\ \ dx.$

$\int x - x^{\frac{2}{5}} + 3e^x - cos\, x\ \ dx = \int x\ dx + \int -x^{\frac{2}{5}}\ dx + \int 3e^x\ dx + \int - cos\, x\ \ dx$
$\int x - x^{\frac{2}{5}} + 3e^x - cos\, x\ \ dx = \frac{x^2}{2} - \int x^{\frac{2}{5}}\ dx + 3 \int e^x\ dx - \int cos\, x\ \ dx$
$\int x - x^{\frac{2}{5}} + 3e^x - cos\, x\ \ dx = \frac{x^2}{2} - \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + 3e^x$ - sen x
$\int x - x^{\frac{2}{5}} + 3e^x - cos\, x\ \ dx = \frac{1}{2} x^2 - \frac{5}{7} x^{\frac{7}{5}} + 3e^x$ - sen x + C.

Ejercicio 3.

Calcular las siguientes integrales utilizando los métodos vistos.

(a) $\int (3x^4 + 5x^2 + 8)^4 (12x^3 + 10x)\, dx$.

$\int (3x^4 + 5x^2 + 8)^4 (12x^3 + 10x)\, dx = \int u^4\, du$ (*)
$\int (3x^4 + 5x^2 + 8)^4 (12x^3 + 10x)\, dx = \frac{u^5}{5}$
$\int (3x^4 + 5x^2 + 8)^4 (12x^3 + 10x)\, dx = \frac{(3x^4+5x^2+8)^5}{5} + C$.

(*) u= $3x^4 + 5x^2 + 8$; du= $(12x^3 + 10x)$ dx.

(b) $\int x \cos x\ dx$.

$\int x \cos x\ dx$= x sen x - $\int sen\ x\ dx$ (*)
$\int x \cos x\ dx$= x sen x - (-cos x)
$\int x \cos x\ dx$= x sen x + cos x + C.

(*) u= x; du= dx; dv= cos x dx; v= sen x.

(c) $\int x^3 \ln x\ dx$.

$\int x^3 \ln x\ dx$= ln x $\frac{x^4}{4}$ - $\int \frac{x^4}{4} \frac{1}{x}\ dx$ (*)
$\int x^3 \ln x\ dx$= $\frac{x^4}{4}$ ln x - $\frac{1}{4} \int x^3\ dx$
$\int x^3 \ln x\ dx$= $\frac{x^4}{4}$ ln x - $\frac{1}{4} \frac{x^4}{4}$
$\int x^3 \ln x\ dx$= $\frac{x^4}{4}$ ln x - $\frac{x^4}{16}$
$\int x^3 \ln x\ dx$= $\frac{x^4}{4}$ (ln x - $\frac{1}{4}$) + C.

(*) u= ln x; du= $\frac{1}{x}$ dx; dv= x^3 dx; v= $\frac{x^4}{4}$.

(d) $\int \cos 5x * 5\, dx$.

$\int \cos 5x * 5\, dx = \int \cos u\ du$ (*)
$\int \cos 5x * 5\, dx$= sen u
$\int \cos 5x * 5\, dx$= sen 5x + C.

(*) u= 5x; du= 5 dx.

(e) $\int \frac{2+e^x}{e^x+2x} dx$.

$\int \frac{2+e^x}{e^x+2x} dx= \int \frac{1}{u} du$ (*)
$\int \frac{2+e^x}{e^x+2x} dx= \ln |u|$
$\int \frac{2+e^x}{e^x+2x} dx= \ln |e^x + 2x|$ + C.

(*) u= e^x + 2x; du= (e^x + 2) dx.

(f) $\int x\sqrt{x-1}\, dx$.

(*) u= $\sqrt{x-1}$; du= $\frac{1}{2\sqrt{x-1}}$ dx.

$\int x\sqrt{x-1}\, dx= \int (u+1)\sqrt{u}\, du$ (*)
$\int x\sqrt{x-1}\, dx= \int u^{\frac{3}{2}} + u^{\frac{1}{2}}\, du$
$\int x\sqrt{x-1}\, dx= \int u^{\frac{3}{2}}\, du + \int u^{\frac{1}{2}}\, du$
$\int x\sqrt{x-1}\, dx= \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}}$
$\int x\sqrt{x-1}\, dx= \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}}$
$\int x\sqrt{x-1}\, dx= 2\,[\frac{(x-1)^{\frac{5}{2}}}{5} + \frac{(x-1)^{\frac{3}{2}}}{3}]$ + C.

(*) u= x - 1; du= dx.

(g) $\int_0^8 \frac{1}{\sqrt{x+1}} dx$.

$\int_0^8 \frac{1}{\sqrt{x+1}} dx= \int_0^8 \frac{1}{\sqrt{u}} dx$ (*)
$\int_0^8 \frac{1}{\sqrt{x+1}} dx= \int_{0+1}^{8+1} u^{\frac{-1}{2}}\, du$
$\int_0^8 \frac{1}{\sqrt{x+1}} dx= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \Big|_{0+1}^{8+1}$
$\int_0^8 \frac{1}{\sqrt{x+1}} dx= 2\,(x+1)^{\frac{1}{2}} \Big|_0^8$
$\int_0^8 \frac{1}{\sqrt{x+1}} dx= 2\,\sqrt{x+1} \Big|_0^8$
$\int_0^8 \frac{1}{\sqrt{x+1}} dx= 2\,(\sqrt{8+1} - \sqrt{0+1})$
$\int_0^8 \frac{1}{\sqrt{x+1}} dx= 2\,(\sqrt{9} - \sqrt{1})$

$\int_0^8 \frac{1}{\sqrt{x+1}} dx$= 2 (3 - 1)
$\int_0^8 \frac{1}{\sqrt{x+1}} dx$= 2 * 2
$\int_0^8 \frac{1}{\sqrt{x+1}} dx$= 4.

(*) u= x + 1; du= dx.

(h) $\int_0^{2\pi} x\ sen\ x\ dx.$

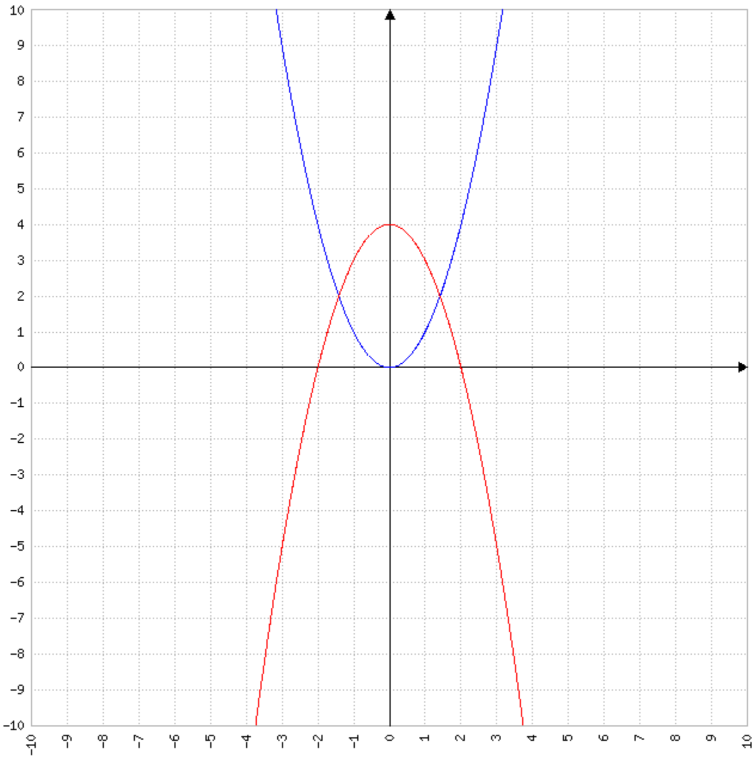
$\int_0^{2\pi} x\ sen\ x\ dx$= x (-cos x) $|_0^{2\pi}$ - $\int_0^{2\pi} -\cos x\ \ dx$ (*)
$\int_0^{2\pi} x\ sen\ x\ dx$= -x cos x $|_0^{2\pi}$ + $\int_0^{2\pi} \cos x\ \ dx$
$\int_0^{2\pi} x\ sen\ x\ dx$= -(2$\pi$ cos 2π - 0 cos 0) + sen x $|_0^{2\pi}$
$\int_0^{2\pi} x\ sen\ x\ dx$= -(2$\pi$ * 1 - 0 * 1) + (sen 2π - sen 0)
$\int_0^{2\pi} x\ sen\ x\ dx$= -(2$\pi$ - 0) + (0 - 0)
$\int_0^{2\pi} x\ sen\ x\ dx$= -2$\pi$ + 0
$\int_0^{2\pi} x\ sen\ x\ dx$= -2$\pi$.

(*) u= x; du= dx; dv= sen x dx; v= -cos x.

Ejercicio 4.

Hallar el área comprendida entre las gráficas de las siguientes pares de funciones:

(a) *f (x)=* x^2 *y g (x)=* -$x^2 + 4$.



f (x)= g (x)

x^2= -x^2 + 4

$x^2 + x^2$= 4

$2x^2$= 4

x^2= $\frac{4}{2}$

x^2= 2

$\sqrt{x^2}$= $\sqrt{2}$

$|x|$= $\sqrt{2}$

x= $\pm$ $\sqrt{2}$.

Intervalo	**(-$\sqrt{2}$, $\sqrt{2}$)**
VP	0
f (x)	0
g (x)	4

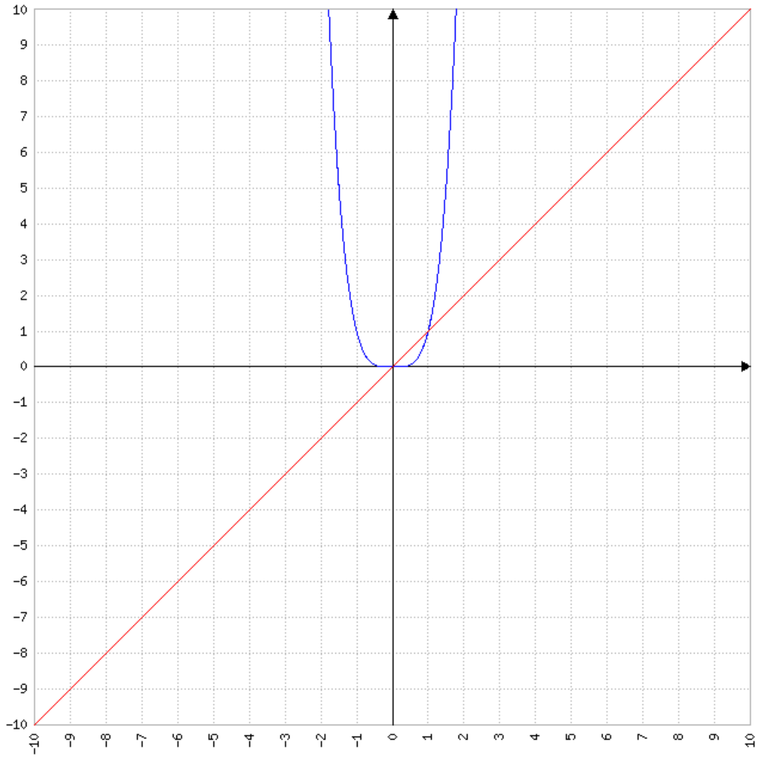
A= $\int_{-\sqrt{2}}^{\sqrt{2}} g\ (x) - f\ (x)\ dx$

A= $\int_{-\sqrt{2}}^{\sqrt{2}} (-x^2 + 4) - x^2\ dx$

A= $\int_{-\sqrt{2}}^{\sqrt{2}} -x^2 + 4 - x^2\ dx$

A= $\int_{-\sqrt{2}}^{\sqrt{2}} -2x^2 + 4\, dx$
A= $\int_{-\sqrt{2}}^{\sqrt{2}} -2x^2\, dx + \int_{-\sqrt{2}}^{\sqrt{2}} 4\, dx$
A= -2 $\int_{-\sqrt{2}}^{\sqrt{2}} x^2\, dx$ + 4 $\int_{-\sqrt{2}}^{\sqrt{2}} 1\, dx$
A= -2 $\frac{x^3}{3} |_{-\sqrt{2}}^{\sqrt{2}}$ + 4x $|_{-\sqrt{2}}^{\sqrt{2}}$
A= $\frac{-2}{3}$ $[(\sqrt{2})^3$ - $(-\sqrt{2})^3]$ + 4 [$\sqrt{2}$ - (-$\sqrt{2}$)]
A= $\frac{-2}{3}$ $[(\sqrt{2})^3 + (\sqrt{2})^3]$ + 4 ($\sqrt{2} + \sqrt{2}$)
A= $\frac{-2}{3}$ * 2 $(\sqrt{2})^3$ + 4 * 2 $\sqrt{2}$
A= $\frac{-4}{3}$ $(\sqrt{2})^2$ $\sqrt{2}$ + 8 $\sqrt{2}$
A= $\frac{-4}{3}$ * 2 $\sqrt{2}$ + 8 $\sqrt{2}$
A= $\frac{-8}{3}$ $\sqrt{2}$ + 8 $\sqrt{2}$
A= $\frac{16}{3}$ $\sqrt{2}$.

(b) *f (x)=* x^4 *y g (x)= x.*



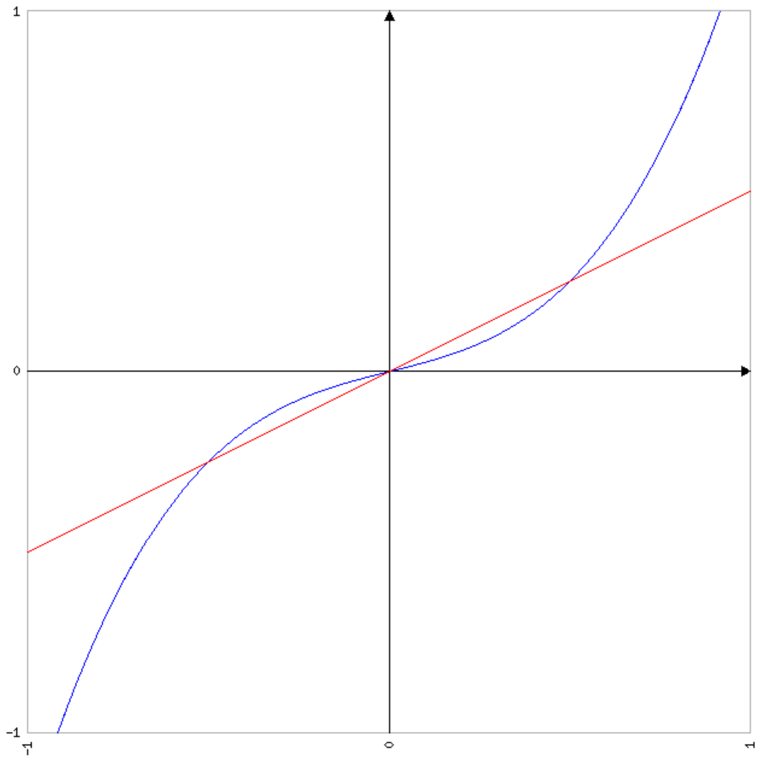
f (x)= g (x)
x^4= x
x^4 - x= 0
x^3 (x - 1)= 0.

x_1= 0; x_2= 1.

Intervalo	(0, 1)
VP	$\frac{1}{2}$
f (x)	$\frac{1}{16}$
g (x)	$\frac{1}{2}$

A= $\int_0^1 g\ (x) - f\ (x)\ dx$
A= $\int_0^1 x - x^4\ dx$
A= $\int_0^1 x\ dx + \int_0^1 -x^4\ dx$
A= $\frac{x^2}{2}\ |_0^1$ - $\int_0^1 x^4\ dx$
A= $\frac{1}{2}$ $(1^2$ - $0^2)$ - $\frac{x^5}{5}\ |_0^1$
A= $\frac{1}{2}$ (1 - 0) - $\frac{1}{5}$ $(1^5$ - $0^5)$
A= $\frac{1}{2}$ * 1 - $\frac{1}{5}$ (1 - 0)
A= $\frac{1}{2}$ - $\frac{1}{5}$ * 1
A= $\frac{1}{2}$ - $\frac{1}{5}$
A= $\frac{3}{10}$.

(c) *f (x)=* $x^3 + \frac{1}{4}x$ *y g (x)=* $\frac{1}{2}x$.



f (x)= g (x)
$x^3 + \frac{1}{4}$ x= $\frac{1}{2}$ x
$x^3 + \frac{1}{4}$ x - $\frac{1}{2}$ x= 0
x^3 - $\frac{1}{4}$ x= 0
x (x^2 - $\frac{1}{4}$)= 0.

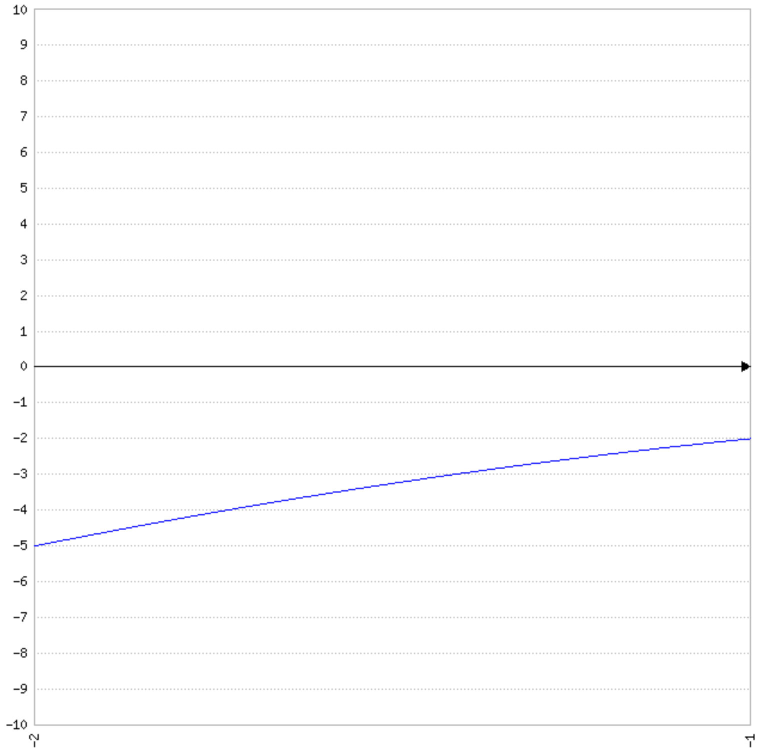
x_1= 0; x_2= $\frac{-1}{2}$; x_3= $\frac{1}{2}$.

Intervalo	**($\frac{-1}{2}$, 0)**	**(0, $\frac{1}{2}$)**
VP	$\frac{-1}{4}$	$\frac{1}{4}$
f (x)	$\frac{-5}{64}$	$\frac{5}{64}$
g (x)	$\frac{-1}{8}$	$\frac{1}{8}$

A= $\int_{\frac{-1}{2}}^{0} f\ (x) - g\ (x)\ dx$ + $\int_{0}^{\frac{1}{2}} g\ (x) - f\ (x)\ dx$
A= $\int_{\frac{-1}{2}}^{0} (x^3 + \frac{1}{4}x) - \frac{1}{2}x\ dx$ + $\int_{0}^{\frac{1}{2}} \frac{1}{2}x - (x^3 + \frac{1}{4}x)\ dx$
A= $\int_{\frac{-1}{2}}^{0} x^3 + \frac{1}{4}x - \frac{1}{2}x\ dx$ + $\int_{0}^{\frac{1}{2}} \frac{1}{2}x - x^3 - \frac{1}{4}x\ dx$
A= $\int_{\frac{-1}{2}}^{0} x^3 - \frac{1}{4}x\ dx$ + $\int_{0}^{\frac{1}{2}} -x^3 + \frac{1}{4}x\ dx$
A= $\int_{\frac{-1}{2}}^{0} x^3\ dx$ + $\int_{\frac{-1}{2}}^{0} \frac{-1}{4}x\ dx$ + $\int_{0}^{\frac{1}{2}} -x^3\ dx$ + $\int_{0}^{\frac{1}{2}} \frac{1}{4}x\ dx$
A= $\int_{\frac{-1}{2}}^{0} x^3\ dx$ - $\frac{1}{4}$ $\int_{\frac{-1}{2}}^{0} x\ dx$ - $\int_{0}^{\frac{1}{2}} x^3\ dx$ + $\frac{1}{4}$ $\int_{0}^{\frac{1}{2}} x\ dx$
A= $\frac{x^4}{4} \Big|_{\frac{-1}{2}}^{0}$ - $\frac{1}{4}\frac{x^2}{2} \Big|_{\frac{-1}{2}}^{0}$ - $\frac{x^4}{4} \Big|_{0}^{\frac{1}{2}}$ + $\frac{1}{4}\frac{x^2}{2} \Big|_{0}^{\frac{1}{2}}$
A= $\frac{1}{4}$ [0^4 - $(\frac{-1}{2})^4$] - $\frac{1}{8}$ [0^2 - $(\frac{-1}{2})^2$] - $\frac{1}{4}$ [$(\frac{1}{2})^4$ - 0^4] + $\frac{1}{8}$ [$(\frac{1}{2})^2$ - 0^2]
A= $\frac{1}{4}$ (0 - $\frac{1}{16}$) - $\frac{1}{8}$ (0 - $\frac{1}{4}$) - $\frac{1}{4}$ ($\frac{1}{16}$ - 0) + $\frac{1}{8}$ ($\frac{1}{4}$ - 0)
A= $\frac{1}{4}$ ($\frac{-1}{16}$) - $\frac{1}{8}$ ($\frac{-1}{4}$) - $\frac{1}{4}\frac{1}{16}$ + $\frac{1}{8}\frac{1}{4}$
A= $\frac{-1}{64}$ + $\frac{1}{32}$ - $\frac{1}{64}$ + $\frac{1}{32}$
A= $\frac{1}{32}$.

Ejercicio 5.

Calcular el área de la región comprendida entre el eje x y el gráfico de la función f (x)= -x^2 - 1 entre -2 ≤ x ≤ -1.



f (x)= 0
-x^2 - 1= 0
x^2 ≠ -1.

Intervalo	(-2, -1)
VP	$\frac{-3}{2}$
f (x)	< 0

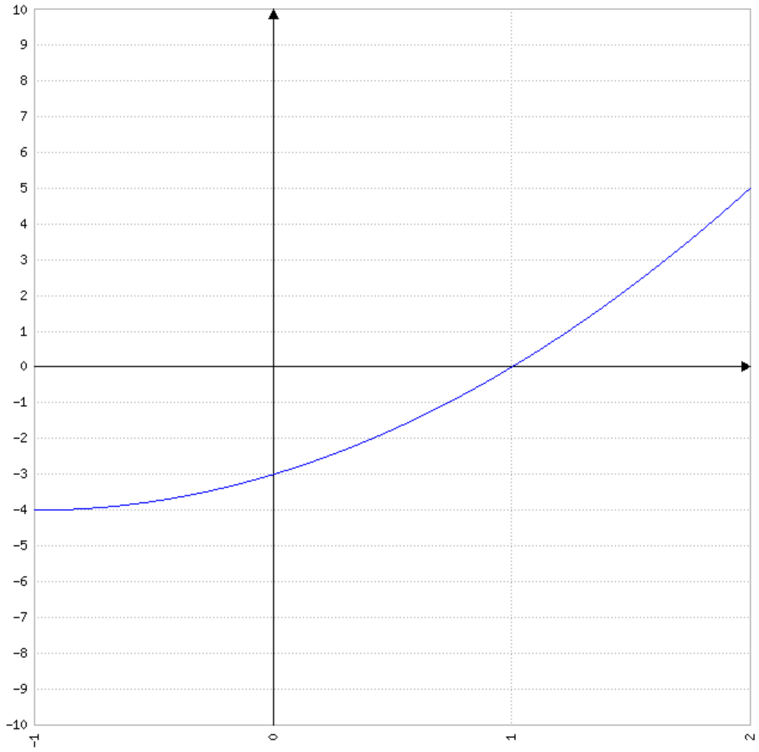
A= -$\int_{-2}^{-1} -x^2 - 1\, dx$
A= -($\int_{-2}^{-1} -x^2\, dx$ + $\int_{-2}^{-1} -1\, dx$)
A= -(-$\int_{-2}^{-1} x^2\, dx$ - $\int_{-2}^{-1} 1\, dx$)
A= -($\frac{-x^3}{3} |_{-2}^{-1}$ - x $|_{-2}^{-1}$)
A= -{$\frac{-1}{3}$ [$(-1)^3$ - $(-2)^3$] - [-1 - (-2)]}
A= -{$\frac{-1}{3}$ [-1 - (-8)] - (-1 + 2)}
A= -[$\frac{-1}{3}$ (-1 + 8) - 1]
A= -($\frac{-1}{3}$ * 7 - 1)
A= -($\frac{-7}{3}$ - 1)

A= -($\frac{-10}{3}$)

A= $\frac{10}{3}$.

Ejercicio 6.

Calcular el área de la región comprendida entre el eje x y el gráfico de la función f (x)= x^2 + 2x - 3 entre -1 ≤ x ≤ 2.



f (x)= 0
x^2 + 2x - 3= 0.

x_1, x_2= $\frac{-2\pm\sqrt{2^2-4*1(-3)}}{2*1}$
x_1, x_2= $\frac{-2\pm\sqrt{4+12}}{2}$
x_1, x_2= $\frac{-2\pm\sqrt{16}}{2}$
x_1, x_2= $\frac{-2\pm 4}{2}$
x_1= $\frac{-2+4}{2}$= $\frac{2}{2}$= 1.
x_2= $\frac{-2-4}{2}$= $\frac{-6}{2}$= -3.

Intervalo	(-1, 1)	(1, 2)
VP	0	$\frac{3}{2}$
f (x)	< 0	> 0

A= $-\int_{-1}^{1} x^2 + 2x - 3\, dx + \int_{1}^{2} x^2 + 2x - 3\, dx$
A= $-(\int_{-1}^{1} x^2\, dx + \int_{-1}^{1} 2x\, dx + \int_{-1}^{1} -3\, dx) + (\int_{1}^{2} x^2\, dx + \int_{1}^{2} 2x\, dx + \int_{1}^{2} -3\, dx)$
A= $-(\frac{x^3}{3}\, |_{-1}^{1} + 2\int_{-1}^{1} x\, dx - 3\int_{-1}^{1} 1\, dx) + (\frac{x^3}{3}\, |_{1}^{2} + 2\int_{1}^{2} x\, dx - 3\int_{1}^{2} 1\, dx)$

A= $-\{\frac{1}{3}\ [1^3 - (-1)^3] + 2\ \frac{x^2}{2}\ |_{-1}^{1} - 3x\ |_{-1}^{1}\} + [\frac{1}{3}\ (2^3 - 1^3) + 2\ \frac{x^2}{2}\ |_{1}^{2} - 3x\ |_{1}^{2}]$

A= $-\{\frac{1}{3}\ [1 - (-1)] + [1^2 - (-1)^2] - 3\ [1 - (-1)]\} + [\frac{1}{3}\ (8 - 1) + (2^2 - 1^2) - 3\ (2 - 1)]$

A= $-[\frac{1}{3}\ (1 + 1) + (1 - 1) - 3\ (1 + 1)] + [\frac{1}{3} * 7 + (4 - 1) - 3 * 1]$

A= $-(\frac{1}{3} * 2 + 0 - 3 * 2) + (\frac{7}{3} + 3 - 3)$

A= $-(\frac{2}{3} + 0 - 6) + \frac{7}{3}$

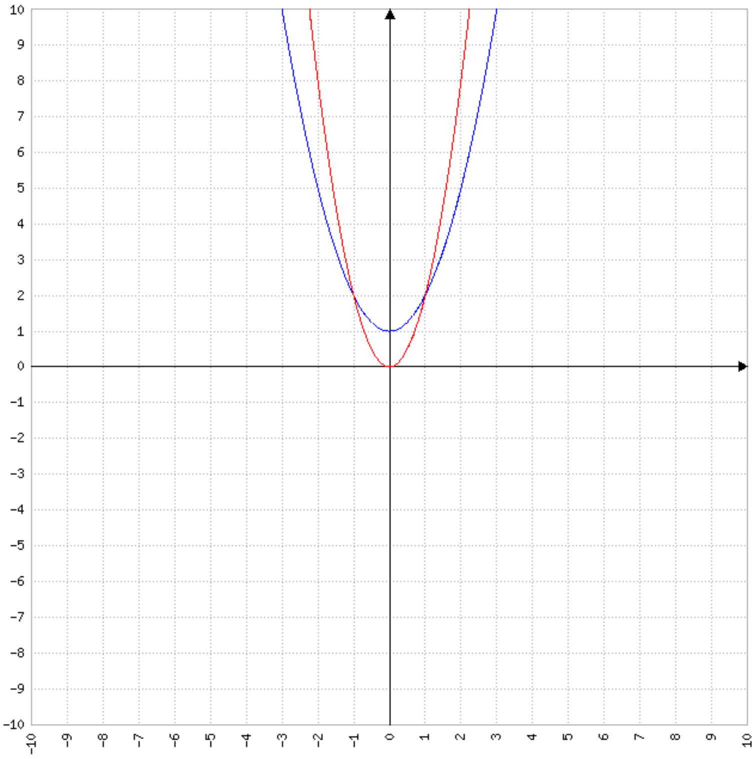
A= $-(\frac{-16}{3}) + \frac{7}{3}$

A= $\frac{16}{3} + \frac{7}{3}$

A= $\frac{23}{3}$.

Ejercicio 7.

Calcular el área de la región comprendida entre los gráficos de f (x)= x^2 + 1 y g (x)= $2x^2$ para $0 \leq x \leq 2$.



f (x)= g (x)
$x^2 + 1 = 2x^2$
$2x^2 - x^2 = 1$
$x^2 = 1$
$\sqrt{x^2} = \sqrt{1}$
$|x| = 1$
x= ± 1.

Intervalo	(-1, 1)
VP	0
f (x)	1
g (x)	0

A= $\int_{-1}^{1} (x^2 + 1) - 2x^2 \, dx$
A= $\int_{-1}^{1} x^2 + 1 - 2x^2 \, dx$
A= $\int_{-1}^{1} -x^2 + 1 \, dx$
A= $\int_{-1}^{1} -x^2 \, dx + \int_{-1}^{1} 1 \, dx$
A= $-\int_{-1}^{1} x^2 \, dx + x \,|_{-1}^{1}$
A= $\frac{-x^3}{3} \,|_{-1}^{1}$ + [1 - (-1)]

A= $\frac{-1}{3}$ [1^3 - $(-1)^3$] + (1 + 1)

A= $\frac{-1}{3}$ [1 - (-1)] + 2

A= $\frac{-1}{3}$ (1 + 1) + 2

A= $\frac{-1}{3}$ * 2 + 2

A= $\frac{-2}{3}$ + 2

A= $\frac{4}{3}$.

Ejercicio 8.

Hallar f (x) sabiendo que f´ (x)= $x + \frac{1}{x^2}$ *y f (1)= 1.*

f (x)= $\int f'(x)\,dx$
f (x)= $\int x + \frac{1}{x^2}\,dx$
f (x)= $\int x\,dx$ + $\int \frac{1}{x^2}\,dx$
f (x)= $\frac{x^2}{2}$ + $\int x^{-2}\,dx$
f (x)= $\frac{x^2}{2}$ + $\frac{x^{-1}}{-1}$
f (x)= $\frac{1}{2}x^2$ - $\frac{1}{x}$ + C.

f (1)= 1
$\frac{1}{2} * 1^2$ - $\frac{1}{1}$ + C= 1
$\frac{1}{2}$ * 1 - 1 + C= 1
$\frac{1}{2}$ - 1 + C= 1
$\frac{-1}{2}$ + C= 1
C= 1 + $\frac{1}{2}$
C= $\frac{3}{2}$.

f (x)= $\frac{1}{2}x^2$ - $\frac{1}{x}$ + $\frac{3}{2}$.

Ejercicio 9.

Sabiendo que f´ (x)= $3x^2$ - 8x + 2 y, además, que f (3)= -4, hallar la función f (x).

f (x)= $\int f'(x)\,dx$
f (x)= $\int 3x^2 - 8x + 2\,dx$
f (x)= $\int 3x^2\,dx$ + $\int -8x\,dx$ + $\int 2\,dx$
f (x)= 3 $\int x^2\,dx$ - 8 $\int x\,dx$ + 2 $\int 1\,dx$
f (x)= 3 $\frac{x^3}{3}$ - 8 $\frac{x^2}{2}$ + 2x
f (x)= x^3 - 4x^2 + 2x + C.

f (3)= -4
3^3 - 4 * 3^2 + 2 * 3 + C= -4
27 - 4 * 9 + 6 + C= -4
27 - 36 + 6 + C= -4
-3 + C= -4
C= -4 + 3
C= -1.

f (x)= x^3 - 4x^2 + 2x - 1.

Ejercicio 10.

Hallar todas las funciones cuya derivada es g´ (x)= x^2 *cos x.*

g (x)= $\int g´(x)\,dx$
g (x)= $\int x^2 \cos x\, dx$
g (x)= x^2 sen x - $\int sen\, x * 2x\, dx$ (*)
g (x)= x^2 sen x - 2 $\int x\, sen\, x\, dx$
g (x)= x^2 sen x - 2 [x (-cos x) - $\int -\cos x\, dx$] (**)
g (x)= x^2 sen x - 2 (-x cos x + $\int \cos x\, dx$)
g (x)= x^2 sen x - 2 (-x cos x + sen x)
g (x)= x^2 sen x - 2x cos x - 2 sen x
g (x)= (x^2 - 2) sen x - 2x cos x + C.

(*) u= x^2; du= 2x dx; dv= cos x dx; v= sen x.
(**) u= x; du= dx; dv= sen x; v= -cos x.

Ejercicio 11.

Sea g´´ (x)= 2x^3 - 4x^7, g´ (1)= -2 y g (0)= 0, hallar la función g (x).

g´ (x)= $\int g´´ (x)\ dx$
g´ (x)= $\int 2x^3 - 4x^7\ dx$
g´ (x)= $\int 2x^3\ dx$ + $\int -4x^7\ dx$
g´ (x)= 2 $\int x^3\ dx$ - 4 $\int x^7\ dx$
g´ (x)= 2 $\frac{x^4}{4}$ - 4 $\frac{x^8}{8}$
g´ (x)= $\frac{1}{2}$ x^4 - $\frac{1}{2}$ x^8
g´ (x)= $\frac{1}{2}$ (x^4 - x^8) + C.

g´ (1)= -2
$\frac{1}{2}$ (1^4 - 1^8) + C= -2
$\frac{1}{2}$ (1 - 1) + C= -2
$\frac{1}{2}$ * 0 + C= -2
0 + C= -2
C= -2.

g´ (x)= $\frac{1}{2}$ (x^4 - x^8) - 2.

g (x)= $\int g´ (x)\ dx$
g (x)= $\int \frac{1}{2}(x^4 - x^8) - 2\ dx$
g (x)= $\int \frac{1}{2}x^4 - \frac{1}{2}x^8 - 2\ dx$
g (x)= $\int \frac{1}{2}x^4\ dx$ + $\int \frac{-1}{2}x^8\ dx$ + $\int -2\ dx$
g (x)= $\frac{1}{2}$ $\int x^4\ dx$ - $\frac{1}{2}$ $\int x^8\ dx$ - 2 $\int 1\ dx$
g (x)= $\frac{1}{2}\frac{x^5}{5}$ - $\frac{1}{2}\frac{x^9}{9}$ - 2x
g (x)= $\frac{1}{10}$ x^5 - $\frac{1}{18}$ x^9 - 2x + C.

g (0)= 0
$\frac{1}{10}$ 0^5 - $\frac{1}{18}$ 0^9 - 2 * 0 + C= 0
$\frac{1}{10}$ * 0 - $\frac{1}{18}$ * 0 - 0 + C= 0
0 - 0 - 0 + C= 0
C= 0.

g (x)= $\frac{1}{10}$ x^5 - $\frac{1}{18}$ x^9 - 2x.