

Trabajo Práctico N° 3:
Expresiones Algebraicas. Ecuaciones, Sistemas de Ecuaciones Lineales y Mixtos.

Ejercicio 1.

Indicar, en cada caso, cuál/cuáles expresiones algebraicas es/son equivalentes a la dada (justificar).

(a) $\frac{2x}{2+x}$.

(i) $\frac{x}{1+x}$.

(ii) $\frac{2}{3}$.

(iii) $\frac{2x}{2+x}$. Esta expresión es equivalente a la dada.

(b) $x^2 x^n$.

(i) x^{2-n} .

(ii) x^{2+n} . Esta expresión es equivalente a la dada.

(iii) $(x^2)^n$.

(c) $\frac{h^n}{h^2}$.

(i) $h^{\frac{n}{2}}$.

(ii) h^{2-n} .

(iii) h^{n-2} . Esta expresión es equivalente a la dada.

(d) $x^2 - x^2 x * 2 + 2$.

(i) $-x^2 + 1$.

(ii) $x^{(2+2)} + 2$.

(iii) $-x^2 + 2$.

(e) $\frac{2}{x^2-5x} + \frac{1}{x-5}$.

(i) $\frac{3}{x^2-5x}$.

(ii) $\frac{2+x}{x(x-5)}$. Esta expresión es equivalente a la dada.

(iii) $\frac{3}{x-5}$.

(f) $3xy^2 - x^2y + 5y(xy)$.

(i) $3(xy)^2 + 5xy + 5y^2$.

(ii) $7xy$.

(iii) $8xy^2 - yx^2$. Esta expresión es equivalente a la dada.

Ejercicio 2.

Resolver justificando cada paso:

(a) $10 - 3x = x - 2$.

$$10 - 3x = x - 2$$

$$x + 3x = 10 + 2$$

$$4x = 12$$

$$x = \frac{12}{4}$$

$$x = 3.$$

(b) $a - x = 3(x - a)$.

$$a - x = 3(x - a)$$

$$a - x = 3x - 3a$$

$$3x + x = a + 3a$$

$$4x = 4a$$

$$x = \frac{4a}{4}$$

$$x = a.$$

(c) $3(2 - x) + 1 = -x + \frac{5}{2}(1 - x) + \frac{x+3}{2}$.

$$3(2 - x) + 1 = -x + \frac{5}{2}(1 - x) + \frac{x+3}{2}$$

$$6 - 3x + 1 = -x + \frac{5}{2} - \frac{5}{2}x + \frac{1}{2}x + \frac{3}{2}$$

$$-3x + 7 = -3x + 4$$

$$7 \neq 4.$$

(d) $\frac{1}{3}x - x = \frac{1}{4}x + 1$.

$$\frac{1}{3}x - x = \frac{1}{4}x + 1$$

$$\frac{1}{3}x - x - \frac{1}{4}x = 1$$

$$\frac{11}{12}x = 1$$

$$x = \frac{1}{\frac{11}{12}}$$

$$x = \frac{12}{11}$$

$$x = 1,0\overline{9}.$$

(e) $5x + 2 = 8x - \frac{1}{2} - 3x.$

$$5x + 2 = 8x - \frac{1}{2} - 3x$$

$$5x - 8x + 3x = \frac{-1}{2} - 2$$

$$0 \neq \frac{-5}{2}$$

Ejercicio 3.

Resolver las ecuaciones e indicar el conjunto numérico al que pertenecen.

(a) $10 = x - 2$.

$$\begin{aligned}10 &= x - 2 \\x &= 10 + 2 \\x &= 12 \in \mathbb{N}.\end{aligned}$$

(b) $x = 3(x - 5)$.

$$\begin{aligned}x &= 3(x - 5) \\x &= 3x - 15 \\3x - x &= 15 \\2x &= 15 \\x &= \frac{15}{2} \\x &= 7,5 \in \mathbb{Q}.\end{aligned}$$

(c) $\frac{3}{2}x - \frac{1}{2} = -x + \frac{5}{2}$.

$$\begin{aligned}\frac{3}{2}x - \frac{1}{2} &= -x + \frac{5}{2} \\\frac{3}{2}x + x &= \frac{5}{2} + \frac{1}{2} \\\frac{5}{2}x &= 6 \\x &= \frac{6}{\frac{5}{2}} \\x &= \frac{12}{5} \\x &= 2,4 \in \mathbb{Q}.\end{aligned}$$

(d) $\sqrt{5} - \frac{1}{3}x = \frac{1}{4}x + 1$.

$$\begin{aligned}\sqrt{5} - \frac{1}{3}x &= \frac{1}{4}x + 1 \\\frac{1}{4}x + \frac{1}{3}x &= \sqrt{5} - 1 \\\frac{7}{12}x &= 1,24 \\x &= \frac{1,24}{\frac{7}{12}} \\x &= 2,118973 \dots \in I.\end{aligned}$$

$$(e) 5\pi x + 2\pi = 8x - \frac{5}{2}$$

$$5\pi x + 2\pi = 8x - \frac{5}{2}$$

$$5\pi x - 8x = \frac{-5}{2}\pi$$

$$x(5\pi - 8) = \frac{-5}{2}\pi$$

$$x = \frac{\frac{-5}{2}\pi}{5\pi - 8}$$

$$x = -1,018943 \dots \in I.$$

$$(f) x + 3 - \frac{2}{3}(x - 1) = \frac{1}{3}(x + 5) + 2.$$

$$x + 3 - \frac{2}{3}(x - 1) = \frac{1}{3}(x + 5) + 2$$

$$x + 3 - \frac{2}{3}x + \frac{2}{3} = \frac{1}{3}x + \frac{5}{3} + 2$$

$$x - \frac{2}{3}x - \frac{1}{3}x = \frac{5}{3} + 2 - 3 - \frac{2}{3}$$

$$0 = 0 \in \mathbb{N}.$$

$$(g) 3(2 - x) + 1 = -x + \frac{5}{2}(1 - x) + \frac{1}{2}(x + 3).$$

$$3(2 - x) + 1 = -x + \frac{5}{2}(1 - x) + \frac{1}{2}(x + 3)$$

$$6 - 3x + 1 = -x + \frac{5}{2} - \frac{5}{2}x + \frac{1}{2}x + \frac{3}{2}$$

$$-3x + x + \frac{5}{2}x - \frac{1}{2}x = \frac{5}{2} + \frac{3}{2} - 6 - 1$$

$$\frac{1}{2}x = -3$$

$$x = \frac{-3}{\frac{1}{2}}$$

$$x = -6 \in \mathbb{Z}.$$

Ejercicio 4.

Resolver:

$$(a) \begin{cases} 3x - y = \frac{1}{2} \Leftrightarrow y = 3x - \frac{1}{2} \\ 2x - 3y = \frac{-5}{6} \Leftrightarrow y = \frac{2}{3}x + \frac{5}{18} \end{cases}$$

$$\begin{aligned} 3x - \frac{1}{2} &= \frac{2}{3}x + \frac{5}{18} \\ 3x - \frac{2}{3}x &= \frac{5}{18} + \frac{1}{2} \\ \frac{7}{3}x &= \frac{7}{9} \\ x &= \frac{9}{7} \\ x^* &= \frac{1}{3}. \end{aligned}$$

$$\begin{aligned} y^* &= 3 \frac{1}{3} - \frac{1}{2} = 1 - \frac{1}{2} = \frac{1}{2}, \\ y^* &= \frac{2}{3} \frac{1}{3} + \frac{5}{18} = \frac{2}{9} + \frac{5}{18} = \frac{1}{2}. \end{aligned}$$

Por lo tanto, existe una única solución: $x^* = \frac{1}{3}$, $y^* = \frac{1}{2}$.

$$(b) \begin{cases} 2x + y = 4 \Leftrightarrow y = -2x + 4 \\ 4x + 2y = 5 \Leftrightarrow y = -2x + \frac{5}{2} \end{cases}$$

$$\begin{aligned} -2x + 4 &= -2x + \frac{5}{2} \\ 4 &\neq \frac{5}{2}. \end{aligned}$$

Por lo tanto, no existe solución.

$$(c) \begin{cases} x = 2y \Leftrightarrow y = \frac{1}{2}x \\ \frac{x}{6} - \frac{y}{3} = 0 \Leftrightarrow y = \frac{1}{2}x \end{cases}$$

Por lo tanto, existen infinitas soluciones, en donde se cumpla $y^* = \frac{1}{2}x^*$.

$$(d) \begin{cases} x - y = 1 \Leftrightarrow x = y + 1 \\ y + z = 1 \Leftrightarrow y = z + 1 \\ z - 3x = 1 \Leftrightarrow z = 3x + 1 \end{cases}$$

$$\begin{aligned}y &= 3x + 1 + 1 \\y &= 3x + 2.\end{aligned}$$

$$\begin{aligned}x &= 3x + 2 + 1 \\x &= 3x + 3 \\3x - x &= -3 \\x^* &= \frac{-3}{2}.\end{aligned}$$

$$\begin{aligned}y &= 3\left(\frac{-3}{2}\right) + 2 \\y &= \frac{-9}{2} + 2 \\y^* &= \frac{-5}{2}.\end{aligned}$$

$$\begin{aligned}z &= 3\left(\frac{-3}{2}\right) + 1 \\z &= \frac{-9}{2} + 1 \\z^* &= \frac{-7}{2}.\end{aligned}$$

Por lo tanto, existe una única solución: $x^* = \frac{-3}{2}$, $y^* = \frac{-5}{2}$, $z^* = \frac{-7}{2}$.

$$(e) \begin{cases} x + y + z = 1 \Leftrightarrow z = -x - y + 1 \\ x - 2y = 5 \Leftrightarrow y = \frac{1}{2}x - \frac{5}{2} \\ -5x + y = -7 \Leftrightarrow y = 5x - 7 \end{cases}.$$

$$\begin{aligned}\frac{1}{2}x - \frac{5}{2} &= 5x - 7 \\5x - \frac{1}{2}x &= \frac{-5}{2} + 7 \\\frac{9}{2}x &= \frac{9}{2} \\x &= \frac{\frac{9}{2}}{\frac{9}{2}} \\x^* &= 1.\end{aligned}$$

$$\begin{aligned}y^* &= 5 * 1 - 7 \\y^* &= 5 - 7 \\y^* &= -2.\end{aligned}$$

$$\begin{aligned}z^* &= -1 - (-2) + 1 \\z^* &= -1 + 2 + 1 \\z^* &= 2.\end{aligned}$$

Por lo tanto, existe una única solución: $x^* = 1$, $y^* = -2$, $z^* = 2$.

$$(f) \begin{cases} x - 2y + 3z = -4 \Leftrightarrow x = 2y - 3z - 4 \\ 3x - 4y + 2z = -1 \Leftrightarrow y = \frac{3}{4}x + \frac{1}{2}z + \frac{1}{4} \\ 2x + y + 2z = 6 \Leftrightarrow z = -x - \frac{1}{2}y + 3 \end{cases}$$

$$\begin{aligned} y &= \frac{3}{4}x + \frac{1}{2}(-x - \frac{1}{2}y + 3) + \frac{1}{4} \\ y &= \frac{3}{4}x - \frac{1}{2}x - \frac{1}{4}y + \frac{3}{2} + \frac{1}{4} \\ y + \frac{1}{4}y &= \frac{1}{4}x + \frac{7}{4} \\ \frac{5}{4}y &= \frac{1}{4}x + \frac{7}{4} \\ y &= \frac{\frac{1}{4}x + \frac{7}{4}}{\frac{5}{4}} \\ y &= \frac{1}{5}x + \frac{7}{5}. \end{aligned}$$

$$\begin{aligned} z &= -x - \frac{1}{2}(\frac{3}{4}x + \frac{1}{2}z + \frac{1}{4}) + 3 \\ z &= -x - \frac{3}{8}x - \frac{1}{4}z - \frac{1}{8} + 3 \\ z + \frac{1}{4}z &= -\frac{11}{8}x + \frac{23}{8} \\ \frac{5}{4}z &= -\frac{11}{8}x + \frac{23}{8} \\ z &= \frac{-\frac{11}{8}x + \frac{23}{8}}{\frac{5}{4}} \\ z &= -\frac{11}{10}x + \frac{23}{10}. \end{aligned}$$

$$\begin{aligned} x &= 2(\frac{1}{5}x + \frac{7}{5}) - 3(-\frac{11}{10}x + \frac{23}{10}) - 4 \\ x &= \frac{2}{5}x + \frac{14}{5} + \frac{33}{10}x - \frac{69}{10} - 4 \\ x - \frac{14}{5}x - \frac{33}{10}x &= \frac{-81}{10} \\ \frac{-27}{10}x &= \frac{-81}{10} \\ x &= \frac{-81}{-27} \\ x &= \frac{10}{10} \\ x^* &= 3. \end{aligned}$$

$$\begin{aligned} y^* &= \frac{1}{5} * 3 + \frac{7}{5} \\ y^* &= \frac{3}{5} + \frac{7}{5} \\ y^* &= 2. \end{aligned}$$

$$\begin{aligned} z^* &= \frac{-11}{10} * 3 + \frac{23}{10} \\ z^* &= \frac{-33}{10} + \frac{23}{10} \\ z^* &= \frac{-10}{10} \\ z^* &= -1. \end{aligned}$$

Por lo tanto, existe una única solución: $x^* = 3, y^* = 2, z^* = -1$.

$$(g) \begin{cases} y = \sqrt{3}x + 4 \\ x^2 + y^2 = 4 \end{cases}$$

$$\begin{aligned} x^2 + (\sqrt{3}x + 4)^2 &= 4 \\ x^2 + (\sqrt{3}x)^2 + 8\sqrt{3}x + 16 &= 4 \\ x^2 + 3x^2 + 8\sqrt{3}x + 16 - 4 &= 0 \\ 4x^2 + 8\sqrt{3}x + 12 &= 0 \\ x^2 + 2\sqrt{3}x + 3 &= 0. \end{aligned}$$

$$\begin{aligned} x_1, x_2 &= \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4*1*3}}{2*1} \\ x_1, x_2 &= \frac{-2\sqrt{3} \pm \sqrt{4*3 - 12}}{2} \\ x_1, x_2 &= \frac{-2\sqrt{3} \pm \sqrt{12 - 12}}{2} \\ x_1, x_2 &= \frac{-2\sqrt{3} \pm \sqrt{0}}{2} \\ x_1, x_2 &= \frac{-2\sqrt{3} \pm 0}{2} \\ x^* &= \frac{-2\sqrt{3}}{2} \\ x^* &= -\sqrt{3}. \end{aligned}$$

$$\begin{aligned} y^* &= \sqrt{3}(-\sqrt{3}) + 4 \\ y^* &= -3 + 4 \\ y^* &= 1. \end{aligned}$$

Por lo tanto, existe una única solución: $x^* = -\sqrt{3}$, $y^* = 1$.

$$(h) \begin{cases} x^2 - y^2 = 2 \\ x - y = 1 \Leftrightarrow y = x - 1 \end{cases}$$

$$\begin{aligned} x^2 - (x - 1)^2 &= 2 \\ x^2 - (x^2 - 2x + 1) &= 2 \\ x^2 - x^2 + 2x - 1 &= 2 \\ 2x - 1 &= 2 \\ 2x &= 2 + 1 \\ 2x &= 3 \\ x^* &= \frac{3}{2}. \end{aligned}$$

$$\begin{aligned} y^* &= \frac{3}{2} - 1 \\ y^* &= \frac{1}{2}. \end{aligned}$$

Por lo tanto, existe una única solución: $x^* = \frac{3}{2}$, $y^* = \frac{1}{2}$.

(i) $\begin{cases} -x + y = 2 \Leftrightarrow y = x + 2 \\ x^2 - 6x + 8 = y \end{cases}$.

$$x^2 - 6x + 8 = x + 2$$

$$x^2 - 6x + 8 - x - 2 = 0$$

$$x^2 - 7x + 6 = 0.$$

$$x_1, x_2 = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$$

$$x_1, x_2 = \frac{7 \pm \sqrt{49 - 24}}{2}$$

$$x_1, x_2 = \frac{7 \pm \sqrt{25}}{2}$$

$$x_1, x_2 = \frac{7 \pm 5}{2}$$

$$x_1^* = \frac{7+5}{2} = \frac{12}{2} = 6.$$

$$x_2^* = \frac{7-5}{2} = \frac{2}{2} = 1.$$

$$y_1^* = 6 + 2 = 8.$$

$$y_2^* = 1 + 2 = 3.$$

Por lo tanto, existen dos soluciones: $x_1^* = 6$, $y_1^* = 8$; $x_2^* = 1$, $y_2^* = 3$.

Ejercicio 5.

Un cartel en una mueblería dice “llevé los dos por \$655”. Si una silla cuesta \$55 más que una banqueta, ¿cuánto cuesta la silla?

$$s + b = 655.$$

$$s = b + 55.$$

$$b + 55 + b = 655$$

$$2b = 655 - 55$$

$$2b = 600$$

$$b = \frac{600}{2}$$

$$b = 300.$$

$$s = 300 + 55$$

$$s = 355.$$

Por lo tanto, una silla cuesta \$355.

Ejercicio 6.

Resolver despejando la incógnita:

(a) $m^2 - 12 = 0$.

$$m^2 - 12 = 0$$

$$m^2 = 12$$

$$\sqrt{m^2} = \sqrt{12}$$

$$|m| = \sqrt{4 * 3}$$

$$|m| = 2\sqrt{3}$$

$$m = \pm 2\sqrt{3}$$

$$m = \pm 2\sqrt{3}.$$

(b) $n^2 + 25 = 0$.

$$n^2 + 25 = 0$$

$$n^2 = -25$$

$$\sqrt{n^2} = \sqrt{-25}$$

$$|n| = \sqrt{-1 * 25}$$

$$|n| = \sqrt{-1} \sqrt{5}$$

$$|n| = 5i$$

$$n = \pm 5i.$$

(c) $3y^2 - 45 = 0$.

$$3y^2 - 45 = 0$$

$$3y^2 = 45$$

$$y^2 = \frac{45}{3}$$

$$y^2 = 15$$

$$\sqrt{y^2} = \sqrt{15}$$

$$|y| = \sqrt{15}$$

$$y = \pm \sqrt{15}.$$

(d) $4u^2 - 9 = 0$.

$$4u^2 - 9 = 0$$

$$4u^2 = 9$$

$$u^2 = \frac{9}{4}$$

$$\sqrt{u^2} = \sqrt{\frac{9}{4}}$$

$$|u| = \frac{3}{2}$$

$$u = \pm \frac{3}{2}.$$

(e) $(d - 3)^2 - \frac{1}{2} = 0.$

$$(d - 3)^2 - \frac{1}{2} = 0$$

$$d^2 - 6d + 9 - \frac{1}{2} = 0$$

$$d^2 - 6d + \frac{17}{2} = 0.$$

$$d_1, d_2 = \frac{-(-6) \pm \sqrt{(-6)^2 - 4*1\frac{17}{2}}}{2*1}$$

$$d_1, d_2 = \frac{6 \pm \sqrt{36 - 34}}{2}$$

$$d_1, d_2 = \frac{6 \pm \sqrt{2}}{2}$$

$$d_1 = \frac{6 + \sqrt{2}}{2} = 3 + \frac{1}{\sqrt{2}}.$$

$$d_2 = \frac{6 - \sqrt{2}}{2} = 3 - \frac{1}{\sqrt{2}}.$$

(f) $(y + 1)^2 - 9 = 0.$

$$(y + 1)^2 - 9 = 0$$

$$y^2 + 2y + 1 - 9 = 0$$

$$y^2 + 2y - 8 = 0.$$

$$y_1, y_2 = \frac{-2 \pm \sqrt{2^2 - 4*1(-8)}}{2*1}$$

$$y_1, y_2 = \frac{-2 \pm \sqrt{4 + 32}}{2}$$

$$y_1, y_2 = \frac{-2 \pm \sqrt{36}}{2}$$

$$y_1, y_2 = \frac{-2 \pm 6}{2}$$

$$y_1 = \frac{-2 + 6}{2} = \frac{4}{2} = 2.$$

$$y_2 = \frac{-2 - 6}{2} = \frac{-8}{2} = -4.$$

(g) $\frac{3}{2} z^2 - \frac{1}{2} = 0.$

$$\frac{3}{2} z^2 - \frac{1}{2} = 0$$

$$\frac{3}{2} z^2 = \frac{1}{2}$$

$$\begin{aligned}z^2 &= \frac{1}{\frac{2}{3}} \\z^2 &= \frac{1}{\frac{2}{3}} \\z^2 &= \sqrt{\frac{1}{\frac{2}{3}}} \\|z| &= \frac{1}{\sqrt{3}} \\z &= \pm \frac{1}{\sqrt{3}}.\end{aligned}$$

(h) $w^2 - 25 = 0$.

$$\begin{aligned}w^2 - 25 &= 0 \\w^2 &= 25 \\\sqrt{w^2} &= \sqrt{25} \\|w| &= 5 \\w &= \pm 5.\end{aligned}$$

(i) $\frac{49}{4} d^2 = I$.

$$\begin{aligned}\frac{49}{4} d^2 &= 1 \\d^2 &= \frac{1}{\frac{49}{4}} \\d^2 &= \frac{4}{49} \\\sqrt{d^2} &= \sqrt{\frac{4}{49}} \\|d| &= \frac{2}{7} \\d &= \pm \frac{2}{7}.\end{aligned}$$

Ejercicio 7.

Resolver sacando factor común:

(a) $12m^2 + m = 0$.

$$\begin{aligned}12m^2 + m &= 0 \\m(12m + 1) &= 0 \\12m + 1 &= 0 \\12m &= -1 \\m &= \frac{-1}{12}.\end{aligned}$$

(b) $9n^2 + 9n = 0$.

$$\begin{aligned}9n^2 + 9n &= 0 \\9n(n + 1) &= 0 \\n + 1 &= 0 \\n &= -1.\end{aligned}$$

(c) $7y^2 = -4y$.

$$\begin{aligned}7y^2 &= -4y \\7y^2 + 4y &= 0 \\y(7y + 4) &= 0 \\7y + 4 &= 0 \\7y &= -4 \\y &= \frac{-4}{7}.\end{aligned}$$

(d) $6u^2 - u = 0$.

$$\begin{aligned}6u^2 - u &= 0 \\u(6u - 1) &= 0 \\6u - 1 &= 0 \\6u &= 1 \\u &= \frac{1}{6}.\end{aligned}$$

(e) $x^2 = 2x$.

$$x^2 = 2x$$

$$\begin{aligned}x^2 - 2x &= 0 \\x(x - 2) &= 0 \\x - 2 &= 0 \\x &= 2.\end{aligned}$$

(f) $\left(\frac{y}{2}\right)^2 - \frac{1}{2}y = 0.$

$$\begin{aligned}\left(\frac{y}{2}\right)^2 - \frac{1}{2}y &= 0 \\\frac{1}{4}y^2 - \frac{1}{2}y &= 0 \\\frac{1}{2}y\left(\frac{1}{2}y - 1\right) &= 0 \\\frac{1}{2}y - 1 &= 0 \\\frac{1}{2}y &= 1 \\y &= \frac{1}{\frac{1}{2}} \\y &= 2.\end{aligned}$$

Ejercicio 8.

Determinar las soluciones de las siguientes ecuaciones cuadráticas:

(a) $(x - 3)^2 = \frac{1}{2}$.

$$(x - 3)^2 = \frac{1}{2}$$

$$x^2 - 6x + 9 - \frac{1}{2} = 0$$

$$x^2 - 6x + \frac{17}{2} = 0.$$

$$x_1, x_2 = \frac{-(-6) \pm \sqrt{(-6)^2 - 4*1*\frac{17}{2}}}{2*1}$$

$$x_1, x_2 = \frac{6 \pm \sqrt{36 - 34}}{2}$$

$$x_1, x_2 = \frac{6 \pm \sqrt{2}}{2}$$

$$x_1 = \frac{6 + \sqrt{2}}{2} = 3 + \frac{1}{\sqrt{2}}$$

$$x_2 = \frac{6 - \sqrt{2}}{2} = 3 - \frac{1}{\sqrt{2}}$$

(b) $(1 - x)^2 = \sqrt{2}$.

$$(1 - x)^2 = \sqrt{2}$$

$$1 - 2x + x^2 - \sqrt{2} = 0$$

$$x^2 - 2x + (1 - \sqrt{2}) = 0.$$

$$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4*1*(1-\sqrt{2})}}{2*1}$$

$$x_1, x_2 = \frac{2 \pm \sqrt{4 - 4 + 4\sqrt{2}}}{2}$$

$$x_1, x_2 = \frac{2 \pm \sqrt{4\sqrt{2}}}{2}$$

$$x_1, x_2 = \frac{2 \pm 2*2^{\frac{1}{4}}}{2}$$

$$x_1 = \frac{2 + 2*2^{\frac{1}{4}}}{2} = 1 + 2^{\frac{1}{4}}$$

$$x_2 = \frac{2 - 2*2^{\frac{1}{4}}}{2} = 1 - 2^{\frac{1}{4}}$$

(c) $(2x + 1)^2 = 4$.

$$(2x + 1)^2 = 4$$

$$4x^2 + 4x + 1 - 4 = 0$$

$$4x^2 + 4x - 3 = 0$$

$$4(x^2 + x - \frac{3}{4}) = 0$$

$$x^2 + x - \frac{3}{4} = 0.$$

$$x_1, x_2 = \frac{-1 \pm \sqrt{1^2 - 4*1(-\frac{3}{4})}}{2*1}$$

$$x_1, x_2 = \frac{-1 \pm \sqrt{1+3}}{2}$$

$$x_1, x_2 = \frac{-1 \pm \sqrt{4}}{2}$$

$$x_1, x_2 = \frac{-1 \pm 2}{2}$$

$$x_1 = \frac{-1+2}{2} = \frac{1}{2}.$$

$$x_2 = \frac{-1-2}{2} = \frac{-3}{2}.$$

$$(d) (3 - 2x)^2 = 0.$$

$$(3 - 2x)^2 = 0$$

$$9 - 12x + 4x^2 = 0$$

$$4x^2 - 12x + 9 = 0$$

$$4(x^2 - 3x + \frac{9}{4}) = 0$$

$$x^2 - 3x + \frac{9}{4} = 0.$$

$$x_1, x_2 = \frac{-(-3) \pm \sqrt{(-3)^2 - 4*1\frac{9}{4}}}{2*1}$$

$$x_1, x_2 = \frac{3 \pm \sqrt{9-9}}{2}$$

$$x_1, x_2 = \frac{3 \pm \sqrt{0}}{2}$$

$$x_1, x_2 = \frac{3 \pm 0}{2}$$

$$x = \frac{3}{2}.$$

Ejercicio 9.

Resolver completando cuadrados:

(a) $x^2 + 6x = 7$.

$$\begin{aligned}x^2 + 6x &= 7 \\x^2 + 6x - 7 &= 0 \\(x + 3)^2 - 16 &= 0 \\(x + 3)^2 &= 16 \\\sqrt{(x + 3)^2} &= \sqrt{16} \\|x + 3| &= 4 \\x_{1,2} + 3 &= \pm 4 \\x_{1,2} &= \pm 4 - 3 \\x_1 &= 1 \\x_2 &= -7.\end{aligned}$$

(b) $x^2 - 8x + 11 = 0$.

$$\begin{aligned}x^2 - 8x + 11 &= 0 \\(x - 4)^2 - 5 &= 0 \\(x - 4)^2 &= 5 \\\sqrt{(x - 4)^2} &= \sqrt{5} \\|x - 4| &= \sqrt{5} \\x_{1,2} - 4 &= \pm \sqrt{5} \\x_{1,2} &= \pm \sqrt{5} + 4 \\x_1 &= \sqrt{5} + 4 \\x_2 &= -\sqrt{5} + 4.\end{aligned}$$

(c) $4x^2 = 12x + 11$.

$$\begin{aligned}4x^2 &= 12x + 11 \\4x^2 - 12x - 11 &= 0 \\4(x^2 - 3x - \frac{11}{4}) &= 0 \\x^2 - 3x - \frac{11}{4} &= 0 \\(x - \frac{3}{2})^2 + \frac{1}{2} &= 0 \\(x - \frac{3}{2})^2 &= -\frac{1}{2} \\\sqrt{(x - \frac{3}{2})^2} &= \sqrt{-\frac{1}{2}} \\|x - \frac{3}{2}| &= \frac{i}{\sqrt{2}}\end{aligned}$$

$$\begin{aligned}x_{1,2} - \frac{3}{2} &= \pm \frac{i}{\sqrt{2}} \\x_{1,2} &= \pm \frac{i}{\sqrt{2}} + \frac{3}{2} \\x_1 &= \frac{i}{\sqrt{2}} + \frac{3}{2} \\x_2 &= \frac{-i}{\sqrt{2}} + \frac{3}{2}.\end{aligned}$$

(d) $x^2 - 10x + 5 = -20.$

$$\begin{aligned}x^2 - 10x + 5 &= -20 \\x^2 - 10x + 5 + 20 &= 0 \\x^2 - 10x + 25 &= 0 \\(x - 5)^2 &= 0 \\\sqrt{(x - 5)^2} &= \sqrt{0} \\|x - 5| &= 0 \\x - 5 &= \pm 0 \\x - 5 &= 0 \\x &= 5.\end{aligned}$$

(e) $(x - 1)(x - 3) = 1.$

$$\begin{aligned}(x - 1)(x - 3) &= 1 \\x^2 - 3x - x + 3 - 1 &= 0 \\x^2 - 4x + 2 &= 0 \\(x - 2)^2 - 2 &= 0 \\(x - 2)^2 &= 2 \\\sqrt{(x - 2)^2} &= \sqrt{2} \\|x - 2| &= \sqrt{2} \\x_{1,2} - 2 &= \pm \sqrt{2} \\x_{1,2} &= \pm \sqrt{2} + 2 \\x_1 &= \sqrt{2} + 2 \\x_2 &= -\sqrt{2} + 2.\end{aligned}$$

(f) $\frac{5}{3}x^2 + x - \frac{2}{3} = 0.$

$$\begin{aligned}\frac{5}{3}x^2 + x - \frac{2}{3} &= 0 \\\frac{5}{3}(x^2 + \frac{3}{5}x - \frac{2}{5}) &= 0 \\x^2 + \frac{3}{5}x - \frac{2}{5} &= 0 \\(x + \frac{3}{10})^2 - \frac{49}{100} &= 0\end{aligned}$$

$$\begin{aligned}(x + \frac{3}{10})^2 &= \frac{49}{100} \\ \sqrt{(x + \frac{3}{10})^2} &= \sqrt{\frac{49}{100}} \\ \left|x + \frac{3}{10}\right| &= \frac{7}{10} \\ x_{1,2} + \frac{3}{10} &= \pm \frac{7}{10} \\ x_{1,2} &= \pm \frac{7}{10} - \frac{3}{10} \\ x_1 &= \frac{7}{10} - \frac{3}{10} = \frac{2}{5} \\ x_2 &= \frac{-7}{10} - \frac{3}{10} = -1.\end{aligned}$$

Ejercicio 10.

Utilizar el discriminante para completar la siguiente tabla:

Ecuación	Discriminante	Cantidad de soluciones
$\frac{x^2}{3} - 2x + 6 = 0$	$(-2)^2 - 4 \cdot \frac{1}{3} \cdot 6 < 0$	0
$2x^2 - 6x + 3 = 0$	$(-6)^2 - 4 \cdot 2 \cdot 3 > 0$	2
$\sqrt{3}x^2 = -x - 2$	$1^2 - 4\sqrt{3} \cdot 2 < 0$	0
$2x^2 = 2x + 1$	$(-2)^2 - 4 \cdot 2 \cdot (-1) > 0$	2
$0,32x^2 - 0,75x - 0,66 = 0$	$(-0,75)^2 - 4 \cdot 0,32 \cdot (-0,66) > 0$	2
$ax^2 = -bx$	$b^2 - 4a \cdot 0 > 0$	2
$x^2 = (a + b)x - ab$	$[-(a + b)]^2 - 4 \cdot 1 \cdot ab \leq 0$	0, 1, 2

(a) $\frac{x^2}{3} - 2x + 6 = 0.$

$$\begin{aligned} \frac{x^2}{3} - 2x + 6 &= 0 \\ \frac{1}{3}(x^2 - 6x + 18) &= 0 \\ x^2 - 6x + 18 &= 0. \end{aligned}$$

$$\begin{aligned} x_1, x_2 &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 18}}{2 \cdot 1} \\ x_1, x_2 &= \frac{6 \pm \sqrt{36 - 72}}{2} \\ x_1, x_2 &= \frac{6 \pm \sqrt{-36}}{2} \\ x_1, x_2 &= \frac{6 \pm \sqrt{-1 \cdot 36}}{2} \\ x_1, x_2 &= \frac{6 \pm \sqrt{-1} \sqrt{36}}{2} \\ x_1, x_2 &= \frac{6 \pm 6i}{2} \\ x_1 &= \frac{6+6i}{2} = 3 + 3i. \\ x_2 &= \frac{6-6i}{2} = 3 - 3i. \end{aligned}$$

(b) $2x^2 - 6x + 3 = 0.$

$$\begin{aligned} 2x^2 - 6x + 3 &= 0 \\ 2(x^2 - 3x + \frac{3}{2}) &= 0 \\ x^2 - 3x + \frac{3}{2} &= 0. \end{aligned}$$

$$x_1, x_2 = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot \frac{3}{2}}}{2 \cdot 1}$$

$$x_1, x_2 = \frac{3 \pm \sqrt{9-6}}{2}$$

$$x_1, x_2 = \frac{3 \pm \sqrt{3}}{2}$$

$$x_1 = \frac{3+\sqrt{3}}{2} = \frac{3}{2} + \frac{\sqrt{3}}{2}$$

$$x_2 = \frac{3-\sqrt{3}}{2} = \frac{3}{2} - \frac{\sqrt{3}}{2}$$

(c) $\sqrt{3}x^2 = -x - 2$

$$\sqrt{3}x^2 = -x - 2$$

$$\sqrt{3}x^2 + x + 2 = 0$$

$$\sqrt{3}(x^2 + \frac{1}{\sqrt{3}}x + \frac{2}{\sqrt{3}}) = 0$$

$$x^2 + \frac{1}{\sqrt{3}}x + \frac{2}{\sqrt{3}} = 0$$

$$x_1, x_2 = \frac{\frac{-1}{\sqrt{3}} \pm \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 - 4 \cdot 1 \cdot \frac{2}{\sqrt{3}}}}{2 \cdot 1}$$

$$x_1, x_2 = \frac{\frac{-1}{\sqrt{3}} \pm \sqrt{\frac{1}{3} - \frac{8}{\sqrt{3}}}}{2}$$

$$x_1, x_2 = \frac{\frac{-1}{\sqrt{3}} \pm \sqrt{\frac{\sqrt{3}-24}{3\sqrt{3}}}}{2}$$

$$x_1, x_2 = \frac{\frac{-1}{\sqrt{3}} \pm \sqrt{\frac{\sqrt{3}-24}{\sqrt{3}\sqrt{3}}}}{2} \notin \mathbb{R}$$

(d) $2x^2 = 2x + 1$

$$2x^2 = 2x + 1$$

$$2x^2 - 2x - 1 = 0$$

$$2(x^2 - x - \frac{1}{2}) = 0$$

$$x^2 - x - \frac{1}{2} = 0$$

$$x_1, x_2 = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-\frac{1}{2})}}{2 \cdot 1}$$

$$x_1, x_2 = \frac{1 \pm \sqrt{1+2}}{2}$$

$$x_1, x_2 = \frac{1 \pm \sqrt{3}}{2}$$

$$x_1 = \frac{1+\sqrt{3}}{2} = \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$x_2 = \frac{1-\sqrt{3}}{2} = \frac{1}{2} - \frac{\sqrt{3}}{2}$$

(e) $0,32x^2 - 0,75x - 0,66 = 0$

$$0,32x^2 - 0,75x - 0,66 = 0.$$

$$\begin{aligned} x_1, x_2 &= \frac{-(-0,75) \pm \sqrt{(-0,75)^2 - 4 * 0,32 * (-0,66)}}{2 * 0,32} \\ x_1, x_2 &= \frac{0,75 \pm \sqrt{0,5625 + 0,8448}}{0,64} \\ x_1, x_2 &= \frac{0,75 \pm \sqrt{0,5625 + 0,8448}}{0,64} \\ x_1, x_2 &= \frac{0,75 \pm \sqrt{1,4073}}{0,64} \\ x_1, x_2 &= \frac{0,75 \pm \sqrt{1,4073}}{0,64} \\ x_1, x_2 &= \frac{0,75 \pm 1,19}{0,64} \\ x_1 &= \frac{0,75 + 1,19}{0,64} = \frac{1,94}{0,64} = 3,03. \\ x_2 &= \frac{0,75 - 1,19}{0,64} = \frac{-0,44}{0,64} = -0,69. \end{aligned}$$

(f) $ax^2 = -bx$.

$$\begin{aligned} ax^2 &= -bx \\ ax^2 + bx &= 0. \end{aligned}$$

$$\begin{aligned} x_1, x_2 &= \frac{-b \pm \sqrt{b^2 - 4a * 0}}{2a} \\ x_1, x_2 &= \frac{-b \pm \sqrt{b^2 - 0}}{2a} \\ x_1, x_2 &= \frac{-b \pm \sqrt{b^2}}{2a}. \end{aligned}$$

Si $b > 0$:

$$\begin{aligned} x_1, x_2 &= \frac{-b \pm b}{2a} \\ x_1 &= \frac{-b + b}{2a} = \frac{0}{2a} = 0. \\ x_2 &= \frac{-b - b}{2a} = \frac{-2b}{2a} = \frac{-b}{a}. \end{aligned}$$

Si $b < 0$:

$$\begin{aligned} x_1, x_2 &= \frac{-b \pm (-b)}{2a} \\ x_1 &= \frac{-b - b}{2a} = \frac{-2b}{2a} = \frac{-b}{a}. \\ x_2 &= \frac{-b - (-b)}{2a} = \frac{-b + b}{2a} = \frac{0}{2a} = 0. \end{aligned}$$

(g) $x^2 = (a + b)x - ab$.

$$x^2 = (a + b)x - ab$$
$$x^2 - (a + b)x + ab = 0.$$

$$x_1, x_2 = \frac{-(a+b) \pm \sqrt{[-(a+b)]^2 - 4*1*ab}}{2*1}$$

$$x_1, x_2 = \frac{a+b \pm \sqrt{(a+b)^2 - 4ab}}{2}$$

$$x_1, x_2 = \frac{a+b \pm \sqrt{a^2 + 2ab + b^2 - 4ab}}{2}$$

$$x_1, x_2 = \frac{a+b \pm \sqrt{a^2 - 2ab + b^2}}{2}.$$

Si $a^2 - 2ab + b^2 < 0$, se tienen 0 soluciones.

Si $a^2 - 2ab + b^2 = 0$, se tiene 1 solución.

Si $a^2 - 2ab + b^2 > 0$, se tienen 2 soluciones.

Ejercicio 11.

Resolver las siguientes ecuaciones, indicar el/los valor/es de x no permitidos:

$$(a) \frac{2}{x^2-4} + \frac{1}{x+2} = \frac{1}{x^2-2x}.$$

$$\begin{aligned} \frac{2}{x^2-4} + \frac{1}{x+2} &= \frac{1}{x^2-2x} \\ \frac{2}{(x+2)(x-2)} + \frac{2}{x+2} &= \frac{1}{x(x-2)} \\ \frac{2+x-2}{(x+2)(x-2)} &= \frac{1}{x(x-2)} \\ \frac{x}{(x+2)(x-2)} &= \frac{1}{x(x-2)} \\ \frac{xx(x-2)}{(x+2)(x-2)} &= 1 \\ \frac{x^2}{x+2} &= 1 \\ x^2 &= x + 2 \\ x^2 - x - 2 &= 0. \end{aligned}$$

$$\begin{aligned} x_1, x_2 &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} \\ x_1, x_2 &= \frac{1 \pm \sqrt{1+8}}{2} \\ x_1, x_2 &= \frac{1 \pm \sqrt{9}}{2} \\ x_1, x_2 &= \frac{1 \pm 3}{2} \\ x_1 &= \frac{1+3}{2} = \frac{4}{2} = 2. \\ x_2 &= \frac{1-3}{2} = \frac{-2}{2} = -1. \end{aligned}$$

$$(b) \frac{1}{x-1} + \frac{1}{x+3} = 1.$$

$$\begin{aligned} \frac{1}{x-1} + \frac{1}{x+3} &= 1 \\ \frac{x+3+x-1}{(x-1)(x+3)} &= 1 \\ \frac{2x+2}{x^2+3x-x-3} &= 1 \\ \frac{2x+2}{x^2+2x-3} &= 1 \\ 2x+2 &= x^2+2x-3 \\ x^2+2x-2x &= 2+3 \\ x^2 &= 5 \\ \sqrt{x^2} &= \sqrt{5} \\ |x| &= \sqrt{5} \\ x &= \pm \sqrt{5}. \end{aligned}$$

$$(c) 1 + \frac{1}{x} = x (1 - \frac{x+1}{x}).$$

$$\begin{aligned} 1 + \frac{1}{x} &= x \left(1 - \frac{x+1}{x}\right) \\ 1 + \frac{1}{x} &= x - x - 1 \\ 1 + \frac{1}{x} &= -1 \\ \frac{1}{x} &= -1 - 1 \\ \frac{1}{x} &= -2 \\ -2x &= 1 \\ x &= \frac{-1}{2}. \end{aligned}$$

$$(d) \frac{6}{x^2-9} = 3.$$

$$\begin{aligned} \frac{6}{x^2-9} &= 3 \\ x^2 - 9 &= \frac{6}{3} \\ x^2 - 9 &= 2 \\ x^2 &= 2 + 9 \\ x^2 &= 11 \\ \sqrt{x^2} &= \sqrt{11} \\ |x| &= \sqrt{11} \\ x &= \pm \sqrt{11}. \end{aligned}$$

$$(e) \frac{2x}{x^2+1} = \frac{1}{x+1}.$$

$$\begin{aligned} \frac{2x}{x^2+1} &= \frac{1}{x+1} \\ 2x(x+1) &= x^2 + 1 \\ 2x^2 + 2x &= x^2 + 1 \\ 2x^2 - x^2 + 2x &= 1 \\ x^2 + 2x - 1 &= 0. \end{aligned}$$

$$\begin{aligned} x_1, x_2 &= \frac{-2 \pm \sqrt{2^2 - 4*1*(-1)}}{2*1} \\ x_1, x_2 &= \frac{-2 \pm \sqrt{4+4}}{2} \\ x_1, x_2 &= \frac{-2 \pm \sqrt{2*4}}{2} \\ x_1, x_2 &= \frac{-2+2\sqrt{2}}{2} \\ x_1 &= \frac{-2+2\sqrt{2}}{2} = -1 + \sqrt{2}. \end{aligned}$$

$$x_2 = \frac{-2-2\sqrt{2}}{2} = -1 - \sqrt{2}.$$

(f) $\frac{3x-3}{x^2-1} = 2x$.

$$\begin{aligned}\frac{3x-3}{x^2-1} &= 2x \\ \frac{3(x-1)}{(x+1)(x-1)} &= 2x \\ \frac{3}{x+1} &= 2x \\ 2x(x+1) &= 3 \\ 2x^2 + 2x &= 3 \\ 2x^2 + 2x - 3 &= 0 \\ 2(x^2 + x - \frac{3}{2}) &= 0 \\ x^2 + x - \frac{3}{2} &= 0.\end{aligned}$$

$$\begin{aligned}x_1, x_2 &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-\frac{3}{2})}}{2 \cdot 1} \\ x_1, x_2 &= \frac{-1 \pm \sqrt{1+6}}{2} \\ x_1, x_2 &= \frac{-1 \pm \sqrt{7}}{2} \\ x_1 &= \frac{-1 + \sqrt{7}}{2} \\ x_2 &= \frac{-1 - \sqrt{7}}{2}.\end{aligned}$$

Ejercicio 12.

De acuerdo a la definición, ¿cuáles de las siguientes expresiones son polinomios?

(a) $P(x) = 7x^4 + 5x - 2.$

Esta expresión es un polinomio.

(b) $Q(x) = \frac{5}{3}x^2 + \ln(2)x.$

Esta expresión es un polinomio.

(c) $P(x) = \frac{1}{3}x^{-2} + 5x^2 - 2.$

Esta expresión no es un polinomio.

(d) $S(x) = x^7 + 5x^{\frac{3}{2}} - 2x^5.$

Esta expresión no es un polinomio.

Ejercicio 13.

Sean $P(x) = 2x - 8x^3 + 5x^2$ y $Q(x) = -x^6 + x - 4x^2 - 2x^7 + 7x^3$. Hallar $Q(x) + P(x)$ y $Q(x) - P(x)$.

$$Q(x) - P(x) = (-x^6 + x - 4x^2 - 2x^7 + 7x^3) - (2x - 8x^3 + 5x^2)$$

$$Q(x) - P(x) = -x^6 + x - 4x^2 - 2x^7 + 7x^3 - 2x + 8x^3 - 5x^2$$

$$Q(x) - P(x) = -x^6 - x - 9x^2 - 2x^7 + 15x^3$$

$$Q(x) - P(x) = -2x^7 - x^6 + 15x^3 - 9x^2 - x.$$

$$Q(x) + P(x) = (-x^6 + x - 4x^2 - 2x^7 + 7x^3) + (2x - 8x^3 + 5x^2)$$

$$Q(x) + P(x) = -x^6 + x - 4x^2 - 2x^7 + 7x^3 + 2x - 8x^3 + 5x^2$$

$$Q(x) + P(x) = -x^6 + 3x + x^2 - 2x^7 - x^3$$

$$Q(x) + P(x) = -2x^7 - x^6 - x^3 + x^2 + 3x.$$

Ejercicio 14.

Sean $S(x) = 4x^3 - 2$, $T(x) = -4x^3 + x$ y $W(x) = 6 - x$. Colocar el símbolo de $<$, $=$, $>$, según corresponda:

(a)

$$\text{gr}(S(x)) > \text{gr}(S(x) + T(x)).$$

(b)

$$\text{gr}(S(x)) > \text{gr}(S(x) + T(x) + W(x)).$$

(c)

$$\text{gr}(S(x)) + \text{gr}(T(x)) > \text{gr}(S(x) + T(x)).$$

(d)

$$\text{gr}(S(x)) = \text{gr}(T(x)).$$

(e)

$$\text{gr}(W(x)) = \text{gr}(S(x) + T(x)).$$

Ejercicio 15.

Hallar el opuesto de $P(x) = x^3 + 8 - (-x^2 + 2x^4)$.

$$P(x) = x^3 + 8 - (-x^2 + 2x^4)$$

$$P(x) = x^3 + 8 + x^2 - 2x^4$$

$$P(x) = -2x^4 + x^3 + x^2 + 8.$$

$$Q(x) = -P(x)$$

$$Q(x) = -(-2x^4 + x^3 + x^2 + 8)$$

$$Q(x) = 2x^4 - x^3 - x^2 - 8.$$

Ejercicio 16.

Dados $P(x) = 2x^6 - 3x^4 + 2x^2 - 4$ y $Q(x) = 8 - 3x^2 - 5x$. Hallar $P(x) * Q(x)$.

$$P(x) * Q(x) = (2x^6 - 3x^4 + 2x^2 - 4)(8 - 3x^2 - 5x)$$

$$P(x) * Q(x) = 16x^6 - 62x^8 - 10x^7 - 24x^4 + 9x^6 + 15x^5 + 16x^2 - 6x^4 - 10x^3 - 32 + 12x^2 + 20x$$

$$P(x) * Q(x) = 25x^6 - 62x^8 - 10x^7 - 30x^4 + 15x^5 + 28x^2 - 10x^3 - 32 + 20x$$

$$P(x) * Q(x) = -62x^8 - 10x^7 + 25x^6 + 15x^5 - 30x^4 - 10x^3 + 28x^2 - 20x - 32.$$

Ejercicio 17.

Decidir si es Verdadero o Falso: “El grado del polinomio producto es siempre mayor que cada uno de los grados de los factores”. Justificar.

Es FALSO, ya que, cuando uno de los dos factores es un monomio, el grado del polinomio producto es igual al grado del factor de mayor grado.

Ejercicio 18.

Hallar el grado, el coeficiente principal y el término independiente del polinomio $W(x) = P(x) * Q(x)$, sabiendo que son ordenados y completos, que sus expresiones comienzan así y que sus coeficientes cumplen con la secuencia que se evidencia en sus primeros términos.

$$P(x) = x^{50} - x^{49} + x^{49} - x^{47} + \dots$$

Λ

$$Q(x) = 2x^{23} + 4x^{22} + 8x^{21} + 16x^{20} \dots$$

$$W(x) = P(x) * Q(x)$$

$$W(x) = (x^{50} - x^{49} + x^{49} - x^{47} + \dots) * (2x^{23} + 4x^{22} + 8x^{21} + 16x^{20} + \dots).$$

$$\text{gr}(W(x)) = 73.$$

Coeficiente principal = 2.

Término independiente = 8388608.

Ejercicio 19.

Sean $S(x) = 2x^3 - x + 2$, $T(x) = x - 3$ y $W(x) = -x^2 - x - 1$. Hallar:

(a) $2 [(S(x) + T(x)) * W(x)]$.

$$2 [(S(x) + T(x)) * W(x)] = 2 [(2x^3 - x + 2) + (x - 3)(-x^2 - x - 1)]$$

$$2 [(S(x) + T(x)) * W(x)] = 2 (2x^3 - x + 2 - x^3 - x^2 - x + 3x^2 + 3x + 3)$$

$$2 [(S(x) + T(x)) * W(x)] = 2 (x^3 + x + 5 + 2x^2)$$

$$2 [(S(x) + T(x)) * W(x)] = 2x^3 + 2x + 4x^2 + 10$$

$$2 [(S(x) + T(x)) * W(x)] = 2x^3 + 4x^2 + 2x + 10.$$

(b) $\frac{1}{3} [T(x) * T(x)] - 4 [W(x) * S(x)]$.

$$\frac{1}{3} [T(x) * T(x)] - 4 [W(x) * S(x)] = \frac{1}{3} [(x - 3)(x - 3)] - 4 [(-x^2 - x - 1)(2x^3 - x + 2)]$$

$$\frac{1}{3} [T(x) * T(x)] - 4 [W(x) * S(x)] = \frac{1}{3} (x^2 - 6x + 9) - 4 (-2x^5 + x^3 - 2x^2 - 2x^4 + x^2 - 2x - 2x^3 + x - 2)$$

$$\frac{1}{3} [T(x) * T(x)] - 4 [W(x) * S(x)] = \frac{1}{3} x^2 - 2x + 3 - 4 (-2x^5 - x^3 - x^2 - 2x^4 - x - 2)$$

$$\frac{1}{3} [T(x) * T(x)] - 4 [W(x) * S(x)] = \frac{1}{3} x^2 - 2x + 3 + 8x^5 + 4x^3 + 4x^2 + 8x^4 + 4x + 8$$

$$\frac{1}{3} [T(x) * T(x)] - 4 [W(x) * S(x)] = \frac{13}{3} x^2 + 2x + 11 + 8x^5 + 4x^3 + 8x^4$$

$$\frac{1}{3} [T(x) * T(x)] - 4 [W(x) * S(x)] = 8x^5 + 8x^4 + 4x^3 + \frac{13}{3} x^2 + 2x + 11.$$

Ejercicio 20.

Sean $P(x) = 2x^7 + 3x^6 + 18x^3 + 29x + 10$ y $Q(x) = 2x^2 + 3x$. Hallar el cociente y el resto de la división entre $P(x)$ y $Q(x)$.

$$\begin{array}{r|l} 2x^7 + 3x^6 + 18x^3 + 29x + 10 & 2x^2 + 3x \\ -(2x^7 + 3x^6 + 0x^3 + 0x + 0) & x^5 + 9x - \frac{27}{2} \\ 18x^3 + 29x + 10 & \\ -(18x^3 + 27x^2 + 0x + 10) & \\ -27x^2 + 29x + 10 & \\ -(-27x^2 - \frac{81}{2}x + 0) & \\ \frac{139}{2}x + 10. & \end{array}$$

$$C(x) = x^5 + 9x - \frac{27}{2}$$

$$R(x) = \frac{139}{2}x + 10.$$

Ejercicio 21.

¿Existe un polinomio $T(x)$ tal que $6x^6 - 9x^4 + 10x^2 - 15 = T(x)(2x^2 - 3)$?

$$T(x) = \frac{6x^6 - 9x^4 + 10x^2 - 15}{2x^2 - 3}.$$

$$\begin{array}{r|l} 6x^6 - 9x^4 + 10x^2 - 15 & | 2x^2 - 3 \\ -(6x^6 - 9x^4 + 0x^2 + 0) & 3x^4 + 5 \\ \hline 10x^2 - 15 & \\ -(10x^2 - 15) & \\ \hline 0. & \end{array}$$

Por lo tanto, existe un polinomio $T(x) = 3x^4 + 5$ tal que cumple con lo solicitado.

Ejercicio 22.

Hallar $S(x)$, si es posible, tal que $9x^5 + x^2 - 5x = (4x^2 - 5) S(x) + (x - 8)$.

$$9x^5 + x^2 - 5x = (4x^2 - 5) S(x) + (x - 8)$$

$$(4x^2 - 5) S(x) = 9x^5 + x^2 - 5x - x + 8$$

$$(4x^2 - 5) S(x) = 9x^5 + x^2 - 6x + 8$$

$$S(x) = \frac{9x^5 + x^2 - 6x + 8}{4x^2 - 5}$$

$$\begin{array}{r} 9x^5 + x^2 - 6x + 8 \\ -(9x^5 - \frac{45}{4}x^3 + 0x^2 + 0x + 0) \\ \hline \frac{-45}{4}x^3 + x^2 - 6x + 8 \\ -(\frac{-45}{4}x^3 + 0x^2 + \frac{225}{16}x + 0) \\ \hline x^2 + \frac{129}{16}x + 8 \\ -(x^2 - \frac{5}{4}) \\ \hline \frac{129}{16}x + \frac{27}{4}. \end{array} \quad \left| \begin{array}{l} 4x^2 - 5 \\ \frac{9}{4}x^3 - \frac{45}{16}x + \frac{1}{4} \end{array} \right.$$

Por lo tanto, no es posible hallar $S(x)$ tal que cumple con lo solicitado.

Ejercicio 23.

Sean $P(x) = x^3 + 2x + 12$, $Q(x) = x - 2$ y $S(x) = x + 2$. Hallar el resto de las divisiones entre (a) $P(x)$ y $Q(x)$; (b) $P(x)$ y $S(x)$. Sacar conclusiones, relacionado con el concepto de raíz.

(a) $\frac{P(x)}{Q(x)}$.

$$\begin{array}{r} x^3 + 2x + 12 \\ -(x^3 - 2x^2 + 0x + 0) \\ \hline 2x^2 + 2x + 12 \\ -(2x^2 - 4x + 0) \\ \hline 6x + 12 \\ -(6x - 12) \\ \hline 24. \end{array}$$

$$R(x) = 24.$$

(b) $\frac{P(x)}{S(x)}$.

$$\begin{array}{r} x^3 + 2x + 12 \\ -(x^3 + 2x^2 + 0x + 0) \\ \hline -2x^2 + 2x + 12 \\ -(-2x^2 - 4x + 0) \\ \hline 6x + 12 \\ -(6x + 12) \\ \hline 0. \end{array}$$

$$R(x) = 0.$$

Ejercicio 24.

Calcular el valor de $k \in \mathbb{R}$ tal que $Q(x) = 3x - 2$ divida a $P(x) = kx^3 + x^2 - k$.

$$\begin{array}{l} kx^3 + x^2 - k \\ -(kx^3 + \frac{2k}{3}x^2 + 0) \quad | \quad 3x - 2 \\ \hline \frac{3-2k}{3}x^2 - k \\ -(\frac{3-2k}{3}x^2 - \frac{6-4k}{9}x + 0) \\ \hline \frac{6-4k}{9}x - k \\ -(\frac{6-4k}{27}x - \frac{12-8k}{27}) \\ \hline \frac{-35}{27}k + \frac{12}{27}. \end{array}$$

$$\frac{-35}{27}k + \frac{12}{27} = 0$$

$$\frac{35}{27}k = \frac{12}{27}$$

$$k = \frac{12}{35}$$

$$k = \frac{12}{35}.$$

Por lo tanto, el valor de k tal que $Q(x)$ divida a $P(x)$ es $\frac{12}{35}$.

Ejercicio 25.

Expresar los siguientes polinomios como producto usando la técnica que corresponda o más de una de ellas:

(a) $P(x) = 2x^4 - x^3 + 6x^2.$

$$P(x) = 2x^4 - x^3 + 6x^2$$

$$P(x) = 2x^2 (x^2 - \frac{3}{2}x + 3).$$

(b) $P(x) = x^6 - x^2.$

$$P(x) = x^6 - x^2$$

$$P(x) = x^2 (x^4 - 1).$$

(c) $P(x) = x^3 - x^2 + x + 1.$

$$P(x) = x^3 - x^2 + x + 1$$

$$P(x) = x (x^2 - x + 1) + 1.$$

(d) $P(x) = 2x^3 - 6x^2 + x - 3.$

$$P(x) = 2x^3 - 6x^2 + x - 3$$

$$P(x) = 2x^2 (x - 3) + (x - 3)$$

$$P(x) = (2x^2 + 1)(x - 3)$$

$$P(x) = 2 (x^2 + \frac{1}{2})(x - 3).$$

(e) $P(x) = x^{10} - x^6 - x^4 + 1.$

$$P(x) = x^{10} - x^6 - x^4 + 1$$

$$P(x) = x^6 (x^4 - 1) - (x^4 + 1).$$

(f) $P(x) = 4x^2 + 4x + 1.$

$$P(x) = 4x^2 + 4x + 1$$

$$P(x) = 4 (x^2 + x + \frac{1}{4})$$

$$P(x) = 4 (x + \frac{1}{2})^2.$$

(g) $P(x) = x^4 - \frac{3}{2}x^2 + \frac{9}{16}$.

$$P(x) = x^4 - \frac{3}{2}x^2 + \frac{9}{16}$$

$$P(x) = x^2 (x^2 - \frac{3}{2}) + \frac{9}{16}$$

$$P(x) = x^2 (x + \sqrt{\frac{3}{2}})(x - \sqrt{\frac{3}{2}}) + \frac{9}{16}.$$

(h) $P(x) = x^5 - x^4 + 6x^3 - 6x^2 + 9x - 9.$

$$P(x) = x^5 - x^4 + 6x^3 - 6x^2 + 9x - 9$$

$$P(x) = x^4(x - 1) + 6x^2(x - 1) + 9(x - 1)$$

$$P(x) = (x^4 + 6x^2 + 9)(x - 1)$$

$$P(x) = (x^2 + 3)^2(x - 1).$$

Ejercicio 26 (Adicional).

Encontrar dos números consecutivos y positivos enteros cuyo producto sea 168.

$$x(x + 1) = 168$$

$$x^2 + x = 168$$

$$x^2 + x - 168 = 0.$$

$$x_1, x_2 = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-168)}}{2 \cdot 1}$$

$$x_1, x_2 = \frac{-1 \pm \sqrt{1 + 672}}{2}$$

$$x_1, x_2 = \frac{-1 \pm \sqrt{673}}{2}$$

$$x_1^* = \frac{-1 + \sqrt{673}}{2} \notin \mathbb{N}.$$

$$x_2^* = \frac{-1 - \sqrt{673}}{2} \notin \mathbb{N}.$$

Por lo tanto, no existen dos números consecutivos y positivos enteros cuyo producto sea 168.

Ejercicio 27 (Adicional).

En cada caso, elegir la opción correcta:

(a) Si $9x + 12 = 15$, entonces:

- (i) $3x + 4 = 5$. Opción correcta.
(ii) $3x + 12 = 5$.
(iii) $9x + 4 = 5$.

(b) Si $6x + 12y - 10 = 0$, entonces:

- (i) $3x + 6y = 0$.
(ii) $3x + 6y = 10$.
(iii) $3x + 6y = 5$. Opción correcta.

Ejercicio 28 (Adicional).

Encontrar la base y la altura de un triángulo cuya área es de $2m^2$ si su base es 3m más larga que su altura.

$$A = \frac{bh}{2}$$
$$2m^2 = \frac{bh}{2}.$$

$$b = h + 3m.$$

$$2m^2 = \frac{(h+3m)h}{2}$$

$$4m^2 = (h + 3m) h$$

$$4m^2 = h^2 + 3h$$

$$h^2 + 3mh - 4m^2 = 0.$$

$$h_1, h_2 = \frac{-3m \pm \sqrt{(3m)^2 - 4*1(-4m^2)}}{2*1}$$

$$h_1, h_2 = \frac{-3m \pm \sqrt{9m^2 + 16m^2}}{2}$$

$$h_1, h_2 = \frac{-3m \pm \sqrt{25m^2}}{2}$$

$$h_1, h_2 = \frac{-3m \pm 5m}{2}$$

$$h_1 = \frac{-3m + 5m}{2} = \frac{2m}{2} = 1m.$$

$$h_2 = \frac{-3m - 5m}{2} = \frac{-8m}{2} = -4m.$$

$$b_1 = 1m + 3m = 4m.$$

$$b_2 = -4m + 3m = -1m.$$

Por lo tanto, la base y la altura son 4 metros y 1 metro, respectivamente.

Ejercicio 29 (Adicional).

La suma de un número y su recíproco es $\frac{10}{3}$. Encontrar el número. (Recíproco de a es $\frac{1}{a}$, cuando $a \neq 0$).

$$\begin{aligned} a + \frac{1}{a} &= \frac{10}{3} \\ a^2 + 1 &= \frac{10}{3} a \\ a^2 + 1 - \frac{10}{3} a &= 0. \end{aligned}$$

$$\begin{aligned} a_1, a_2 &= \frac{-\left(\frac{-10}{3}\right) \pm \sqrt{\left(\frac{-10}{3}\right)^2 - 4*1*1}}{2*1} \\ a_1, a_2 &= \frac{\frac{10}{3} \pm \sqrt{\frac{100}{9} - 4}}{2} \\ a_1, a_2 &= \frac{\frac{10}{3} \pm \sqrt{\frac{64}{9}}}{2} \\ a_1, a_2 &= \frac{\frac{10}{3} \pm \frac{8}{3}}{2} \\ a_1 &= \frac{\frac{10}{3} + \frac{8}{3}}{2} = \frac{\frac{18}{3}}{2} = \frac{6}{2} = 3. \\ a_2 &= \frac{\frac{10}{3} - \frac{8}{3}}{2} = \frac{\frac{2}{3}}{2} = \frac{1}{3}. \end{aligned}$$

Por lo tanto, los números son 3 y $\frac{1}{3}$.

Ejercicio 30 (Adicional).

Cuarenta alumnos deben ser distribuidos para prácticas de Linux o de Java. En cada grupo de Linux, hay 8 alumnos, mientras que, en los de Java, 2; el número de grupos de Java supera en 10 a los de Linux. ¿Cuántos grupos de Linux y cuántos de Java se realizarán?

$$40 = 8l + 2j.$$
$$j = l + 10.$$

$$40 = 8l + 2(l + 10)$$

$$40 = 8l + 2l + 20$$

$$10l = 40 - 20$$

$$10l = 20$$

$$l = \frac{20}{10}$$

$$l^* = 2.$$

$$j^* = 2 + 10$$
$$j^* = 12.$$

Por lo tanto, se realizarán 2 grupos de Linux y 12 grupos de Java.

Ejercicio 31 (Adicional).

Dos personas están acomodando una gran cantidad de sillas en un patio de manera de formar un cuadrado. Una sugiere una manera, pero le sobran 39 sillas. La otra, entonces, propone sumarle una silla más a cada fila, pero le faltan 50 sillas. ¿Cuántas sillas tenían?

$$x^2 = s - 39.$$

$$(x + 1)^2 = s + 50.$$

$$s = x^2 + 39.$$

$$x^2 + 2x + 1 = x^2 + 39 + 50$$

$$2x + 1 = 89$$

$$2x = 89 - 1$$

$$2x = 88$$

$$x = \frac{88}{2}$$

$$x^* = 44.$$

$$s^* = 44^2 + 39$$

$$s^* = 1936 + 39$$

$$s^* = 1975.$$

Por lo tanto, tenían 1975 sillas.

Ejercicio 32 (Adicional).

Encontrar todos los valores de k tales que $P(x)$ sea divisible por el polinomio lineal dado en cada caso:

(a) $P(x) = kx^3 + x^2 + k^2x + 11; x + 2.$

$$\begin{aligned}
 & kx^3 + x^2 + k^2x + 11 && | x + 2 \\
 & -(kx^3 + 2kx^2 + 0x + 0) && kx^2 + (1 - 2k)x - 2(1 - 2k) \\
 & (1 - 2k)x^2 + k^2x + 11 && \\
 & -[(1 - 2k)x^2 + 2x(1 - 2k) + 0] && \\
 & -2x(1 - 2k) + 11 && \\
 & -[-2(1 - 2k) - 4(1 - 2k)] && \\
 & 11 + 4(1 - 2k). &&
 \end{aligned}$$

$$11 + 4(1 - 2k) = 0$$

$$11 + 4 - 8k = 0$$

$$15 - 8k = 0$$

$$8k = 15$$

$$k = \frac{15}{8}.$$

Por lo tanto, el valor de k tal que $P(x)$ sea divisible por el polinomio lineal dado es $\frac{15}{8}$.

(b) $P(x) = k^2x^3 - 4kx + 3; x - 2.$

$$\begin{aligned}
 & k^2x^3 - 4kx + 3 && | x - 2 \\
 & -(k^2x^3 - 2k^2x^2 + 0x + 0) && k^2x^2 + 2k^2x + (4k^2 + 4k) \\
 & 2k^2x^2 - 4kx + 3 && \\
 & -(2k^2x^2 - 4k^2x + 0) && \\
 & (4k^2 - 4k)x + 3 && \\
 & -[(4k^2 - 4k) - 2(4k^2 - 4k)] && \\
 & 3 + 2(4k^2 - 4k). &&
 \end{aligned}$$

$$3 + 2(4k^2 - 4k) = 0$$

$$3 + 8k^2 - 8k = 0$$

$$8(k^2 - k + \frac{3}{8}) = 0$$

$$k^2 - k + \frac{3}{8} = 0.$$

$$k_1, k_2 = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 * 1 * \frac{3}{8}}}{2 * 1}$$

$$k_1, k_2 = \frac{1 \pm \sqrt{1 - \frac{3}{2}}}{2}$$

$$k_1, k_2 = \frac{1 \pm \sqrt{\frac{-1}{2}}}{2}$$

$$k_1, k_2 = \frac{1 \pm \sqrt{-1\frac{1}{2}}}{2}$$

$$k_1, k_2 = \frac{1 \pm \sqrt{-1}\sqrt{\frac{1}{2}}}{2}$$

$$k_1, k_2 = \frac{1 \pm \frac{i}{\sqrt{2}}}{2}$$

$$k_1 = \frac{1 + \frac{i}{\sqrt{2}}}{2} = \frac{1}{2} + \frac{i}{2^2}$$

$$k_2 = \frac{1 - \frac{i}{\sqrt{2}}}{2} = \frac{1}{2} - \frac{i}{2^2}$$

Por lo tanto, los valores de k tal que P(x) sea divisible por el polinomio lineal dado son $(\frac{1}{2} + \frac{i}{2^2})$ y $(\frac{1}{2} - \frac{i}{2^2})$.