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Trabajo Práctico N° 2: Límites.

Ejercicio 1.

Calcular los límites de las siguientes funciones, si existen.

(a) *f (x)=* $\begin{cases} x^2, si\ x > 2 \\ 1, si\ x \leq 2 \end{cases}$. *Calcular* $\lim_{x \to 2^+} f\ (x)$ *y* $\lim_{x \to 2^-} f\ (x)$.

$\lim_{x \to 2^-} f\ (x)= \lim_{x \to 2^-} 1= 1.$

$\lim_{x \to 2^+} f\ (x)= \lim_{x \to 2^+} x^2= 2^2= 4.$

Por lo tanto, ya que $\lim_{x \to 2^-} f\ (x) \neq \lim_{x \to 2^+} f\ (x)$, $\nexists \lim_{x \to 2} f\ (x)$.

(b) *Sea g la función valor absoluto, g (x)=* $|x|= \begin{cases} x, si\ x \geq 0 \\ -x, si\ x < 0 \end{cases}$. *Calcular* $\lim_{x \to 0^+} g\ (x)$ *y* $\lim_{x \to 0^-} g\ (x)$. *¿Qué se puede decir respecto al* $\lim_{x \to 0} g\ (x)$*?*

$\lim_{x \to 0^-} g\ (x)= \lim_{x \to 0^-} -x= 0.$

$\lim_{x \to 0^+} g\ (x)= \lim_{x \to 0^+} x= 0.$

Por lo tanto, ya que $\lim_{x \to 0^-} g\ (x)= \lim_{x \to 0^+} g\ (x)$, $\exists \lim_{x \to 0} g\ (x)$.

Ejercicio 2.

Dada la función f (x)= $\begin{cases} \frac{x-1}{x^2-1}, si\ x \neq 1 \\ 3, si\ x = 1 \end{cases}$*, calcular* $\lim_{x \to 1^+} f(x)$ *y* $\lim_{x \to 1^-} f(x)$.

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} \frac{x-1}{x^2-1} = \frac{1-1}{1^2-1} = \frac{0}{1-1} = (\frac{0}{0})$.
$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} \frac{x-1}{(x+1)(x-1)}$
$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$.

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{x-1}{x^2-1} = \frac{1-1}{1^2-1} = \frac{0}{1-1} = (\frac{0}{0})$.
$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{x-1}{(x+1)(x-1)}$
$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$.

Ejercicio 3.

Calcular, si existen, los siguientes límites:

(a) $\underset{x \to 0}{lim} \frac{\sqrt{x^2+9}-3}{x^2}$.

$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \frac{\sqrt{0^2+9}-3}{0^2} = \frac{\sqrt{0+9}-3}{0} = \frac{\sqrt{9}-3}{0} = \frac{3-3}{0} = (\frac{0}{0})$.

$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3}$

$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \to 0} \frac{x^2+9+3\sqrt{x^2+9}-3\sqrt{x^2+9}-9}{x^2(\sqrt{x^2+9}+3)}$

$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \to 0} \frac{x^2}{x^2(\sqrt{x^2+9}+3)}$

$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \lim_{x \to 0} \frac{1}{\sqrt{x^2+9}+3} = \frac{1}{\sqrt{0^2+9}+3} = \frac{1}{\sqrt{0+9}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$.

(b) $\underset{x \to 1}{lim} \frac{x^2-2x+1}{x^2-3x+2}$.

$\lim_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2} = \frac{1^2-2*1+1}{1^2-3*1+2} = \frac{1-2+1}{1-3+2} = (\frac{0}{0})$.

$\lim_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2} = \lim_{x \to 1} \frac{(x-1)^2}{(x-2)(x-1)}$ (*) y (**)

$\lim_{x \to 1} \frac{x^2-2x+1}{x^2-3x+2} = \lim_{x \to 1} \frac{x-1}{x-2} = \frac{1-1}{1-2} = \frac{0}{-1} = 0$.

(*) $x_1, x_2 = \frac{-(-2)\pm\sqrt{(-2)^2-4*1*1}}{2*1}$

$x_1, x_2 = \frac{2\pm\sqrt{4-4}}{2}$

$x_1, x_2 = \frac{2\pm\sqrt{0}}{2}$

$x_1, x_2 = \frac{2\pm0}{2}$

$x_1 = \frac{2+0}{2} = \frac{2}{2} = 1$.

$x_2 = \frac{2-0}{2} = \frac{2}{2} = 1$.

(**) $x_1, x_2 = \frac{-(-3)\pm\sqrt{(-3)^2-4*1*2}}{2*1}$

$x_1, x_2 = \frac{3\pm\sqrt{9-8}}{2}$

$x_1, x_2 = \frac{3\pm\sqrt{1}}{2}$

$x_1, x_2 = \frac{3\pm1}{2}$

$x_1 = \frac{3+1}{2} = \frac{4}{2} = 2$.

$x_2 = \frac{3-1}{2} = \frac{2}{2} = 1$.

Ejercicio 4.

Calcular los siguientes límites:

(a) $\lim_{x \to 3} \frac{x+2}{x+3}$.

$\lim_{x \to 3} \frac{x+2}{x+3} = \frac{3+2}{3+3} = \frac{5}{6}$.

(b) $\lim_{x \to 1} x^3 + 5x^2 + 10$.

$\lim_{x \to 1} x^3 + 5x^2 + 10 = 1^3 + 5 * 1^2 + 10 = 1 + 5 * 1 + 10 = 1 + 5 + 10 = 16$.

(c) $\lim_{x \to -2} \frac{x^3+2x^2-1}{5-3x}$.

$\lim_{x \to -2} \frac{x^3+2x^2-1}{5-3x} = \frac{(-2)^3+2(-2)^2-1}{5-3(-2)} = \frac{-8+2*4-1}{5+6} = \frac{-8+8-1}{11} = \frac{-1}{11}$.

(d) $\lim_{x \to \pi} sen\ (x - \pi)$.

$\lim_{x \to \pi} sen\ (x - \pi) = \text{sen}\ (\pi - \pi) = \text{sen}\ 0 = 0$.

(e) $\lim_{x \to 2} \frac{\sqrt[3]{x^2+2}}{x}$.

$\lim_{x \to 2} \frac{\sqrt[3]{x^2+2}}{x} = \frac{\sqrt[3]{2^2+2}}{2} = \frac{\sqrt[3]{4+2}}{2} = \frac{\sqrt[3]{6}}{2}$.

(f) $\lim_{x \to 1} ln \frac{2}{x^3+1}$.

$\lim_{x \to 1} \ln \frac{2}{x^3+1} = \ln \frac{2}{1^3+1} = \ln \frac{2}{1+1} = \ln \frac{2}{2} = \ln 1 = 0$.

(g) $\lim_{x \to 0} \frac{ln(x+1)}{e^x}$.

$\lim\limits_{x \to 0} \frac{\ln(x+1)}{e^x} = \frac{\ln(0+1)}{e^0} = \frac{\ln 1}{1} = \frac{0}{1} = 0.$

(h) $\lim\limits_{x \to 0} 3\cos x^2 \, (1+x)^4.$

$\lim\limits_{x \to 0} 3\cos x^2 \, (1+x)^4 = 3 \cos 0^2 \, (1+0)^4 = 3 \cos 0 * 1^4 = 3 * 1 * 1 = 3.$

(i) $\lim\limits_{x \to 3} |x-3|.$

$\lim\limits_{x \to 3} |x-3| = |3-3| = |0| = 0.$

(j) $\lim\limits_{x \to 3} \frac{|x-3|}{x}.$

$\lim\limits_{x \to 3} \frac{|x-3|}{x} = \frac{|3-3|}{3} = \frac{|0|}{3} = \frac{0}{3} = 0.$

(k) $\lim\limits_{x \to 0} \frac{x}{x^2+x}.$

$\lim\limits_{x \to 0} \frac{x}{x^2+x} = \frac{0}{0^2+0} = \frac{0}{0+0} = (\frac{0}{0}).$
$\lim\limits_{x \to 0} \frac{x}{x^2+x} = \lim\limits_{x \to 0} \frac{x}{x(x+1)}$
$\lim\limits_{x \to 0} \frac{x}{x^2+x} = \lim\limits_{x \to 0} \frac{1}{x+1} = \frac{1}{0+1} = \frac{1}{1} = 1.$

(l) $\lim\limits_{x \to 2} \frac{x^2-x-2}{x^2-4}.$

$\lim\limits_{x \to 2} \frac{x^2-x-2}{x^2-4} = \frac{2^2-2-2}{2^2-4} = \frac{4-2-2}{4-4} = (\frac{0}{0}).$
$\lim\limits_{x \to 2} \frac{x^2-x-2}{x^2-4} = \lim\limits_{x \to 2} \frac{(x-2)(x+1)}{(x+2)(x-2)}$ (*)
$\lim\limits_{x \to 2} \frac{x^2-x-2}{x^2-4} = \lim\limits_{x \to 2} \frac{x+1}{x+2} = \frac{2+1}{2+2} = \frac{3}{4}.$

(*) $x_1, x_2 = \frac{-(-1) \pm \sqrt{(-1)^2 - 4*1(-2)}}{2*1}$
$x_1, x_2 = \frac{1 \pm \sqrt{1+8}}{2}$
$x_1, x_2 = \frac{1 \pm \sqrt{9}}{2}$

$x_1, x_2 = \frac{1 \pm 3}{2}$
$x_1 = \frac{1+3}{2} = \frac{4}{2} = 2.$
$x_2 = \frac{1-3}{2} = \frac{-2}{2} = -1.$

(m) $\lim_{x \to -1} \frac{x^2-1}{x^2+3x+2}.$

$\lim_{x \to -1} \frac{x^2-1}{x^2+3x+2} = \frac{(-1)^2-1}{(-1)^2+3(-1)+2} = \frac{1-1}{1-3+2} = (\frac{0}{0}).$
$\lim_{x \to -1} \frac{x^2-1}{x^2+3x+2} = \lim_{x \to -1} \frac{(x+1)(x-1)}{(x-2)(x-1)}$ (*)
$\lim_{x \to -1} \frac{x^2-1}{x^2+3x+2} = \lim_{x \to -1} \frac{x+1}{x-2} = \frac{-1+1}{-1-2} = \frac{0}{-3} = 0.$

(*) $x_1, x_2 = \frac{-(-3) \pm \sqrt{(-3)^2-4*1*2}}{2*1}$
$x_1, x_2 = \frac{3 \pm \sqrt{9-8}}{2}$
$x_1, x_2 = \frac{3 \pm \sqrt{1}}{2}$
$x_1, x_2 = \frac{3 \pm 1}{2}$
$x_1 = \frac{3+1}{2} = \frac{4}{2} = 2.$
$x_2 = \frac{3-1}{2} = \frac{2}{2} = 1.$

(n) $\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1}.$

$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \frac{1-1}{\sqrt{1}-1} = \frac{0}{1-1} = (\frac{0}{0}).$
$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} \frac{\sqrt{x}+1}{\sqrt{x}+1}$
$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \to 1} \frac{(x-1)(\sqrt{x}+1)}{x+\sqrt{x}-\sqrt{x}-1}$
$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \to 1} \frac{(x-1)(\sqrt{x}+1)}{x-1}$
$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \to 1} \sqrt{x} + 1 = \sqrt{1} + 1 = 1 + 1 = 2.$

(ñ) $\lim_{x \to 0} \frac{x^3}{x^2+x}.$

$\lim_{x \to 0} \frac{x^3}{x^2+x} = \frac{0^3}{0^2+0} = \frac{0}{0+0} = (\frac{0}{0}).$
$\lim_{x \to 0} \frac{x^3}{x^2+x} = \lim_{x \to 0} \frac{x^3}{x(x+1)}$
$\lim_{x \to 0} \frac{x^3}{x^2+x} = \lim_{x \to 0} \frac{x^2}{x+1} = \frac{0^2}{0+1} = \frac{0}{1} = 0.$

(o) $\lim_{x \to 2} \frac{x+2}{2-x}$.

$\lim_{x \to 2} \frac{x+1}{2-x} = \frac{2+1}{2-2} = \frac{3}{0} = +\infty$.

(p) $\lim_{x \to 3} \frac{x}{x-3}$.

$\lim_{x \to 3} \frac{x}{x-3} = \frac{3}{3-3} = \frac{3}{0} = +\infty$.

(q) $\lim_{x \to -2} \frac{x}{|x+2|}$.

$\lim_{x \to -2} \frac{x}{|x+2|} = \frac{-2}{|-2+2|} = \frac{-2}{|0|} = \frac{-2}{0} = -\infty$.

Ejercicio 5.

Calcular los siguientes límites al infinito:

(a) $\lim_{x\to -\infty} x - 3x^4$.

$\lim_{x\to -\infty} x - 3x^4 = -\infty$.

(b) $\lim_{x\to -\infty} \frac{5}{x} - \frac{3}{x}$.

$\lim_{x\to -\infty} \frac{5}{x} - \frac{3}{x} = 0 - 0 = 0$.

(c) $\lim_{x\to +\infty} \frac{1}{x^2+3x-1}$.

$\lim_{x\to +\infty} \frac{1}{x^2+3x-1} = 0$.

(d) $\lim_{x\to +\infty} \frac{3x^2+2x-16}{x^2-x-2}$.

$\lim_{x\to +\infty} \frac{3x^2+2x-16}{x^2-x-2} = (\frac{\infty}{\infty})$.

$\lim_{x\to +\infty} \frac{3x^2+2x-16}{x^2-x-2} = \lim_{x\to +\infty} \frac{x^2(3+\frac{2}{x}-\frac{16}{x^2})}{x^2(1-\frac{1}{x}-\frac{2}{x^2})}$

$\lim_{x\to +\infty} \frac{3x^2+2x-16}{x^2-x-2} = \lim_{x\to +\infty} \frac{3+\frac{2}{x}-\frac{16}{x^2}}{1-\frac{1}{x}-\frac{2}{x^2}} = \frac{3+0-0}{1-0-0} = \frac{3}{1} = 3$.

(e) $\lim_{x\to -\infty} \frac{x^2-2x-3}{x^3-x^2-6x-2}$.

$\lim_{x\to -\infty} \frac{x^2-2x-3}{x^3-x^2-6x-2} = 0$.

(f) $\lim_{x\to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}}$.

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^4(1-\frac{2}{x^2})}{\sqrt{x^4(1-\frac{6}{x^3})}}$$

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^4(1-\frac{2}{x^2})}{\sqrt{x^4}\sqrt{1-\frac{6}{x^3}}}$$

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^4(1-\frac{2}{x^2})}{x^2\sqrt{1-\frac{6}{x^3}}}$$

$$\lim_{x \to -\infty} \frac{x^4-2x^2}{\sqrt{x^4-6x}} = \frac{x^2(1-\frac{2}{x^2})}{\sqrt{1-\frac{6}{x^3}}} = +\infty.$$

Ejercicio 6.

Calcular los siguientes límites, si es que existen:

(a) *Dada h (x)=* $\begin{cases} \sqrt{x-4}, si\ x > 4 \\ 8-2x, si\ x < 4 \end{cases}$. *Calcular* $\lim\limits_{x \to 4} h\ (x)$.

$\lim\limits_{x \to 4^-} h\ (x)$= $\lim\limits_{x \to 4^-} 8 - 2 * 4$= 8 - 8= 0.

$\lim\limits_{x \to 4^+} h\ (x)$= $\lim\limits_{x \to 4^+} \sqrt{x-4}$= $\sqrt{4-4}$= $\sqrt{0}$= 0.

Por lo tanto, ya que $\lim\limits_{x \to 4^-} h\ (x)$= $\lim\limits_{x \to 4^+} h\ (x)$= 0, entonces, $\lim\limits_{x \to 4} h\ (x)$= 0.

(b) *Sea f (x)=* $\begin{cases} \frac{x}{e^x}, si\ x \leq 0 \\ x^2, si\ x > 0 \end{cases}$. *Calcular* $\lim\limits_{x \to 0} f\ (x)$ *y* $\lim\limits_{x \to -1} f\ (x)$.

$\lim\limits_{x \to 0^-} f\ (x)$= $\lim\limits_{x \to 0^-} \frac{x}{e^x}$= $\frac{0}{e^0}$= $\frac{0}{1}$= 0.

$\lim\limits_{x \to 0^+} f\ (x)$= $\lim\limits_{x \to 0^+} x^2$= 0^2= 0.

Por lo tanto, ya que $\lim\limits_{x \to 0^-} f\ (x)$= $\lim\limits_{x \to 0^+} f\ (x)$= 0, entonces, $\lim\limits_{x \to 0} f\ (x)$= 0.

$\lim\limits_{x \to -1} f\ (x)$= $\lim\limits_{x \to -1} x^2$= $(-1)^2$= 1.

Por lo tanto, $\lim\limits_{x \to -1} f\ (x)$= 1.

(c) $\lim\limits_{x \to +\infty} \frac{(\ln x)^2}{x}$.

$\lim\limits_{x \to +\infty} \frac{(\ln x)^2}{x}$= 0.

(d) $\lim\limits_{x \to +\infty} \frac{3x^2+2x+1}{e^x}$.

$\lim\limits_{x \to +\infty} \frac{3x^2+2x+1}{e^x}$= 0.

(e) $\lim\limits_{x \to 0^+} x \ln x$.

$\lim_{x \to 0^+} x \ln x = \lim_{y \to +\infty} \frac{1}{y} \ln \frac{1}{y}$

$\lim_{x \to 0^+} x \ln x = \lim_{y \to +\infty} \frac{\ln \frac{1}{y}}{y}$

$\lim_{x \to 0^+} x \ln x = \lim_{y \to +\infty} \frac{\ln 1 - \ln y}{y}$

$\lim_{x \to 0^+} x \ln x = \lim_{y \to +\infty} \frac{0 - \ln y}{y}$

$\lim_{x \to 0^+} x \ln x = \lim_{y \to +\infty} \frac{-\ln y}{y} = 0.$

Ejercicio 7.

Calcular las asíntotas verticales y horizontales, si existen, de las funciones dadas a continuación:

(a) $f(x)=\frac{x^2}{x+1}$.

$\lim_{x \to -1^-} f(x)= \lim_{x \to -1^-} \frac{x^2}{x+1}= \frac{(-1)^2}{-1+1}= \frac{1}{0}= -\infty$.

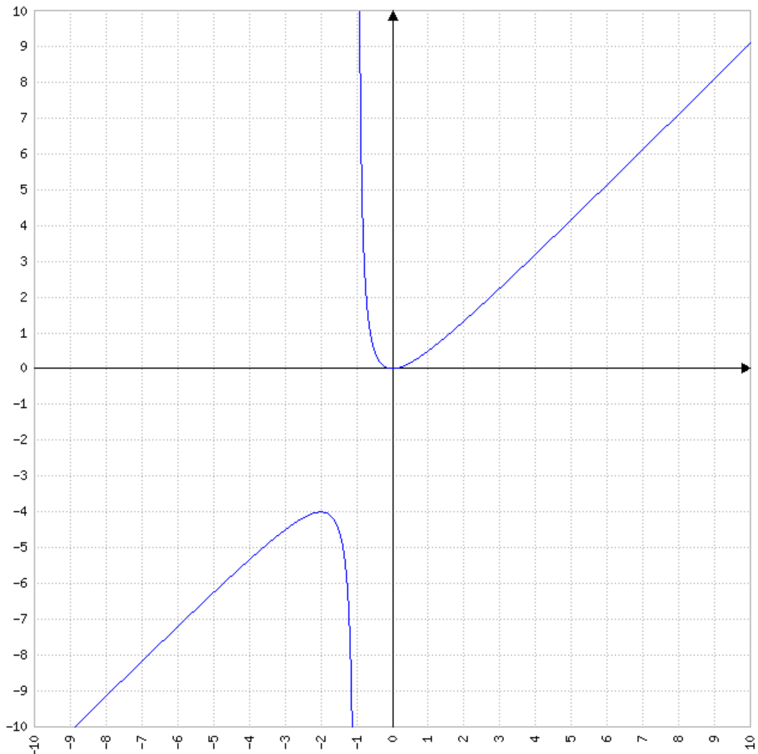
$\lim_{x \to -1^+} f(x)= \lim_{x \to -1^+} \frac{x^2}{x+1}= \frac{(-1)^2}{-1+1}= \frac{1}{0}= +\infty$.

Por lo tanto, f (x) tiene un asíntota vertical en x= -1.

$\lim_{x \to -\infty} f(x)= \lim_{x \to -\infty} \frac{x^2}{x+1}= -\infty$.

$\lim_{x \to +\infty} f(x)= \lim_{x \to +\infty} \frac{x^2}{x+1}= +\infty$.

Por lo tanto, f (x) no tiene asíntotas horizontales.



(b) $g(x)=\frac{4x^2+2x-2}{3x-1}$.

$$\lim_{x \to \frac{1}{3}^-} g\ (x) = \lim_{x \to \frac{1}{3}^-} \frac{4x^2+2x-2}{3x-1} = \frac{4(\frac{1}{3})^2+2\frac{1}{3}-2}{3(\frac{1}{3})-1} = \frac{4\frac{1}{9}+\frac{2}{3}-2}{1-1} = \frac{\frac{4}{9}+\frac{2}{3}-2}{0} = \frac{\frac{-8}{9}}{0} = +\infty.$$

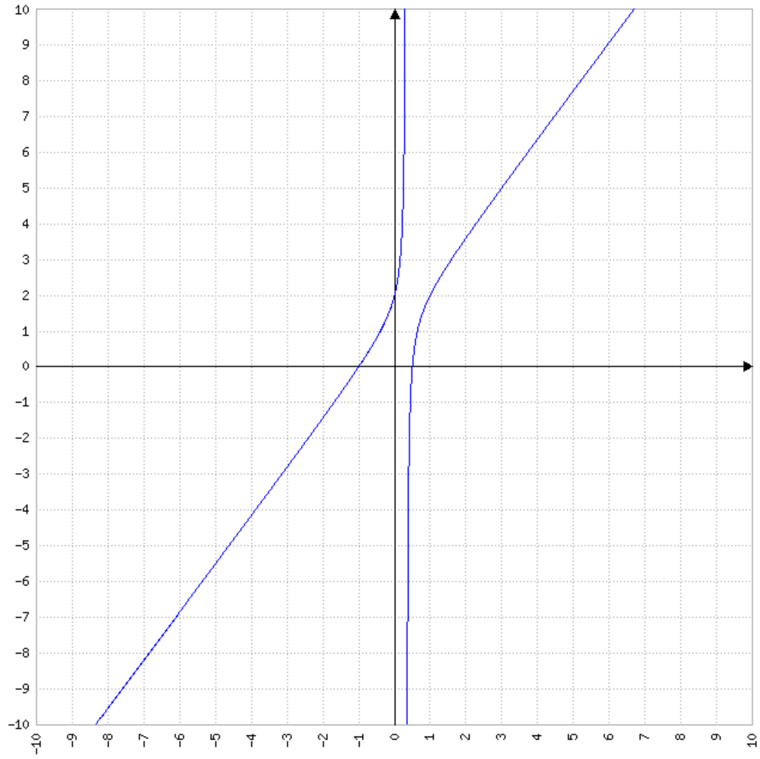
$$\lim_{x \to \frac{1}{3}^+} g\ (x) = \lim_{x \to \frac{1}{3}^+} \frac{4x^2+2x-2}{3x-1} = \frac{4(\frac{1}{3})^2+2\frac{1}{3}-2}{3(\frac{1}{3})-1} = \frac{4\frac{1}{9}+\frac{2}{3}-2}{1-1} = \frac{\frac{4}{9}+\frac{2}{3}-2}{0} = \frac{\frac{-8}{9}}{0} = -\infty.$$

Por lo tanto, g (x) tiene un asíntota vertical en x= $\frac{1}{3}$.

$$\lim_{x \to -\infty} g\ (x) = \lim_{x \to -\infty} \frac{4x^2+2x-2}{3x-1} = -\infty.$$

$$\lim_{x \to +\infty} g\ (x) = \lim_{x \to +\infty} \frac{4x^2+2x-2}{3x-1} = +\infty.$$

Por lo tanto, g (x) no tiene asíntotas horizontales.



(c) $h\ (x) = \frac{x-2}{x^2-4x+4}$.

$$x_1,\ x_2 = \frac{-(-4) \pm \sqrt{(-4)^2-4*1*4}}{2*1}$$
$$x_1,\ x_2 = \frac{4 \pm \sqrt{16-16}}{2}$$
$$x_1,\ x_2 = \frac{4 \pm \sqrt{0}}{2}$$
$$x_1,\ x_2 = \frac{4 \pm 0}{2}$$

$x_1 = \frac{4+0}{2} = \frac{4}{2} = 2.$
$x_2 = \frac{4-0}{2} = \frac{4}{2} = 2.$

h (x)= $\frac{x-2}{(x-2)^2}$
h (x)= $\frac{1}{x-2}$

$\lim_{x \to 2^-} h(x) = \lim_{x \to 2^-} \frac{1}{x-2} = \frac{1}{2-2} = \frac{1}{0} =$ -∞.

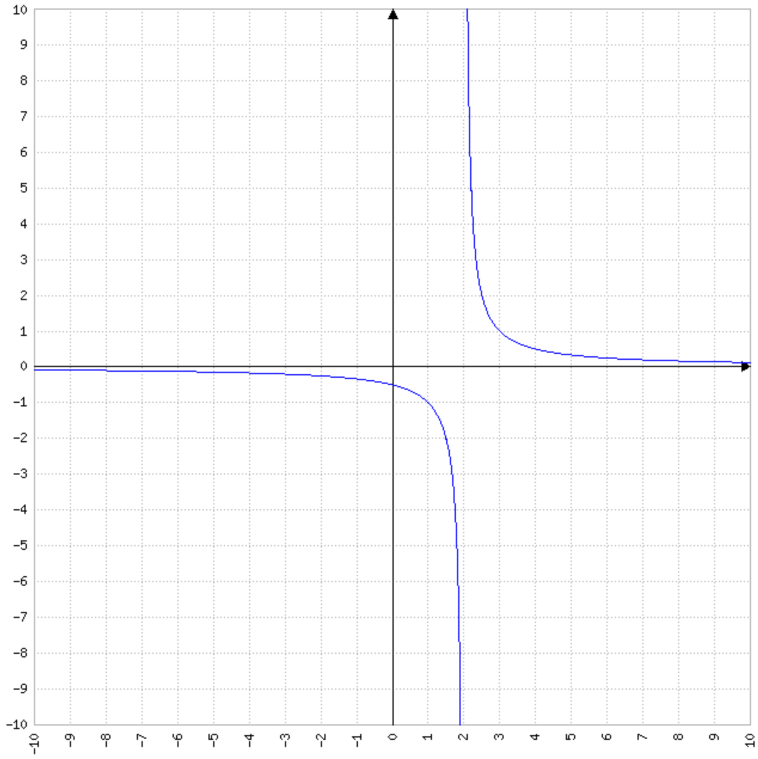
$\lim_{x \to 2^+} h(x) = \lim_{x \to 2^+} \frac{1}{x-2} = \frac{1}{2-2} = \frac{1}{0} = +\infty.$

Por lo tanto, h (x) tiene un asíntota vertical en x= 2.

$\lim_{x \to -\infty} h(x) = \lim_{x \to -\infty} \frac{1}{x-2} = 0.$

$\lim_{x \to +\infty} h(x) = \lim_{x \to +\infty} \frac{1}{x-2} = 0.$

Por lo tanto, h (x) tiene un asíntota horizontal en y= 0.



(d) *k (x)*= $\begin{cases} \frac{x}{x^2-1}, si\ x < 2 \\ 3x^3 - 2x, si\ x \geq 2 \end{cases}$.

$\lim_{x \to -1^-} k(x) = \lim_{x \to -1^-} \frac{x}{x^2-1} = \frac{1}{(-1)^2-1} = \frac{1}{1-1} = \frac{1}{0} = -\infty.$
$\lim_{x \to -1^+} k(x) = \lim_{x \to -1^+} \frac{x}{x^2-1} = \frac{1}{(-1)^2-1} = \frac{1}{1-1} = \frac{1}{0} = +\infty.$

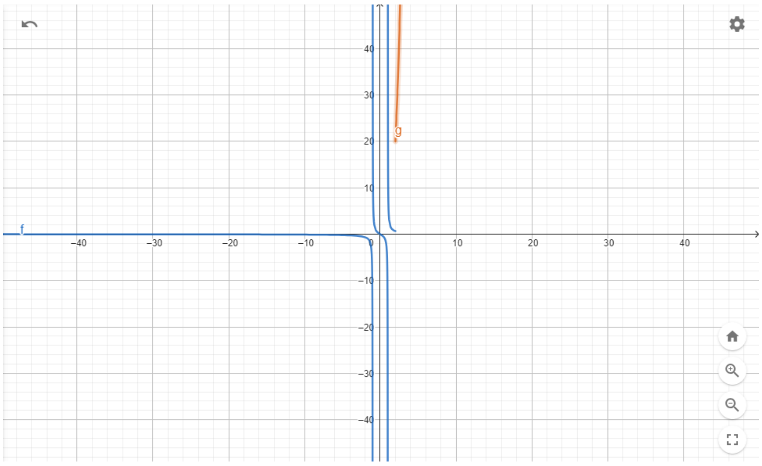
$\lim_{x \to 1^-} k(x) = \lim_{x \to 1^-} \frac{x}{x^2-1} = \frac{1}{1^2-1} = \frac{1}{1-1} = \frac{1}{0} = -\infty.$
$\lim_{x \to 1^+} k(x) = \lim_{x \to 1^+} \frac{x}{x^2-1} = \frac{1}{1^2-1} = \frac{1}{1-1} = \frac{1}{0} = +\infty.$

Por lo tanto, k (x) tiene asíntotas verticales en x= -1 y x= 1.

$\lim_{x \to -\infty} k(x) = \lim_{x \to -\infty} \frac{x}{x^2-1} = 0.$

$\lim_{x \to +\infty} k(x) = \lim_{x \to +\infty} 3x^3 - 2x = +\infty.$

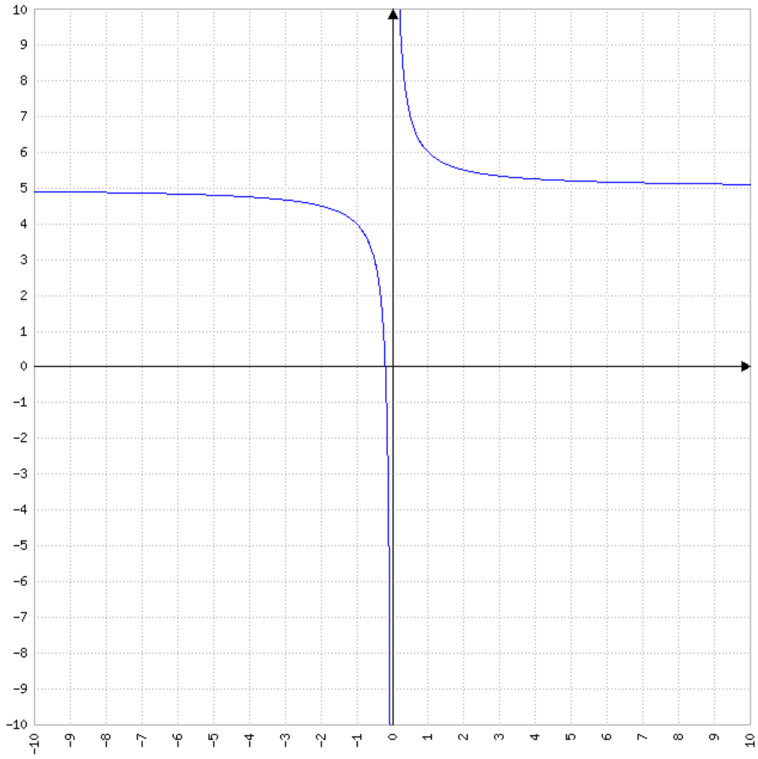
Por lo tanto, k (x) tiene un asíntota horizontal en y= 0.



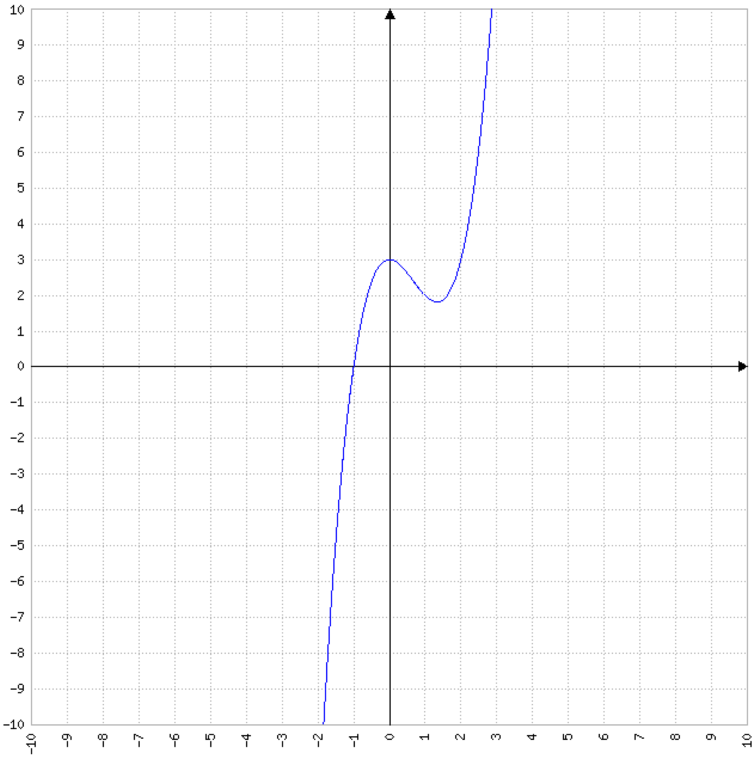
Ejercicio 8.

Utilizar GeoGebra para verificar, gráficamente, el comportamiento de las funciones de los Ejemplos (11) y (15).

Ejemplo 11:



Ejemplo 15:


10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10