$$\{X_1, \dots, X_n\} \overset{iid}{\sim} f(x;\theta) \longrightarrow \bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$\mu = E(X)$

$\uparrow$

- $E\{\bar{X}_n\} = \mu$
- $Var(\bar{X}_n) = \frac{Var(X)}{n}$
- $\underline{n \gg 0} \Rightarrow \bar{X}_n \sim_a N\left(\mu; \frac{Var(X)}{n}\right)$

Fact: $\underline{FN}$:

$$\{X_1, \dots, X_n\} \overset{iid}{\sim} Bern(\theta) \qquad T_n = \overbrace{\sum_{i=1}^{n} X_i}^{\sum_{i=1}^{n} t(X_i)} \qquad t(X) = X$$

$$X_1 \sim \underline{f(x;\theta)} = \mathbb{1}_{\{0,1\}}(x)\, \theta^x (1-\theta)^{1-x} = h(x)c(\theta)e^{w(\theta)t(x)}$$

$$\underline{X} \sim f(\underline{x};\theta) = \prod_{i=1}^{n} \mathbb{1}_{\{0,1\}}(x_i)\, \theta^{x_i} (1-\theta)^{1-x_i}$$

$$= \underbrace{\left(\prod_{i=1}^{n} \mathbb{1}_{\{0,1\}}(x_i)\right)}_{\uparrow\, h(\underline{x})} \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{n - \sum_{i=1}^{n} x_i}$$

$$= h(\underline{x}) \underbrace{(1-\theta)^n \left(\frac{\theta}{1-\theta}\right)^{\sum_{i=1}^{n} x_i}}_{g(T_n;\theta)}$$

$\underline{X} = \{X_1, \dots, X_n\} \overset{iid}{\sim} \text{Unif}[0, \theta)$ $\qquad T_n = ?$ suf. ?

$X_1 \sim f(x;\theta) = \mathbb{1}^{(x)}_{\{[0,\theta]\}} \dfrac{1}{\theta}$

$\underline{X} \sim f(\underline{x};\theta) = \prod_{i=1}^{n} \mathbb{1}^{(x_i)}_{\{[0,\theta]\}} \dfrac{1}{\theta}$

$$\prod_{i=1}^{n} \mathbb{1}^{(x_i)}_{\{[0,\theta]\}} = \begin{cases} 1 & \iff \begin{array}{l} X_{(1)}(\bar{x}) = \min\{X_1,\dots,X_n\} \geq 0 \text{ y} \\ X_{(n)}(\bar{x}) = \underbrace{\max\{X_1,\dots,X_n\}}_{T_n} \leq \theta \end{array} \\ 0 & \text{en otro caso} \end{cases}$$

$$= \mathbb{1}^{(\underline{X})}_{\{X_{(1)}(\bar{x}) \geq 0\}} \mathbb{1}^{(\underline{X})}_{\{X_{(n)} \leq \theta\}}$$

$$\underline{X} \sim f(\underline{x};\theta) = \underbrace{\mathbb{1}^{(\underline{X})}_{\{X_{(1)} \geq 0\}}}_{h(\underline{x})} \underbrace{\mathbb{1}^{(\underline{X})}_{\{X_{(n)} \leq \theta\}} \frac{1}{\theta^n}}_{g(T_n;\theta)}$$

$X \sim \text{Geom}(\theta)$

$$f(x;\theta) = \mathbb{1}^{(x)}_{\{\mathbb{N}\}} \theta (1-\theta)^{x-1} = \mathbb{1}^{(x)}_{\{\mathbb{N}\}} \frac{\theta}{1-\theta} (1-\theta)^x$$

$$= \underbrace{\mathbb{1}^{(x)}_{\{\mathbb{N}\}}}_{h(x)} \underbrace{\frac{\theta}{1-\theta}}_{c(\theta)} \exp\{\underbrace{x}_{} \underbrace{\ln(1-\theta)}_{\omega(\theta)}\}$$

$t(x) = x$

$\{X_1, \dots, X_n\} \overset{iid}{\sim} \text{Geom}(\theta)$

$$T_n = \sum_{i=1}^{n} X_i$$

$\{X_1 = 1, X_2 = 1, X_3 = 0\}$ $\quad X \sim \text{Bern}(\theta)$

$$f(x; \theta) = \theta^x (1-\theta)^{1-x}$$

$$L(\theta) = f(x_1 = 1; \theta) f(x_2 = 1; \theta) f(x_3 = 0; \theta)$$

$$= \theta^1 (1-\theta)^0 \; \theta^1 (1-\theta)^0 \; \theta^0 (1-\theta)^1$$

$$L(\theta) = \theta^2 - \theta^3$$

$$\frac{L(\theta = 1/2)}{L(\theta = 2/3)} = \frac{\left(\frac{1}{2}\right)^2 - \left(1/2\right)^3}{\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3} = 0.88 < 1$$

Métodos Momentos:

$\underline{X} = \{X_1, \ldots, X_n\} \overset{iid}{\sim} \mathrm{Bern}(\theta)$

$$\mu_1(\theta) = E(X) = \theta$$

$$M_1(\underline{X}) = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}_n$$

$$\mu_1 = M_1 \Rightarrow \boxed{\tilde{\theta}_n = \bar{X}_n}$$

Normal: $\{X_1, \ldots, X_n\} \overset{iid}{\sim} N(\underbrace{\mu, \sigma^2}_{\theta})$

$$\mu_1(\theta) = E(X) = \mu$$

$$\mu_2(\theta) = E(X^2) = \sigma^2 + \mu^2$$

$$M_1 = \bar{X}_n$$

$$M_2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 = \frac{n-1}{n} S_n^2 + \bar{X}_n^2$$

$$\rightarrow \frac{n-1}{n} S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2 = \left(\frac{1}{n}\sum_{i=1}^{n} X_i^2\right) - \bar{X}_n^2$$

$\mu_1 = M_1 \Rightarrow \boxed{\tilde{\mu}_n = \bar{X}_n}$

$\mu_2 = M_2 \Rightarrow \sigma^2 + \mu^2 = \frac{n-1}{n} S_n^2 + \bar{X}_n^2$

$\sigma^2 + \cancel{\bar{X}_n^2} = \frac{n-1}{n} S_n^2 + \cancel{\bar{X}_n^2}$

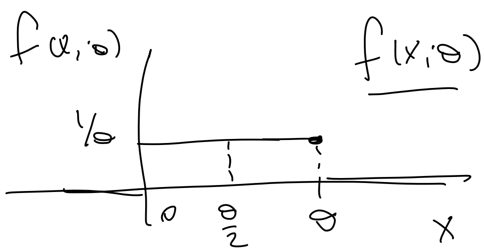
$\tilde{\sigma}_n^2 = \frac{n-1}{n} S_n^2 = \frac{1}{n}\sum_{i=1}^{n} (X_i - \bar{X}_n)^2$

$X \sim \text{Unif}(0, \theta)$

$\{X_1, \ldots, X_n\} \overset{iid}{\sim} \text{Unif}(\theta) \quad T_n = X_{(n)}$

$\mu_1(\theta) = \frac{\theta}{2}$

$M_1 = \bar{X}_n$



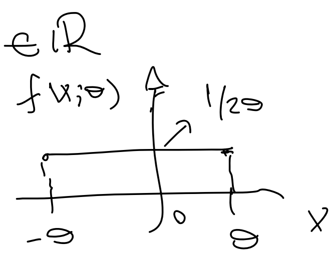
$f(x;\theta) = \begin{cases} \frac{1}{\theta} & \text{si } x \in (0,\theta) \\ 0 & \text{otro caso} \end{cases}$

$\mu_1 = M_1 \Rightarrow \frac{\theta}{2} = \bar{X}_n$

$\boxed{\tilde{\theta}_n = 2\bar{X}_n}$

$X \sim \text{Unif}(\underbrace{-\theta}_{a} ; \underbrace{\theta}_{b})$

$\theta \in \mathbb{R}$



$E(X) = 0 = \mu_1(\theta)$

$$\boxed{E(X^2) = \frac{\theta^2}{3} = \mu_2(\theta)} \qquad M_2 = \frac{1}{n}\sum_{i=1}^{n} X^2$$

$$\mu_2 = M_2 = \frac{\theta^2}{3} = \frac{1}{n}\sum_{i=1}^{n} X_i^2$$

$$\boxed{\tilde{\theta}_n = \sqrt{\frac{3}{n}\sum_{i=1}^{n} X_i^2}}$$