Problem 1

First, we review the model Hamilton (1989) uses. After some calulations, Hamilton uses the following specification for GNP growth (given by variable y_t)

$$y_t = \alpha_0 + \alpha_1 S_t + z_t \tag{1}$$

Where S_t is a first order, two state Markov process and z_t is used to adress the AR structure of GNP growth (we'll discuss that in a moment).
In this case, S_t is the unobserved state variable. It can only take two values: $S_t = 1$ or $S_t = 0$. In this case, as $\alpha_1 > 0$, $S_t = 1$ is the 'expansion' state, whereas $S_t = 0$ is 'recession'. We notice that the constant in the process conditional on $S_t = 0$ is α_0, and coditional in $S_t =!$ the constant is given by $\alpha_0 + \alpha_1$, a higher value. We denote $P(S_t = 0|S_{t-1} = 0) = q$ and $P(S_t = 1|S_{t-1} = 1) = p$ as the transition probabilities.
On the other hand, z_t is taken to be an AR(4) process to model the structure of GNP growth. That is, z_t is given by:

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \phi_3 z_{t-3} + \phi_4 z_{t-4} + \varepsilon_t \tag{2}$$

With $\varepsilon_t \sim N(0, \sigma^2)$. Note that by specifying the model this way, we are saying that the coefficients in the AR part **do not switch** between states. Variance of innovations is also constant among regimes. This could be changed if we wanted to. However, note that this type of models is highly sensitive to changes in what we are allowing to switch among regimes.
In order to estimate the model, we use the Gauss code *EX1.g*, which is an adaptation of the original Hamiltion's code [1]. The structure of the code is similar to the one before: data is loaded, starting values are set, and optimization funtions are called.

Estimated parameters are printed in GAUSS as This can be written more neatly in a table. Filetered and smoothed probs are given in the figure below.

[1] This one is called *maxseek.g* and can be found in the folder given

```
Vector is reparameterized to report final results as follows
Means for each state:
       1.1635158       -0.35881273
Autoregressive coefficients:
     0.013487767      -0.057518296      -0.24698395       -0.21292052
Variances:
      0.59136706

Matrix of Markov transition probabilities:

      0.90408497        0.24532961
     0.095915032        0.75467039

Ergodic probs for full state vector:
      0.48030944       0.050956378       0.013827360       0.042535017       0.013827360       0.0014669547       0.011542166       0.035

Ergodic probs for primitive states:
      0.71892590        0.28107410

Log likelihood:
      -60.882447
for vector of coefficients parameterized as follows,
       1.1635158       -0.35881273       0.013487767      -0.057518296       -0.24698395       -0.21292052        0.59136706        0.90
the standard errors are
     0.074707737        0.26602279        0.13261991        0.13888266        0.10800718        0.11257879        0.10355399        0.037
```

Coefficent	Estimation	SE
α_0	-0.35881273	0.26602279
$\alpha_0 + \alpha_1$	1.1635158	0.074707737
ϕ_1	0.013487767	0.13261991
ϕ_2	-0.057518296	0.13888266
ϕ_3	-0.24698395	0.10800718
ϕ_4	-0.21292052	0.11257879
σ^2	0.59136706	0.10355399
p	0.90408497	0.037761445
q	0.75467039	0.096536454

Cuadro 1: Replication of Table 1 in Hamiltion(1989)

It's important to know the difference between both kinds of probabilities. **Filtered** probabilities are given as a direct product of the filter. These are given by $P(S_t = 0|y_t, y_{t-1}, ...)$. That is, **we use information available only up to** t. This is the type of inference we could do in real time, recursively, to estimate the probabilities of being in a given state today.

On the other hand, **smoothed** probabilities are computed based on all sample information, that is, $P(S_t = 0|y_T, y_{T-1}, ..)$. Note that in this case, we condition on all sample observations, not only in those previous to t. These two types of probabilities can be compared to assess whether the model can accurately predict which the current state is. If both types of probabilities are similar, then the model can be thought to be performing well, because adding future

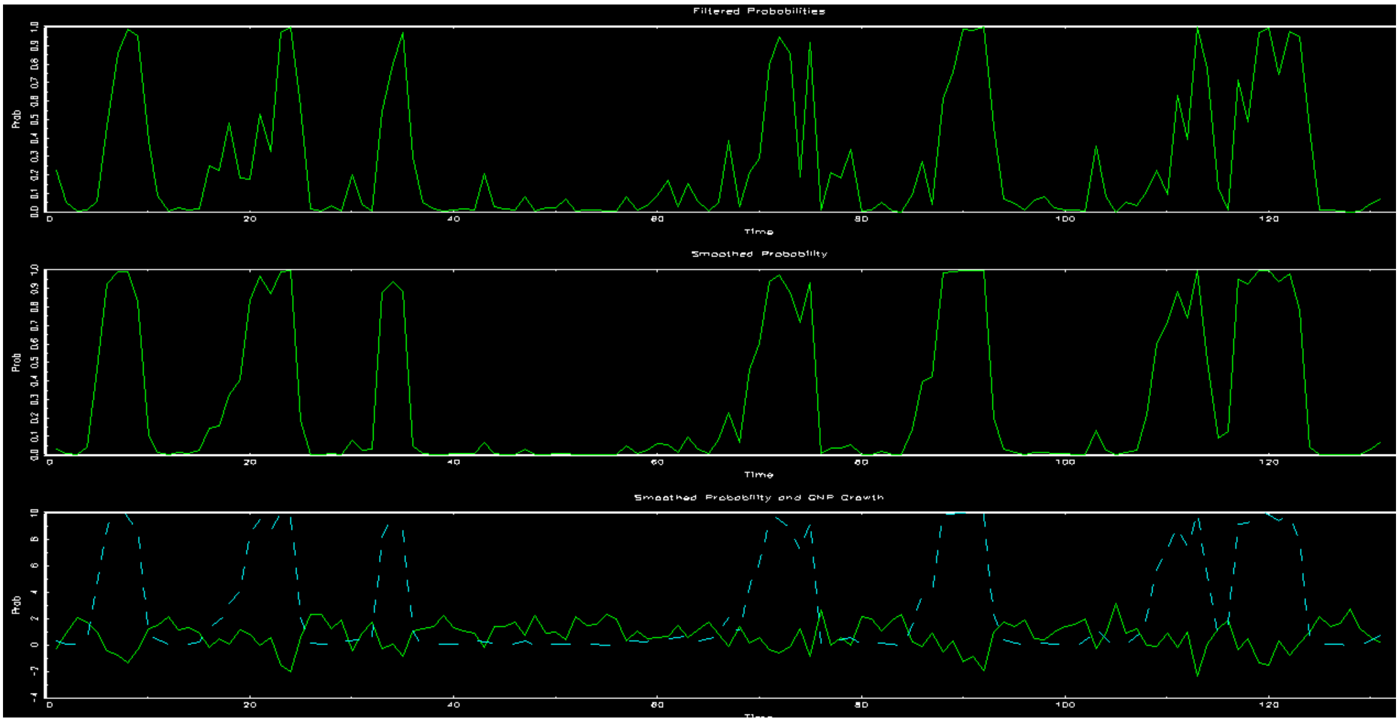


Figura 1: Filtered and smoothed probabilities

information does not alter the probabilities substantially. On the contrary, were the two types of probabilities appreciably different, then the model would not be performing very well.

In this case, we see that both types seem similar, identifying the same recessions. Remeber that, with this type of models, we can identify recessions by a rule of thumb such as "We identify that the economy is in recession in period t whenever $P(S_t = 0|y_t, y_{t-1}...) > 0{,}5$ (we could use smoothed probbilities as well. This would make more sense if our objective is to identify historical recessions as in Hamilton(1989)). The table show the results for Hamilton's paper.

TABLE II

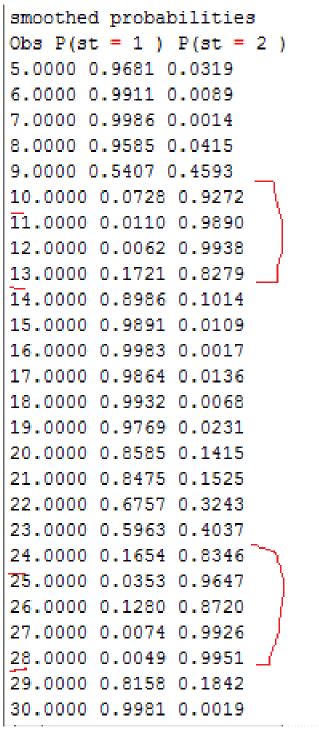
Alternative Dating of U.S. Business Cycle Peaks and Troughs as Determined by (1) NBER, and (2) Probability of Being in Recession Greater than 0.5 as Determined from Full-Sample Smoother

NBER		Smoother	
Peak	Trough	Peak	Trough
1953 : III	1954 : II	1953 : III	1954 : II
1957 : III	1958 : II	1957 : I	1958 : I
1960 : II	1961 : I	1960 : II	1960 : IV
1969 : IV	1970 : IV	1969 : III	1970 : IV
1973 : IV	1975 : I	1974 : I	1975 : I
1980 : I	1980 : III	1979 : II	1980 : III
1981 : III	1982 : IV	1981 : II	1982 : IV

Figura 2: Identificated recessions in Hamilton (1989)

We can achive the same results by seeing the Smoothed probability series, which is printed when we run the code. We literally read the series and manually identify the values higher than

0,5



```
smoothed probabilities
Obs P(st = 1 ) P(st = 2 )
5.0000 0.9681 0.0319
6.0000 0.9911 0.0089
7.0000 0.9986 0.0014
8.0000 0.9585 0.0415
9.0000 0.5407 0.4593
10.0000 0.0728 0.9272
11.0000 0.0110 0.9890
12.0000 0.0062 0.9938
13.0000 0.1721 0.8279
14.0000 0.8986 0.1014
15.0000 0.9891 0.0109
16.0000 0.9983 0.0017
17.0000 0.9864 0.0136
18.0000 0.9932 0.0068
19.0000 0.9769 0.0231
20.0000 0.8585 0.1415
21.0000 0.8475 0.1525
22.0000 0.6757 0.3243
23.0000 0.5963 0.4037
24.0000 0.1654 0.8346
25.0000 0.0353 0.9647
26.0000 0.1280 0.8720
27.0000 0.0074 0.9926
28.0000 0.0049 0.9951
29.0000 0.8158 0.1842
30.0000 0.9981 0.0019
```

Figura 3: Identificating recessions from estimation output

Beginning	(as date)	End	(as date)
10	3Q53	13	2Q54
24	1Q57	28	1Q58
37	2Q60	39	4Q60
74	3Q69	79	4Q70
92	1Q74	96	1Q75
113	2Q79	118	3Q80
121	2Q81	127	4Q82

Cuadro 2: Our identification for recessions

Eviews

In order to do this in Eviews, we have to go to *Quick/Estimate Equation* and select the *Switching regression* option. We'll get a window where we specify the model we want. In this case, we want to explain variable g (GNP growth) with a switching constant, and 4 AR terms which are constant between regimes. Therefore, we specify the model as shown in the figure.

We want regime variances to be non-switching, therefore we don't tick the corresponding box. We choose *Markov* in regime type, and just use a constant as a probability regressor (we could include exogenous variables to predict the regime switch). In the options window, we can control optimization settings. In this case, I'll choose a bigger number of initial random starts (perhaps 200) to increase the change of getting a good set of initial values. We can also ask Eviews to iterate longer for each random start.

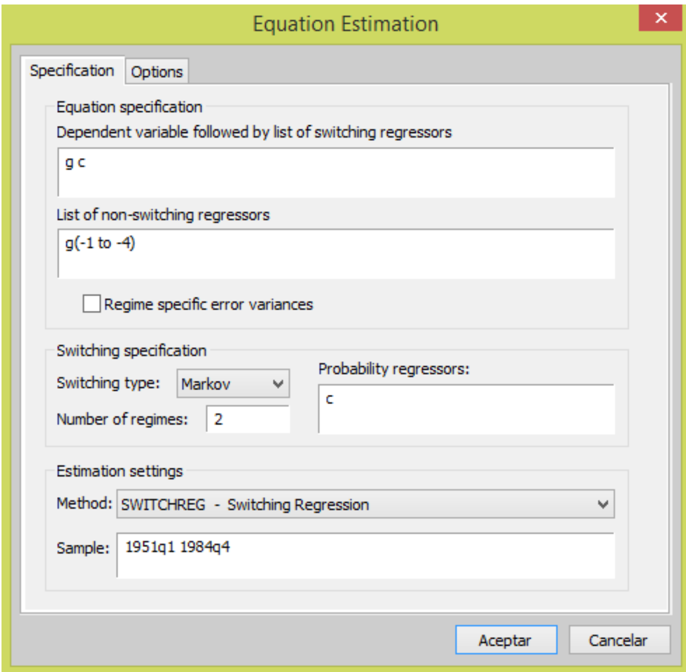


Figura 4: Estimation specification

Estiamtion output is shown in the figure below.

We can see the estimated constants for each regime, and the AR terms. To get the transtion probs, we go to *View/Regime Results/Transition Probabilities* to get the transition matrix. We can also plot regime probabilities by clicking *View/Regime Results/Regime Probabilities*

We can see that the estimated parameters are similar to those in Hamilton. To find the business cicles, we can use the same method as before. In order to do that, we would need smoothed probabilities. To create that series, we may click in *Proc/Make Regime Resuts/Make regimes probabilities results*. This will create two series: the probabilities of being in each state. In our estimation, $S_t = 2$ is 'recession', therefore, we should check which intervals the probability of being in $S_t = 2$ was higher than 0,5.

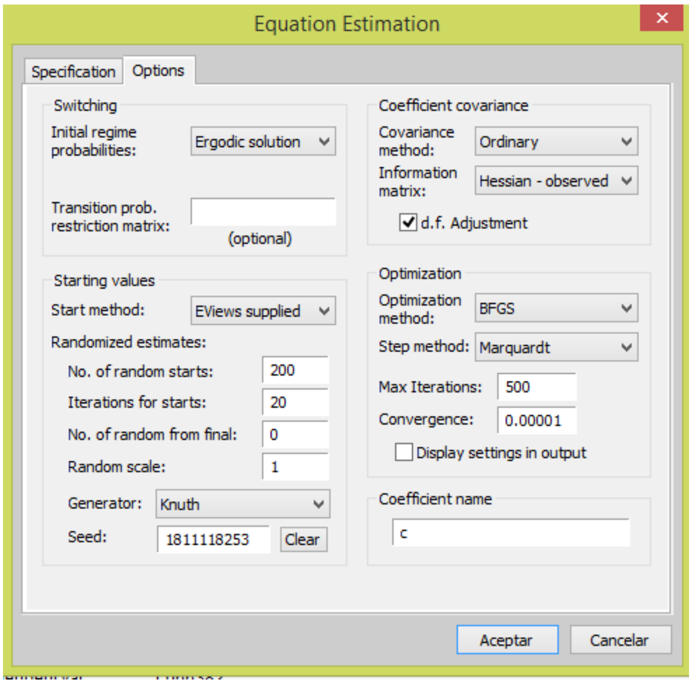


Figura 5: Estimation options

Dependent Variable: G
Method: Markov Switching Regression (BFGS / Marquardt steps)
Date: 05/07/20 Time: 09:02
Sample (adjusted): 1952Q2 1984Q4
Included observations: 131 after adjustments
Number of states: 2
Initial probabilities obtained from ergodic solution
Standard errors & covariance computed using observed Hessian
Random search: 200 starting values with 20 iterations using 1 standard deviation (rng=kn, seed=1811118253)
Convergence achieved after 12 iterations

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Regime 1				
C	1.112971	0.193821	5.742246	0.0000
Regime 2				
C	-0.447392	0.278644	-1.605602	0.1084
Common				
G(-1)	0.111763	0.099572	1.122432	0.2617
G(-2)	0.064701	0.084419	0.766428	0.4434
G(-3)	-0.126221	0.083188	-1.517298	0.1292
G(-4)	-0.135633	0.084268	-1.609536	0.1075
LOG(SIGMA)	-0.236864	0.082602	-2.867515	0.0041
Transition Matrix Parameters				
P11-C	2.345034	0.518428	4.523352	0.0000
P21-C	-0.700118	0.634413	-1.103567	0.2698

Mean dependent var	0.719835	S.D. dependent var	1.066382
S.E. of regression	1.002124	Sum squared resid	124.5274
Durbin-Watson stat	1.988752	Log likelihood	-180.1844
Akaike info criterion	2.888311	Schwarz criterion	3.085843
Hannan-Quinn criter.	2.968577		

Figura 6: Estimation output

We see that identifaction of recessions differ slightly from before, but its qualitatively identical.

Equation: UNTITLED
Date: 05/07/20 Time: 09:05
Transition summary: Constant Markov transition probabilities and expected durations
Sample (adjusted): 1952Q2 1984Q4
Included observations: 131 after adjustments

Constant transition probabilities:
P(i, k) = P(s(t) = k | s(t-1) = i)
(row = i / column = j)

	1	2
1	0.912539	0.087461
2	0.331786	0.668214

Constant expected durations:

1	2
11.43363	3.013989

Figura 7: Transition probabilities (Problem 1)

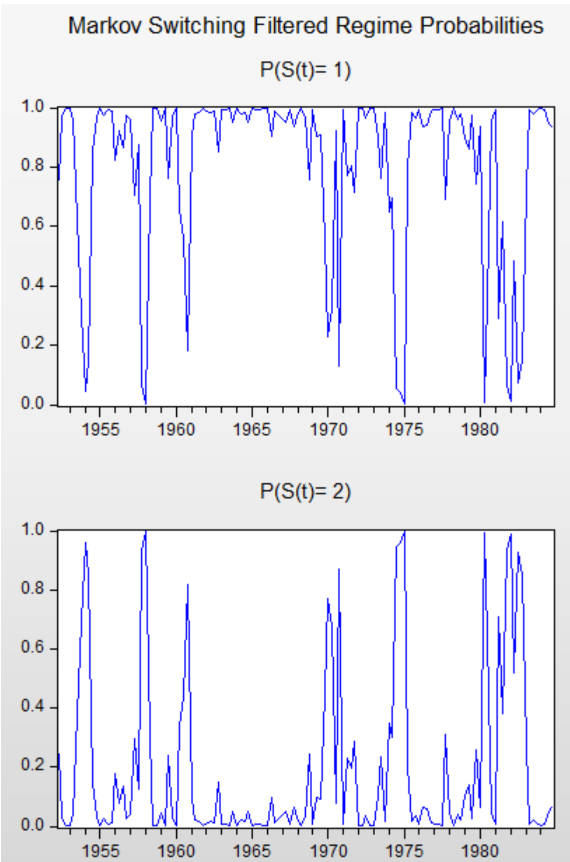


Figura 8: Filtered probabilities (Problem 1)

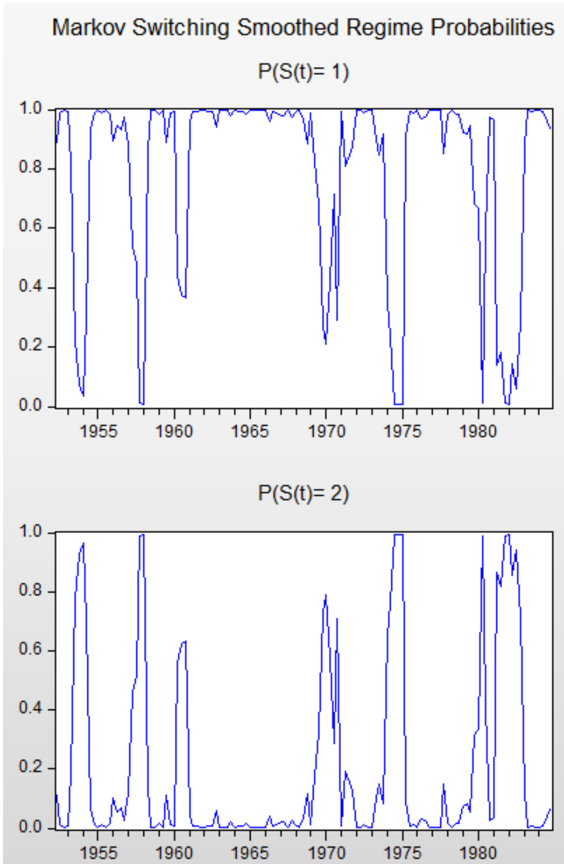


Figura 9: Smoothed probabilities (Problem 1)

Problem 2

In this case, we'll use the code *EX1AR.g*, which is practically the same as before, but it loads different data (Argentina Industrial Production Index (IPI)). We proceed the same way as before. Estimated parameters are shown in the table. Filtered and smoothed probabilities are shown in the graph. Recessions infered form filtered probs are shown in the second table.

Beginning	End
3Q53	2Q54
3Q57	1Q58
2Q60	4Q60
4Q69	4Q70
1Q74	1Q75
2Q80	3Q80
2Q81	4Q82

Cuadro 3: Our identification for recessions with Eviews (Problem 1)

Coefficent	Estimation	SE
α_0	-1.3561484	0.12422302
$\alpha_0 + \alpha_1$	0.72273491	0.23052093
ϕ_1	-0.46851926	0.080949784
ϕ_2	-0.099299241	0.088757844
ϕ_3	0.074577172	0.10040041
ϕ_4	0.098393558	0.082183766
σ^2	2.6050418	0.35296044
p	0.94515804	0.031230593
q	0.79182751	0.090883024

Cuadro 4: Estimated parameters

Beginning	(as date)	End	(as date)
7	Jul95	9	Sept95
43	Jul98	55	Jul99
61	Jan00	66	Jun00
73	Jan01	87	Mar02
111	Mar04	112	Abr04
166	Oct08	170	Feb09
207	Mar12	209	May12

Cuadro 5: Industrial recessions according to our model

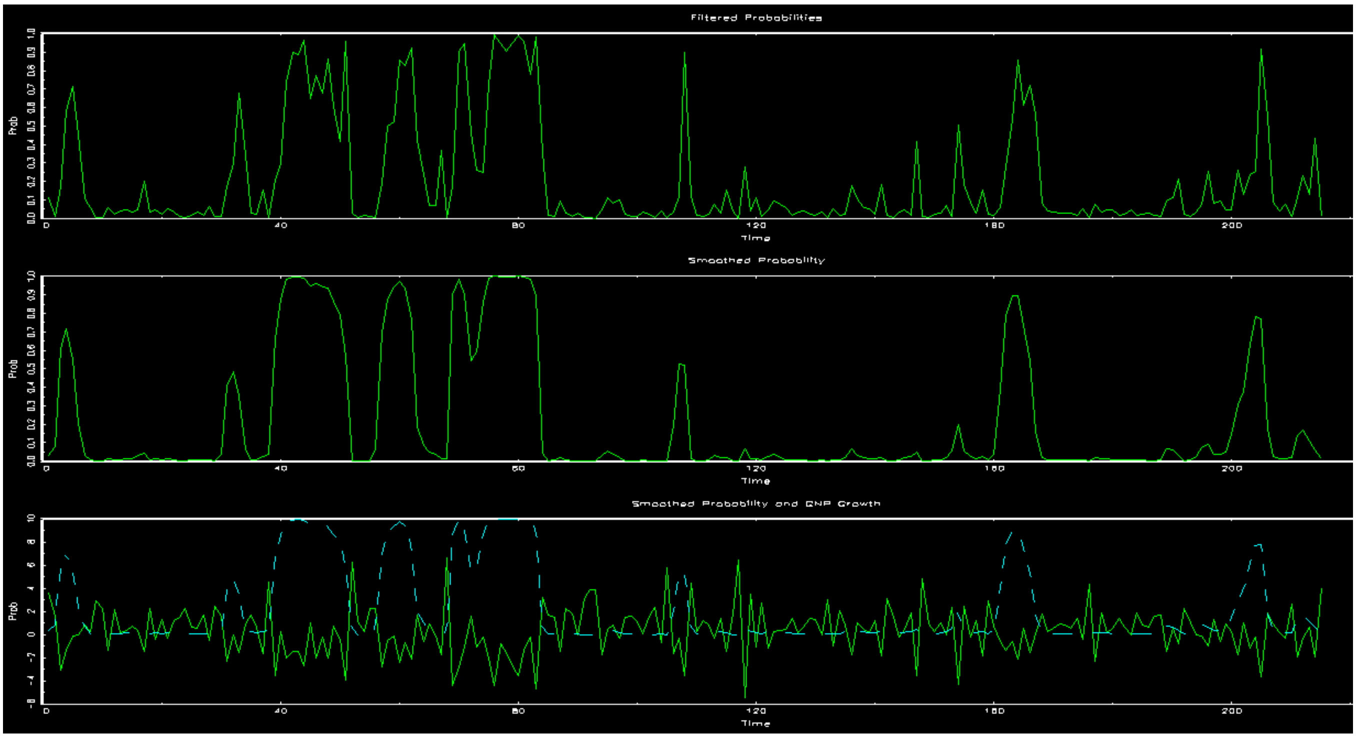


Figura 10: Estimated probabilities for each regime

Problem 3

In order to estimate this, we'll the file *US3S.g*. The code is very much the same, but with 3 states. In this case, we just assume one lag. Estimation output is shown in the figure. We can

```
Vector is reparameterized to report final results as follows
Means for each state:
-1.0677 0.2863 1.1728
Autoregressive coefficients:
0.0861
Variances:
0.5807

Matrix of Markov transition probabilities:

0.5012 0.2568 0.0000
0.0000 0.7432 0.0835
0.4988 0.0000 0.9165

Ergodic probs for full state vector:
0.0562 0.0000 0.0559 0.0559 0.1619 0.0000 0.0000 0.0559 0.6140

Ergodic probs for primitive states:
0.1122 0.2178 0.6700
```

Figura 11: Estimation output

see from the means that $S_t = 1$ correspond to recession, $S_t = 2$ to stagnation and $S_t = 3$ to expansion. Note that the transition matrix can be written as:

$$\begin{pmatrix} P(S_t = 1|S_t = 1) & P(S_t = 1|S_t = 2) & P(S_t = 1|S_t = 3) \\ P(S_t = 2|S_t = 1) & P(S_t = 2|S_t = 2) & P(S_t = 2|S_t = 3) \\ P(S_t = 3|S_t = 1) & P(S_t = 3|S_t = 2) & P(S_t = 3|S_t = 3) \end{pmatrix} = \begin{pmatrix} 0{,}5012 & 0{,}2568 & 0{,}0000 \\ 0{,}0000 & 0{,}7432 & 0{,}0835 \\ 0{,}4988 & 0{,}0000 & 0{,}9165 \end{pmatrix}$$

Note that some transition probabilities are equal to zero. For example, $P(S_t = 2|S_t = 1) = 0$, that means that, when exiting a recession, economy goes directly to expansion. We can also deduct that the economy never transitions from stagnation to expansion, and that it never transition from expansion directly to recession. That is, we find that the transition cycle can be summarized as *Expansion* $\rightarrow$ *Stagnation* $\rightarrow$ *Recession* $\rightarrow$ *Expansion*. If we keep our criteria

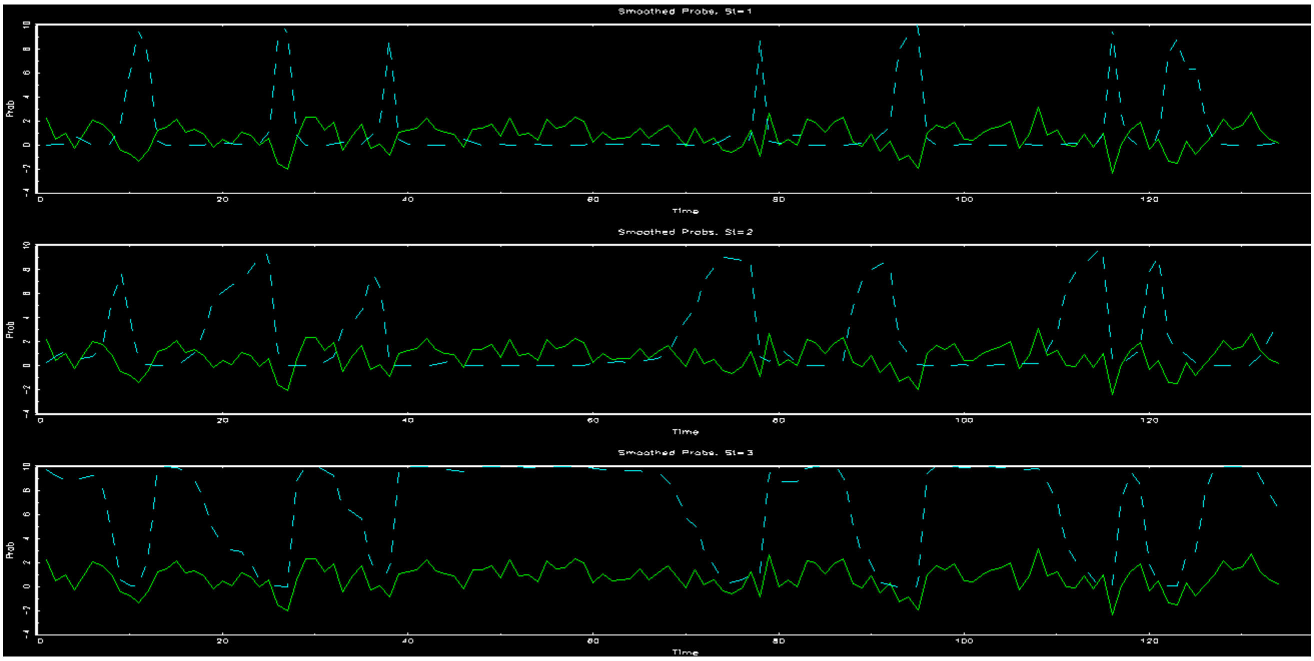


Figura 12: Smoothed Probabilities

of finding recession where $P(S_t = 1) > 0{,}5$ then the recession dates are If we compare these results to those found in exercise 1, we see that all of the recession periods found here where included in recessions found in the first excercise, but lasting less time according to the current model. Now, some of the quarters previously found as recessions are classified as mild growth instead.

Eviews

We can do the same with Eviews. Proceding the same way as before, but estimating 3 regimes and just one lag in the AR terms. In the options term, we use a lot of initial randomized starting

Beginning	(as date)	End	(as date)
11	3Q53	13	1Q54
27	3Q57	28	4Q58
39	3Q60	39	3Q60
79	3Q70	79	3Q70
94	2Q74	96	4Q74
117	2Q80	117	2Q80
123	1Q82	126	3Q82

Cuadro 6: Our identification for recessions

values in order to get a better chance of converge in the algorithm.

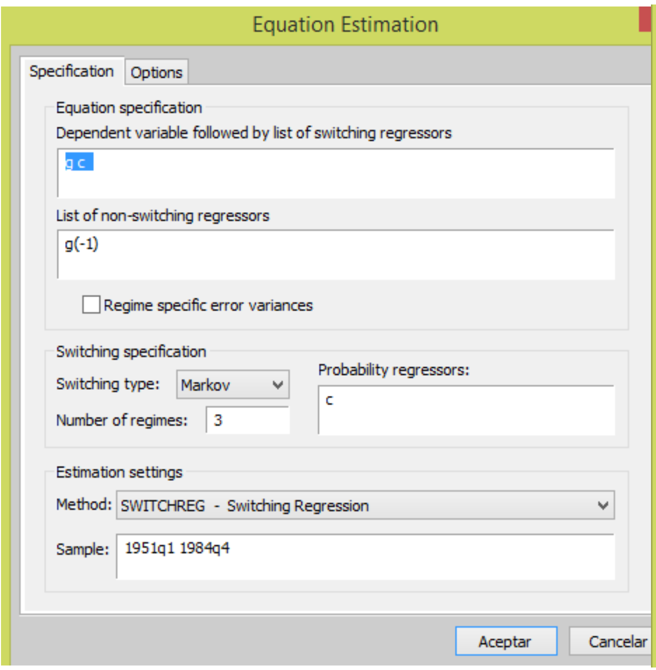


Figura 13: Estimation settings

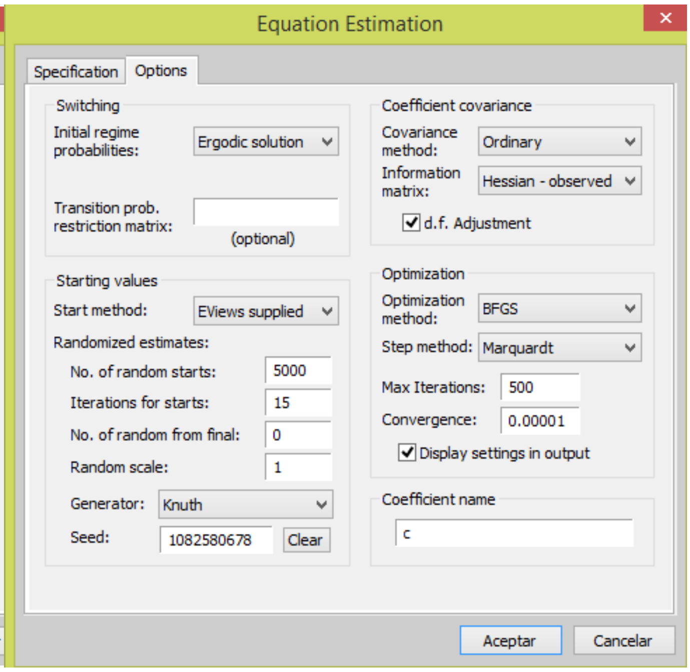


Figura 14: Estimation options

Nevertheless, estimation output shown in the figure seems to have some issues.

What's going on? The problem here is not the optimization algorithm actually. Note that estimated coefficients are the same as in GAUSS. The problem here is that estimated transtion probabilities for some states are zero, which is a bound for the parameter set. As we know from previous courese, maximum likelihood properties assume that we have an interior solution. Were not be the case, then we can't use the Hessian to compute standard errors. GAUSS output issues a warning about that. Therefore, the problem is not Eviews, but the fact we are estimating an unrestricted model when we should estimate a restricted one. In order to fix thix,

Dependent Variable: G
Method: Markov Switching Regression (BFGS / Marquardt steps)
Date: 05/07/20 Time: 10:02
Sample (adjusted): 1951Q3 1984Q4
Included observations: 134 after adjustments
Number of states: 3
Initial probabilities obtained from ergodic solution
Standard errors & covariance computed using observed Hessian
Random search: 5000 starting values with 15 iterations using 1 standard deviation (rng=kn, seed=1082580678)
Estimation settings: tol= 1.0e-05, derivs=analytic (linear)
Initial Values: C(1)=0.26140, C(2)=-1.14165, C(3)=1.21443, C(4)=0.08273, C(5)=-0.34644, C(6)=1.56676, C(7)=0.18898, C(8)=-2.06149, C(9)= -0.06501, C(10)=-1.59875, C(11)=-11352.7
Failure to improve objective (non-zero gradients) after 37 iterations
WARNING: Singular covariance - coefficients are not unique

Variable	Coefficient	Std. Error	z-Statistic	Prob.
		Regime 1		
C	0.233920	NA	NA	NA
		Regime 2		
C	-1.119395	NA	NA	NA
		Regime 3		
C	1.070939	NA	NA	NA
		Common		
G(-1)	0.095024	NA	NA	NA
LOG(SIGMA)	-0.271906	NA	NA	NA

Figura 15: Estimation output

```
Log likelihood:
-60.6718
Negative of Hessian is not positive definite
Either you have not found local maximum, or else estimates are up against boundary condition.
In latter case, impose the restricted params rather than estimate them to calculate standard errors
-------------------------------
```

Figura 16: GAUSS warning

we will estimate a restricted model, imposing restrictions in the transition matrix. From what we've done, we know that we have to restrict elements $(2,1)$, $(3,2)$ and $(1,3)$ to zero. To do that in Eviews, we have to define the matrix shown in the figure. In order to do so, we have to go to *Object/New object* then we select *Matrix-Vector-Coef* form the list. We choose a 3 rows, 3 columns matrix. Then we manually complete the cells in order to get what we need. The *NA* cells indicate that those coefficents need to be estimated, whereas the 0's indicate the restrictions. We leave a cell without a zero because Eviews crashes if we want to use the full

	C1	C2	C3
	Last updated: 05/07/20 - 10:35		
R1	NA	NA	NA
R2	0.000000	NA	NA
R3	NA	0.000000	NA

Figura 17: Restriction Matrix

restricted version. Neverthelss, we will show that the estimated coeficient is zero anyway. Once we type what we need, we click on *Name* to store the matrix in the data base. With this matrix generated, we have to reestimate the model, but with this restriction matrix. In order to do so, we specify everything as before, but add m (I called the matrix m)as a restriction matrix in the options as shown in the figure. We estimate and get the output shown in the figure.

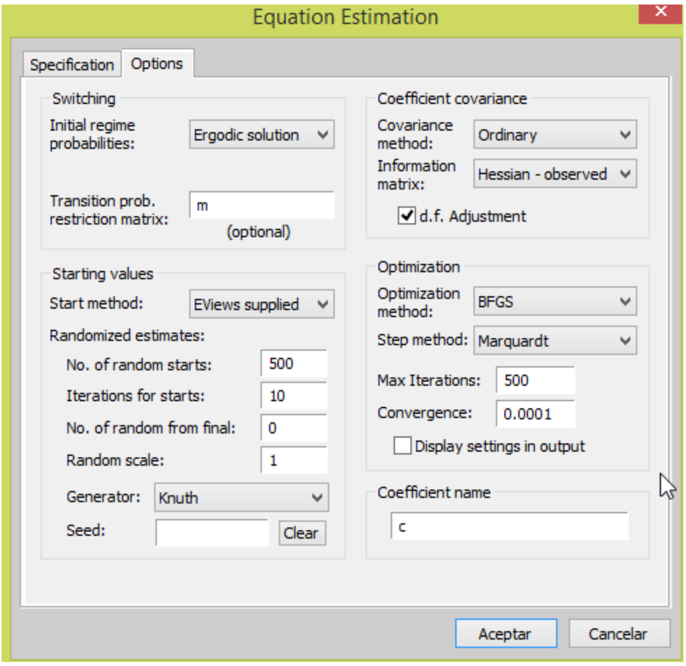


Figura 18: Options for restricted estimation

Dependent Variable: G
Method: Markov Switching Regression (BFGS / Marquardt steps)
Date: 05/07/20 Time: 10:36
Sample (adjusted): 1951Q3 1984Q4
Included observations: 134 after adjustments
Number of states: 3
Fixed probability matrix: M
Initial probabilities obtained from ergodic solution
Standard errors & covariance computed using observed Hessian
Random search: 25 starting values with 10 iterations using 1 standard
deviation (rng=kn, seed=555233790)
Convergence achieved after 42 iterations

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Regime 1				
C	0.233921	0.263977	0.886143	0.3755
Regime 2				
C	-1.119393	0.397603	-2.815358	0.0049
Regime 3				
C	1.070940	0.182461	5.869407	0.0000
Common				
G(-1)	0.095023	0.101715	0.934213	0.3502
LOG(SIGMA)	-0.271906	0.079916	-3.402400	0.0007

Figura 19: Estimation output

We see that estimated coefficents are the same as before, but we get the SE calculated from the Hessian. Estimated transition matrix is shown in the figure [2]. We note that the matrix is

Equation: EQ01
Date: 05/07/20 Time: 10:43
Transition summary: Constant Markov transition probabilities and expected durations
Sample (adjusted): 1951Q3 1984Q4
Included observations: 134 after adjustments

Constant transition probabilities:
P(i, k) = P(s(t) = k | s(t-1) = i)
(row = i / column = j)

	1	2	3
1	0.746512	0.253488	1.79E-09
2	0.000000	0.454878	0.545122
3	0.083241	0.000000	0.916759

Constant expected durations:

	1	2	3
	3.944959	1.834452	12.01336

Figura 20: Transition matrix

the same we've estimated in GAUSS (note that regimes are in different order). Filtered and smoothed probs are shown in the figure. We note that in this case, the difference among the two types of probs are bigger than we are used to. Nevertheless, results are pretty much the same as in GAUSS.

[2]The fact that an estimated probability is 0 and Eviews does not present problems remains an open question to me

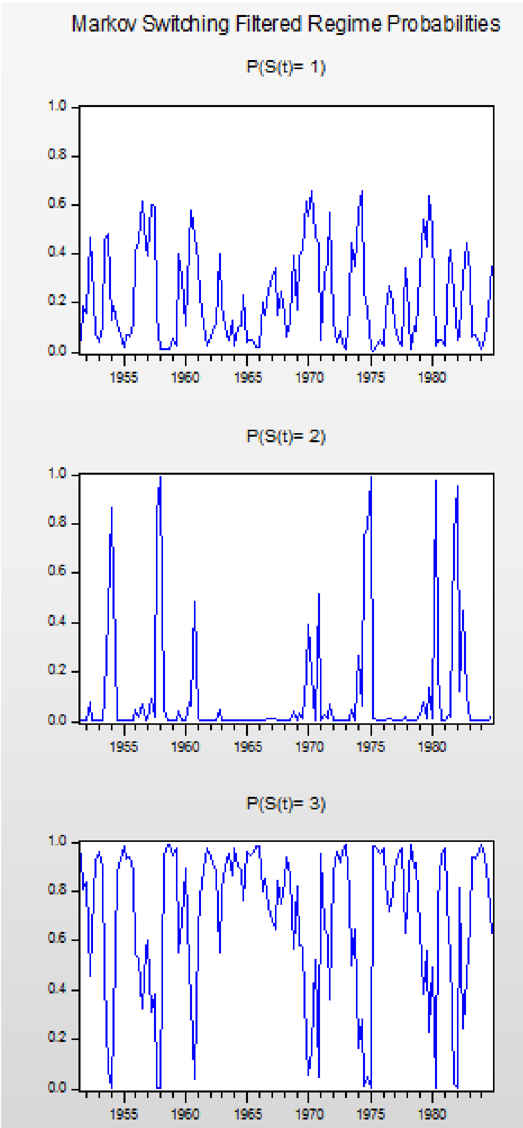


Figura 21: Filtered probs

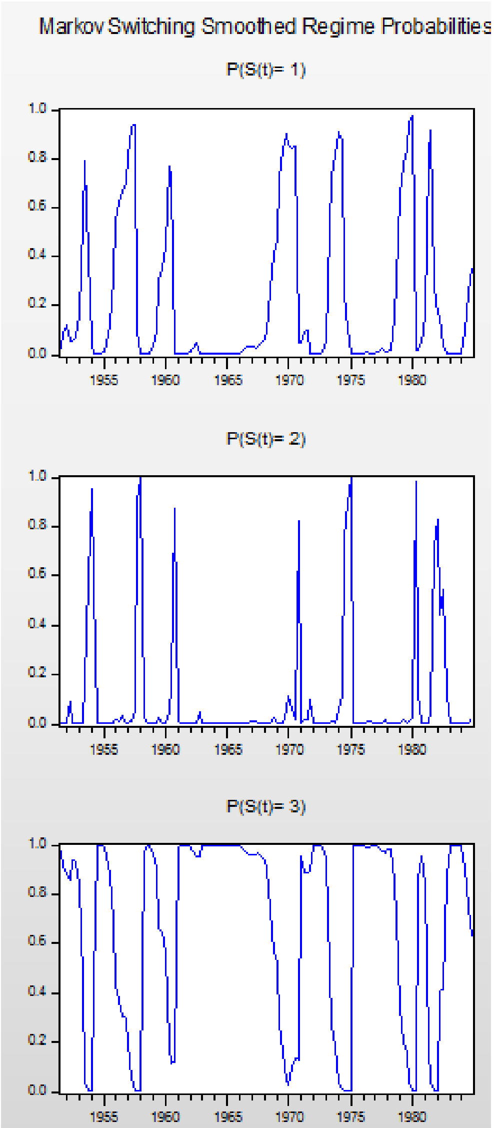


Figura 22: Smoothed probs