PS7
Time Series

Problem 1

1)

We test stationarity by means of ADF, as we did in earlier Problem Sets. We reject the null of unit root in specifications which include a constant and/or a trend.

2)

To generate the demeaned series, we run OLS with a constant only. As we know, the estimated constant is the mean of the series. Therefore, the residuals of the estimation is what we need. After estimation, we go to *Proc/Make Residual Series* and generate the demeaned series. I'll name it *ipdemean* but you can call it as you like. We then estimate an AR(4) as we did in PS0. Estimation output is shown below.

3)

In order to find the theoretical State-Space (SS) representation for an AR(4), we start from the AR(4) specification (as deviations from the long run mean, provided the process is stationary). We write the model in terms of y_{t+1} in order to match the usual Kalman Filter notation.

$$(y_{t+1} - \mu) = \phi_1(y_t - \mu) + \phi_2(y_{t-1} - \mu) + \phi_3(y_{t-2} - \mu) + \phi_4(y_{t-3} - \mu) + \varepsilon_{t+1} \tag{1}$$

Where $\mu = E(y_t)$. We can rewrite this as an SS.

State equation

$$\underbrace{\begin{bmatrix} y_{t+1} - \mu \\ y_t - \mu \\ y_{t-1} - \mu \\ y_{t-2} - \mu \end{bmatrix}}_{\xi_{t+1}} = \underbrace{\begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & \phi_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} y_t - \mu \\ y_{t-1} - \mu \\ y_{t-2} - \mu \\ y_{t-3} - \mu \end{bmatrix}}_{\xi_t} + \underbrace{\begin{bmatrix} \varepsilon_{t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{v_{t+1}} \tag{2}$$

Dependent Variable: IPDEMEAN
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 05/12/20 Time: 18:29
Sample: 1995M01 2013M03
Included observations: 219
Convergence achieved after 13 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	-0.256251	0.056497	-4.535645	0.0000
AR(2)	0.121923	0.064100	1.902070	0.0585
AR(3)	0.217001	0.068412	3.171966	0.0017
AR(4)	0.155073	0.077735	1.994896	0.0473
SIGMASQ	3.871819	0.314821	12.29846	0.0000

R-squared	0.116582	Mean dependent var	-1.48E-16
Adjusted R-squared	0.100070	S.D. dependent var	2.098304
S.E. of regression	1.990548	Akaike info criterion	4.238528
Sum squared resid	847.9284	Schwarz criterion	4.315904
Log likelihood	-459.1188	Hannan-Quinn criter.	4.269778
Durbin-Watson stat	1.994651		

Inverted AR Roots	.73	-.20+.57i	-.20-.57i	-.59

Figura 1: Estimation output for AR(4)

Observation equation

$$y_t - \mu = + \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_t - \mu \\ y_{t-1} - \mu \\ y_{t-2} - \mu \\ y_{t-3} - \mu \end{pmatrix} \tag{3}$$

In order to estimate this in Eviews, we have to creat an SS object. In order to do so, we click on *New Object* and create an new SSpace. We will get a blank window. There we have to type the system as it is shown in the figure.

We take a deeper look in how to specify the system. First of all, we write *signal* in order to tell Eviews that the first equation is one of the observed equations. Tell Eviews that it is equal to sv1, that is, the first state variable. In order to translate the system to Eviews, we well use the follow notation:

$$\begin{aligned} y_t - \mu &\Rightarrow sv1 \\ y_{t-1} - \mu &\Rightarrow sv2 \\ y_{t-2} - \mu &\Rightarrow sv3 \\ y_{t-3} - \mu &\Rightarrow sv4 \end{aligned}$$

That is, we are defining 4 “unobserved” states that we will try to recover from the filter. With

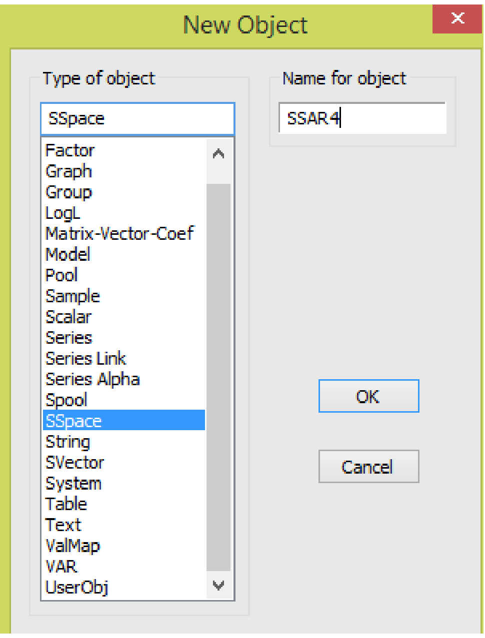


Figura 2: Create New SS

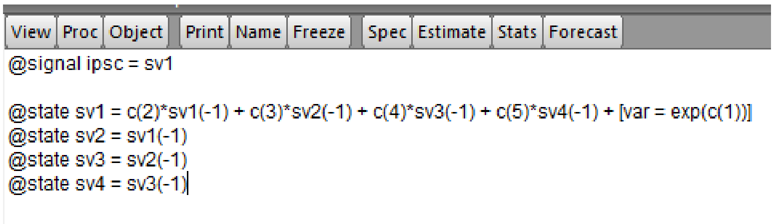


Figura 3: How to write the system

this in mind, we proceed to type the State equations. The first one is the actual AR(4) equation. Note that the term *[var = exp(c(1))]* is telling Eviews that there is a random innovation in that equation. The remaining equations are mere identities (that come from the way we wrote this).

In order to estimate, we just have to click in *Estimate* and “OK”. The output is shown in the figure. Note that the estimated parameters are the same as those estimated the usual way.

Problem 2

1)

In order to generate the variable, we proceed as we've done before: go to *Quick/Generate series* and enter the equation given.

2)

We write the teorical model we want to estimate. We define $\xi_t = i_t - \pi_t^e - \mu$ where $\mu = E(i_t - \pi^e)$. We write it this way in order to guarantee that $E(\xi_t) = 0$ which is an standard assumption.

The State Equation is given by an AR(1)

$$\xi_{t+1} = \phi \xi_t + v_{t+1} \tag{4}$$

Sspace: SSAR4
Method: Maximum likelihood (BFGS / Marquardt steps)
Date: 05/12/20 Time: 19:06
Sample: 1995M01 2013M03
Included observations: 219
Convergence achieved after 0 iterations
Coefficient covariance computed using outer product of gradients

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	1.353724	0.081311	16.64873	0.0000
C(2)	-0.256251	0.056497	-4.535645	0.0000
C(3)	0.121923	0.064100	1.902070	0.0572
C(4)	0.217001	0.068412	3.171966	0.0015
C(5)	0.155073	0.077735	1.994896	0.0461
	Final State	**Root MSE**	**z-Statistic**	**Prob.**
SV1	-1.249866	1.967694	-0.635193	0.5253
SV2	3.795431	0.000000	NA	0.0000
SV3	-2.209993	0.000000	NA	0.0000
SV4	0.388493	0.000000	NA	0.0000

Log likelihood	-459.1188	Akaike info criterion	4.238528
Parameters	5	Schwarz criterion	4.315904
Diffuse priors	0	Hannan-Quinn criter.	4.269778

Figura 4: Estimation output SS system

Where v_{t+1} is an innovation. On the other hand, the observation (signal) equation is given by the ex-post real interest rate

$$i_t - \pi_t = (i_t - \pi_t^e) + (\pi_t^e - \pi_t) = \mu + \xi_t + w_t \tag{5}$$

Where w_t is an innovation in the observation (signal) equation. This two-equation State-Space has the exact same form as the generic one used in class, with

$$F = \phi, \quad y_t = i_t - \pi_t$$
$$A'x_t = \mu, \quad H = 1, \quad w_t = (\pi_t^e - \pi_t)$$

To estimate this in Eviews, we have to create the SSpace object and type the equations as shown in the figure. Note that we have specfied innovations in a different way as we did in Problem 1. Both ways are valid. The way used here is more flexible because it would allow us to estimate a non-zero covariance between innovations. If we wanted to do this, we just would have to add an extra parameter. Estimation output is shown in the figure.

3)

The ex-ante interest rate series is estimated as the unobserved state by the filter. In order to see the graph, we click on *View/State Views/Graph State series.* There we can choose what

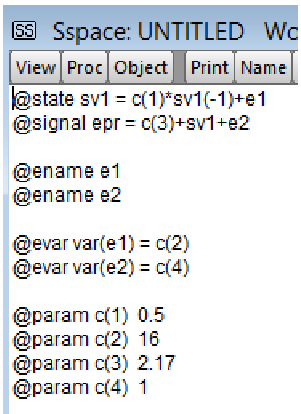
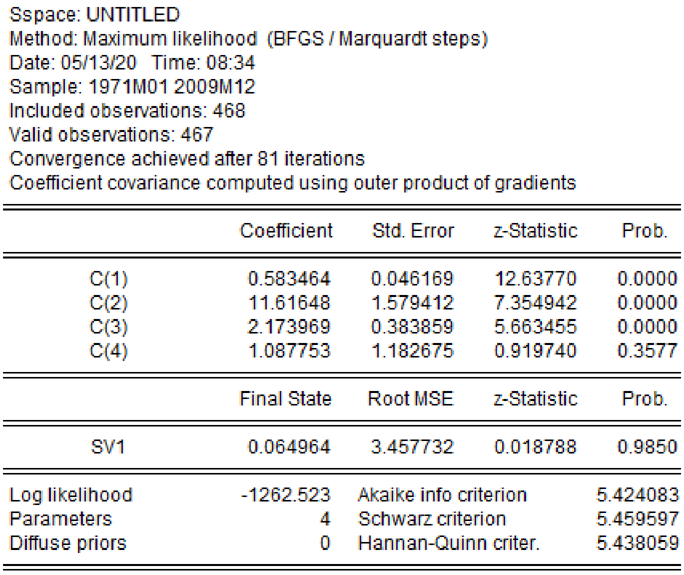


Figura 5: Specification of the SS


Sspace: UNTITLED
Method: Maximum likelihood (BFGS / Marquardt steps)
Date: 05/13/20 Time: 08:34
Sample: 1971M01 2009M12
Included observations: 468
Valid observations: 467
Convergence achieved after 81 iterations
Coefficient covariance computed using outer product of gradients

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.583464	0.046169	12.63770	0.0000
C(2)	11.61648	1.579412	7.354942	0.0000
C(3)	2.173969	0.383859	5.663455	0.0000
C(4)	1.087753	1.182675	0.919740	0.3577
	Final State	Root MSE	z-Statistic	Prob.
SV1	0.064964	3.457732	0.018788	0.9850

Log likelihood	-1262.523	Akaike info criterion	5.424083
Parameters	4	Schwarz criterion	5.459597
Diffuse priors	0	Hannan-Quinn criter.	5.438059

Figura 6: Estimation output

type of estimation of the state we want to plot[1]. We start plotting the one-step ahead states.

To calculate the mean and the variance of the ex-ante rate, note that is follows an AR(1), so

$$E(\xi_{t+1}) = E(i_t - \pi_t^e - \mu) = 0$$

Therefore $E(i_t - \pi_t^e) = \mu$, which we estimated as 2,174 . On the the other hand, as it is an AR(1), we know that the long-run variance is given by $\dfrac{\sigma^2}{1-\phi^2}$. In our estimation, this is given by

$$\frac{c(2)}{1-c(1)^2} = 17{,}611$$

In order to look at the ex-post real interest rate series, we open the series and click on *View/Descriptive statistics and tests/stats table*. There we see that the mean is 2,174 and the variance is $4{,}33^2 = 18{,}75$. It is natural that the ex-post interest turned out to be more volatile, because we estimated the equation $y_t = \mu + \xi_t + w_t$, therefore, the ex-post rate is equal to the ex-ante one plus an innovation, which increases variance.

Problem 3

In order to work with this, we have to rewrite the system in a wat that matches our notation. One key problem is that the state variable ς_t follows an AR(2). This can fixed by writing the system in matrix form as we did in Problem 1. **Eviews won't allow you to add lags grater**

[1] One step ahead: $\hat{\xi}_{t|t-1}$
Filtered: $\hat{\xi}_{t|t}$
Smoothed: $\hat{\xi}_{t|T}$

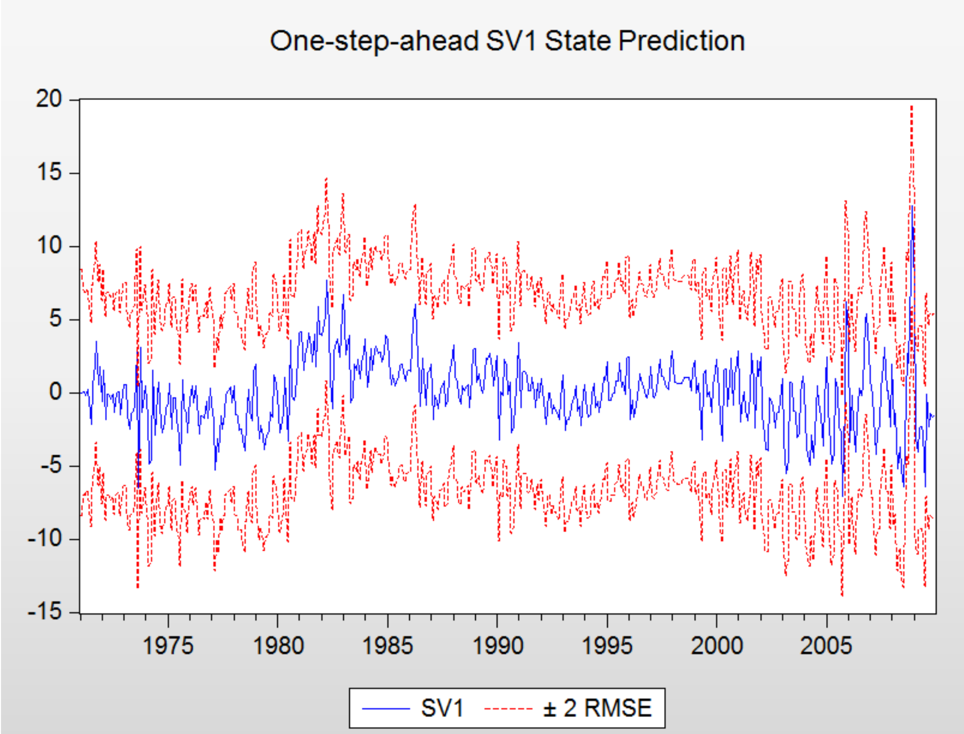


Figura 7: One-step ahead estimated ex-ante real interest rates

than 1 in state equations. With this in mind we write the system as

$$\begin{pmatrix} \varsigma_{t+1} \\ \varsigma_t \end{pmatrix} = \begin{pmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \varsigma_t \\ \varsigma_{t-1} \end{pmatrix} + \begin{pmatrix} \upsilon_t \\ 0 \end{pmatrix} \tag{6}$$

The signal equations are already written in a suitable way. In order to estimate the system, we create the SSpace as before, and type the equations in Eviews [2].

```
Sspace: UNTITLED   Workfile: BCYCLE::B
View | Proc | Object | Print | Name | Freeze | Spec | Estimat
@state sv1 = c(1)*sv1(-1)+c(2)*sv2(-1)+[var=1]
@state sv2 = sv1(-1)

@signal gdp = c(3)*sv1+e2
@signal un = c(4)*sv1+e3
@signal em = c(5)*sv1+e4
@signal sales = c(6)*sv1+e5
@signal inc = c(7)*sv1+e6
@signal prod = c(8)*sv1+e7

@ename e2 e3 e4 e5 e6 e7

@evar var(e2) = c(9)
@evar var(e3) = c(10)
@evar var(e4) = c(11)
@evar var(e5) = c(12)
@evar var(e6) = c(13)
@evar var(e7) = c(14)
```

Figura 8: Specification of the SS

```
@param c(3) 0.0917
@param c(4) -0.06
@param c(5) 0.088
@param c(6) 0.0674
@param c(7) 0.0233
@param c(8) 0.0988
@param c(1) 1.89
@param c(2) -0.9
@param c(9) 0.88
@param c(10) 0.04
@param c(11) 0.6
@param c(12) 8.39
@param c(13) 9.6
@param c(14) 14.63
```

Figura 9: Initial values

Estimation output is shown in the figure.

[2] Initial values are taken from the book cited

Sspace: SS01
Method: Maximum likelihood (BFGS / Marquardt steps)
Date: 05/13/20 Time: 09:19
Sample: 1981M01 2009M09
Included observations: 345
Convergence achieved after 22 iterations
Coefficient covariance computed using outer product of gradients

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	1.895199	0.035258	53.75178	0.0000
C(2)	-0.912455	0.034315	-26.59043	0.0000
C(3)	0.091694	0.013587	6.748768	0.0000
C(4)	-0.058360	0.008264	-7.062171	0.0000
C(5)	0.083645	0.012406	6.742374	0.0000
C(6)	0.064437	0.011163	5.772468	0.0000
C(7)	0.022735	0.010623	2.140280	0.0323
C(8)	0.093428	0.019021	4.911804	0.0000
C(9)	0.879179	0.088768	9.904230	0.0000
C(10)	0.044341	0.004243	10.45021	0.0000
C(11)	0.599140	0.051349	11.66793	0.0000
C(12)	8.357483	0.759629	11.00206	0.0000
C(13)	9.598550	1.162177	8.259115	0.0000
C(14)	14.62478	1.111254	13.16061	0.0000

	Final State	Root MSE	z-Statistic	Prob.
SV1	-24.51169	3.245214	-7.553181	0.0000
SV2	-25.78544	2.269478	-11.36183	0.0000

Log likelihood	-3633.442	Akaike info criterion	21.14459
Parameters	14	Schwarz criterion	21.30056
Diffuse priors	0	Hannan-Quinn criter.	21.20670

Figura 10: Estimation output

In this case, we are interested in the state of the business cycle. For this, we need the smoothed estimation of the state, $\hat{\xi}_{t|T}$ [3]. In order to generate this from estimation, we click on *Proc/Make State Series* and generate the smoothed state series. Note that this command will create two series, because technically we have two state variables: $sv1 = \varsigma_t$ and $sv2 = \varsigma_{t-1}$. Naturally, we will use $sv1$.

We now have to compute the indexes to compare. We go to *Quick/Generate Series* and generate the *bc* series as required. We then create the series *bcm*, which is the demeaned version of *bc*.

Note that this can also by done by typing

*series bc = 100 * (d (log (coin) ; 0; 12))*

series bcm = bc - @mean (bc)

In the command bar. Once we've created this, we follow the hint and rescale sv1. We go to *Quick/Generate Series* and rescale it as required. With this, we can compare the DOC index and ours. This is shown in the graph.

Problem 4

Our only problem here is that we have a constant in the State equation (the AR process of α_t). Therefore, we can add a fictitious state variable that will be the constant. Using that idea,

[3] The term *smoothed* here is used in a similar way is in Markov Switching procedures

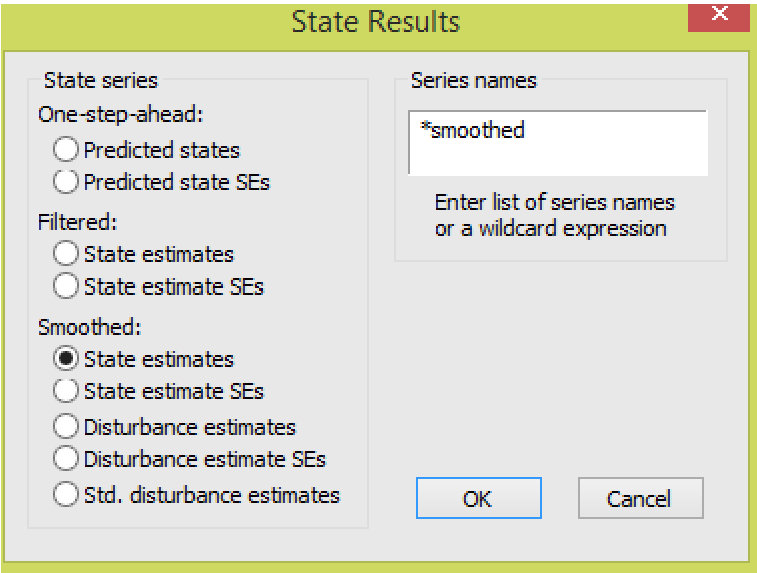


Figura 11: Generate the smoothed state series

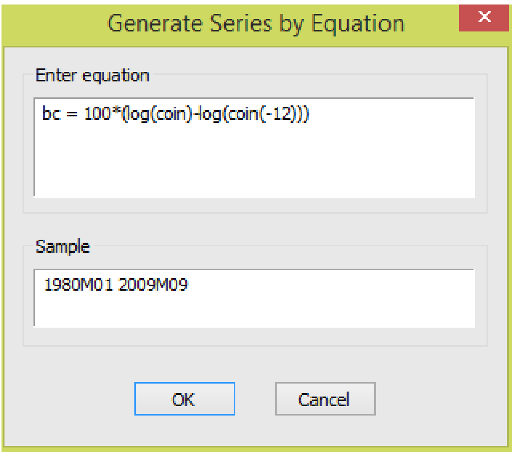


Figura 12: Generate *bc* series

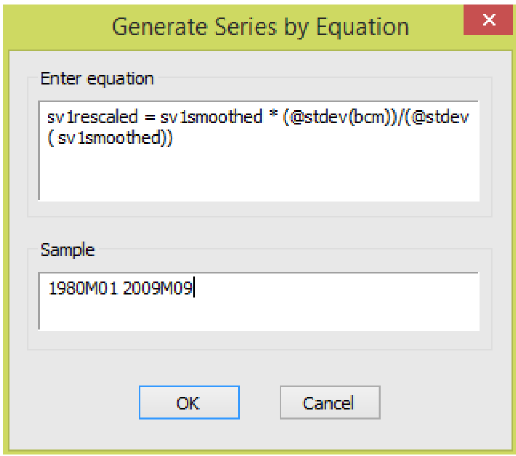


Figura 13: Rescale *sv1* as asked

we can write the state equation as

$$\begin{pmatrix} sv1_t \\ \alpha_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \mu & \phi \end{pmatrix} \begin{pmatrix} sv1_{t-1} \\ \alpha_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ u_t \end{pmatrix} \tag{7}$$

In this case, we will initializate $sv1_t$ in $sv1_0 = 1$, therefore, it will be constant forerver. With this notation, the signal equation can be written as:

$$\log(\frac{m_t}{p_t y_t}) = \begin{pmatrix} \alpha_0 & i_t \end{pmatrix} \begin{pmatrix} sv1_t \\ \alpha_t \end{pmatrix} + \varepsilon_t \tag{8}$$

Where is important to remember that we will set things to make $sv1_t = 1$ for all t.

Now we have to specify the SSpace in Eviews. In this case we have to be careful with the following: we have added an artificial state variable $sv1_t$ which **by construction** has unit root. WE may have also problems in α_t as we will see later on. Therefore, we have to provide starting values for the filter. This happens because, by default, the filter is initialized with $\xi_{1|0} = E[\xi_t]$

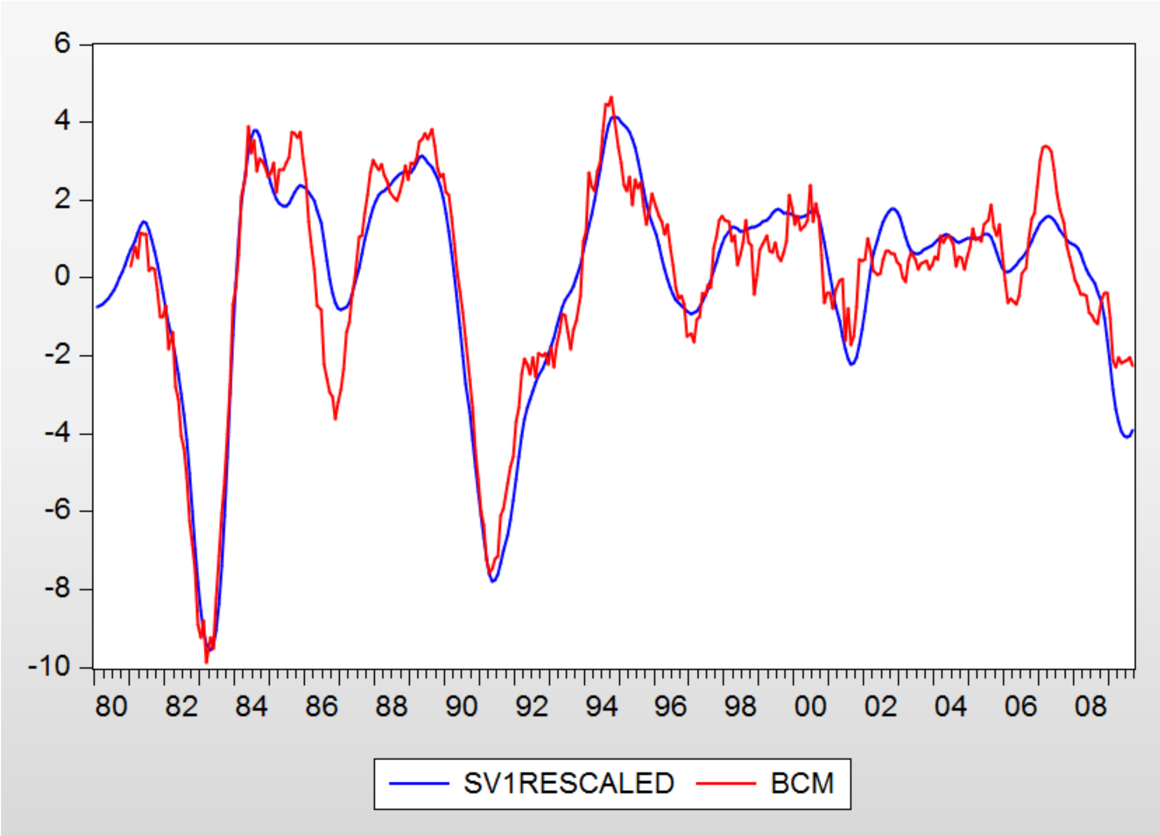


Figura 14: Concident indexes

and $P_{1|0} = E(\xi_{t+1}\xi'_{t+1})$. But if we have unit root, $\xi_{1|0} = E[\xi_t]$ doesn't exists, as well may be $P_{1|0} = E(\xi_{t+1}\xi'_{t+1})$. Therefore we should provide stating values ourselves. In order to do so, we have to create first the matrix with which we want to initialize the filter.

In order to do so, we have to go to *Object/New Object/Matrix-vector-coef.* We have to create a 2×1 vector for the initial states and a 2×2 matrix for the initial mean square error. We define them with the values shown in the figure below. For the intial state, we intialize the constant in 1 because we know that it has to be constant, and the semi-elasticity in $-0{,}5$ because it as widely used value. For the matrix, *Object/New Object/Matrix-vector-coef.* **There, we have to define a symetric matrix**. In this case, for $P_{1|0}$, which encodes our uncertainty, we assign 0 variance (that is, total certainty) to the value of the constant. That's why we have a 0 in the $(1, 1)$ element of the matrix. Covariances are also zero as well, because we know that $sv1$ is a constant. On the other hand, the 4 in the $(2, 2)$ element encodes our uncertainty about the initial value of α. It is perhaps too big.

We also have to take into account the fact that **SSpace equation specifictation does not allow us to use operators with the variables**. That is, we can't write $\log(\frac{m_t}{p_t y_t})$ directly in the prompt. Therefore, we have to create the variable before we estimate. In order to do so, we proceed as we always have: *Quick/Generate Series* and create the series. I'll call it *demreal.* With that in mind, we proceed to specify the SSpace. With commands *@mprior* and *@vprior* we tell eviews to use that vector and symetric matrix as initial conditions. Initial values for

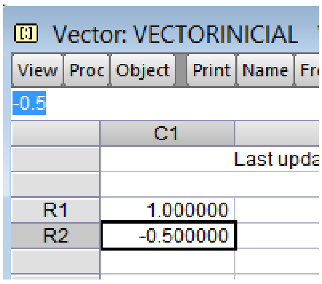


Figura 15: Initial state

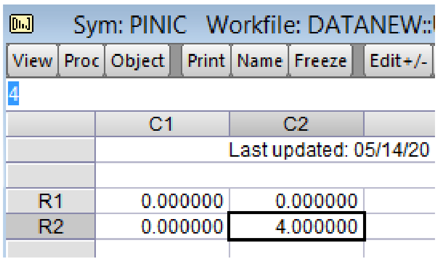


Figura 16: $P_{1|0}$

```
@state sv1 = sv1(-1)
@state sv2 = c(1)*sv1(-1)+c(2)*sv2(-1)+[var = exp(c(3))]

@signal demreal = c(4)*sv1 + sv2*interestrate+[var = exp(c(5))]

@mprior vectorinicial
@vprior pinic

@param c(1) -0.4 c(2) 0.9 c(3) -2 c(4) 18 c(5) -2
```

Figura 17: SSpace equations

parameter optimization come from earlier, simpler estimations. For example, in order to get an initial value for the constant on the parameter of the semi-elasticity, you could run plain OLS and use the estimated coefficients.

Estimation output and the values of α_t are shown in the figures below. We don't show the estimate of *sv1* because it is trivially a constant

Sspace: UNTITLED
Method: Maximum likelihood (BFGS / Marquardt steps)
Date: 05/14/20 Time: 09:28
Sample: 1960Q1 2018Q1
Included observations: 233
User prior mean: VECTORINICIAL
User prior variance: PINIC
Convergence achieved after 21 iterations
Coefficient covariance computed using outer product of gradients

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-7.54E-05	0.000940	-0.080151	0.9361
C(2)	0.984332	0.012671	77.68433	0.0000
C(3)	-9.325152	0.286128	-32.59089	0.0000
C(4)	18.75150	0.008844	2120.228	0.0000
C(5)	-5.024448	0.081739	-61.46941	0.0000

	Final State	Root MSE	z-Statistic	Prob.
SV1	1.000000	0.000000	NA	0.0000
SV2	0.141295	0.027076	5.218364	0.0000

Log likelihood	190.6927	Akaike info criterion	-1.593929
Parameters	5	Schwarz criterion	-1.519872
Diffuse priors	0	Hannan-Quinn criter.	-1.564066

Figura 18: Estimation output

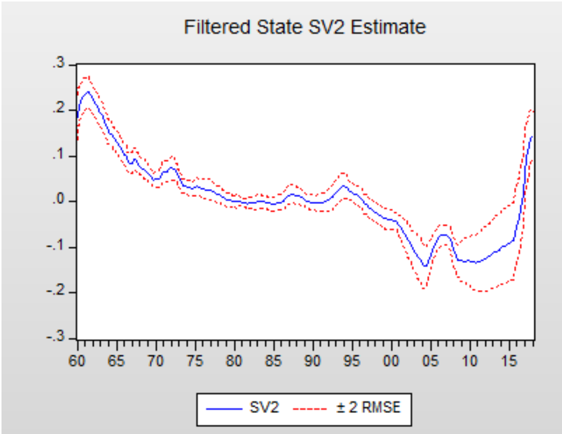


Figura 19: Filtered estimation of α_t

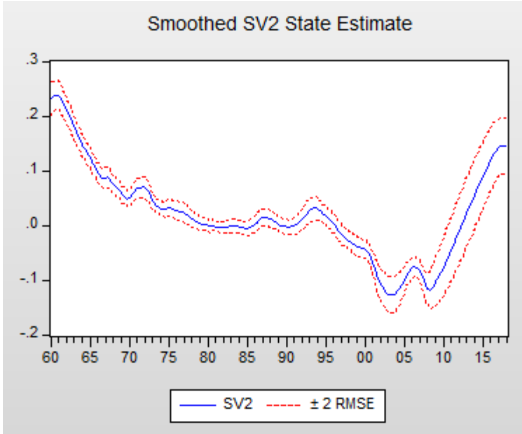


Figura 20: Smoothed values of α_t

Several things can be noted. First, the estimated α_t is positive for many periods in the sample, which is a big issue, given that economic theory tells us that money demand is inversely related to interest rates. On the other hand, note that $\hat{\phi} = 0{,}98$, which is close to unit root. This could be showing that, actually, there were many structural breaks, therefore, **money demand doesn't seem to be so stable after all**.