

## **Trabajo Práctico N° 1:** **Vectores y Espacios Vectoriales.**

### **Ejercicio 1.**

*Dados los vectores  $u = (1, 2)$ ,  $v = (-1, 3)$  y  $w = (-1, -2)$ , calcular analítica y gráficamente las siguientes operaciones.*

**(a)**

$$u + v = (1, 2) + (-1, 3)$$

$$u + v = (0, 5).$$

**Gráfico.**

**(b)**

$$v + w = (-1, 3) + (-1, -2)$$

$$v + w = (-2, 1).$$

**Gráfico.**

**(c)**

$$3u + 3v = 3(1, 2) + 3(-1, 3)$$

$$3u + 3v = (3, 6) + (-3, 9)$$

$$3u + 3v = (0, 15).$$

**Gráfico.**

**(d)**

$$3(u + v) = 3[(1, 2) + (-1, 3)]$$

$$3(u + v) = 3(0, 5)$$

$$3(u + v) = (0, 15).$$

**Gráfico.**

**(e)**

$$(u + v) + w = [(1, 2) + (-1, 3)] + (-1, 2)$$

$$(u + v) + w = (0, 5) + (-1, 2)$$

$$(u + v) + w = (-1, 7).$$

Gráfico.

(f)

$$u + (v + w) = (1, 2) + [(-1, 3) + (-1, -2)]$$

$$u + (v + w) = (1, 2) + (-2, 1)$$

$$u + (v + w) = (-1, 3).$$

Gráfico.

(g)

$$u - v = (1, 2) - (-1, 3)$$

$$u - v = (4, -1).$$

Gráfico.

(h)

$$u + (v - w) = (1, 2) + [(-1, 3) - (-1, -2)]$$

$$u + (v - w) = (1, 2) + (0, 5)$$

$$u + (v - w) = (1, 7).$$

Gráfico.

(i)

$$\frac{5}{4}u + \frac{1}{2}v - \frac{3}{2}w = \frac{5}{4}(1, 2) + \frac{1}{2}(-1, 3) - \frac{3}{2}(-1, -2)$$

$$\frac{5}{4}u + \frac{1}{2}v - \frac{3}{2}w = \left(\frac{5}{4}, \frac{5}{2}\right) + \left(\frac{-1}{2}, \frac{3}{2}\right) + \left(\frac{3}{2}, 3\right)$$

$$\frac{5}{4}u + \frac{1}{2}v - \frac{3}{2}w = \left(\frac{9}{4}, 7\right).$$

Gráfico.

## **Ejercicio 2.**

Sea  $w = (1, 3) \in \mathbb{R}^2$  un vector. Graficar en el plano.

(a)  $L = \{tw : t \in \mathbb{R}\}$ .

Gráfico.

(b)  $L = \{tw : t \in \mathbb{R}_{\geq 0}\}$ .

Gráfico.

(c)  $L = \{tw : t \in \mathbb{R}, 0 \leq t \leq 1\}$ .

Gráfico.

**Ejercicio 3.**

Dados los vectores  $u = (0, 1, 2)$ ,  $v = (1, 1, 0)$  y  $w = (-1, 1, 1)$ , calcular las operaciones:

(a)

$$\begin{aligned}u + v &= (0, 1, 2) + (1, 1, 0) \\u + v &= (1, 2, 2).\end{aligned}$$

(b)

$$\begin{aligned}u + v + w &= (0, 1, 2) + (1, 1, 0) + (-1, 1, 1) \\u + v + w &= (0, 3, 3).\end{aligned}$$

(c)

$$\begin{aligned}u - v &= (0, 1, 2) - (1, 1, 0) \\u - v &= (-1, 0, 2).\end{aligned}$$

(d)

$$\begin{aligned}2u &= 2(0, 1, 2) \\2u &= (0, 2, 4).\end{aligned}$$

(e)

$$\begin{aligned}-3v &= -3(1, 1, 0) \\-3v &= (-3, -3, 0).\end{aligned}$$

(f)

$$\begin{aligned}-v + \frac{2}{3}w &= -(1, 1, 0) + \frac{2}{3}(-1, 1, 1) \\-v + \frac{2}{3}w &= (-1, -1, 0) + \left(\frac{-2}{3}, \frac{2}{3}, \frac{2}{3}\right) \\-v + \frac{2}{3}w &= \left(\frac{-5}{3}, \frac{-1}{3}, \frac{2}{3}\right).\end{aligned}$$

**Ejercicio 4.**

Hallar  $x$  e  $y$  para que los vectores  $v$  y  $w$  resulten iguales.

(a)  $v = (x, 3)$  y  $w = (2, x + y)$ .

$$v = w$$

$$(x, 3) = (2, x + y).$$

$$\begin{cases} x = 2 \\ 3 = x + y \end{cases}$$

$$x + y = 3$$

$$y = -x + 3$$

$$y = -2 + 3$$

$$y = 1.$$

Por lo tanto, los valores de  $x$  e  $y$  para que los vectores  $v$  y  $w$  resulten iguales son  $(2, 1)$ .

(b)  $v = (4, y)$  y  $w = x(2, 3)$ .

$$v = w$$

$$(4, y) = x(2, 3)$$

$$(4, y) = (2x, 3x).$$

$$\begin{cases} 4 = 2x \\ y = 3x \end{cases}$$

$$x = \frac{4}{2}$$

$$x = 2.$$

$$y = 3 \cdot 2$$

$$y = 6.$$

Por lo tanto, los valores de  $x$  e  $y$  para que los vectores  $v$  y  $w$  resulten iguales son  $(2, 6)$ .

(c)  $v = x(3, 2)$  y  $w = 2(y, -1)$ .

$$v = w$$

$$x(3, 2) = 2(y, -1)$$

$$(3x, 2x) = (2y, -2).$$

$$\begin{cases} 3x = 2y \\ 2x = -2 \end{cases}$$

$$x = \frac{-2}{2}$$

$$x = -1.$$

$$2y = 3x$$

$$y = \frac{3}{2}x$$

$$y = \frac{3}{2}(-1)$$

$$y = \frac{-3}{2}.$$

Por lo tanto, los valores de  $x$  e  $y$  para que los vectores  $v$  y  $w$  resulten iguales son  $(-1, \frac{-3}{2})$ .

(d)  $v = x(2, y)$  y  $w = y(1, -2)$ .

$$v = w$$

$$x(2, y) = y(1, -2)$$

$$(2x, xy) = (y, -2y).$$

$$\begin{cases} 2x = y \\ xy = -2y \end{cases}$$

$$x2x = -2 * 2x$$

$$2x^2 = -4x$$

$$x^2 = \frac{-4}{2}x$$

$$x^2 = -2x$$

$$x^2 + 2x = 0$$

$$x(x + 2) = 0.$$

$$x_1 = 0.$$

$$x_2 = -2.$$

$$y_1 = 2 * 0 = 0.$$

$$y_2 = 2(-2) = -4.$$

Por lo tanto, los valores de  $x$  e  $y$  para que los vectores  $v$  y  $w$  resulten iguales son  $(0, 0)$  y  $(-2, -4)$ .

## Ejercicio 5

Normalizar los siguientes vectores.

(a)  $u = (-3, 1, -2, 4, -5)$ .

$$\begin{aligned} u' &= \frac{u}{\|u\|} \\ u' &= \frac{1}{\sqrt{(-3)^2 + 1^2 + (-2)^2 + 4^2 + (-5)^2}} (-3, 1, -2, 4, -5) \\ u' &= \frac{1}{\sqrt{9+1+4+16+25}} (-3, 1, -2, 4, -5) \\ u' &= \frac{1}{\sqrt{55}} (-3, 1, -2, 4, -5) \\ u' &= \left( \frac{-3}{\sqrt{55}}, \frac{1}{\sqrt{55}}, \frac{-2}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{-5}{\sqrt{55}} \right) \\ u' &= \left( \frac{-3\sqrt{55}}{55}, \frac{\sqrt{55}}{55}, \frac{-2\sqrt{55}}{55}, \frac{4\sqrt{55}}{55}, \frac{-5\sqrt{55}}{55} \right) \\ u' &= \left( \frac{-3\sqrt{55}}{55}, \frac{\sqrt{55}}{55}, \frac{-2\sqrt{55}}{55}, \frac{4\sqrt{55}}{55}, \frac{-\sqrt{55}}{11} \right). \end{aligned}$$

(b)  $v = (4, -2, -3, 8)$ .

$$\begin{aligned} v' &= \frac{v}{\|v\|} \\ v' &= \frac{1}{\sqrt{4^2 + (-2)^2 + (-3)^2 + 8^2}} (4, -2, -3, 8) \\ v' &= \frac{1}{\sqrt{16+4+9+64}} (4, -2, -3, 8) \\ v' &= \frac{1}{\sqrt{93}} (4, -2, -3, 8) \\ v' &= \left( \frac{4}{\sqrt{93}}, \frac{-2}{\sqrt{93}}, \frac{-3}{\sqrt{93}}, \frac{8}{\sqrt{93}} \right) \\ v' &= \left( \frac{4\sqrt{93}}{93}, \frac{-2\sqrt{93}}{93}, \frac{-3\sqrt{93}}{93}, \frac{8\sqrt{93}}{93} \right) \\ v' &= \left( \frac{4\sqrt{93}}{93}, \frac{-2\sqrt{93}}{93}, \frac{-\sqrt{93}}{31}, \frac{8\sqrt{93}}{93} \right). \end{aligned}$$

(c)  $w = \left( \frac{1}{2}, \frac{2}{3}, \frac{-1}{4} \right)$ .

$$\begin{aligned} w' &= \frac{w}{\|w\|} \\ w' &= \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{4}\right)^2}} \left( \frac{1}{2}, \frac{2}{3}, \frac{-1}{4} \right) \\ w' &= \frac{1}{\sqrt{\frac{1}{4} + \frac{4}{9} + \frac{1}{16}}} \left( \frac{1}{2}, \frac{2}{3}, \frac{-1}{4} \right) \\ w' &= \frac{1}{\sqrt{\frac{109}{144}}} \left( \frac{1}{2}, \frac{2}{3}, \frac{-1}{4} \right) \\ w' &= \frac{1}{\frac{\sqrt{109}}{12}} \left( \frac{1}{2}, \frac{2}{3}, \frac{-1}{4} \right) \end{aligned}$$

$$\begin{aligned}
 w' &= \frac{12}{\sqrt{109}} \left( \frac{1}{2}, \frac{2}{3}, \frac{-1}{4} \right) \\
 w' &= \left( \frac{6}{\sqrt{109}}, \frac{8}{\sqrt{109}}, \frac{-3}{\sqrt{109}} \right) \\
 w' &= \left( \frac{6\sqrt{109}}{109}, \frac{8\sqrt{109}}{109}, \frac{-3\sqrt{109}}{4 \cdot 109} \right) \\
 w' &= \left( \frac{6\sqrt{109}}{109}, \frac{8\sqrt{109}}{109}, \frac{-3\sqrt{109}}{109} \right).
 \end{aligned}$$



**Ejercicio 6.**

Determinar todos los valores de  $k \in \mathbb{R}$  que verifican:

(a) El vector  $u = (4, k)$  tiene norma 5.

$$\begin{aligned} \|u\| &= 5 \\ \sqrt{4^2 + k^2} &= 5 \\ \sqrt{16 + k^2} &= 5 \\ 16 + k^2 &= 5^2 \\ 16 + k^2 &= 25 \\ k^2 &= 25 - 16 \\ k^2 &= 9 \\ \sqrt{k^2} &= \sqrt{9} \\ |k| &= 3 \\ k &= \pm 3. \end{aligned}$$

(b) El vector  $v = (1, k, 0)$  tiene norma 2.

$$\begin{aligned} \|v\| &= 2 \\ \sqrt{1^2 + k^2 + 0^2} &= 2 \\ \sqrt{1 + k^2 + 0} &= 2 \\ \sqrt{1 + k^2} &= 2 \\ 1 + k^2 &= 2^2 \\ 1 + k^2 &= 4 \\ k^2 &= 4 - 1 \\ k^2 &= 3 \\ \sqrt{k^2} &= \sqrt{3} \\ |k| &= \sqrt{3} \\ k &= \pm \sqrt{3}. \end{aligned}$$

(c) El vector  $w = k(2, 2, 1)$  tiene norma 1.

$$\begin{aligned} \|w\| &= 1 \\ \sqrt{(2k)^2 + (2k)^2 + 1^2} &= 1 \\ \sqrt{4k^2 + 4k^2 + 1} &= 1 \\ \sqrt{8k^2 + 1} &= 1 \\ 8k^2 + 1 &= 1^2 \\ 8k^2 + 1 &= 1 \\ 8k^2 &= 1 - 1 \\ 8k^2 &= 0 \\ k^2 &= \frac{0}{8} \end{aligned}$$

$$k^2 = 0$$

$$\sqrt{k^2} = \sqrt{0}$$

$$|k| = 0$$

$$k = \pm 0$$

$$k = 0.$$

(d) El vector  $z = (1, k, -2, 5)$  tiene norma  $\sqrt{39}$ .

$$\|z\| = \sqrt{39}$$

$$\sqrt{1^2 + k^2 + (-2)^2 + 5^2} = \sqrt{39}$$

$$\sqrt{1 + k^2 + 4 + 25} = \sqrt{39}$$

$$\sqrt{30 + k^2} = \sqrt{39}$$

$$30 + k^2 = (\sqrt{39})^2$$

$$30 + k^2 = 39$$

$$k^2 = 39 - 30$$

$$k^2 = 9$$

$$\sqrt{k^2} = \sqrt{9}$$

$$|k| = 3$$

$$k = \pm 3.$$

**Ejercicio 7.**

Dados los vectores  $v = (1, -2, 2)$ ,  $w = (2, 0, 3)$  y  $z = (4, 4, 4)$ , realizar las operaciones.

(a)

$$\begin{aligned} v * w &= (1, -2, 2) * (2, 0, 3) \\ v * w &= 1 * 2 - 2 * 0 + 2 * 3 \\ v * w &= 2 - 0 + 6 \\ v * w &= 8. \end{aligned}$$

(b)

$$\begin{aligned} w * v &= (2, 0, 3) * (1, -2, 2) \\ w * v &= 2 * 1 + 0 * (-2) + 3 * 2 \\ w * v &= 2 + 0 + 6 \\ w * v &= 8. \end{aligned}$$

(c)

$$\begin{aligned} (v + w) * z &= [(1, -2, 2) + (2, 0, 3)] * (4, 4, 4) \\ (v + w) * z &= (3, -2, 5) * (4, 4, 4) \\ (v + w) * z &= 3 * 4 - 2 * 4 + 5 * 4 \\ (v + w) * z &= 12 - 8 + 20 \\ (v + w) * z &= 24. \end{aligned}$$

(d)

$$\begin{aligned} (v * z) + (w * z) &= (1, -2, 2) * (4, 4, 4) + (2, 0, 3) * (4, 4, 4) \\ (v * z) + (w * z) &= [(1, -2, 2) + (2, 0, 3)] * (4, 4, 4) \\ (v * z) + (w * z) &= (3, -2, 5) * (4, 4, 4) \\ (v * z) + (w * z) &= 3 * 4 - 2 * 4 + 5 * 4 \\ (v * z) + (w * z) &= 12 - 8 + 20 \\ (v * z) + (w * z) &= 24. \end{aligned}$$

(e)

$$\begin{aligned} 3v * w &= 3 (1, -2, 2) * (2, 0, 3) \\ 3v * w &= (3, -6, 12) * (2, 0, 3) \\ 3v * w &= 3 * 2 - 6 * 0 + 12 * 3 \\ 3v * w &= 6 - 0 + 36 \end{aligned}$$

$$3v * w = 42.$$

**(f)**

$$v * 3w = (1, -2, 2) * 3(2, 0, 3)$$

$$v * 3w = (1, -2, 2) * (6, 0, 9)$$

$$v * 3w = 1 * 6 - 2 * 0 + 2 * 9$$

$$v * 3w = 6 - 0 + 18$$

$$v * 3w = 24.$$

**(g)**

$$3(v * w) = 3[(1, -2, 2) * (2, 0, 3)]$$

$$3(v * w) = 3(1 * 2 - 2 * 0 + 2 * 3)$$

$$3(v * w) = 3(2 - 0 + 6)$$

$$3(v * w) = 3 * 8$$

$$3(v * w) = 24.$$

**(h)**

$$v * v = (1, -2, 2) * (1, -2, 2)$$

$$v * v = 1 * 1 - 2 * (-2) + 2 * 2$$

$$v * v = 1 - 4 + 4$$

$$v * v = 1.$$

**(i)**

$$w * w = (2, 0, 3) * (2, 0, 3)$$

$$w * w = 2 * 2 + 0 * 0 + 3 * 3$$

$$w * w = 4 + 0 + 9$$

$$w * w = 13.$$

**Ejercicio 8.**

En cada uno de los siguientes casos, calcular el ángulo entre los vectores  $u$  y  $v$ .

(a)  $u = (1, 1)$  y  $v = (1, -1)$ .

$$u \cdot v = (1, 1) \cdot (1, -1)$$

$$u \cdot v = 1 \cdot 1 + 1 \cdot (-1)$$

$$u \cdot v = 1 - 1$$

$$u \cdot v = 0.$$

$$\varphi = \arccos \left( \frac{\cos(\varphi)}{\|u\| \|v\|} \right)$$

$$\varphi = \arccos(0)$$

$$\varphi = 90^\circ.$$

Por lo tanto,  $u$  y  $v$  son ortogonales.

(b)  $u = (3, -1, 2)$  y  $v = (4, 3, -1)$ .

$$u \cdot v = (3, -1, 2) \cdot (4, 3, -1)$$

$$u \cdot v = 3 \cdot 4 - 1 \cdot 3 + 2 \cdot (-1)$$

$$u \cdot v = 12 - 3 - 2$$

$$u \cdot v = 7.$$

$$\|u\| = \sqrt{3^2 + (-1)^2 + 2^2}$$

$$\|u\| = \sqrt{9 + 1 + 4}$$

$$\|u\| = \sqrt{14}.$$

$$\|v\| = \sqrt{4^2 + 3^2 + (-1)^2}$$

$$\|v\| = \sqrt{16 + 9 + 1}$$

$$\|v\| = \sqrt{26}.$$

$$\varphi = \arccos \left( \frac{u \cdot v}{\|u\| \|v\|} \right)$$

$$\varphi = \arccos \left( \frac{7}{\sqrt{14} \sqrt{26}} \right)$$

$$\varphi = \arccos \left( \frac{7}{\sqrt{14 \cdot 26}} \right)$$

$$\varphi = \arccos \left( \frac{7}{\sqrt{390}} \right)$$

$$\varphi = 69,24^\circ.$$

Por lo tanto,  $u$  y  $v$  no son ortogonales.

(c)  $u = (1, -2, 3)$  y  $v = (2, 5, 4)$ .

$$\begin{aligned} u \cdot v &= (1, -2, 3) \cdot (2, 5, 4) \\ u \cdot v &= 1 \cdot 2 - 2 \cdot 5 + 3 \cdot 4 \\ u \cdot v &= 2 - 10 + 12 \\ u \cdot v &= 4. \end{aligned}$$

$$\begin{aligned} \|u\| &= \sqrt{1^2 + (-2)^2 + 3^2} \\ \|u\| &= \sqrt{1 + 4 + 9} \\ \|u\| &= \sqrt{14}. \end{aligned}$$

$$\begin{aligned} \|v\| &= \sqrt{2^2 + 5^2 + 4^2} \\ \|v\| &= \sqrt{4 + 25 + 16} \\ \|v\| &= \sqrt{45}. \end{aligned}$$

$$\begin{aligned} \varphi &= \arccos \left( \frac{u \cdot v}{\|u\| \|v\|} \right) \\ \varphi &= \arccos \left( \frac{4}{\sqrt{14} \sqrt{45}} \right) \\ \varphi &= \arccos \left( \frac{4}{\sqrt{14 \cdot 45}} \right) \\ \varphi &= \arccos \left( \frac{4}{\sqrt{630}} \right) \\ \varphi &= 80,6^\circ. \end{aligned}$$

Por lo tanto,  $u$  y  $v$  no son ortogonales.

**Ejercicio 9.**

Se consideran los vectores de  $\mathbb{R}^3$   $u = (1, -3, 2)$  y  $v = (2, -1, 1)$ .

(a) Escribir al vector  $w = (1, 7, -4)$  como combinación lineal de  $u$  y  $v$ .

$$w = au + bv$$

$$(1, 7, -4) = a(1, -3, 2) + b(2, -1, 1)$$

$$(1, 7, -4) = (a, -3a, 2a) + (2b, -b, b)$$

$$(1, 7, -4) = (a + 2b, -3a - b, 2a + b).$$

$$\begin{cases} 1 = a + 2b \\ 7 = -3a - b \\ -4 = 2a + b \end{cases}$$

$$2b = 1 - a$$

$$b = \frac{1-a}{2}$$

$$b = \frac{1}{2} - \frac{1}{2}a.$$

$$7 = -3a - \left(\frac{1}{2} - \frac{1}{2}a\right)$$

$$7 = -3a - \frac{1}{2} + \frac{1}{2}a$$

$$7 = \frac{-5}{2}a - \frac{1}{2}$$

$$\frac{5}{2}a = \frac{-1}{2} - 7$$

$$\frac{5}{2}a = \frac{-15}{2}$$

$$a = \frac{-15}{5}$$

$$a = -3.$$

$$b = \frac{1}{2} - \frac{1}{2}(-3)$$

$$b = \frac{1}{2} + \frac{3}{2}$$

$$b = 2.$$

$$-4 = 2(-3) + 2$$

$$-4 = -6 + 2$$

$$-4 = -4.$$

Por lo tanto, se escribe al vector  $w$  como combinación lineal de  $u$  y  $v$  de la siguiente manera:

$$w = -3u + 2v$$

$$(1, 7, -4) = -3(1, -3, 2) + 2(2, -1, 1).$$

(b) Escribir al vector  $z = (2, -5, 4)$  como combinación lineal de  $u$  y  $v$ .

$$z = au + bv$$

$$(2, -5, 4) = a(1, -3, 2) + b(2, -1, 1)$$

$$(2, -5, 4) = (a, -3a, 2a) + (2b, -b, b)$$

$$(2, -5, 4) = (a + 2b, -3a - b, 2a + b).$$

$$\begin{cases} 2 = a + 2b \\ -5 = -3a - b. \\ 4 = 2a + b \end{cases}$$

$$2b = 2 - a$$

$$b = \frac{2-a}{2}$$

$$b = 1 - \frac{1}{2}a.$$

$$-5 = -3a - (1 - \frac{1}{2}a)$$

$$-5 = -3a - 1 + \frac{1}{2}a$$

$$-5 = -\frac{5}{2}a - 1$$

$$\frac{5}{2}a = -1 + 5$$

$$\frac{5}{2}a = 4$$

$$a = \frac{4}{\frac{5}{2}}$$

$$a = \frac{8}{5}.$$

$$b = 1 - \frac{1}{2} \cdot \frac{8}{5}$$

$$b = 1 - \frac{4}{5}$$

$$b = \frac{1}{5}.$$

$$4 = 2 \cdot \frac{8}{5} + \frac{1}{5}$$

$$4 = \frac{16}{5} + \frac{1}{5}$$

$$4 \neq \frac{17}{5}.$$

Por lo tanto, no es posible escribir al vector  $z$  como combinación lineal de  $u$  y  $v$ .

(c) ¿Para qué valores de  $k$  el vector  $y = (1, k, 5)$  es una combinación lineal de  $u$  y  $v$ ?

$$y = au + bv$$

$$(1, k, 5) = a(1, -3, 2) + b(2, -1, 1)$$

$$(1, k, 5) = (a, -3a, 2a) + (2b, -b, b)$$

$$(1, k, 5) = (a + 2b, -3a - b, 2a + b).$$



$$\begin{cases} 1 = a + 2b \\ k = -3a - b \\ 5 = 2a + b \end{cases}$$

$$2b = 1 - a$$

$$b = \frac{1-a}{2}$$

$$b = \frac{1}{2} - \frac{1}{2}a.$$

$$5 = 2a + \frac{1}{2} - \frac{1}{2}a$$

$$5 = \frac{3}{2}a + \frac{1}{2}$$

$$\frac{3}{2}a = 5 - \frac{1}{2}$$

$$\frac{3}{2}a = \frac{9}{2}$$

$$a = \frac{\frac{9}{2}}{\frac{3}{2}}$$

$$a = 3.$$

$$b = \frac{1}{2} - \frac{1}{2} * 3$$

$$b = \frac{1}{2} - \frac{3}{2}$$

$$b = -1.$$

$$k = -3 * 3 - (-1)$$

$$k = -9 + 1$$

$$k = -8.$$

Por lo tanto, el vector  $y$  es una combinación lineal de  $u$  y  $v$  para  $k = -8$ .

**Ejercicio 10.**

Estudiar si el conjunto de vectores  $S = \{(2, 1, 0), (3, 1, 1), (3, 2, -1)\}$  es una base  $\mathbb{R}^3$ .

$$a(2, 1, 0) + b(3, 1, 1) + c(3, 2, -1) = (0, 0, 0)$$

$$(2a, a, 0) + (3b, b, b) + (3c, 2c, -c) = (0, 0, 0)$$

$$(2a + 3b + 3c, a + b + 2c, b - c) = (0, 0, 0).$$

$$\begin{cases} 2a + 3b + 3c = 0 \\ a + b + 2c = 0 \\ b - c = 0 \end{cases}.$$

$$b = c.$$

$$a + c + 2c = 0$$

$$a + 3c = 0$$

$$a = -3c.$$

$$2(-3c) + 3c + 3c = 0$$

$$-6c + 6c = 0$$

$$0 = 0.$$

Por lo tanto,  $S$  no es linealmente independiente y, entonces, no es una base de  $\mathbb{R}^3$ .

**Ejercicio 11.**

Encontrar un sistema generador del subespacio de  $\mathbb{R}^4$  definido por:

$$T = \{(x, y, z, t) \in \mathbb{R}^4 : x + 2z - 3t + 2y = 0\}.$$

$$x = -2z + 3t - 2y.$$

$$(x, y, z, t) = (-2z + 3t - 2y, y, z, t)$$

$$(x, y, z, t) = y(-2, 1, 0, 0) + z(-2, 0, 1, 0) + t(3, 0, 0, 1).$$

$$T = \langle (-2, 1, 0, 0), (-2, 0, 1, 0), (3, 0, 0, 1) \rangle.$$

## Ejercicio 12.

Determinar si los siguientes conjuntos son bases de  $\mathbb{R}^3$ .

(a)  $\{(1, 1, 4), (0, 2, 1), (3, 1, 9)\}$ .

$$a(1, 1, 4) + b(0, 2, 1) + c(3, 1, 9) = (0, 0, 0)$$

$$(a, a, 4a) + (0, 2b, b) + (3c, c, 9c) = (0, 0, 0)$$

$$(a + 3c, a + 2b + c, 4a + b + 9c) = (0, 0, 0).$$

$$\begin{cases} a + 3c = 0 \\ a + 2b + c = 0 \\ 4a + b + 9c = 0 \end{cases}$$

$$3c = -a$$

$$c = \frac{-1}{3} a.$$

$$a + 2b - \frac{1}{3} a = 0$$

$$\frac{2}{3} a + 2b = 0$$

$$2b = \frac{-2}{3} a$$

$$b = \frac{\frac{-2}{3}}{2} a$$

$$b = \frac{-1}{3} a.$$

$$4a - \frac{1}{3} a + 9 \left( \frac{-1}{3} a \right) = 0$$

$$4a - \frac{1}{3} a - 3a = 0$$

$$\frac{2}{3} a = 0$$

$$a = \frac{0}{\frac{2}{3}}$$

$$a = 0.$$

$$b = \frac{-1}{3} * 0$$

$$b = 0.$$

$$c = \frac{-1}{3} * 0$$

$$c = 0.$$

$$a = b = c = 0.$$

$$a(1, 1, 4) + b(0, 2, 1) + c(3, 1, 9) = (x, y, z)$$

$$(a, a, 4a) + (0, 2b, b) + (3c, c, 9c) = (x, y, z)$$

$$(a + 3c, a + 2b + c, 4a + b + 9c) = (x, y, z).$$

$$\begin{cases} a + 3c = x \\ a + 2b + c = y \\ 4a + b + 9c = z \end{cases}$$

$$3c = x - a$$

$$c = \frac{x-a}{3}$$

$$c = \frac{1}{3}x - \frac{1}{3}a.$$

$$a + 2b + \frac{1}{3}x - \frac{1}{3}a = y$$

$$\frac{2}{3}a + 2b + \frac{1}{3}x = y$$

$$2b = y - \frac{2}{3}a - \frac{1}{3}x$$

$$b = \frac{y - \frac{2}{3}a - \frac{1}{3}x}{2}$$

$$b = \frac{1}{2}y - \frac{1}{3}a - \frac{1}{6}x.$$

$$4a + \frac{1}{2}y - \frac{1}{3}a - \frac{1}{6}x + 9\left(\frac{1}{3}x - \frac{1}{3}a\right) = z$$

$$4a + \frac{1}{2}y - \frac{1}{3}a - \frac{1}{6}x + 3x - 3a = z$$

$$\frac{2}{3}a + \frac{17}{6}x + \frac{1}{2}y = z$$

$$\frac{2}{3}a = \frac{-17}{6}x - \frac{1}{2}y + z$$

$$a = \frac{\frac{-17}{6}x - \frac{1}{2}y + z}{\frac{2}{3}}$$

$$a = \frac{-51}{12}x - \frac{3}{4}y + \frac{3}{2}z$$

$$a = \frac{-51x - 9y + 18z}{12}.$$

$$b = \frac{1}{2}y - \frac{1}{3}\left(\frac{-51}{12}x - \frac{3}{4}y + \frac{3}{2}z\right) - \frac{1}{6}x$$

$$b = \frac{1}{2}y + \frac{17}{12}x + \frac{1}{4}y - \frac{1}{2}z - \frac{1}{6}x$$

$$b = \frac{15}{12}x + \frac{3}{4}y - \frac{1}{2}z$$

$$b = \frac{15x + 9y - 6z}{12}.$$

$$c = \frac{1}{3}x - \frac{1}{3}\left(\frac{-51}{12}x - \frac{3}{4}y + \frac{3}{2}z\right)$$

$$c = \frac{1}{3}x - \frac{17}{12}x + \frac{1}{4}y - \frac{1}{2}z$$

$$c = \frac{-13}{12}x + \frac{1}{4}y - \frac{1}{2}z$$

$$c = \frac{-13x + 3y - 6z}{12}.$$

Por lo tanto, este conjunto es base de  $\mathbb{R}^3$ .

(b)  $\{(1, 0, 1), (1, 1, 0)\}$ .

Este conjunto no es base de  $\mathbb{R}^3$ , ya que  $\dim(\mathbb{R}^3) = 3$ .

(c)  $\{(2, 1, 1), (2, 2, 1), (2, 2, -1)\}$ .

$$\begin{aligned} a(2, 1, 1) + b(2, 2, 1) + c(2, 2, -1) &= (0, 0, 0) \\ (2a, a, a) + (2b, 2b, b) + (2c, 2c, -c) &= (0, 0, 0) \\ (2a + 2b + 2c, a + 2b + 2c, a + b - c) &= (0, 0, 0). \end{aligned}$$

$$\begin{cases} 2a + 2b + 2c = 0 \\ a + 2b + 2c = 0 \\ a + b - c = 0 \end{cases}.$$

$$\begin{aligned} 2(a + b + c) &= 0 \\ a + b + c &= \frac{0}{2} \\ a + b + c &= 0 \\ c &= -a - b. \end{aligned}$$

$$\begin{aligned} a + 2b + 2(-a - b) &= 0 \\ a + 2b - 2a - 2b &= 0 \\ -a &= 0 \\ a &= \frac{0}{-1} \\ a &= 0. \end{aligned}$$

$$\begin{aligned} c &= -0 - b \\ c &= -b. \end{aligned}$$

$$\begin{aligned} 0 + b - (-b) &= 0 \\ b + b &= 0 \\ 2b &= 0 \\ b &= \frac{0}{2} \\ b &= 0. \end{aligned}$$

$$\begin{aligned} c &= -0 \\ c &= 0. \end{aligned}$$

$$a = b = c = 0.$$

$$\begin{aligned} a(2, 1, 1) + b(2, 2, 1) + c(2, 2, -1) &= (x, y, z) \\ (2a, a, a) + (2b, 2b, b) + (2c, 2c, -c) &= (x, y, z) \\ (2a + 2b + 2c, a + 2b + 2c, a + b - c) &= (x, y, z). \end{aligned}$$

$$\begin{cases} 2a + 2b + 2c = x \\ a + 2b + 2c = y \\ a + b - c = z \end{cases}.$$

$$\begin{aligned} 2(a + b + c) &= x \\ a + b + c &= \frac{1}{2}x \end{aligned}$$

$$c = \frac{1}{2}x - a - b.$$

$$a + 2b + 2\left(\frac{1}{2}x - a - b\right) = y$$

$$a + 2b + x - 2a - 2b = y$$

$$-a + x = y$$

$$a = x - y.$$

$$c = \frac{1}{2}x - (x - y) - b$$

$$c = \frac{1}{2}x - x + y - b$$

$$c = \frac{-1}{2}x + y - b.$$

$$x - y + b - \left(\frac{-1}{2}x + y - b\right) = z$$

$$x - y + b + \frac{1}{2}x - y + b = z$$

$$\frac{3}{2}x - 2y + 2b = z$$

$$2b = \frac{-3}{2}x + 2y + z$$

$$b = \frac{\frac{-3}{2}x + 2y + z}{2}$$

$$b = \frac{-3}{4}x + y + \frac{1}{2}z$$

$$b = \frac{-3x + 4y + 2z}{4}.$$

$$c = \frac{-1}{2}x + y - \left(\frac{-3}{4}x + y + \frac{1}{2}z\right)$$

$$c = \frac{-1}{2}x + y + \frac{3}{4}x - y - \frac{1}{2}z$$

$$c = \frac{1}{4}x - \frac{1}{2}z$$

$$c = \frac{x - 2z}{4}.$$

Por lo tanto, este conjunto es base de  $\mathbb{R}^3$ .

$$(d) \{(1, 2, 1), (1, 3, 1), (1, 4, 1), (1, 5, 1)\}.$$

Este conjunto no es base de  $\mathbb{R}^3$ , ya que es linealmente dependiente.

$$(e) \{(1, 1, 1), (-2, 1, 0), (-1, 0, 1)\}.$$

$$a(1, 1, 1) + b(-2, 1, 0) + c(-1, 0, 1) = (0, 0, 0)$$

$$(a, a, a) + (-2b, b, 0) + (-c, 0, c) = (0, 0, 0)$$

$$(a - 2b - c, a + b, a + c) = (0, 0, 0).$$

$$\begin{cases} a - 2b - c = 0 \\ a + b = 0 \\ a + c = 0 \end{cases}.$$

$$b = -a.$$

$$c = -a.$$

$$a - 2(-a) - (-a) = 0$$

$$a + 2a + a = 0$$

$$4a = 0$$

$$a = \frac{0}{4}$$

$$a = 0.$$

$$b = -0$$

$$b = -0.$$

$$c = -0$$

$$c = 0.$$

$$a = b = c = 0.$$

$$a(1, 1, 1) + b(-2, 1, 0) + c(-1, 0, 1) = (x, y, z)$$

$$(a, a, a) + (-2b, b, 0) + (-c, 0, c) = (x, y, z)$$

$$(a - 2b - c, a + b, a + c) = (x, y, z).$$

$$\begin{cases} a - 2b - c = x \\ a + b = y \\ a + c = z \end{cases}.$$

$$b = y - a.$$

$$c = z - a.$$

$$a - 2(y - a) - (z - a) = x$$

$$a - 2y + 2a - z + a = x$$

$$4a - 2y - z = x$$

$$2a = x + 2y + z$$

$$a = \frac{x + 2y + z}{4}$$

$$a = \frac{1}{4}x + \frac{1}{2}y + \frac{1}{4}z$$

$$a = \frac{x + 2y + z}{4}.$$

$$b = y - \left(\frac{1}{4}x + \frac{1}{2}y + \frac{1}{4}z\right)$$

$$b = y - \frac{1}{4}x - \frac{1}{2}y - \frac{1}{4}z$$

$$b = \frac{-1}{4}x + \frac{1}{2}y - \frac{1}{4}z$$

$$b = \frac{-x + 2y - z}{4}.$$

$$c = z - \left(\frac{1}{4}x + \frac{1}{2}y + \frac{1}{4}z\right)$$

$$c = z - \frac{1}{4}x - \frac{1}{2}y - \frac{1}{4}z$$



$$c = \frac{-1}{4}x - \frac{1}{2}y + \frac{3}{4}z$$
$$c = \frac{-x-2y+3z}{4}.$$

Por lo tanto, este conjunto es base de  $\mathbb{R}^3$ .

**Ejercicio 13 (\*).**

Sean  $B = \{(2, 1, 1), (1, -1, 3), v\}$  una base de  $\mathbb{R}^3$ . Hallar  $v$  sabiendo que las coordenadas del vector  $(1, -2, 5)$  en la base  $B$  son  $(2, -1, 3)$ .

Se sabe que:

$$B = \{(2, 1, 1), (1, -1, 3), v\}.$$

Entonces, se tiene:

$$[(1, -2, 5)]_B = (2, -1, 3).$$

Operando, se llega a:

$$(1, -2, 5) = 2(2, 1, 1) - (1, -1, 3) + 3v$$

$$(1, -2, 5) = (4, 2, 2) - (1, -1, 3) + 3v$$

$$(1, -2, 5) = (3, 3, -1) + 3v$$

$$3v = (1, -2, 5) - (3, 3, -1)$$

$$3v = (-2, -5, 6)$$

$$v = \frac{1}{3}(-2, -5, 6)$$

$$v = \left(\frac{-2}{3}, \frac{-5}{3}, 2\right).$$

Por lo tanto,  $v = \left(\frac{-2}{3}, \frac{-5}{3}, 2\right)$ .

**Ejercicio 14.**

Sean  $B = \{(-1, 4, 2), v, (0, 0, -1)\}$  y  $B' = \{w, (1, -1, 1), (-1, 0, 2)\}$  bases de  $\mathbb{R}^3$ . Hallar  $v$  y  $w$  sabiendo que las coordenadas de  $v$  en la base de  $B'$  son  $(1, 2, 3)$  y que las coordenadas de  $w$  en la base  $B$  son  $(1, 2, 3)$ .

$$B = \{(-1, 4, 2), v, (0, 0, -1)\}.$$

$$B' = \{w, (1, -1, 1), (-1, 0, 2)\}.$$

$$[v]_{B'} = (1, 2, 3).$$

$$[w]_B = (1, 2, 3).$$

$$v = w + 2(1, -1, 1) + 3(-1, 0, 2)$$

$$v = w + (2, -2, 2) + (-3, 0, 6)$$

$$v = w + (-1, -2, 8).$$

$$w = (-1, 4, 2) + 2v + 3(0, 0, -1)$$

$$w = (-1, 4, 2) + 2v + (0, 0, -3)$$

$$w = 2v + (-1, 4, -1).$$

$$v = 2v + (-1, 4, -1) + (-1, -2, 8)$$

$$v = 2v + (-2, 2, 7)$$

$$2v - v = -(-2, 2, 7)$$

$$v = (2, -2, -7).$$

$$w = 2(2, -2, -7) + (-1, 4, -1)$$

$$w = (4, -4, -14) + (-1, 4, -1)$$

$$w = (3, 0, -15).$$

**Ejercicio 15.**

Probar que los siguientes conjuntos son subespacios:

(a)  $W = \{(x, y, z): x = y = z\}$  de  $\mathbb{R}^3$ .

W es cerrado para la suma:

$$\begin{aligned} (x_1, y_1, z_1), (x_2, y_2, z_2) \in W &\Rightarrow x_1 = y_1 = z_1 \wedge x_2 = y_2 = z_2 \\ &\Rightarrow x_1 + x_2 = y_1 + y_2 = z_1 + z_2 \\ &\Rightarrow (x_1 + x_2, y_1 + y_2, z_1 + z_2) \in W \\ &\Rightarrow (x_1, y_1, z_1) + (x_2, y_2, z_2) \in W. \end{aligned}$$

W es cerrado para la multiplicación de escalares:

$$\begin{aligned} (x, y, z) \in W &\Rightarrow x = y = z \\ &\Rightarrow \forall k \in \mathbb{R}, kx = ky = kz \\ &\Rightarrow (kx, ky, kz) \in W \\ &\Rightarrow k(x, y, z) \in W. \end{aligned}$$

Por lo tanto, el conjunto es un subespacio.

(b)  $W = \{(x, y, z, t): x = z, y = t\}$  de  $\mathbb{R}^4$ .

W es cerrado para la suma:

$$\begin{aligned} (x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2) \in W &\Rightarrow x_1 = z_1, y_1 = t_1 \wedge x_2 = z_2, y_2 = t_2 \\ &\Rightarrow x_1 + x_2 = z_1 + z_2, y_1 + y_2 = t_1 + t_2 \\ &\Rightarrow (x_1 + x_2, y_1 + y_2, z_1 + z_2, t_1 + t_2) \in W \\ &\Rightarrow (x_1, y_1, z_1, t_1) + (x_2, y_2, z_2, t_2) \in W. \end{aligned}$$

W es cerrado para la multiplicación de escalares:

$$\begin{aligned} (x, y, z, t) \in W &\Rightarrow x = z, y = t \\ &\Rightarrow \forall k \in \mathbb{R}, kx = kz, ky = kt \\ &\Rightarrow (kx, ky, kz, kt) \in W \\ &\Rightarrow k(x, y, z, t) \in W. \end{aligned}$$

Por lo tanto, el conjunto es un subespacio.

(c)  $W = \{(x, y, z, t): 2y + 3z = 0\}$  de  $\mathbb{R}^4$ .

W es cerrado para la suma:

$$(x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2) \in W \Rightarrow 2y_1 + 3z_1 = 0 \wedge 2y_2 + 3z_2 = 0$$

$$\begin{aligned}
&\Rightarrow 2y_1 + 2y_2 + 3z_1 + 3z_2 = 0 \\
&\Rightarrow 2(y_1 + y_2) + 3(z_1 + z_2) = 0 \\
&\Rightarrow (x_1 + x_2, y_1 + y_2, z_1 + z_2, t_1 + t_2) \in W \\
&\Rightarrow (x_1, y_1, z_1, t_1) + (x_2, y_2, z_2, t_2) \in W.
\end{aligned}$$

W es cerrado para la multiplicación de escalares:

$$\begin{aligned}
(x, y, z, t) \in W &\Rightarrow 2y + 3z = 0 \\
&\Rightarrow \forall k \in \mathbb{R}, k(2y + 3z) = 0 \\
&\Rightarrow \forall k \in \mathbb{R}, 2ky + 3kz = 0 \\
&\Rightarrow (kx, ky, kz, kt) \in W \\
&\Rightarrow k(x, y, z, t) \in W.
\end{aligned}$$

Por lo tanto, el conjunto es un subespacio.

**Ejercicio 16.**

Una compañía petrolera puede convertir un barril de crudo en tres clases distintas de combustible. Sin aditivos de plomo, sus producciones de las tres clases de combustible a partir de un barril de crudo vienen dadas por el vector  $(2, 2, 4)$ . Con el máximo de aditivos de plomo permitidos legalmente, las producciones son  $(5, 0, 3)$ . Se supone que los efectos de los aditivos de plomo son proporcionales, es decir, que al usar una fracción del máximo permitido  $(0 \leq \alpha \leq 1)$  se tiene la producción:

$$(1 - \alpha)(2, 2, 4) + \alpha(5, 0, 3).$$

Determinar si es posible que la compañía produzca los siguientes vectores:

(a)  $(\frac{5}{2}, 1, \frac{7}{2})$ .

$$(\frac{5}{2}, 1, \frac{7}{2}) = (1 - \alpha)(2, 2, 4) + \alpha(5, 0, 3)$$

$$(\frac{5}{2}, 1, \frac{7}{2}) = (2, 2, 4) + (-2\alpha, -2\alpha, -4\alpha) + (5\alpha, 0, 3\alpha)$$

$$(\frac{5}{2}, 1, \frac{7}{2}) = (2 + 3\alpha, 2 - 2\alpha, 4 - \alpha).$$

$$\begin{cases} \frac{5}{2} = 2 + 3\alpha \\ 1 = 2 - 2\alpha \\ \frac{7}{2} = 4 - \alpha \end{cases}$$

$$3\alpha = \frac{5}{2} - 2$$

$$3\alpha = \frac{1}{2}$$

$$\alpha = \frac{\frac{1}{2}}{3}$$

$$\alpha = \frac{1}{6}.$$

$$2\alpha = 2 - 1$$

$$2\alpha = 1$$

$$\alpha = \frac{1}{2}.$$

$$\alpha = 4 - \frac{7}{2}$$

$$\alpha = \frac{1}{2}.$$

Por lo tanto, no es posible que la compañía produzca este vector.

(b)  $(\frac{9}{2}, \frac{1}{3}, \frac{19}{6})$ .

$$(\frac{9}{2}, \frac{1}{3}, \frac{19}{6}) = (1 - \alpha)(2, 2, 4) + \alpha(5, 0, 3)$$

$$\left(\frac{9}{2}, \frac{1}{3}, \frac{19}{6}\right) = (2, 2, 4) + (-2\alpha, -2\alpha, -4\alpha) + (5\alpha, 0, 3\alpha)$$

$$\left(\frac{9}{2}, \frac{1}{3}, \frac{19}{6}\right) = (2 + 3\alpha, 2 - 2\alpha, 4 - \alpha).$$

$$\begin{cases} \frac{9}{2} = 2 + 3\alpha \\ \frac{1}{3} = 2 - 2\alpha \\ \frac{19}{6} = 4 - \alpha \end{cases}$$

$$3\alpha = \frac{9}{2} - 2$$

$$3\alpha = \frac{5}{2}$$

$$\alpha = \frac{\frac{5}{2}}{3}$$

$$\alpha = \frac{5}{6}.$$

$$2\alpha = 2 - \frac{1}{3}$$

$$2\alpha = \frac{5}{3}$$

$$\alpha = \frac{\frac{5}{3}}{2}$$

$$\alpha = \frac{5}{6}.$$

$$\alpha = 4 - \frac{19}{6}$$

$$\alpha = \frac{5}{6}.$$

Por lo tanto, sí es posible que la compañía produzca este vector.

(c)  $(1, 6, 9)$ .

$$(1, 6, 9) = (1 - \alpha)(2, 2, 4) + \alpha(5, 0, 3)$$

$$(1, 6, 9) = (2, 2, 4) + (-2\alpha, -2\alpha, -4\alpha) + (5\alpha, 0, 3\alpha)$$

$$(1, 6, 9) = (2 + 3\alpha, 2 - 2\alpha, 4 - \alpha).$$

$$\begin{cases} 1 = 2 + 3\alpha \\ 6 = 2 - 2\alpha \\ 9 = 4 - \alpha \end{cases}$$

$$3\alpha = 1 - 2$$

$$3\alpha = -1$$

$$\alpha = \frac{-1}{3}.$$

$$2\alpha = 6 - \frac{1}{3}$$

$$2\alpha = \frac{17}{3}$$

$$\alpha = \frac{\frac{17}{3}}{2}$$
$$\alpha = \frac{17}{6}.$$

$$\alpha = 4 - 9$$

$$\alpha = -5.$$

Por lo tanto, no es posible que la compañía produzca este vector.



**Ejercicio 17.**

Considerar el conjunto:

$$V = \{(w, td, ti, P, GP) : P = w, td + ti = GP\},$$

donde  $w$  es el crecimiento de los salarios nominales,  $td$  es el crecimiento de los impuestos directos,  $ti$  es el crecimiento de los impuestos indirectos,  $P$  es el crecimiento de los precios y  $GP$  es el crecimiento del gasto público.

(a) Mostrar que  $V$  es un subespacio vectorial de  $\mathbb{R}^5$ .

$V$  es cerrado para la suma:

$$\begin{aligned} (w_1, td_1, ti_1, P_1, GP_1), (w_2, td_2, ti_2, P_2, GP_2) \in V &\Rightarrow P_1 = w_1, td_1 + ti_1 = GP_1 \wedge P_2 = w_2, \\ td_2 + ti_2 = GP_2 & \\ \Rightarrow P_1 + P_2 = w_1 + w_2, (td_1 + td_2) + (ti_1 + ti_2) = GP_1 + GP_2 & \\ \Rightarrow (w_1 + w_2, td_1 + td_2, ti_1 + ti_2, P_1 + P_2, GP_1 + GP_2) \in V & \\ \Rightarrow (w_1, td_1, ti_1, P_1, GP_1) + (w_2, td_2, ti_2, P_2, GP_2) \in V. & \end{aligned}$$

$V$  es cerrado para la multiplicación de escalares:

$$\begin{aligned} (w, td, ti, P, GP) \in V &\Rightarrow P = w, td + ti = GP \\ \Rightarrow \forall k \in \mathbb{R}, kP = kw, k(td + ti) = kGP & \\ \Rightarrow \forall k \in \mathbb{R}, kP = kw, ktd + kti = kGP & \\ \Rightarrow (kw, ktd, kti, kP, kGP) \in V & \\ \Rightarrow k(w, td, ti, P, GP) \in V. & \end{aligned}$$

Por lo tanto,  $V$  es un subespacio vectorial de  $\mathbb{R}^5$ .

(b) Hallar una base y la dimensión de  $V$ . Dar una interpretación económica del resultado.

$$(P, td, ti, P, td + ti) = P(1, 0, 0, 1, 0) + td(0, 1, 0, 0, 1) + ti(0, 0, 1, 0, 1).$$

$$B_V = \{(1, 0, 0, 1, 0), (0, 1, 0, 0, 1), (0, 0, 1, 0, 1)\}.$$

$$\dim(V) = 3.$$