Time Series
Problem Set 3 - Answer Key

Problem 1

Consider the standar linear regression model

$$\underbrace{y_t}_{1\times 1} = \underbrace{x_t^T}_{1\times k}\underbrace{\beta}_{k\times 1} + \underbrace{u_t}_{1\times 1}$$

for x_t a $(k \times 1)$ of explanatory variables. As usual assume that the explanatory variables are uncorrelated with the residual (u_t):

$$E(x_t u_t) = 0$$

a. Show that the Ordinary Least Squares estimator of β has the same expression that the Generalized Method of Moments (GMM) estimador of β

b. Find the asymptotic distribution of $\widehat{\beta}_{GMM}$. Suppose that

$$E\left[u_t u_{t-v} x_t x_t^T\right] = \begin{cases} \sigma^2 E(x_t x_t^T) \text{ si } v = 0 \\ \mathbf{0} \text{ si } v \neq 0 \end{cases}$$

c. What would happen if the assumption in (b) does not hold?

a. Remember that $\hat{\beta}_{OLS} = \left(\sum_{i=1}^{T} x_t x_t^T\right)^{-1}\left(\sum_{i=1}^{T} x_t y_t\right)$.
Ir order to find the GMM estimator of β, we first need to find the orthogonality conditions

$$\begin{aligned} E\left[x_t u_t\right] &= 0 \\ E\left[x_t u_t\right] &= E\left[x_t\left(y_t - x_t^T\beta\right)\right] = 0 \end{aligned}$$

This result implies that

$$h\left(\beta; y_t; x_t\right) = \left(x_t y_t - x_t x_t^T\beta\right)$$

Lets define $w_t = (y_t; x_t)$.

$$g\left(\beta; w_1; ...; w_T\right) = \frac{1}{T}\sum_{i=1}^{T} h\left(\beta; y_t; x_t\right) = \frac{1}{T}\sum_{i=1}^{T}\left(x_t y_t - x_t x_t^T\beta\right)$$

Since the number of unknown coefficients is equal to te number of orthogonality conditions, $\hat{\beta}_{GMM}$ is the value that sets the sample average of $h\left(\beta; y_t; x_t\right)$ equal to zero.

$$g\left(\hat{\beta}; w_1; ...; w_T\right) = 0 \iff \frac{1}{T}\sum_{i=1}^{T} x_t y_t = \frac{1}{T}\sum_{i=1}^{T} x_t x_t^T\hat{\beta} \iff \left(\sum_{i=1}^{T} x_t x_t^T\right)^{-1}\left(\sum_{i=1}^{T} x_t y_t\right) = \hat{\beta}_{GMM} = \hat{\beta}_{OLS}$$

b. Rembmer that

$$
\begin{aligned}
\sqrt{T}\left(\hat{\beta}_{GMM}-\beta\right) \xrightarrow{D} N\left(0;V\right)\ con\ V &= \left(D^{T}S^{-1}D\right)^{-1} \\
\hat{\beta}_{GMM} &\approx N\left(0;\frac{V}{T}\right)
\end{aligned}
$$

With

$$
\begin{aligned}
D &\equiv \lim_{T\to\infty}\frac{\partial g\left(\hat{\beta}_{GMM};Y_1;...;Y_T\right)}{\partial\hat{\beta}_{GMM}} \\
\frac{\partial g\left(\hat{\beta}_{GMM};Y_1;...;Y_T\right)}{\partial\hat{\beta}_{GMM}} &= \frac{1}{T}\sum_{i=1}^{T}x_t x_t^T \\
D &\equiv \lim_{T\to\infty}\frac{\partial g\left(\hat{\beta}_{GMM};Y_1;...;Y_T\right)}{\partial\hat{\beta}_{GMM}} = E\left[x_t x_t^T\right]
\end{aligned}
$$

$$
\begin{aligned}
S &\equiv \sum_{v=-\infty}^{\infty}\Gamma_v\ with\ \Gamma_v \equiv E\left[h\left(\beta;w_t\right)h\left(\beta;w_{t-v}\right)^T\right] \\
&= E\left[x_t\left(y_t-x_t^T\beta\right)\left(y_{t-v}-x_{t-v}^T\beta\right)x_{t-v}^T\right] \\
&= E\left[x_t u_t u_{t-v}x_{t-v}^T\right] \\
&= \left[u_t u_{t-v}x_t x_{t-v}^T\right] \\
&= \left\{\begin{array}{c} \sigma^2 E\left[x_t x_t^T\right]\ \text{si}\ v=0 \\ 0\ \text{si}\ v\neq 0 \end{array}\right.
\end{aligned}
$$

Wich implies that

$$
V=\left(E\left[x_t x_t^T\right]^T\frac{1}{\sigma^2}E\left[x_t x_t^T\right]^{-1}E\left[x_t x_t^T\right]\right)^{-1}=\left(E\left[x_t x_t^T\right]^T\frac{1}{\sigma^2}\right)^{-1}=\sigma^2 E\left[x_t x_t^T\right]^{-1}
$$

.

c. If assumption of homocedasticity is not valid, the GMM estimator will no longer be efficient.

Problem 2

Consider the standar linear regression model

$$
\underbrace{y_t}_{1\times 1}=\underbrace{x_t^T}_{1\times k}\underbrace{\beta}_{k\times 1}+\underbrace{u_t}_{1\times 1}
$$

for x_t a $(k\times 1)$ of explanatory variables. Assume that

$$
E(x_t u_t)\neq 0
$$

Let z_t be a $(k\times 1)$ vector of valid instruments:

$$
E(z_t u_t)=0
$$

a. Show that the GMM estimator of β is equal to the Instrumental Variables Estimator of β ($\widehat{\beta}_{GMM} = \widehat{\beta}_{IV}$)

b. Assume now that z_t is a $(r \times 1)$ vector with $r > k$. Show that the GMM estimator of β is equal to the Two Stage Least Square Estimator of β . ($\widehat{\beta}_{GMM} = \widehat{\beta}_{2SLS}$)

a. Lets consider the linear regression model

$$\underbrace{y_t}_{1\times 1} = \underbrace{x_t^T}_{1\times k}\underbrace{\beta}_{k\times 1} + \underbrace{u_t}_{1\times 1}$$

Since

$$E\left[x_t u_t\right] \neq 0$$

We need to find k instruments $\underbrace{z_t}_{k\times 1}$

Since z_t are valid instruments, now

$$E\left[z_t u_t\right] = 0$$

Wich implies

$$E\left[z_t\left(y_t - x_t^T\beta\right)\right] = 0$$

Lets call

$$w_t \equiv (y_t; x_t; z_t)$$

$$h\left(\beta; w_t\right) = z_t\left(y_t - x_t^T\beta\right)$$

$$g\left(\beta; w_t\right) = \frac{1}{T}\sum_{i=1}^{T} h\left(\beta; w_t\right)$$

Since the number of othogonality conditions is equal to the number of parameters to estimate, to find

$\hat{\beta}_{GMM}$

$$\begin{aligned} g\left(\hat{\beta}_{GMM}; w_t\right) &= 0 \iff \frac{1}{T}\sum_{i=1}^{T} z_t\left(y_t - x_t^T\hat{\beta}_{GMM}\right) = 0 \\ \sum_{i=1}^{T} z_t y_t &= \sum_{i=1}^{T} z_t x_t^T\hat{\beta}_{GMM} \iff \left(\sum_{i=1}^{T} z_t x_t^T\right)^{-1}\left(\sum_{i=1}^{T} z_t y_t\right) = \hat{\beta}_{GMM} \end{aligned}$$

b. Suposse now that $z_{t\ r\times 1}$ with $r > k$ (i.e. the number of instruments is larger than the number of parameters to estimate)

The orthogonality conditions are

$$E\left[z_t u_t\right] = 0$$

Lets call

$$w_t \equiv (y_t; x_t; z_t)$$

$$h(\beta; w_t) = z_t\left(y_t - x_t^T\beta\right)$$

$$g(\beta; w_t) = \frac{1}{T}\sum_{i=1}^{T} h(\beta; w_t)$$

Since the number of number of othogonality conditions is larger than the number of parameters to estimate, we cannot find $\hat{\beta}_{GMM}$ by setting

$$g\left(\hat{\beta}_{MGM}; w_t\right) = 0$$

Instead we need to minimize

$$Q(\beta; w_1; ...; w_T) \equiv g(\beta; w_1; ...; w_T)^T S^{-1} g(\beta; w_1; ...; w_T)$$

The FOC are

$$\left[\frac{\partial g(\beta; w_1; ...; w_T)}{\partial\beta}\right]^T S^{-1} g(\beta; w_1; ...; w_T) = 0$$

with

$$S \equiv \sum_{v=-\infty}^{\infty} \Gamma_v$$

and

$$\Gamma_v \equiv E\left[h(\beta; w_t)\, h(\beta; w_{t-v})^T\right]$$

Since

$$\Gamma_v = E\left[z_t u_t u_{t-v} z_{t-v}^T\right] = E\left[u_t u_{t-v} z_t z_{t-v}^T\right] = \begin{cases} \sigma^2 E\left[z_t z_t^T\right] \text{ si } v = 0 \\ 0 \text{ si } v \neq 0 \end{cases}$$

$\hat{\beta}_{GMM}$ satisfies

$$\left[\frac{\partial g(\beta; w_1; ...; w_T)}{\partial\beta}\left(\hat{\beta}_{MGM}\right)\right]^T \widehat{S}^{-1} g\left(\hat{\beta}_{MGM}; w_1; ...; w_T\right) = 0$$

with

$$\begin{aligned} \widehat{S} &= \hat{\sigma}^2 \frac{1}{T}\sum_{i=1}^{T}\left(z_t z_t^T\right) \\ \hat{\sigma}^2 &= \frac{1}{T}\sum_{i=1}^{T}\left(y_t - x_t^T\hat{\beta}_{OLS}\right)^2 \end{aligned}$$

$$g(\beta; w_1; ...; w_T) = \frac{1}{T}\sum_{i=1}^{T} z_t y_t - \frac{1}{T}\sum_{i=1}^{T} z_t x_t^T\beta \implies \frac{\partial g(\beta; w_1; ...; w_T)}{\partial\beta}\left(\hat{\beta}_{GMM}\right) = -\frac{1}{T}\sum_{i=1}^{T} z_t x_t^T$$

Then the FOC are

$$
\begin{aligned}
-\frac{1}{T}\left(\sum_{i=1}^{T} z_t x_t^T\right)\frac{1}{\hat{\sigma}^2}\left(\sum_{i=1}^{T} z_t z_t^T\right)^{-1}\frac{1}{T}\sum_{i=1}^{T}\left(z_t y_t - z_t x_t^T\hat{\beta}_{GMM}\right) &= 0\\
\left(\sum_{i=1}^{T} x_t^T z_t\right)\left(\sum_{i=1}^{T} z_t z_t^T\right)^{-1}\sum_{i=1}^{T}\left(z_t y_t - z_t x_t^T\hat{\beta}_{GMM}\right) &= 0\\
\left(\sum_{i=1}^{T} x_t^T z_t\right)\left(\sum_{i=1}^{T} z_t z_t^T\right)^{-1}\sum_{i=1}^{T}\left(z_t y_t - z_t x_t^T\hat{\beta}_{GMM}\right) &= 0
\end{aligned}
$$

If we identify

$$
\begin{aligned}
A &\equiv \left(\sum_{i=1}^{T} x_t^T z_t\right)\left(\sum_{i=1}^{T} z_t z_t^T\right)^{-1}\\
A^T &= \left[\left(\sum_{i=1}^{T} z_t z_t^T\right)^{-1}\right]^T\left(\sum_{i=1}^{T} x_t^T z_t\right)^T = \left[\left(\sum_{i=1}^{T} z_t z_t^T\right)^T\right]^{-1}\left(\sum_{i=1}^{T} x_t^T z_t\right)^T\\
&= \left(\sum_{i=1}^{T} z_t z_t^T\right)^{-1}\left(\sum_{i=1}^{T} z_t x_t^T\right)
\end{aligned}
$$

$\hat{\delta}^T = \left(\sum_{i=1}^{T} z_t z_t^T\right)^{-1}\left(\sum_{i=1}^{T} z_t x_t^T\right)$ is the matrix which rows represent the coefficients from an OLS regression of z_t on x_t

Then

$$
\hat{\delta}^T\sum_{i=1}^{T}\left(z_t y_t - z_t x_t^T\hat{\beta}_{GMM}\right) = \sum_{i=1}^{T}\hat{\delta}^T z_t y_t - \sum_{i=1}^{T}\hat{\delta}^T z_t x_t^T\hat{\beta}_{GMM} = 0 \iff \sum_{i=1}^{T}\widehat{z}_t y_t = \sum_{i=1}^{T}\widehat{z}_t x_t^T\hat{\beta}_{GMM}
$$

$$
\left(\sum_{i=1}^{T}\widehat{z}_t x_t^T\right)^{-1}\left(\sum_{i=1}^{T}\widehat{z}_t y_t\right) = \hat{\beta}_{GMM} = \hat{\beta}_{2SLS}
$$

which equals to 2SLS.

Problem 3

Consider simple consumption-based asset pricing model. The conditions to maximize the utility of the representative agent are

$$
\begin{aligned}
E\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(1+r_{f,t+1}) \mid I_t^*\right] &= 1\\
E\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(r_{j,t+1}-r_{f,t+1}) \mid I_t^*\right] &= 0 \text{ con } j = 1,2,3..,10
\end{aligned}
$$

where $r_{f,t}$ is the risklees asset return in t, C_t is the consumption level in t and $r_{j,t}$ are the returns of the atlernative risky assets in t.. I_t^* stands for the information set a time t.

Based on this conditions, the author estimates the unknow parameters β (intertemporal discount factor) and γ (risk aversion coefficient)

Based on this conditions, open apm.wf1 and estimate by GMM the unknow parameters β (intertemporal discount factor) and γ (risk aversion coeficient), using identity weighting matrix and sequential updating. Are these models valid under Hansen's J-Test Hypothesis?

Solution

First of all, we must write de system of equations for the Eviews' GMM estimation. Go to .ºbject/New Object/System". Name the system and press .ºK".

In the empty window that appears we must write the system as given in the FOC above. As E-views treats every equation entered as being an expected value, it is only necessary to enter the expression inside the expected value operator:

C(1)*(cons^(-C(2)))*(1+rf)-1=0
C(1)*(cons^(-C(2)))*(r1-rf)=0
C(1)*(cons^(-C(2)))*(r2-rf)=0
C(1)*(cons^(-C(2)))*(r3-rf)=0
C(1)*(cons^(-C(2)))*(r4-rf)=0
C(1)*(cons^(-C(2)))*(r5-rf)=0
C(1)*(cons^(-C(2)))*(r6-rf)=0
C(1)*(cons^(-C(2)))*(r7-rf)=0
C(1)*(cons^(-C(2)))*(r8-rf)=0
C(1)*(cons^(-C(2)))*(r9-rf)=0
C(1)*(cons^(-C(2)))*(r10-rf)=0

Note that C(1) corresponds to β and C(2) corresponds to γ, both parametres we want to estimate with GMM.

In this sense, we must state our instrumental variables. Many models include lags of the consumption ratio and of the return on assets (see Hansen and Singleton (1982)), yet in this example no lags are taken. This example is taken from Verbeek (2014) , '*A guide to Modern Econometrics*' . Therefore our instrumental variables shall only be the constants. **NOTE THAT THIS IS NOT THE USUAL WAY TO PROCEED. TYPICALLY, WE SHOULD INCLUDE LAGS OF THE VARIABLES AS INSTRUMENTS, AS WE'LL DO IN THE FOLLOWING EXERCISE.**. So after the system we write:

"@inst c"

By doing this, we are test just the unconditional mean, that is, whether

$$
\begin{aligned}
E\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(1+r_{f,t+1})\right] &= 1 \\
E\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(r_{j,t+1}-r_{f,t+1})\right] &= 0 \text{ con } j = 1,2,3..,10
\end{aligned}
$$

holds or not. Note that this is a necesarry but not sufficent condition for the model to hold [1]

Next we go to 'estimate'. At 'estimation method' we choose 'GMM - Time Series (HAC)'. Then, on 'Estimation Settings' we leave 'Add lagged regressors...' unchecked.

The assignment asks us to estimate the GMM model both with one step estimation and an Identity Weighting Matrix, and Iterated GMM. For the first case we check on Ïdentity weighting matrix". For the

Figura 1: How the system is written in Eviews

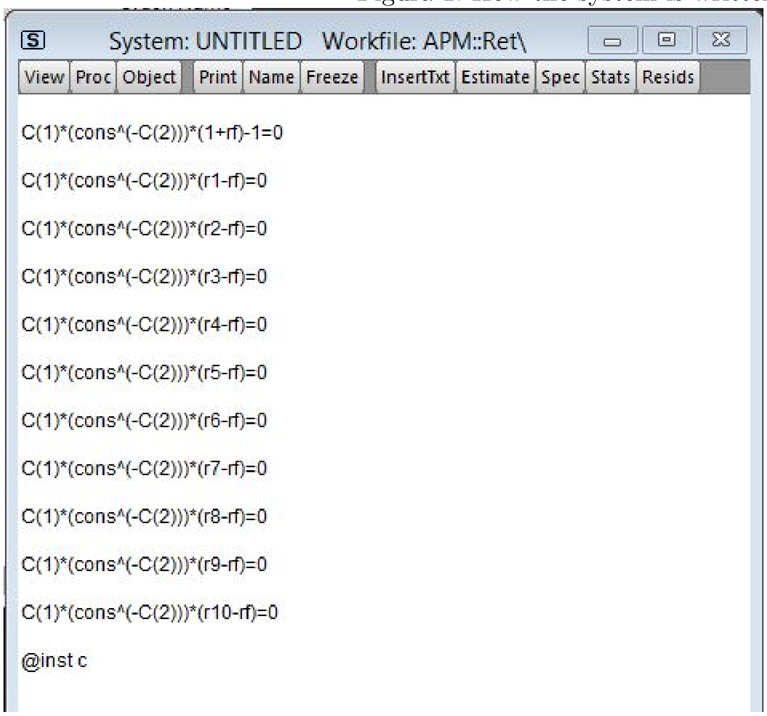


Figura 2: Estimation specifications

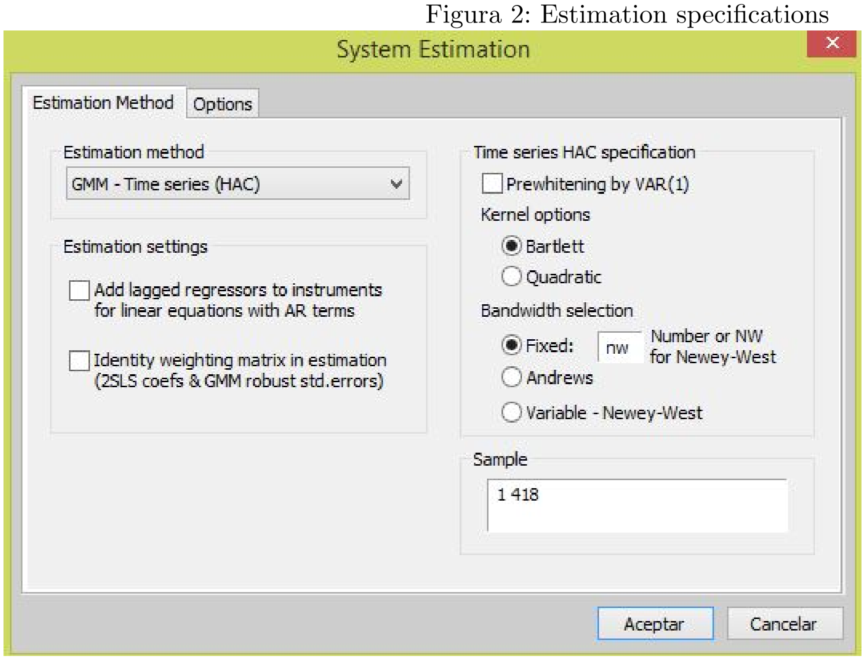


Figura 3: Estimation specifications: Identity weighting matrix

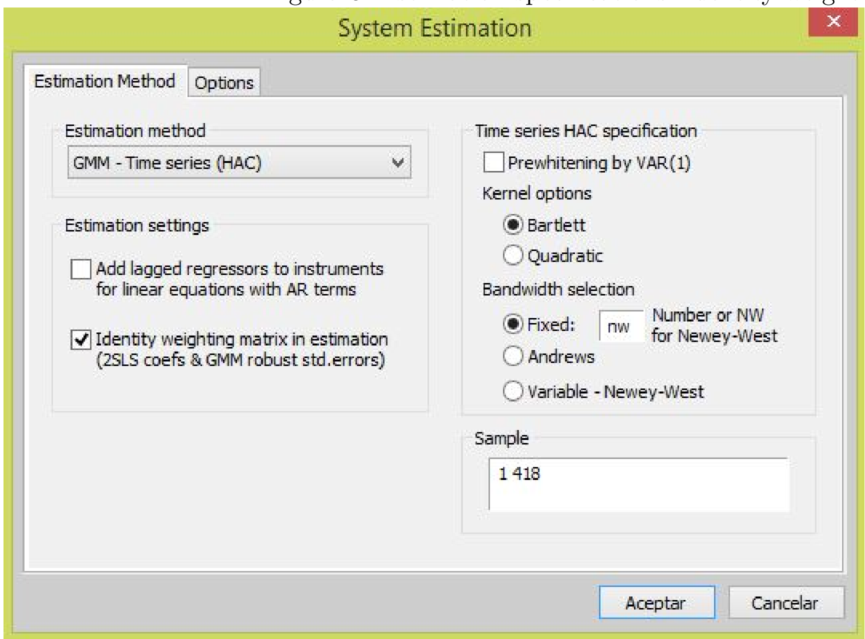
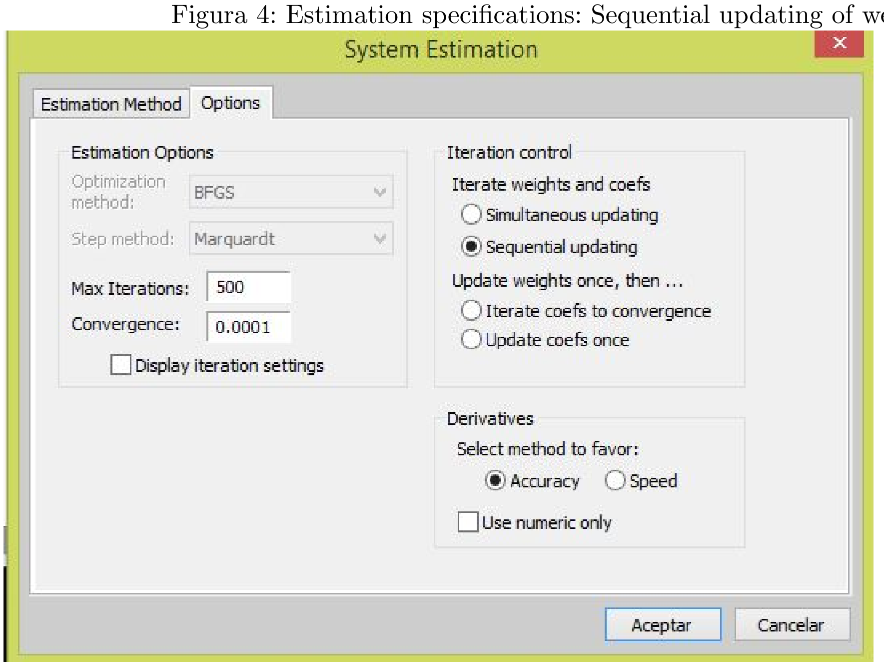


Figura 4: Estimation specifications: Sequential updating of weightning matrix



second case we uncheck Ïdentity weighting matrix”, and in options select ”sequential updatingünder the Ïterate Weights...”heading.

Then we estimate. Results of the two different specifications are shown below.

Figura 5: Estimation output: Identity weighting matrix

System: UNTITLED
Estimation Method: Generalized Method of Moments
Date: 04/14/20 Time: 13:34
Sample: 1 418
Included observations: 418
Total system (balanced) observations 4598
Identity matrix estimation weights - 2SLS coefs with GMM standard errors
Kernel: Bartlett, Bandwidth: Fixed (6), No prewhitening
Convergence achieved after 5 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.699606	59.99942	0.011660	0.9907
C(2)	91.40973	16221.32	0.005635	0.9955
Determinant residual covariance		1.54E-36		
J-statistic		0.001140		

For the second case we uncheck Ïdentity weighting matrix”, and in options select ”sequential updatingünder the Ïterate Weights...”heading.

Figura 6: Estimation output: Sequential updating of weightning matrix

System: UNTITLED
Estimation Method: Generalized Method of Moments
Date: 04/14/20 Time: 13:27
Sample: 1 418
Included observations: 418
Total system (balanced) observations 4598
Kernel: Bartlett, Bandwidth: Fixed (6), No prewhitening
Sequential weighting matrix & coefficient iteration
Convergence achieved after: 8 weight matrices, 29 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.836037	0.112194	7.451719	0.0000
C(2)	58.34707	33.73132	1.729759	0.0837
Determinant residual covariance		6.30E-38		
J-statistic		0.015619		

Note that the estimated coefficents are totally different from usual calibrations. The problem could arise from the fact that we use only the constant as an instrument. If we add lagged consumption, for example, the results would be considerably different.

Note also that the standard errors under Identity weightning are huge. We are not able to reject anything. This is because we didn't use a weightning matrix close to optimal. The software gives us the estimated coefficients, its standard errors and below a sort J-Statistic. Note however that this statistic is not what we need. Actually we must multiply its value by the number of observations. In this case we have 418 observations. In addition, we have $r = 11$ restrictions and $k = 2$ parameters. Therefore, the calculated J-Statistics has a χ^2_9 distribution.

To calculate the actual J-Statistic and its p-value, we first store the estimated system. To do so, go to *Name* and save with some name. In my case, I've stored it as SYS01. To compute the actual statistic write in the command window:

[1]PROOF: Note that, by the Law of Iterated Expectations, $E[|I_t^*] = 0 \Rightarrow E[] = 0$. Therefore, if we can reject that the unconditional mean equations hold, then we reject the conditionals to hold as well

```
scalar j=SYS01.@regobs*SYS01.@jstat
scalar j_p=1-@cchisq(j,9)
```

The first command computes the statistic as the product of the number of observations in the system (SYS01.@regobs) and the reported J-stat in Eviews (SYS01.@jstat) . The second command computes the p-value of the statistic, as 1- the CDF of a χ^2 with the right number of degrees of freedom.

Hansen's overidentification test is as follows:

$H_0 : E(h(X_t, \theta_0)) = 0$
$H_1 : E(h(X_t, \theta_0)) \neq 0$

Under H_0 we have:

$$J = \frac{1}{T}\left[\sum_{t=1}^{T} g\left(X_t, \hat{\theta}\right)\right]' \hat{W} \left[\sum_{t=1}^{T} g\left(X_t, \hat{\theta}\right)\right] \xrightarrow{L} \chi^2_{r-k}$$

H_0 is rejected if $J >$ 0,95 percentile of χ^2_{r-k}.
H_0 is rejected if $J >$ 0,95 percentile of χ^2_{r-k}.

In the first case (Identity weights) we are not able to reject the null: the p-value is 0,99 . This is because we used Identity weights, which are far away from optimal. That generates huge standard errors, leading to an inability to reject the null.

In the second case (sequential updating) it is 6,52 and its p-value 0,683. Note that the p-value is smaller, but still we're not able to reject the null. Therefore the null hypothesis cannot be rejected with a 95 % confidence.

Finally, it is worth mentioning that, even though the estimated β's may be economically relevant, the estimated γ's are definitely not. As stated by the author, these estimations are "huge and imprecise". This issue may arise form the fact that we didn't used lags as intruments, and also problems with the model.

More instruments

For example, we can use first lags as instruments. We also use 'param' to set initial conditions.

The estimation results are as shown in the figures. Note that the estimated parameters are radically different.

Note that estimated coefficents are radically different. Note also that in the case of Sequential updating, the estimated γ is negative, against the theory.

Figura 7: System specification with more instruments

```
C(1)*(cons^(-C(2)))*(1+rf)-1=0

C(1)*(cons^(-C(2)))*(r1-rf)=0

C(1)*(cons^(-C(2)))*(r2-rf)=0

C(1)*(cons^(-C(2)))*(r3-rf)=0

C(1)*(cons^(-C(2)))*(r4-rf)=0

C(1)*(cons^(-C(2)))*(r5-rf)=0

C(1)*(cons^(-C(2)))*(r6-rf)=0

C(1)*(cons^(-C(2)))*(r7-rf)=0

C(1)*(cons^(-C(2)))*(r8-rf)=0

C(1)*(cons^(-C(2)))*(r9-rf)=0

C(1)*(cons^(-C(2)))*(r10-rf)=0

@inst c cons(-1) r1(-1) r2(-1) r3(-1) r4(-1) r5(-1) r6(-1)  r7(-1) r8(-1) r9(-1)  r10(-1)

param  c(1) 0.9 c(2) 4
```

Figura 8: Estimation output with Identity Weightning

System: SYS01
Estimation Method: Generalized Method of Moments
Date: 04/14/20   Time: 15:10
Sample: 2 418
Included observations: 417
Total system (balanced) observations 4587
Identity matrix estimation weights - 2SLS coefs with GMM standard errors
Kernel: Bartlett, Bandwidth: Fixed (6), No prewhitening
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.991251	1.362274	0.727644	0.4669
C(2)	0.543492	615.3747	0.000883	0.9993
Determinant residual covariance		1.01E-42		
J-statistic		18.93655		

Figura 9: Estimation output with Sequential updating

System: SYS01
Estimation Method: Generalized Method of Moments
Date: 04/14/20   Time: 15:16
Sample: 2 418
Included observations: 417
Total system (balanced) observations 4587
Kernel: Bartlett, Bandwidth: Fixed (6), No prewhitening
Simultaneous weighting matrix & coefficient iteration
Convergence achieved after: 157 weight matrices, 158 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.997793	0.000403	2474.161	0.0000
C(2)	-1.611413	0.168661	-9.554129	0.0000
Determinant residual covariance		2.73E-42		
J-statistic		0.146724		

Problem 4

1) Estimate the following model through GMM equation (not system)::

$$0 = E\left[\beta\left(\frac{C_{t+2}}{C_t}\right)^{-\gamma} Z_t R_{t+2} - Z_t\right] \quad (1)$$

where the instruments are $Z_t = \left(1, \frac{C_t}{C_{t-2}}, R_t, \frac{H_t}{H_{t-2}}, \frac{W_t}{W_{t-2}}\right)$.

Use a bandwidth of 4 and simultaneous updating of weighting matrix and coefficients.

Hint: write the equation as an orthogonality condition between residuals and the instrumental variables.

2) Find the J-Statistic. Is the model correctly specified according to this sample?
3) Fix β as 0.9, 0.95 and 1. Estimate a new γ in each case and find the J-statistics.
4) Re-estimate (β, γ) now using the new list of instruments, $\bar{Z}_t = (Z_t, Z_{t-1}, Z_{t-2})$. Note that the constant should appear only once. Do you find any change in the results? Perform Hansen's test.

Solution

1)

In order to estimate the model through GMM equation estimation in Eviews, the equation should be re-written as an orthogonality condition:

$$0 = E\left[\left(\beta\left(\frac{C_{t+2}}{C_t}\right)^{-\gamma} R_{t+2} - 1\right) Z_t\right]$$

In Eviews we go to Quick/Estimate Equation... In Method we choose GMM and a new window appears. First we write the model as we have just stated. Then we state the instrument variables. Be aware of including a constant. We select time series (HAC) weighting matrix and choose a fixed bandwith of 4. Then we estimate.

The result is shown in the image

2)

The J-statistic is 8,2609 and its corresponding p-value would be $0,0409$. Note that in this case, the J-stat and p-values are the ones we want to use, in sharp contrast to the case of system estimation. Therefore we reject the null hypothesis at the usual $5\,\%$ level of significance. This means that we have strong evidence against the null hipothesis. Therefore, the model is not correct according to this sample.

3)

In order to do so, we replace the specified values in the estimated equation. We keep the bandwith of 4, chosen by us.

$\beta =$ 0,9

Note that the estimated γ is negative.

Figura 10: We specify the equation and estimation settings

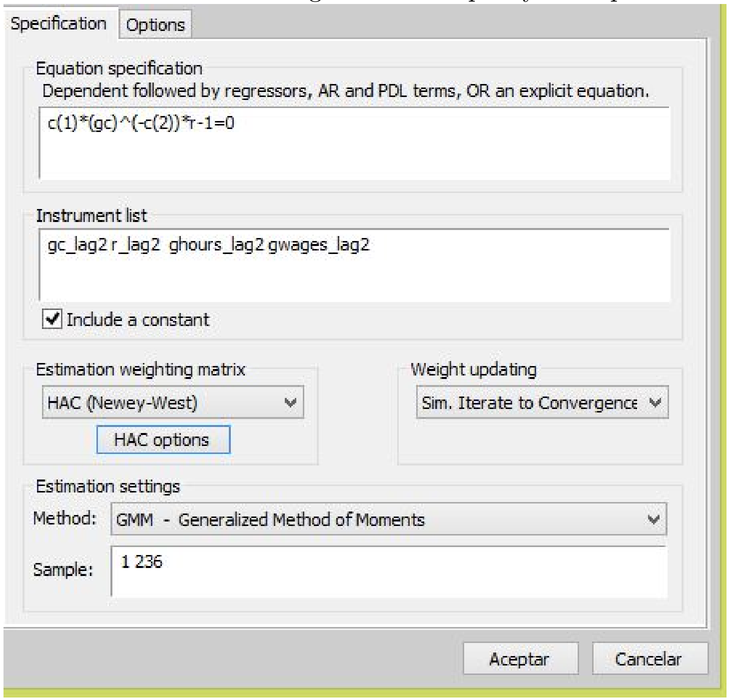
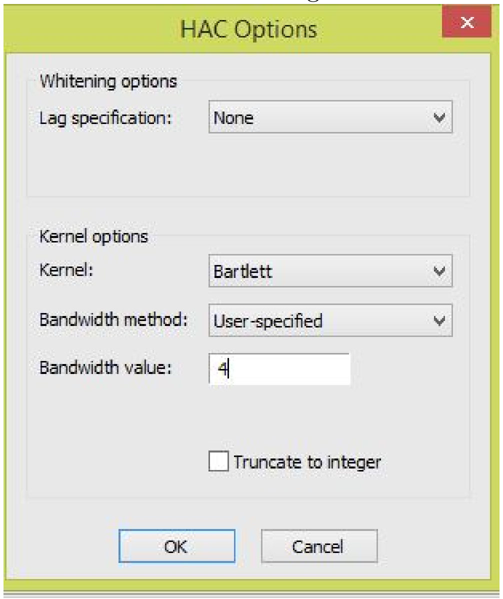


Figura 11: Set the bandwith to 4 by clicking 'HAC options'



$\beta = 0{,}95$

We proceed as before. Thus, only the estimation output is shown

$\beta = 0{,}999$

We proceed as before. Thus, only the estimation output is shown
Note that the estimated γ is negative.

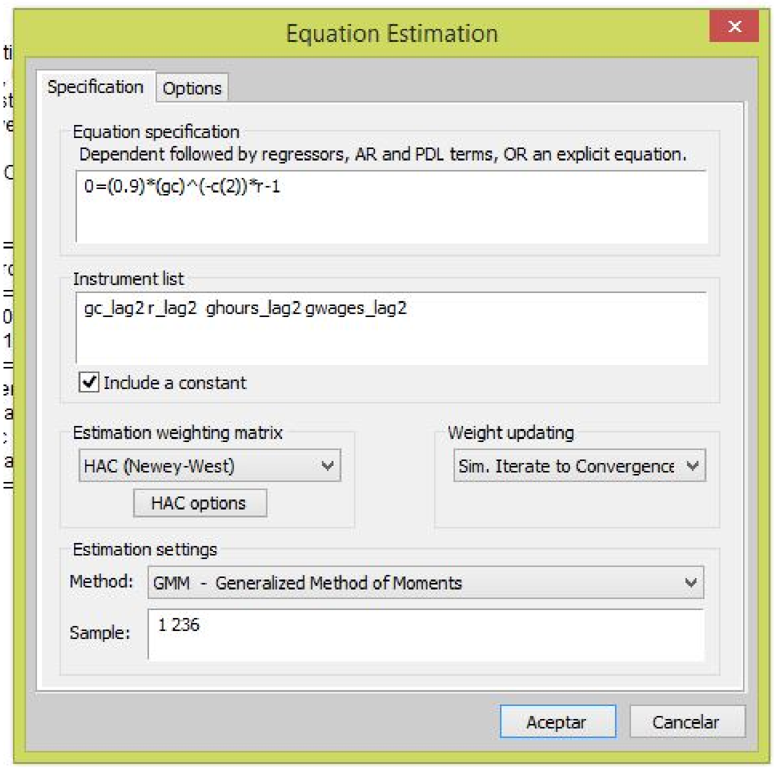
Figura 12: Estimation output

Dependent Variable: Implicit Equation
Method: Generalized Method of Moments
Date: 04/15/20 Time: 18:58
Sample: 1 236
Included observations: 236
Sequential 1-step weighting matrix & coefficient iteration
Estimation weighting matrix: HAC (Bartlett kernel, User bandwidth = 4.0000)
Standard errors & covariance computed using estimation weighting matrix
Convergence achieved after 7 iterations
0-(C(1)*(GC)^(-C(2))*R-1)
Instrument specification: GC_LAG2 R_LAG2 GHOURS_LAG2
GWAGES_LAG2
Constant added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.008969	0.003986	253.1224	0.0000
C(2)	0.819536	0.222050	3.690779	0.0003

Mean dependent var	0.000000	S.D. dependent var	0.000000
S.E. of regression	0.017032	Sum squared resid	0.067879
Durbin-Watson stat	0.570561	J-statistic	7.987351
Instrument rank	5	Prob(J-statistic)	0.046274

Figura 13: Estimation spec for $\beta = 0{,}9$



Now we have one coefficient to be estimated and 5 intruments. Therefore we have $r - k = 4$ overidentifying restrictions. The J-statistics and its p-values are shown in the table below.

In all of the three cases the null hypothesis is rejected. Therefore the modified model is not valid for this sample.

Figura 14: Estimation output for $\beta = 0{,}9$

Dependent Variable: Implicit Equation
Method: Generalized Method of Moments
Date: 04/15/20 Time: 19:07
Sample: 1 236
Included observations: 236
Simultaneous weighting matrix & coefficient iteration
Estimation weighting matrix: HAC (Bartlett kernel, User bandwidth = 4.0000)
Standard errors & covariance computed using estimation weighting matrix
Convergence achieved after 8 coefficient and 7 weight iterations
0-((0.9)*(GC)^(-C(2))*R-1)
Instrument specification: GC_LAG2 R_LAG2 GHOURS_LAG2
GWAGES_LAG2
Constant added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C(2)	-4.846893	0.266654	-18.17671	0.0000

Mean dependent var	0.000000	S.D. dependent var	0.000000
S.E. of regression	0.058318	Sum squared resid	0.799228
Durbin-Watson stat	0.605678	J-statistic	14.58462
Instrument rank	5	Prob(J-statistic)	0.005645

Figura 15: Estimation output for $\beta = 0{,}95$

Dependent Variable: Implicit Equation
Method: Generalized Method of Moments
Date: 04/15/20 Time: 19:10
Sample: 1 236
Included observations: 236
Simultaneous weighting matrix & coefficient iteration
Estimation weighting matrix: HAC (Bartlett kernel, User bandwidth = 4.0000)
Standard errors & covariance computed using estimation weighting matrix
Convergence achieved after 8 coefficient and 7 weight iterations
0-((0.95)*(GC)^(-C(2))*R-1)
Instrument specification: GC_LAG2 R_LAG2 GHOURS_LAG2
GWAGES_LAG2
Constant added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C(2)	-2.221786	0.135920	-16.34633	0.0000

Mean dependent var	0.000000	S.D. dependent var	0.000000
S.E. of regression	0.028986	Sum squared resid	0.197448
Durbin-Watson stat	0.498835	J-statistic	14.69389
Instrument rank	5	Prob(J-statistic)	0.005380

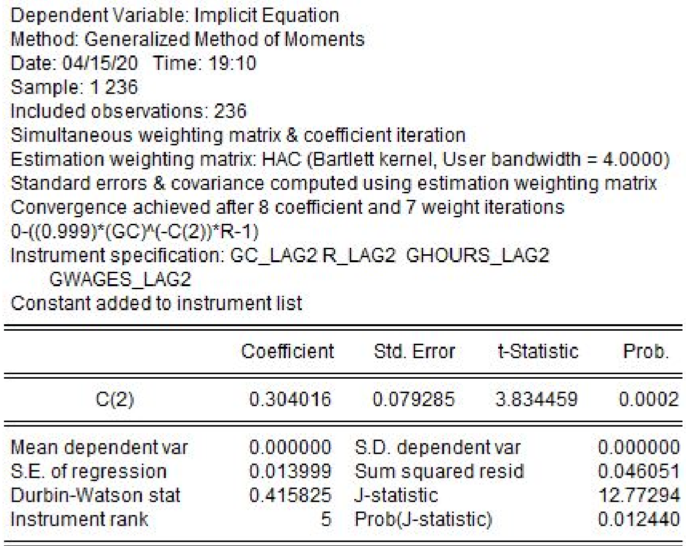
4)

We use more lags as instruments. The estimation settings and estimation output are shown in the figures below.

Here we have $T = 234$, $r = 13, k = 2, r - k = 11$. The J-statistic is 13.19476 and its corresponding p-value is 0.280787. Therefore the null hypothesis cannot be rejected, and the model is valid for this sample. This is beacuse we added a lot more of orthogonality conditions. By doing so, the power of the test is lower than in the less-instruments case.

Here we can notice that whilst the β estimate suffered little change, the γ estimate fell significantly.

Figura 16: Estimation output for $\beta = 0{,}999$



Dependent Variable: Implicit Equation
Method: Generalized Method of Moments
Date: 04/15/20 Time: 19:10
Sample: 1 236
Included observations: 236
Simultaneous weighting matrix & coefficient iteration
Estimation weighting matrix: HAC (Bartlett kernel, User bandwidth = 4.0000)
Standard errors & covariance computed using estimation weighting matrix
Convergence achieved after 8 coefficient and 7 weight iterations
0-((0.999)*(GC)^(-C(2))*R-1)
Instrument specification: GC_LAG2 R_LAG2 GHOURS_LAG2
GWAGES_LAG2
Constant added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C(2)	0.304016	0.079285	3.834459	0.0002

Mean dependent var	0.000000	S.D. dependent var	0.000000
S.E. of regression	0.013999	Sum squared resid	0.046051
Durbin-Watson stat	0.415825	J-statistic	12.77294
Instrument rank	5	Prob(J-statistic)	0.012440

β	J-Statistic	p-value
$\beta = 0,9$	14,58462	0,005645
$\beta = 0,95$	14,69368	0,005381
$\beta = 0{,}999$	12,77296	0,012440

Figura 17: Estiamtion setting: more lags added as instruments

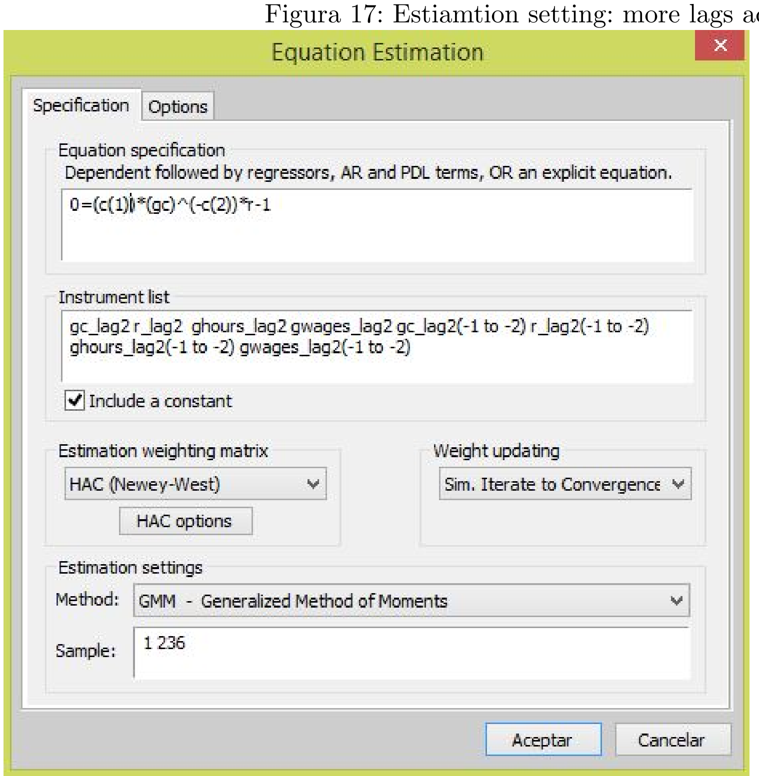


Figura 18: Estiamtion output: more lags added as instruments

Dependent Variable: Implicit Equation
Method: Generalized Method of Moments
Date: 04/15/20 Time: 19:18
Sample (adjusted): 3 236
Included observations: 234 after adjustments
Simultaneous weighting matrix & coefficient iteration
Estimation weighting matrix: HAC (Bartlett kernel, User bandwidth = 4.0000)
Standard errors & covariance computed using estimation weighting matrix
Convergence achieved after 11 coefficient and 10 weight iterations
0-((C(1))*(GC)^(-C(2))*R-1)
Instrument specification: GC_LAG2 R_LAG2 GHOURS_LAG2
GWAGES_LAG2 GC_LAG2(-1 TO -2) R_LAG2(-1 TO -2)
GHOURS_LAG2(-1 TO -2) GWAGES_LAG2(-1 TO -2)
Constant added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.005734	0.002857	351.9959	0.0000
C(2)	0.560405	0.155398	3.606250	0.0004

Mean dependent var	0.000000	S.D. dependent var	0.000000
S.E. of regression	0.015277	Sum squared resid	0.054144
Durbin-Watson stat	0.506665	J-statistic	13.19476
Instrument rank	13	Prob(J-statistic)	0.280787