Time Series
Problem Set 3

Problem 1

Consider the standar linear regression model

$$\underbrace{y_t}_{1\times 1} = \underbrace{x_t^T}_{1\times k}\underbrace{\beta}_{k\times 1} + \underbrace{u_t}_{1\times 1}$$

for x_t a $(k \times 1)$ of explanatory variables. As usual assume that the explanatory variables are uncorrelated with the residual (u_t):

$$E(x_t u_t) = 0$$

a. Show that the Ordinary Least Squares estimator of β has the same expression that the Generalized Method of Moments (GMM) estimador of β

b. Find the asymptotic distribution of $\widehat{\beta}_{GMM}$. Suppose that

$$E\left[u_t u_{t-v} x_t x_t^T\right] = \begin{cases} \sigma^2 E(x_t x_t^T) \text{ si } v = 0 \\ \mathbf{0} \text{ si } v \neq 0 \end{cases}$$

c. What would happen if the assumption in (b) does not hold?

Problem 2

Consider the standar linear regression model

$$\underbrace{y_t}_{1\times 1} = \underbrace{x_t^T}_{1\times k}\underbrace{\beta}_{k\times 1} + \underbrace{u_t}_{1\times 1}$$

for x_t a $(k \times 1)$ of explanatory variables. Assume that

$$E(x_t u_t) \neq 0$$

Let z_t be a $(k \times 1)$ vector of valid instruments:

$$E(z_t u_t) = 0$$

a. Show that the GMM estimator of β is equal to the Instrumental Variables Estimator of β ($\widehat{\beta}_{GMM} = \widehat{\beta}_{IV}$)

b. Assume now that z_t is a $(r \times 1)$ vector with $r > k$. Show that the GMM estimator of β is equal to the Two Stage Least Square Estimator of β . ($\widehat{\beta}_{GMM} = \widehat{\beta}_{2SLS}$)

Problem 3

This exercise is taken form an example of Verbeek(2014) *'A guide to modern econometrics'* . Consider simple consumption-based asset pricing model. The conditions to maximize the utility of the representative agent are

$$
\begin{aligned}
E\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(1+r_{f,t+1}) \mid I_t^*\right] &= 1 \\
E\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(r_{j,t+1}-r_{f,t+1}) \mid I_t^*\right] &= 0 \text{ con } j=1,2,3..,10
\end{aligned}
$$

where $r_{f,t}$ is the risklees asset return in t, C_t is the consumption level in t and $r_{j,t}$ are the returns of the atlernative risky assets in t.. I_t^* stands for the information set a time t.

Based on this conditions, the author estimates the unknow parameters β (intertemporal discount factor) and γ (risk aversion coefficient)

Based on this conditions, open apm.wf1 and estimate by GMM the unknow parameters β (intertemporal discount factor) and γ (risk aversion coeficient), using identity weighting matrix and sequential updating. Are these models valid under Hansen's J-Test Hypothesis?

Problem 4

1) Estimate the following model through GMM equation (not system):

$$
0 = E\left[\beta\left(\frac{C_{t+2}}{C_t}\right)^{-\gamma} Z_t R_{t+2} - Z_t\right]
$$

where the instruments are $Z_t = \left(1, \frac{C_t}{C_{t-2}}, R_t, \frac{H_t}{H_{t-2}}, \frac{W_t}{W_{t-2}}\right)$.

Use a bandwidth of 4 and simultaneous updating of weighting matrix and coefficients.

Hint: write the equation as an orthogonality condition between residuals and the instrumental variables.

2) Find the J-Statistic. Is the model correctly specified according to this sample?

3) Fix β as 0.9, 0.95 and 0.999. Estimate a new γ in each case and find the J-statistics.

4) Re-estimate (β, γ) now using the new list of instruments, $\bar{Z}_t = (Z_t, Z_{t-1}, Z_{t-2})$. Note that the constant should appear only once. Do you find any change in the results? Perform Hansen's test.