Time Series

Problem Set 2 - VAR

Problem 1 - Long-Run Multiplier

Consider the following bivariate system

$$\begin{cases} y_t = b_{10} - b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt} \\ z_t = b_{20} - b_{22}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt} \end{cases}$$

where it is assumed that (1) both y_t and z_t are stationary; (2) ε_{yt} and ε_{zt} are white noise disturbances with standar deviations of σ_y and σ_z respectively, and $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{yt}\}$ are uncorrelated white noise disturbances.

a. Rewrite the system as an VAR(1) process:

$$W_t = A_0 + A_1 W_{t-1} + e_t$$

Here $W_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$; $A_0 = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix}$; $A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $e_t = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$

b. Find the expression for each element of A_0, A_1 and e_t.

c. Name the variance/covariance matrix of $\begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$, Δ. Compute Λ, the variance/covariance matrix of e_t. Compare Λ with Δ.

d. What is the condition required for stationarity?

e. Suppose that the condition found in (d) is met, rewritte the expression found in (a) as a vector MA(∞).

f. Use the moving average representation found in (e) to express W_t in terms of the $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{yt}\}$ secuences. Use this expression to find the instantaneous impact of a unit change in ε_{yt} on y_t. Find the expresión for the long-run multiplier.

g. Decompose the n-step-ahead forcast error variance of y_{t+n} into the proportions due to each shock ($\{\varepsilon_{yt}\}$ and $\{\varepsilon_{yt}\}$)

Problem 2 - IRF and LR Effects

Let y_t represent output and x_t, an economic policy instrument (for instance, money supply). Assume they evolve according to the following bivariate VAR(1) process:

$$\begin{aligned}
\begin{bmatrix} y_t \\ x_t \end{bmatrix} &= \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} \\
\begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} &= \begin{bmatrix} 1 & \theta \\ \phi & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \\
E\left\{ \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \right\} &= 0_2 \\
E\left\{ \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \begin{bmatrix} \varepsilon_{ys} \\ \varepsilon_{xs} \end{bmatrix}^T \right\} &= I(t=s)\Sigma_\varepsilon \\
0 &< a_1 < 1
\end{aligned}$$

i) Under what conditions can u_{xt} be interpreted as a policy innovation?

ii) It can be showed that the impulse-response function on y_t of ε_{xt} is given by

$$\varphi_{xt}(0) = \theta$$
$$\varphi_{xt}(s) = a_1^{s-1}(a_1\theta + a_2) \qquad s \geq 1$$

But the parameter θ cannot be estimated from a sample, and so without further restrictions this function is not identified. Assume that policy shocks affect output with a one-period lag. What is the Impulse Response Function in that case?

iii) What about when policy shocks are neutral (ineffective) in the long-run?

v) And when policy is not responsive to contemporaneous output shocks?

Problem 3 - US Data

The VAR.wf1 data set contains quarterly U.S. data on the rate of price inflation (π_t), the unemployment rate (u_t), and the interest rate (R_t), specifically, the federal founds rate from the first quarter of 1960 to the fourth quarter of 2000.

a. Use this data set and estimate a trivariate VAR(4).

b. Do you have evidence that the estimated VAR is stable?

c. Is there evidence that the number of lags should be 4? That all lags should be included?

d. Suppose that de VAR(4) specification is correct. Execute a Granger Causality test. What are your conclusions?

e. Obtain the impulse-response functions.

f. Estimate the proportions of the n-step ahead forecast error variance for every variable. . For $n = 1, 4, 8, 12$

Problem 4 - Testing Rational Expectations Hypothesis

Review the lecture notes on Testing Rational expectations Hypothesis.

The E-Views Workfile "uncovered" contains quarterly series from Q3 1978 to Q3 2008 for: USD/GBP exchange rate, 3 month GBP libor rate and 3 month USD Treasury Bill rate.

Perform a hypothesis test to asses wether the uncovered interest rate parity holds.

Problem 5 - Supply and Demand Shocks

Read the paper "The Dynamic Effects of Agregate Demand and Supply Disturbances" by Blanchard and Quah (1989). As covered in the lecture notes, the authors offered a different approach to use structural information to identify the estimated reduced form VAR. We will follow the authors by recognizing the existence of supply and demand disturbances which both affect the output and the unemployment rate in the short-run, but the demand shocks does not affect the output in the long term.

In Eviews terminology, $C_{1,1} = 0$, where $C_{1,1}$ is the upper left-handed element of the $C_{k\times k}$ matrix given by $C = \hat{\Psi}_{\infty}A^{-1}B$, where matrix A and B are given by $A\epsilon_t = Bu_t$ (see Eviews guide II).

The E-Views Workfile "bc" contains quarterly series from Q2 1948 to Q4 2007 for US Output (in Billion Dollars) and Unemployment rate. Unemployment rate is measured monthly, this is a 3 month average.

i) Check wether GDP growth and average unemployment are stationary. Argue if this assumption is plausible.

ii) Estimate a VAR for GDP growth and average unemployment choosing the number of lags based on relevant criteria. The error term in the reduced form VAR will be a linear combination of supply and demand schocks.

iii) For the preferred VAR in part ii) compute impulse response functions and variance decomposition of forecasts for the two possible cholesky orderings.

iv) For the preferred VAR in part ii) estimate a structural var with the restriction that the long run effect of a demand schock in GDP growth is zero. Compare the results with the ones you obtained in iii)