Clase 6/5 a

Ejercicio 1

$y_{t-1} = 7 + y_{t-2}$

a) $\Delta y_t = 7 \rightarrow y_t - y_{t-1} = 7 \rightarrow y_t = 7 + y_{t-1}$

$= 7 + (7 + y_{t-2}) = 2 \cdot 7 + y_{t-2} = 2 \cdot 7 + 7 + y_{t-3}$

$= 3 \cdot 7 + y_{t-3} = \dots = 7t + y_{t-t} = \boxed{7t + C = y_t}$

$\Delta y_t = \Delta(7t + c) = (7t + \cancel{c}) - (7(t-1) + \cancel{c})$

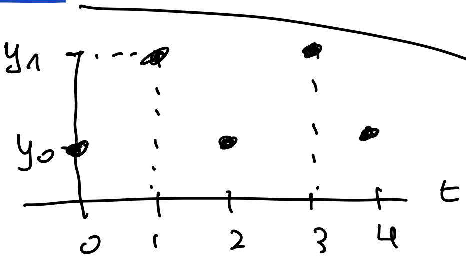
$= 7t - (7t - 7) = 7t - 7t + 7 = 7$

c) $\Delta y_t = 2y_t - 9 \rightarrow y_t - y_{t-1} = 2y_t - 9 \rightarrow 9 = 2y_t - y_t + y_{t-1}$

$\Rightarrow 9 - y_{t-1} = y_t \rightarrow y_t = 9 - (9 - y_{t-2}) = y_{t-2}$

$y_0 = C$

$y_1 = 9 - C$



$$y_t = \begin{cases} 9 - C & \text{para } t \text{ impar} \\ C & \text{para } t \text{ par} \end{cases}$$

Ejercicio 2

a) $\begin{cases} y_t = y_{t-1} + 1 \\ y_0 = 10 \end{cases} \rightarrow y_t = y_{t-1} + 1 = (y_{t-2} + 1) + 1$

$\dots = t + y_0$

$\Rightarrow \boxed{y_t = t + 10}$

c) $\begin{cases} 2y_t - y_{t-1} = 6 \\ y_0 = 7 \end{cases} \rightarrow 2y_t = 6 + y_{t-1} \rightarrow y_t = 3 + \frac{y_{t-1}}{2}$

$\rightarrow y_t = 3 + \frac{1}{2}\left(3 + \frac{1}{2} y_{t-2}\right)$

$= 3\left(1+\frac{1}{2}\right) + \frac{1}{4} y_{t-2}$

$y_t = 3\left(1+\frac{1}{2}\right) + \frac{1}{4}\left(3 + \frac{1}{2} y_{t-3}\right)$

$= 3\left(1+\frac{1}{2}+\frac{1}{4}\right) + \frac{1}{8} y_{t-3}$

$y_t = 3\left(\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}\right) + \frac{1}{2^3} y_{t-3}$

$\vdots$

$y_t = 3\underbrace{\left(\frac{1}{2^0}+\frac{1}{2^1}+\dots+\frac{1}{2^{t-1}}\right)}_{S} + \frac{1}{2^t} \overset{y_0}{y_{t-t}}$

$y_t = 3\left(2 - \frac{1}{2^{t-1}}\right) + \frac{y_0}{2^t}, \quad y_0 = 7$

$y_t = 6 - \frac{3}{2^{t-1}} + \frac{7}{2^t}$

$= 6 - \frac{\overset{=6}{3\cdot 2}}{2^t} + \frac{7}{2^t}$

$y_t = 6 + \frac{1}{2^t}$ $\quad \left(y_t = 6 + \frac{y_0 - 6}{2^t}\right)$

CA :

$\frac{1}{2^0} + \frac{1}{2^1} + \dots + \frac{1}{2^{t-1}} = S$

$1 + \left(\frac{1}{2^1} + \frac{1}{2^2} + \dots + \frac{1}{2^{t-1}}\right) = S$

$1 + \frac{1}{2}\left(\frac{1}{2^0} + \frac{1}{2^1} + \dots + \frac{1}{2^{t-2}}\right) = S$

$S = 1 + \frac{1}{2}\left(\underbrace{\frac{1}{2^0} + \frac{1}{2^1} + \dots + \frac{1}{2^{t-2}} + \frac{1}{2^{t-1}}}_{S} - \frac{1}{2^{t-1}}\right)$

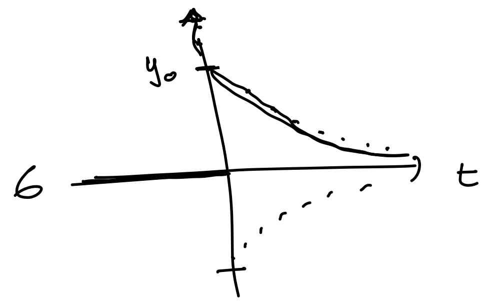
$S = 1 + \frac{1}{2}\left(S - \frac{1}{2^{t-1}}\right)$

$S = 1 + \frac{S}{2} - \frac{1}{2^t}$

$S - \frac{S}{2} = \frac{S}{2} = 1 - \frac{1}{2^t}$

$\boxed{S = 2 - \frac{1}{2^{t-1}}}$

$+ \frac{1}{2}\frac{1}{2^{t-1}} = \frac{1}{2 \cdot 2^{t-1}} = \frac{1}{2^t}$

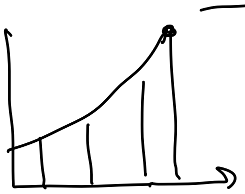


b) $\begin{cases} y_t + 3y_{t-1} = 4 \\ y_0 = 4 \end{cases} \longrightarrow y_t = 4 - 3y_{t-1}$

$$S = a^0 + a^1 + a^2 + \dots + a^{t-1}$$
$$= 1 + a(a^0 + a^1 + \dots + a^{t-2})$$
$$= 1 + a(\underbrace{a^0 + a^1 + \dots + a^{t-2} + a^{t-1}}_{S} - a^{t-1})$$
$$\Rightarrow \quad S = 1 + a(S - a^{t-1})$$
$$S = 1 + aS - a^t$$
$$(1-a)S = 1 - a^t \qquad a \neq 1$$
$$\boxed{S = \frac{1-a^t}{1-a}}$$

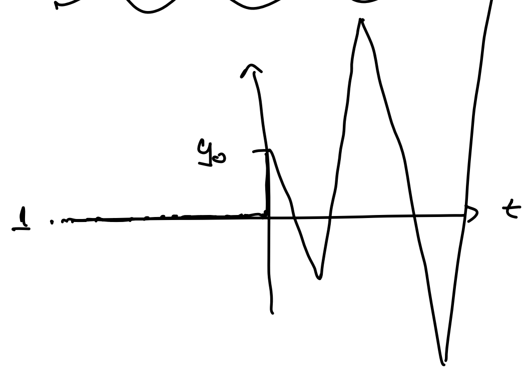
si $|a|<1$ $\quad a^t \longrightarrow 0$

$\Rightarrow S \rightarrow \frac{1}{1-a}$



si $a>1$

$S \rightarrow +\infty$



$$y_t = 4 - 3(4 - 3y_{t-2})$$
$$= 4 - 3\cdot 4 + 3^2 y_{t-2}$$
$$= 4 - 3\cdot 4 + 3^2(4 - 3y_{t-3})$$
$$= 4 - 3\cdot 4 + 3^2\cdot 4 - 3^3 y_{t-3}$$
$$= 4(1 + (-3) + (-3)^2) + (-3)^3 y_{t-3}$$
$$= 4((-3)^0 + (-3)^1 + (-3)^2) + (-3)^3(4 + (-3)y_{t-4})$$
$$= 4((-3)^0 + (-3)^1 + (-3)^2 + (-3)^3) + (-3)^4 y_{t-4}$$
$$\vdots$$
$$= 4((-3)^0 + (-3)^1 + \dots + (-3)^{t-1}) + (-3)^t y_0$$
$$= 4\,\frac{1-(-3)^t}{1-(-3)} + (-3)^t y_0$$
$a = -3$
$$= \cancel{4}\,\frac{1-(-3)^t}{\cancel{4}} + (-3)^t y_0$$

$$y_t = 1 + (y_0 - 1)(-3)^t \qquad y_0 = 4$$
$$y_t = 1 + (3)(-3)^t$$
$$= 1 + 3(-1)^t 3^t$$
$$\boxed{y_t = 1 + (-1)^t 3^{t+1}}$$

d) $\begin{cases} y_t = 0{,}2\, y_{t-1} + 4 \\ y_0 = 4 \end{cases}$

$$S = a^0 + a^1 + \dots + a^{t-1}$$

$$\Rightarrow S = \frac{1-a^t}{1-a}$$

$$a = \frac{1}{5} \to S = \frac{1-(1/5)^t}{1-1/5}$$

$$= \frac{1-(1/5)^t}{4/5}\,\frac{5^t}{5^t}$$

$$= \frac{5^t - 1}{4/5\; 5^t} = \frac{(5^t-1)5}{4 \cdot 5^t}$$

$$\to y_t = 4 + \frac{1}{5}\left(4 + \frac{1}{5} y_{t-2}\right)$$

$$= 4\left(1 + \frac{1}{5}\right) + \frac{1}{5^2} y_{t-2}$$

$$= 4\left(1 + \frac{1}{5}\right) + \frac{1}{5^2}\left(4 + \frac{1}{5} y_{t-3}\right)$$

$$= 4\left(1 + \frac{1}{5} + \frac{1}{5^2}\right) + \frac{1}{5^3} y_{t-3}$$

$$\vdots$$

$$y_t = 4\left(\frac{1}{5^0} + \frac{1}{5^1} + \dots + \frac{1}{5^{t-1}}\right) + \frac{1}{5^t} y_0$$

$$y_t = \cancel{4}\,\frac{(5^t-1)5}{\cancel{4} \cdot 5^t} + \frac{1}{5^t} y_0$$

$$y_t = \frac{(5^t-1)5 + y_0}{5^t} \qquad y_0 = 4$$

$$y_t = \frac{5^{t+1} - 5 + 4}{5^t}$$

$$y_t = 5 - \frac{1}{5^t}$$

Ejercicio 3

a) $\begin{cases} y_t - \frac{1}{3} y_{t-1} = 6 \\ y_0 = 1 \end{cases}$

$$a = 1/3 \to |a| < 1$$

$$S = \frac{1-a^t}{1-a} = \frac{1-(1/3)^t}{1-1/3}$$

$$= \frac{1-(1/3)^t}{2/3}\,\frac{3^t}{3^t}$$

$$= \frac{3^t - 1}{2/3\; 3^t} = \frac{(3^t-1)3}{2 \cdot 3^t}$$

$$\to y_t = 6 + \frac{1}{3} y_{t-1}$$

$$y_t = 6 + \frac{1}{3}\left(6 + \frac{1}{3} y_{t-2}\right)$$

$$= 6\left(1 + \frac{1}{3^1}\right) + \frac{1}{3^2} y_{t-2}$$

$$\vdots$$

$$= 6\left(1 + \dots + \frac{1}{3^{t-1}}\right) + \frac{1}{3^t} y_0$$

$$= \cancel{6}^{\,3}\,\frac{(3^t-1)3}{\cancel{2} \cdot 3^t} + \frac{1}{3^t} y_0$$

$$= \frac{(3^t-1)3^2}{3^t} + \frac{1}{3^t} y_0$$

$y_t = 9 - \frac{8}{3^t}$

$\underset{y_0=1}{\sim} = \frac{3^{t+2} - 3^2 + y_0}{3^t} = 9 + \frac{y_0 - 9}{3^t} = y_t$

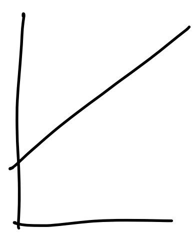
$y_t \xrightarrow{t\to\infty} 9$ converge

d) $\begin{cases} y_t - y_{t-1} = 3 \\ y_0 = 5 \end{cases} \longrightarrow y_t = 3 + y_{t-1} = 3 + (3 + y_{t-2})$

$= \dots = 3t + y_0$

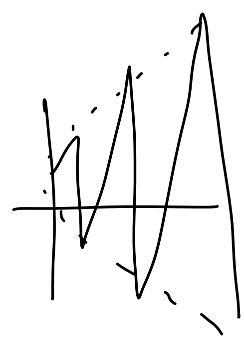
$\xrightarrow{y_0=5} y_t = 3t + 5$

$y_t \xrightarrow{t\to\infty} \infty$ no converge
no oscila

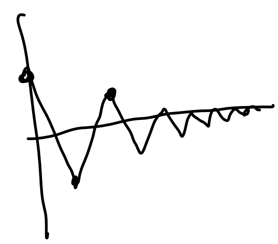


b) $\begin{cases} y_t + 2y_{t-1} = 9 \\ y_0 = 4 \end{cases} \longrightarrow y_t = 9 - 2y_{t-1}$

qué nos va a dar?



c) $\begin{cases} 2y_t + \frac{1}{4}y_{t-1} = 5 \\ y_0 = 5 \end{cases} \longrightarrow y_t = \frac{5}{2} - \frac{1}{8}y_{t-1}$



$y_t = \frac{5}{2} - \frac{1}{8}\left(\frac{5}{2} - \frac{1}{8}y_{t-2}\right)$

$= \frac{5}{2}\left(\left(-\frac{1}{8}\right)^0 + \left(-\frac{1}{8}\right)^1\right) + \left(-\frac{1}{8}\right)^2 y_{t-2}$

$= \frac{5}{2}\left(\left(-\frac{1}{8}\right)^0 + \left(-\frac{1}{8}\right)^1 + \dots + \left(-\frac{1}{8}\right)^{t-1}\right) + \left(-\frac{1}{8}\right)^t y_0$

$= \frac{5}{\cancel{2}}\,\overset{4}{\cancel{8}}\,\frac{8^t - (-1)^t}{9 \cdot 8^t} + \left(-\frac{1}{8}\right)^t y_0$

$= \frac{20\left(8^t - (-1)^t\right)}{9 \cdot 8^t} + \frac{(-1)^t y_0\, 9}{8^t \cdot 9}$

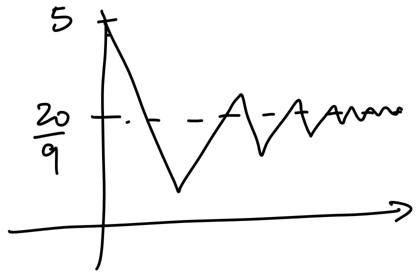
$\left\{ \begin{array}{l} S = \frac{1-a^t}{1-a} \\ a = -1/8 \\ S = \frac{1-(-1/8)^t}{9/8} \\ = \frac{8^t - (-1)^t}{9/8 \; 8^t} \\ = \frac{8^{t+1} - (-1)^t 8}{9 \cdot 8^t} \end{array} \right.$

$$= \frac{20\cdot 8^t - 20(-1)^t + 9(-1)^t y_0}{9\cdot 8^t}$$

$$= \frac{20}{9} + \frac{(-1)^t(9y_0-20)}{9\cdot 8^t}, \quad y_0 = 5$$

$$y_t = \frac{20}{9} + \frac{25}{9}\left(\frac{-1}{8}\right)^t$$

$$y_t \xrightarrow{t\to\infty} \frac{20}{9}$$



Ejercicio 4

c) $2y_{t+2} + y_{t+1} - y_t = 10$; assume $\hat{y}_t$ es sol.

$y_t^h / \; 2y_{t+2}^h + y_{t+1}^h - y_t^h = 0 \Rightarrow \hat{y}_t + y_t^h$ tb es sol.

Homogéneas: $y_t^h = r^t \rightarrow 2r^{t+2} + r^{t+1} - r^t = 0$ (homogénea)

$$\Rightarrow r^t(2r^2+r-1) = 0 \rightarrow r_{1,2} = \frac{-1\pm\sqrt{1+8}}{4}$$

$$\rightarrow r_{1,2} = \frac{-1\pm 3}{4} \begin{cases} r_1 = -1 \rightarrow y_t^{h_1} = (-1)^t \\ r_2 = 1/2 \rightarrow y_t^{h_2} = (1/2)^t \end{cases}$$

$$\Rightarrow y_t^h = c_1(-1)^t + c_2(1/2)^t$$

Particular: Propongo: $y_t = k \in \mathbb{R}$

$\xrightarrow{\text{reemplazo}} 2k + k - k = 10 \rightarrow k = 5 \rightarrow y_t^P = 5$

General: $y_t = 5 + c_1(-1)^t + c_2(1/2)^t$