Clase 2 (15/3)

Ejercicio 9

$u = (1,-3,2)\ ,\ v = (2,-1,1)$

c) $k /\ y = (1,k,5)$

$$y = a u + b v$$

$$(1,k,5) = a(1,-3,2) + b(2,-1,1)$$

$$= (a,-3a,2a) + (2b,-b,b)$$

$$(1,k,5) = (a+2b, -b-3a, 2a+b)$$

$$\begin{cases} 1 = a+2b \longrightarrow a = 1-2b \ (*) \xrightarrow{b=-1} a = 1-2(-1) \Rightarrow a = 3 \\ k = -b-3a \ (**) \\ 5 = 2a+b \xrightarrow[(*)]{} 5 = 2(1-2b)+b = 2-4b+b = 2-3b \end{cases}$$

$$\Rightarrow b = -1$$

$$(**) \rightarrow k = -(-1) - 3(3) = 1-9 \Rightarrow k = 8$$

Comentario $\Longrightarrow$

$$y = \underline{a u} + b v$$

$$y - a u = \underline{b} v$$

$$\frac{1}{b} y - \frac{a}{b} u = v$$

$$\Rightarrow \boxed{k_1 y + k_2 u = v}$$

si son l.d. $\{u_1, u_2, \ldots, u_n\} \longrightarrow a_i \neq 0 \,/\, \sum_{i=1}^{n} a_i u_i = 0$

$$u_j \Rightarrow \sum_{i \neq j}^{n} a_i u_i + a_j u_j = 0 \longrightarrow u_j = \frac{1}{a_j} \sum_{i \neq j}^{n} a_i u_i$$

b) $u = (1, -3, 2), \quad v = (2, -1, 1)$

$z = (2, -5, 4)$

$z = a\, u + b\, v$

$$\Rightarrow (2, -5, 4) = a\,(1, -3, 2) + b\,(2, -1, 1)$$

$$= (a + 2b, -3a - b, 2a + b)$$

$$\Rightarrow \begin{cases} 2 = a + 2b \Rightarrow a = 2 - 2b \xrightarrow{b = 1/5} a = \frac{10}{5} - 2\left(\frac{1}{5}\right) = 8/5 = a \\ -5 = -3a - b \Longrightarrow -5 = -3(2 - 2b) - b = -6 + 5b \rightarrow b = 1/5 \\ 4 = 2a + b \longrightarrow 4 \neq 2\,\frac{8}{5} + \frac{1}{5} = \frac{17}{5} \end{cases}$$

$\Rightarrow \nexists\, a, b \,/\, z = au + bv$

Ejercicio 10

$S = \{(2, 1, 0), (3, 1, 1), (3, 2, -1)\}$

Son l.i.? $\quad a\,(2, 1, 0) + b\,(3, 1, 1) + c\,(3, 2, -1) = 0$

si son l.i. $\Longrightarrow a = b = c = 0$

$$(2a, a, 0) + (3b, b, b) + (3c, 2c, -c) = 0$$

$$(2a + 3b + 3c, a + b + 2c, b - c) = (0, 0, 0)$$

$$\begin{cases} b - c = 0 \Rightarrow b = c \\ 2a + 3b + 3c = 0 \xrightarrow{b = c} 2a + 6c = 0 \rightarrow a = -3c \\ a + b + 2c = 0 \longrightarrow (-3c) + c + 2c = 0 \end{cases}$$

satisfaga $\forall c$

$c=1,\ a=-3,\ b=1$

$\Rightarrow$ no son l.i. $\Rightarrow$ S no es base

Si fuesen l.i., el siguiente paso sería encontrar (a,b,c) / $\forall (x,y,z)\in\mathbb{R}^3$

$$(x,y,z)=a s_1+b s_2+c s_3$$

$$\{s_1,s_2,s_3\}=S$$

Ejercicio 11

$$T=\{(x,y,z,t)\in\mathbb{R}^4 : x+2z-3t+2y=0\}$$

si $(x,y,z,t)\in T \Rightarrow x=3t-2z-2y$

$\rightarrow (x,y,z,t)=(3t-2z-2y,\ y,\ z,\ t)$

$$=y(-2,1,0,0)+z(-2,0,1,0)+t(3,0,0,1)$$

$$T=\langle(-2,1,0,0),(-2,0,1,0),(3,0,0,1)\rangle$$

$$y(-2,1,0,0)+z(-2,0,1,0)+t(3,0,0,1)$$

$$=(-2y,y,0,0)+(-2z,0,z,0)+(3t,0,0,t)$$

$$=(3t-2z-2y,\ y,\ z,\ t)$$

$= y(-2,1,0,0) + z(-2,0,1,0) + t(3,0,0,1)$

Ejercicio 12

b) $\{(1,0,1),(1,1,0)\}$ ← no es base de $\mathbb{R}^3$

$\dim(\mathbb{R}^3) = 3$

d) $\{(1,2,1),\ (1,3,1),\ (1,4,1),\ (1,5,1)\}$

← tampoco

Si fuera l.i. tendría dimensión 4

No es l.i.

e) $\{(1,1,1),(-2,1,0),(-1,0,1)\}$

$a(1,1,1) + b(-2,1,0) + c(-1,0,1) = 0 \overset{\text{q.v.q.}}{\Longrightarrow} a=b=c=0$

$\Rightarrow (a-2b-c,\ a+b,\ a+c) = (0,0,0)$

$$\left\{\begin{array}{l} a+c=0 \Rightarrow c=-a \\ a+b=0 \rightarrow b=-a \\ a-2(-a)-(-a)=0 \Rightarrow a+2a+a=0 \end{array}\right.$$

$\Rightarrow 4a=0 \Rightarrow a=0$

$\Rightarrow b=c=0 \Rightarrow$ son l.i.

Tomemos $(x,y,z)\in\mathbb{R}^3$

$(x,y,z) = a(1,1,1) + b(-2,1,0) + c(-1,0,1)$

$(x,y,z) = (a-2b-c,\ a+b,\ a+c)$

$$\begin{cases} z = a + c \Rightarrow z - a = c \\ y = a + b \Rightarrow y - a = b \\ x = a - 2b - c \Longrightarrow x = a - 2(y-a) - (z-a) \end{cases}$$

$$x = a - 2y + 2a - z + a$$

$$x + 2y + z = 4a$$

$$a = \frac{x+2y+z}{4}, \quad b = y - \frac{x+2y+z}{4}, \quad c = z - \frac{x+2y+z}{4}$$

Ejercicio 14

$$B = \{(-1,4,2), v, (0,0,-1)\}$$

$$B' = \{w, (1,-1,1), (-1,0,2)\}$$

$\rightarrow [v]_{B'} = (1,2,3)$

$\rightarrow [w]_B = (1,2,3)$

$$w = 1 \cdot (-1,4,2) + 2 \cdot v + 3 \cdot (0,0,-1) \quad (1)$$

$$v = 1 \cdot w + 2 \cdot (1,-1,1) + 3 \cdot (-1,0,2) \quad (2)$$

de (2) $\Rightarrow$ $v = w + (-1,-2,8)$

lo reemplazo en (1):

$$w = (-1,4,2) + 2[w + (-1,-2,8)] + (0,0,-3)$$

Conmutatividad + asociatividad

$$w = (-1,4,2) + 2(-1,-2,8) + (0,0,-3) + 2w$$

$$w = (-3,0,15) + 2w$$

$\exists (-w) \,/\, w + (-w) = 0$

$$w + (-2w) = (-3,0,15) + \cancel{2w + (-2w)}$$

$$-w = (-3,0,15)$$

$$w = (3,0,-15)$$

$$\Rightarrow v = (3,0,-15) + (-1,-2,8)$$

$$v = (2,-2,-7)$$

Ejercicio 15

a) $W = \{(x,y,z) : x = y = z\} \subset \mathbb{R}^3$: cerrado en · de escalares, cerrado en + de vectores

$$\begin{cases} w \in W \Rightarrow \forall k \in \mathbb{R},\ kw \in W \quad \leftarrow \text{cerrado en · de escalares} \\ w_1, w_2 \in W \Rightarrow w_1 + w_2 \in W \quad \leftarrow \text{cerrado en + de vectores} \end{cases}$$

- $(x,y,z) \in W \Rightarrow x = y = z \Rightarrow \forall k \in \mathbb{R},\ kx = ky = kz$
  $\Rightarrow (kx, ky, kz) \in W \Rightarrow k(x,y,z) \in W$

- $(x_1,y_1,z_1), (x_2,y_2,z_2) \in W \Rightarrow x_1 = y_1 = z_1,\ x_2 = y_2 = z_2$
  $\Rightarrow x_1 + x_2 = y_1 + y_2 = z_1 + z_2 \Rightarrow (x_1+x_2, y_1+y_2, z_1+z_2) \in W$
  $\Rightarrow (x_1,y_1,z_1) + (x_2,y_2,z_2) \in W \quad \square$

c) $W = \{(x,y,z,t) \in \mathbb{R}^4 : 2y + 3z = 0\}$

- pdq: $\forall w \in W \Rightarrow \forall k \in \mathbb{R},\ kw \in W$

$(x,y,z,t) \in \mathbb{R}^4 \Rightarrow 2y + 3z = 0 \Rightarrow \forall k \in \mathbb{R},\ (2y+3z)k = 0$

$\Rightarrow 2(yk) + 3(zk) = 0 \Rightarrow (kx, ky, kz, kt) \in W$

$\Rightarrow k(x,y,z,t) \in W$

- $\underline{qvq}$: $\forall\, w_1, w_2 \in W,\quad w_1+w_2 \in W$

$(x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2) \in W \Rightarrow 2y_1+3z_1=0,$
$2y_2+3z_2=0$

$\Rightarrow 2y_1+2y_2+3z_1+3z_2=0$

$\Rightarrow 2(y_1+y_2)+3(z_1+z_2)=0$

$\Rightarrow (x_1+x_2, y_1+y_2, z_1+z_2, t_1+t_2) \in W$

$\Rightarrow (x_1, y_1, z_1, t_1)+(x_2, y_2, z_2, t_2) \in W \quad \square$

b) $W=\{(x,y,z,t): x=z,\ y=t\}$

- $(x,y,z,t) \in W \Rightarrow x=z,\ y=t \Rightarrow kx=kz,\ ky=kt$

$\Rightarrow (kx, ky, kz, kt) \in W \Rightarrow k(x,y,z,t) \in W$

- $(x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2) \in W \Rightarrow$

$\underline{x_1=z_1},\ \underline{y_1=t_1},\quad \underline{x_2=z_2},\ \underline{y_2=t_2}$

$\Rightarrow x_1+x_2=z_1+z_2,\quad y_1+y_2=t_1+t_2$

$\Rightarrow (x_1+x_2, y_1+y_2, z_1+z_2, t_1+t_2) \in W$

$\Rightarrow (x_1, y_1, z_1, t_1)+(x_2, y_2, z_2, t_2) \in W \quad \square$

Ejercicio 16

$(1-\alpha)(2,2,4)+\alpha(5,0,3)$

$=(2+3\alpha, 2-2\alpha, 4-\alpha)$

a) $(5/2,\ 1,\ 7/2)$

$\underline{(5/2,\ 1,\ 7/2)}=(2+3\alpha,\ 2-2\alpha,\ 4-\alpha)$

$$\left\{\begin{array}{l} 4-\alpha=7/2 \longrightarrow \alpha=1/2 \\ 1=2-2\alpha \longrightarrow \alpha=1/2 \\ 5/2=2+3\alpha \longrightarrow \alpha=1/6 \end{array}\right\}$$ Absurdo $(1/2 \neq 1/6)$ $\Rightarrow$ el vector no se puede producir

Ejercicio 17

$V=\{(\omega, td, ti, P, GP) : P=\omega, \; td+ti=GP\}$

a) • $(\omega, td, ti, P, GP) \in V \Rightarrow P=\omega, \; td+ti=GP$

$\Rightarrow \forall k, \; kP=k\omega, \; ktd+kti=kGP$

$\Rightarrow (k\omega, ktd, kti, kP, kGP) \in V$

$\Rightarrow k(\omega, td, ti, P, GP) \in V$

• $(\omega_1, td_1, ti_1, P_1, GP_1), (\omega_2, td_2, ti_2, P_2, GP_2) \in V$

$\Rightarrow P_1=\omega_1, \; td_1+ti_1=GP_1 \wedge P_2=\omega_2, \; td_2+ti_2=GP_2$

$\Rightarrow P_1+P_2=\omega_1+\omega_2, \; (td_1+td_2)+(ti_1+ti_2)=GP_1+GP_2$

$\Rightarrow (\omega_1+\omega_2, td_1+td_2, ti_1+ti_2, P_1+P_2, GP_1+GP_2) \in V$

$\Rightarrow (\omega_1, td_1, ti_1, P_1, GP_1)+(\omega_2, td_2, ti_2, P_2, GP_2) \in V \; \square$

b) $(\omega, td, ti, P, GP) \in V \Rightarrow \omega=P, \; td+ti=GP$

$$\left\{\begin{array}{l} (\omega, td, ti, P, GP) = (P, td, ti, P, td+ti) \\ = P(1,0,0,1,0)+td(0,1,0,0,1) \\ \quad +ti(0,0,1,0,1) \end{array}\right.$$

$B_V=\{(1,0,0,1,0), (0,1,0,0,1), (0,0,1,0,1)\}$

$\dim(B_V) = 3$

$V = \langle (1,0,0,1,0), (0,1,0,0,1), (0,0,1,0,1) \rangle$