Time Series

Problem Set 4 - Answer Key

Problem 1

1)

The authors argue that time series models of asset prices must measure risk and its movement over time, and include it as a determinant of price. Increases in the expected rate of return of assets as they become riskier are identified as risk premium. The authors analyze time varying risk premia. Their main claim is that "time varying premia on different term debt instruments can be well modeled as risk premia where the risk is due to unanticipated interest rate movements and is measured by the conditional variance of the one period holding yield."

They begin by assuming there are only two assets in the economy: (i) risk free with price=1, perfectly elastically supplied with a return rate r, (ii) risky, with price p and yields a random total return q (measured in units of the riskless asset), which has a mean θ and variance ϕ. Agents have wealth W measured in units of risk free asset, which all invested in both assets: $W = p.s + x$, where s is the share of risky asset.

Thus, the excess return per dollar invested in the risky asset is: $y = \frac{q}{p} - r$

$\Rightarrow E(y) = \mu = \frac{\theta}{p} - r \qquad V(y) = \sigma^2 = \frac{\phi}{p^2}$

Under constant risk aversion, the expected utility may be expressed as $EU = 2E(qs + rx) - bV(qs + rx)$, and is maximized by choosing $sp = \mu/(b\sigma^2)$.

Assume that ϕ varies in time (hence σ^2 varies in time) and that the number of shares and the return of the riskless asset are fixed. Then we have:

$\mu = \frac{-r+\sqrt{r^2+4bs\sigma^2\theta}}{2}$

So, if ϕ varies in time but θ, r and s do not, we should expect a positive relation between mean and variance, though not proportional.

ARCH-M

The authors intorduce an ARCH in mean model to allow the conditional variance to affect the expected return on a portfolio.

The initial model is: $y_t = \mu_t + \varepsilon_t \qquad Var(\varepsilon_t | I_t) = h_t^2$ (I_t is the information set at time t)

where μ_t is the risk premium, y_t is the excess holding yield on a long bond relative to a one-period treasury bill and ε_t is the difference between ex-ante and ex-post rate of return (an unforecastable event in efficient markets).

They assume that the risk of holding a long bond is not diversifiable, an then only variance matters.

Taking θ as the mean of the risky asset, we can model it as a linear function of the standard deviation h_t: $\theta_t = \beta + \delta h_t$.

The authors give two explanations for a non-zero β: (i) linearization of a non-linear function (ii) a preferred habitat argument.

Furthermore, they choose the standard deviation for the ARCH-M model instead of the variance, as they assume that changes in the variance are reflected less than proportionally in the mean ($\log h_t$ could also be used).

Finally, as in Mandelbrot (volatility clustering), the authors argue that the conditional variance depends on previous innovations in the following way:

$$h_t^2 = \alpha_0 + \alpha_t \sum_{i=1}^{p} w_i \varepsilon_{t-i}^2$$

Thus, the conditional variance is a weighted sum of squared past innovations.

As an application, the authors model the risk premia associated the excess holding yield between a 6 month T-Bill (R_t) and a 3 month T-Bill (r_t). They model an agent who borrows at the 3 month T-Bill and lends at the 6 month T-Bill. Following the theory above, one way to see it is as follows. Both R_t and r_t are quarterly rates. The agent has two options: he can invest \$1 in R_t (lend) or in r_t. At $t+1$, he would have would have either $(1+R_t)$ dollars or $(1+r_t)$ dollars. However, his lending commitment is for 6 months, thus he would have to borrow in $t+1$ for a three-month period at a r_{t+1} rate. Thus, the return of his investment in the second period would be $\frac{1+R_t}{1+r_{t+1}}$. In that case, the actual return of his investment would be

$(1+R_t)\,\frac{1+R_t}{1+r_{t+1}} = \frac{(1+R_t)^2}{1+r_{t+1}}$. So, the excess return would be $\frac{(1+R_t)^2}{1+r_{t+1}} - (1+r_t)$. See the analogy with $y = \frac{q}{p} - r$.

In reality, it is highly improbable that the excess return will remain relatively constant. Rather, we will see a time varying risk premia, which will depend on the volatility of the interest rates.

2)

We go to *Quick / Estimate Equation* and type *y c* in the prompt in order to get an estimate of y on only a constant.

Our estimated constant, that is, the mean of the excess holding yield in our sample period, is 0.073 (significant at 5 %), lower that the one estimated in the paper, 0.142. Difference in estimates are due to the use of a different data set.

3)

Once we estimated the model, we click on *View/ Residual Diagnostics / Correlogram Squared Residuals* in order to get the correlogram of squared residuals

The required correlogram is shown in the image below

We read this the same way as when we worked with ARMA models. Remember that this correlogram shows the correlations of **squared** residuales, thus, it is giving us information about the time structure of variance. We see that correlations of initial lags are significantly different from zero. Therefore, we should model a time-variant variance somehow. in this case, we use an ARCH specification as the authors do.

4)

We start particular to general, thus, we first estimate an ARCH(1). To do so, go to *Quick/Estimate equation* and in 'Estimation method' choose *ARCH*.

We specify the mean equation with just a constant. We'll change this later. Starting by an ARCH(1), we get the output shown below

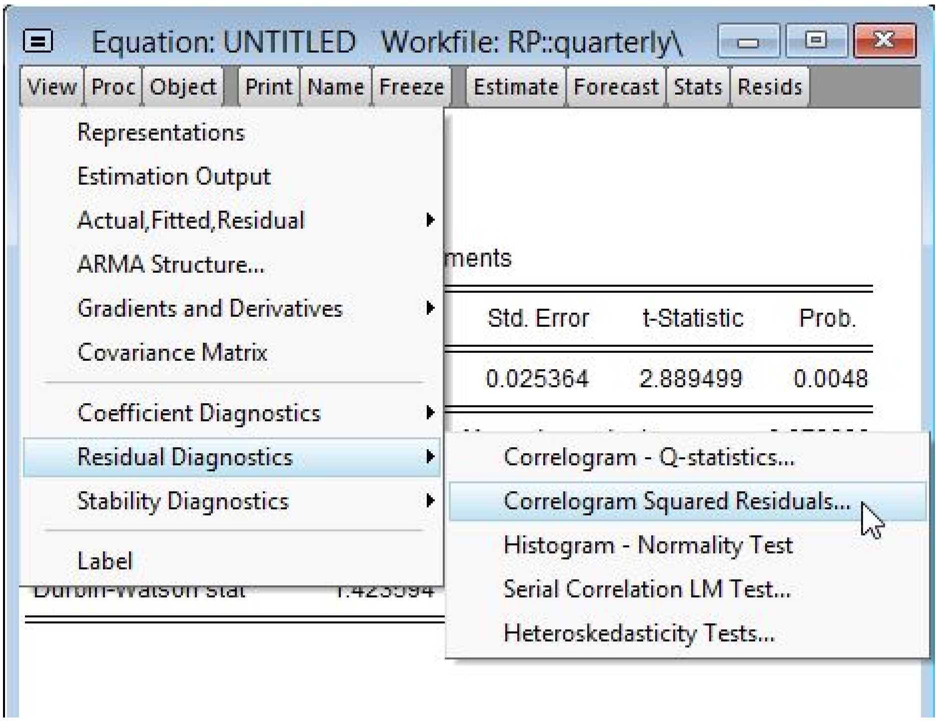
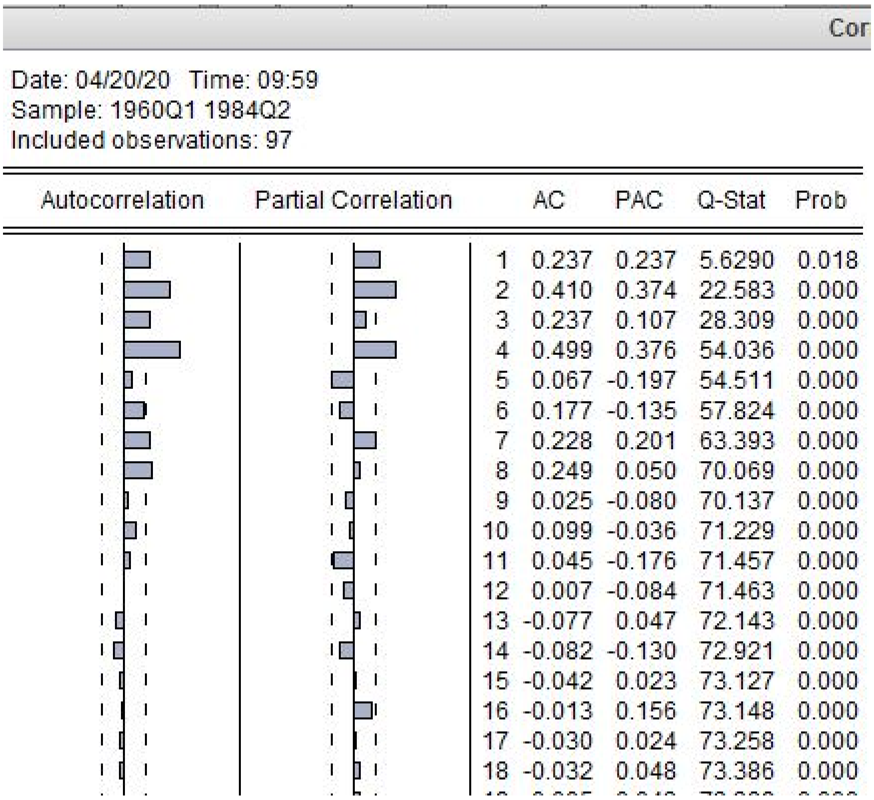


Figura 1: How to get the correlgram of squared residuals in Eviews



Date: 04/20/20 Time: 09:59
Sample: 1960Q1 1984Q2
Included observations: 97

	AC	PAC	Q-Stat	Prob
1	0.237	0.237	5.6290	0.018
2	0.410	0.374	22.583	0.000
3	0.237	0.107	28.309	0.000
4	0.499	0.376	54.036	0.000
5	0.067	-0.197	54.511	0.000
6	0.177	-0.135	57.824	0.000
7	0.228	0.201	63.393	0.000
8	0.249	0.050	70.069	0.000
9	0.025	-0.080	70.137	0.000
10	0.099	-0.036	71.229	0.000
11	0.045	-0.176	71.457	0.000
12	0.007	-0.084	71.463	0.000
13	-0.077	0.047	72.143	0.000
14	-0.082	-0.130	72.921	0.000
15	-0.042	0.023	73.127	0.000
16	-0.013	0.156	73.148	0.000
17	-0.030	0.024	73.258	0.000
18	-0.032	0.048	73.386	0.000

Figura 2: Correlogram of squared residuals

By studying the correlogram, we see that squared resiudals are not ’clean’, i.e. we need to add more structure to the model. Adding more terms, we see that an ARCH(4) models all the conditional heteroskedasticity, with no correlation left in the squared residuals. Nevertheless,

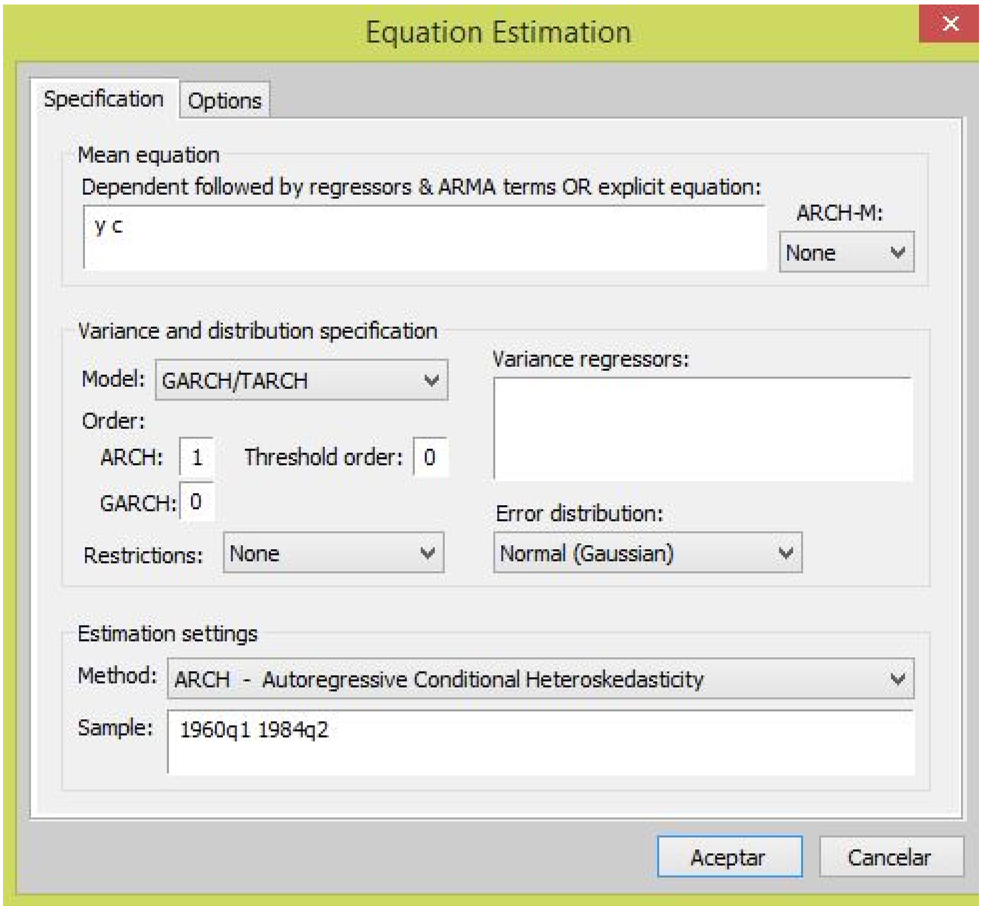


Figura 3: Estimation window for an ARCH model

Dependent Variable: Y
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 04/20/20 Time: 10:10
Sample (adjusted): 1960Q1 1984Q1
Included observations: 97 after adjustments
Convergence achieved after 13 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.048077	0.025239	1.904866	0.0568
		Variance Equation		
C	0.031924	0.004700	6.791843	0.0000
RESID(-1)^2	0.678067	0.281854	2.405740	0.0161

R-squared	-0.010293	Mean dependent var	0.073290
Adjusted R-squared	-0.010293	S.D. dependent var	0.249810
S.E. of regression	0.251092	Akaike info criterion	-0.063467
Sum squared resid	6.052548	Schwarz criterion	0.016163
Log likelihood	6.078147	Hannan-Quinn criter.	-0.031268
Durbin-Watson stat	1.409091		

Figura 4: Estimation output for an ARCH(1) model

we note that, with this formulation, coeficients of 2nd and 3rd lags in the variance equation are not significant. Note that the second lag of the conditional variance specification has a negative coefficient. Clearly this violates the ARCH modelling condition that every coefficient must be non-negative to avoid possibilities of negative variances, not validating our model. The way the authors avoided having this sign problem was to model the conditional variance with decreasing coefficients. That is, the volatility of three periods ago cannot have higher weight in explaining current volatility than the past period has. This goes hand to hand with Mandelbrot's findings of volatility clustering.

Due to the problem mentioned above, we check that we do not get a negative conditional variance, by clicking in *View/Garch Graph/ Conditional Variance*

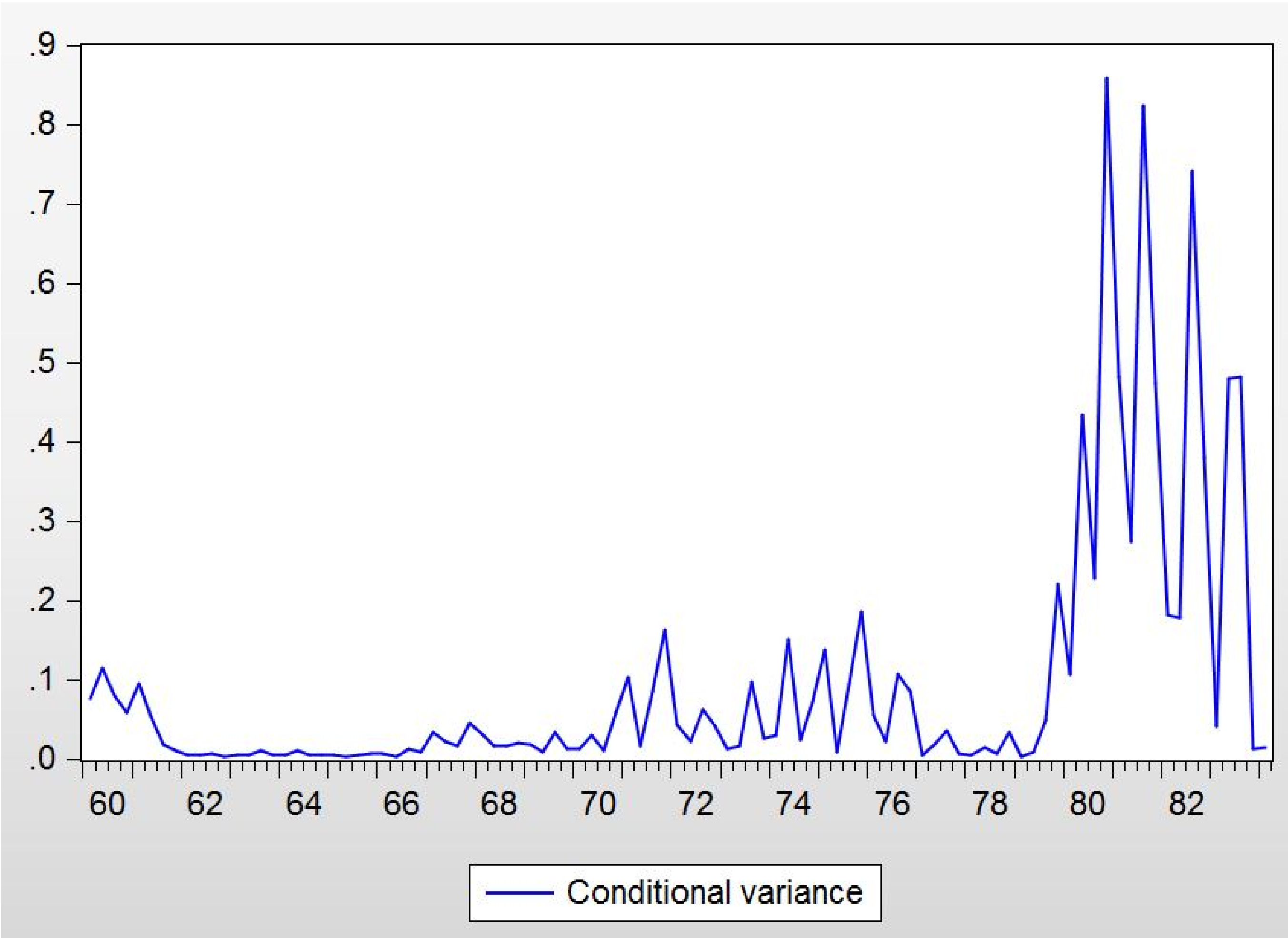


Figura 5: Conditional variance

Despite having a negative coefficient, we don't get an estimated negative conditional variance.

Another problem with our estimation is that **there may be a regime shift in our sam-**

ple, that we seem to estimate as a ARCH effects (see lecture notes).

To adress this problem, we estimate the model for different sub-samples. For example, we divide the sample in three 8-year periods (60-68, 68-76, 76-84). For the first sub-sample, we found ARCH effects to be correctly modelled by an ARCH(1) specification. For the second sub-sample, no ARCH effects are needed at all. The same is true for the third sub-sample. Therefore, we could argue that, by estimating an ARCH for the whole sample, **we would be incorrectly treating changes in the variance as ARCH effects, when actually these changes might be structural breaks.** [1] The overall message can be stated as **BE CAREFUL WITH POTENTIAL STRUCTURAL BREAKS**

5)

Just to show an application (it is actually wrong to proceed this way given what we said in the last paragraph), we estimate the ARCH-M model. That is, we include the conditional standard deviation in the mean equation. This can be intuitively derived as it follows: usual asset pricing models (such as CAPM) state that the following relation approximately holds between returns of risky and risk-free assets

$$r_{i,t} = r_f + \lambda \cdot \hat{\sigma}$$

Where $r_{i,t}$ is the return of the risky asset, r_f is the risk-free rate, λ is the price of risk, and σ is the amount of non-diversifiable risk in the asset. With this equation, the spread (approximately y_t) is given by $\lambda \cdot \hat{\sigma}$. The paper adds a constant because, for several reasons (such as unequal tax treatment), that relation might not hold exactly in practice. [2]

Therefore, the authors estimate an ARCH-M model, using the conditional standard deviation in the mean equation. In their notation:

$$y_t = \beta + \delta h_t$$

Where h_t is the time-varying standard deviation.

In order to estimate an ARCH-M, we select that option in estimation settings. We use an

[1] Using sightly diffrent structural sub-samples, we get no ARCH effects for all of them.

[2] This can be derived analiticly in several ways. See, for example, Huang and Litzemberger, *Foundations for financial economics, Chapter 5. The result is in equation 4.15.5*

ARCH(4) as they do in the paper.

In this specification, only the lag of the conditional variance equation are significant at 5 %

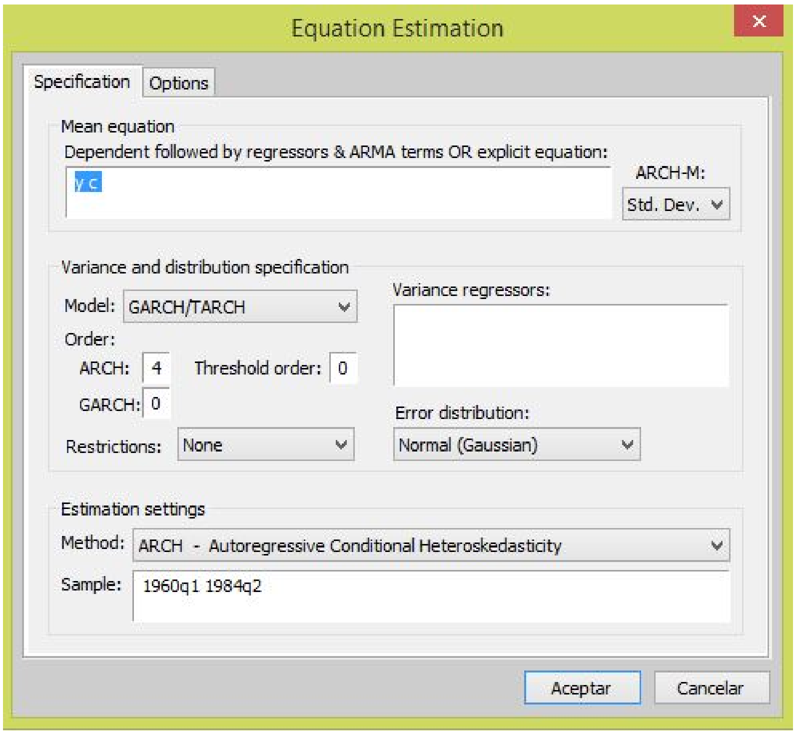
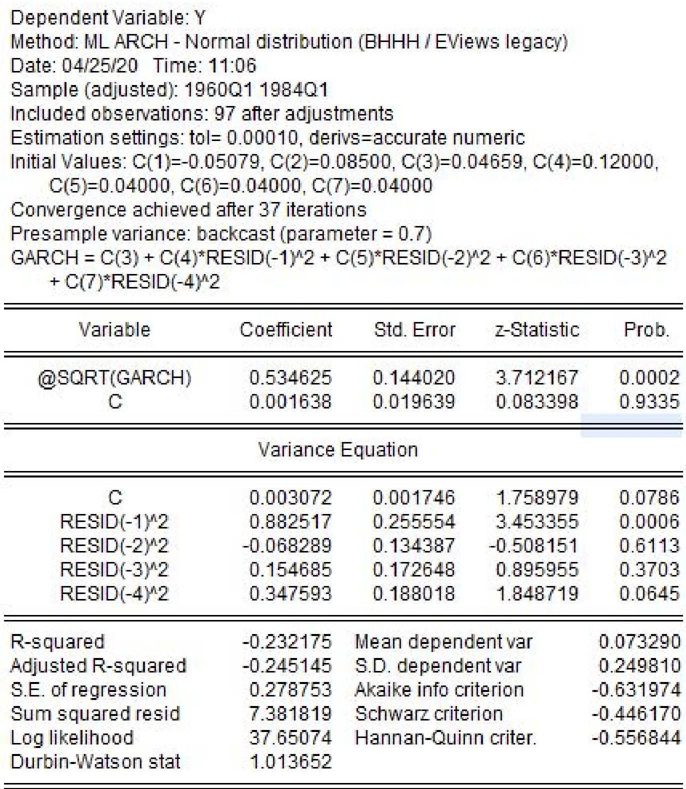


Figura 6: ARCH-M specification



Dependent Variable: Y
Method: ML ARCH - Normal distribution (BHHH / EViews legacy)
Date: 04/25/20 Time: 11:06
Sample (adjusted): 1960Q1 1984Q1
Included observations: 97 after adjustments
Estimation settings: tol= 0.00010, derivs=accurate numeric
Initial Values: C(1)=-0.05079, C(2)=0.08500, C(3)=0.04659, C(4)=0.12000,
C(5)=0.04000, C(6)=0.04000, C(7)=0.04000
Convergence achieved after 37 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*RESID(-3)^2
+ C(7)*RESID(-4)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.534625	0.144020	3.712167	0.0002
C	0.001638	0.019639	0.083398	0.9335
		Variance Equation		
C	0.003072	0.001746	1.758979	0.0786
RESID(-1)^2	0.882517	0.255554	3.453355	0.0006
RESID(-2)^2	-0.068289	0.134387	-0.508151	0.6113
RESID(-3)^2	0.154685	0.172648	0.895955	0.3703
RESID(-4)^2	0.347593	0.188018	1.848719	0.0645

R-squared	-0.232175	Mean dependent var	0.073290
Adjusted R-squared	-0.245145	S.D. dependent var	0.249810
S.E. of regression	0.278753	Akaike info criterion	-0.631974
Sum squared resid	7.381819	Schwarz criterion	-0.446170
Log likelihood	37.65074	Hannan-Quinn criter.	-0.556844
Durbin-Watson stat	1.013652		

Figura 7: ARCH-M estimation output

level. More importantly, in the mean equation, the constant is no longer significant while the coefficient associated to the standard deviation is significant at 5 % level. This indicates the presence of a time varying risk premia in the excess yield, as claimed by the authors [3]. Note that the estimate of the constant suffers dramatic changes when the ARCH term is incorporated into the mean equation.

Also note that once again we have a negative coefficient in the conditional variance estimation. Thus, it is worth checking that we do not have negative values for this sample:

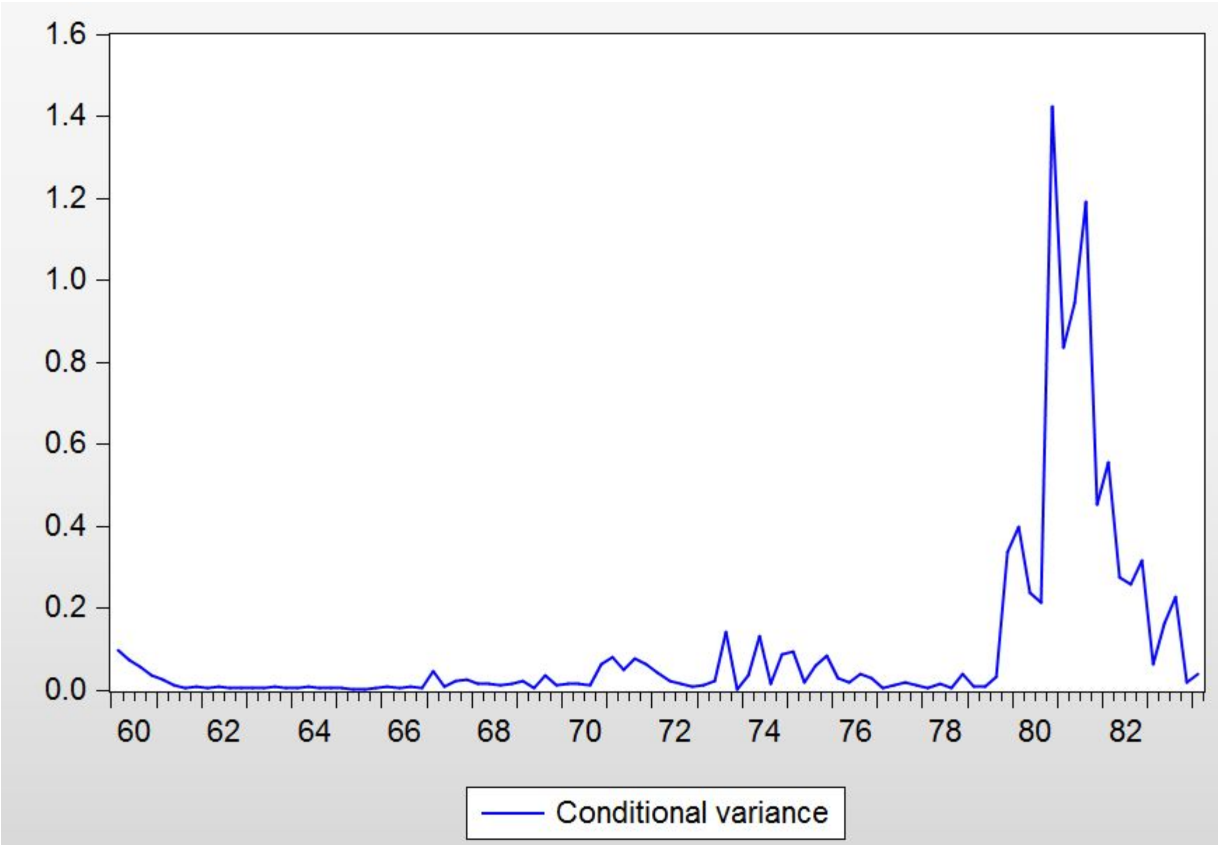


Figura 8: Estimated conditional variance

Another huge problem with this specification is that the squared residuals process is explosive, that is, the lag polinomial has a root with is strictly less than one (that is, an eigenvalue strictly greater than one if we wrote the process as a VAR(1)).

The authors also use the logarithm of the standard deviation (and actually prefer this specification). To do this, we just need to use the *Log(Var)* option when specifying the ARCH-M.

[3]Once again, we know that all the estimations in this part are wrong beacuse we are not taking into account structural breaks

Results are qualitatively similar to those if the previous specification.

6)

Figure 1 in the paper plots the excess holding yield (y_t) and the estimated risk premium ($\hat{\delta}h_t$ or $\hat{\delta}log(h_t)$) depending on how we specified the model.

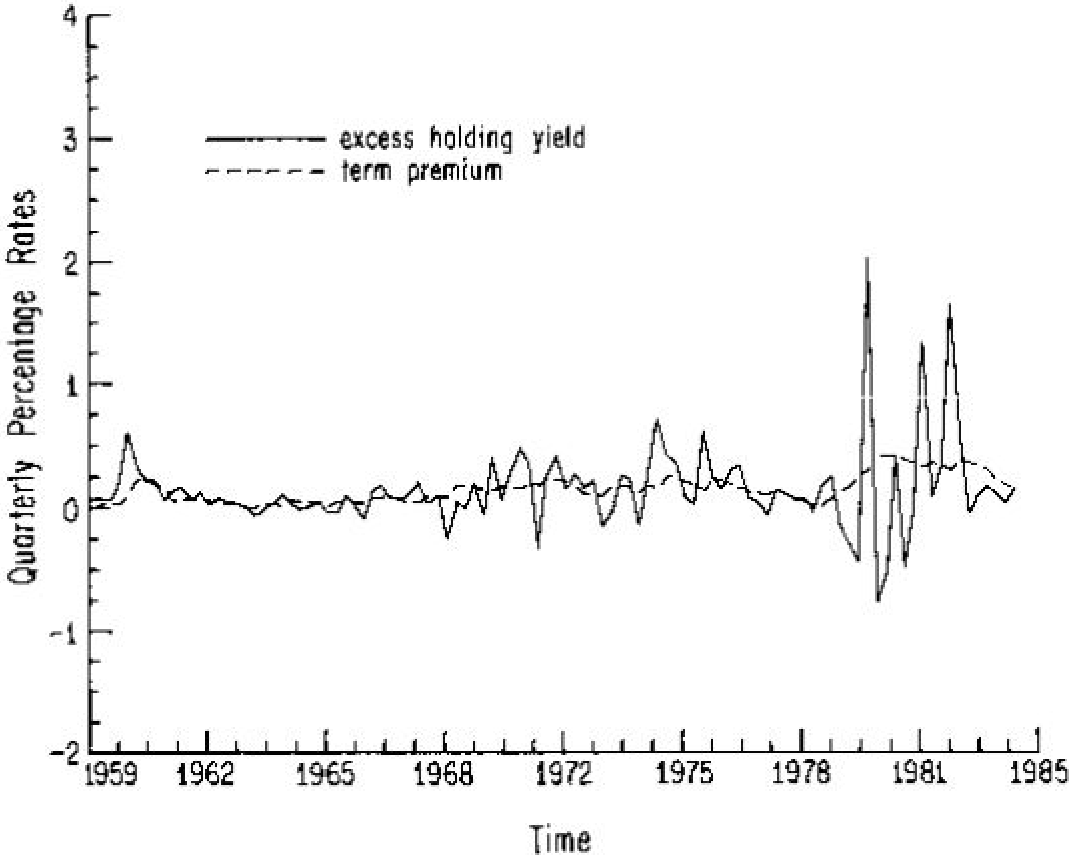


FIGURE 1—Excess hold yield of 6 month Treasury Bills and estimated risk premia.

Figura 9: Engle, Lilen, Robbins (1987), Figure 1

To create our replication of this series, we need to generate the risk premium. In order to do so, we need the conditional standard deviation series. We can create this by clicking in *Proc/Make Garch Variance Series.* With this series created (I've named *condvar* for no reason at all), we can compute the estimated risk premium by going to *Quick/generate series* and imputing the formula with the estimated coefficient.

Then we plot both series together to have the desired figure.

We note that our figure is qualitatively similar to the original one in the paper.

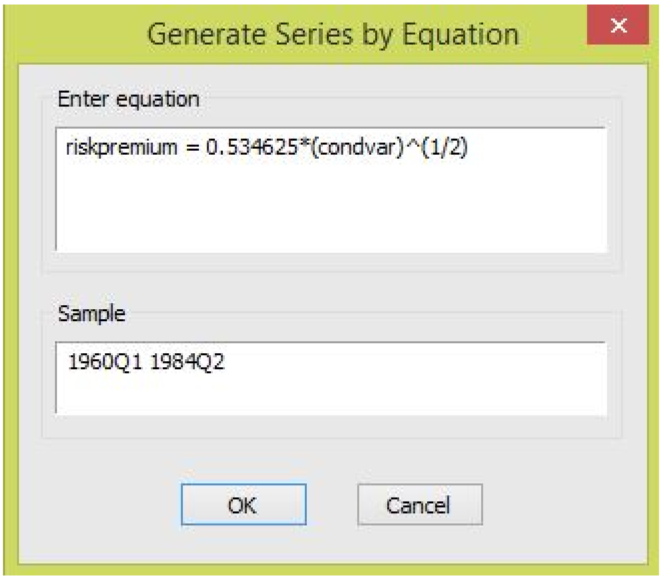


Figura 10: Generate risk-premium series

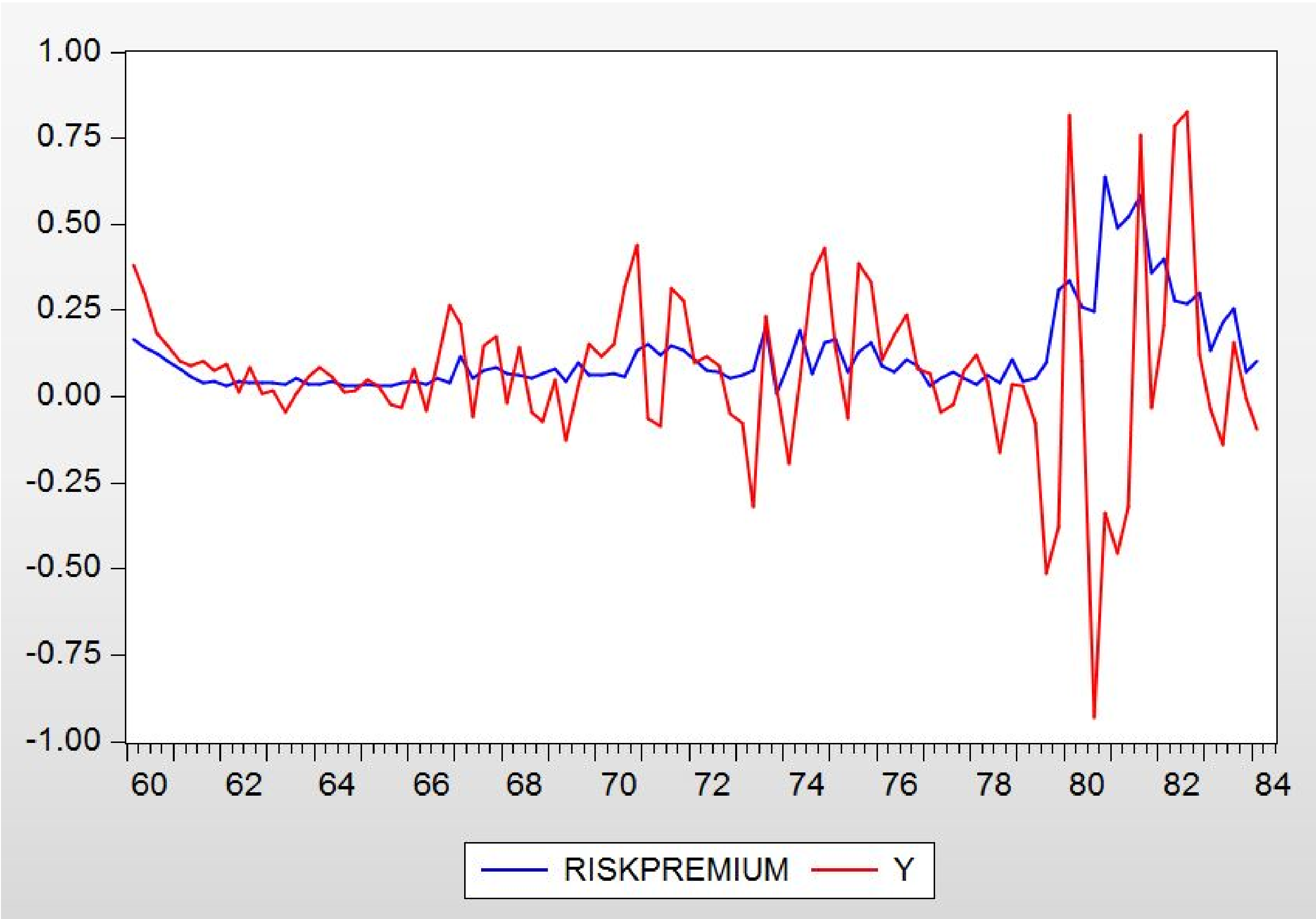


Figura 11: Figure 1 replication

7)

To start with, we have different sources of data, thus having a source of divergence. Secondly, we have an alternative modelization for the conditional variance. The model in the paper would be:

$$h_t^2 = \gamma + \alpha \sum_{\tau=1}^{4} w_\tau \varepsilon_{t-\tau}^2 \quad w_\tau = (5-\tau)/10 \qquad \tau = 1, ..., 4$$

In our case it is:

$$h_t^2 = \gamma + \sum_{\tau=1}^{4} \gamma_\tau \varepsilon_{t-\tau}^2$$

The reason why the authors take that modelization of the conditional variance is that then the estimated coefficients associated to each lagged squared innovation is decreasing in higher orders. In practice, they are taking a decreasing weighted average of the lagged squared innovations.

Problem 2

1)

We go to *Quick/Estimate equation* and, as we did in PS0, estimate the model with constant and AR(1), AR(6) terms. The estimation output is given by:

All estimated parameters are significant at usual 5 % confidence levels. The correlograms show that we've modelled corerctly the autocorrelation structure in the mean equation.

2)

Correlogram of squared residuals shows the presence of GARCH effects. We can also test explicitly for ARCH effects. By clicking in *Quick/residual diagnostics/hetorskedasticity tests/ARCH* after estimation, we can conduct a test for the existence of ARCH effects. With just 1 lag we reject the null of no ARCH effects. Therefore, we have to model the conditional variance.

Adopting a 'particular to general' strategy, we start specifying an ARCH(1) model. In most cases, with either ARCH(1) or GARCH(1,1) specfication we will be able to correctly model the stochastic properties of the conditional variance.
We start with an ARCH(1) specification with normal (gaussian) errors. It doesn't seem to clean the squared residuals series. We check whether this is still true if we consider Student's t errors.

Dependent Variable: RETURNS
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 04/20/20 Time: 11:31
Sample: 7001 7928
Included observations: 928
Convergence achieved after 10 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000535	0.000247	2.161871	0.0309
AR(1)	0.149074	0.028290	5.269558	0.0000
AR(6)	-0.087692	0.029054	-3.018290	0.0026
SIGMASQ	4.94E-05	1.67E-06	29.60092	0.0000

R-squared	0.029235	Mean dependent var	0.000537
Adjusted R-squared	0.026083	S.D. dependent var	0.007134
S.E. of regression	0.007040	Akaike info criterion	-7.069984
Sum squared resid	0.045798	Schwarz criterion	-7.049152
Log likelihood	3284.473	Hannan-Quinn criter.	-7.062038
F-statistic	9.275649	Durbin-Watson stat	1.999637
Prob(F-statistic)	0.000005		

Inverted AR Roots	.60-.33i	.60+.33i	.02+.66i	.02-.66i
	-.55-.33i	-.55+.33i		

Figura 12: ARMA estimation output

Date: 04/20/20 Time: 11:34
Sample: 7001 7928
Included observations: 928

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.108	0.108	10.818	0.001
		2	0.197	0.187	46.930	0.000
		3	0.080	0.044	52.864	0.000
		4	0.087	0.041	59.861	0.000
		5	0.147	0.120	80.043	0.000
		6	0.041	-0.005	81.632	0.000
		7	0.044	-0.012	83.459	0.000
		8	0.081	0.062	89.673	0.000
		9	0.061	0.031	93.132	0.000
		10	0.084	0.036	99.687	0.000
		11	0.093	0.064	107.78	0.000
		12	0.020	-0.025	108.15	0.000
		13	0.127	0.083	123.36	0.000
		14	0.122	0.096	137.35	0.000
		15	[illegible]	[illegible]	[illegible]	[illegible]

Figura 13: Correlogram of squared residual

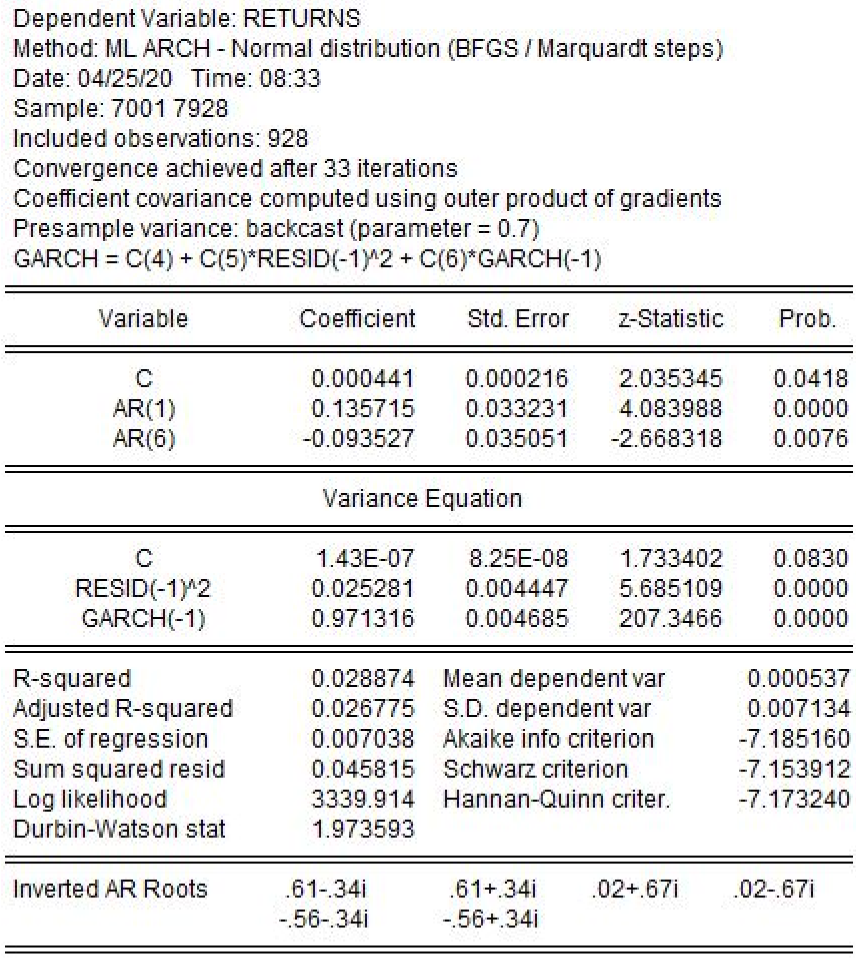
Heteroskedasticity Test: ARCH

F-statistic	10.87893	Prob. F(1,925)	0.0010
Obs*R-squared	10.77571	Prob. Chi-Square(1)	0.0010

Figura 14: Testing for ARCH effects

It's important to check specifications with both types of innovations, specially while working with financial series. Several evidence sources detect non-normalities in innovations of financial time series, specially in the form of 'fat-tails' in the distributions. As we know form previous coureses, Student's t distribution has fatter tails than the Normal distribution.

We next consider a GARCH(1,1) spec with Normal errors. The estimation output is shown in the figure below. In this case, model seems to fit the rate fairly well. Correlograms of sq.



Dependent Variable: RETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 04/25/20 Time: 08:33
Sample: 7001 7928
Included observations: 928
Convergence achieved after 33 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000441	0.000216	2.035345	0.0418
AR(1)	0.135715	0.033231	4.083988	0.0000
AR(6)	-0.093527	0.035051	-2.668318	0.0076
		Variance Equation		
C	1.43E-07	8.25E-08	1.733402	0.0830
RESID(-1)^2	0.025281	0.004447	5.685109	0.0000
GARCH(-1)	0.971316	0.004685	207.3466	0.0000

R-squared	0.028874	Mean dependent var	0.000537
Adjusted R-squared	0.026775	S.D. dependent var	0.007134
S.E. of regression	0.007038	Akaike info criterion	-7.185160
Sum squared resid	0.045815	Schwarz criterion	-7.153912
Log likelihood	3339.914	Hannan-Quinn criter.	-7.173240
Durbin-Watson stat	1.973593		

Inverted AR Roots	.61-.34i	.61+.34i	.02+.67i	.02-.67i
	-.56-.34i	-.56+.34i		

Figura 15: GARCH(1,1) with gaussian innovations

residuals seem clean.

We could also try to change the distribution of errors to t's.

Estimation with the specified options leads to a failure of the optimization algorithm. We change it as it is shown in the figure.

On optimization algorithms. It is likely that we experience problems with regards to optimization during the following lectures. If it happens, we should try to change either stating values, optimization algorithms, step methods, or all of them. One important thing is that in many cases Eviews take as staring values whatever there is in the coefficient vector, c. By

Date: 04/25/20 Time: 08:36
Sample: 7001 7928
Included observations: 928

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.028	0.028	0.7360	0.391
		2	0.052	0.051	3.2247	0.199
		3	-0.025	-0.028	3.8254	0.281
		4	-0.009	-0.010	3.8968	0.420
		5	0.025	0.029	4.5005	0.480
		6	-0.014	-0.016	4.6972	0.583
		7	-0.058	-0.061	7.8192	0.349
		8	0.024	0.030	8.3426	0.401
		9	-0.021	-0.016	8.7439	0.461
		10	0.049	0.044	11.044	0.354
		11	0.002	0.002	11.046	0.439
		12	-0.037	-0.040	12.357	0.417
		13	0.024	0.025	12.894	0.456
		14	0.065	0.068	16.837	0.265
		15	0.022	0.013	17.286	0.302
		16	-0.011	-0.021	17.407	0.360
		17	-0.033	-0.021	18.413	0.363
		18	-0.003	-0.003	18.424	0.428
		19	0.012	0.009	18.552	0.486
		20	-0.042	-0.041	20.204	0.445
		21	-0.006	0.000	20.233	0.507
		22	-0.025	-0.015	20.823	0.532

Figura 16: GARCH(1,1) with gaussian innovations: correlogram of sq. residuals

changing this values by hand, we can alter the starting values. This is the reason why in some cases we fail to estimate something because of the optimization failing, then we run something else (an OLS regression perhaps), next we try to estimate the first model again and surprisingly we succeed in the second attempt. Further details on what optimization algorithms actually do can be found in Martin, Hurn and Harris (2013) *Econometric modelling with time series*. As a general feature, the problem often lies in starting values, not in the optimization algorithm, but if something doesn't work, we should try many strategies to make it work.

The estimated output of the GARCH(1,1) with t-errors is shown in the figure below. We see that, apart from the constant in the variance equation, everything else has p-values lower than 5 %. The correlograms of residuals and squared residuals are also clean.

We can test the presence of ARCH effects as we did before, with ARCH-LM tests. To do so, we click in *Quick/residual diagnostics/ARCH-LM*. We try with 1 and 2 lags. We are not able to reject the null. Aditional terms in the variance equation don't result significant.

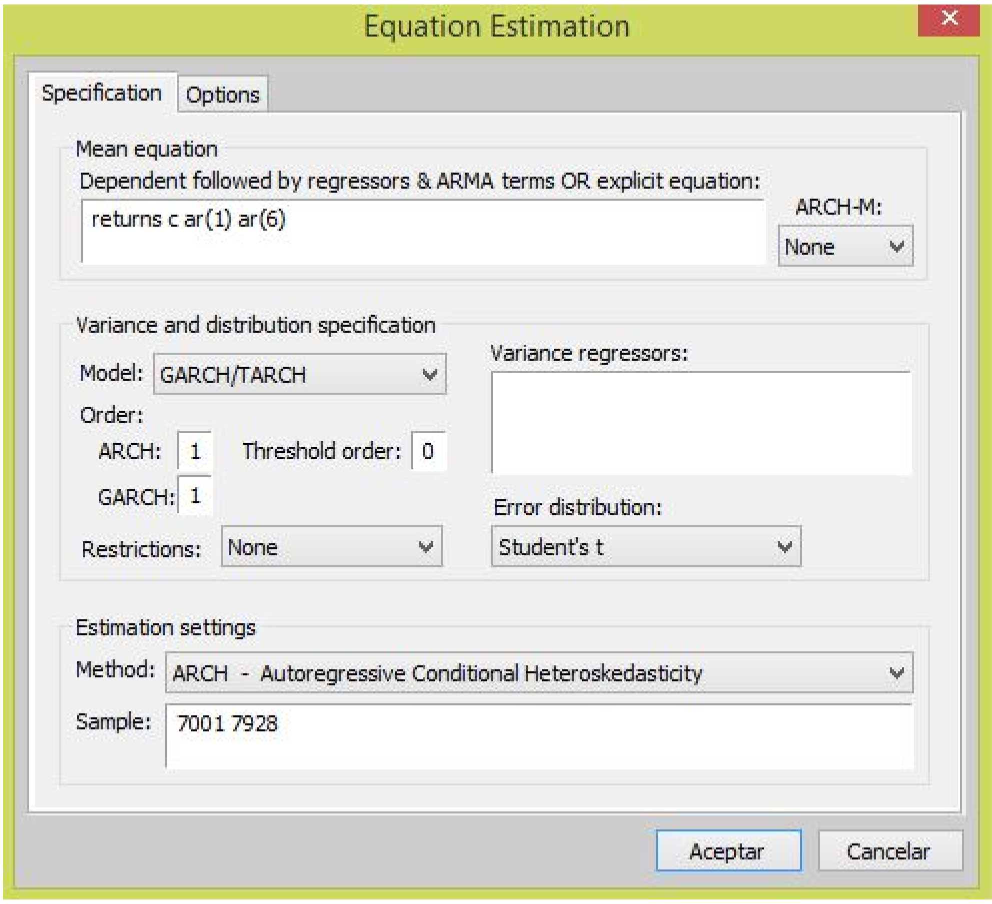


Figura 17: GARCH(1,1) spec with t errors

As we had several potential ARCH/GARCH specfications, we can compare them either by regressions and coefficients tests (Pagan-Schwert (1989)) or by forecast precision criteria. See lecture notes for a more detailed explanation.

In our case based on forecast criteria, we will keep the gaussian errors as the prefered GARCH specification. Note that the assumptions with regards to the distribution of errors are key to correctly specify the model.

3)

As exposed in the lecture notes, EGARCH models are usefull to test for leverage effects. Recall form lecture notes that leverage effects 'refers to the tendency for stock prices to be negatively correlated with changes in stock volatility'. In particular, we will use a EGARCH(1,1), as it seems to model properly the conditional variances (using the same metrics as before). The

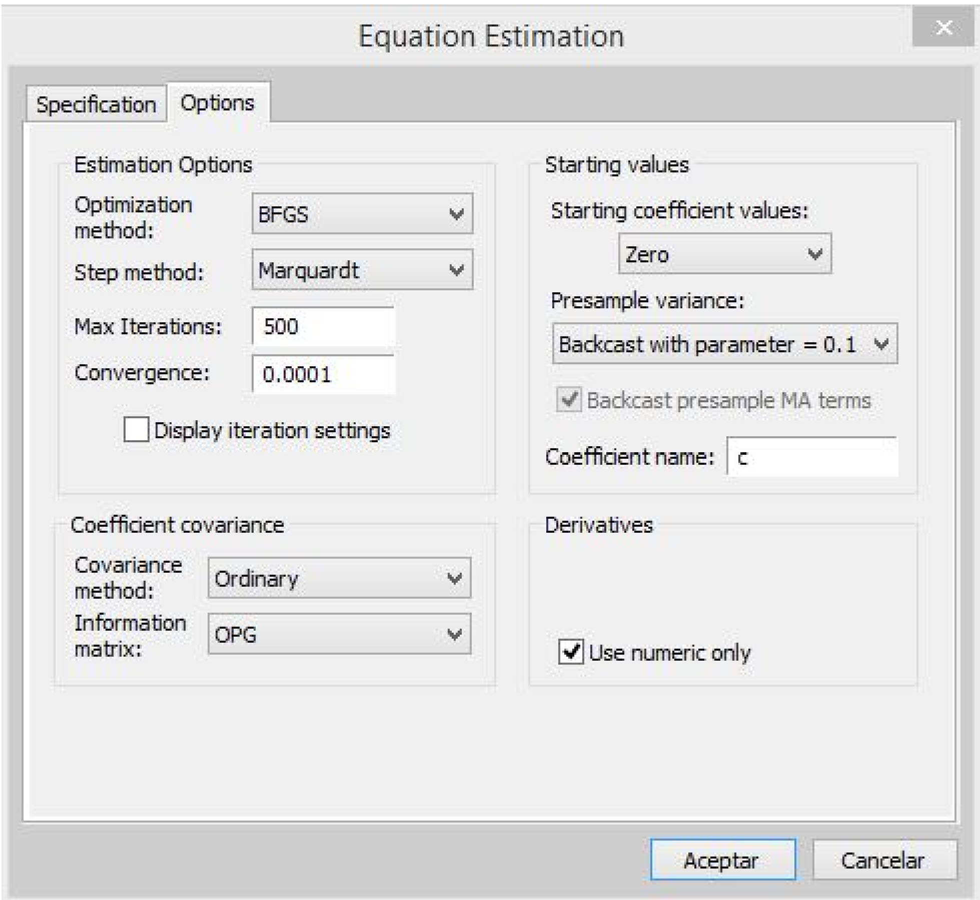


Figura 18: GARCH(1,1) spec with t errors

specification in this case is given by:

$$\ln \sigma_t^2 = \alpha_0 + \beta_1 \ln \sigma_{t-1}^2 + \gamma_0 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \delta \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right) \tag{1}$$

Note that, with this specification, shocks are allowed to have a diferential impact on variance depending on its sign. If $\delta < 0$, then negative values of ε_{t-1} will increase variance more than postive ones, giving us information on the existence of leverage effects. Estimation output of this model in Eviews (in order to do so, we have to select the EGARCH option in the 'Model' menu when specifying the variance equation) is shown in the figure below.

Eviews tells us what each coefficient is in the equation written just over the estimated figures. With this in mind, we see that $\hat{\delta}$ corresponds to *c(6)*, which is the coefficient that multiplies the *RESID(-1)* term. We find this coefficient to be negative and significantly different from zero. We therefore have evidence in favor of leverage effects.

Dependent Variable: RETURNS
Method: ML ARCH - Student's t distribution (OPG - BHHH / Marquardt steps)
Date: 04/20/20 Time: 12:02
Sample: 7001 7928
Included observations: 928
Convergence achieved after 21 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.1)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000577	0.000187	3.081800	0.0021
AR(1)	0.107870	0.031236	3.453355	0.0006
AR(6)	-0.088680	0.031764	-2.791864	0.0052
		Variance Equation		
C	1.91E-07	2.05E-07	0.931261	0.3517
RESID(-1)^2	0.040718	0.011127	3.659241	0.0003
GARCH(-1)	0.957183	0.011568	82.74401	0.0000
T-DIST. DOF	5.310534	0.880280	6.032781	0.0000

R-squared	0.027504	Mean dependent var	0.000537
Adjusted R-squared	0.025401	S.D. dependent var	0.007134
S.E. of regression	0.007043	Akaike info criterion	-7.247684
Sum squared resid	0.045879	Schwarz criterion	-7.211228
Log likelihood	3369.926	Hannan-Quinn criter.	-7.233778
Durbin-Watson stat	1.919541		

Inverted AR Roots	.60+.33i	.60-.33i	.02-.67i	.02+.67i
	-.56+.33i	-.56-.33i		

Figura 19: GARCH(1,1) with t errors estimation output

4)

As we did before, we specfiy an ARCH-M model by selecting it in the appropiate menu. We try with different specfications, but none shows the standard deviation to be signficant. We show as an example a GARCH(1,1) with ARCH-M effects We note that the estimated coefficient is negative (we would expect it to be positive) and not significant. Therefore, we don't find evidence to include the St. dev. in the mean equation.

Problem 3

1)

The series are already generated in the workfile. We can also use the *dlog* command.

Date: 04/20/20 Time: 12:09
Sample: 7001 7928
Included observations: 928

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.019	0.019	0.3191	0.572
		2	0.025	0.024	0.8823	0.643
		3	-0.030	-0.031	1.7395	0.628
		4	-0.017	-0.016	2.0015	0.735
		5	0.026	0.028	2.6252	0.758
		6	-0.005	-0.006	2.6520	0.851
		7	-0.058	-0.060	5.7506	0.569
		8	0.013	0.017	5.9092	0.657
		9	-0.021	-0.018	6.3289	0.707
		10	0.046	0.042	8.3269	0.597
		11	0.016	0.014	8.5534	0.663
		12	-0.036	-0.037	9.8024	0.633
		13	0.009	0.010	9.8718	0.704
		14	0.048	0.050	12.008	0.606
		15	0.021	0.016	12.425	0.647

Figura 20: Correlogram of sq. residuals

2)

We do that by going to *Sample* and selecting the specified range

3)

We estimate the model as we've done before, by going to *Quick/Estimate Equation*, and writing *dsp c ar(1)*. The estimation output is shown in the figure below.

We check the correlograms. These are shown below.

Correlogram of residuals show that we are modelling the conditional mean correctly. Nevertheless, the other one shows that, for longer lags (such as 10th or 11th lag) we have some unmodeled structure (see Q-stats.) Therefore, we should model the conditional variance. Note that if we do the ARCH-LM tests, we are unable to reject the null of no Arch effects. We guide ourselves with the correlogram in this case.

4)

Yes. We've answered that in the preceding paragraph.

Dependent Variable: RETURNS
Method: ML ARCH - Student's t distribution (OPG - BHHH / Marquardt steps)
Date: 04/20/20 Time: 12:28
Sample: 7001 7928
Included observations: 928
Convergence achieved after 28 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.1)
LOG(GARCH) = C(4) + C(5)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(6)
*RESID(-1)/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000440	0.000187	2.356528	0.0184
AR(1)	0.103348	0.031138	3.319075	0.0009
AR(6)	-0.084021	0.031376	-2.677846	0.0074
Variance Equation				
C(4)	-0.178666	0.074785	-2.389072	0.0169
C(5)	0.116153	0.027747	4.186107	0.0000
C(6)	-0.035456	0.017240	-2.056676	0.0397
C(7)	0.990716	0.006709	147.6703	0.0000
T-DIST. DOF	5.332234	0.904841	5.893009	0.0000

R-squared	0.026962	Mean dependent var	0.000537
Adjusted R-squared	0.024858	S.D. dependent var	0.007134
S.E. of regression	0.007045	Akaike info criterion	-7.242095
Sum squared resid	0.045905	Schwarz criterion	-7.200431
Log likelihood	3368.332	Hannan-Quinn criter.	-7.226203
Durbin-Watson stat	1.910256		

Inverted AR Roots	.59+.33i	.59-.33i	.02-.66i	.02+.66i
	-.56+.33i	-.56-.33i		

Figura 21: EGARCH estimation output

5)

We estimate the GARCH(1,1) as before. Nothing is said to us about the distribution of residuals, so we use normally distributed ones. After playing for a while with the optimization options, we get the estimation output as shown below. Despite the constant, GARCH coefficients are all significant. Correlogram of squared residuals seems clean now.

6)

To add variance regressors, we have to list the variables in the appropiate box in the specification window. Estimation output is shown in the figure.

We see that the estimated coefficient of the added variable is significant at the usual levels

Dependent Variable: RETURNS
Method: ML ARCH - Student's t distribution (OPG - BHHH / Marquardt steps)
Date: 04/20/20 Time: 12:36
Sample: 7001 7928
Included observations: 928
Convergence achieved after 20 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.1)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	-0.052415	0.110818	-0.472985	0.6362
C	0.000892	0.000689	1.293973	0.1957
AR(1)	0.109156	0.031136	3.505762	0.0005
AR(6)	-0.088417	0.031743	-2.785385	0.0053
		Variance Equation		
C	1.87E-07	2.01E-07	0.926988	0.3539
RESID(-1)^2	0.039863	0.010886	3.661894	0.0003
GARCH(-1)	0.958026	0.011309	84.71182	0.0000
T-DIST. DOF	5.300789	0.880857	6.017762	0.0000

R-squared	0.026966	Mean dependent var	0.000537
Adjusted R-squared	0.023807	S.D. dependent var	0.007134
S.E. of regression	0.007048	Akaike info criterion	-7.245759
Sum squared resid	0.045905	Schwarz criterion	-7.204095
Log likelihood	3370.032	Hannan-Quinn criter.	-7.229866
Durbin-Watson stat	1.921421		

Inverted AR Roots	.60+.33i	.60-.33i	.02-.67i	.02+.67i
	-.56+.33i	-.56-.33i		

Figura 22: GARCH(1,1) with ARCH-M

7)

The sq. residuals correlogram is shown in the figure.

Dependent Variable: DSP
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 04/20/20 Time: 19:06
Sample: 1/02/1985 7/31/1987
Included observations: 652
Convergence achieved after 6 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000987	0.000371	2.663352	0.0079
AR(1)	0.087684	0.036086	2.429879	0.0154
SIGMASQ	7.01E-05	2.62E-06	26.78104	0.0000

R-squared	0.007687	Mean dependent var	0.000989
Adjusted R-squared	0.004629	S.D. dependent var	0.008410
S.E. of regression	0.008391	Akaike info criterion	-6.718742
Sum squared resid	0.045694	Schwarz criterion	-6.698128
Log likelihood	2193.310	Hannan-Quinn criter.	-6.710747
F-statistic	2.513715	Durbin-Watson stat	1.991889
Prob(F-statistic)	0.081755		

Inverted AR Roots	.09

Figura 23: Estimation output for AR(1)

Date: 04/20/20 Time: 19:14
Sample: 1/02/1985 7/31/1987
Included observations: 652
Q-statistic probabilities adjusted for 1 ARMA term

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.002	0.002	0.0039	
		2	-0.030	-0.030	0.6023	0.438
		3	0.013	0.013	0.7097	0.701
		4	-0.051	-0.052	2.4389	0.486
		5	-0.027	-0.026	2.9339	0.569
		6	-0.034	-0.037	3.6933	0.594
		7	0.001	0.001	3.6946	0.718
		8	0.002	-0.003	3.6963	0.814
		9	0.010	0.008	3.7633	0.878
		10	0.062	0.058	6.3001	0.710
		11	-0.032	-0.034	6.9843	0.727
		12	0.003	0.006	6.9909	0.800
		13	0.030	0.028	7.5902	0.816
		14	-0.068	-0.061	10.636	0.641
		15	-0.044	-0.042	11.933	0.612
		16	-0.000	-0.002	11.933	0.684
		17	0.014	0.014	12.069	0.739
		18	0.004	-0.000	12.077	0.795

Figura 24: Correlgram of residuals

Problem 4

1)

We have the indexes in levels, but we need the returns. In order to get them, we log-difference the data and multiply that by 100 to make units match.

Date: 04/20/20 Time: 19:14
Sample: 1/02/1985 7/31/1987
Included observations: 652

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.041	0.041	1.0924	0.296
		2	0.023	0.021	1.4368	0.488
		3	-0.003	-0.004	1.4415	0.696
		4	0.070	0.070	4.7113	0.318
		5	0.034	0.028	5.4590	0.362
		6	0.067	0.062	8.4551	0.207
		7	0.053	0.048	10.331	0.171
		8	-0.009	-0.020	10.390	0.239
		9	0.013	0.009	10.495	0.312
		10	0.100	0.092	17.125	0.072
		11	0.075	0.058	20.877	0.035
		12	0.019	0.007	21.113	0.049
		13	-0.014	-0.022	21.241	0.068
		14	0.022	0.011	21.573	0.088
		15	0.072	0.060	25.029	0.050
		16	0.012	-0.011	25.122	0.068
		17	0.031	0.014	25.768	0.079
		18	-0.027	-0.034	26.242	0.094
		19	0.029	0.024	26.813	0.109
		20	-0.022	-0.035	27.151	0.131
		21	0.026	0.001	27.598	0.152

Figura 25: Correlgram of squared residuals

Dependent Variable: DSP
Method: ML ARCH - Normal distribution (OPG - BHHH / Marquardt steps)
Date: 04/20/20 Time: 19:26
Sample: 1/02/1985 7/31/1987
Included observations: 652
Convergence achieved after 37 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: unconditional
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000923	0.000348	2.649437	0.0081
AR(1)	0.086869	0.041981	2.069222	0.0385
		Variance Equation		
C	5.67E-07	4.22E-07	1.342746	0.1794
RESID(-1)^2	0.027806	0.010075	2.759801	0.0058
GARCH(-1)	0.964597	0.013251	72.79199	0.0000

R-squared	0.007382	Mean dependent var	0.000989
Adjusted R-squared	0.005855	S.D. dependent var	0.008410
S.E. of regression	0.008386	Akaike info criterion	-6.752681
Sum squared resid	0.045708	Schwarz criterion	-6.718325
Log likelihood	2206.374	Hannan-Quinn criter.	-6.739357
Durbin-Watson stat	1.989854		

Inverted AR Roots	.09

Figura 26: Estimation output of GARCH(1,1)

To create the log-difference series, we may go yo *Quick/Generate Series* and then we input something like

$$diffdow = 100*d(log(dow))=$$

Date: 04/20/20 Time: 19:31
Sample: 1/02/1985 7/31/1987
Included observations: 652

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.003	-0.003	0.0080	0.929
		2	-0.024	-0.024	0.4007	0.818
		3	-0.031	-0.032	1.0452	0.790
		4	0.025	0.024	1.4578	0.834
		5	0.001	-0.000	1.4589	0.918
		6	0.023	0.023	1.8066	0.937
		7	0.014	0.016	1.9438	0.963
		8	-0.032	-0.032	2.6334	0.955
		9	0.001	0.003	2.6343	0.977
		10	0.037	0.036	3.5521	0.965
		11	0.050	0.048	5.2286	0.920
		12	-0.029	-0.026	5.7800	0.927
		13	-0.029	-0.026	6.3510	0.932
		14	-0.002	-0.001	6.3525	0.957
		15	0.050	0.046	8.0024	0.924
		16	0.018	0.016	8.2205	0.942
		17	-0.011	-0.011	8.2990	0.960
		18	-0.050	-0.045	9.9528	0.933
		19	0.000	0.003	9.9528	0.954
		20	-0.056	-0.062	12.056	0.914
		21	-0.008	-0.018	12.102	0.937
		22	-0.012	-0.015	12.195	0.953

Figura 27: Correlgram of sq. resid of GARCH(1,1) model

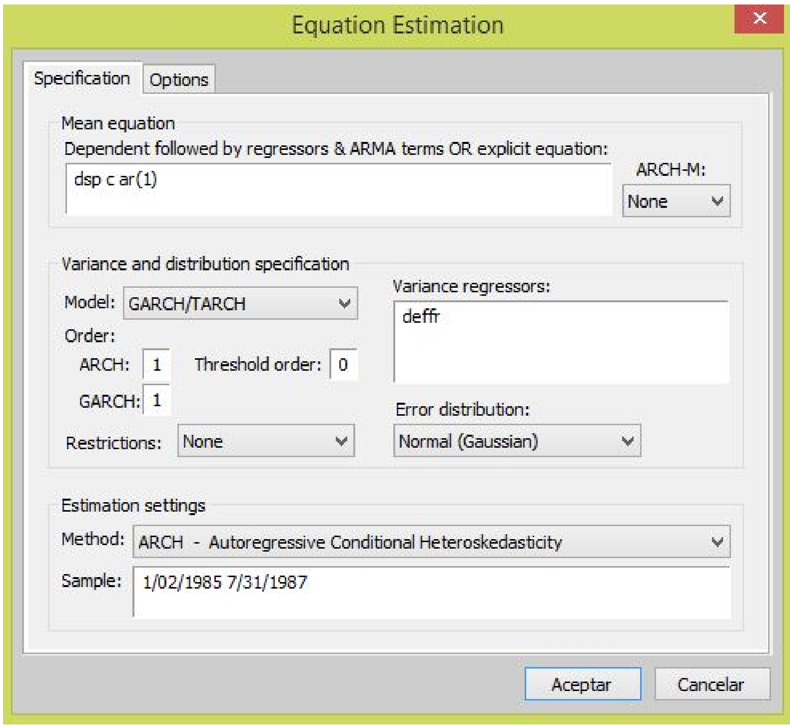


Figura 28: Adding variance regressors

We have to do this for all three series. We have this already created in the data-base.

We now proceed to estimate the proposed models. All of them have only a constant in the mean specfication.

Dependent Variable: DSP
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 05/19/20 Time: 15:30
Sample: 1/02/1985 7/31/1987
Included observations: 652
Convergence achieved after 51 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1) + C(6)*DEFFR

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000878	0.000343	2.562615	0.0104
AR(1)	0.090659	0.042533	2.131481	0.0330
		Variance Equation		
C	4.92E-07	3.77E-07	1.304383	0.1921
RESID(-1)^2	0.029843	0.009870	3.023555	0.0025
GARCH(-1)	0.963040	0.011635	82.77056	0.0000
DEFFR	-1.50E-05	6.92E-06	-2.169978	0.0300

R-squared	0.007270	Mean dependent var	0.000989
Adjusted R-squared	0.005742	S.D. dependent var	0.008410
S.E. of regression	0.008386	Akaike info criterion	-6.751846
Sum squared resid	0.045713	Schwarz criterion	-6.710619
Log likelihood	2207.102	Hannan-Quinn criter.	-6.735858
Durbin-Watson stat	1.997008		
Inverted AR Roots	.09		

Figura 29: GARCH(1,1) with variance regressor: estimation output

Correlogra

Date: 05/19/20 Time: 15:32
Sample: 1/02/1985 7/31/1987
Included observations: 652

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.006	-0.006	0.0265	0.871
		2	-0.022	-0.022	0.3544	0.838
		3	-0.030	-0.030	0.9432	0.815
		4	0.032	0.031	1.5998	0.809
		5	-0.004	-0.005	1.6090	0.900
		6	0.024	0.025	1.9923	0.920
		7	0.010	0.012	2.0557	0.957
		8	-0.033	-0.033	2.7570	0.949
		9	0.002	0.004	2.7609	0.973
		10	0.036	0.034	3.6145	0.963
		11	0.047	0.046	5.1108	0.926
		12	-0.028	-0.024	5.6412	0.933
		13	-0.023	-0.020	5.9813	0.947
		14	0.001	0.002	5.9822	0.967
		15	0.048	0.044	7.5092	0.942
		16	0.018	0.017	7.7240	0.957
		17	-0.012	-0.011	7.8151	0.970
		18	-0.049	-0.044	9.3996	0.950
		19	-0.002	-0.000	9.4012	0.966
		20	-0.055	-0.061	11.437	0.934
		21	-0.011	-0.020	11.521	0.952

Figura 30: GARCH(1,1) with deffr as variance regressor: correlogram of sq. residuals

It seems to be clean. Therefore, it can be a good idea to use the difference of the rates in order to explain the variance of returns.

2)

We show the results only for the Nikkei equation, but the procedure is identical for all of them. Once we have estimated the model, we click on *View/Garch Graph/Conditional Variance*.

Dependent Variable: DFTSE
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 04/21/20 Time: 08:33
Sample (adjusted): 1/06/1989 12/31/2007
Included observations: 4952 after adjustments
Convergence achieved after 10 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.031095	0.012866	2.416857	0.0157
		Variance Equation		
C	0.739176	0.013278	55.66806	0.0000
RESID(-1)^2	0.255320	0.017288	14.76863	0.0000

R-squared	-0.000028	Mean dependent var	0.025800
Adjusted R-squared	-0.000028	S.D. dependent var	0.992133
S.E. of regression	0.992147	Akaike info criterion	2.757234
Sum squared resid	4873.546	Schwarz criterion	2.761177
Log likelihood	-6823.912	Hannan-Quinn criter.	2.758617
Durbin-Watson stat	1.988342		

Figura 31: ARCH(1) in FTSE returns: estimation output

Dependent Variable: DFTSE
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 04/21/20 Time: 08:35
Sample (adjusted): 1/06/1989 12/31/2007
Included observations: 4952 after adjustments
Convergence achieved after 26 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.041611	0.011327	3.673523	0.0002
		Variance Equation		
C	0.013223	0.002332	5.670690	0.0000
RESID(-1)^2	0.080033	0.006341	12.62246	0.0000
GARCH(-1)	0.906335	0.007472	121.3023	0.0000

R-squared	-0.000254	Mean dependent var	0.025800
Adjusted R-squared	-0.000254	S.D. dependent var	0.992133
S.E. of regression	0.992259	Akaike info criterion	2.565382
Sum squared resid	4874.644	Schwarz criterion	2.570638
Log likelihood	-6347.885	Hannan-Quinn criter.	2.567225
Durbin-Watson stat	1.987894		

Figura 32: GARCH(1,1) in FTSE returns: estimation output

Then we get the following plot. Note that here we can see several spikes in conditional variance, therefore, we won't gere rid of the GARCH structure by only taking subsamples as we did in exercise 1 with interest rates. In this case, the series truly seem to have GARCH structure rather than structural breaks.

Dependent Variable: DDOW
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 04/21/20 Time: 08:37
Sample (adjusted): 1/06/1989 12/31/2007
Included observations: 4952 after adjustments
Convergence achieved after 9 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.046040	0.013289	3.464498	0.0005
Variance Equation				
C	0.746985	0.010441	71.54264	0.0000
RESID(-1)^2	0.195518	0.011887	16.44780	0.0000

R-squared	-0.000102	Mean dependent var	0.036368
Adjusted R-squared	-0.000102	S.D. dependent var	0.958679
S.E. of regression	0.958727	Akaike info criterion	2.712092
Sum squared resid	4550.752	Schwarz criterion	2.716034
Log likelihood	-6712.139	Hannan-Quinn criter.	2.713474
Durbin-Watson stat	2.012831		

Figura 33: ARCH(1) in Dow returns: estimation output

Dependent Variable: DDOW
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 04/21/20 Time: 08:37
Sample (adjusted): 1/06/1989 12/31/2007
Included observations: 4952 after adjustments
Convergence achieved after 31 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.051217	0.011607	4.412403	0.0000
Variance Equation				
C	0.010476	0.001291	8.114738	0.0000
RESID(-1)^2	0.054828	0.003612	15.18033	0.0000
GARCH(-1)	0.934496	0.004434	210.7765	0.0000

R-squared	-0.000240	Mean dependent var	0.036368
Adjusted R-squared	-0.000240	S.D. dependent var	0.958679
S.E. of regression	0.958794	Akaike info criterion	2.552328
Sum squared resid	4551.381	Schwarz criterion	2.557584
Log likelihood	-6315.564	Hannan-Quinn criter.	2.554171
Durbin-Watson stat	2.012553		

Figura 34: GARCH(1,1) in Dow returns: estimation output

Problem 5

As brief review, the BEKK model is a multivariate generalization from the univarate GARCH. The specification for the time-varying covariance matrix is

$$V_t = CC' + Au_{t-1}u_{t-1}'A' + DV_{t-1}D' \qquad (2)$$

Dependent Variable: DNIKKEI
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 04/21/20 Time: 08:38
Sample (adjusted): 1/06/1989 12/31/2007
Included observations: 4952 after adjustments
Convergence achieved after 9 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000630	0.018870	0.033400	0.9734
		Variance Equation		
C	1.585070	0.026215	60.46508	0.0000
RESID(-1)^2	0.183161	0.011617	15.76609	0.0000

R-squared	-0.000107	Mean dependent var	-0.013711
Adjusted R-squared	-0.000107	S.D. dependent var	1.386959
S.E. of regression	1.387033	Akaike info criterion	3.458689
Sum squared resid	9525.029	Schwarz criterion	3.462631
Log likelihood	-8560.714	Hannan-Quinn criter.	3.460071
Durbin-Watson stat	2.035562		

Figura 35: ARCH(1) in Nikkei returns: estimation output

Dependent Variable: DNIKKEI
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 04/21/20 Time: 08:38
Sample (adjusted): 1/06/1989 12/31/2007
Included observations: 4952 after adjustments
Convergence achieved after 44 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.037760	0.015505	2.435427	0.0149
		Variance Equation		
C	0.023580	0.002571	9.172554	0.0000
RESID(-1)^2	0.086864	0.005114	16.98638	0.0000
GARCH(-1)	0.904975	0.005189	174.4092	0.0000

R-squared	-0.001377	Mean dependent var	-0.013711
Adjusted R-squared	-0.001377	S.D. dependent var	1.386959
S.E. of regression	1.387913	Akaike info criterion	3.304482
Sum squared resid	9537.129	Schwarz criterion	3.309739
Log likelihood	-8177.898	Hannan-Quinn criter.	3.306326
Durbin-Watson stat	2.032980		

Figura 36: GARCH(1,1) in Nikkei returns: estimation output

Where V_t is the variance matrix of the errors in the mean (vectorial) equation. The advantage of this specfication is that, as everything is specified in quadratic form, variance matrix is always positive definite, so we don't have the problem of possible 'negative variances'.

For the two variables case, the model is given by:

$$V_t = \begin{bmatrix} c_{1,1} & 0 \\ c_{2,1} & c_{2,2} \end{bmatrix} \begin{bmatrix} c_{1,1} & c_{2,1} \\ 0 & c_{2,2} \end{bmatrix} +$$

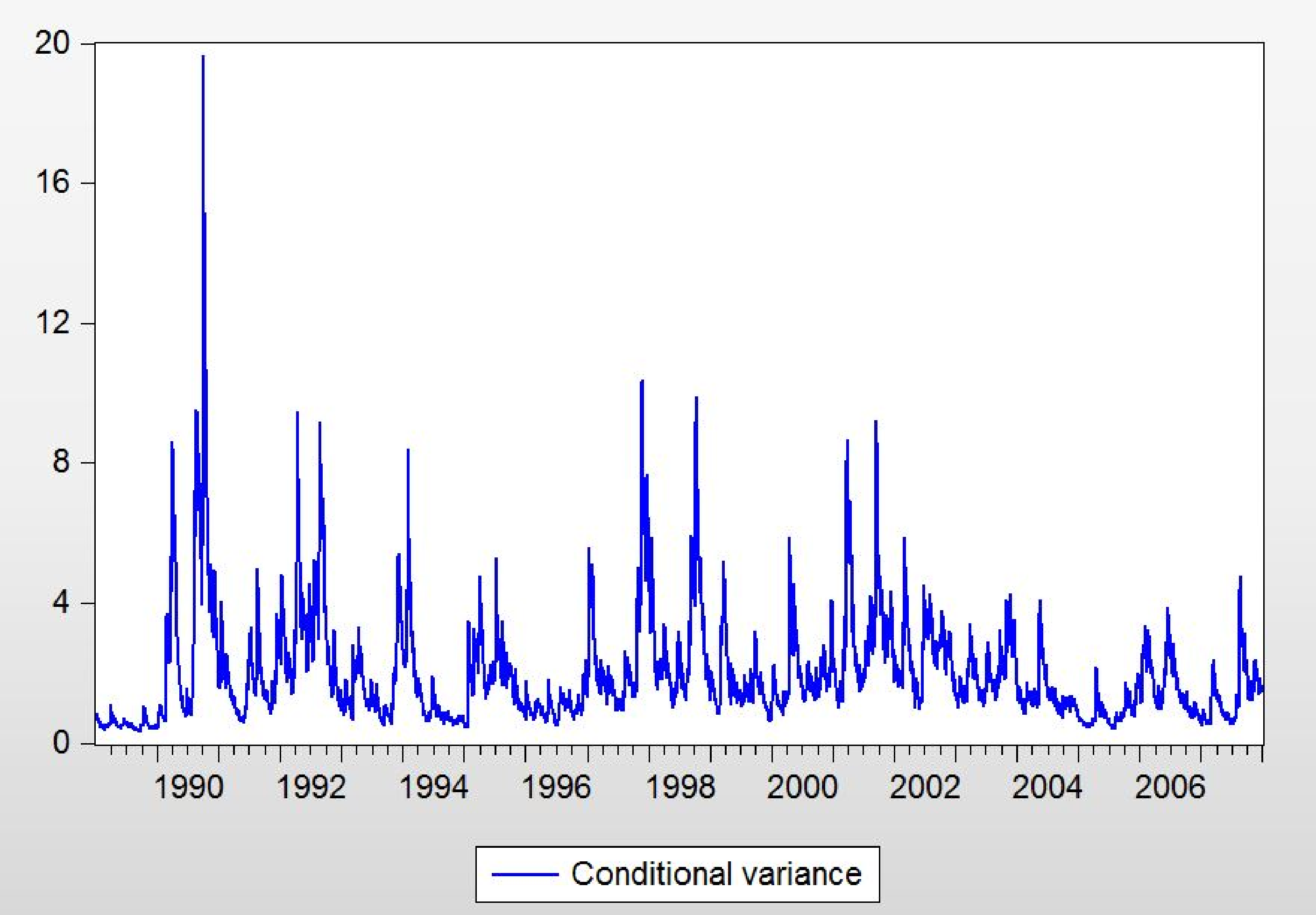


Figura 37: GARCH(1,1) in Nikkei returns: Conditional variance

$$+\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \begin{bmatrix} u^2_{1,t-1} & u_{1,t-1}u_{2,t-1} \\ u_{1,t-1}u_{2,t-1} & u^2_{2,t-1} \end{bmatrix} \begin{bmatrix} a_{1,1} & a_{2,1} \\ a_{1,2} & a_{2,2} \end{bmatrix}$$
$$+\begin{bmatrix} d_{1,1} & d_{1,2} \\ d_{2,1} & d_{2,2} \end{bmatrix} \begin{bmatrix} \sigma^2_{x,t-1} & \sigma_{yx,t-1} \\ \sigma_{yx,t-1} & \sigma^2_{y,t-1} \end{bmatrix} \begin{bmatrix} d_{1,1} & d_{2,1} \\ d_{1,2} & d_{2,2} \end{bmatrix}$$

1)

The variables are created as before. In the worfile, those are already made. They're called *dm* (1 month) and dy (3 months). By multiplying by 100, we get them in basis points (remember that 100 basis points equals 1 % of interest).

2)

The model we estimate is a diagonal BEKK, that is, the form of matrices A and D are restricted to be diagonal. We get this by including the restriction that $a_{1,2} = a_{2,1} = d_{1,2} = d_{2,1} = 0$. In order to do so, we choose both *dy* and *dm* right click and select *Open/As system*. A window appears, we let things as they are and press 'OK'. Then we should have something like the figure below. Then, we go to *Estimate*. We choose ARCH as estimation method, and

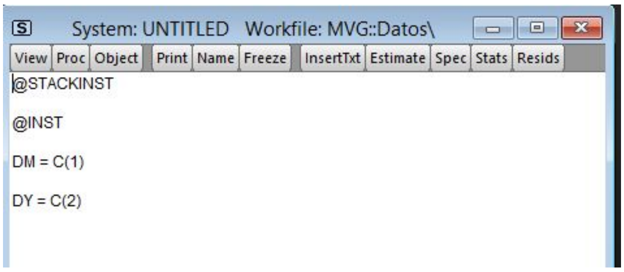


Figura 38: System specification

select 'Diagonal BEKK' in model type. We press OK and wait for the optimization algorithm

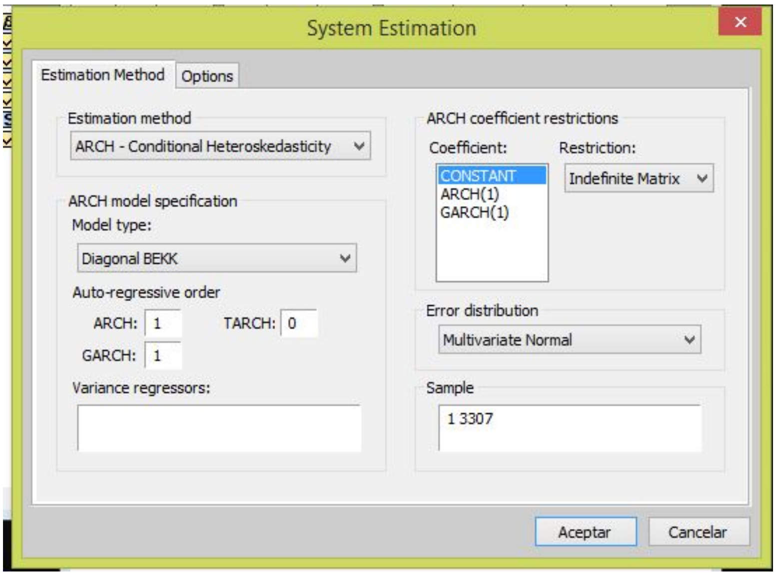


Figura 39: System estimation

to stop. We get the following output. The first two coefficents are the constants in the mean equation. The rest of the coefficents are the variance equation ones. To know excatly what they are, we can scroll down to the bottom of the output. We'll get what we need there. We can see estimated coefficients in a clearer notation.

With the estimated model, we can get the variance/covariance series. In order to do so, we may

System: UNTITLED
Estimation Method: ARCH Maximum Likelihood (BFGS / Marquardt steps)
Covariance specification: Diagonal BEKK
Date: 04/21/20 Time: 10:23
Sample: 2 3307
Included observations: 3306
Total system (balanced) observations 6612
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 25 iterations
Coefficient covariance computed using outer product of gradients

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.073981	0.073323	1.008976	0.3130
C(2)	0.041677	0.085278	0.488725	0.6250
	Variance Equation Coefficients			
C(3)	1.473938	0.093537	15.75774	0.0000
C(4)	0.946703	0.056875	16.64521	0.0000
C(5)	0.765546	0.070045	10.92937	0.0000
C(6)	0.400770	0.006490	61.75561	0.0000
C(7)	0.213575	0.006746	31.66127	0.0000
C(8)	0.899650	0.003551	253.3234	0.0000
C(9)	0.964119	0.002169	444.4545	0.0000

Log likelihood	-19219.63	Schwarz criterion	11.64918
Avg. log likelihood	-2.906780	Hannan-Quinn criter.	11.63851
Akaike info criterion	11.63256		

Figura 40: BEKK output

Covariance specification: Diagonal BEKK
GARCH = M + A1*RESID(-1)*RESID(-1)'*A1 + B1*GARCH(-1)*B1
M is an indefinite matrix
A1 is a diagonal matrix
B1 is a diagonal matrix

Transformed Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	1.473938	0.093537	15.75774	0.0000
M(1,2)	0.946703	0.056875	16.64521	0.0000
M(2,2)	0.765546	0.070045	10.92937	0.0000
A1(1,1)	0.400770	0.006490	61.75561	0.0000
A1(2,2)	0.213575	0.006746	31.66127	0.0000
B1(1,1)	0.899650	0.003551	253.3234	0.0000
B1(2,2)	0.964119	0.002169	444.4545	0.0000

Figura 41: BEKK Variance specification and coefficents (bottom of output)

click in *View/Conditional Covariance*. We can choose different forms of output, but, in this case, I'll leave the options as they are. The obtained graph is shown below.

We note that the restriction of diagonal A and D matrix doesn't turn into constant covariances.

If we wanted to estimate an unrestricted model, there are some MATLAB codes we can use. I'll just show the output figures. These show the estimated conditional covariance for retricted (diagonal BEKK) and unsrestricted BEKK models.

Note that the graphs are qualitatively similar. Furthermore, the second graph (diagonal BEKK) is identical to the one we've obtained in Eviews. The code also performs a LM test on the res-

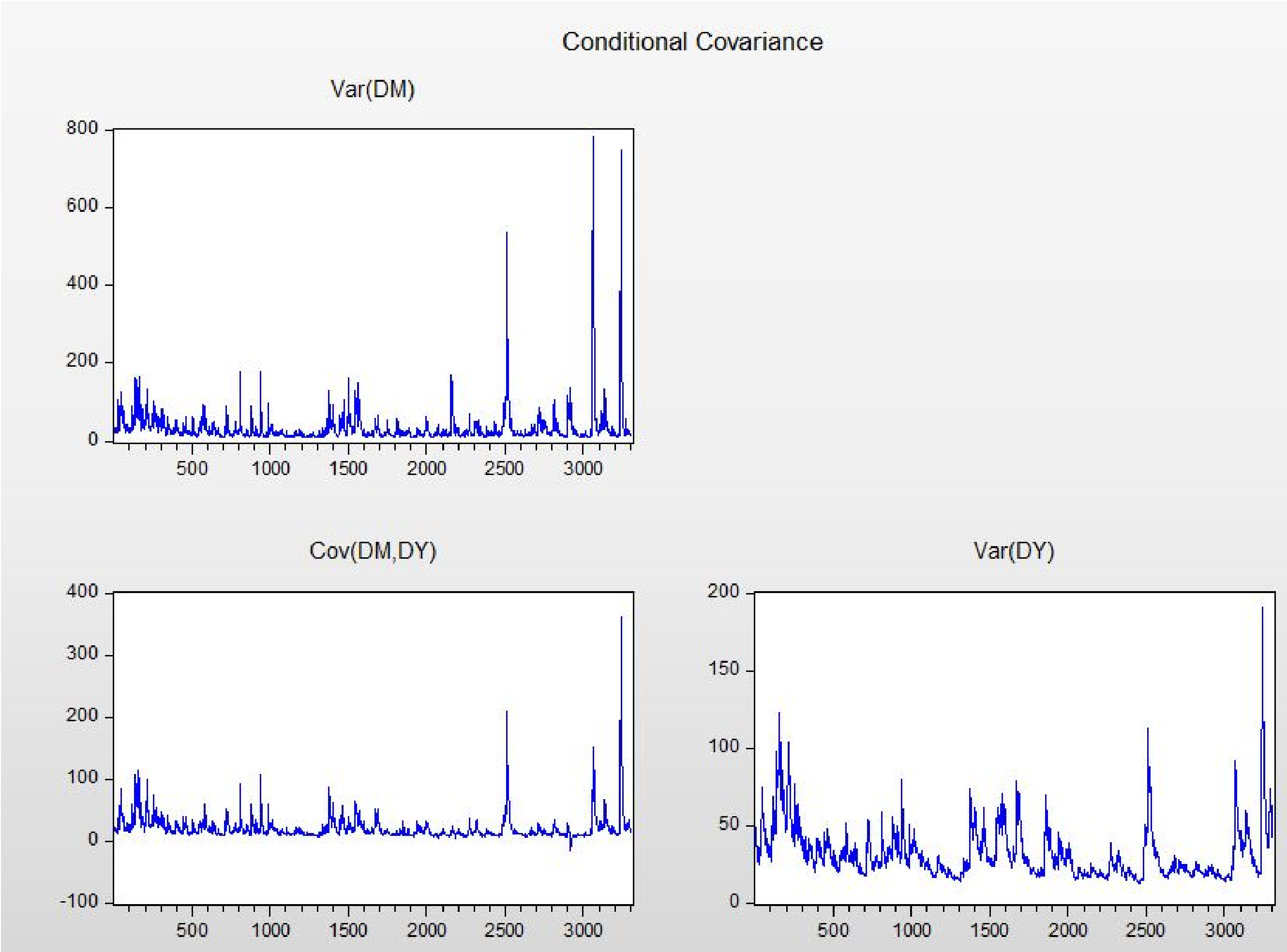


Figura 42: Conditional covariance

triction. The resulting statistic and p-value are printed in MATLAB command bar. We can see that we reject the null of diagonal A and D matrices.

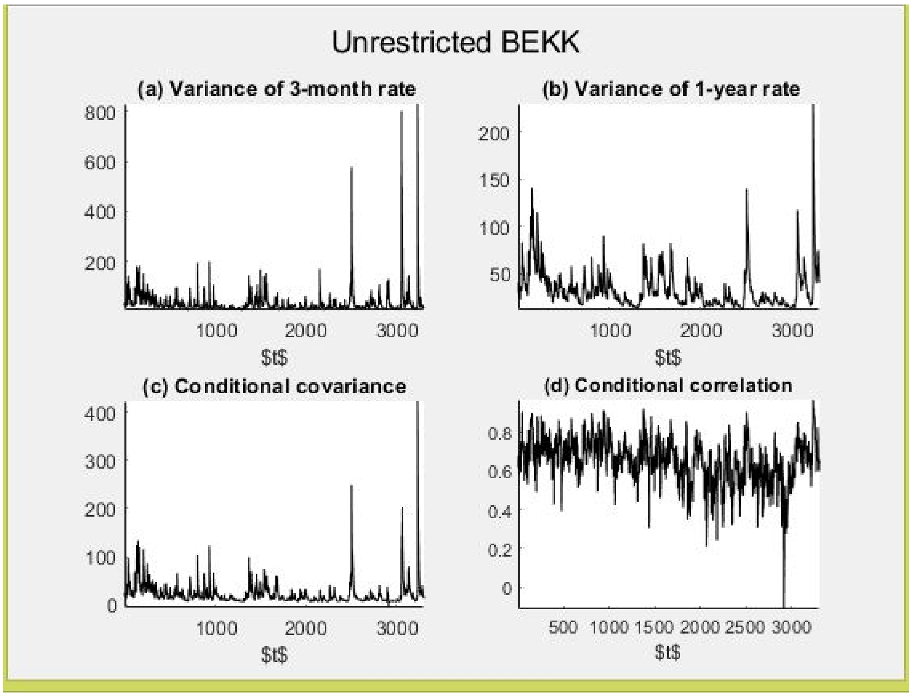


Figura 43: Conditional covariance: unrestricted BEKK

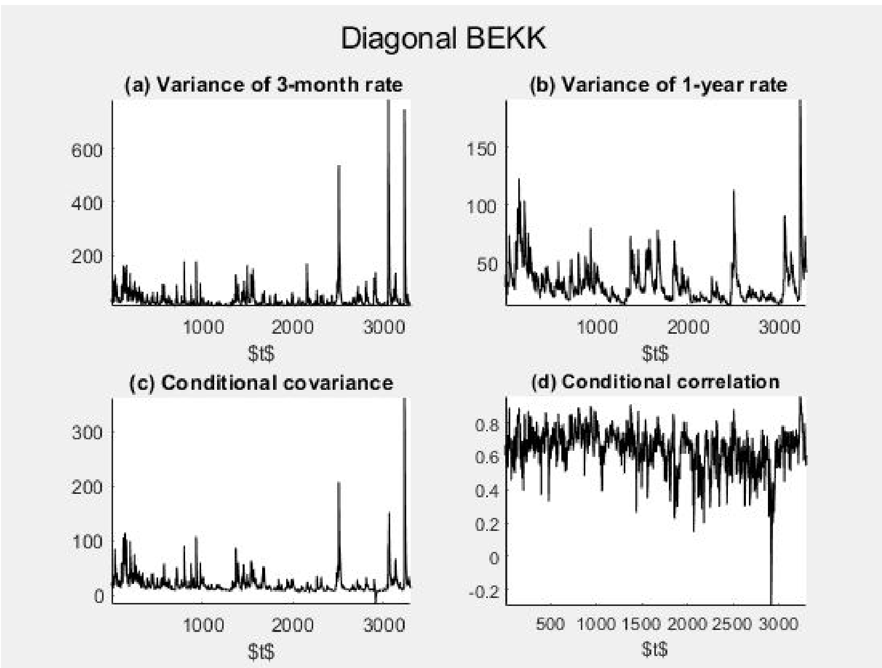


Figura 44: Conditional covariance: diagonal BEKK