Resumen: $\underline{X} = \{X_1, \ldots, X_n\} \overset{iid}{\sim} f(x;\theta)$ y $T_n(\underline{X})$

* Suficiencia
  * FN: $\prod_{i=1}^{n} f(x_i;\theta) = h(x)\, g(T(x);\theta)$
  * f e fam. exponencial
    $f(x;\theta) = h(x)c(\theta)e^{w(\theta)t(x)} \Rightarrow T_n = \sum_{i=1}^{n} t(x_i)$

  ↳ T_n contiene toda la info sobre θ.

* Verosimilitud: Dada $\underline{x}$ = "Datos" $\Rightarrow \theta_1$ más factible que θ_2
  si $L(\theta_1) / L(\theta_2) > 1$.

* Invarianza

Métodos para construir Estimadores
- Momentos
- Máxima Veros.

① Momentos: $X \sim f(x;\theta)$ con $\theta \in \mathbb{R}^k$

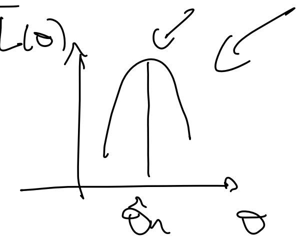
para $j = 1, \ldots, k$: $\boxed{\mu_j(\theta) = E(X^j) = \frac{1}{n}\sum_{i=1}^{n} X_i^j = M_j}$

$\boxed{\tilde{\theta}_n = (\tilde{\theta}_1, \ldots, \tilde{\theta}_k)}$

② Máxima Verosimilitud:

Dado una datos: $\underline{x} = \{X_1 = x_1, \ldots, X_n = x_n\}$

Estimación Máximo verosimil $\hat{\theta}_n = \arg\max L(\theta)$



$\underline{Ej\ 1}$: $X \sim Bern(\theta)$ $\quad \underline{\{X_1, \dots, X_n\} \overset{iid}{\sim} X}$

$$L(\theta) = \prod_{i=1}^{n} \underline{\theta^{X_i}(1-\theta)^{1-X_i}}$$

$$L(\theta) = \underline{\theta^{\sum_{i=1}^{n} X_i}(1-\theta)^{n-\sum_{i=1}^{n} X_i}} \qquad T_n = \sum_{i=1}^{n} X_i$$

$$\ell(\theta) = \ln L(\theta) = T_n \ln(\theta) + (n-T_n)\ln(1-\theta)$$

$$\ell'_\theta: \ T_n \frac{1}{\theta} + (n-T_n)\frac{1}{1-\theta}(-1) = 0$$

$$\theta(n-T_n) = (1-\theta)T_n$$

$$\boxed{\theta n = T_n} \Rightarrow \boxed{\hat{\theta}_n = \frac{T_n}{n} = \bar{X}_n}$$

$$\ell''_\theta: -\frac{T_n}{\theta^2} - \frac{(n-T_n)}{(1-\theta)^2} < 0 \quad \forall \theta \in (0,1)$$

$\underline{Ej\ 2}$: $\{X_1, \dots, X_n\} \overset{iid}{\sim} N(\underline{\mu}; \sigma^2 = 1)$

$$L(\mu) = \prod_{i=1}^{n} \underline{\frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(x_i-\mu)^2\right\}}.$$

$$= \left(\frac{1}{\sqrt{2\pi}}\right)^n \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2\right\}$$

$$l(\mu) = c - \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$l'_{\mu}: \sum_{i=1}^{n}(x_i-\mu) = 0$$

$$\sum_{i=1}^{n} x_i - n\mu = 0$$

$$\longrightarrow \boxed{\hat{\mu}_n = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

$$l''_{\mu} = -n < 0 \quad \forall \mu$$

Reg. lineal: (1) $Y = \beta_0 + \beta_1 X + \varepsilon \leftarrow$

(2) $\varepsilon \sim N(0, \sigma^2)$; $\varepsilon \perp X$

$\longrightarrow \cdot\ Y|X \sim N(\beta_0 + \beta_1 X, \sigma^2)$

(3) $\boxed{X \sim f_x(x)} \leftarrow$

$\theta = (\beta_0, \beta_1, \sigma^2)$.

Reg. Logística: $P(A \cap B) = P(A|B)P(B)$

$$\longrightarrow \quad f_{Y,X}(x,y;\theta) = f_{Y|X}(y;\theta)\, f_X(x)$$

$$\{(X_1,Y_1) \dots (X_n,Y_n)\} \overset{iid}{\sim} f_{Y,X}(x,y;\theta)$$

$$L(\theta) = \prod_{i=1}^{n} f_{Y,X}(x_i,y_i;\theta) = \prod_{i=1}^{n} \underline{f_{Y|X_i}(y_i;\theta)}\, f_X(x_i)$$

$$L(\theta) = \prod_{i=1}^{n} \left[\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{1}{2\sigma^2} (y_i - \beta_0 - \beta_1 x_i)^2 \right\} \right] f_X(x_i)$$

$$\underline{l(\theta)} = n \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) \underline{- \frac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2} + \ln\left[\prod_{i=1}^{n} f_X(x_i)\right]$$

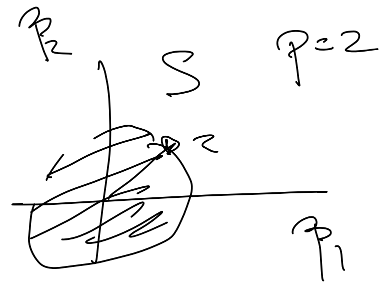
$$S(\theta) = \begin{bmatrix} \frac{\partial}{\partial \beta_0} l(\theta) = \frac{1}{\sigma^2} \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) \\ \frac{\partial}{\partial \beta_1} l(\theta) = \quad x \\ \frac{\partial}{\partial \sigma^2} l(\theta) = \quad x \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \qquad \hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\hat{\sigma}^2_n = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p + \varepsilon \qquad p >> n$$

$$\sum_{i=1}^{p} \beta_i^2 \leq t$$



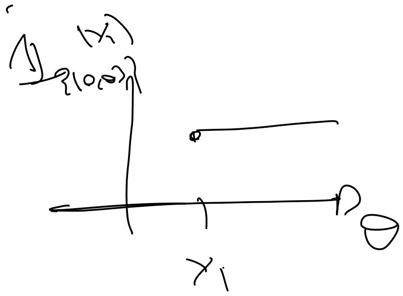
$$\{X_1, \dots, X_n\} \overset{iid}{\sim} Unif(0, \theta)$$

$$L(\theta) = \prod_{i=1}^{n} \frac{1}{\theta} \mathbb{1}_{\{[0,\theta]\}}(X_i) = \frac{1}{\theta^n} \prod_{i=1}^{n} \mathbb{1}_{\{[0,\theta]\}}(X_i)$$

Dec. exp.

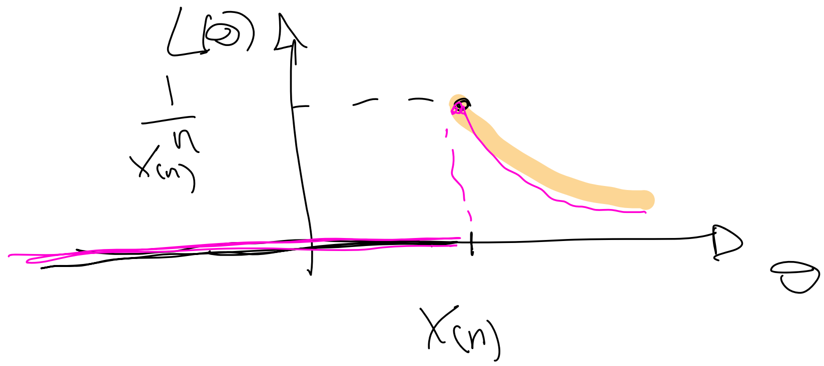
θ.

$$\mathbb{1}_{\{[0,\theta]\}}(X_1) = \begin{cases} 1 & \text{si } \theta \geq X_1 \\ 0 & \text{en otro caso} \end{cases}$$



$$\prod_{i=1}^{n} \mathbb{1}_{\{[0,\theta]\}}(X_i) = \begin{cases} 1 & \text{si } \theta \geq X_1, \theta \geq X_2, \dots, \theta \geq X_n \\ 0 & \text{en otro caso} \end{cases}$$

$$\rightarrow \theta \geq \max\{X_1, \dots, X_n\} = X_{(n)}$$



$$\hat{\Theta}_n = \max\{X_1, \ldots, X_n\}$$ ✓

$$\boxed{\tilde{\Theta}_n = 2\bar{X}_n}$$ ← Estimador de Momentos (No involucra al estadístico suficiente)

Invariancia:

$$X \sim N(\mu; \sigma_0^2)$$

$$\psi = P(X \geq c) = \int_c^\infty f(x, \mu; \sigma_0^2)\,dx$$

$$\hat{\mu}_n = \bar{X}_n$$ → Estimador máximo verosímil de μ

$$\hat{\psi}_n = \int_c^\infty \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{1}{2\sigma_0^2}(x - \bar{X}_n)^2\right)dx$$

$$\hat{\psi}_n = \psi(\hat{\mu}_n)$$

Newton-Raphson:

$X \sim t_1$ y param loc θ.

$$L(\theta) = \prod_{i=1}^{n} \frac{1}{\sqrt{\pi}\,\Gamma(1/2)} \frac{1}{(1+(x_i-\theta)^2)}$$

$$\ell(\theta) = cte + \sum_{i=1}^{n} \ln\left(\frac{1}{1+(x_i-\theta)^2}\right)$$

$$S(\theta) = \sum_{i=1}^{n} \frac{2(x_i-\theta)}{(1+(x_i-\theta)^2)} = 0$$

$$\ell''_\theta = H(\theta) = 2\sum_{i=1}^{n} \left[\frac{2(x_i-\theta)^2}{(1+(x_i-\theta)^2)^2} - \frac{1}{(1+(x_i-\theta)^2)}\right]$$

$\frac{\partial}{\partial\theta} S(\theta)$.

$\hat{\theta}_n^{(0)}$

$$\boxed{\hat{\theta}_n^{(k)} = \hat{\theta}_n^{(k-1)} - \frac{S(\hat{\theta}_n^{(k-1)})}{H(\hat{\theta}_n^{(k-1)})}}$$

Riesgo

Descomp ECM : W_n ; θ

$$ECM(W_n;\theta) = E\{(W_n-\theta)^2\}$$

$$= E\{[(W_n - E(W_n)) - (\theta - E(W_n))]^2\}$$

$$= E\{\underbrace{[\theta - E(W_n)]^2}_{(1)} - \underbrace{\cancel{2(W_n - E(W_n))(\theta - E(W_n))}}_{(2)} + \underbrace{[W_n - E(W_n)]^2}_{(3)}\}$$

$$= \underbrace{[\theta - E(W_n)]^2}_{Sesgo_\theta(W_n)^2} + \underbrace{E\{(W_n - E(W_n))^2\}}_{Var_\theta(W_n)}$$

$X \sim N(\mu;\sigma^2)$ $\qquad \hat{\mu}_n = \bar{X}_n \qquad \mu = E(X)$

$$ECM(\hat{\mu}_n;\mu) = \underbrace{[E(\bar{X}_n) - \mu]^2}_{0} + \underbrace{Var(\bar{X}_n)}_{\sigma^2/n}$$

$$ECM(\hat{\mu}_n; \mu) = \sigma^2/n$$