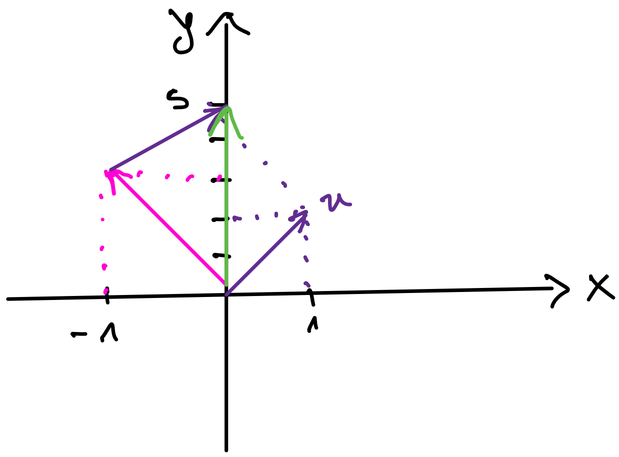
Clase 1 (11/3)

TP 1

Ejercicio 1

$u=(1,2),\ v=(-1,3),\ \omega=(-1,-2)$

a) $u+v = (1,2)+(-1,3) = (0,5)$



i) $u\,\frac{5}{4} + \frac{1}{2} v - \frac{3}{2} w =$

$\frac{5}{4}(1,2) + \frac{1}{2}(-1,3) - \frac{3}{2}(-1,-2)$

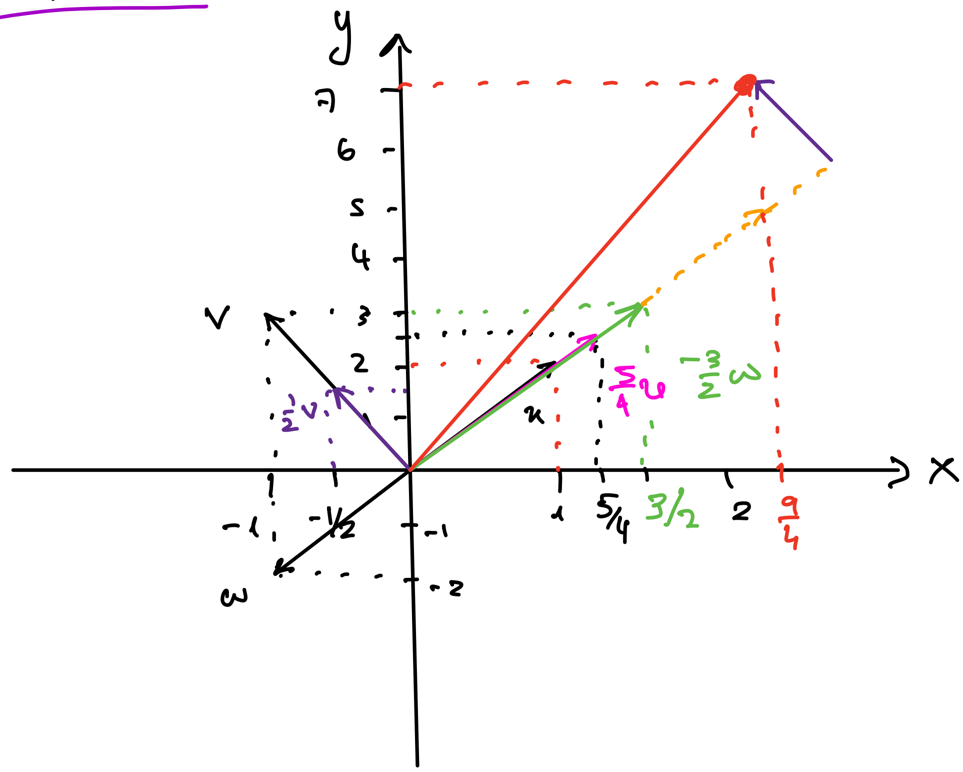
$= \left(\frac{5}{4}, \frac{5}{2}\right) + \left(-\frac{1}{2}, \frac{3}{2}\right) - \left(-\frac{3}{2}, -3\right)$

$= \left(\frac{5}{4}, \frac{5}{2}\right) + \left(-\frac{1}{2}, \frac{3}{2}\right) + \left(\frac{3}{2}, 3\right)$

$= \left(\frac{5}{4} - \frac{1}{2} + \frac{3}{2}, \frac{5}{2} + \frac{3}{2} + 3\right)$

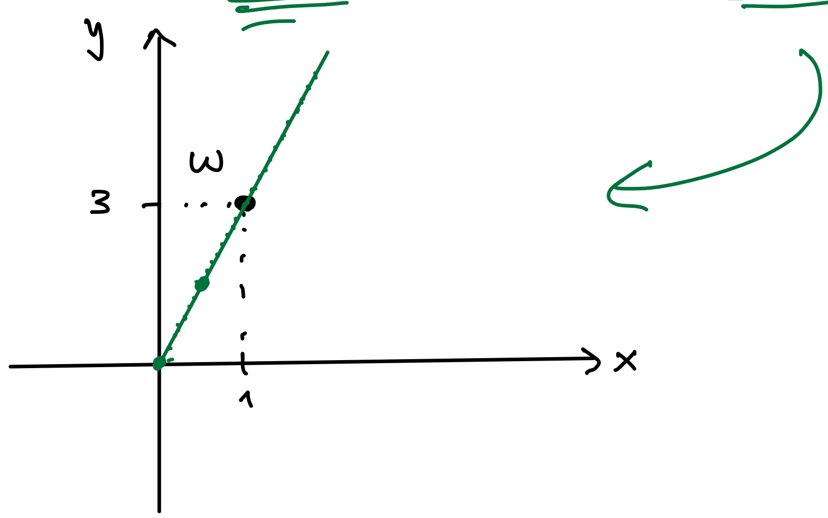
$= \left(\frac{5-2+6}{4}, \frac{5+3+6}{2}\right)$

$$= \left(\frac{9}{4}, 7\right)$$



Ejercicio 2 b) $w = (1, 3)$

$$\mathcal{L} = \{tw : t \in \mathbb{R}_{\geq 0}\} \Rightarrow tw = (t, 3t)$$



Ejercicio 4

Encontrar $x, y \,/\, v = w$

a) $v = (x, 3)$, $\quad w = (2, x+y)$

$$v = w \Rightarrow (x, 3) = (2, x+y) \Rightarrow \begin{cases} x = 2 \\ 3 = x + y \end{cases}$$

$$\Rightarrow 3 = 2 + y \Rightarrow \boxed{y = 1}$$

d) $v = x(2, y) = (2x, yx)$; $\quad w = y(1, -2) = (y, -2y)$

$$v = w \Rightarrow (2x, yx) = (y, -2y) \Rightarrow \begin{cases} 2x = y \\ yx = -2y \end{cases}$$

$$\Rightarrow (\cancel{2}x)\, x = -2(\cancel{2}x) \Rightarrow x^2 = -2x \Rightarrow x^2 + 2x = 0$$

$$\Rightarrow x(x+2) = 0$$

$$\Rightarrow x = 0 \lor x = -2$$

lo meto en

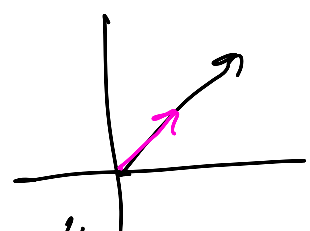
$$a \cdot b = 0 \Rightarrow a = 0 \lor b = 0$$

$x = 0 \Rightarrow y = 0$

$x = -2 \Rightarrow y = -4$

Ejercicio 5

a) $u = (-3, 1, -2, 4, -5)$



$$\|u\| \longrightarrow u' = \frac{u}{\|u\|} \Rightarrow \|u'\| = \left\| \frac{u}{\|u\|} \right\| = \frac{1}{\cancel{\|u\|}} \cancel{\|u\|}$$

$$\|u\| = \sqrt{(-3)^2 + 1^2 + (-2)^2 + 4^2 + (-5)^2}$$

$$= \sqrt{9+1+4+16+25}$$

$$= \sqrt{55}$$

$$u' = \frac{u}{\|u\|} = \frac{1}{\sqrt{55}}(-3, 1, -2, 4, -5)$$

$$= \left(\frac{-3}{\sqrt{55}}, \frac{1}{\sqrt{55}}, \frac{-2}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{-5}{\sqrt{55}} \right)$$

$$= \left(-3\,\frac{\sqrt{55}}{55}, \frac{\sqrt{55}}{55}, -2\,\frac{\sqrt{55}}{55}, \frac{4\sqrt{55}}{55}, -\frac{\sqrt{55}}{11} \right)$$

Ejercicio 6

Encontrar $k \in \mathbb{R}$

a) $u = (4, k) \wedge \underline{\underline{\|u\| = 5}}$

$$\|u\| = \sqrt{4^2 + k^2} = 5$$

$$\downarrow$$

$$4^2 + k^2 = 5^2$$

$$16 + k^2 = 25$$

$$k^2 = 9$$

$$k = \pm 3$$

Ejercicio 7

$v = (1, -2, 2),\ w = (2, 0, 3),\ z = (4, 4, 4)$

d) $(v \cdot z) + (w \cdot z)$

$$= \left((1,-2,2)\cdot \underline{(4,4,4)} + (2,0,3)\cdot \underline{(4,4,4)} \right) \text{ distr.}$$

$$= \left[(1,-2,2) + (2,0,3) \right] \cdot (4,4,4)$$

$$= (3,-2,5)\cdot(4,4,4)$$

$$= 3\cdot 4 + (-2)\cdot 4 + 5\cdot 4$$

$$= 12 - 8 + 20 = \underline{\underline{24}} \in \mathbb{R}$$

$$\cdot : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$$

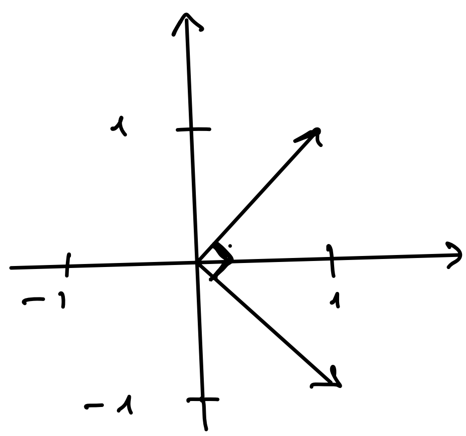
Ejercicio 8 Calcular el ángulo

a) $u = (1,1)$, $v = (1,-1)$

$$u\cdot v = (1,1)\cdot(1,-1) = 1\cdot 1 + (-1)\cdot 1 = 1 - 1 = \underline{\underline{0}}$$

$$\cos(\varphi) = \frac{u\cdot v}{\|u\|\,\|v\|} = 0 \Rightarrow \varphi = \arccos(0)$$

$$\underline{\underline{\varphi = \pi/2}}$$



$\Rightarrow$ ángulos ortogonales

b) $u = (3,-1,2)$, $v = (4,3,-1)$

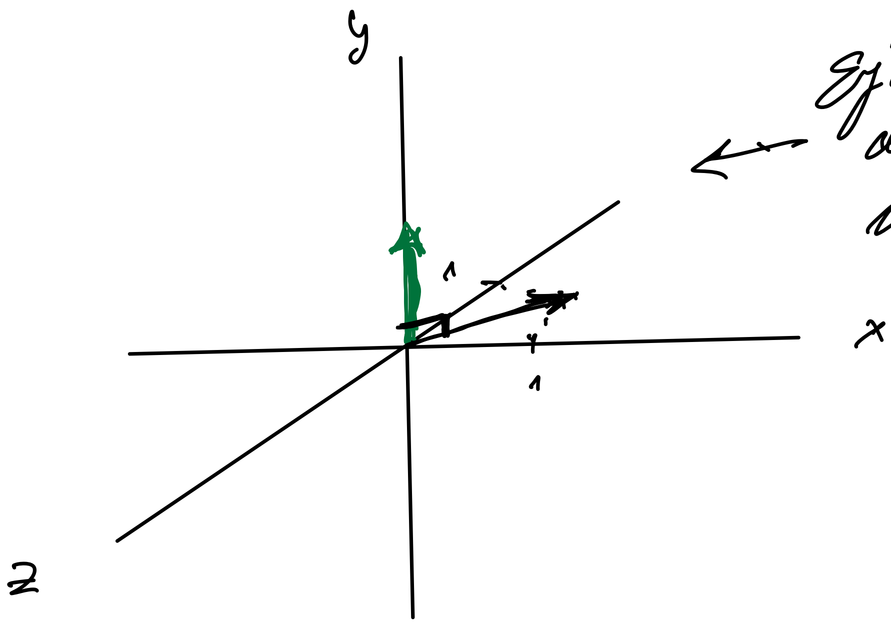
$$u\cdot v = 3\cdot 4 + (-1)\cdot 3 + 2\cdot(-1) = 12 - 3 - 2 = 7$$

$$\|u\| = \sqrt{3^2+(-1)^2+2^2} = \sqrt{9+1+4} = \sqrt{14}$$

$$\|v\| = \sqrt{4^2+3^2+(-1)^2} = \sqrt{16+9+1} = \sqrt{26}$$

$$\cos(\varphi) = \frac{u \cdot v}{\|u\|\,\|v\|} = \frac{7}{\sqrt{14}\sqrt{26}} = \frac{7}{\sqrt{390}}$$

$$\Rightarrow \quad \varphi = \arccos\left(\frac{7}{\sqrt{390}}\right) \neq \frac{\pi}{2} \rightarrow$$ no son ortogonales



Ejemplo de ortogonal en 3-D

Ejercicio 9 $\quad u = (1,-3,2),\ v = (2,-1,1)$

a) $\quad w = (1,7,-4)$

$\quad w = a u + b v$

$$\Rightarrow \quad (1,7,-4) = a(1,-3,2) + b(2,-1,1)$$

$$= (a,-3a,2a) + (2b,-b,b)$$

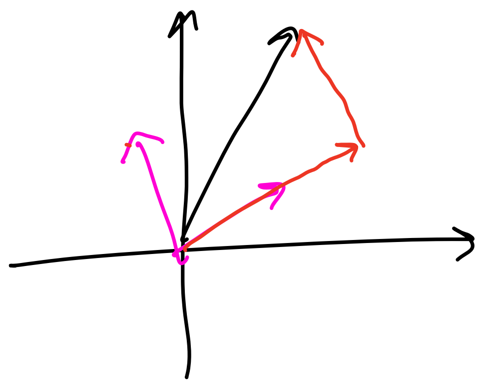
$$(1,7,-4) = (a+2b, -3a-b, 2a+b)$$

$\{$ $\qquad b=2$

$$\Rightarrow \begin{cases} 1 = a + 2b \longrightarrow a = 1 - 2b \longrightarrow a = 1 - 4 \Rightarrow a = -3 \\ 7 = -3a - b \longrightarrow 7 = -3(1-2b) - b = -3 + \underbrace{6b - b}_{5b} \\ -4 = 2a + b \qquad \Rightarrow 10 = 5b \Rightarrow b = 2 \end{cases}$$

$$\Downarrow$$

$$-4 = 2\cdot(-3) + 2 = -6 + 2 = -4 \checkmark$$



Ejercicio 5 normalizar

b) $v = (4, -2, -3, 8)$

$$\|v\| = \sqrt{4^2 + (-2)^2 + (-3)^2 + 8^2} =$$

$$= \sqrt{16 + 4 + 9 + 64}$$

$$= \sqrt{93}$$

$$\Rightarrow v' = \frac{v}{\|v\|} = \frac{1}{\sqrt{93}}(4, -2, -3, 8)$$

$$= \left(\frac{4}{\sqrt{93}}, \frac{-2}{\sqrt{93}}, \frac{-3}{\sqrt{93}}, \frac{8}{\sqrt{93}}\right)$$

c) $w = \left(\frac{1}{2}, \frac{2}{3}, -\frac{1}{4}\right)$

$$\|w\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{4}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{4}{9} + \frac{1}{16}}$$

$$= \sqrt{\frac{36+64+9}{144}} = \sqrt{\frac{109}{144}} = \frac{\sqrt{109}}{12}$$

$$\Rightarrow w' = \frac{w}{\|w\|} = \frac{12}{\sqrt{109}} \left(\frac{1}{2}, \frac{2}{3}, \frac{-1}{4} \right)$$

$$= \left(\frac{6}{\sqrt{109}}, \frac{8}{\sqrt{109}}, \frac{-3}{\sqrt{109}} \right) \quad //$$

Ejercicio 3

$u = (0,1,2)$, $v = (1,1,0)$
$w = (-1,1,1)$

f) $-v + \frac{2}{3} w = -(1,1,0) + \frac{2}{3}(-1,1,1)$

$$= (-1,-1,0) + \left(-\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$$

$$= \left(-1-\frac{2}{3}, -1+\frac{2}{3}, \frac{2}{3}\right)$$

$$= \left(\frac{-3-2}{3}, \frac{-3+2}{3}, \frac{2}{3}\right)$$

$$= \left(\frac{-5}{3}, \frac{-1}{3}, \frac{2}{3}\right)$$

b) $u+v+w = (0,1,2) + (1,1,0) + (-1,1,1)$

$$= (0+1+(-1), 1+1+1, 2+0+1)$$

$$= (0,3,3)$$

Ejercicio 6

d) $z = (1, k, -2, 5) \;/\; \|z\| = \sqrt{39}$

$$\|z\| = \sqrt{1^2 + k^2 + (-2)^2 + 5^2} = \sqrt{39}$$

$$\Rightarrow \sqrt{1 + k^2 + 4 + 25} = \sqrt{39}$$

$$\Rightarrow \quad \sqrt{30 + k^2} = \sqrt{39}$$

$$30 + k^2 = 39$$

$$k^2 = 9 \Longrightarrow \boxed{k = \pm 3}$$