Problem Set 0
Procesos Estacionarios y No Estacionarios

Ejercicio 1

Derive las funciones teóricas de autocorrelación y autocorrelación parcial de los siguientes procesos estocásticos:AR(1) ,MA(1) ,AR(2) ,MA(2) y ARMA$(1,1)$.

Explique como puede utilizar estas funciones para identificar el proceso estocástico que sigue una serie.

Ejercicio 2

Explique en que consiste la metodología de Box-Jenkins.

Ejercicio 3

Recuerde la definición del R^2 :

$$R^2 = 1 - \frac{\sum_{t=1}^{T} \widehat{\varepsilon}_t^2}{\sum_{t=1}^{T} (y_t - \overline{y})^2}$$

1. Para un estimador consistente de los parámetros del proceso $\{y_t\}_{t=-\infty}^{+\infty}$ el $p\lim R^2 = 1 - \frac{\sigma_\varepsilon^2}{\gamma_0}$. Calcule esta expresión cuando$\{y_t\}_{t=-\infty}^{+\infty}$ sigue un AR(1) y cuando sigue un MA(1) .
2. Argumente que comparar los R^2 de los modelos no puede ser un mecanismo válido para la selección de modelos.
3. Proponga algún criterio de selección de modelos. De acuerdo a estos, ¿cuándo un modelo es preferido a otro?

Ejercicio 4

Abra el archivo generatedata.wf1 (utilizando Eviews). Este archivo contiene cinco series generadas por computadora.

1. Utilice la ACF y PACF muestral para identificar cada uno de los procesos.
2. Utilice la metodología de Box-Jenkins para confirmar sus conjeturas del inciso (a) .

Ejercicio 5

Repita el inciso 4.a para la serie "returns" del archivo returns.wf1

Ejercicio 6

En este ejercicio vamos a modelar y forcastear el PBI de Estados Unidos. Utilice el archivo ej7.wf1 el cual contiene datos para el logaritmo del producto en términos reales.

Asumamos que $\ln(GNP)$ tiene un tendencia lineal determinística.

1. ¿Cómo especificaría el modelo para el logaritmo del PBI? Estime una tendencia lineal para la muestra.
2. Quite la tendencia a los datos de producto. ¿Qué tipo de serie tiene?
3. Identifique y estime el proceso correcto.
4. Compute un forecast dinámico para 2008Q1 en adelante para la serie sin la tendencia.
5. Compute un forecast dinámico para 2008Q1 en adelante para la serie original.
6. Calcule el RMSE, MAE y MAPE.

Time Series

Problem Set 2 - VAR

Problem 1 - Long-Run Multiplier

Consider the following bivariate system

$$\begin{cases} y_t = b_{10} - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt} \\ z_t = b_{20} - b_{22} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt} \end{cases}$$

where it is assumed that (1) both y_t and z_t are stationary; (2) ε_{yt} and ε_{zt} are white noise disturbances with standar deviations of σ_y and σ_z respectively, and $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{yt}\}$ are uncorrelated white noise disturbances.

a. Rewrite the system as an VAR(1) process:

$$W_t = A_0 + A_1 W_{t-1} + e_t$$

Here $W_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$; $A_0 = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix}$; $A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $e_t = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$

b. Find the expression for each element of A_0, A_1 and e_t.

c. Name the variance/covariance matrix of $\begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$, Δ. Compute Λ, the variance/covariance matrix of e_t. Compare Λ with Δ.

d. What is the condition required for stationarity?

e. Suppose that the condition found in (d) is met, rewritte the expression found in (a) as a vector MA(∞).

f. Use the moving average representation found in (e) to express W_t in terms of the $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{yt}\}$ secuences. Use this expression to find the instantaneous impact of a unit change in ε_{yt} on y_t. Find the expresión for the long-run multiplier.

g. Decompose the n-step-ahead forcast error variance of y_{t+n} into the proportions due to each shock ($\{\varepsilon_{yt}\}$ and $\{\varepsilon_{yt}\}$)

Problem 2 - IRF and LR Effects

Let y_t represent output and x_t, an economic policy instrument (for instance, money supply). Assume they evolve according to the following bivariate VAR(1) process:

$$\begin{aligned} \begin{bmatrix} y_t \\ x_t \end{bmatrix} &= \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} \\ \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} &= \begin{bmatrix} 1 & \theta \\ \phi & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \\ E\left\{ \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \right\} &= 0_2 \\ E\left\{ \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \begin{bmatrix} \varepsilon_{ys} \\ \varepsilon_{xs} \end{bmatrix}^T \right\} &= I(t=s)\Sigma_\varepsilon \\ 0 &< a_1 < 1 \end{aligned}$$

i) Under what conditions can u_{xt} be interpreted as a policy innovation?

ii) It can be showed that the impulse-response function on y_t of ε_{xt} is given by

$$\varphi_{xt}(0) = \theta$$
$$\varphi_{xt}(s) = a_1^{s-1}\left(a_1\theta + a_2\right) \qquad s \geq 1$$

But the parameter θ cannot be estimated from a sample, and so without further restrictions this function is not identified. Assume that policy shocks affect output with a one-period lag. What is the Impulse Response Function in that case?

iii) What about when policy shocks are neutral (ineffective) in the long-run?

v) And when policy is not responsive to contemporaneous output shocks?

Problem 3 - US Data

The VAR.wf1 data set contains quarterly U.S. data on the rate of price inflation (π_t), the unemployment rate (u_t), and the interest rate (R_t), specifically, the federal founds rate from the first quarter of 1960 to the fourth quarter of 2000.

a. Use this data set and estimate a trivariate VAR(4).

b. Do you have evidence that the estimated VAR is stable?

c. Is there evidence that the number of lags should be 4? That all lags should be included?

d. Suppose that de VAR(4) specification is correct. Execute a Granger Causality test. What are your conclusions?

e. Obtain the impulse-response functions.

f. Estimate the proportions of the n-step ahead forecast error variance for every variable. . For $n = 1, 4, 8, 12$

Problem 4 - Testing Rational Expectations Hypothesis

Review the lecture notes on Testing Rational expectations Hypothesis.

The E-Views Workfile "uncovered" contains quarterly series from Q3 1978 to Q3 2008 for: USD/GBP exchange rate, 3 month GBP libor rate and 3 month USD Treasury Bill rate.

Perform a hypothesis test to asses wether the uncovered interest rate parity holds.

Problem 5 - Supply and Demand Shocks

Read the paper "The Dynamic Effects of Agregate Demand and Supply Disturbances" by Blanchard and Quah (1989). As covered in the lecture notes, the authors offered a different approach to use structural information to identify the estimated reduced form VAR. We will follow the authors by recognizing the existence of supply and demand disturbances which both affect the output and the unemployment rate in the short-run, but the demand shocks does not affect the output in the long term.

In Eviews terminology, $C_{1,1} = 0$, where $C_{1,1}$ is the upper left-handed element of the $C_{k\times k}$ matrix given by $C = \hat{\Psi}_\infty A^{-1}B$, where matrix A and B are given by $A\epsilon_t = Bu_t$ (see Eviews guide II).

The E-Views Workfile "bc" contains quarterly series from Q2 1948 to Q4 2007 for US Output (in Billion Dollars) and Unemployment rate. Unemployment rate is measured monthly, this is a 3 month average.

i) Check wether GDP growth and average unemployment are stationary. Argue if this assumption is plausible.

ii) Estimate a VAR for GDP growth and average unemployment choosing the number of lags based on relevant criteria. The error term in the reduced form VAR will be a linear combination of supply and demand schocks.

iii) For the preferred VAR in part ii) compute impulse response functions and variance decomposition of forecasts for the two possible cholesky orderings.

iv) For the preferred VAR in part ii) estimate a structural var with the restriction that the long run effect of a demand schock in GDP growth is zero. Compare the results with the ones you obtained in iii)

Problem Set 2
Cointegración

Ejercicio 1

Asuma que el siguiente VAR(p) para n variado para el proceso $\{y_t\}_{t=-\infty}^{+\infty}$

$$y_t = c + \Phi_1 y_{t-1} + ... + \Phi_p y_{t-p} + \varepsilon_t$$

y asuma que el proceso contiene exactamente una raiz unitaria. Encuentre la representación VMA(∞) y la VECM.

Ejercicio 2

Considere el siguiente ejemplo

$$\begin{aligned} y_t + \beta x_t &= u_{1t} \\ y_t + \alpha x_t &= u_{2t} \end{aligned}$$

donde

$$\begin{aligned} u_{1t} &= 0.2u_{1t-1} + 0.8u_{1t-2} + \varepsilon_{1t} \\ u_{2t} &= \rho u_{2t-1} + 0.5u_{2t-2} + \varepsilon_{2t} \end{aligned}$$

1. ¿Cuál es el orden de integración de y_t y x_t?
2. Bajo que condiciones son y_t y x_t cointegradas?
3. Encuentre la representación MA y ECM (asumiendo que las variables cointegran)

Ejercicio 3

Abra el archivo pv.wf1.

1. Chequee si las variables precios y dividendos son integradas.
2. Regrese los real stock prices contra los real dividends y una constante. Chequee el orden de integración de los residuos. Según el modelo teórico ¿cuál debería ser el grado de integración?

Ejercicio 4

Para este ejercicio va a necesitar el archivo bond.wf1. El mismo contiene tasas de interés de 3,6 y 12 meses.

1. Encuentre el orden de integración de estas series.
2. Chequee si las variables cointegran.
3. Estime un ECM para estas variables.

Ejercicio 5

Para este ejercicio va a necesitar el archivo termduffie.wf1.

1. Estime un ECM para las tasas de interés. Recuerde que para ello primero tiene que encontrar el número de lags.
2. ¿Las variables cointegran?
3. ¿Cuántas relaciones de cointegración hay?
4. El modelo teórico nos da una relación especifica sobre los vectores de cointegración. La misma establece que la relación debería ser 1 -1. Testee esta restricción.

Time Series
Problem Set 3

Problem 1

Consider the standar linear regression model

$$\underbrace{y_t}_{1\times 1} = \underbrace{x_t^T}_{1\times k}\underbrace{\beta}_{k\times 1} + \underbrace{u_t}_{1\times 1}$$

for x_t a $(k \times 1)$ of explanatory variables. As usual assume that the explanatory variables are uncorrelated with the residual (u_t):

$$E(x_t u_t) = 0$$

a. Show that the Ordinary Least Squares estimator of β has the same expression that the Generalized Method of Moments (GMM) estimador of β

b. Find the asymptotic distribution of $\widehat{\beta}_{GMM}$. Suppose that

$$E\left[u_t u_{t-v} x_t x_t^T\right] = \begin{cases} \sigma^2 E(x_t x_t^T) \text{ si } v = 0 \\ \mathbf{0} \text{ si } v \neq 0 \end{cases}$$

c. What would happen if the assumption in (b) does not hold?

Problem 2

Consider the standar linear regression model

$$\underbrace{y_t}_{1\times 1} = \underbrace{x_t^T}_{1\times k}\underbrace{\beta}_{k\times 1} + \underbrace{u_t}_{1\times 1}$$

for x_t a $(k \times 1)$ of explanatory variables. Assume that

$$E(x_t u_t) \neq 0$$

Let z_t be a $(k \times 1)$ vector of valid instruments:

$$E(z_t u_t) = 0$$

a. Show that the GMM estimator of β is equal to the Instrumental Variables Estimator of β ($\widehat{\beta}_{GMM} = \widehat{\beta}_{IV}$)

b. Assume now that z_t is a $(r \times 1)$ vector with $r > k$. Show that the GMM estimator of β is equal to the Two Stage Least Square Estimator of β . ($\widehat{\beta}_{GMM} = \widehat{\beta}_{2SLS}$)

Problem 3

This exercise is taken form an example of Verbeek(2014) *'A guide to modern econometrics'* . Consider simple consumption-based asset pricing model. The conditions to maximize the utility of the representative agent are

$$\begin{aligned} E\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(1+r_{f,t+1}) \mid I_t^*\right] &= 1 \\ E\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(r_{j,t+1}-r_{f,t+1}) \mid I_t^*\right] &= 0 \text{ con } j=1,2,3..,10 \end{aligned}$$

where $r_{f,t}$ is the risklees asset return in t, C_t is the consumption level in t and $r_{j,t}$ are the returns of the atlernative risky assets in t.. I_t^* stands for the information set a time t.

Based on this conditions, the author estimates the unknow parameters β (intertemporal discount factor) and γ (risk aversion coefficient)

Based on this conditions, open apm.wf1 and estimate by GMM the unknow parameters β (intertemporal discount factor) and γ (risk aversion coeficient), using identity weighting matrix and sequential updating. Are these models valid under Hansen's J-Test Hypothesis?

Problem 4

1) Estimate the following model through GMM equation (not system):

$$0 = E\left[\beta\left(\frac{C_{t+2}}{C_t}\right)^{-\gamma} Z_t R_{t+2} - Z_t\right]$$

where the instruments are $Z_t = \left(1, \frac{C_t}{C_{t-2}}, R_t, \frac{H_t}{H_{t-2}}, \frac{W_t}{W_{t-2}}\right)$.

Use a bandwidth of 4 and simultaneous updating of weighting matrix and coefficients.

Hint: write the equation as an orthogonality condition between residuals and the instrumental variables.

2) Find the J-Statistic. Is the model correctly specified according to this sample?

3) Fix β as 0.9, 0.95 and 0.999. Estimate a new γ in each case and find the J-statistics.

4) Re-estimate (β, γ) now using the new list of instruments, $\bar{Z}_t = (Z_t, Z_{t-1}, Z_{t-2})$. Note that the constant should appear only once. Do you find any change in the results? Perform Hansen's test.

Problem 1

Download Engle, Lilien, Robins: 'Estimating Time Varying Risk Premia in the Term Structure: The Arch-M Model', Econometrica, 1987. We shall follow some of the estimations of the paper. Use the 'RP.WF1' Eviews file

1. Explain the general model and the application of excess yield holding.
2. Regress 'y' against a constant. Compare with the one estimated in the paper.
3. Evaluate the existence of ARCH effects through a correlogram and an heteroskedastic test.
4. Estimate an appropiate ARCH model. Is this specification correct when we take into account possible structural breaks in the DGP?
5. Estimate an ARCH-M. Specify the ARCH in mean as done in the paper
6. Compare the results with Figure 1
7. Which are the main differences between your modelizations and the one in the paper?

Problem 2

Using the e-views file named returns.wf1:

1. Estimate an AR(1) AR(6) with a constant.
2. Find the prefered GARCH(p; q) model for the series.
3. Explain wheter you find evidence of leverage effects.
4. Explain whether you think GARCH-M models are appropiate for this data.

Problem 3

Using the var_reg.wf1 workfile:

1. Create the series of log-differences for the S&P 500 and the difference of the EFFR (Effective Federal Funds Rate).

   Note: To do so, write in the command window:

   series dsp = log (sp) - log (sp (-1))

   series deffr = effr - effr (-1)

2. Limit the sample to 1/01/1985 - 7/31/1987.

3. Estimate an AR(1) model for dsp. What can you say about the residuals?

4. Is there evidence of ARCH effects?

5. Estimate a GARCH(1; 1) model. Do the new squared standarized residuals present autocorrelation?

6. Estimate a GARCH(1; 1) model with deffr as a variance regressor. Is it statistically significant?

7. Analyze the new squared residuals.

Problem 4

Using the workfile *rep.wf1*

1. Replicate the first three columns of Table 20.2 from 'Econometric Modelling with Time Series'. Martin, Hurn, Harris, Cambridge University Press, 2013.

2. Plot the estimated conditional variance with the GARCH(1; 1) model.

You may find tables and figures here:

http://www.cambridge.org/features/econmodelling/figuresandtables.htm

The table we need is

Problem 5

Open the workfile MVG.wf1.

Table 20.2

Maximum likelihood estimates of GARCH(1,1) and GARCH(0,1) = ARCH(1) models of equity returns. Standard errors in parentheses are based on the Hessian. The sample size is $T = 4952$.

Index	α_0	α_1	β_1	$T \ln L$	Unconditional Variance (Theoretical)	Unconditional Variance (Empirical)
FTSE	0.013	0.079	0.907	-6348.796	0.964	0.984
	(0.003)	(0.008)	(0.012)			
	0.740	0.255	0.000	-6824.029	0.993	0.984
	(0.020)	(0.023)				
DOW	0.009	0.051	0.940	-6316.263	0.975	0.919
	(0.002)	(0.006)	(0.008)			
	0.748	0.195	0.000	-6712.500	0.929	0.919
	(0.019)	(0.022)				
NIKKEI	0.026	0.088	0.903	-8187.337	2.728	1.924
	(0.005)	(0.008)	(0.008)			
	1.587	0.182		-8561.058	1.940	1.924
	(0.040)	(0.020)				

1. Create the variables in differences of the 3 months and 1 year US yield, both expressed in percentages (muultiplied by 100).

2. Estimate a diagonal BEKK.

3. Plot the conditional covariances and the correlation.

PS5 - Time Series

Problem 1

Using the GAUSS L-E-STAR, and quarterly american data contained in the files *cpiqnew*, *gdpq* and *tb3qnew*:

1. Estimate a LSTAR model of the interest rate with 4 lags and student's t distribution, with equal degrees of freedom for both states. Present the estimated coefficients in a table with its S.E and t-statistic.
2. Analyze the standardized residuals and the squared standardized residuals.
3. Plot the estimated threshold value and the mixing functions.
4. Repeat 1-3 but with student's t distribution and no constraints on d.o.f.
5. Repeat 1-3 but with gaussian distribution.

Problem 2

Using the file rate.wf1, estimate an LSTAR model with 4 lags in Eviews. Compare it to the one estimated in the preceding exercise. What are the differences?

Problem 3

Open the files L-E-STAR-SDT and ttstand22.

1. What model specification are we working with?

2. Close the ttstand22 file and run L-E-STAR-SDT, under the logistic specification, using 4 lags, and assuming student's t distribution with distinct d.o.f. in each state

3. Compare the results with the previous exercise

Problem 4

Read the paper from Dueker, Psaradakis, Sola and Spagnolo (2012). Using the Gauss file newthret, and quarterly american data contained in the files cpiqnew gdpq and tb3qnew, we will replicate columns 3 and 4 from Table 3.

1. Write down the SDC- STAR(4) and RC - STARX(4) specifications.

2. Estimate a SDC - STAR(4) model.

3. Plot the threshold, the mixing functions and the variables.

4. Estimate a RC - STARX(4) model and plot

Problem 1

1. Replicate table 1 from Hamilton(1989) paper

TABLE I

MAXIMUM LIKELIHOOD ESTIMATES OF PARAMETERS AND ASYMPTOTIC STANDARD ERRORS BASED ON DATA FOR U.S. REAL GNP, $t = 1952:\text{II}$ TO $1984:\text{IV}$

Parameter	Estimate	Standard error
α_1	1.522	0.2636
α_0	−0.3577	0.2651
p	0.9049	0.03740
q	0.7550	0.09656
σ	0.7690	0.06676
ϕ_1	0.014	0.120
σ_2	−0.058	0.137
ϕ_3	−0.247	0.107
ϕ_4	−0.213	0.110

2. Plot the filtered probabilities and the smoothed probabilities.

3. Find the business cycle dates. Do you arrive to the same conclusions as Hamilton?

4. Repeat 1-3 with Eviews, using the file *gnp_hamilton*

Problem 2

Using Argentina's monthly Industrial Production, s.a., period Jan-95 to Mar- 13:

1. Estimate the same model as in exercise 1.

2. Plot the probabilities and the real growth rates.

3. Find the business cycle dates.

Problem 3

Using the same data on US GNP as in exercise 1:

1. Estimate the same model as in exercise 1, but now assuming three states in the economy and one lag.

2. Plot the smoothed probabilities and the GNP real growth in a same graph.

3. Which S_t corresponds to each state (i.e recession, stagnation or expansion)?

4. Find the business cycle dates and compare with exercise 1

PS7

Time Series

Problem 1

We will work with Argentina's industrial production growth rate. The series IP was genereted as a log-difference multiplied by 100. Source: IADB. The series is seasonally adjusted according to the source.

1. Open the Eviews file "argipm.wf1" and analyze stationarity of "IP".
2. Generate de demeaned series for IP and estimate an AR(4) by OLS.
3. Estimate an AR(4) by means of a state-space representation. (Read Eviews Guide (chapter "State Space Models and the Kalman Filter") for further details)

Problem 2

We will work with an adaptation of an exercise obtained from "Econometric Modelling with Time Series". The model is based on Fama and Gibbons (1982):

1. Using the data contained in the file named *exante.wf1* generate an ex-post real interest rate and call it "epr". Compute the ex post real interest rate as $y_t = i_t - 1200 * (\log p_t - \log p_{t-12})$
2. Estimate a state space model where the state variable is the ex-ante real interest rate and the signal variable is the ex-post real interest rate. Assume the state variable follows an AR(1) process. Use as initial values $\phi_0 = 0{,}5$, $\mu_0 = 2{,}17$, $\sigma^2_{v0} = 16$, $\sigma^2_{w0} = 1$.
3. Compare the unconditional mean and variance of the ex ante real rate with the sample mean and variance of the ex post real rate

Problem 3

Excercise based on "Econometric Modelling with Time Series". Use the *"byclicle.wf1"* file. Many researchers are interested in analyzing the state of the business cycle. To this end, the US Department of Commerce publishes a coincident index. Stock and Watson (1991) developed a model in which the comovements of various macroeconomic variables have a common element that can be captured by a single latent variable. That is, using the Kalman Filter, they created their own coincident index, which could be compare with DOC's index. We will be working with data from Australia: GDP, Unemployment (un), Employment (em), Sales, Income (inc)and Production($prod$). We shall use the model

$$\varsigma_{t+1} = \phi_1 \varsigma_t + \phi_2 \varsigma_{t-1} + v_t \qquad v_t \sim N(0,1)$$
$$y_{it} = \theta_i \varsigma_t + w_{it} \qquad v_t \sim N(0, \sigma_i^2) \qquad i = 1,..,6$$

Note that $y_{it} = 100(\log(x_{i,t}) - \log(x_{i,t-12}))$ and were demeaned. Estimate the model above. Estimate the business cycle using the smoothed conditional mean $\varsigma_{t|T}$ and the alternative based on the coincident index ($bc_t = 100(log(coin_t) - log(coin_{t-12}))$). Compare the turnings points given by both estimators.
HINT: rescale $\varsigma_{t|T}$ to have the same sample variance as bc_t by multiplying $\varsigma_{t|T}$ by the ratio of the standard deviation of bc_t to the standard deviation of $\varsigma_{t|T}$

Problem 4

The file *datanew.wf1* contains US data for the period 1960q1 - 2018q1 . Use it to estimate the following equation

$$\log(\frac{m_t}{p_t y_t}) = \alpha_0 + \alpha_t i_t + \varepsilon_t$$
$$\alpha_t = \mu + \phi \alpha_{t-1} + u_t$$

Where α_t is the semi-elasticity of the demand for money which is assumed to be a time-varying parameter that follows an AR process.

1. Write the system as an State-Space with our usual notation.
2. Estimate the parameters and α_t using the Kalman Filter
3. Based on your results, comment on the stability of the semi elasticity of the demand for money.

PS8 - Markov Chain Monte Carlo
Time Series

Problem 1

In this exercise we will replicate the table 7.2 from Kim and Nelson "State - Space Models with Regime Switching". We will use the codes mle.ato and gbs.ato, which are in the folder *Ch7*. The model is given by

$$\begin{aligned} y_t &= \beta_0 + \beta_1 x_t + e_t \\ e_t &= \phi e_{t-1} + v_t, v_t \overset{\text{iid}}{\sim} N\left(0, \sigma^2\right) \\ x_t &= 0{,}5 x_{t-1} + w_t, w_t \overset{\text{iid}}{\sim} N(0,1) \end{aligned} \tag{1}$$

The data were generated using the parameters values $\beta_0 = 0$, $\beta_1 = 1$, $\phi = 0{,}5$ y $\sigma^2 = 1$. The file "gbs_ato.dta" contains the generated data.
Use Gibbs Sampling to obtain the posterior distributions. Report their mean, SD and median. Compare these results from those obtained from classical ML estimation.

Problem 2

Consider the application of Gibbs-sampling to Bayesian estimation of a univariate autoregression. The data set we employ is the real US GDP series for the period 1952:II-1995:III. Consider the following linear AR(4) model for the first differences of the log real GDP (that is, GDP growth):

$$\begin{aligned} y_t =& \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \phi_4 y_{t-4} + \varepsilon_t \\ &\varepsilon_t \sim N(0, \sigma^2) \end{aligned}$$

Perform Bayesian analysis of this model by means of Gibbs - Sampling methods. For this purpose, use the code *gbs_ar4* . Report mean, standard error and median of the posterior distributions. Compare this results with MLE estimates (use the code *mle_ar4* for this purpose). All the codes are in the "*Ch7*" folder.

Problem 3

Using the real US GDP series for the period 1952:II-1995:III, perform Bayesian analysis of the following Markov-Switching model for GDP growth:

$$\begin{aligned} y_t =& \mu_{S_t} + e_t \\ e_t \sim& N(0, \sigma^2) \\ \mu_{S_t} =& \mu_0 + \mu_1 S_t \\ P(S_t = 1|S_{t-1} = 1) = q,& \quad P(S_t = 0|S_{t-1} = 0) = p \end{aligned}$$

That is, we have a two state MS model, with no AR terms, and gaussian errors with constant variance. Use the code *gibs_ms0* (which can be found in folder "Ch9") to perform the analysis. In particular, report standard error and median of the posterior distributions. Plot also the estimated probabilities of being in each state.