

PS8 - Markov Chain Monte Carlo

Time Series

Problem 1

In this exercise we will replicate the table 7.2 from Kim and Nelson “State - Space Models with Regime Switching”. We will use the codes `mle.ato` and `gbs.ato`, which are in the folder *Ch7*. The model is given by

$$\begin{aligned}y_t &= \beta_0 + \beta_1 x_t + e_t \\e_t &= \phi e_{t-1} + v_t, v_t \stackrel{\text{iid}}{\sim} N(0, \sigma^2) \\x_t &= 0,5x_{t-1} + w_t, w_t \stackrel{\text{iid}}{\sim} N(0, 1)\end{aligned}\tag{1}$$

The data were generated using the parameters values $\beta_0 = 0$, $\beta_1 = 1$, $\phi = 0,5$ y $\sigma^2 = 1$. The file “`gbs.ato.dta`” contains the generated data.

Use Gibbs Sampling to obtain the posterior distributions. Report their mean, SD and median. Compare these results from those obtained from classical ML estimation.

Problem 2

Consider the application of Gibbs-sampling to Bayesian estimation of a univariate autoregression. The data set we employ is the real US GDP series for the period 1952:II-1995:III. Consider the following linear AR(4) model for the first differences of the log real GDP (that is, GDP growth):

$$\begin{aligned}y_t &= \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \phi_4 y_{t-4} + \varepsilon_t \\ \varepsilon_t &\sim N(0, \sigma^2)\end{aligned}$$

Perform Bayesian analysis of this model by means of Gibbs - Sampling methods. For this purpose, use the code `gbs.ar4`. Report mean, standard error and median of the posterior distributions. Compare this results with MLE estimates (use the code `mle.ar4` for this purpose). All the codes are in the “*Ch7*” folder.

Problem 3

Using the real US GDP series for the period 1952:II-1995:III, perform Bayesian analysis of the following Markov-Switching model for GDP growth:

$$y_t = \mu_{S_t} + e_t$$

$$e_t \sim N(0, \sigma^2)$$

$$\mu_{S_t} = \mu_0 + \mu_1 S_t$$

$$P(S_t = 1 | S_{t-1} = 1) = q, \quad P(S_t = 0 | S_{t-1} = 0) = p$$

That is, we have a two state MS model, with no AR terms, and gaussian errors with constant variance. Use the code *gibs.ms0* (which can be found in folder “Ch9”) to perform the analysis. In particular, report standard error and median of the posterior distributions. Plot also the estimated probabilities of being in each state.