Time Series

Problem Set 2 - Answer Key

Problem 1 - Long-Run Multiplier

$$\begin{aligned} y_t &= b_{10} - b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt} \\ z_t &= b_{20} - b_{22}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt} \end{aligned}$$

Using matrix notation

$$\underbrace{\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}}_{\equiv B_1} \underbrace{\begin{bmatrix} y_t \\ z_t \end{bmatrix}}_{\equiv w_t} = \begin{bmatrix} y_t + b_{12}z_t \\ z_t + b_{22}y_t \end{bmatrix} = \underbrace{\begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}}_{\equiv B_0} + \underbrace{\begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}}_{\equiv \Gamma} \underbrace{\begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix}}_{\equiv w_{t-1}} + \underbrace{\begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}}_{\equiv \varepsilon_t}$$

$$W_t = \underbrace{B_1^{-1}B_0}_{\equiv A_0} + \underbrace{B_1^{-1}\Gamma}_{\equiv A_1} W_{t-1} + \underbrace{B_1^{-1}\varepsilon_t}_{\equiv e_t}$$

b.

$A_0 = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} = \frac{1}{1-b_{12}b_{21}} \begin{bmatrix} b_{10} - b_{21}b_{20} \\ -b_{12}b_{10} + b_{20} \end{bmatrix}$

$A_1 = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} = \frac{1}{1-b_{12}b_{21}} \begin{bmatrix} \gamma_{11} - b_{21}\gamma_{21} & \gamma_{12} - b_{12}\gamma_{12} \\ -b_{12}\gamma_{11} + \gamma_{21} & -b_{12}\gamma_{12} + \gamma_{22} \end{bmatrix}$

$e_t = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} = \frac{1}{1-b_{12}b_{21}} \begin{bmatrix} \varepsilon_{yt} - b_{21}\varepsilon_{zt} \\ \varepsilon_{zt} - b_{12}\varepsilon_{yt} \end{bmatrix} \equiv \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$

c. Since ε_{yt} and ε_{yt} are independent $\Delta = \begin{bmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix}$

Lets now find Λ

$$E\left[e_t\right] = E\left[\frac{1}{1-b_{12}b_{21}} \times \begin{bmatrix} \varepsilon_{yt} - b_{21}\varepsilon_{zt} \\ \varepsilon_{zt} - b_{12}\varepsilon_{yt} \end{bmatrix}\right] = \frac{1}{1-b_{12}b_{21}} \times \begin{bmatrix} E\left[\varepsilon_{yt}\right] - b_{21}E\left[\varepsilon_{zt}\right] \\ E\left[\varepsilon_{zt}\right] - b_{12}E\left[\varepsilon_{yt}\right] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

To do it, we find the covariance matrix of e_t

$$\Lambda = E(e_t e_t') = E[B_1^{-1}\varepsilon_t \varepsilon'(B_1^{-1})'] = B_1^{-1}E[\varepsilon_t \varepsilon_t'](B_1^{-1})'$$

But we know that $E[\varepsilon_t \varepsilon_t'] = \Delta = diag(\sigma_y^2, \sigma_z^2)$. Therefore:

$$\Lambda = B_1^{-1}\Delta(B_1^{-1})'$$

Writing the terms explicitly

$$\Lambda = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix} \begin{bmatrix} 1 & b_{21} \\ b_{12} & 1 \end{bmatrix}^{-1}$$

$$\Lambda = \frac{1}{(1-b_{12}b_{21})^2}\begin{bmatrix} 1 & -b_{21} \\ -b_{12} & 1 \end{bmatrix}\begin{bmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix}\begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix}$$

$$\Lambda = \frac{1}{(1-b_{12}b_{21})^2}\begin{bmatrix} 1 & -b_{21} \\ -b_{12} & 1 \end{bmatrix}\begin{bmatrix} \sigma_y^2 & -b_{12}\sigma_y^2 \\ -b_{21}\sigma_z^2 & \sigma_z^2 \end{bmatrix}$$

$$\Lambda = \frac{1}{\left(1-b_{12}b_{21}\right)^2}\begin{bmatrix} \sigma_y^2 + b_{21}^2\sigma_z^2 & -\left(b_{12}\sigma_y^2 + b_{21}\sigma_z^2\right) \\ -\left(b_{12}\sigma_y^2 + b_{21}\sigma_z^2\right) & \sigma_z^2 + b_{12}^2\sigma_y^2 \end{bmatrix}$$

Note that Δ is diagonal whereas Λ is not. The fact that we can't estimate the structural form directly, but only the reduced form derived in a), makes the residuals in the estimable equation to be related.

d. The process W_t will be stationary if the solutions of the following equation $|I - xA_1| = 0$ are such that $|x_1| > 1$ and $|x_2| > 1$. That is, we need the eigenvalues of A_1 to be all strictly less than one in absolute value.

e.

The procedure here is analogous to that with univariate time series: we rewrite the system using the lag operator, and then we operate.

$$\begin{aligned} W_t &= A_0 + A_1 W_{t-1} + e_t \\ E\left[W_t\right] &= A_0 + A_1 E\left[W_{t-1}\right] \implies \mu = \left[I - A_1\right]^{-1} A_0. \end{aligned}$$

We operate to write the system in term of deviations from the unconditional mean. Then we use the lag operator.

$$W_t = [I - A_1][I - A_1]^{-1}A_0 + A_1 W_{t-1} + e_t$$
$$W_t - [I - A_1]^{-1}A_0 = A_1[W_{t-1} - [I - A_1]^{-1}A_0] + e_t$$

Define $\tilde{W}_t = W_t - [I - A_1]^{-1}A_0$ (the deviation from the long-run mean). Then our system can be written as:

$$\tilde{W}_t = A_1 \tilde{W_{t-1}} + e_t$$

Using the lag operator

$$\tilde{W}_t = A_1 L \tilde{W}_t + e_t$$
$$[I - A_1 L]\tilde{W}_t = e_t$$
$$\tilde{W}_t = [I - A_1 L]^{-1} e_t$$

But $[I - A_1 L]^{-1} = \sum_{i=0}^{\infty}(A_1 L)^i$. Therefore

$$\tilde{W}_t = \sum_{i=0}^{\infty}(A_1 L)^i e_t = \sum_{i=0}^{\infty}(A_1)^i e_{t-i} \tag{1}$$

Replacing $\tilde{W}_t$, we get

$$W_t = \mu + \sum_{i=0}^{\infty}(A_1)^i e_{t-i} \tag{2}$$

Where $\mu = [I - A_1]^{-1} A_0$ is the unconditional mean.

Another way is to proceed recursively.

$$\begin{aligned} W_t &= A_0 + A_1 W_{t-1} + e_t \\ W_t &= A_0 + A_1 (A_0 + A_1 W_{t-2} + e_{t-1}) + e_t \\ W_t &= A_0 + A_1 A_0 + A_1^2 W_{t-2} + A_1 e_{t-1} + e_t \\ W_t &= A_0 + A_1 A_0 + A_1^2 (A_0 + A_1 W_{t-3} + e_{t-2}) + A_1 e_{t-1} + e_t \\ W_t &= A_0 + A_1 A_0 + A_1^2 A_0 + A_1^3 W_{t-3} + A_1^2 e_{t-2} + A_1 e_{t-1} + e_t \end{aligned}$$

Recursively

$$W_t = \left[\sum_{i=0}^{n} A_1^i\right] A_0 + A_1^{n+1} \times W_{t-(n+1)} + \left[\sum_{i=0}^{n} A_1^i e_{t-i}\right]$$

If $n \to \infty$:

$$W_t = \mu + \left[\sum_{i=0}^{\infty} A_1^i e_{t-i}\right]$$

since $\lim_{n\to\infty} \left[\sum_{i=0}^{n} A_1^i\right] = [I - A_1]^{-1}$.

f) Using that $e_t = B_1^{-1}\varepsilon_t$

$$\begin{aligned} W_t &= \mu + \left[\sum_{i=0}^{\infty} A_1^i\right] e_{t-i} \\ W_t &= \begin{bmatrix} \mu_y \\ \mu_z \end{bmatrix} + \sum_{i=0}^{\infty} A_1^i \frac{1}{1 - b_{12}b_{21}} \begin{bmatrix} 1 & -b_{21} \\ -b_{12} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt-i} \\ \varepsilon_{zt-i} \end{bmatrix} \\ W_t &= \mu + \sum_{i=0}^{\infty} \phi(i) \begin{bmatrix} \varepsilon_{yt-i} \\ \varepsilon_{zt-i} \end{bmatrix} \end{aligned}$$

Where $\phi(i) = A_1^i B_1^{-1}$ This implies that the instantaneous impact of a unit change in ε_{yt} and ε_{zt} on y_t and z_t are

$$\begin{aligned} \frac{\partial y_t}{\partial \varepsilon_{yt}} &= \phi_{11}(0) \\ \frac{\partial y_t}{\partial \varepsilon_{zt}} &= \phi_{12}(0) \\ \frac{\partial z_t}{\partial \varepsilon_{yt}} &= \phi_{21}(0) \\ \frac{\partial z_t}{\partial \varepsilon_{zt}} &= \phi_{22}(0) \end{aligned}$$

The long-term multiplier (LTM) is given by

$$\begin{aligned} \sum_{i=0}^{\infty} \phi_{11}(i) &= LTM\ of\ \varepsilon_{yt}\ on\ y \\ \sum_{i=0}^{\infty} \phi_{12}(i) &= LTM\ of\ \varepsilon_{zt}\ on\ y \\ \sum_{i=0}^{\infty} \phi_{21}(i) &= LTM\ of\ \varepsilon_{yt}\ on\ z \\ \sum_{i=0}^{\infty} \phi_{22}(i) &= LTM\ of\ \varepsilon_{zt}\ on\ z \end{aligned}$$

g.

$$W_t = A_0 + A_1 W_{t-1} + e_t.$$

The best prediction of W_{t+1}, give the information available in t

$$E_t\left[W_{t+1}\right] = A_0 + A_1 W_t$$

The one step ahead forcast error (FE_1) is:

$$EP_1 = W_{t+1} - E_t\left[W_{t+1}\right] = e_{t+1}$$

To find the two step ahead forcast error (FE_1) we first find

$$W_{t+2} = A_0 + A_1 W_{t+1} + e_{t+2} = A_0 + A_1\left(A_0 + A_1 W_t + e_{t+1}\right) + e_{t+2} = \left(I + A_1\right) A_0 + A_1^2 W_t + A_1 e_{t+1} + e_{t+2}$$

$$EP_2 = A_1 e_{t+1} + e_{t+2}$$

Recursively,

$$\begin{aligned} W_{t+n} &= \left[\sum_{i=0}^{n-1} A_1^i\right] A_0 + A_1^n W_t + \sum_{i=0}^{n-1} A_1^i e_{t+n-i} \\ EP_n &= \sum_{i=0}^{n-1} A_1^i e_{t+n-i} = \sum_{i=0}^{n-1} \phi(i) \begin{bmatrix} \varepsilon_{y,t+n-i} \\ \varepsilon_{z,t+n-i} \end{bmatrix} \end{aligned}$$

We need to find the proportion of the variance of EP_n $\left(\sigma_y(n)^2\right)$ that corresponds to each shock

$$\begin{aligned} \sigma_y(n)^2 &= \phi_{11}(0)\sigma_y^2 + \phi_{12}(0)\sigma_z^2 + \phi_{11}(1)\sigma_y^2 + \phi_{12}(1)\sigma_z^2 + ... + \phi_{11}(n-1)\sigma_y^2 + \phi_{12}(n-1)\sigma_z^2 \\ &= \left\{\phi_{11}(0) + \phi_{11}(1) + ... + \phi_{11}(n-1)\right\}\sigma_y^2 + \left\{\phi_{12}(0) + \phi_{12}(1) + ... + \phi_{12}(n-1)\right\}\sigma_z^2 \end{aligned}$$

Finally

$$1 = \frac{\left[\sum_{i=0}^{n-1} \phi_{11}(i)\right]\sigma_y^2}{\sigma_y(n)^2} + \frac{\left[\sum_{i=0}^{n-1} \phi_{12}(i)\right]\sigma_z^2}{\sigma_y(n)^2}$$

Problem 2 - IRF and LR Effects

Let y_t represent output and x_t, an economic policy instrument (for instance, money supply). Assume they evolve according to the following bivariate VAR(1) process:

$$\begin{aligned} \begin{bmatrix} y_t \\ x_t \end{bmatrix} &= \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} \\ \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} &= \begin{bmatrix} 1 & \theta \\ \phi & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \\ E\left\{\begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix}\right\} &= 0_2 \\ E\left\{\begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \begin{bmatrix} \varepsilon_{ys} \\ \varepsilon_{xs} \end{bmatrix}^T\right\} &= I(t=s)\Sigma_\varepsilon \\ 0 &< a_1 < 1 \end{aligned}$$

i) The true policy innovation is actually ε_{xt} rather than u_{xt}.

$u_{xt} = \phi\varepsilon_{yt} + \varepsilon_{xt}$ is actually a linear combination of policy and activity innovations. If $u_{xt} = \varepsilon_{xt}$ for all t then u_{xt} would represent policy innovations. This restriction is equivalent to the linear restriction $\phi = 0$:

$$u_{xt} = \varepsilon_{xt} \Leftrightarrow \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} = \begin{bmatrix} 1 & \theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix}$$

ii) This hypothesis is equivalent to the following linear restriction:

$$\begin{aligned} \varphi_{xt}(0) &= 0 \\ \theta &= 0 \end{aligned}$$

Now,

$$\begin{aligned} \begin{bmatrix} y_t \\ x_t \end{bmatrix} &= \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} \\ \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ \phi & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} = \begin{bmatrix} \varepsilon_{yt} \\ \phi\varepsilon_{yt} + \varepsilon_{xt} \end{bmatrix} \\ &\Rightarrow \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} = \begin{bmatrix} \varepsilon_{yt} \\ \phi u_{yt} + \varepsilon_{xt} \end{bmatrix} \\ \varphi_{xt}(0) &= 0 \\ \varphi_{xt}(s) &= a_1^{s-1} a_2 = \psi_{xt}(s) \quad s \geq 1 \end{aligned}$$

which is exactly identified.

iii) This second hypothesis is about the long-run multiplier:

$$\begin{aligned} \varphi_{xt}^{LR} &= \sum_{s=0}^{\infty} \varphi_{xt}(s) \\ \varphi_{xt}^{LR} &= \theta + \sum_{s=1}^{\infty} a_1^{s-1} (a_1\theta + a_2) \\ \varphi_{xt}^{LR} &= \theta + (a_1\theta + a_2) \sum_{s=1}^{\infty} a_1^{s-1} \\ \varphi_{xt}^{LR} &= \theta + (a_1\theta + a_2) \sum_{s=0}^{\infty} a_1^{s} \\ \varphi_{xt}^{LR} &= \theta + \frac{a_1\theta + a_2}{1 - a_1} \Leftrightarrow 0 < a_1 < 1 \end{aligned}$$

And can be formulated as:

$$\begin{aligned} \varphi_{xt}^{LR} &= 0 \\ \theta + \frac{a_1\theta + a_2}{1 - a_1} &= 0 \\ \frac{a_1\theta + a_2}{1 - a_1} &= -\theta \\ a_1\theta + a_2 &= -(1 - a_1)\theta = -\theta + \theta a_1 \\ &\Rightarrow a_2 = -\theta \end{aligned}$$

Now, we have:

$$
\begin{aligned}
\begin{bmatrix} y_t \\ x_t \end{bmatrix} &= \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} \\
\begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} &= \begin{bmatrix} 1 & -a_2 \\ \phi & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} = \begin{bmatrix} \varepsilon_{yt} - a_2\varepsilon_{xt} \\ \phi\varepsilon_{yt} + \varepsilon_{xt} \end{bmatrix} \\
&\Rightarrow \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} = \begin{bmatrix} \varepsilon_{yt} - a_2\varepsilon_{xt} \\ \phi\varepsilon_{yt} + \varepsilon_{xt} \end{bmatrix} \\
u_{xt} &= \phi\varepsilon_{yt} + \varepsilon_{xt} = \phi\left(u_{yt} + a_2\varepsilon_{xt}\right) + \varepsilon_{xt} \\
u_{xt} &= \phi u_{yt} + (1 + \phi a_2)\varepsilon_{xt} \\
u_{xt} &= \phi u_{yt} + \nu_{xt} \\
\varphi_{xt}(0) &= \theta = -a_2 \\
\varphi_{xt}(s) &= a_1^{s-1}\left(a_1\theta + a_2\right) = a_1^{s-1}\left(a_2 - a_1 a_2\right) \\
&= a_1^{s-1}\left(1 - a_1\right) a_2
\end{aligned}
$$

Again, this function is identified.

v) This last hypothesis is equivalent to saying that the policy shock is an innovation:

$$
\begin{aligned}
u_{xt} &= \varepsilon_{xt} \Leftrightarrow \phi = 0 \\
\begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} &= \begin{bmatrix} 1 & \theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix}
\end{aligned}
$$

Under this hypothesis, θ can be estimated in two-stages by means of:

$$\widehat{u}_{yt} = \theta \widehat{u}_{xt} + \nu_t$$

Let $\widehat{\theta}$ be this two-stage estimator. Then,

$$
\begin{aligned}
\widehat{\varphi}_{xt}(0) &= \widehat{\theta} \\
\widehat{\varphi}_{xt}(s) &= a_1^{s-1}\left(a_1\widehat{\theta} + a_2\right)
\end{aligned}
$$

Problem 3 - US Data

The VAR.wf1 data set contains quarterly U.S. data on the rate of price inflation (π_t), the unemployment rate (u_t), and the interest rate (R_t), specifically, the federal founds rate from the first quarter of 1960 to the fourth quarter of 2000.

a. Use this data set and estimate a trivariate VAR(4).

b. Do you have evidence that the estimated VAR is stable?

c. Is there evidence that the number of lags should be 4? That all lags should be included?

d. Suppose that de VAR(4) specification is correct. Execute a Granger Causality test. What are your conclusions?

e. Obtain the impulse-response functions.

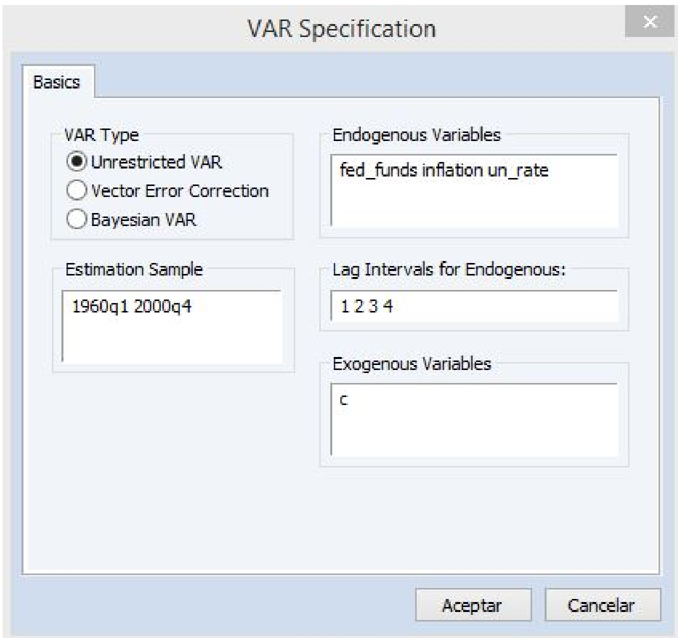
f. Estimate the proportions of the n-step ahead forecast error variance for every variable. . For $n = 1, 4, 8, 12$

Solution

The following results are obtain using E-views 9.

a. Use this data set and estimate a trivariate VAR(4).

To do so, we first select the series in Eviews. Then we right click on them. We choose the option 'open → as VAR'. We get the following window:



We change the lags to 4. We get the estimation of the VAR:

How do we read this? In the first image (left side), the coeficient estimated in the cell (FED_FUNDS(-2),INFLATION) corresponds to the coeficient of the second lag of the Fed funds variable in the inflation equation.

For the image on the right side:

'Determinant resid covariance' is given by the following expression

$$\left|\widehat{\Omega}\right| = \det \frac{1}{T}\sum_t \widehat{\epsilon}_t \widehat{\epsilon}_t'$$

And 'Determinant resid covariance (dof adj.)' is given by the following expression

$$\left|\widehat{\Omega}\right| = \det \frac{1}{T-p}\sum_t \widehat{\epsilon}_t \widehat{\epsilon}_t'$$

Where p is the number of parameters per equation in the VAR

The Akaike information criteria (AIC) and Schwarz criteria (SC) are calculated by

$$\begin{aligned} AIC &= -\frac{2l}{T} + \frac{2n}{T} \\ SC &= -\frac{2l}{T} + \frac{n \ln T}{T} \end{aligned}$$

Vector Autoregression Estimates
Date: 03/20/20 Time: 10:40
Sample: 1960Q1 2000Q4
Included observations: 164
Standard errors in ( ) & t-statistics in [ ]

	FED_FUNDS	INFLATION	UN_RATE
FED_FUNDS(-1)	0.951822	0.145892	-0.003278
	(0.08946)	(0.09961)	(0.02431)
	[10.6402]	[1.46469]	[-0.13483]
FED_FUNDS(-2)	-0.377919	-0.130586	0.051549
	(0.11959)	(0.13316)	(0.03250)
	[-3.16023]	[-0.98070]	[1.58624]
FED_FUNDS(-3)	0.339691	0.033849	-0.021843
	(0.11751)	(0.13084)	(0.03193)
	[2.89081]	[0.25871]	[-0.68403]
FED_FUNDS(-4)	0.016384	-0.043354	0.006579
	(0.08971)	(0.09989)	(0.02438)
	[0.18262]	[-0.43402]	[0.26986]
INFLATION(-1)	0.071722	0.628822	0.034913
	(0.07224)	(0.08043)	(0.01963)
	[0.99288]	[7.81790]	[1.77853]
INFLATION(-2)	0.218889	0.057046	-0.020791
	(0.08373)	(0.09323)	(0.02275)
	[2.61414]	[0.61186]	[-0.91372]
INFLATION(-3)	-0.094985	0.124316	0.013631
	(0.08483)	(0.09446)	(0.02305)
	[-1.11970]	[1.31612]	[0.59129]
INFLATION(-4)	-0.023874	0.185934	-0.023574
	(0.07621)	(0.08486)	(0.02071)
	[-0.31327]	[2.19118]	[-1.13832]

R-squared	0.929802	0.856851	0.976944
Adj. R-squared	0.924223	0.845475	0.975112
Sum sq. resids	115.9785	143.7920	8.564735
S.E. equation	0.876396	0.975841	0.238160
F-statistic	166.6716	75.32036	533.1944
Log likelihood	-204.2961	-221.9230	9.375563
Akaike AIC	2.649952	2.864915	0.044200
Schwarz SC	2.895673	3.110636	0.289922
Mean dependent	6.590061	3.876840	5.959756
S.D. dependent	3.183704	2.482443	1.509639

Determinant resid covariance (dof adj.)	0.032520
Determinant resid covariance	0.025383
Log likelihood	-396.8777
Akaike information criterion	5.315582
Schwarz criterion	6.052745

Where l is the value of the log likelihood function (assuming multivariate normal distribution)

$$\begin{aligned}
l &= -\frac{T}{2}\left[k\left(1+\ln 2\pi\right)+\ln\left|\widehat{\Omega}\right|\right] \\
n &= k(d+pk) \\
n &= \text{Total number of estimated parameters} \\
k &= \text{Number of endogenous variables} \\
d &= \text{Number of exogenous variables} \\
p &= \text{Number of Lags}
\end{aligned}$$

b. Do you have evidence that the estimated VAR is stable?

After the estimation click on:
View/Lag Structure/AR Roots Table
View/Lag Structure/AR Roots Graph (see graph on the following page)

There is evidence that the VAR is stable: no eigenvalue is grater than one.

c. Is there evidence that the number of lags should be 4? That all lags should be included?

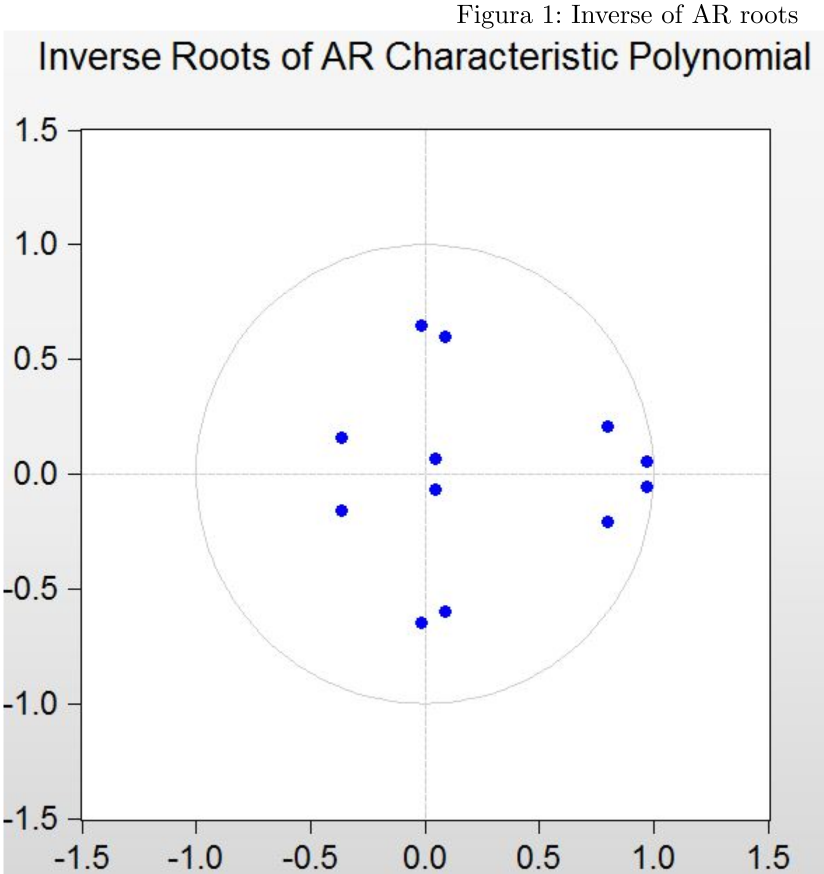
After the estimation click on:

Roots of Characteristic Polynomial
Endogenous variables: FED_FUNDS INFLATION U...
Exogenous variables: C
Lag specification: 1 2 3 4
Date: 03/20/20 Time: 10:46

Root	Modulus
0.968998 - 0.054265i	0.970516
0.968998 + 0.054265i	0.970516
0.797153 - 0.207849i	0.823805
0.797153 + 0.207849i	0.823805
-0.016504 - 0.649848i	0.650057
-0.016504 + 0.649848i	0.650057
0.086889 - 0.599927i	0.606187
0.086889 + 0.599927i	0.606187
-0.361531 - 0.157693i	0.394426
-0.361531 + 0.157693i	0.394426
0.048367 - 0.068255i	0.083654
0.048367 + 0.068255i	0.083654

No root lies outside the unit circle.
VAR satisfies the stability condition.

Figura 1: Inverse of AR roots



View/Lag Structure/Lag Length Criteria (view Figure 2)

LR Is the result of the sequential modified likelihood ratio test.Starting from the maximun lag, test the hypothesis that the coefficients on lag l are jointly zero using the χ^2 statistic.

Figura 2: Lag length criteria

VAR Lag Order Selection Criteria
Endogenous variables: FED_FUNDS INFLATION UN_RATE
Exogenous variables: C
Date: 03/20/20 Time: 10:51
Sample: 1960Q1 2000Q4
Included observations: 164

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-1046.822	NA	72.89951	12.80271	12.85942	12.82573
1	-469.5361	1126.412	0.071275	5.872392	6.099211	5.964472
2	-423.6145	87.92304	0.045441	5.422128	5.819062*	5.583269
3	-403.7559	37.29541	0.039817	5.289706	5.856755	5.519907*
4	-396.8777	12.66595	0.040883	5.315582	6.052745	5.614843
5	-390.5333	11.45098	0.042265	5.347967	6.255245	5.716287
6	-371.9818	32.80443	0.037666	5.231485	6.308878	5.668866
7	-367.2503	8.193613	0.039747	5.283540	6.531047	5.789981
8	-357.8853	15.87476	0.039662	5.279089	6.696711	5.854590
9	-344.3915	22.38003	0.037658	5.224286	6.812023	5.868847
10	-333.5120	17.64598	0.036940	5.201366	6.959217	5.914987
11	-320.9297	19.94755*	0.035520*	5.157679*	7.085645	5.940360
12	-315.0014	9.181549	0.037076	5.195140	7.293220	6.046881

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

$$LR = (T - m)\left[\ln|\Omega_{l-1}| - \ln|\Omega_l|\right] \sim \chi^2_{(k^2)}$$

where m is the number of parameters per equation under the alternative. This criteria chooses the eleven lags.

FPE Final Prediction Error Information Criterion

$$FPE = \left|\widehat{\Omega}\right| \frac{1+\frac{n}{T}}{1-\frac{n}{T}}$$

This Criteria chooses a structure with eleven lags.

The AIC criteria chooses a structure with 11 lags, meanwhile the SC criteria chooses a structure with two lags.

HQ Hannan-Quinn Informtation Criterion

$$HQ = T\ln\left|\widehat{\Omega}\right| + 2n\ln(\ln T)$$

The HQ criteria chooses a structure with three lags.

How do we know what criterion to use? See Luktepohl (2007), *New introduction to Multiple Times Series Analysis*, Chapter 4, for an extensive review on information criteria. It's recommendable to make the selection of which criterion to use **before** the estimation of the VAR. One posible variable to take into account is sample size. This is important due to the fact that AIC and FPE **are not consistent** (that is, they choose a wrong order of the VAR with positive probability when the sample size grows to infinity). Given this fact, it may be better to use HQ or SC when the sample size is large.
Another important thing to have into account is what the model is going to be used for: AIC and FPE are

based on forecasting error. Therefore, the order selected by these criteria could be better for forecasting even if the model is misspecified.

d. Suppose that de VAR(4) specification is correct. Execute a Granger Causality test. What are your conclusions?

After the estimation click on:
View/LagStructure/GrangerCausality
The results are given in Figure 3

Figura 3: Granger causality tests

VAR Granger Causality/Block Exogeneity Wald Tests
Date: 03/20/20 Time: 11:14
Sample: 1960Q1 2000Q4
Included observations: 164

Dependent variable: FED_FUNDS

Excluded	Chi-sq	df	Prob.
INFLATION	23.93924	4	0.0001
UN_RATE	30.95839	4	0.0000
All	43.47565	8	0.0000

Dependent variable: INFLATION

Excluded	Chi-sq	df	Prob.
FED_FUNDS	2.160230	4	0.7063
UN_RATE	14.58041	4	0.0057
All	29.78474	8	0.0002

Dependent variable: UN_RATE

Excluded	Chi-sq	df	Prob.
FED_FUNDS	15.19879	4	0.0043
INFLATION	3.860157	4	0.4253
All	34.74778	8	0.0000

Now, what are we testing in this case? Keep in mind that Granger causality is a different concept from what we normally understand as 'causality'. See lecture notes for an example (stock prices and dividends). In this case, we are only testing if the coefficients of lagged values of other endogeneous variables are jointly different form zero. For example, in our estimation, a VAR(4), the equation for Fed Funds rate looks like:

$$R_t = a_0 + \sum_{i=1}^{4} \phi_{u,i} u_{t-i} + \sum_{i=1}^{4} \phi_{\pi,i} \pi_{t-i} + \nu_t$$

Where $\phi_{u,i}$ is the coefficient of the i-th lag of unemployment in the interest rate equation, whereas $\phi_{\pi,i}$ the coefficient of the i-th lag of inflation.

In the first set of tests (the ones with 'FED_FUNDS' as dependant variable), we are working with the mentioned equation. The H_0 being tested are:

- When the excluded variable is INFLATION, we are testing $H_0 : \phi_{\pi,i} = 0 \quad \forall\, i$
- When the excluded variable is UN_RATE, we are testing $H_0 : \phi_{u,i} = 0 \quad \forall\, i$
- When the excluded variable are all, we are testing $H_0 : \phi_{u,i} = 0 \wedge \phi_{\pi,i} = 0 \quad \forall i$

This are all tests of Granger causation: if, for example, $\phi_{\pi,i} = 0 \quad \forall\, i$ holds, then no lagged value of inflation helps to explain the interest rate. Were this the case, then we would say that inflation does not Granger cause interest rate.

The same logic applies to the other two equations

Actual interpretation of results. We see that

- In the interest rate equation, we reject all null hypothesis being tested. Therefore, we say that **both unemployment and inflation Granger cause the Fed Funds Rate**
- In the inflation equation, we see that we can't reject the null of the interest rate Granger causing inflation. Nevertheless, we reject the null for unemployment. Therefore, we conclude that, in ur estimations **unemployment Granger causes inflation**, whereas **there is no strong statistical evidence of interest rate Granger causing inflation**
- In the unemployment equation, we see that *interest rate Granger causes unemployment*, whereas **there is no statistical evidence of inflation Granger causing unemployment**

e. Obtain the impulse-response functions.

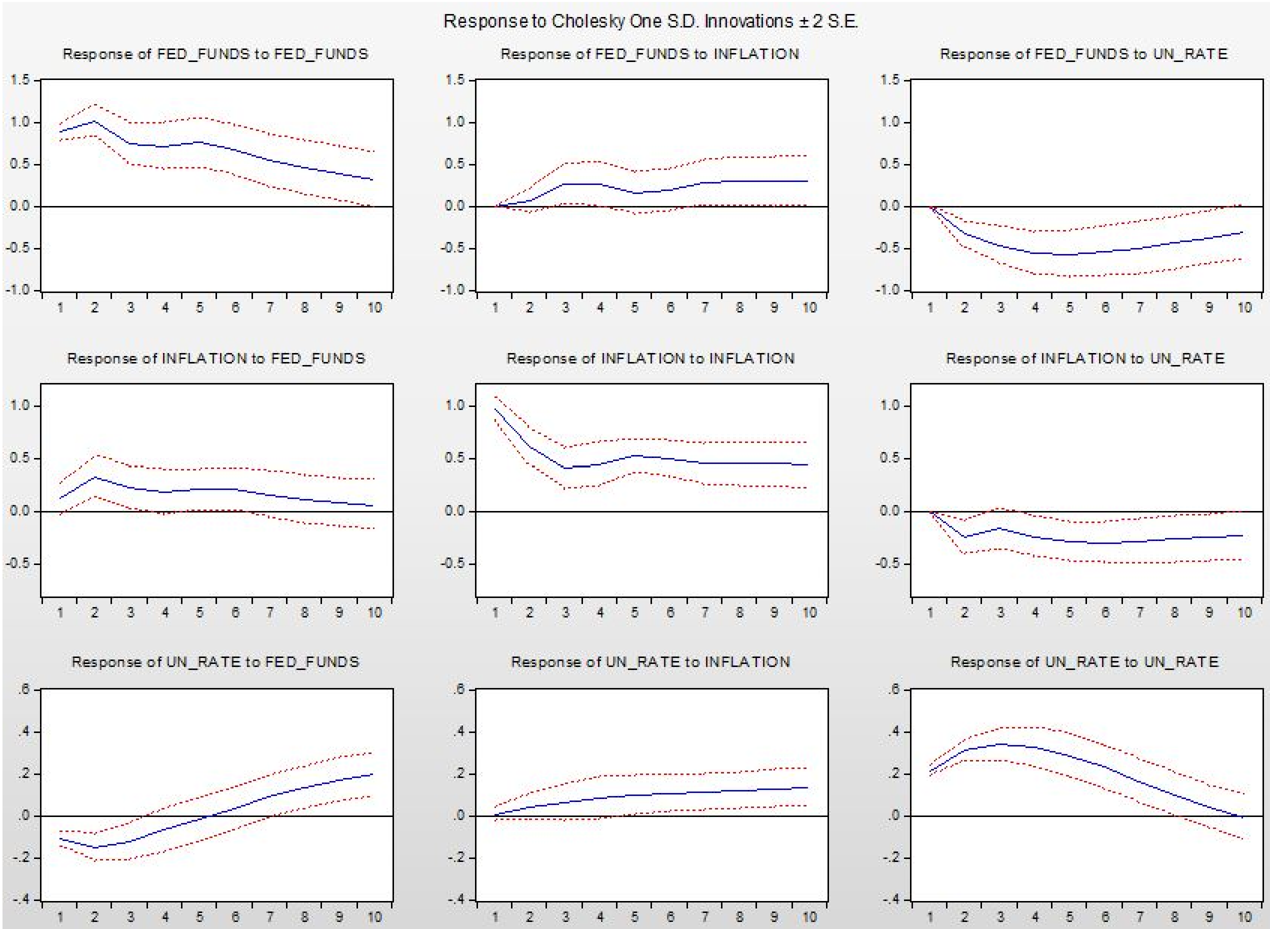
After the estimation click on:
View/ Impulse Response . The results are plotted on Figure 4
IMPORTANT: The impulse response functions being plotted are the responses to an **orthogonalized** shock. To see why this is important, see Lecture Notes. But, making long things short, the problem here is that errors in the equations tend to be correlated. Therefore, a typical impulse response function (that is, to simulate the system when there is a shock in only one equation in time t, and no other shock in the system (both in other equations and other time periods)) has an statiscal problem: in reality, shocks in one variable are asociated with shocks in other varibles. This problem arises because we can't estimate directly structural errors (which could be assumed to be uncorelated, see Excersises 1 and 2), but only reduced form ones, which tend to be different lineal combinations of structural errors. Eviews 9 User Guide II explains this in a short, clear way (page 631).
The important point to be made here is that impulse response functions, when calculated using this decomposition, **change when the order of the variables change**. This is explained in Lecture Notes. In the ordening used, the first variable has immediate impact on the other variables, whereas the second variable has immediate impact on the third one but not in the second one, and so on. Several orderings should be tried and/or reported keeping in mind what different orderings mean.

The impulse responses plotted used the ordering that we were using (for no reason at all). If we wanted to try a different order , such as *Unemployment, inflation, interest rate* [1], we can change this in the '*impulse definition Cholesky ordening* box.
The results are on Figure 6
f. Estimate the proportions of the n-step ahead forecast error variance for every variable. . For $n = 1, 4, 8, 12$

After the estimation click on:
View/ Variance Decomposition/ Table
Again, we have to select the ordering. We use the same as in the last example. The results are on Figure 8

[1] for example, we could think that unemployment shocks are real shocks uncorrelated with anything, therefore we put it first. Then, inflation shocks may have an immediate impact on interest rate.

Figura 4: Impulse response



Problem 4 - Testing Rational Expectations Hypothesis

The E-Views Workfile 'uncovered.WF1' contains quarterly series from Q3 1978 to Q3 2008 for: USD/GBP exchange rate, 3 month GBP libor rate and 3 month USD Treasury Bill rate. Perform a hypothesis test to asses wether the uncovered interest rate parity holds.

Solution

Let i_t, i_i^* dep_{t+1} be the interest rate on the 3 month USD t bill, the 3 month libor rate in GBP and USD/GBP percent change from t to $t+1$. Under risk neutrality, the following must hold

$$i_t - i_i^* = E_t(dep_{t+1})$$

To test this hypothesis, we estimate a VAR for both process, $i_t - i_i^*$ and dep_{t+1}. As stated in the lecture notes, rational expectations models impose nonlinear cross equation restrictions beetween the parameters of the model. This restrictions can be tested using a Likelihood ratio test (or a Wald test; remember that are asymptotically equivalent) under the null hypothesis that the model is true. It is important to correctly specify de Lag structure of the VAR model. If not the test will not be completely valid.

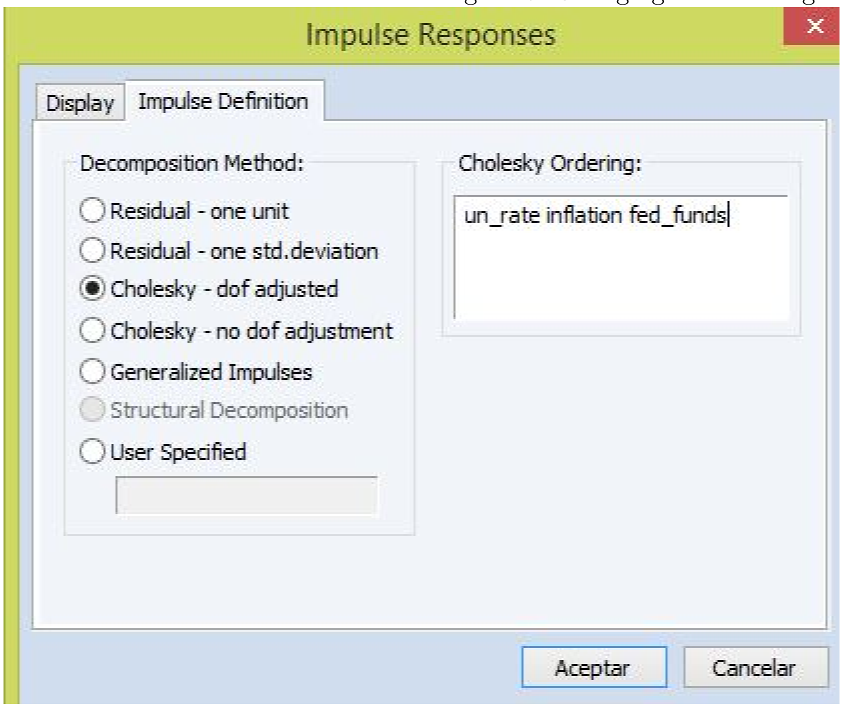
To find the order of the VAR we begin guessing a VAR of order one. Again we are working "from specific to general". Let's name $i_t - i_i^*$ as "dif"[2]

To estimate this VAR, go to "Quick", "Estimate VAR", then type 1 in the "lags intervals" and then specify the "endogenous" series which in this case are: dev dif.

Estimation results of the VAR(1)are in Figure 10

[2]We generate this in Eviews with *quick _ Generate Series...* then we type in the box: *dif = tbill - libor*

Figura 5: Changing the ordering



The order of the VAR has to be inferred using the selection criteria. Go to view/lag structure/lag length criteria. The resuts are on Figure 11

So the VAR(1) model is the preferred model for every selection criterion. So we keep working with the VAR(1). A word of caution. Although some of the estimated parameters in the VAR(1) are not statistically different from zero, it is not correct to reestimate the model without them and then test the expectations hypothesis. If we eliminate this parameters we are adding another restriction. So, we have to work with all parameters although some of them are not statistically different from zero.

The VAR (1) model can be written as

$$\begin{bmatrix} i_t - i_i^* \\ dep_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} i_{t-1} - i_{t-1}^* \\ dep_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

Let's define $x_t = i_t - i_i^*$ and $y_t = dep_t$. Remember that in our estimated model some parameters where not statistically different from zero. By the expectations hypothesis

$$x_t = E_t\left(y_{t+1}\right)$$

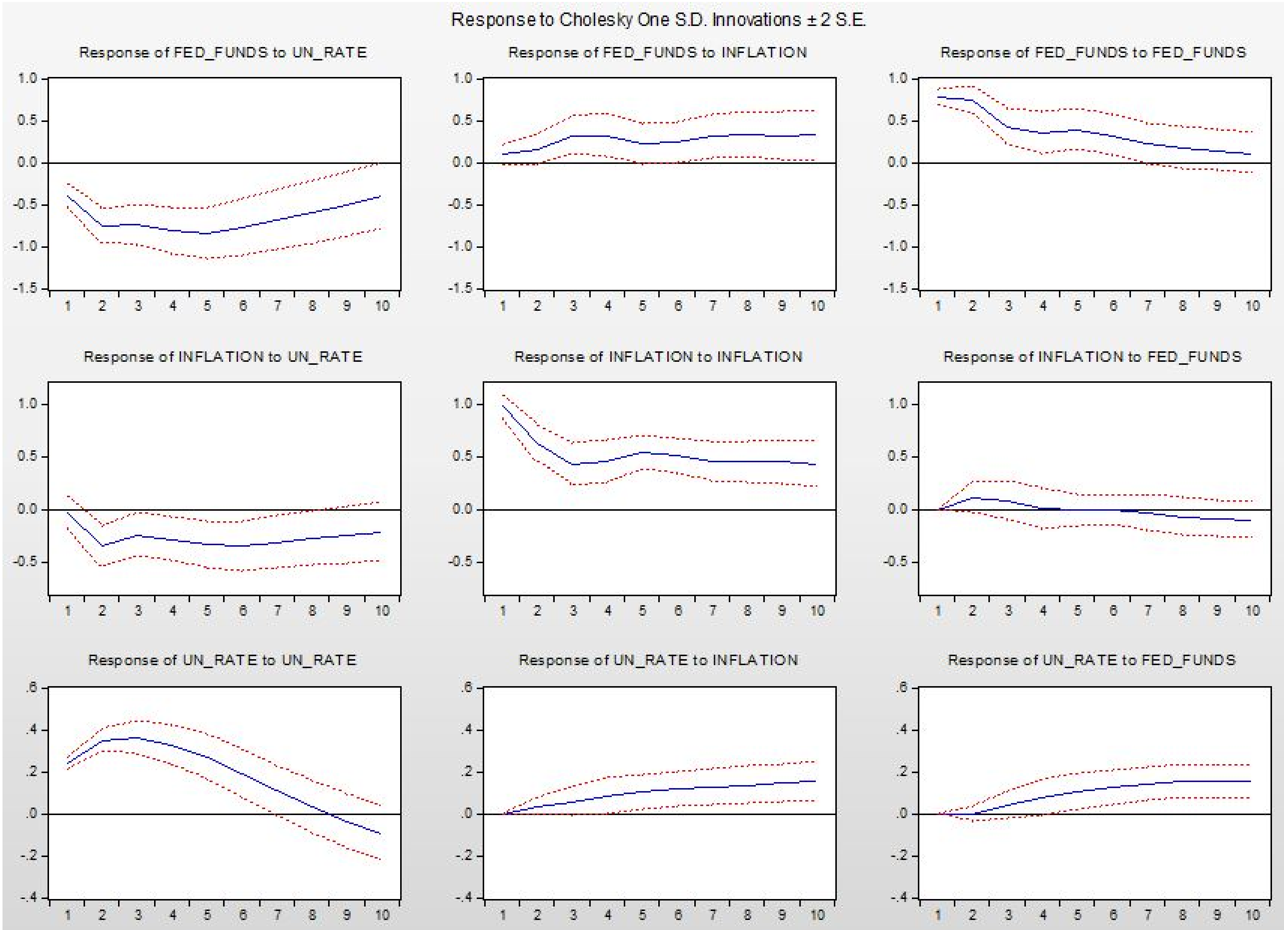
and by the law of iterated expectations

$$E_{t-1}(x_t) = E_{t-1}\left(y_{t+1}\right)$$

In terms of the bivariate VAR(1) that we estimated, the last equation can be expressed as

$$\begin{aligned} E_{t-1}(x_t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} E_{t-1}\left\{ \begin{bmatrix} x_t \\ y_t \end{bmatrix} \right\} \\ &= \begin{bmatrix} 1 & 0 \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + A \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} \right\} \end{aligned}$$

Figura 6: IR with different ordering



and

$$
\begin{aligned}
E_{t-1}(y_{t+1}) &= \begin{bmatrix} 0 & 1 \end{bmatrix} E_{t-1}\left\{ \begin{bmatrix} x_{t+1} \\ y_{t+1} \end{bmatrix} \right\} \\
&= \begin{bmatrix} 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + A \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + A^2 \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} \right\}
\end{aligned}
$$

So the expectations hypothesis, implies that the following must hold

$$
\begin{bmatrix} 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + A \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + A^2 \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} \right\} = \begin{bmatrix} 1 & 0 \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + A \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} \right\}
$$

that yields a pair of equations

$$
\begin{bmatrix} 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + A \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \right\} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}
$$

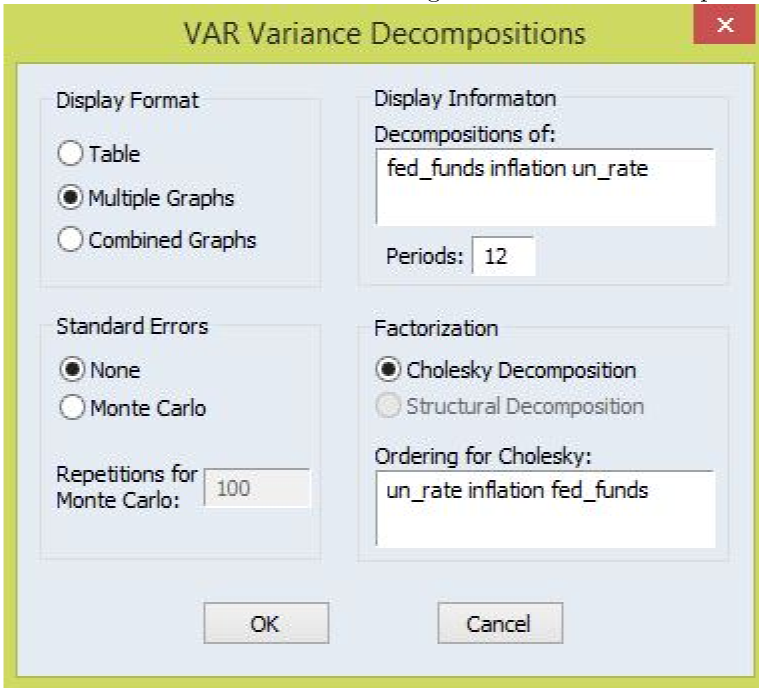
$$
\begin{bmatrix} 0 & 1 \end{bmatrix} A^2 \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} A \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix}
$$

and we arrive to the three restrictions that have to be tested

$$
c_2 + c_1 a_{21} + c_2 a_{22} = c_1
$$

$$
\begin{aligned}
a_{11}a_{21} + a_{21}a_{22} &= a_{11} \\
a_{12}a_{21} + a_{22}^2 &= a_{12}
\end{aligned}
$$

Figura 7: Variance decomposition box



Now to empirically test this restrictions, once you estimated de VAR(1) in Eviews go to 'Proc/Make a System/Order By variable". We get the screen on Figure 12.

Then estimate (hitting the 'estimate' button on the bar) by OLS and you will get the same result of estimation than in the estimation output at the beginning of the exercise (see Lecture Notes and Hamilton on this. The key is that conditional maximum likelihood estimation of a gaussian VAR is precisely OLS). The result of the estimation is given in Figure 13

Then go to View/Coefficient Diagnostics/ Wald Coefficient test and introduce the 3 restrictions above. Remember that

$$\begin{aligned} c(1) &= a_{22} \\ c(2) &= a_{21} \\ c(3) &= c_2 \\ c(4) &= a_{12} \\ c(5) &= a_{11} \\ c(6) &= c_1 \end{aligned}$$

so the restrictions that we have to introduce are

$$\begin{aligned} c(3) + c(6)c(2) + c(3)c(1) = c(6) \\ c(5)c(2) + c(2)c(1) &= c(5) \\ c(4)c(2) + c(1)^2 &= c(4) \end{aligned}$$

The output of the test is shown in Figure 14

The probability of the data sample under the Null of the Expectations Hypothesis is approximately zero. So we reject the null. One of the reasons for rejecting the null, could be, the existence of a constant or time varying risk premium.

Appendix to Exercise 9

Figura 8: IR with different ordering

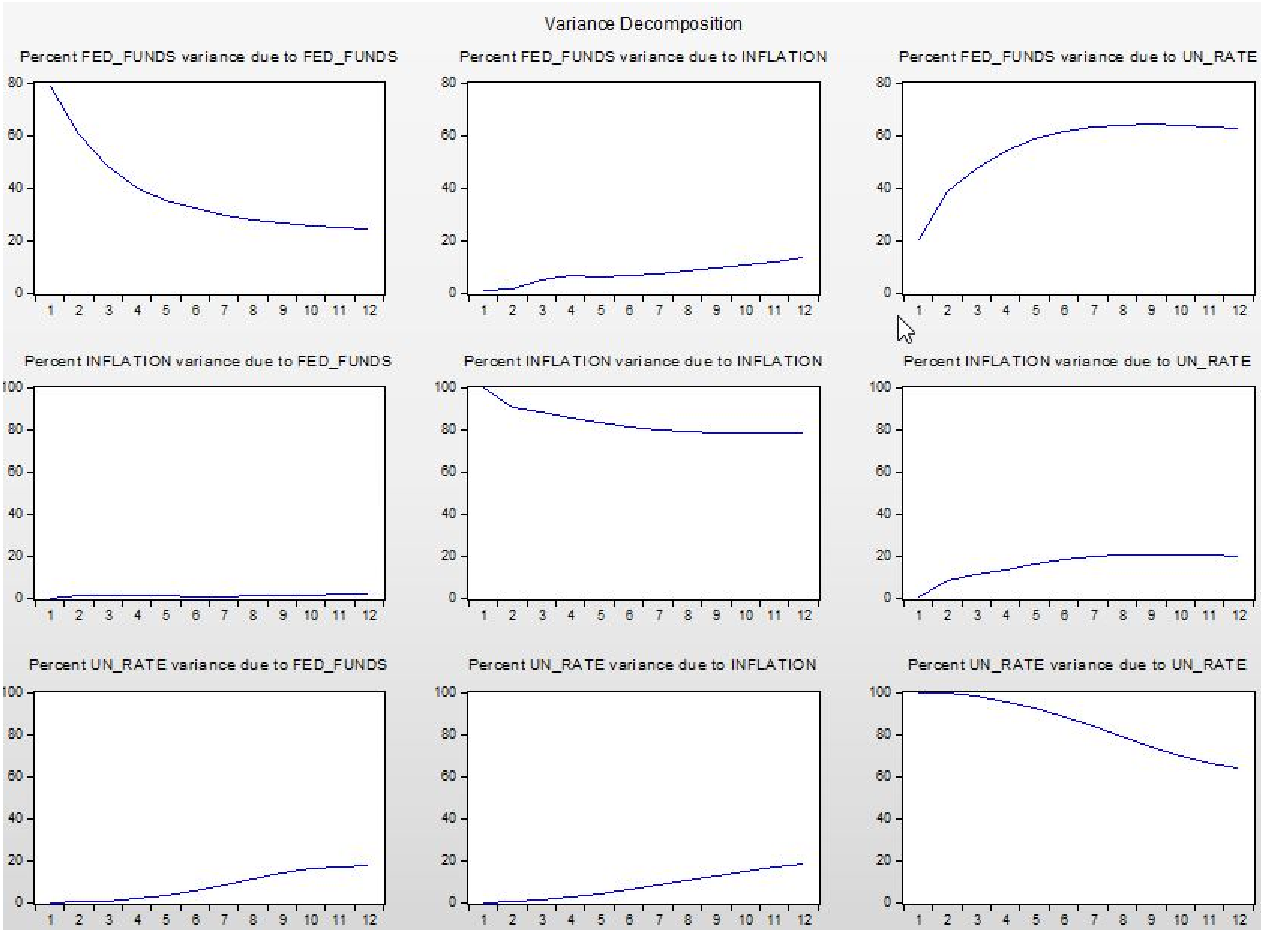


Figura 9: Estimating a VAR(1)

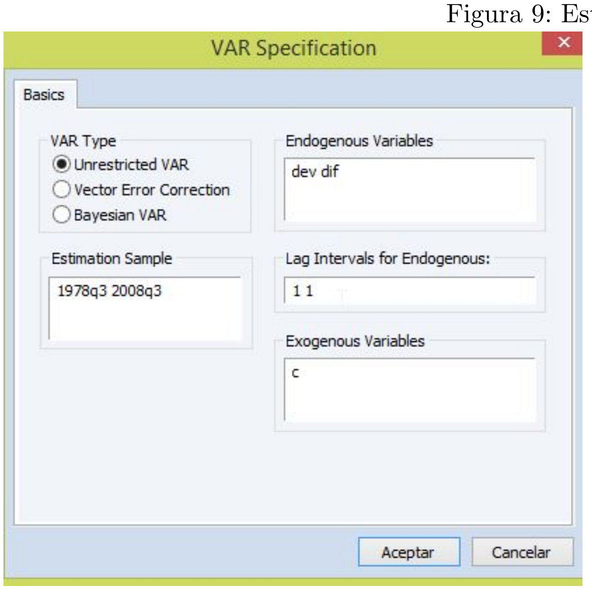


Figura 10: Estimating a VAR(1):results

Vector Autoregression Estimates
Date: 03/20/20 Time: 13:22
Sample (adjusted): 1979Q1 2008Q3
Included observations: 119 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	DEV	DIF
DEV(-1)	0.135950	3.295955
	(0.09410)	(2.23411)
	[1.44474]	[1.47529]
DIF(-1)	-0.001626	0.877313
	(0.00205)	(0.04858)
	[-0.79450]	[18.0580]
C	-0.004223	-0.363375
	(0.00720)	(0.17103)
	[-0.58627]	[-2.12467]
R-squared	0.028957	0.742502
Adj. R-squared	0.012215	0.738063
Sum sq. resids	0.269962	152.1697
S.E. equation	0.048242	1.145342
F-statistic	1.729598	167.2446
Log likelihood	193.4178	-183.4831
Akaike AIC	-3.200299	3.134170
Schwarz SC	-3.130237	3.204232
Mean dependent	0.000376	-2.804538
S.D. dependent	0.048539	2.237878
Determinant resid covariance (dof adj.)		0.002753
Determinant resid covariance		0.002616
Log likelihood		16.08406
Akaike information criterion		-0.169480
Schwarz criterion		-0.029356

Figura 11: Lag length criteria

VAR Lag Order Selection Criteria
Endogenous variables: DEV DIF
Exogenous variables: C
Date: 03/20/20 Time: 13:24
Sample: 1978Q3 2008Q3
Included observations: 108

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-61.04368	NA	0.011017	1.167475	1.217145	1.187614
1	35.67398	188.0621	0.001979*	-0.549518*	-0.400511*	-0.489101*
2	37.83351	4.119109	0.002048	-0.515435	-0.267090	-0.414740
3	39.90820	3.880437	0.002122	-0.479781	-0.132098	-0.338808
4	41.91734	3.683434	0.002203	-0.442914	0.004108	-0.261663
5	44.10407	3.928007	0.002280	-0.409335	0.137025	-0.187805
6	44.70026	1.048847	0.002430	-0.346301	0.299397	-0.084494
7	51.22911	11.24414*	0.002322	-0.393132	0.351905	-0.091046
8	54.26930	5.123286	0.002368	-0.375357	0.469017	-0.032994
9	56.33963	3.412199	0.002459	-0.339623	0.604090	0.043019
10	58.05334	2.760975	0.002572	-0.297284	0.745767	0.125635
11	59.87371	2.865395	0.002687	-0.256920	0.885469	0.206277
12	61.09602	1.878739	0.002839	-0.205482	1.036246	0.297994

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Figura 12: System estimation

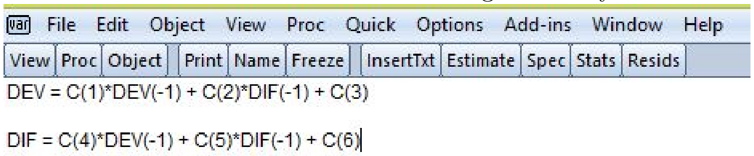


Figura 13: System estimation

System: UNTITLED
Estimation Method: Least Squares
Date: 03/20/20 Time: 13:36
Sample: 1979Q1 2008Q3
Included observations: 119
Total system (balanced) observations 238

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.135950	0.094101	1.444736	0.1499
C(2)	-0.001626	0.002046	-0.794497	0.4277
C(3)	-0.004223	0.007204	-0.586266	0.5583
C(4)	3.295955	2.234110	1.475288	0.1415
C(5)	0.877313	0.048583	18.05796	0.0000
C(6)	-0.363375	0.171026	-2.124674	0.0347

Determinant residual covariance 0.002616

Equation: DEV = C(1)*DEV(-1) + C(2)*DIF(-1) + C(3)
Observations: 119

R-squared	0.028957	Mean dependent var	0.000376
Adjusted R-squared	0.012215	S.D. dependent var	0.048539
S.E. of regression	0.048242	Sum squared resid	0.269962
Durbin-Watson stat	1.974702		

Equation: DIF = C(4)*DEV(-1) + C(5)*DIF(-1) + C(6)
Observations: 119

R-squared	0.742502	Mean dependent var	-2.804538
Adjusted R-squared	0.738063	S.D. dependent var	2.237878
S.E. of regression	1.145342	Sum squared resid	152.1697
Durbin-Watson stat	1.717915		

Some of the computations

$$\begin{pmatrix} 0 & 1 \end{pmatrix}\left(\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}\right) = c_2 + c_1 a_{21} + c_2 a_{22}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = c_1$$

and

$$\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^2 = \begin{pmatrix} a_{11}a_{21} + a_{21}a_{22} & a_{12}a_{21} + a_{22}^2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \end{pmatrix}$$

Figura 14: Testing Rational expectations hipothesis

Wald Test:
System: {%system}

Test Statistic	Value	df	Probability
Chi-square	1043.866	3	0.0000

Null Hypothesis: C(3)+C(6)*C(2)+C(3)*C(1) = C(6), C(5) *C(2)+C(2)*C(1) = C(5) , C(4)*C(2)+C(1)*C(1) =C (4)
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(3) + C(1)*C(3) - C(6) + C(2)*...	0.359168	0.171471
C(1)*C(2) - C(5) + C(2)*C(5)	-0.878960	0.048705
C(1)^2 - C(4) + C(2)*C(4)	-3.282831	2.237917

Delta method computed using analytic derivatives.

Problem 5 - Supply and Demand Shocks

Read the paper "The Dynamic Effects of Agregate Demand and Supply Disturbances" by Blanchard and Quah (1989). As covered in the lecture notes, the authors offered a different approach to use structural information to identify the estimated reduced form VAR. We will follow the authors by recognizing the existence of supply and demand disturbances which both affect the output and the unemployment rate in the short-run, but the demand shocks does not affect the output in the long term.

In Eviews terminology, $C_{1,1} = 0$, where $C_{1,1}$ is the upper left-handed element of the $C_{k\times k}$ matrix given by $C = \hat{\Psi}_{\infty} A^{-1} B$, where matrix A and B are given by $A\epsilon_t = Bu_t$ (see Eviews guide II, on page 474).

The E-Views Workfile 'bc' contains quarterly series from Q2 1948 to Q4 2007 for US Output (in Billion Dollars) and Unemployment rate. Unemployment rate is measured monthly, this is a 3 month average.

i) Check wether GDP growth and average unemployment are stationary. Argue if this assumption is plausible.

ii) Estimate a VAR for GDP growth and average unemployment choosing the number of lags based on relevant criteria. The error term in the reduced form VAR will be a linear combination of supply and demand schocks.

iii) For the preferred VAR in part ii) compute impulse response functions and variance decomposition of forecasts for the two possible cholesky orderings.

iv) For the preferred VAR in part ii) estimate a structural var with the restriction that the long run effect of a demand schock in GDP growth is zero. Compare the results with the ones you obtained in iii)

Solution

i) We compute the Agumented Dickey-Fuller test for both variables, under the three specifications: (i) Intercept, (ii) Intercept and Trend, (iii) None. In all three cases the test rejects the unit root hypothesis for both Unemployment and GDP growth under the three different specifications. The results of the ADF test are reported next . To compute the test go to, Quick ", "Series StatisticsUnit Rot Test". We present the results with intercept for both series.

We could perform further tests. But the results would be the same.

In Blanchard Quah (1989) the authors find a small but steady increase in average unemployment rate as well as a decline (structural break) in GDP growth since 1970's. On theoretical grounds, they argue that

Figura 15: Unit Root test for GDP growth

Null Hypothesis: DLGP has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.316211	0.0000
Test critical values:	1% level	-3.457747	
	5% level	-2.873492	
	10% level	-2.573215	

*MacKinnon (1996) one-sided p-values.

Figura 16: Unit Root test for unemployment

Augmented Dickey-Fuller Unit Root Test on U

Null Hypothesis: U has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.295221	0.0038
Test critical values:	1% level	-3.997083	
	5% level	-3.428819	
	10% level	-3.137851	

*MacKinnon (1996) one-sided p-values.

unemployment may be non-stationary and affected permanently by supply and demand shocks. These facts may be predicted by models of Hysteresis in the labour market.

To handle this issues they work with four different specifications for the time series used to estimate the VAR. They use two different specifications for each series, and then combine them. For Unemployment they work with the series in levels and with the series after being subtracted a deterministic time trend. For GDP growth they work with the original series and a series after being subtracted a structural break in the 1970's.

The results of the different specifications are similar for Impulse Response Functions but slightly different for Variance Decompositions. We will continue working with Unemployment in levels and GDP Growth. Recall also that their sample was smaller than ours.

ii) We work from specific to general. We start estimating a VAR(1) for both variables, and run a Lag Length Criteria. To do this, go to "Quick/."Estimate VAR", write the dependent variables and lags "1 1". We estimate and then go to View/Lag Structure/Lag length.... We choose 12 lags. The results are shown in Figure

The lag length criteria test has different results. A VAR(2) will be preferred with Schwartz criteria of parsimony. For forecasting purposes, a parsimonious model is preferred.

Other criteria favour a VAR(12) model. These are: sequential Likelihood Ratio tests, Final Prediction Error and Akaike criteria.We also perform lag exclusion tests: the 12-th lag results signifiacative at the 5 % level. The model may be overparametrized for forecasting purposes or testing restrictions between parameters but not for impulse response[3].

iii) We first use the ordering *(GDP growth, unemployment)*. The IRF are given in Figure 19. The other ordering is represented in Figure 20

[3]In the paper a model with 8 lags is estimated.

Figura 17: Lag Length Criteria

VAR Lag Order Selection Criteria
Endogenous variables: DLGP U
Exogenous variables: C
Date: 04/03/20 Time: 08:44
Sample: 1948Q2 2008Q1
Included observations: 224

Lag	LogL	LR	FPE	AIC	SC	HQ
0	1350.436	NA	2.02e-08	-12.03960	-12.00914	-12.02731
1	1726.724	742.4985	7.29e-10	-15.36361	-15.27223	-15.32672
2	1769.642	83.91888	5.15e-10	-15.71109	-15.55878*	-15.64961
3	1779.043	18.21400	4.91e-10	-15.75931	-15.54608	-15.67324*
4	1783.723	8.984576	4.88e-10	-15.76538	-15.49123	-15.65472
5	1790.787	13.43348	4.75e-10	-15.79274	-15.45766	-15.65749
6	1796.855	11.43185	4.66e-10	-15.81120	-15.41521	-15.65136
7	1797.281	0.795620	4.81e-10	-15.77929	-15.32238	-15.59486
8	1802.302	9.279721	4.77e-10	-15.78841	-15.27057	-15.57938
9	1814.135	21.65812	4.45e-10	-15.85835	-15.27958	-15.6[illegible]73
10	1819.956	10.55151	4.38e-10	-15.87461	-15.23492	-15.61640
11	1822.412	4.406982	4.44e-10	-15.86082	-15.16021	-15.57802
12	1832.687	18.25767*	4.20e-10*	-15.91685*	-15.15532	-15.60946
13	1833.864	2.070033	4.31e-10	-15.89165	-15.06920	-15.55966
14	1837.694	6.667865	4.32e-10	-15.89013	-15.00675	-15.53355
15	1838.429	1.266153	4.45e-10	-15.86097	-14.91668	-15.47981

We can observe that results are very similar for both specifications. This result is not anomalous. When we compute the correlation of the residual of the reduced form VAR estimated in part (ii), we find that is less that 5 % (-0.26). Enders (1995) argues that when the correlation between residuals in the reduced form VAR are less than 20 % the results from different orderings will be similar.

Also in the first table we observe that the contemporaneous response of GDP growth to an Unemployment shock is zero (upper right quadrant). The same in the second table (lower left quadrant), but for the response on Unemployment to a GDP growth schock. This is the short run restriction we imposed with the Cholesky ordering.
Also we observe that responses die out in 20 or 25 quarters. This because the estimated VAR is stationary. Now we present the results of Variance Decomposition for Both orderings. In this case, the results are qualitatively similar, but there are some differences on the quantitave results. More specifically, the unemployment orthogonalized shock explains a larger share of the variance of unemployment when it is put first in the ordering.

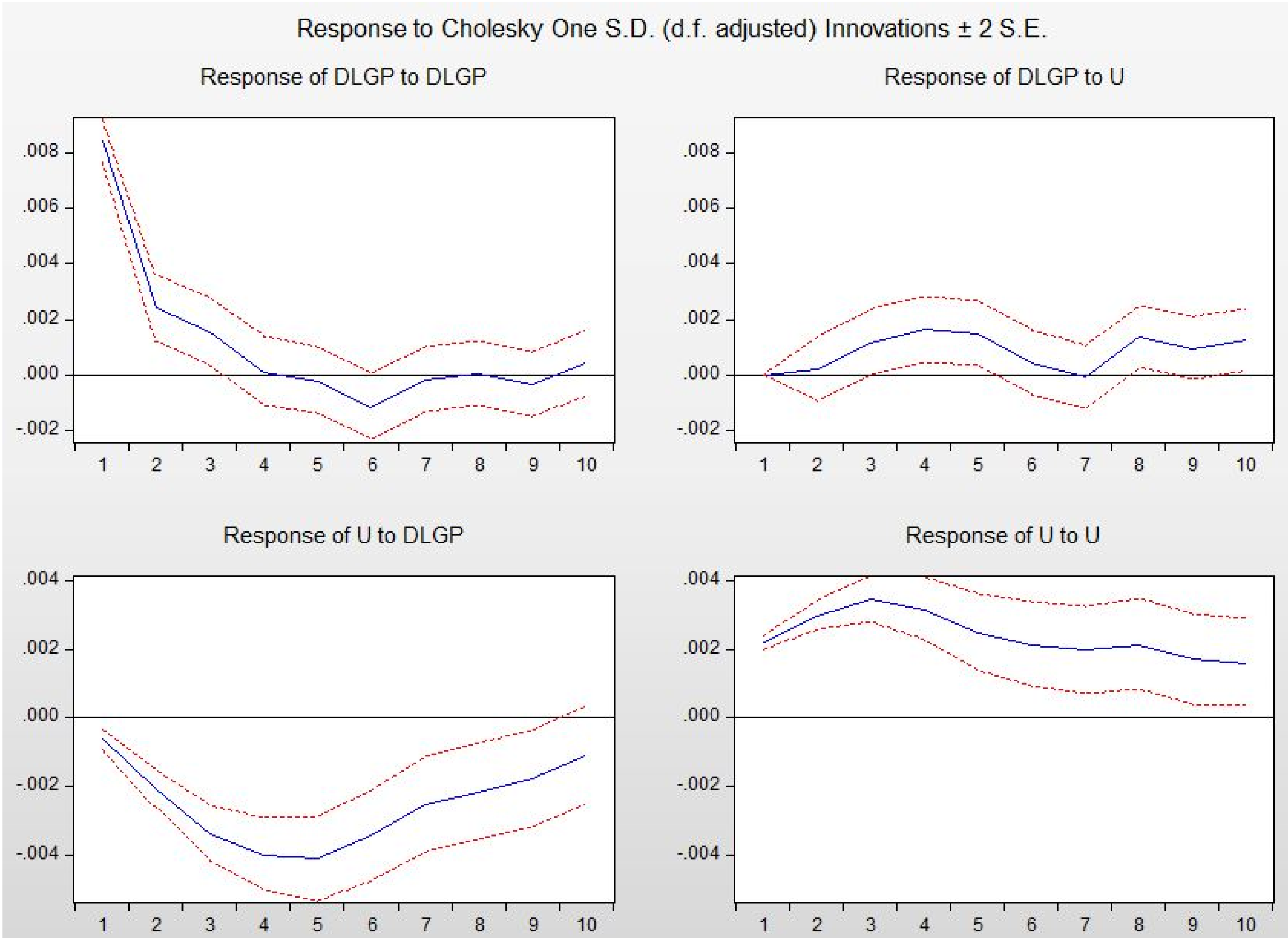
For example, in the upper left graph we observe how changes in the reduced form residual of both variables affect GDP growth forecasting error, in the ordering DLGDP and U. We can see that most of the variance of the forecast error is explained by reduced form shock of GDP growth.

Figura 18: Lag Exclusion tests

Numbers in [ ] are p-values

	DLGP	U	Joint
Lag 1	16.99860 [0.0002]	562.8596 [0.0000]	595.4981 [0.0000]
Lag 2	4.971750 [0.0833]	10.62305 [0.0049]	12.80107 [0.0123]
Lag 3	0.301598 [0.8600]	5.711044 [0.0575]	7.039073 [0.1338]
Lag 4	1.402367 [0.4960]	0.345919 [0.8412]	1.681203 [0.7941]
Lag 5	1.496930 [0.4731]	13.66844 [0.0011]	13.73682 [0.0082]
Lag 6	0.476621 [0.7880]	5.511690 [0.0636]	6.787453 [0.1476]
Lag 7	4.930788 [0.0850]	0.939696 [0.6251]	5.791618 [0.2153]
Lag 8	3.924975 [0.1405]	4.334811 [0.1145]	11.01234 [0.0264]
Lag 9	2.814594 [0.2448]	20.64381 [0.0000]	28.59444 [0.0000]
Lag 10	1.932396 [0.3805]	3.447306 [0.1784]	5.193745 [0.2680]
Lag 11	0.170305 [0.9184]	0.816012 [0.6650]	1.051908 [0.9018]
Lag 12	8.240944 [0.0162]	17.39475 [0.0002]	20.72033 [0.0004]
df	2	2	4

Figura 19: IRF with ordering GDP growth-unemployment



iv) Now we will impose a long run restriction. A demand shock will not have effect on GDP growth in the long run.
Let u be the structural residual of the VAR. Residuals in the reduced form VAR are

$$\begin{array}{l} \varepsilon_{1t} = a_1 u_{2t} + u_{1t} \\ \varepsilon_{2t} = a_2 u_{1t} + u_{2t} \end{array} \Leftrightarrow A\varepsilon = Bu$$

where A is the Identity matrix.

In a bivariate reduced form VAR(p) we will estimate $2+4p+3$ parameters. Two constants, $4p$ for the p lags, and the covariance matrix of the reduced form VAR. In the structural model we have $2+4p+4$. In the covariance matrix we have four parameters instead of three. So, we have to impose one restriction, as a necessary condition for Identification. Lets define u_1 and u_2 as the demand and supply shock. In the model presented in the paper, demand shock is viewed as a Money stock shock and supply shock is viewed as a technology shock.

Now we will estimate the structural model with the restrictions that the LR multiplier of a demand shock is zero. In the output of the VAR (12) you estimated in part ii) go to 'Proc _ Estimate Structural Factorization' and you will arrive to the window shown.

Figura 20: IRF with ordering unemployment-GDP growth

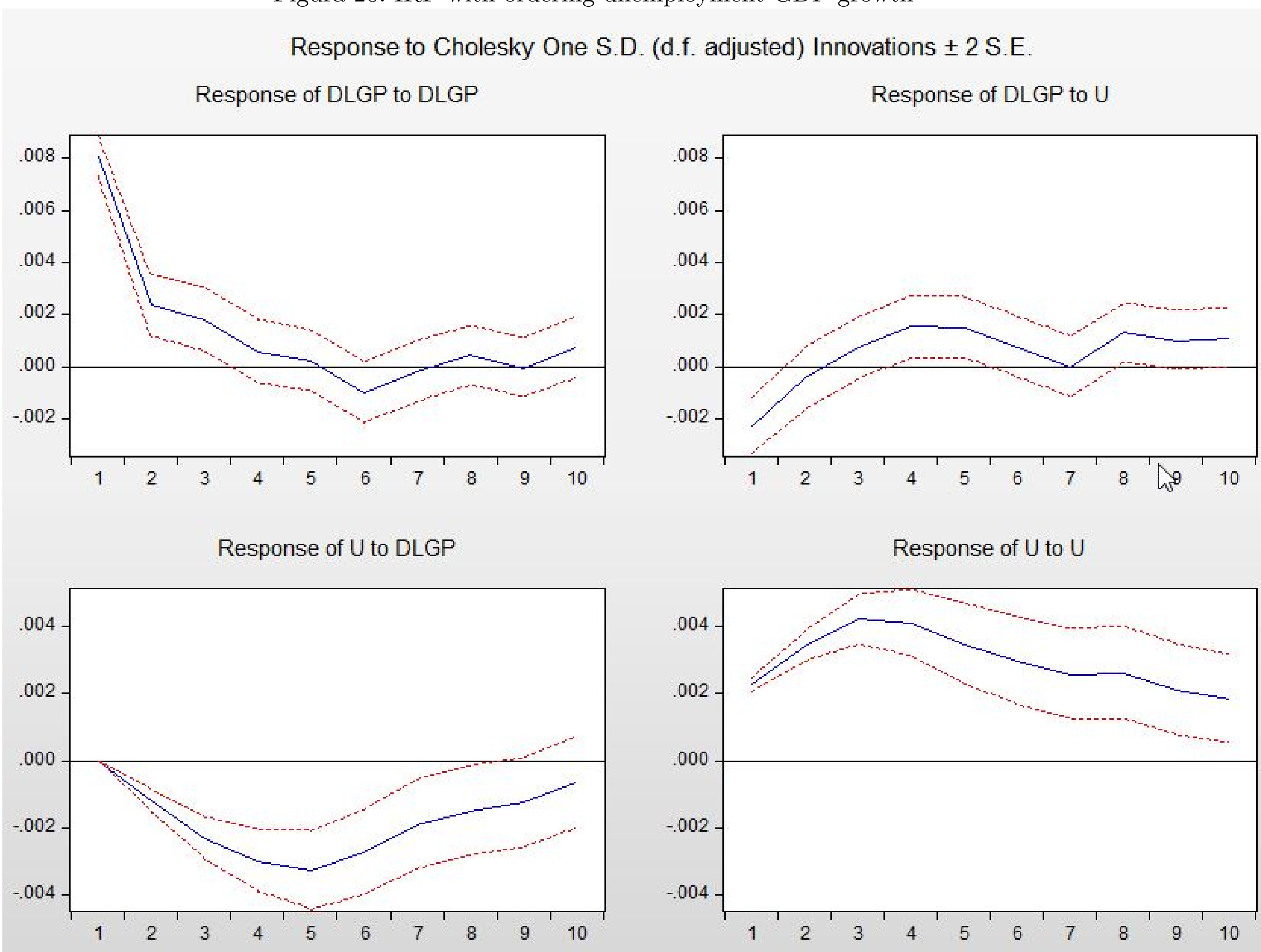


Figura 21: Variance decomposition with ordering unemployment-GDP growth

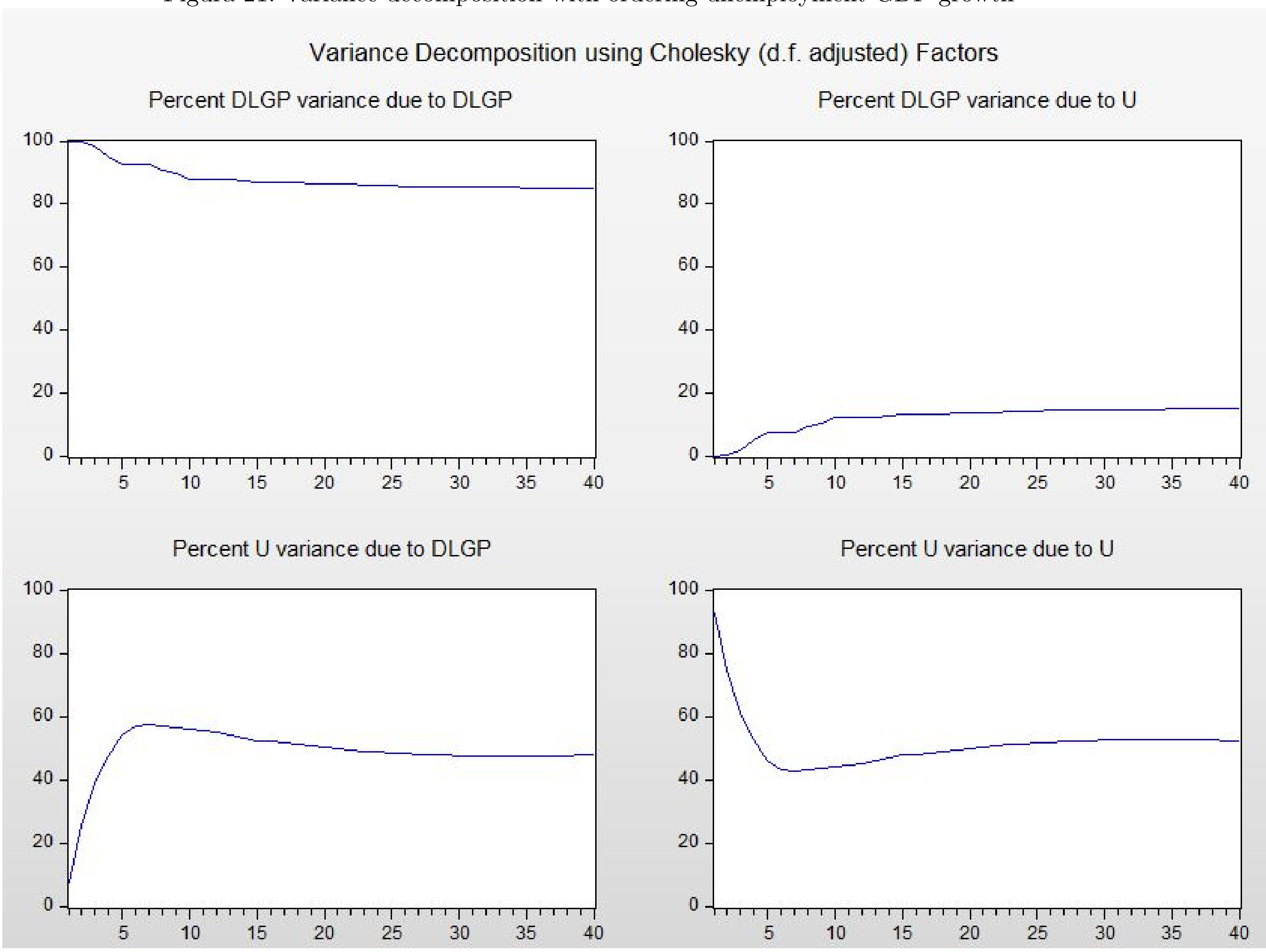
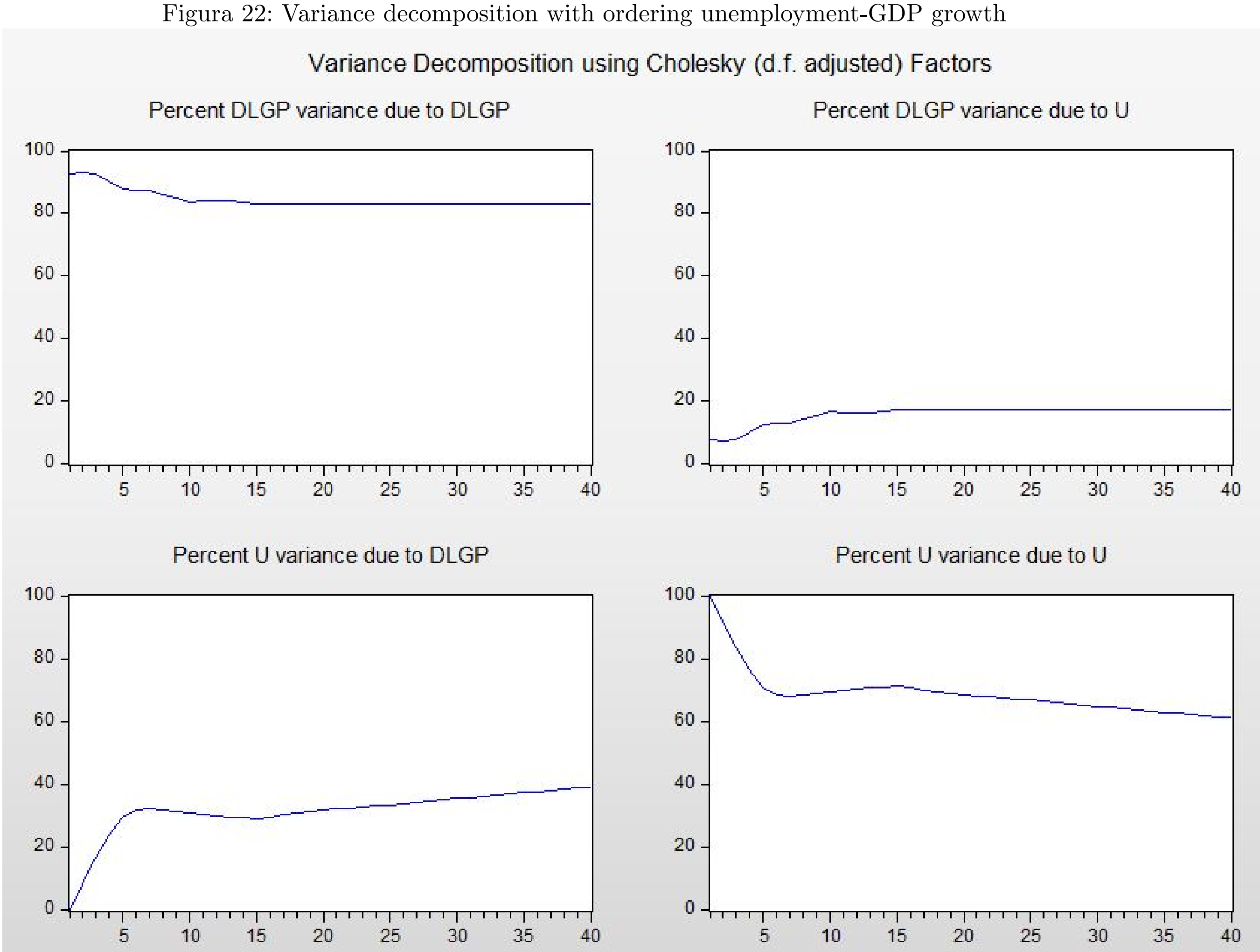
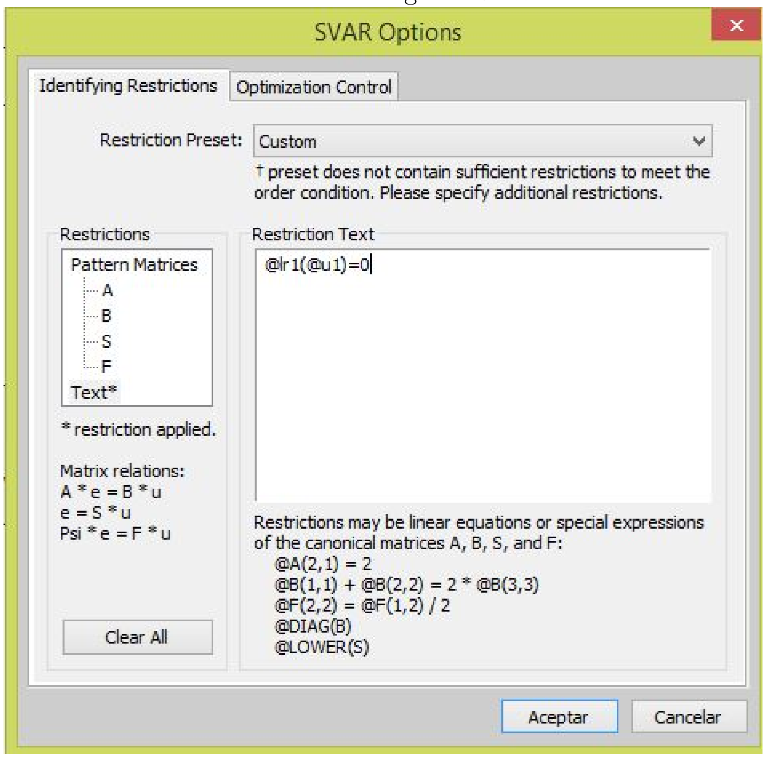


Figura 22: Variance decomposition with ordering unemployment-GDP growth



Then we will define the restriction in text (also you can program a matrix, see the exercise on Peersan's Oil Prices shock model). To define the long run restriction type "@lr1(@u1)=0". Typing *u1* you are telling to Eviews, that shock 1 is the demand shock (you will have to remember this to read impulse responses). We type "*lr1*"because GDP Growth is the first variable.

Figura 23: Window for estimate the SVAR



Estimation output is presented in the next table

The $c(i,j)$ coefficient in the estimated matrix C is the long run response of variable i to a shock in structural shock j .

$$C = \widehat{\Psi}(\infty)A^{-1}B = (I - \widehat{c} - \widehat{A_1} - ... - \widehat{A_{12}})^{-1}A^{-1}B$$

$c(1) = 0$ was the imposed restriction. This says, that the long run response of GDP growth to a Shock in demand is zero.

Coefficients in matrix A and B are estimated by maximum likelihood assuming normal distribuition for structural shocks. The parameters in $\widehat{c}, \widehat{A_1}, ..., \widehat{A_{12}}$ are the ones we estimated in part (ii).

To compute impulse response, in the estimation output of the structural factorization, go to 'Impulse'. Now you will have available the option structural factorization. Select it. The output for the impulse response is :

Results are very different from the ones we obtained in part (iii) and have different interpretations. Impulses do not exhibit the triangular ordering imposed in Cholesky factorization (note that all variables respond on impact to the shock). A shock in demand or supply affects contemporaneously output and unemployment. When we imposed a cholesky ordering, it was like imposing the restriction that a_1 or a_2 were equal to zero.

We can also plot the acumulated response (which was directly estimated before). Note that the acumulated effect of the shock 1 (demand shock) on GDP growth rate is zero.

Variance decomposition results are reported next. Again, after the estimation of the structural process, we go to "ViewVariance Decomposition.[a]nd choose Structural factorization again:

Figura 24: Estimation output of the SVAR

Structural VAR Estimates
Date: 04/03/20 Time: 08:47
Sample (adjusted): 1951Q3 2008Q1
Included observations: 227 after adjustments
Estimation method: Maximum likelihood via Newton-Raphson (analytic derivatives)
Convergence achieved after 11 iterations
Structural VAR is just-identified

Model: e = Phi*Fu where E[uu']=I
F =

0	C(2)
C(1)	C(3)

including the restriction(s)
@LR1(@U1)=0

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-0.048330	0.002268	-21.30727	0.0000
C(2)	0.037998	0.001783	21.30727	0.0000
C(3)	0.070049	0.004593	15.25041	0.0000
Log likelihood	1830.061			

Estimated S matrix:

0.005300	0.006545
-0.002089	0.000903

Estimated F matrix:

0.000000	0.037998
-0.048330	0.070049

Again results differ sharply from the variance decomposition with a Cholesky decomposition.

Figura 25: IRF of the SVAR

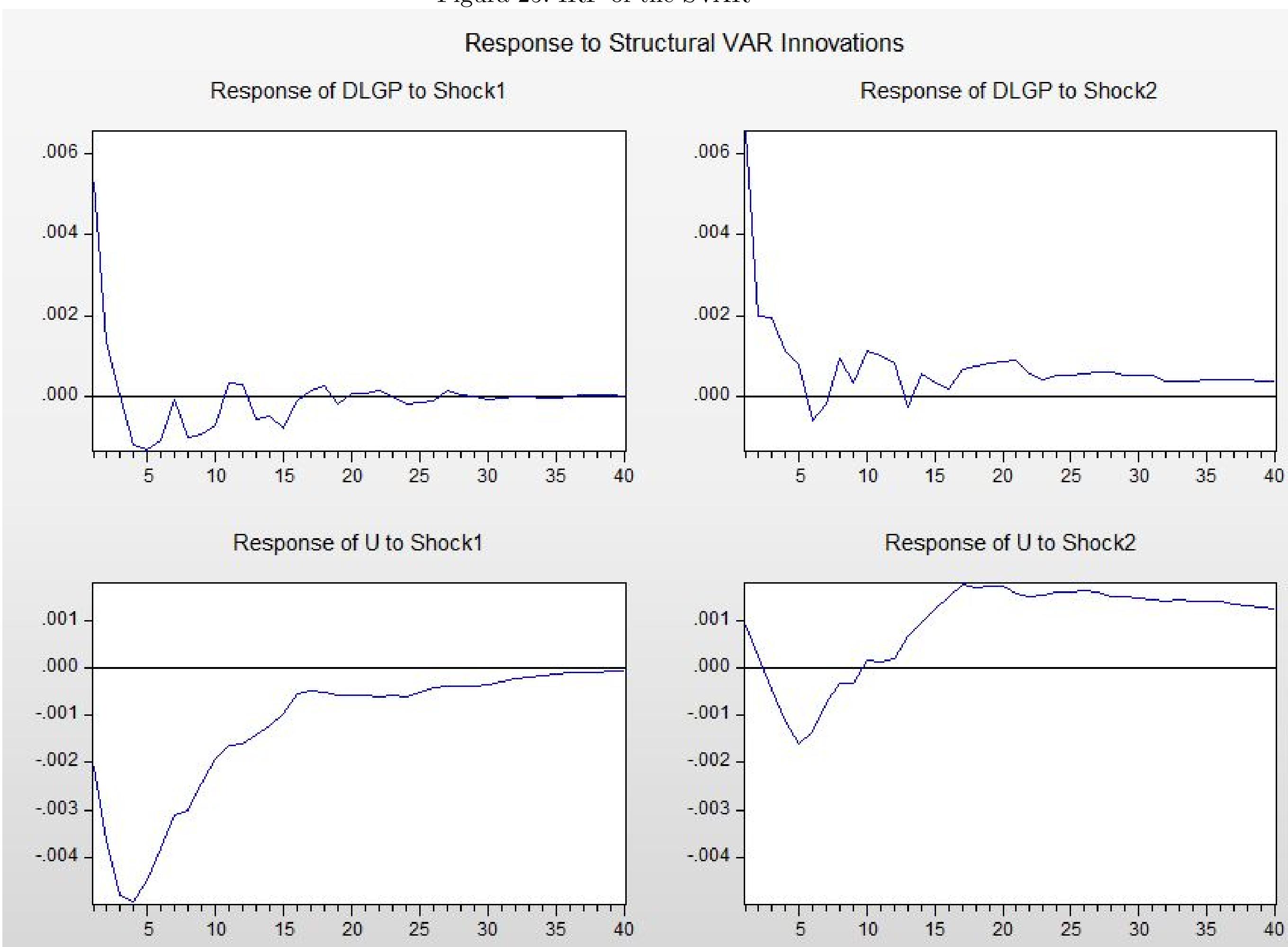


Figura 26: Acumulated IRF of the SVAR

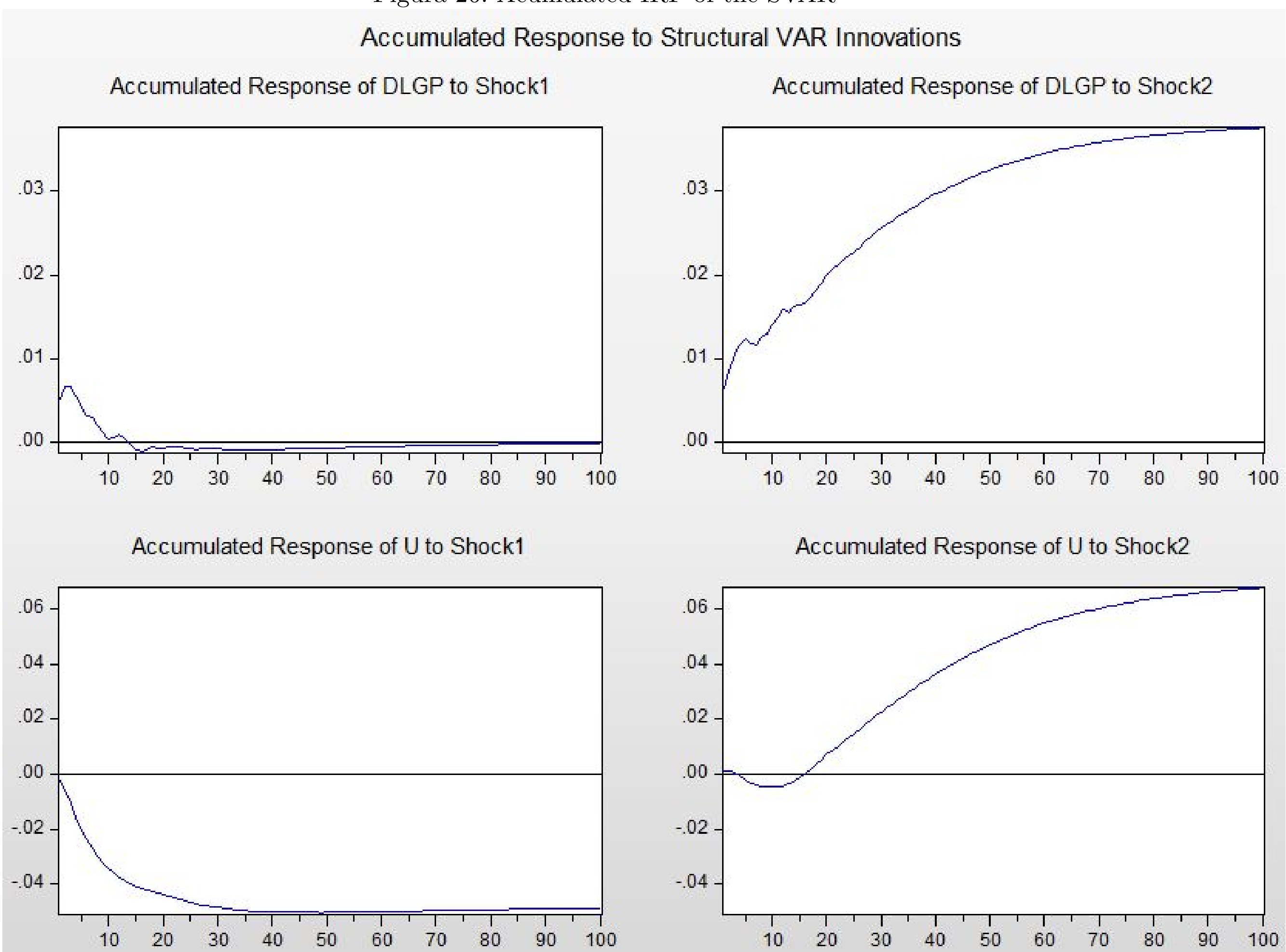


Figura 27: Variance decomposition of the SVAR