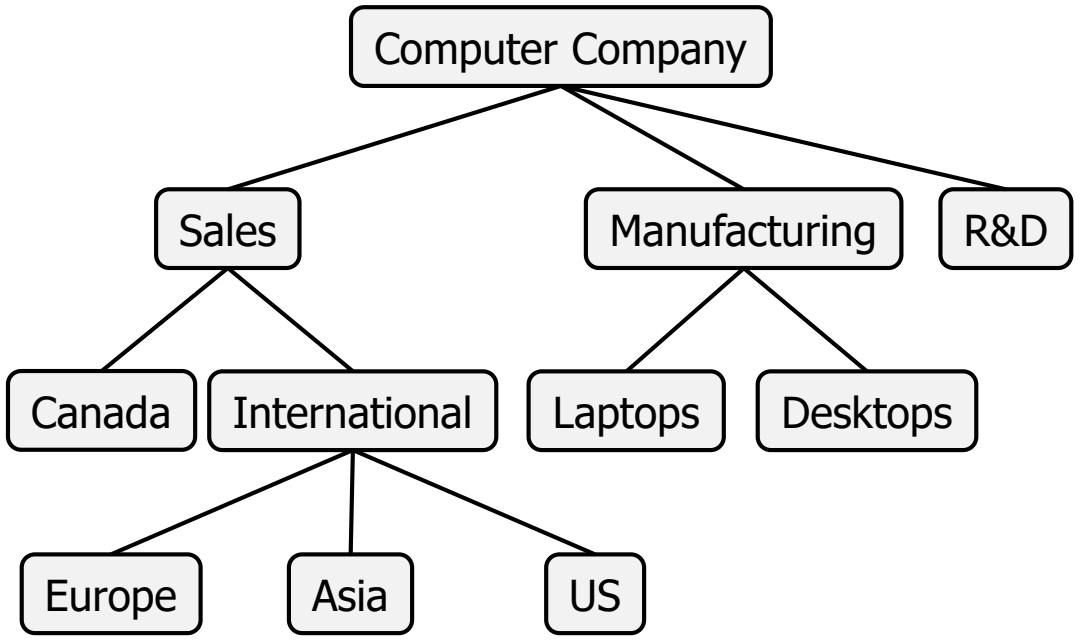
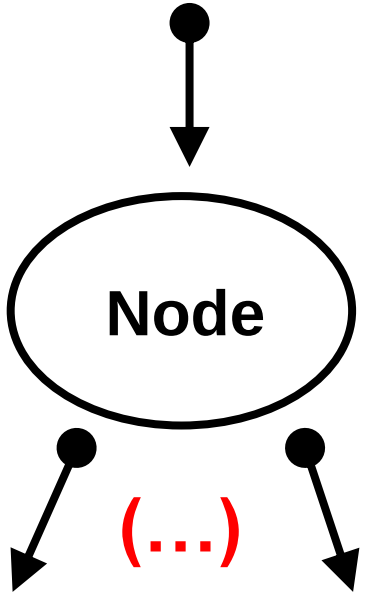
Introduction to Trees

- **A tree is an abstract model of a non-linear, hierarchical collection with a "one to many" relationships**

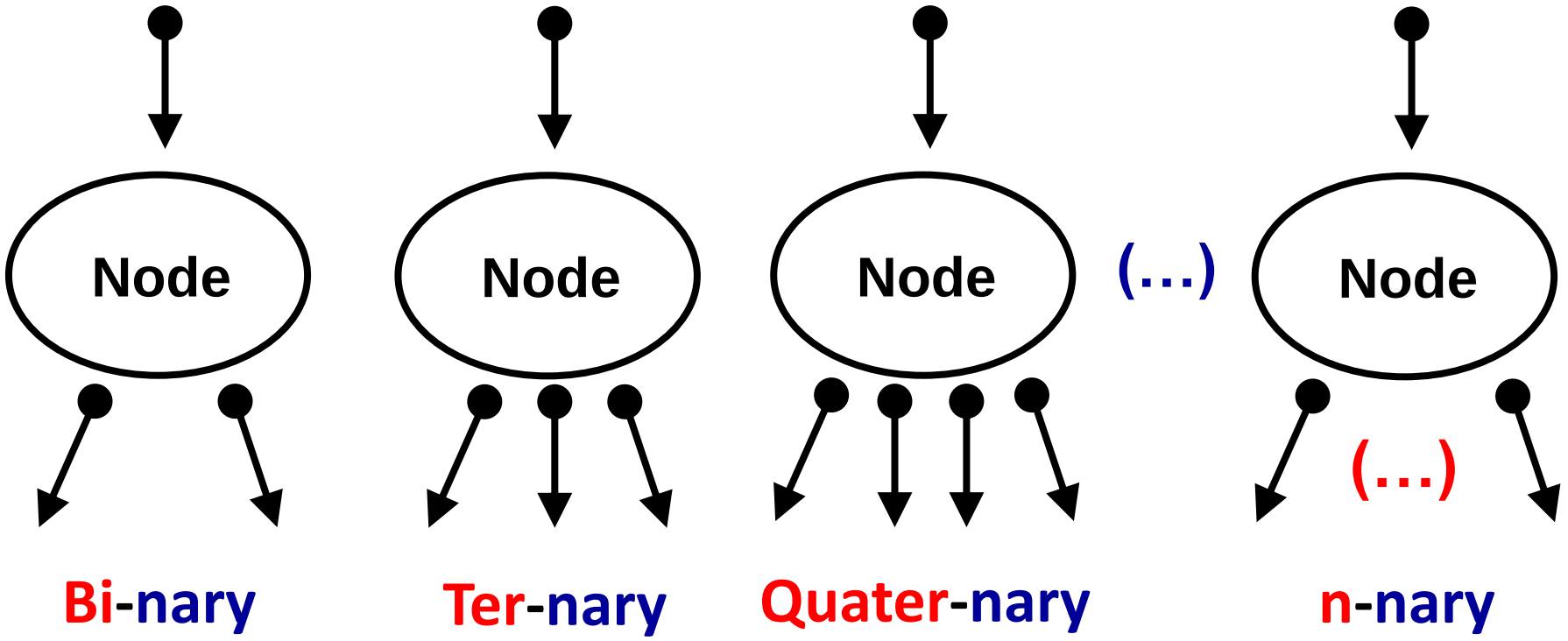
- **A tree consists of nodes with a parent-child relation**

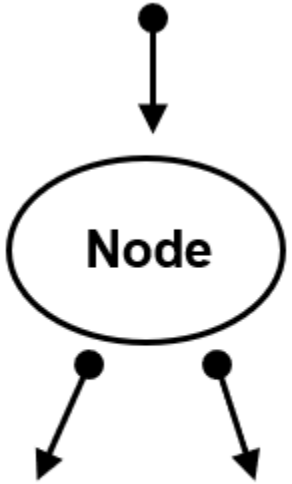




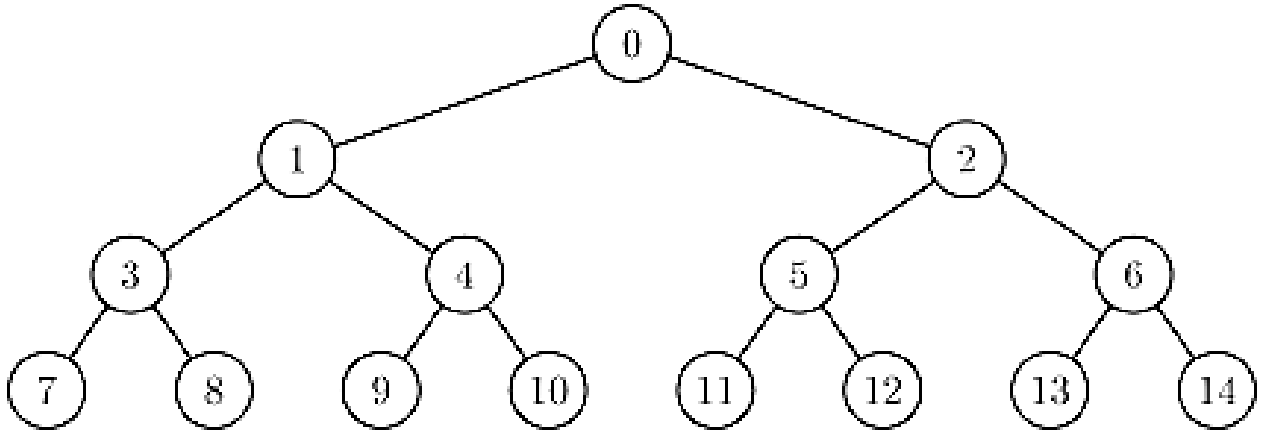
"one to many" links

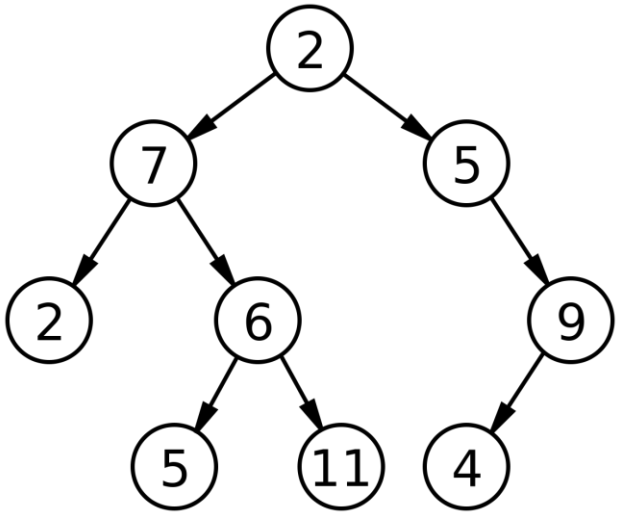
Tree Classification

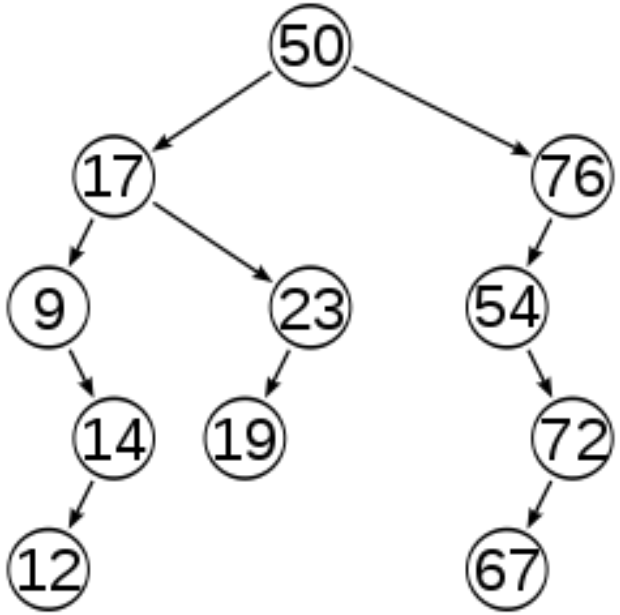


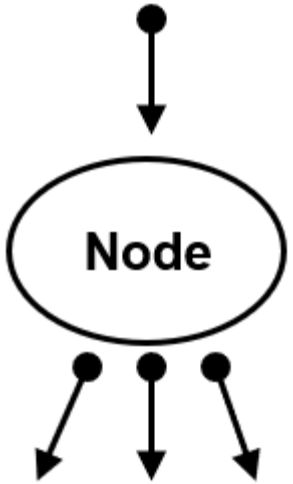

Node

Bi-nary

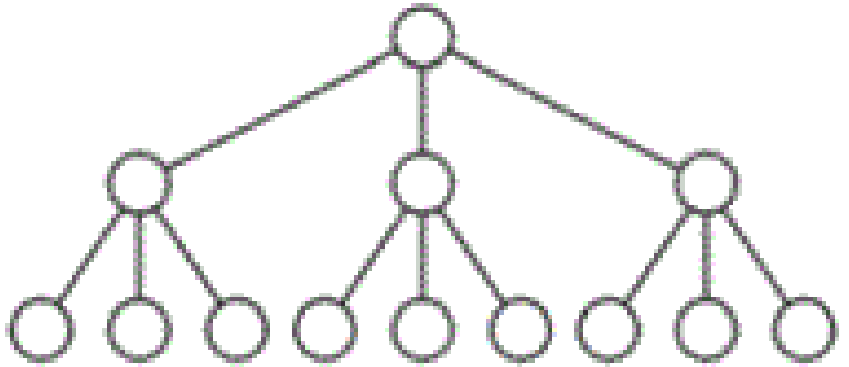

0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

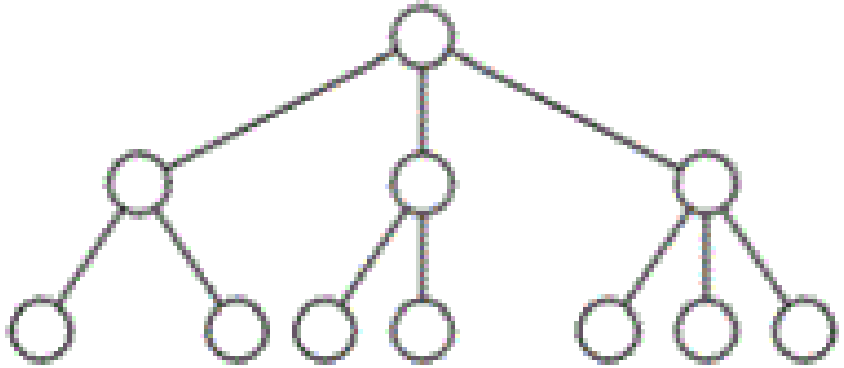

2
7
5
2
6
9
5
11
4

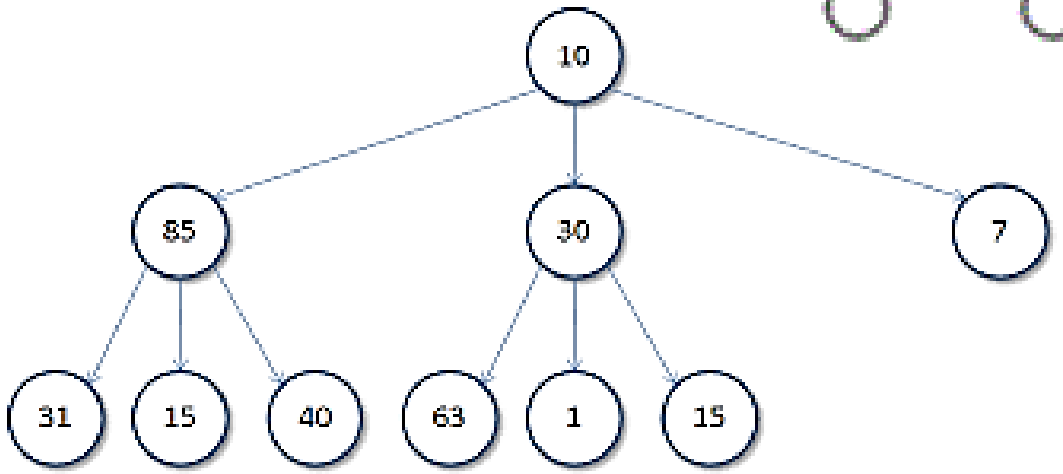

50
17
76
9
23
54
14
19
72
12
67



Ter-nary







Visual Representation of a Tree

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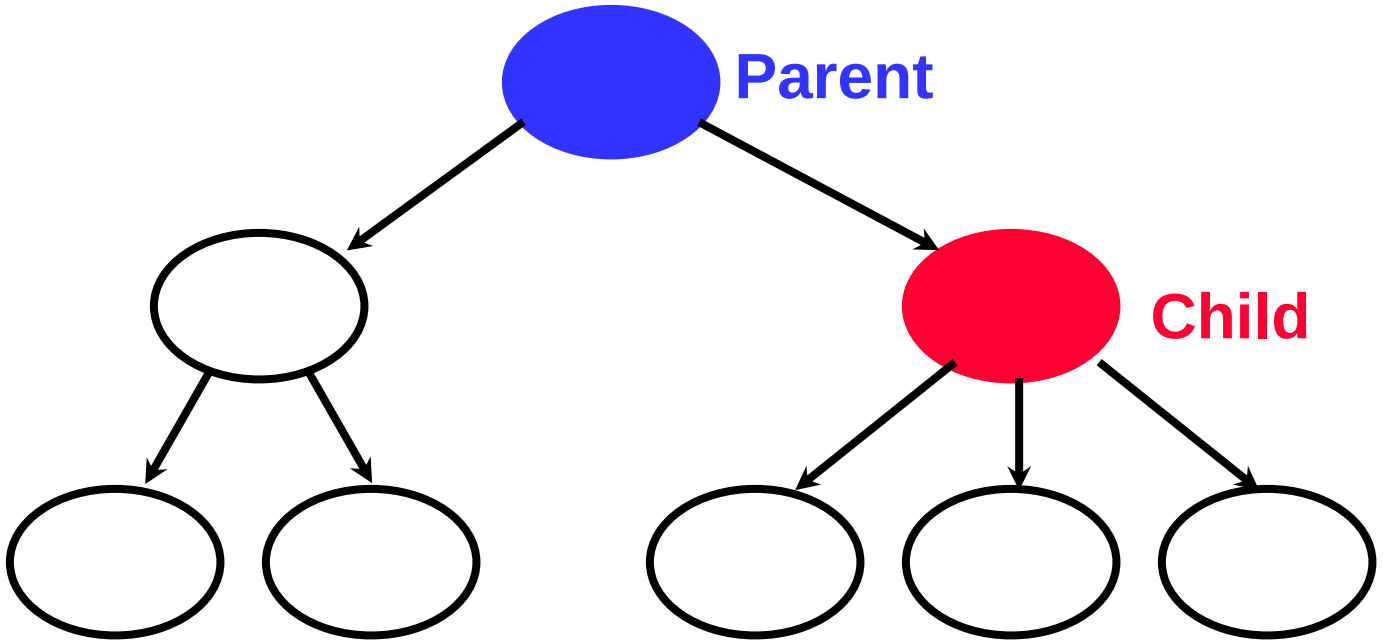
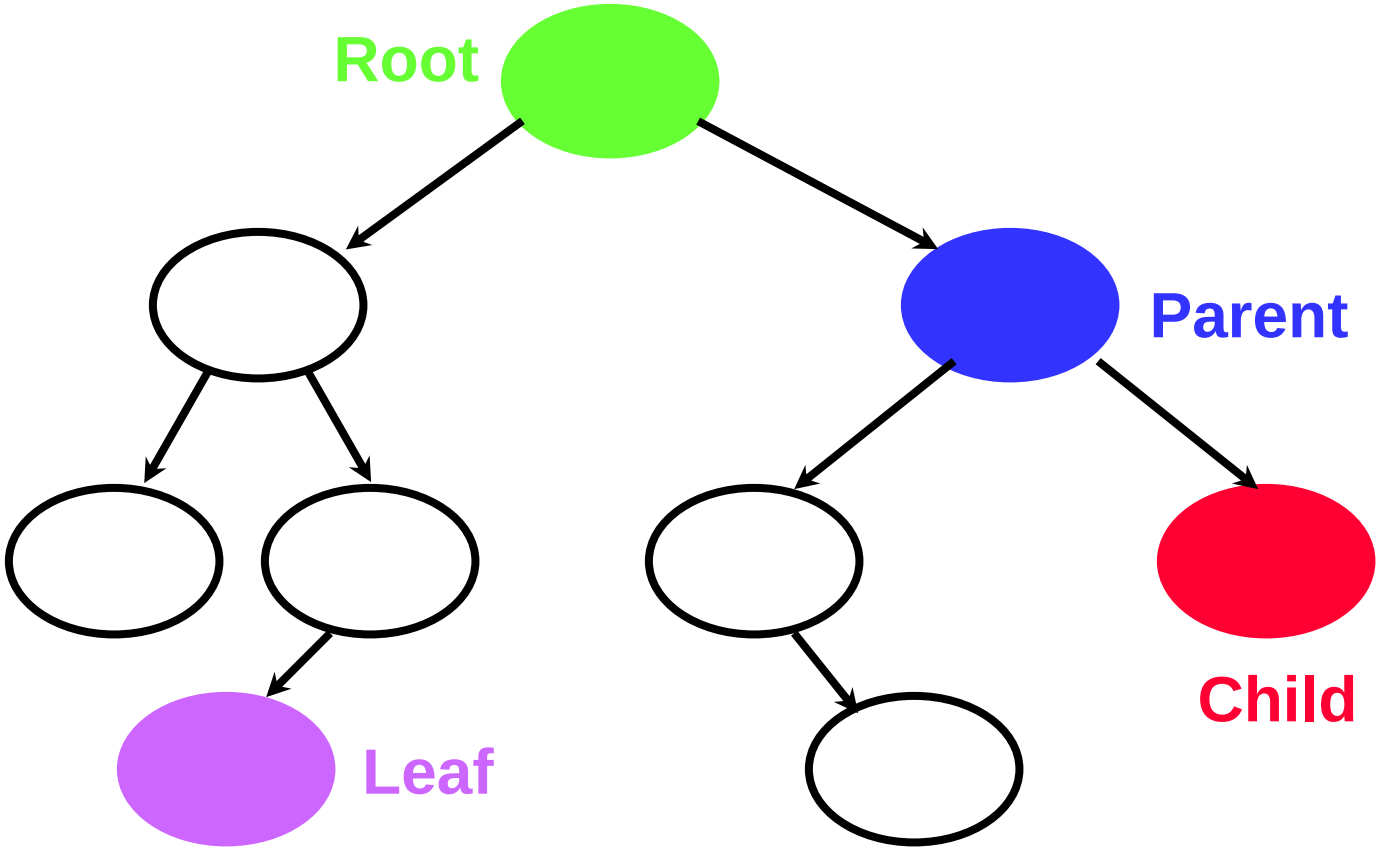
Trees allow each node to have multiple successors



Tree Terminology



Tree Terminology

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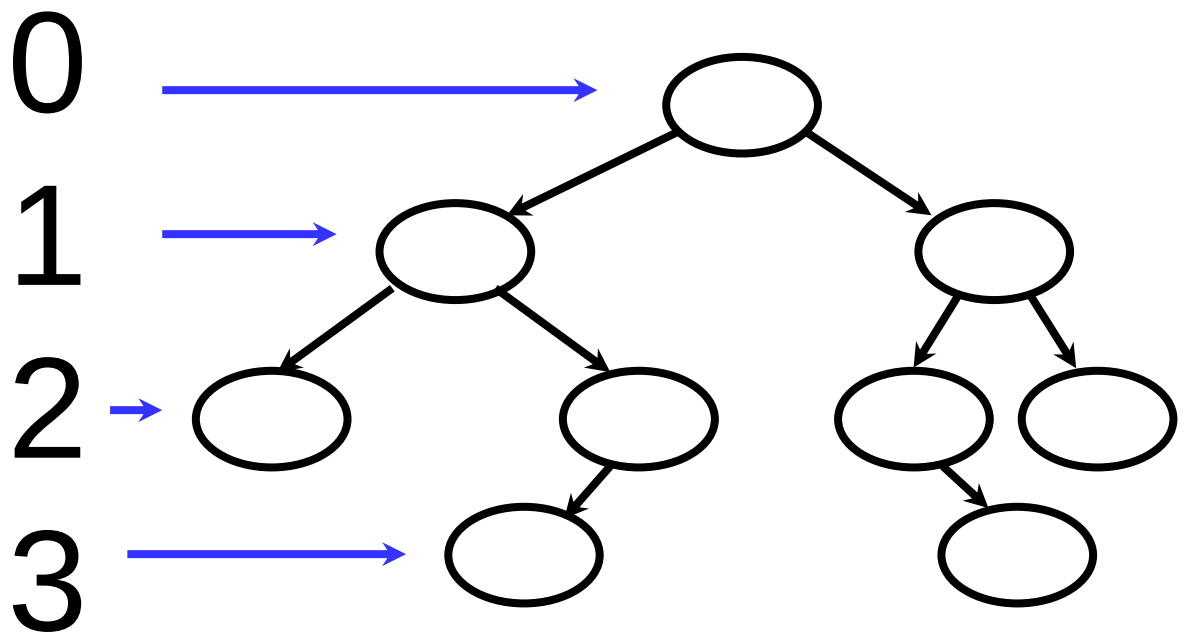
- The first node is called the root.
- Successors are called children
- A parent node points to a child node.
- Nodes with no children are called leaves.

Node Depth in a Tree

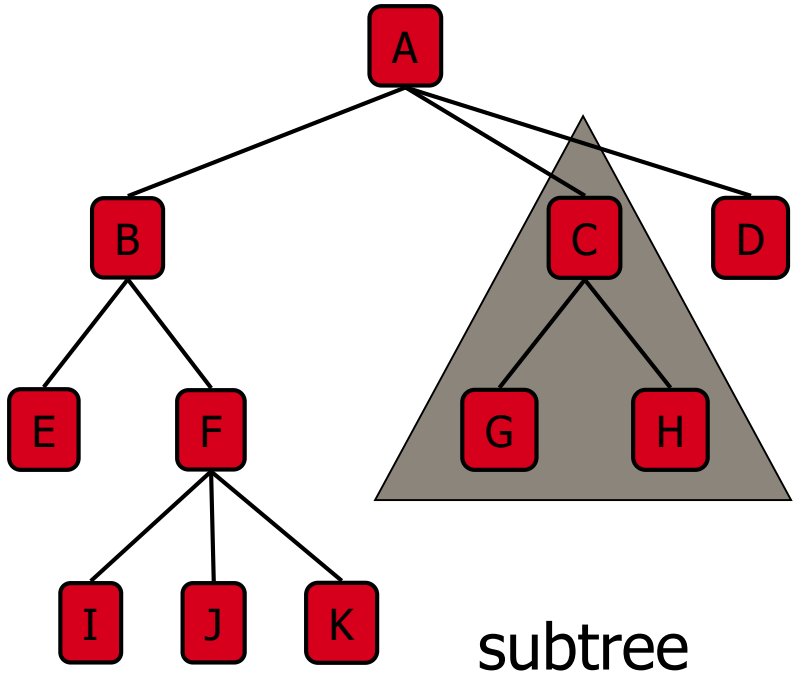
- **Measures how far down a node is in the tree.**



Tree Terminology

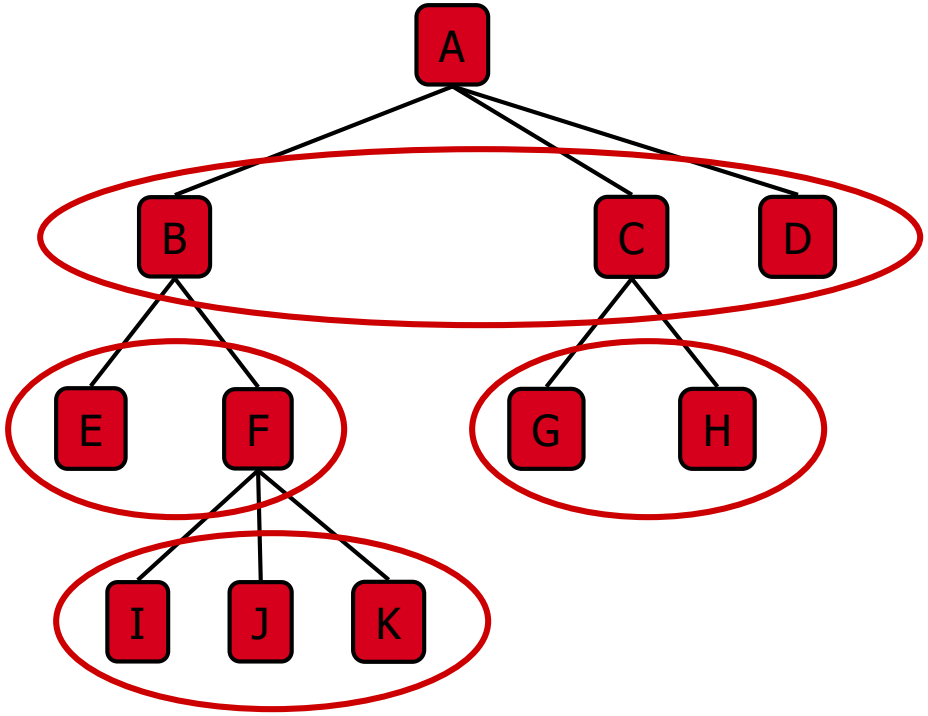
- **Root:** node without parent (A)
- **Internal node:** node with at least one child (A, B, C, F)
- **External node (a.k.a. leaf):** node without children (E, I, J, K, G, H, D)
- **Ancestors of a node:** parent, grandparent, grand-grandparent, etc.
- **Height of a tree:** maximum depth of any node
- **Descendant of a node:** child, grandchild, grand-grandchild, etc.

- **Subtree:** tree consisting of a node and its descendants



Tree Terminology → Nodes

- All children of a given node are *siblings* of each other

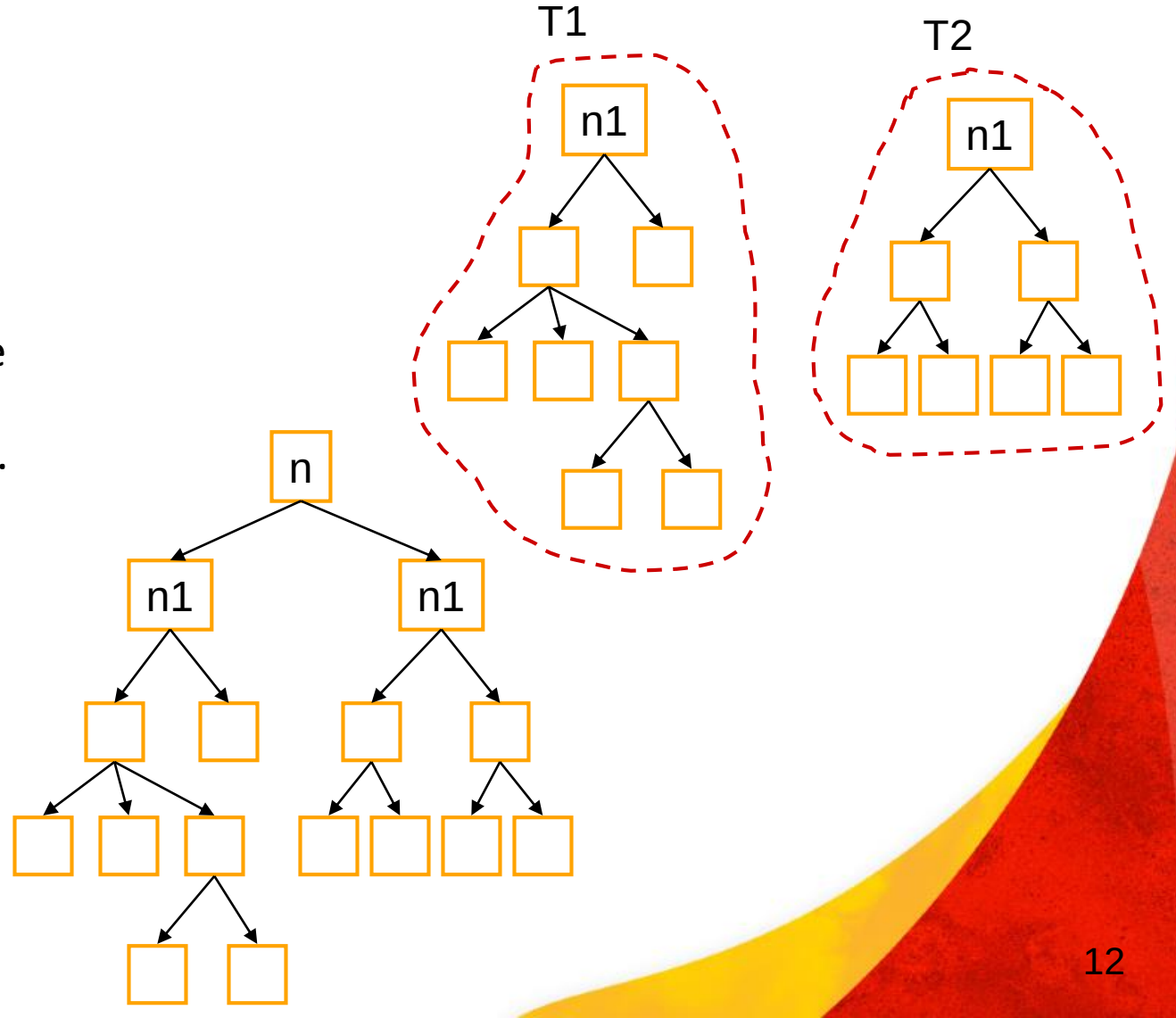


Formal Definition of (any kind of) Tree

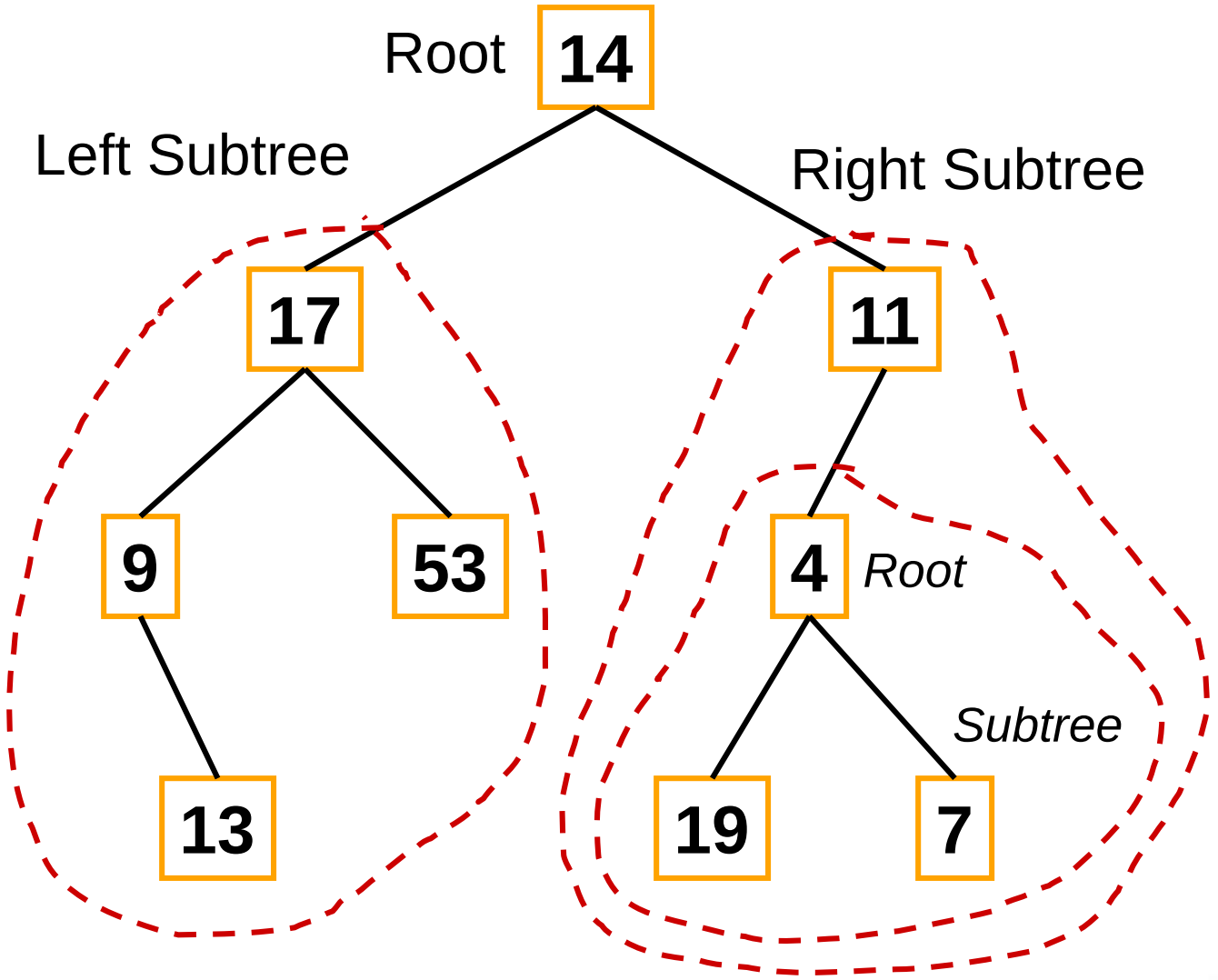
1. A single node by itself is a tree. It is also the root of the tree.

2. If n is a node and T1,T2,...Tk are trees with roots n1,n2,..nk then we can make a new tree by making n the parent of n1,n2,..nk. In this tree, n is the root and T1,T2,...Tk are subtrees.

- All trees are defined recursively.



Subtrees


Root
14
Left Subtree
Right Subtree
17
11
9
53
4
Root
Subtree
13
19
7

Binary tree, leaf, internal and external nodes

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- A tree in which no node may have more than two children is called a BINARY TREE.

- A node which has no children is a LEAF since it is found on the outside of the tree.

- Nodes with children are INTERNAL NODES.

Binary Trees

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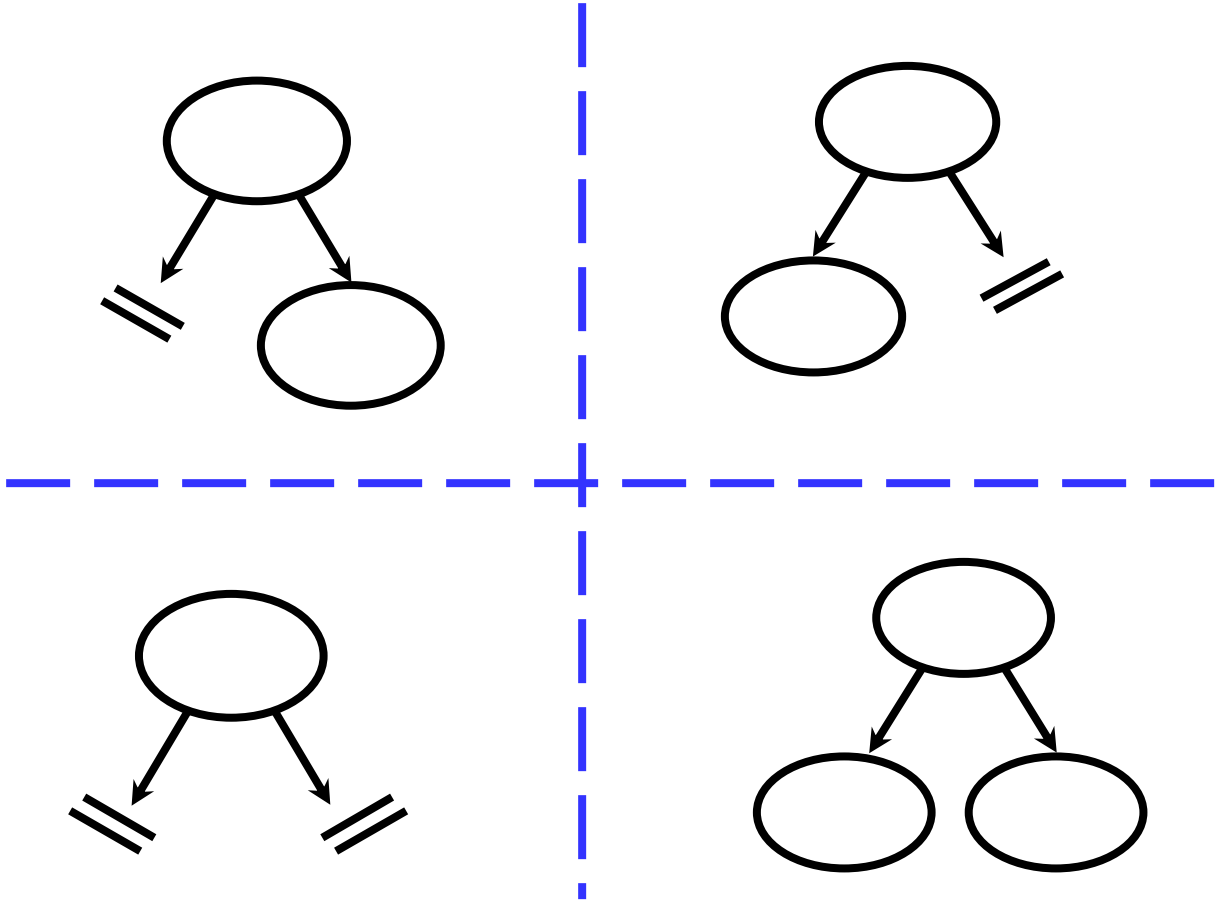
Binary trees can have at most two successors.



Binary Tree Example

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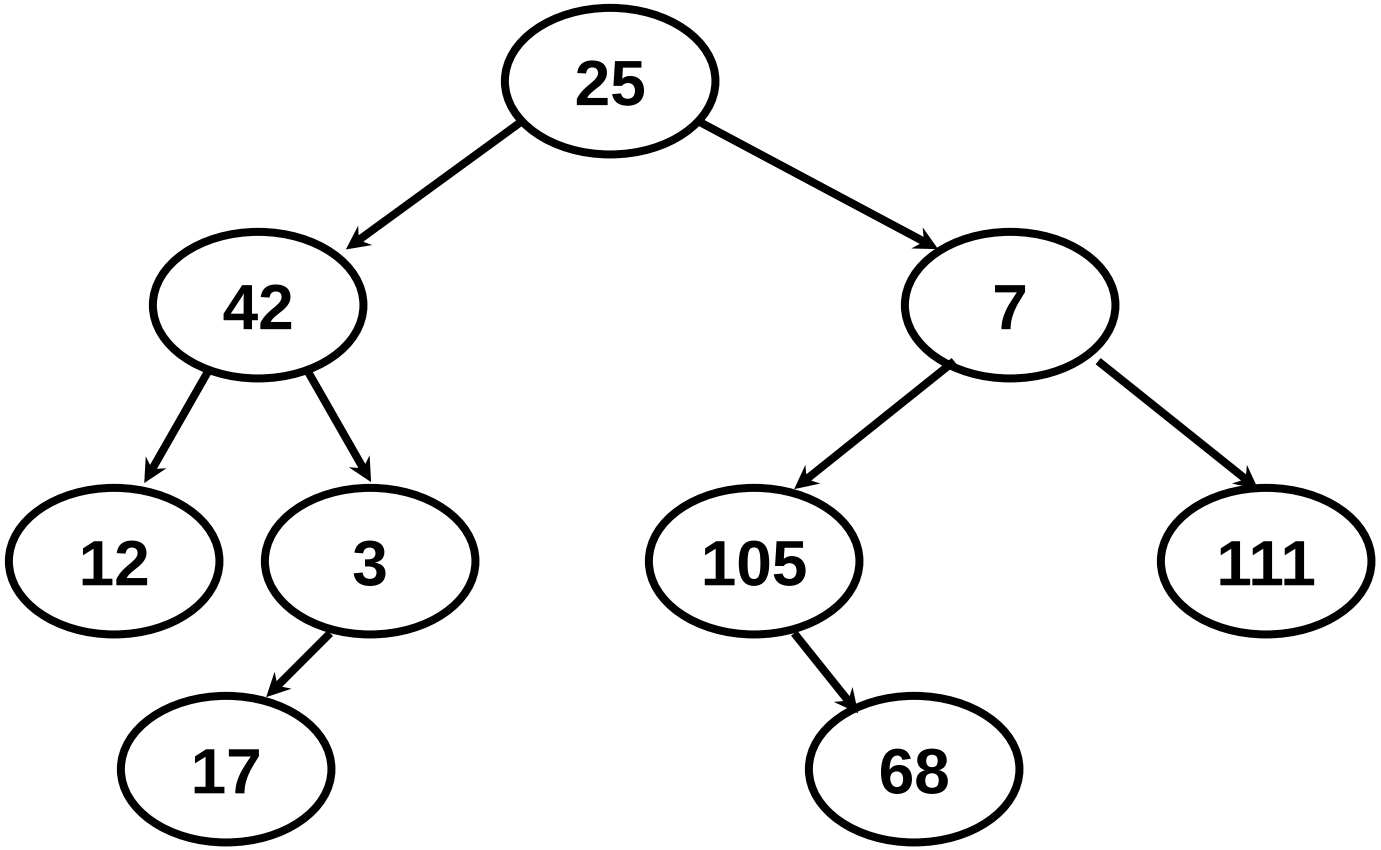
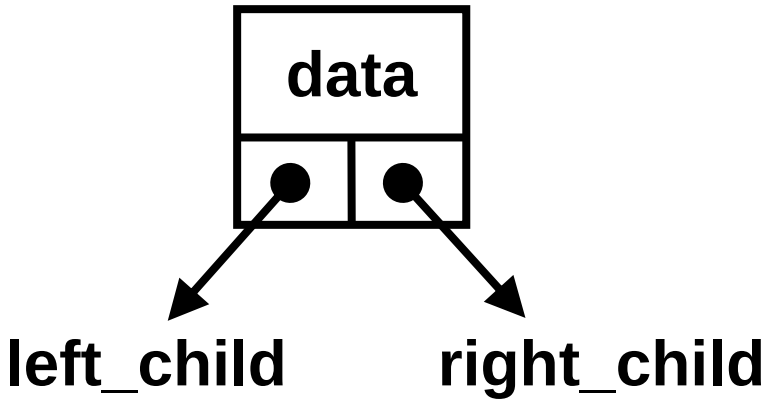
No imposed ordering.



Structure of a Binary Tree Node



Ordered Trees

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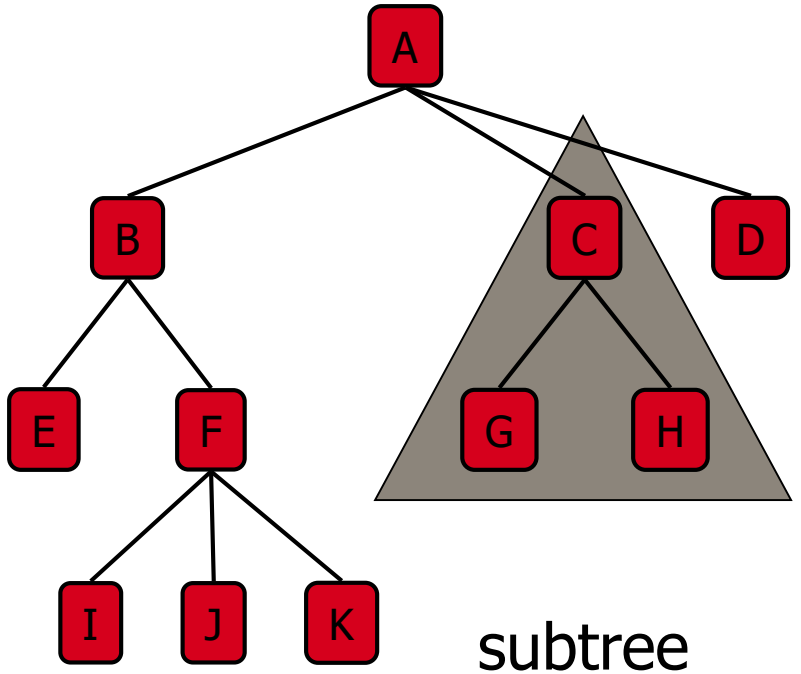
- As well as the parent-child relationship, an ordering may exist between siblings, for example, the left child must be smaller than the right child, or the left child is the first sibling. If it does then the tree is said to be ORDERED.

- In the case of an ordered binary tree the two children are distinguished as the left subtree and the right subtree.

Tree Terminology

- Root: node without parent (A)
- Internal node: node with at least one child (A, B, C, F)
- External node (a.k.a. leaf): node without children (E, I, J, K, G, H, D)
- Ancestors of a node: parent, grandparent, grand-grandparent, etc.
- Depth of a node: number of ancestors
- **Height of a tree:** maximum depth of any node
- Descendant of a node: child, grandchild, grand-grandchild, etc.

- Subtree: tree consisting of a node and its descendants



Implementation of Binary Trees

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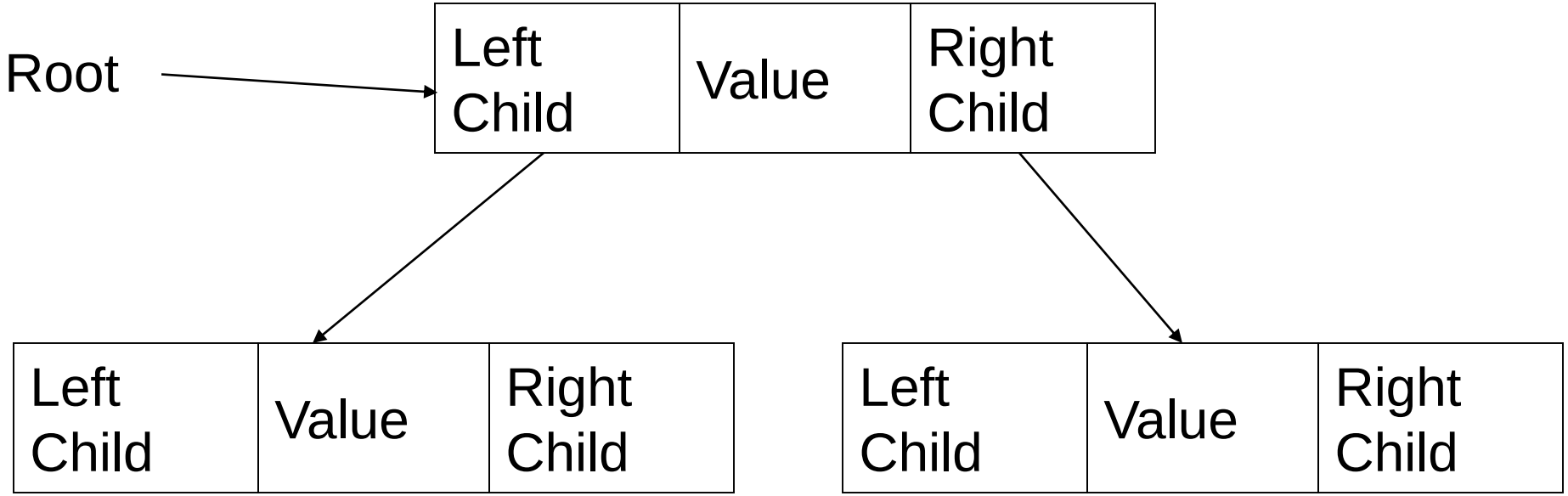
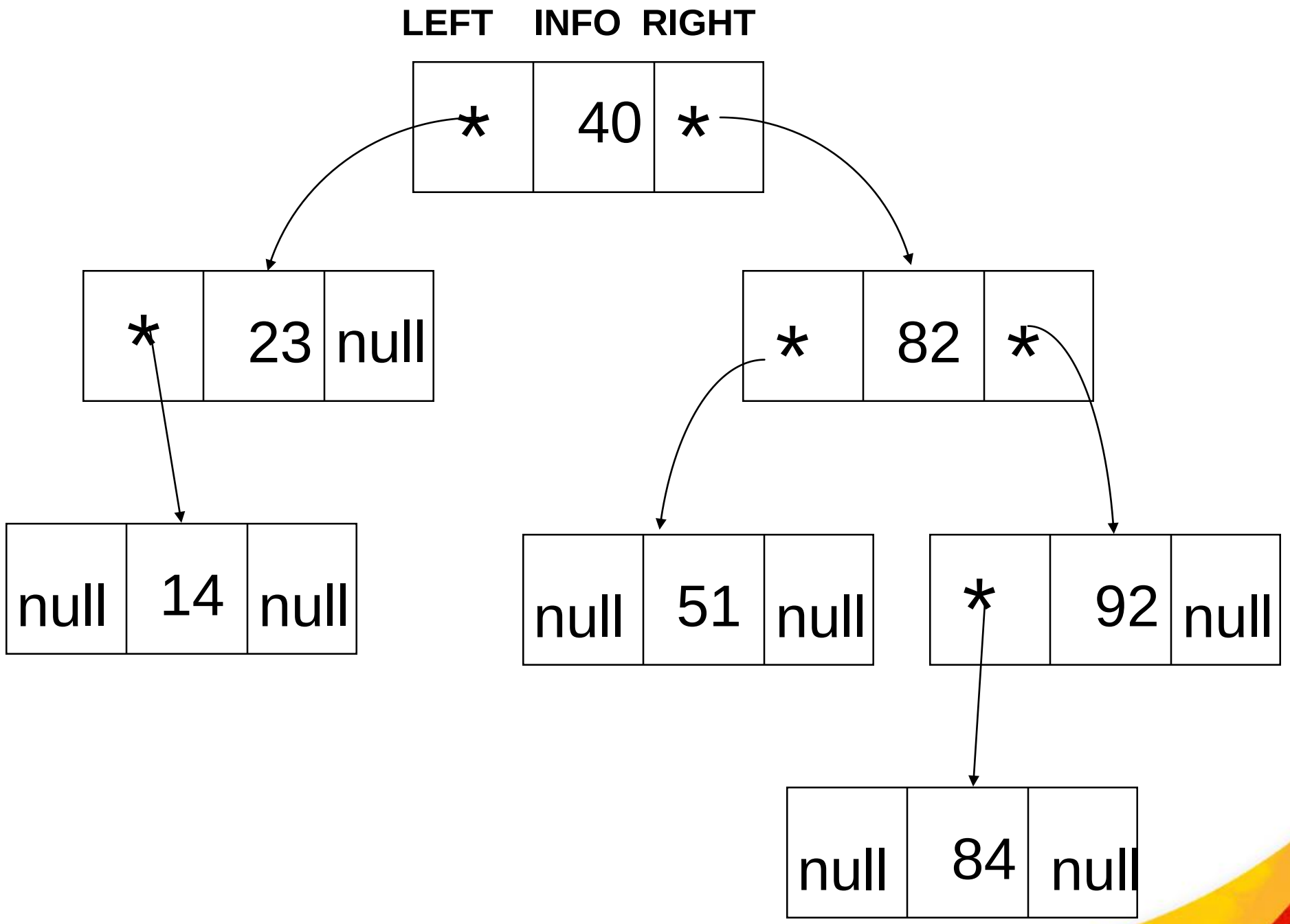
- The basic element of a binary tree is the node.

- This normally requires at least three fields; two to hold pointers to the two subtrees and one to hold the information represented by the node.

Implementing a Binary Tree with Pointers (Data members of TreeNode)



As in the List implementation, a tree is made up of nodes of a helper class. In this case a tree will be made up of TreeNodes. The Tree class will have a root data member that points to the root TreeNode of the tree.


LEFT INFO RIGHT
* 40 *
* 23 null
* 82 *
null 14 null
null 51 null
* 92 null
null 84 null