Trajectory Planning for Underactuated Mechatronic Systems With Unactuated Mechanical Energy Limits: A Power Regulation Perspective

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***Abstract*—In numerous mechatronic systems, there exists a special class where certain states do *not* have directly corresponding control inputs, known as underactuated systems. Currently, several planning and control methods have been developed to ensure the safety of underactuated mechatronic systems by enforcing state constraints, including position and velocity constraints. However, the safety of a system is often closely related to its energy levels. To this end, this paper takes the passivity of underactuated mechatronic systems as a starting point and proposes a trajectory planning approach to limit the unactuated mechanical energy. Specifically, the mechanical energy constraint is first converted into a power constraint, and a power flow valve is designed to ensure them. Through these analyses and designs, the proposed planning approach effectively filters the originally given reference trajectory of the actuated states, which eliminates the power transmission from the actuated subsystem to the unactuated subsystem that violates the passivity. Therefore, the limitation of the unactuated mechanical energy is achieved. Furthermore, the feasibility of the proposed planning approach is verified by numerical simulation on a tower crane.**

***Index Terms*—Underactuated systems, trajectory planning, passivity-based control, energy constraints, power valve.**

I. Introduction

Underactuated mechatronic systems are characterized in the fact that they have fewer independent control inputs than system states [1, 2]. This distinction gives rise to actuated states (with corresponding independent control inputs) and unactuated states (without corresponding independent control inputs). Nevertheless, this does not mean that the unactuated states are uncontrollable. Due to the dynamic coupling (usually nonlinear [3]), the unactuated states are inevitably influenced by the actuated states, which is the physical explanation for why they remain controllable despite lacking direct control inputs. However, it is precisely this dynamic coupling also makes it challenging to control more system states (both actuated and unactuated) with fewer control inputs.

In order to ensure that both the actuated and unactuated states converge to their desired positions, a series of control and planning methods have been developed for underactuated mechatronic systems [4, 5], including backstepping control [6], sliding mode control [7], neural network control [8], etc. Among these methods, passivity-based control (PBC) theory has played a significant role [9]. PBC is a fundamental control strategy derived from an energy perspective, which leverages the passivity property of dynamical systems to ensure stability. Due to its energy-based nature, in other words, the energy function of the system may be a natural Lyapunov function, PBC provides a physically interpretable and intuitive approach to control design, making it a powerful tool for engineering applications [10, 11]. For example, Michel *et al.* propose a passivity-based variable stiffness control method for dynamical systems [12], which utilizes the concept of an energy tank to ensure the passivity of the closed-loop system during interaction with the environment and to achieve asymptotic stability in free motion. In [13], interconnection and damping

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assignment passivity-based control (IDA-PBC) is applied to achieve energy-shaping stabilization of a robotic manipulator under input constraints. Specifically, Xin *et al.*, from an energy perspective, conduct a global swing-up analysis of a soft robot with an actuated constant curvature soft pendulum and an unactuated rotational base joint [14]. For underactuated three-dimensional overhead cranes, a nonlinear controller with injected active damping is proposed in [15], which effectively ensures payload oscillation suppression and robustness.

There are also some noteworthy results on trajectory planning for underactuated mechatronic systems [16, 17]. Reference [18] achieves rest-to-rest trajectory planning for underactuated cable-driven parallel robots under given motion time and predefined path geometry. Huang *et al.* design an open loop method for tower cranes, which suppresses both the swing and twisting of the payload by smoothing reference trajectories online [19]. Also, a trajectory planning algorithm based on offline optimization is reported in [20] and validated on a real tower crane, which enables the payload to follow predefined waypoints. Additionally, time-optimal trajectory planning methods are developed in [21] and [22] for double-pendulum tower cranes and rotary cranes, respectively, where both state constraints and obstacle avoidance are implemented.

From the above literature review, it can be found that although some control/planning methods designed for underactuated mechatronic systems consider position and even velocity constraints [16, 23], their design or solution is relatively complex. In fact, from the perspective of safety assurance, the most direct approach is to prevent injecting too much energy into the system. Since position and velocity are related to gravitational potential energy and kinetic energy, respectively, it is more effective to directly constrain the unactuated mechanical energy rather than separately limiting the position and velocity of the unactuated states. Motivated by this idea, this paper proposes a trajectory planning approach based on PBC, which guarantees unactuated energy constraints for underactuated systems. The advantages of the proposed approach are as follows:

1) The passivity of the unactuated subsystem is proved, from which a "virtual force" representing the power transfer from the actuated subsystem to the unactuated subsystem is identified and utilized for trajectory planning.
2) From the perspective of power regulation, a power valve is designed to control the power flow into the unactuated subsystem, thereby simultaneously realizing both unactuated power constraints and energy constraints.
3) The proposed trajectory planning approach features a simple structure with an analytical formulation, which does not require solving complex optimization problems. This ensures high computational efficiency and is suitable for online applications.

The remainder of this paper is structured as follows. Section II introduces the research problem and planning objective. Section III presents the main theoretical results. Section IV provides simulation results to validate the proposed planning approach. Section V and Section VI discuss its advantages and limitations and conclude the paper, respectively.

II. Problem Statement

The study case in this paper is a class of underactuated mechatronic systems, which can be generally described by the Lagrange equation as

$$\underbrace{\begin{bmatrix} m_{aa} & m_{au} \\ m_{ua} & m_{uu} \end{bmatrix}}_{M(\boldsymbol{q})} \underbrace{\begin{bmatrix} \ddot{\boldsymbol{q}}_a \\ \ddot{\boldsymbol{q}}_u \end{bmatrix}}_{\ddot{\boldsymbol{q}}} + \underbrace{\begin{bmatrix} c_{aa} & c_{au} \\ c_{ua} & c_{uu} \end{bmatrix}}_{C(\boldsymbol{q},\dot{\boldsymbol{q}})} \underbrace{\begin{bmatrix} \dot{\boldsymbol{q}}_a \\ \dot{\boldsymbol{q}}_u \end{bmatrix}}_{\dot{\boldsymbol{q}}} + \underbrace{\begin{bmatrix} \boldsymbol{g}_a \\ \boldsymbol{g}_u \end{bmatrix}}_{\boldsymbol{G}(\boldsymbol{q})} = \underbrace{\begin{bmatrix} \boldsymbol{h} \\ \boldsymbol{0} \end{bmatrix}}_{\boldsymbol{H}}, \quad (1)$$

where $M(\boldsymbol{q}) \in \Re^{n\times n}$ and $C(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \Re^{n\times n}$ represent the inertia matrix and centripetal-Coriolis matrix, respectively, $\boldsymbol{G}(\boldsymbol{q}) \in \Re^n$ and $\boldsymbol{H} \in \Re^n$ represent the gravity vector and control input vector, respectively, and $\boldsymbol{q} = [\boldsymbol{q}_a, \boldsymbol{q}_u]^\top \in \Re^n$ is the state vector. As can be seen from the dynamics (1), the characteristic of underactuated systems is that, compared with fully-actuated systems, some states (namely unactuated states) have no direct corresponding control inputs. Nevertheless, due to the dynamic coupling between the actuated state $\boldsymbol{q}_a \in \Re^{n-u}$ and the unactuated state $\boldsymbol{q}_u \in \Re^u$, the motion of $\boldsymbol{q}_u$ can still be *indirectly* driven by $\boldsymbol{q}_a$. At the same time, it is noted that since $\boldsymbol{q}_a$ is directly driven by the control input $\boldsymbol{h} \in \Re^{n-u}$, the reference trajectory is typically designed for $\boldsymbol{q}_a$. However, there are some implicit challenges in the trajectory planning of underactuated mechatronic systems. For example, if one simply considers $\boldsymbol{q}_a$ to reach its desired position $\boldsymbol{q}_{ad}$, due to inappropriate planning velocity $\dot{\boldsymbol{q}}_a$ or acceleration $\ddot{\boldsymbol{q}}_a$, it is likely that the unactuated state will deviate from its equilibrium position to a large extent, making it difficult to be stabilized for a long time.

In practical applications of underactuated mechatronic systems, since excessive position and velocity pose potential safety risks, it is crucial to limit them. In fact, from a physical perspective, position determines potential energy, while velocity determines kinetic energy. Therefore, to ensure safety, directly limiting the energy of underactuated mechatronic systems is actually a more effective approach. Specifically, the total mechanical energy of underactuated mechatronic systems can be divided into kinetic energy E_k and gravitational potential energy U as

$$\begin{aligned} E_k &= \frac{1}{2}\dot{\boldsymbol{q}}^\top M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &= \frac{1}{2}\dot{\boldsymbol{q}}_a^\top m_{aa}\dot{\boldsymbol{q}}_a + \frac{1}{2}\dot{\boldsymbol{q}}_u^\top m_{uu}\dot{\boldsymbol{q}}_u + \dot{\boldsymbol{q}}_a^\top m_{au}\dot{\boldsymbol{q}}_u, \\ U &= U_a + U_u, \end{aligned} \quad (2)$$

where the three terms in E_k are referred to as the actuated kinetic energy, unactuated kinetic energy, and coupling kinetic energy, respectively, U_a and U_u represent the gravitational potential energy of the actuated and unactuated states, respectively. Although the specific expressions of potential energies are not provided, it is known that they satisfy the property $\partial U/\partial \boldsymbol{q} = \boldsymbol{G}$, $\partial U_u/\partial \boldsymbol{q}_u = \boldsymbol{g}_u$, and $\partial U_a/\partial \boldsymbol{q}_a = \boldsymbol{g}_a$ [24].

Next, we define the actuated mechanical energy E_a and the unactuated mechanical energy E_u as

$$\begin{aligned} E_a &\triangleq \frac{1}{2}\dot{\boldsymbol{q}}_a^\top m_{aa}\dot{\boldsymbol{q}}_a + \dot{\boldsymbol{q}}_a^\top m_{au}\dot{\boldsymbol{q}}_u + U_a, \\ E_u &\triangleq \frac{1}{2}\dot{\boldsymbol{q}}_u^\top m_{uu}\dot{\boldsymbol{q}}_u + U_u. \end{aligned} \tag{3}$$

It is noted that all terms in E_a are related to the actuated states, which, as mentioned above, have direct corresponding control inputs, making E_a relatively easier to control. In contrast, the focus of this paper is to generate feasible motion trajectories for the actuated states online, which can effectively limit E_u, i.e.,

$$E_u(t) \le E_{u\,\max}, \quad \forall t \ge 0, \tag{4}$$

where $E_{u\,\max}$ is the preset upper bound of E_u.

III. Proposed Trajectory Planning Approach

A. Passivity-Based Control Theory

The trajectory planning approach proposed in this paper is formulated by the PBC theory. The main idea of PBC is that a system is passive if it does not generate energy internally, meaning that the total energy dissipation is always greater than or equal to the energy supplied by external inputs. This property allows for a systematic design of controllers that inherently respect physical constraints and ensure stability. It can be summarized as the following lemma:

Lemma 1: [25] A system is passive if the energy absorbed in any period of time $[t_0, t_1]$ is greater than or equal to the stored energy added during the same period of time, i.e.,

$$\int_{t_0}^{t_1} \boldsymbol{y}^\top \boldsymbol{u}\ \mathrm{d}t \ge V\left(\boldsymbol{x}\left(t_1\right)\right) - V\left(\boldsymbol{x}\left(t_0\right)\right), \tag{5}$$

where V is a (scalar) storage function associated with the system state vector $\boldsymbol{x}$, the symbols $\boldsymbol{y}$ and $\boldsymbol{u}$ represent the output and input vectors, respectively. Furthermore, $P \triangleq \boldsymbol{y}^\top \boldsymbol{u}$ is defined as the input power, and its sign indicates whether the system is injected or absorbed energy. Under the conventional notation, $P > 0$ indicates an active system and $P \le 0$ indicates a passive one.

B. Passivity of Unactuated Subsystems

The following actuated subsystem and unactuated subsystem can be obtained by expanding (1) as

$$m_{aa}\ddot{\boldsymbol{q}}_a + m_{au}\ddot{\boldsymbol{q}}_u + c_{aa}\dot{\boldsymbol{q}}_a + c_{au}\dot{\boldsymbol{q}}_u + \boldsymbol{g}_a = \boldsymbol{h}, \tag{6}$$

$$m_{ua}\ddot{\boldsymbol{q}}_a + m_{uu}\ddot{\boldsymbol{q}}_u + c_{ua}\dot{\boldsymbol{q}}_a + c_{uu}\dot{\boldsymbol{q}}_u + \boldsymbol{g}_u = \boldsymbol{0}. \tag{7}$$

Here, the unactuated subsystem (7) will be used to complete trajectory planning for underactuated mechatronic systems. First, we will show the passivity of the unactuated subsystem.

Taking the derivative of the unactuated mechanical energy E_u with respect to time yields that

$$\begin{aligned} \dot{E}_u &= \dot{\boldsymbol{q}}_u^\top m_{uu}\ddot{\boldsymbol{q}}_u + \frac{1}{2}\dot{\boldsymbol{q}}_u^\top \dot{m}_{uu}\dot{\boldsymbol{q}}_u + \dot{U}_u \\ &= \dot{\boldsymbol{q}}_u^\top \left(m_{uu}\ddot{\boldsymbol{q}}_u + \frac{1}{2}\dot{m}_{uu}\dot{\boldsymbol{q}}_u + \frac{\partial U_u}{\partial \boldsymbol{q}_u} \right) \\ &= \dot{\boldsymbol{q}}_u^\top \left(m_{uu}\ddot{\boldsymbol{q}}_u + c_{uu}\dot{\boldsymbol{q}}_u + \boldsymbol{g}_u \right), \end{aligned} \tag{8}$$

where the skew-symmetric property of $\dot{m}_{uu}/2 - c_{uu}$ is used. Further, combining equation (7) and (8), one has that

$$\begin{aligned} \dot{E}_u &= \dot{\boldsymbol{q}}_u^\top \boldsymbol{F}_u, \\ \boldsymbol{F}_u &\triangleq -m_{ua}\ddot{\boldsymbol{q}}_a - c_{ua}\dot{\boldsymbol{q}}_a, \end{aligned} \tag{9}$$

where $\boldsymbol{F}_u$ is defined as a "virtual force" that does work on the unactuated subsystem (7). Obviously, the integral of $\dot{E}_u = \dot{\boldsymbol{q}}_u^\top \boldsymbol{F}_u$ satisfies equation (5), indicating that the unactuated subsystem is a passive system. According to Lemma 1, the "virtual force" $\boldsymbol{F}_u$ can be used to regulate the unactuated mechanical energy E_u.

C. Trajectory Planning via Power Flow Regulation

There are countless trajectories for planning the actuated state $\boldsymbol{q}_a$ to its desired position $\boldsymbol{q}_{ad}$. However, without considering the specific implementation, $\boldsymbol{F}_u$ is likely to inject excessive energy into the unactuated subsystem. In other words, the unactuated subsystem may extract too much energy from the actuated subsystem, which manifests as an increase in unactuated mechanical energy E_u.

To limit E_u, the passivity of the unactuated subsystem indicates that it is feasible to properly design $\boldsymbol{F}_u$, or more specifically, to plan an appropriate reference trajectory for $\boldsymbol{q}_a$. The planning result is presented here directly as

$$\begin{aligned} \ddot{\boldsymbol{q}}_a^\star &= \beta\ddot{\boldsymbol{q}}_a + (\beta - 1)m_{ua}^{-1}c_{ua}\dot{\boldsymbol{q}}_a, \\ \beta &= \begin{cases} P_{u\,\max}/P_u & \text{if } P_u > P_{u\,\max} \\ 1 & \text{otherwise} \end{cases}, \end{aligned} \tag{10}$$

where $\ddot{\boldsymbol{q}}_a^\star$ is the planned acceleration that ensures the energy constraint (4), and β is a valve function used to regulate the power $P_u \triangleq \dot{\boldsymbol{q}}_u^\top \boldsymbol{F}_u$, aiming to prevent P_u from exceeding its upper bound $P_{u\,\max} > 0$. That is to convert the original energy constraint in (4) into a power constraint:

$$P_u \le P_{u\,\max} = \frac{E_{u\,\max} - E_u(k)}{T_s}, \tag{11}$$

where T_s is the sampling period of the control system, and k denotes the k-th sampling period. The following Theorem 1 and its proof guarantee the effectiveness of the proposed planning approach in limiting the unactuated mechanical energy.

Theorem 1: The actuated state trajectory generated by the planning acceleration $\ddot{\boldsymbol{q}}_a^\star$ in (10) ensures that the unactuated mechanical energy does not exceed its constrained upper bound, i.e., the planning objective (4) holds.

Proof: Using the first-order forward Euler approximation to $P_u = \dot{E}_u$ results in

$$E_u(k+1) = E_u(k) + P_u(k)T_s. \tag{12}$$

Then, substituting (11) into (12) naturally derives $E_u(k+1) \le E_{u\,\max}$. Therefore, the following only needs to show that the proposed planning approach guarantees the power constraint

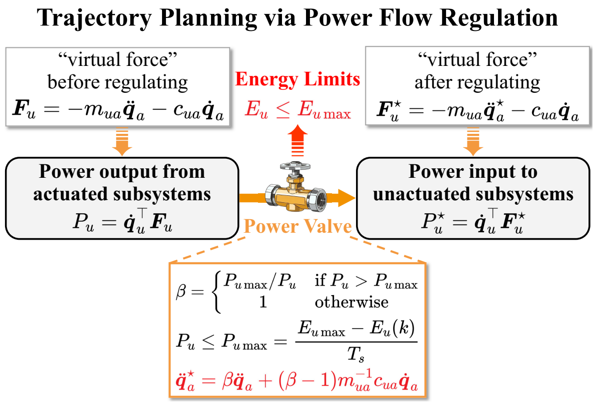

Figure 1: Framework of the proposed trajectory planning approach based on power flow regulation.

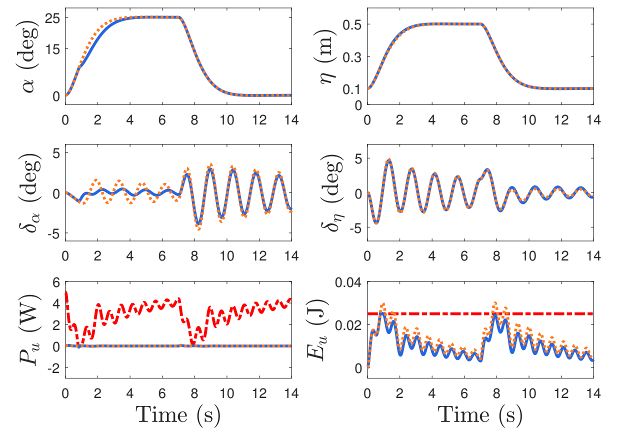

Figure 2: Simulation results: Original trajectories (orange dotted lines), proposed planning approach (blue solid lines), and power/energy upper limits (red dotted-dashed lines).

(11). Next, under the planned $\ddot{q}_a^\star$, the virtual force and its power are further calculated by using (10) as

$$\begin{aligned} F_u^\star &= -m_{ua}\ddot{q}_a^\star - c_{ua}\dot{q}_a \\ &= -\beta m_{ua}\ddot{q}_a - \beta c_{ua}\dot{q}_a \\ &= \beta F_u, \end{aligned} \tag{13}$$

$$P_u^\star = \dot{q}_u^\top F_u^\star = \beta P_u \le P_{u\max}. \tag{14}$$

At this point, it can be concluded that by regulating the power flow from the actuated subsystem to the unactuated subsystem, the proposed planning approach achieves the limitation of the unactuated mechanical energy. ■

Remark 1: The proposed approach can be regarded as a trajectory filter, which safely filters any given reference trajectory of the actuated state to ensure the constraint on unactuated mechanical energy. The physical essence lies in the power flow regulation, which passifies a potential violation of the unactuated subsystem's passivity. Figure 1 illustrates the main framework of the proposed trajectory planning approach based on power flow regulation.

IV. Simulation Results

In this section, a common underactuated mechatronic system, i.e., a tower crane, is used as an example to illustrate the effectiveness of the proposed trajectory planning approach in limiting unactuated mechanical energy. For crane systems, it is of great concern for the safety of the suspended payload with respect to the surrounding environment.

The tower crane has a total of four degrees of freedom: the actuated trolley displacement η, the actuated jib rotation angle α, and the unactuated payload radial swing angle δ_η and tangential swing angle δ_α. The specific dynamic model can be found in [26]. The simulation environment is MATLAB/Simulink 2020b, with a fixed-step solver ($T_s = 5$ ms) set to automatic mode. The other parameters are the trolley mass $M_t = 7$ kg, the payload mass $m = 1$ kg, the jib inertia moment $I = 6.8$ kg $\cdot$ m, and the rope length $l = 0.5$ m.

The initial positions of the trolley and the jib are $\eta_0 = 0.1$ m and $\alpha_0 = 0$ deg, respectively, and the desired positions are specified by the user as

$$\eta_d = \begin{cases} 0.5 \text{ m} & \text{if } t \le 7 \text{ s} \\ 0 \text{ m} & \text{if } t > 7 \text{ s} \end{cases}, \quad \alpha_d = \begin{cases} 25 \text{ deg} & \text{if } t \le 7 \text{ s} \\ 0 \text{ deg} & \text{if } t > 7 \text{ s} \end{cases}. \tag{15}$$

The purpose is to simulate the entire working cycle of a tower crane, where the material is lifted from one point, transported to another, and then returned. The acceleration signal of the original reference trajectory $q_a(t) = [\alpha(t), \eta(t)]^\top$ is generated using a classical proportional-derivative (PD) algorithm:

$$\ddot{q}_a = -T_p(q_a - q_{ad}) - T_d\dot{q}_a, \tag{16}$$

where $T_p = \mathrm{diag}(2,2)$ and $T_d = \mathrm{diag}(2.5, 2.5)$ are selected. The energy upper limit is set to $E_{u\max} = 0.025$ J. It is noted that the proposed approach has *no* parameters that need to be adjusted, which is also its advantage over other planning algorithms.

The simulation results are shown in Figure 2. It can be observed that the original reference trajectory of the actuated states injected excessive energy into the unactuated subsystem, which causes the unactuated mechanical energy E_u to exceed its prescribed limit $E_{u\max}$. In contrast, the proposed trajectory planning approach effectively constrained E_u within $E_{u\max}$ by regulating the input power P_u to remain below the power limit $P_{u\max}$. This fact is also reflected in the filtering effect of the proposed approach on the original reference trajectories of the actuated states. Specifically, it filters out reference trajectories, or power injections, that violate the passivity of the unactuated subsystem. As a result, the payload swing angles are smaller and slower compared with the unfiltered case, which is particularly evident in δ_α.

V. Discussions

Utilizing the passivity to design controllers is an effective and widespread approach, as passivity analysis provides strong theoretical support for stability and energy management. In this work, we propose a trajectory planning approach based on the passivity properties of underactuated mechatronic systems. By regulating the power flow from the actuated subsystem to the unactuated subsystem, the proposed approach effectively limits the unactuated mechanical energy, thereby enhancing system safety. However, the current version is designed under the assumption of a known model. When there are parameter uncertainties or structural variations, the performance may degrade. Additionally, the current power valve is relatively straightforward, which may result in frequent switching operations. Therefore, future research could focus on optimizing the power regulation strategy, such as introducing a smoother power adjustment scheme to reduce switching frequency and improve system stability. Moreover, integrating predictive control concepts could enable the power valve to allocate power more efficiently based on anticipated future states.

VI. Conclusions

This paper has proposed a trajectory planning approach for underactuated mechatronic systems, which effectively enhances system safety by limiting the unactuated mechanical energy. First, based on system dynamics analysis, the passivity of the unactuated subsystem is proven. Then, within the framework of PBC theory, a power flow valve is designed to dissipate power inputs that may potentially violate the passivity of the unactuated subsystem. By converting the constraint on unactuated mechanical energy into the unactuated power constraint, the proposed approach achieves precise energy regulation while generating trajectory planning results. Finally, numerical simulations on a tower crane validate the effectiveness of the proposed trajectory planning approach and demonstrate its feasibility for underactuated systems. In the future, we will further optimize its robustness, particularly its ability to handle model uncertainties, thereby enhancing its applicability in complex engineering environments.

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