Oscillations Damping Control of Variable Cable Length Pendulum Systems by Gain Adaptive MPC

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Variable cable length pendulum systems are typical underactuated systems that are widely present in various hoisting equipments in practice. However, unlike general underactuated systems, there is only velocity (not acceleration) coupling between the cable subsystem and payload subsystem, which sometimes leads to the payload subsystem being uncontrollable. Therefore, the control problem of variable cable length pendulum systems is very challenging. Existing studies on variable cable length pendulum systems usually design trajectories for the cable length to achieve damping oscillations. However, trajectory planning methods do not take into account the actuator constraints of the actual system, especially the physical constraint that the acceleration of the cable lengthening cannot exceed the gravitational acceleration. To this end, a gain adaptive model predictive control (MPC) is designed in this paper for variable cable length pendulum systems. In order to take full advantage of the coupling between the two subsystems, the proposed MPC is built on the original nonlinear dynamic model. The designed gain adaptive algorithm ensures the convergence speed of the damping oscillations. Numerical simulations verify the control and oscillations elimination performance of the proposed gain adaptive MPC.

Keywords: Variable cable length pendulum systems, model predictive control (MPC), gain adaptive, damping oscillations.

1. Introduction

Variable cable length pendulum systems are widely used in practice, especially in various hoisting systems, including crane systems [1–6], unmanned aerial vehicle-suspended payload system [7], etc. One of the main control tasks of these hoisting systems is to prevent payload oscillations as much as possible. However, most studies only consider the case that the cable length is fixed [8–12], and oscillations elimination by utilizing variable length cable has rarely been investigated.

Variable cable pendulum systems have only one control input and the payload oscillation angle cannot be directly controlled, which are typical underactuated systems [13–18]. However, variable cable length pendulum systems differ from general underactuated systems where there is only velocity but no acceleration coupling between the actuated subsystem (i.e., the cable subsystem) and the unactuated subsystem (i.e., the payload subsystem), which indicates that their coupling is quite weak. In particular, when the oscillation angle angular velocity is 0, no matter how the cable length changes, the oscillation angle is not controllable, which reveals that variable cable length pendulum systems are "strong" underactuated systems. Variable cable length pendulum systems are somewhat similar to swing systems and the difference is that the swing system is for swinging up, while the variable cable length pendulum system is for preventing oscillations. An intuitive judgment is that, since swing systems can oscillate higher and higher through changes in body center of gravity [19], there must also be a way to dampen the oscillation only by changing the cable length.

However, achieving automatic oscillations elimination only by changing the cable length is a very difficult problem [20]. Variable cable length pendulum systems are not only "strong" underactuated systems but also nonlinear systems [21, 22], and the nonlinearities in the systems are not always unfavorable for the control objective of oscillations elimination. If the variable cable length pendulum dynamics are linearized, the dynamic coupling between the cable subsystem and the payload subsystem can hardly be covered, and the payload oscillation angle is almost impossible to control indirectly by the cable. Therefore, linearization methods are difficult to apply in variable cable pendulum systems.

The studies in [20] have shown that the Coriolis force is the only way to couple the two subsystems. It is well known that the Coriolis force is a velocity-dependent force, which also corresponds to the fact that only velocity coupling exists in variable cable length pendulum systems. However, the Coriolis force is not always conducive to oscillations elimination. When the cable length changes, the Coriolis force sometimes will dampen the oscillations, and sometimes will amplify the oscillations. Therefore, the timing of changing the cable length must be strictly controlled to make the damping effect greater than the amplification effect and finally realize oscillations elimination. Most existing studies on the control of variable cable pendulum systems are open-loop methods.

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Reference [23] designs a feasible anti-oscillation trajectory for the cable length. In order to avoid the disturbance sensitivity of the trajectory planning methods, reference [24] develops a time-delay control algorithm based on [23] to improve system robustness. In addition, an energy-based control law was developed in [25] to achieve the desired oscillation motion.

However, these methods ignore the physical fact that the force acting on the cable cannot be thrust, that is, the acceleration of the cable lengthening cannot exceed the gravitational acceleration. To this end, a closed-loop control method of gain adaptive based on model predictive control (MPC) is proposed for variable cable length pendulum systems in this paper. The main advantages are summarized as follows:

(1) It is a control method with forward-looking prediction. By using the embedded integrator, it can guarantee the tracking performance with good robustness, and the cable length positioning and the payload oscillations damping can be achieved simultaneously.

(2) Actuator constraints, including control input constraints and incremental constraints are taken into account, which theoretically avoids applying thrust to the system and is beneficial to practical applications.

(3) The designed gain adaptive algorithm realizes that the cable length has enough anti-oscillation actions even in the case of small oscillation angles, which ensures the speed of eliminating the oscillation angle.

The rest of the paper is organized as follows. Section 2 clarifies the control problem and research motivations. The gain adaptive MPC for variable cable pendulum systems is illustrated in Section 3. In Section 4, some simulation results verify the effectiveness of the proposed control method. The conclusions are summarized in Section 5.

2. Motivations

The diagram of variable cable length pendulum systems is shown in Fig. 1, where a cable is connected to a payload, and the change of cable length can be controlled by the motor rotation. By utilizing the Lagrangian modeling technique, the dynamic equations of variable cable length pendulum systems can be described as follows:

$$m\ddot{l} - ml\dot{\phi}^2 - mg\cos\phi = F_u, \tag{1}$$

$$ml^2\ddot{\phi} + 2ml\dot{\phi}\dot{l} + mgl\sin\phi = 0, \tag{2}$$

where (1) represents the cable subsystem, (2) represents the payload subsystem, m and g stand for the payload mass and gravitational acceleration, respectively, l and ϕ denote the cable length and oscillation angle, respectively, and F_u is the control input.

It is noted that variable cable length pendulum systems have two state variables l and ϕ, but only one control input F_u acting on the cable subsystem, that is, ϕ has no direct control input. Therefore, variable cable length pendulum systems are typical underactuated systems. However, variable cable length pendulum systems have their particularity with general underactuated systems. It can be observed from (1) and (2) that there is only velocity but no acceleration coupling between the cable subsystem and the payload subsystem, which means that the coupling between the two subsystems is rather

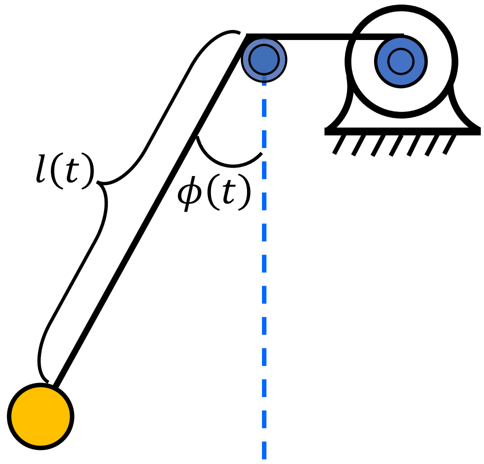

Fig. 1. Variable cable length pendulum systems diagram.

weak. In particular, the payload subsystem (2) can be rewritten as

$$\ddot{\phi} + \frac{2\dot{\phi}\dot{l}}{l} + \frac{g}{l}\sin\phi = 0, \tag{3}$$

where $\frac{g}{l}\sin\phi$ is the restoring force, which keeps the pendulum oscillating, and $\frac{2\dot{\phi}\dot{l}}{l}$ is the Coriolis force, which is the *only* possible force controlling the payload subsystem. However, when either $\dot{\phi}$ or $\dot{l}$ is 0, the Coriolis force will be 0. In other words, we cannot expect any change in cable length to affect the payload oscillation angle. At this point, the payload subsystem is *not* controllable. In conclusion, the control of variable cable length pendulum systems is difficult.

In the work of [23], it is proved that Coriolis force can be used to damp the pendulum oscillation, and the key is to adjust the cable length at the right time. Specifically, when the cable length is extended, i.e., $\dot{l} > 0$, the Coriolis force is opposite to the oscillation motion direction, which will dampen the oscillation. On the other hand, when the cable is shortened, i.e., $\dot{l} < 0$, the Coriolis force acts in the same direction as the oscillation, which will amplify the oscillation. Therefore, the key idea to suppress oscillations is to extend the cable length ($\dot{l} > 0$) when $|\phi|$ is close to its maximum value, i.e., close to $\dot{\phi} = 0$, and to shorten the cable length ($\dot{l} < 0$) when $\phi \approx 0$, i.e., close to the inflection point of the pendulum motion. In this way, the damping effect of Coriolis force (close to $\dot{\phi} = 0$) is greater than its amplifying effect (close to the turning point) during the whole cycle.

Consequently, based on the ideas above, it is proposed and studied in [20] and [23] to eliminate the oscillation by using the cable length change signal, which is double the frequency of the oscillations. In this way, sometimes the oscillations will be amplified, sometimes will be suppressed, but the suppression effect will be greater than the amplification effect, and finally eliminate the oscillations. On this basis, the time-delay control algorithm is developed in [24]. However, these methods are all trajectory planning for the cable length, and do not provide a direct solution to the problem from the perspective of control. More importantly, the cable can only be subjected to tension, which means that $F_{u\max} = 0$ and the cable length is extended only by gravity. In other words, the maximum acceleration of the cable lengthening will not exceed the gravitational acceleration, i.e., $\ddot{l} \le g$. However, these trajectory planning methods are likely to violate this physical constraint, making it difficult to implement in practice. What's more, they are usually difficult to adapt to changing working conditions and actuator constraints.

Therefore, to solve the above problems, from the perspective of control rather than trajectory planning, it is necessary to design a closed-loop control method for variable cable length pendulum systems. The gain adaptive MPC is proposed in this paper to achieve the following objectives:

(1) **Positioning performance:** Control the cable length to the desired position and simultaneously suppress payload oscillations, i.e., $\lim_{t\to\infty} l(t) = l_d$, $\lim_{t\to\infty} \phi(t) = 0$.

(2) **Actuator constraints:** Limit the control input and control input increment to the allowable ranges, i.e., $F_{u\min} \le F_u \le F_{u\max}$ and $|\Delta F_u| \le \Delta F_{u\max}$.

3. Model Predictive Control Design

In this section, we propose a gain adaptive MPC for variable cable length pendulum systems to achieve the above control objectives.

3.1 Prediction Model

For linear MPC, the prediction model is usually derived based on the linearized system dynamics model. By defining $\boldsymbol{x} = [l, \phi, \dot{l}, \dot{\phi}]$, the variable cable length pendulum model (1) and (2) can be linearized as

$$\dot{\boldsymbol{x}} = \bar{A}\boldsymbol{x} + \bar{\boldsymbol{B}}F_u, \tag{4}$$

where

$$\bar{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{g}{l} & 0 & 0 \end{bmatrix}, \quad \bar{\boldsymbol{B}} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m} \\ 0 \end{bmatrix}. \tag{5}$$

However, in this way, one can see that l-subsystem and ϕ-subsystem are almost completely decoupled:

$$\begin{cases} \ddot{l} = \frac{1}{m}F_u \\ \ddot{\phi} = -\frac{g}{l}\phi \end{cases}, \tag{6}$$

which means that the pendulum is *not* controllable based on the linear systems controllability criterion. In other words, it is not possible to achieve oscillations elimination with the linearized dynamic model (4).

Thus, based on (1) and (2) without any linearization operation, the following state-space model is established:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{B}F_u + \boldsymbol{F}_d, \tag{7}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \dot{\phi}^2 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\dot{\phi}}{l} & -\frac{\dot{l}}{l} \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m} \\ 0 \end{bmatrix},$$

$$\boldsymbol{F}_d = \begin{bmatrix} 0 \\ 0 \\ g\cos\phi \\ -\frac{g}{l}\sin\phi \end{bmatrix}. \tag{8}$$

To facilitate the construction of the MPC, it is necessary to discretize the continuous system (7) by utilizing the forward Euler method as

$$\begin{aligned} \boldsymbol{x}(k+1) &= A_p\boldsymbol{x}(k) + \boldsymbol{B}_pF_u(k) + \boldsymbol{F}_{pd}, \\ \boldsymbol{y}(k) &= C_p\boldsymbol{x}(k), \quad C_p = I_{4\times4}, \end{aligned} \tag{9}$$

where $A_p = I_{4\times4} + TA$, $\boldsymbol{B}_p = T\boldsymbol{B}$, $\boldsymbol{F}_{pd} = T\boldsymbol{F}_d$, T is the sampling period, and $\boldsymbol{y}(k)$ represents the system output. Then, by performing difference operations on (9), the following discrete model is obtained:

$$\Delta\boldsymbol{x}(k+1) = A_p\Delta\boldsymbol{x}(k) + \boldsymbol{B}_p\Delta F_u(k) + \boldsymbol{F}_{pd}, \tag{10}$$

$$\begin{aligned} \Delta\boldsymbol{y}(k+1) &= C_p\Delta\boldsymbol{x}(k+1) \\ &= C_pA_p\Delta\boldsymbol{x}(k) + C_pB_p\Delta F_u(k) + C_p\boldsymbol{F}_{pd}, \end{aligned} \tag{11}$$

where $\Delta\boldsymbol{x}(k) = \boldsymbol{x}(k) - \boldsymbol{x}(k-1)$, $\Delta\boldsymbol{y}(k) = \boldsymbol{y}(k) - \boldsymbol{y}(k-1)$, and $\Delta F_u(k) = F_u(k) - F_u(k-1)$.

Although the prediction model can be derived from the differential dynamic model (10) and (11) theoretically, we introduce an embedded integrator by defining $\boldsymbol{x}_e(k) = [\Delta\boldsymbol{x}(k)^\top, \boldsymbol{y}(k)^\top]^\top$ to eliminate the steady-state errors and improve tracking performance. Consequently, a generalized discrete augmented model according to (10) and (11) can be written as

$$\begin{aligned} \boldsymbol{x}_e(k+1) &= A_e\boldsymbol{x}_e(k) + \boldsymbol{B}_e\Delta F_u(k) + \boldsymbol{F}_{ed}, \\ \boldsymbol{y}(k) &= C_e\boldsymbol{x}_e(k), \end{aligned} \tag{12}$$

where

$$A_e = \begin{bmatrix} A_p & O_{4\times4} \\ C_pA_p & I_{4\times4} \end{bmatrix}, \quad B_e = \begin{bmatrix} B_p \\ C_pB_p \end{bmatrix},$$

$$\boldsymbol{F}_{ed} = \begin{bmatrix} \boldsymbol{F}_{pd} \\ O_{4\times4} \end{bmatrix}, \quad C_e = \begin{bmatrix} O_{4\times4} & I_{4\times4} \end{bmatrix}. \tag{13}$$

Therefore, by using the augmented model (12) to iteratively calculate the prediction model as

$$\boldsymbol{Y} = F\boldsymbol{x}_e(k) + \Phi_u\Delta\boldsymbol{U} + \Phi_d\boldsymbol{D}, \tag{14}$$

where

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{y}(k+1 \mid k) \\ \boldsymbol{y}(k+2 \mid k) \\ \vdots \\ \boldsymbol{y}\left(k+N_p \mid k\right) \end{bmatrix}, \quad F = \begin{bmatrix} A_p \\ A_p^2 \\ \vdots \\ A_p^{N_p} \end{bmatrix},$$

$$\Delta\boldsymbol{U} = \begin{bmatrix} \Delta F_u(k) \\ \Delta F_u(k+1) \\ \vdots \\ \Delta F_u(k+N_c-1) \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} \boldsymbol{F}_{pd}(k) \\ \boldsymbol{F}_{pd}(k+1) \\ \vdots \\ \boldsymbol{F}_{pd}(k+N_c-1) \end{bmatrix},$$

$$\Phi_u = \begin{bmatrix} B_p & & & \\ A_pB_p & B_p & & \\ \vdots & & \ddots & \\ A_p^{N_p-1}B_p & A_p^{N_p-2}B_p & \cdots & A_p^{N_p-N_c}B_p \end{bmatrix},$$

$$\Phi_d = \begin{bmatrix} I & & & \\ A_p & I & & \\ \vdots & & \ddots & \\ A_p^{N_p-1} & A_p^{N_p-2} & \cdots & A_p^{N_p-N_c} \end{bmatrix}, \tag{15}$$

$\boldsymbol{Y}$ denotes the output prediction vector, $\Delta\boldsymbol{U}$ is the future control increment vector, and N_p and N_c stand for the prediction and the control horizon respectively with $N_c \le N_p$.

3.2 Control Input Solution

Variable cable length pendulum systems' desired output vector is $\boldsymbol{y}_d = C_p \boldsymbol{x}_d = [l_d, 0, 0, 0]^\top$. To improve control quality, a reference trajectory $\boldsymbol{s}(k)$ with a time-varying soften factor $\rho(t)$ is defined as

$$\begin{aligned} \boldsymbol{s}(k) &= \rho(t)\boldsymbol{y}(k-1) + (1-\rho(t))\boldsymbol{y}_d, \\ \rho(t) &= \rho_0 \exp\left(-\varrho \cdot (kT)^2\right), \end{aligned} \tag{16}$$

where $\rho_0 \in (0, 1]$ denotes the initial soften factor and $\varrho > 0$ is the exponential convergence time factor. Based on (14) and (16), the tracking target $\boldsymbol{S}$ of the prediction model output $\boldsymbol{Y}$ can be represented as follows:

$$\boldsymbol{S} = \left[\overbrace{\begin{matrix} \boldsymbol{s}^\top(k) & \boldsymbol{s}^\top(k) & \cdots & \boldsymbol{s}^\top(k) \end{matrix}}^{N_p} \right]^\top. \tag{17}$$

Then, the cost function is defined as

$$\begin{aligned} J =& (\boldsymbol{S}-\boldsymbol{Y})^\top \bar{Q} (\boldsymbol{S}-\boldsymbol{Y}) + \Delta\boldsymbol{U}^\top \bar{R} \Delta\boldsymbol{U} \\ =& [\boldsymbol{S} - F\boldsymbol{x}_e(k) - \Phi_u \Delta\boldsymbol{U} - \Phi_d \boldsymbol{D}]^\top \bar{Q} \times \\ & [\boldsymbol{S} - F\boldsymbol{x}_e(k) - \Phi_u \Delta\boldsymbol{U} - \Phi_d \boldsymbol{D}] + \Delta\boldsymbol{U}^\top \bar{R} \Delta\boldsymbol{U} \\ =& \Delta\boldsymbol{U}^\top \left(\Phi_u^\top \bar{Q} \Phi_u + \bar{R}\right) \Delta\boldsymbol{U} - 2\Delta\boldsymbol{U}^\top \Phi_u^\top \bar{Q} \times \\ & [\boldsymbol{S} - F\boldsymbol{x}_e(k) - \Phi_d \boldsymbol{D}] \\ & + [\boldsymbol{S} - F\boldsymbol{x}_e(k) - \Phi_d \boldsymbol{D}]^\top \bar{Q} \times \\ & [\boldsymbol{S} - F\boldsymbol{x}_e(k) - \Phi_d \boldsymbol{D}], \end{aligned} \tag{18}$$

where $\bar{Q}$ and $\bar{R}$ denote the positive definite weight matrices, expressed as

$$\begin{aligned} \bar{Q} &= \text{diag}\{\overbrace{Q\ Q\ \cdots\ Q}^{N_p}\}, \quad Q = \text{diag}\{\bar{q}_1\ \bar{q}_2\ \bar{q}_3\ \bar{q}_4\}, \\ \bar{R} &= \text{diag}\{\overbrace{\bar{r}\ \bar{r}\ \cdots\ \bar{r}}^{N_c}\}. \end{aligned} \tag{19}$$

Note that the last term of (18) is a constant term, whose value does not affect the optimization of J. Therefore, it is discarded in the following optimization problem:

$$\begin{aligned} \min_{\Delta\boldsymbol{U}} J &= \frac{1}{2} \Delta\boldsymbol{U}^\top \psi \Delta\boldsymbol{U} + \Delta\boldsymbol{U}^\top \Omega, \\ \psi &= 2\left(\Phi_u^\top \bar{Q} \Phi_u + \bar{R}\right), \\ \Omega &= -2\Phi_u^\top \bar{Q} [\boldsymbol{S} - F\boldsymbol{x}_e(k) - \Phi_d \boldsymbol{D}]. \end{aligned} \tag{20}$$

Obviously, (20) is a typical quadratic programming problem, and the optimal $\Delta\boldsymbol{U}$ can be solved by using some maturity solving algorithms, e.g., the "`quadprog`" algorithm in MATLAB.

3.3 Actuator Constraints and Gain Adaptive

For practical variable cable length pendulum systems, owing to the physical constraints of actuators, we need to keep the control inputs within the allowed ranges. In particular, the force acting on the cable can only be *pulling* and not pushing, which means $F_{u\max} = 0$. In addition, the control input increment also need to be bounded in the automatic control of variable cable length pendulum systems.

To convert the control input and incremental constraints into the constraints on $\Delta\boldsymbol{U}$, the following inequalities are constructed:

$$W\Delta\boldsymbol{U} \le \boldsymbol{\beta}, \quad W\Delta\boldsymbol{U} \le \boldsymbol{\gamma}, \tag{21}$$

where

$$\begin{aligned} W &= \overbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \\ -1 & 0 & \cdots & 0 \end{bmatrix}}^{N_c}, \\ \boldsymbol{\beta} &= \begin{bmatrix} F_{u\min} - F_u(k-1) \\ F_{u\max} - F_u(k-1) \end{bmatrix}, \\ \boldsymbol{\gamma} &= \begin{bmatrix} \Delta F_{u\max} \\ \Delta F_{u\max} \end{bmatrix}. \end{aligned} \tag{22}$$

Therefore, the optimization problem becomes

$$\begin{aligned} &\min_{\Delta\boldsymbol{U}} J = \frac{1}{2} \Delta\boldsymbol{U}^\top \psi \Delta\boldsymbol{U} + \Delta\boldsymbol{U}^\top \Omega, \\ &\text{subject to} \quad W\Delta\boldsymbol{U} \le \boldsymbol{\beta}, \quad W\Delta\boldsymbol{U} \le \boldsymbol{\gamma}. \end{aligned} \tag{23}$$

Then, the optimal $\Delta\boldsymbol{U}$ is calculated to minimize J and only the first element of $\Delta\boldsymbol{U}$ is applied to the system.

Repeating these steps, the cable length can be controlled to its desired position and the oscillation will be eliminated. Seemingly, the problem has been solved. However, there is a slight drawback currently. With the process of oscillations elimination, the oscillation angle and angular velocity will become smaller and smaller, which means that the coupling between the cable and the payload is also getting weaker and weaker. If the weights in Q are constant, the oscillations elimination actions on the cable will become smaller and smaller. Hence, the oscillations elimination speed will become slower and slower. To better eliminate the oscillation through the change of cable length, gain adaptive is necessary.

By the analysis above, it is obvious that the gain $\bar{q}_2$ corresponding to the oscillation angle should increase as the oscillation angle decreases. However, how and how fast $\bar{q}_2$ increases is an open problem. In this paper, we present a heuristic algorithm as follows:

$$\bar{q}_2(k) = \frac{J_\Theta(k-1)J(k)}{N_p \Theta^\top(k)\Theta(k)J(k-1)}, \tag{24}$$

where

$$\begin{aligned} J_\Theta(k) &= N_p \bar{q}_2(k) \Theta^\top(k)\Theta(k), \\ \Theta(k) &= \begin{bmatrix} \phi(k+1 \mid k) \\ \phi(k+2 \mid k) \\ \vdots \\ \phi\left(k+N_p \mid k\right) \end{bmatrix}. \end{aligned} \tag{25}$$

The proposed gain adaptive algorithm ensures that the ratio of the cost corresponding to the oscillation angle to the total cost remains constant in each control period, i.e., the following fact holds:

$$\frac{J_\Theta(k)}{J(k)} = \frac{J_\Theta(k-1)}{J(k-1)}, \tag{26}$$

which ensures that even when the oscillation angle is small, the cable can still have a large enough oscillations elimination actions to accelerate the convergence of the pendulum.

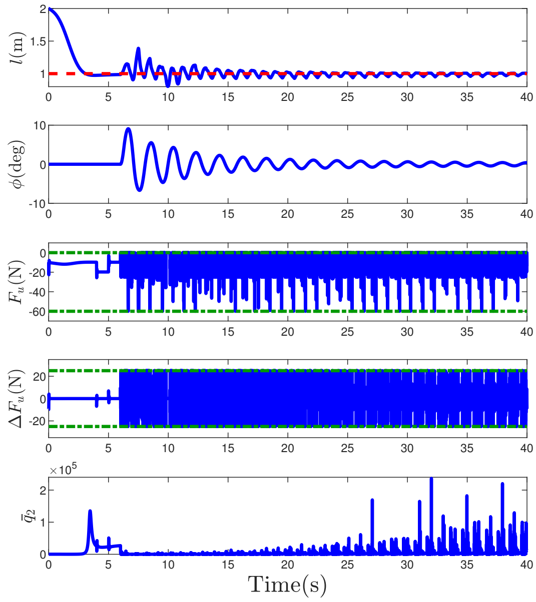

Fig. 2. Scenario 1 with gain adaptive MPC. The dynamic response (blue solid lines), the preset constraints (green dotted-dashed lines), and the desired position (red dashed line).

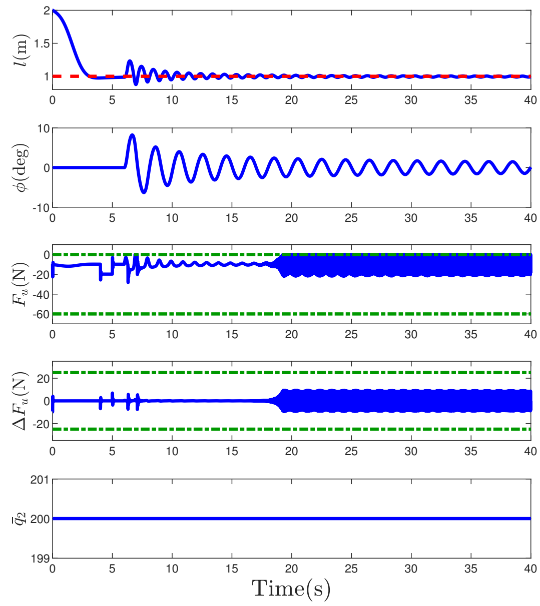

Fig. 3. Scenario 1 with fixed gains MPC. The dynamic response (blue solid lines), the preset constraints (green dotted-dashed lines), and the desired position (red dashed line).

4. Simulation Results

In this section, two different scenarios are considered to verify the control performance of the proposed gain adaptive MPC.

In order to verify the performance of the proposed control method, some simulation tests are carried out in the MATLAB/Simulink environment, and the control period is selected as $T = 5$ ms. At the same time, the parameters and initial/desired positions of the variable cable length pendulum systems are set as

$$m = 1 \text{ kg},\ g = 9.8 \text{ m/s}^2,$$
$$l(0) = 2 \text{ m},\ l_d = 1 \text{ m},\ \phi(0) = 0 \text{ deg},\ \phi_d = 0 \text{ deg}. \quad (27)$$

The controller gains/parameters and constraints are set as

$$\bar{q}_1 = 1,\ \bar{q}_2(0) = 200,\ \bar{q}_3 = 0.01,\ \bar{q}_4 = 0.01,$$
$$F_{u\min} = -60 \text{ N},\ F_{u\max} = 0 \text{ N},\ \Delta F_{u\max} = 25 \text{ N}. \quad (28)$$

In order to more comprehensively verify the effectiveness of the proposed method, we arrange the following two scenarios for simulation:

(1) **Scenario 1: External disturbances.** The external disturbance $d_a = 10$ N is applied to the cable system at 4 s–5 s, and the external disturbance $d_u = 2$ N is applied to the payload subsystem at 6 s–6.3 s. Meanwhile, the MPC with the fixed gain $\bar{q}_2 = 200$ is chosen for comparison.

(2) **Scenario 2: Initial oscillation angle and model uncertainties.** The initial oscillation angle is reset as $\theta(0) = 10$ deg. The payload mass is set as 0.8 kg in the controller, which is 20% less than the actual mass.

For Scenario 1, the simulation results are presented in Fig. 2 and Fig. 3. Both the fixed gains MPC and gain-scheduling MPC can effectively deal with external disturbances, which ensure that the cable length does not deviate from the desired position and eliminate oscillations. The actuator constraints are also fully guaranteed. However, by comparing Fig. 2 and Fig. 3, it can be found that for the MPC with fixed gains, the variation range of the cable length becomes smaller and smaller, which leads to lower and lower anti-oscillation efficiency. In contrast, the MPC with gain adaptive has enough movement even when the oscillation angle is small, which ensures the efficiency of oscillations elimination. The last subplot in Fig. 2 shows the change of $\bar{q}_2$ under the gain adaptive algorithm (24). Similarly, even for variable cable length pendulum systems with initial oscillation angle and model

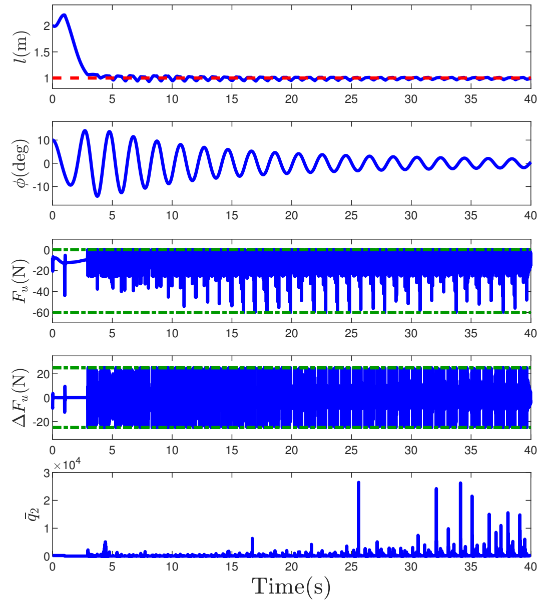

Fig. 4. Scenario 2 with gain adaptive MPC. The dynamic response (blue solid lines), the preset constraints (green dotted-dashed lines), and the desired position (red dashed line).

uncertainties in Scenario 2, Fig. 4 shows that the proposed gain adaptive MPC can quickly realize cable length positioning and oscillations elimination. In addition, the control input and control input increment are kept within the preset ranges.

5. Conclusion

In this paper, a gain adaptive MPC is designed for variable cable length pendulum systems to achieve oscillation elimination using cable length variation. Compared with existing trajectory planning methods, it has the advantage of considering the actuator constraints of variable cable length pendulum systems, including control input constraints and incremental constraints, especially the physical fact that the push force cannot be applied to the cable. In particular, a heuristic gain adaptive algorithm is designed to keep the ratio of pendulum angle loss to total loss constant during each constraint solution, which speeds up the oscillation angle convergence. The simulation results of the two scenarios verify the effectiveness of the proposed method. In the future, we will concentrate on constraining the cable length variation amplitude and optimizing the proposed gain adaptive algorithm.

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