Invariant Form of Slicing and Shift Conditions

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Under the transformation

$$t' = t'(t)$$
$$x^{a'} = x^{a'}(t, x^a)$$

and vice versa

$$t = t(t')$$
$$x^a = x^a(t', x^{a'})$$

We have $\mathring{\alpha}$ transforms as

$$\mathring{\alpha}' = \mathring{\alpha}(\frac{\partial t}{\partial t'})^2$$

where

$$\mathring{\alpha} = (\dot{\alpha} - \beta^a \partial_a \alpha) - \frac{\alpha}{\bar{\alpha}}(\dot{\bar{\alpha}} - \bar{\beta}^a \partial_a \bar{\alpha})$$

And $\mathring{\beta}^a$ transforms as

$$\mathring{\beta}^{a'} = \mathring{\beta}^a \frac{\partial x^{a'}}{\partial x^a}(\frac{\partial t}{\partial t'})^2$$

where

$$\mathring{\beta}^a = \Delta\dot{\beta}^a - \frac{\Delta\beta^a}{\bar{\alpha}}(\dot{\bar{\alpha}} - \bar{\beta}^a \partial_a \bar{\alpha}) + \sigma\beta^c \bar{D}_c \beta^a + (1-\sigma)\beta^c \bar{D}_c \bar{\beta}^a - (1+\sigma)\bar{\beta}^c \bar{D}_c \beta^a + \sigma\bar{\beta}^c \bar{D}_c \bar{\beta}^a$$

Therefore, the invariant form of 1 + log slicing should be

$$\mathring{\alpha} = -2\alpha\bar{\alpha}K$$

and the invariant form of gamma-driver shift condition should be

$$\mathring{\beta}^a = \frac{3}{4}\sqrt{\frac{g}{\bar{g}}}^{2/3} \bar{\alpha}^2 (\Delta\Gamma^a{}_{bc} g^{bc} + \frac{1}{3} g^{ab} \Delta\Gamma^c{}_{bc}) - \eta\bar{\alpha}\Delta\beta^a$$

where

$$\Delta\Gamma^a{}_{bc} = \Gamma^a{}_{bc} - \bar{\Gamma}^a{}_{bc}$$
$$\Delta\beta^a = \beta^a - \bar{\beta}^a$$

Under the transformation mentioned above, we should evolve the transformed lapse and shift along with time as following

$$\partial_{t'}\alpha' = \beta^{a'}\partial_{a'}\alpha' + \frac{\alpha'}{\bar{\alpha}'}(\partial_{t'}\bar{\alpha}' - \bar{\beta}^{a'}\partial_{a'}\bar{\alpha}') - 2\alpha'\bar{\alpha}'K'$$

$$\begin{aligned}\partial_{t'}\beta^{a'} = \partial_{t'}\bar{\beta}^{a'} + \frac{\Delta\beta^{a'}}{\bar{\alpha}'}(\partial_{t'}\bar{\alpha}' - \bar{\beta}^{a'}\partial_{a'}\bar{\alpha}') - \sigma\beta^{c'}\bar{D}_{c'}\beta^{a'} - (1-\sigma)\beta^{c'}\bar{D}_{c'}\bar{\beta}^{a'} + (1+\sigma)\bar{\beta}^{c'}\bar{D}_{c'}\beta^{a'} - \sigma\bar{\beta}^{c'}\bar{D}_{c'}\bar{\beta}^{a'} \\ + \frac{3}{4}\sqrt{\frac{g'}{\bar{g}'}}^{2/3}\bar{\alpha}'^2(\Delta\Gamma^{a'}{}_{b'c'}g^{b'c'} + \frac{1}{3}g^{a'b'}\Delta\Gamma^{c'}{}_{b'c'}) - \eta\bar{\alpha}'\Delta\beta^{a'}\end{aligned}$$

where

$$\begin{aligned}\bar{g}' &= \bar{g}(\frac{\partial x}{\partial x'})^2 \\ \bar{\Gamma}^{a'}{}_{b'c'} &= \bar{\Gamma}^{a}{}_{bc}\frac{\partial x^{a'}}{\partial x^a}\frac{\partial x^b}{\partial x^{b'}}\frac{\partial x^c}{\partial x^{c'}} + \frac{\partial x^{a'}}{\partial x^d}\frac{\partial^2 x^d}{\partial x^{b'}\partial x^{c'}} \\ \bar{\alpha}' &= \bar{\alpha}\frac{\partial t}{\partial t'} \\ \bar{\beta}^{a'} &= \bar{\beta}^a\frac{\partial x^{a'}}{\partial x^a}\frac{\partial t}{\partial t'} + \frac{\partial x^{a'}}{\partial x^a}\frac{\partial x^a}{\partial t'}\end{aligned}$$