Rural-Urban-Suburban Spatial Equilibrium Model

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Abstract

We want to assess the equilibrium distribution of people in this spatial model. The model aims to study how people decide where to locate themselves due to factors such as income, real estate prices, natural resources, productivity levels, and others that would influence their decisions. The model therefore borrows features of the spatial equilibrium optimization model to define the flows that will govern the dynamical system. Whereas other spatial equilibrium models are generally individual agent based and focus on individual optimizations, the RUS model seeks to provide a lens for analyzing the aggregation of individual decisions that lead to changes movements between the rural and urban, the urban and suburban, and the urban and slum areas.

1 Motivation

Urbanization has become a growing urbanization trend in the twenthy-first century. Driving urbanization is the migration of rural workers to the cities to look for jobs either because the city offers higher wages or because ecological depletion has made their livelihoods as farmers difficult.
By incorporating these and many other factors that would cause a resident of a state $(R, U,\ or\ S)$ to relocate into wages, real estate prices, and income for each location, we aim to see how the population would shift spatially over time.

We would then be able to analyze what happens if we change parameters of interest such as what would happen if ecological depletion worsened; would this drive more people to move to the urban area because it would offer higher wages, and if so, would the real estate price increase so much due to the influx that people would be driven to move to the suburbs or, worse yet, slums?

In addition, we will attempt to simply model changes in real estate prices due to changes in population and different amounts of housing supply. For housing that is more compact, we assume that adjacent neighborhoods have an effect on real estate pricing and we assess the changes in dynamics if the extent of this effect were to change. The motivation is to model the preferences of individuals in certain neighborhoods for their neighbors.

Finally, we assess the effect of technology on the ability to move out of the city and work in more satellite offices in suburban areas. Similarly, we study what occurs when there are changes in agricultural productivity.

2 Assumptions and Variables

2.1 Model Assumptions

We assume that people make decisions to move in order to maximize their disposible income after paying for housing. Since this model is not focused on modeling real estate prices, we will assume that the cost to rent and to purchase averages to the same effective cost so that we have one pricing function.

Housing prices are determined by distance from the city center with a known ceiling price p_0 for houses in the city center, as well as from the population at that distance.
We assume that there is no construction and therefore a known constant supply of houses h in each area (R, U, S); this h, which can be interpreted as the capacity for each group type within a location, is assumed to be the same for all groups in all areas to simplify our differential equations later.
The price is adjusted by an amenity level factor δ, where $0 \leq \delta \leq 1$ with 1 being the highest level of amenity and 0 being slum quality. Assume that the amenity level is constant for each socioeconomic group in all locations except for the slums, ie the wealthy live in housing with $\delta = 1$ and the poor live in housing with $0 < \delta < 1$ in all locations except for the slums.

In the city, although realistically lower income groups do not compete for the same housing as higher income groups, the existence of more affluent residents will drive up real estate prices. However, from the perspective of the wealthy, a higher population in general will raise property values due to increased competition but more lower income residents in the city will generally drive property levels down, as seen in historical redlining in the US.[6]

Therefore we need two real estate pricing equations for the city, one for each socioeconomic level:

For U_1, $p_{U_1}(d = 0, U_1, \alpha U_2) = p_0 + U_1 + \alpha U_2$
For U_2, $p_{U_2}(d = 0, \beta U1, U2) = p_0 + \beta U_1 + U_2$

Where α, β are the factors representing the cross-effect of the other income group on the property prices for this group with $0 < \alpha < 1$ and $-1 < \beta < 1$.

Assume that for the other areas the real estate price for each socioeconomic level is only affected by the potential residents, not taking into account the effect of the other group. This is because we assume that the rural area is spacious enough that the two groups can be isolated from each other; in the model, in the suburbs the poor is not present so S_2 prices are not affected by non-S_2 populations.

Assume that there is no income mobility.

2.2 Variables and Parameters

In our model we assume that for each settlement type, there are two distinct socioeconomic groups. Within the rural area in the periphery of the city, there are artisans/ subsistence farmers R_1 which are less financially affluent than the industrial farmers R_2.

Within the city, there is the urban poor U_1 and urban wealthy U_2 which together comprise the urban population.
The income level for each group depends on the industry: for the rural area, it is directly dependent on natural resources as an input; for the urban area, it is education-based skills. Let y_p be the income for U_1 and y_w be the income for U_2.

3 Explanation of Flows between States

This section explains the justification for movement between our four states: rural, urban, slum, suburb. We assume that there is two way flow from rural to urban to suburban or slum, but not directly from rural to suburban or slum area.

3.1 Rural to Urban Migration

Why move from the countryside to the city?
Inhabitants of rural areas on the periphery of cities are more directly dependent upon natural resources. For simplicity, we assume that their wages are directly tied to resources and are therefore sensitive to ecological degradation.

Rural artisans move if their income minus housing is higher than income of the urban poor minus housing, since we assume that they cannot directly become wealthy (can add social mobility factor later). Assume the lower income people all live in similar housing (δ *constant*) across locations.
$\frac{dR_1}{dt} = y_a(k,\gamma) - \delta p_{R_1}(d_r, R_1) - (y_p - \delta p_{U_1}(d = 0, U_1, \alpha U_2))$

$y_a(k,\gamma)$, $\frac{\partial y_a}{\partial k} > 0$

Similarly, for the wealthy industrial farmers, income is also dependent on k.
$\frac{dR_2}{dt} = y_I(k,\gamma) - p_{R_2}(d_r, R_2) - (y_w - p_{U_2}(d = 0, \beta U_1, U_2))$
$y_I(k,\gamma) > y_a(k,\gamma)$

where
$y_a = artisan\ farmer\ income$
$y_I = industrial\ farmer\ income$
$k = natural\ resources\ factor$
$\gamma = industrial\ productivity\ factor,\ explained\ under\ section\ on\ Functional\ Form$
$\delta_r = lower\ rural\ amenity\ housing\ factor$
$d_r = distance\ from\ rural\ area\ to\ city,\ constant$

We treat k as an exogenous constant for simplicity, but it is probably affected by the size of the urban and suburban populations and their use of land.

3.2 Urban Population

The flows for U_1 and U_2 are due to conservation of population since we assume that there is no income mobility in this simplified model.

$\bar{N_1} = R_1 + U_1 + S_1,\ \bar{N_2} = R_2 + U_2 + S_2$
$\frac{d\bar{N_1}}{dt} = \frac{dR_1}{dt} + \frac{dU_1}{dt} + \frac{dS_1}{dt} = 0$ since $\bar{N_1},\ \bar{N_2}$ are assumed to be known constants.

By symmetry, the same is true for group 2. We obtain the flows for U_1 and U_2:

$\frac{dU_1}{dt} = -\frac{dR_1}{dt} - \frac{dS_1}{dt}$
$\frac{dU_2}{dt} = -\frac{dR_2}{dt} - \frac{dS_2}{dt}$

We can simplify our model to four unknowns by substituting in for U1 and U2:
$U_1 = \bar{N_1} - R_1 - S_1$
$U_2 = \bar{N_2} - R_2 - S_2$

3.3 Urban to Slum Migration

Why does urban to slum migration occur?

Assume that the urban poor never accrue savings and are very sensitive to changes in rent. Specifically, if rent goes above their income level, they must evacuate and migrate to a slum, ie if $p_{U_2}(d = 0, U_1, \alpha U_2) > y_p$.

$\frac{dS_1}{dt} = p_{U_1}(d = 0, U_1, \alpha U_2) - y_p$

3.4 Urban to Suburban Migration

Suppose in a simple case that there is one city and one suburb distance d from the city center. We might adjust this to be continuous later.

$U_2 \rightarrow S_2$: In our model, only the wealthy urban group has the option of moving to the suburbs.

Why move from the city to the suburbs? From the perspective of the individual, it makes sense to move if income is higher than cost because we assume that the agent wants to maximize expendable income. Assume that due to technological advances, jobs for the skilled U2 group can be outsourced such that wages in the suburbs for the S2 group can approach and equal U2 wages, y_w.

$\frac{dS2}{dt} = \tau * y_w - p_{S2}(d_s,\ S2) - t(d_s)\ -\ (y_w - p_{U2}(d = 0,\ \beta U1, U2)))$

where
$\tau = technology\ coefficient,\ 0 \leq \tau \leq 1$
$d_s = distance\ from\ city\ center\ to\ suburb,\ constant$
$c = transportation\ cost\ per\ unit\ distance$
$p_{S_2}(d_s,\ S_2) = price\ of\ home\ in\ suburb,\ affected\ by\ current\ density$
$p_{U_2}(d = 0, \beta U_1, U_2) = price\ of\ home\ in\ city,\ affected\ by\ current\ density$

4 Functional Form

4.1 Real Estate Pricing

Since our model depends heavily on housing prices, we focus on the functional form for it:

$p(d, n_t) = p_0 * \triangle p(d, n_t)$ where p_0 is the real estate price at the city center and n_t is the number of people in the location at time t.

$\frac{\partial p}{\partial d} < 0$, $\frac{\partial p}{\partial n} > 0$

For each area, suppose we know the number of pre-existing homes h. We assume that this supply is perfectly inelastic (constant). Then if the number of people in the area exceed the housing supply, price will rise and the opposite will occur if the demand is less than supply.

Therefore we normalize so that change in price at any time is due to an imbalance between the population in that area at that time and the available housing in that area: $\frac{n(t)-h}{h}$. This is the percentage change in the pricing. If the housing supply at that time perfectly meets demand, price will remain the same, but if $n(t) > h$, price will increase by a percentage equal to the fraction of the population not accomodated over the total stock. Therefore price as a function of population is $1 + \frac{n(t)-h}{h} = \frac{n(t)}{h}$

We assume that housing prices monotonically decreases with distance. Price is bounded at $0 < p(d) < p_0$ so for simplicity we assume a ratio $\frac{d}{d_{max}}$ such that $p(d) = p_0(1 - \frac{d}{d_{max}})$ and d_{max} is the boundary of our 1d spatial model. We assume that $0 \leq d < d_{max}$, or that none of our areas under consideration actually reach the boundary so that the housing price never reaches zero. We normalize $d_{max} = 1$ and safely assume that the distance from the city to the suburb is less than the distance from the city to the rural area: $d_s < d_r < d_{max}$.

Our combined price equation is $p(d, n(t)) = p_0(1 - \frac{d}{d_{max}})(\frac{n(t)}{h})$

4.2 Rural Income and Natural Resources

We focus on the income of rural inhabitants because theirs is directly dependent on the quantity of available natural resources k and our model seeks to take into account spatial distribution as a result of ecological depletion.

We assume that subsistence farming yields constant returns to scale with k. However, industrial farmers R_2 are able to achieve increasing returns to scale[1] because they utilize technology that cuts labor costs and increases productivity with increasingly technological equipment[2].

$y(k, \gamma) = \gamma k$

$1 \leq \gamma < \gamma_{max}$ where γ_{max} should arguably be in the single digits since there should be a reasonable upper bound to the extent that new technology can make production more efficient and a lower bound to the cost of labor, which is at minimum zero, and the cost of equipment and fertilizer, which should be a significantly large value.

For the subsistence farmers R_1, we assume that they use little to no technologically advanced tools and therefore have $\gamma = 1$. For the industrial farmers R_2, $1 < \gamma < \gamma_{max}$.

4.3 Urban Income and Skill-based Productivity

For those living in the city, income should not necessarily be the same as in the rural area. Instead of being directly dependent on natural resources, it should rely on skills derived from an education. The functional form is again the same, but what we now attempt to model in a simple way is the increasing return to income due to a education factor ψ. We assume that U_1 is less educated and have a shared ψ for simplicity, normalized to 1. For group U_2, $\psi > 1$. Although the US income distribution monotonically increases with income, the curve is first concave, then convex. However, we use the functional form $y_w = \psi y_p$ for simplicity, where y_p is the income of group U_1 and is our assumed base pay.

5 Flow Equations

Conservation of population: We assume that the group 1 and group 2 populations $\bar{N}_1, \bar{N}_2$ are known and fixed:
$\bar{N} = \bar{N}_1 + \bar{N}_2$
Since there is assumed to be no mobility between socioeconomic levels, we can substitute U_1 and U_2:
$U_1 = \bar{N}_1 - R_1 - S_1$
$U_2 = \bar{N}_2 - R_2 - S_2$

$\frac{dR_1}{dt} = y_a(k, \gamma = 1) - \delta p_{R_1}(d_r, R_1) - (y_p - \delta p_{U_1}(d = 0, U_1 + \alpha U_2))$
With functional form, $\frac{dR_1}{dt} = k - p_0(1 - d_r)(\frac{R_1}{h}) - (y_p - \delta p_0(\frac{U_1+\alpha U_2}{h}))$
Substituting in U_1, U_2 and rearranging to group coefficients together:
$\rightarrow \frac{dR_1}{dt} = -\frac{p_0}{h}(\delta R_1 + \alpha R_2 + \delta S_1 + \alpha S_2) + (k - y_p + \frac{p_0}{h}[h(1 - d_r) + \delta \bar{N}_1 + \alpha \bar{N}_2])$

$\frac{dR_2}{dt} = y_I - p_{R_2}(d_r, R_2) - (y_w - p_{U_2}(d = 0, \beta U_1 + U_2))$
$\frac{dR2}{dt} = \gamma k - p_0(1 - d_r)(\frac{R_2}{h}) - (y_w - p_0(\frac{\beta U_1+U_2}{h}))$
$\rightarrow \frac{dR2}{dt} = -\frac{p_0}{h}(\beta R_1 + R_2 + \beta S_1 + S_2) + (\gamma k - \psi y_p + \frac{p_0}{h}[h(1 - \frac{d_r}{d_{max}}) + \beta \bar{N}_1 + \bar{N}_2])$

$\frac{dS1}{dt} = p_{U1}(d = 0, U_1 + \alpha U_2) - y_p$
$\frac{dS1}{dt} = p_0(\frac{U_1+\alpha U_2}{h}) - y_p$
$\rightarrow \frac{dS1}{dt} = -\frac{p_0}{h}(R_1 + \alpha R_2 + S_1 + \alpha S_2) + (\frac{p_0}{h}[\bar{N}_1 + \alpha \bar{N}_2] - y_p)$

$\frac{dS2}{dt} = \tau * y_w - p_{S2}(d_s,\ S2) - t(d_s)\ -\ (y_w - p_{U2}(d = 0,\ \beta U_1 + U_2)))$
$\frac{dS2}{dt} = \tau * \psi y_p - p_0(1 - d_s)(\frac{S_2}{h}) - cd_s\ -\ (\psi y_p - p_0(\frac{\beta U_1+U_2}{h})))$
$\rightarrow \frac{dS2}{dt} = -\frac{p_0}{h}(\beta R_1 + R_2 + \beta S_1 + [h(1 - d_s) + 1]S_2) + (\tau - 1)\psi y_p - cd_s + \frac{p_0}{h}[h(1 - d_s) + \beta \bar{N}_1 + \bar{N}_2]$

We arrange the differential equations into matrix form: $\frac{d\overrightarrow{x}}{dt} = A\overrightarrow{x} + \overrightarrow{b}$.

$$\frac{d}{dt}\begin{bmatrix} R_1 \\ R_2 \\ S_1 \\ S_2 \end{bmatrix} =$$
$$-\frac{p_0}{h}\begin{bmatrix} \delta & 1 & \delta & \alpha \\ \beta & 1 & \beta & 1 \\ 1 & \alpha & 1 & \alpha \\ \beta & 1 & \beta & h(1-d_s)+1) \end{bmatrix}\begin{bmatrix} R_1 \\ R_2 \\ S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} k - y_p + \frac{p_0}{h}[h(1-d_r) + \delta \bar{N}_1 + \alpha \bar{N}_2] \\ \gamma k - \psi y_p + \frac{p_0}{h}[h(1-d_r) + \beta \bar{N}_1 + \bar{N}_2] \\ \frac{p_0}{h}[\bar{N}_1 + \alpha \bar{N}_2] - y_p \\ (\tau - 1)\psi y_p - cd_s + \frac{p_0}{h}[h(1-d_s) + \beta \bar{N}_1 + \bar{N}_2] \end{bmatrix}$$

5.1 Parameter Fitting

Our list of parameters summarized from above for reference:

$k = natural\ resources\ factor$
$\gamma = industrial\ productivity\ factor,\ explained\ under\ "Functional\ Form",\ 1 \leq \gamma < \gamma_{max}$
$\delta_r = lower\ rural\ amenity\ housing\ factor,\ 0 < \delta_r < 1$
$\tau = technology\ coefficient,\ 0 \leq \tau \leq 1$
$d_r = distance\ from\ rural\ area\ to\ city,\ d_s < d_r < d_{max} = 1$
$d_s = distance\ from\ city\ center\ to\ suburb$
$c = transportation\ cost\ per\ unit\ distance$
$p_0 = ceiling\ price\ for\ housing\ as\ function\ of\ solely\ distance$
$h = housing\ stock\ in\ R, U, S$

$\psi = skill\ factor\ in\ city$
$\alpha = cross - effect\ of\ U_2\ on\ p_{U_1},\ 0 < \alpha < 1$
$\beta = cross - effect\ of\ U_1\ on\ p_{U_2},\ -1 < \beta < 1$

Because there are too many parameters to analyze the equilibrium results with all of them, we will set the values for most of them and focus on analyzing differences in distributions due to variations in
$k,\ \alpha,\ \beta,\ \tau,\ and\ \gamma.$

There is already significant literature on productivity factors so we will not analyze ψ, the skill-based productivity factor that we assume directly translates into a higher wage $y_w = \psi y_p$.

For fitting ψ to real data, we take the income of people ages 15 and older in the US in 2009[3] and let the 80th percentile with income of \$50,000 be representative of U_2; let the 10th percentile with income of \$20,000 be representative of U_1. Given this information, we will use $\psi = 2.5$ since $\psi = \frac{y_w}{y_p} = \frac{5}{2}$.

The average amount that people spend on housing in the US is about 30% of their before tax income.[5] Therefore our p_0 should be approximately $p_0 = .3y_w = .75y_p$. [1]Our transportation cost cd from living in the suburbs should be $cd_s \approx .15y_w = .4y_p = .6p_0$.
Rural income should be on the same order of magnitude as urban income, so assign $y_a = k$ a value relatively close to y_p.
We set $d_s = .5$. Let $c = 2$ so that $cd_s = 1$, $p_0 = 1.7$, and $y_p = 2.5$.

For the housing stock h in the real estate markets (for all but the slums), we can begin by assuming that total housing stock $5h$ should be approximately equal to the overall population N. Therefore begin by setting $h = N/5$.

Our fixed parameter values:
$\psi = 2.5$
$d_s = 0.5$
$d_r = 0.8$
$\delta = 0.5$
$c = 2$
$p_0 = 1.7$
$y_p = 2.5$
$\bar{N}_1 = 2$
$\bar{N}_2 = 3$

Our parameters of interest (fixed unless one being analyzed):
$h = 100$
$\alpha = .3$
$\beta = -.6$
$k = 3$
$\tau = .4$
$\gamma = 3$

6 Equilibria and Analysis

Our solution to $\vec{x}' = A\vec{x} + \vec{b}$ will be inhomogeneous due to the constant $\vec{b}$.
The equation converges or has a stable equilibrium if and only if all of the eigenvalues of A have a negative real part since the λ's are the growth rates for the exponential functions for our populations.

[1]

Since our parameter values are not in real units but normalized, we use ratios to set appropriate values for them relative to each other.

Since our A matrix does not have eigenvalues that produce a stable equilibrium, we turn to Matlab to study the dynamics of the changes within a thirty year timeframe instead of solving analytically for the equilibrium distribution. Although the calibration of the parameters do not carry significant interpretability in themselves, the directional changes in populations and the rate of change can shed insight into trends in movement as key parameters change.

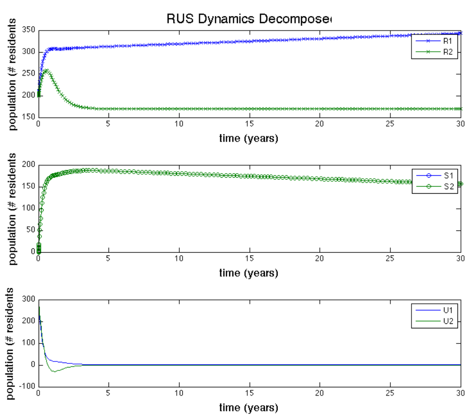
6.1 Results

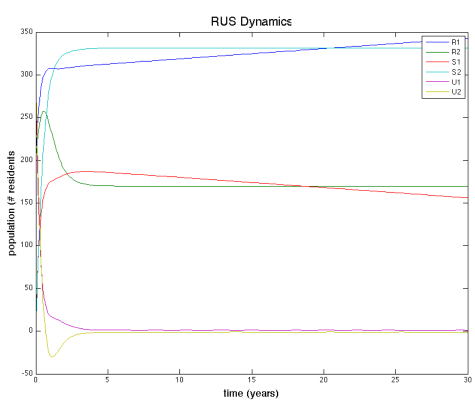
For the following results, we assumed the same parameters as those above in section 4.1.

Initial conditions: $N_0^1 = 500,\ N_0^2 = 500,\ R_0^1 = 200,\ R_0^2 = 200,\ S_0^1 = 0,\ S_0^2 = 0$

6.1.1 Assess changes with respect to housing stock h

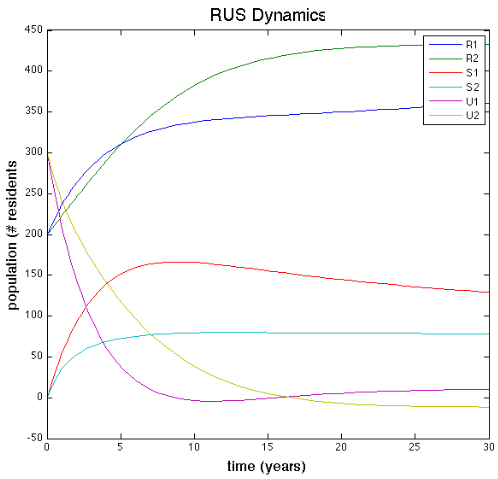
$h = 1$





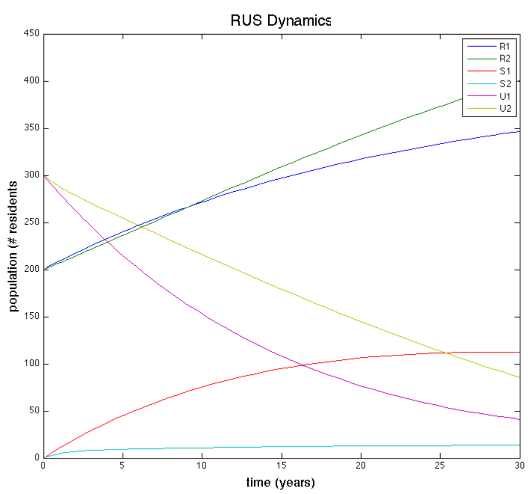
With very scarce housing, the subsistence farming group actually grows while the industrial farming group falls, probably because the lower income group does not have a channel like the suburbs to filter to when urban house prices skyrocket due to tight supply. The urban population quickly plummets, probably because urban housing prices become the highest. Interestsingly, the industrial farming population spikes upward before falling while the urban wealthy population drops, probably to move to the suburbs, then prices in R_2 probably rise enough to drive people from industrial farming to the suburbs too.
The slum population rises predictably due to scarce housing, but at a slower rate than the suburban population.

$h = 10$



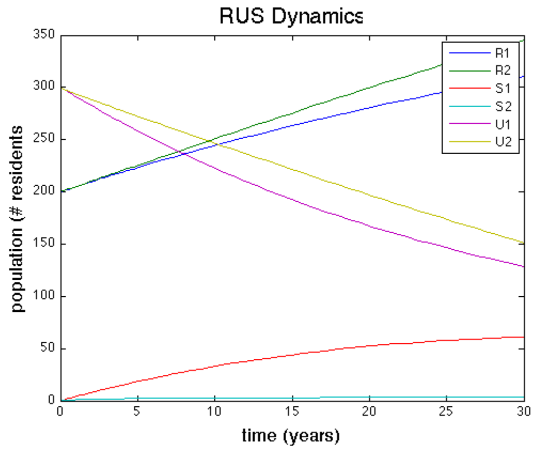
With a slightly increased housing supply, similar trends as above are observed but they occur at a slower rate.
A noticeable difference is that now the slum population is higher than the suburban population. Industrial farmers increase more than subsistence farmers as well. Everyone in U_2 still eventually moves out.

$h = 50$



Given our fixed parameters, without a shortage of housing there is little incentive to move to the suburbs, due in part to a higher cost of transportation and lower salary. We see that increasing housing from 10 to 50 units per area decreases prices sufficiently so that the S_2 population is significantly reduced and close to zero. All groups show similar behavior to $h = 10$ except at a slower rate.

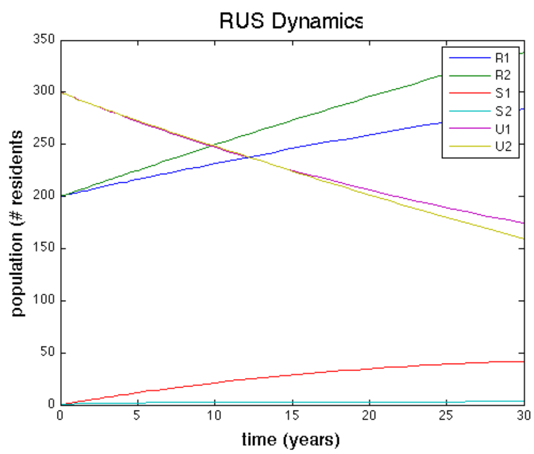
$h = 100$



Results are again very similar but again at a slower rate. Since h is a denominator in the ODEs, increasing it decreases the rate of change, or the eigenvalues of our matrix A.

6.1.2 Assess changes with respect to α, the cross-effect of U_2 on P_{U_1}

$\alpha = 0$: No cross-effect

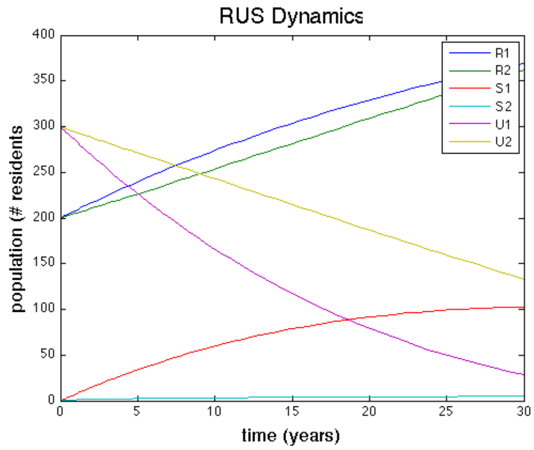


With $|\beta| > |\alpha|$, the urban poor falls at a slower but comparable rate than the urban wealthy. The rural population increases over time, with the industrial farming group increasing at a faster rate. This is likely because prices are most affordable for U_1 when there is no cross-effect α

$\alpha = .3$: Some cross-effect

See $h = 100$

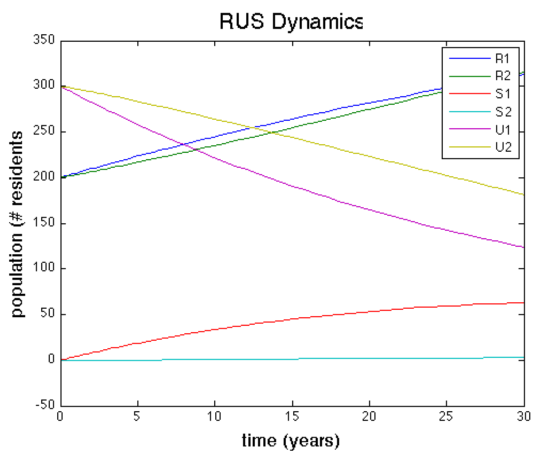
$\alpha = 1$: Complete cross-effect, U_2 population has high appreciative value on real estate prices for the U_1 population



However, the relationship between group 1 and group two is now reversed because housing price has risen for the lower income group. U_1 falls while the slum population rises, suggesting how gentrification in cities could increase the number of residents in slum-like dwellings.

6.1.3 Assess changes with respect to β, the cross-effect of U_1 on P_{U_2}

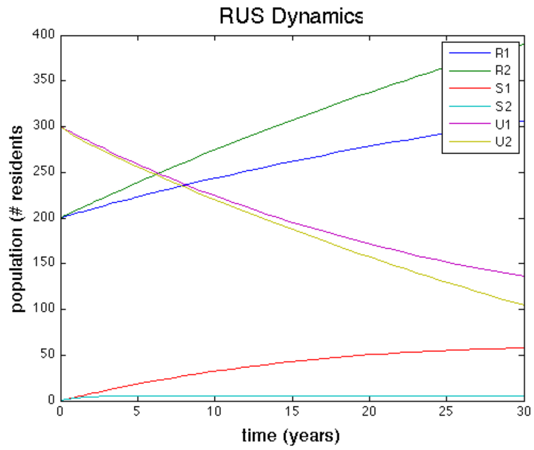
$\beta = -1$: Full cross-effect, U_1 population has high depreciative value on real estate prices for the U_2 population



With high $|\beta|$, price is lowered for the U_2 group, which is why $U_2 > U_1$. This is a less intuitive result but not necessarily representative of reality because we may have omitted a factor that adds a preference for/against U_1.

$\beta = -0.6$: Partial cross-effect
See h = 100

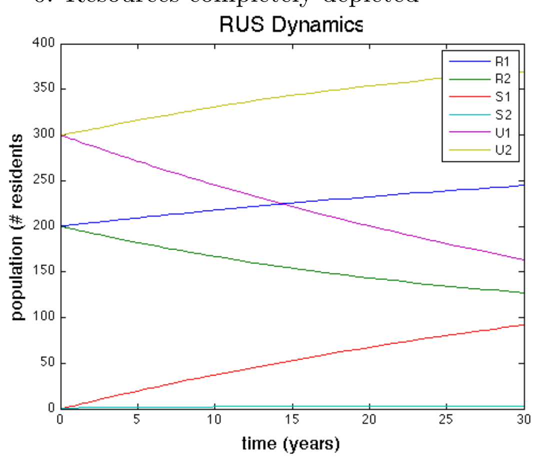
$\beta = 0$: No cross-effect



The same trend is observed without any cross-effect, except at a quicker rate.

6.1.4 Assess changes with respect to k, the amount of available resources

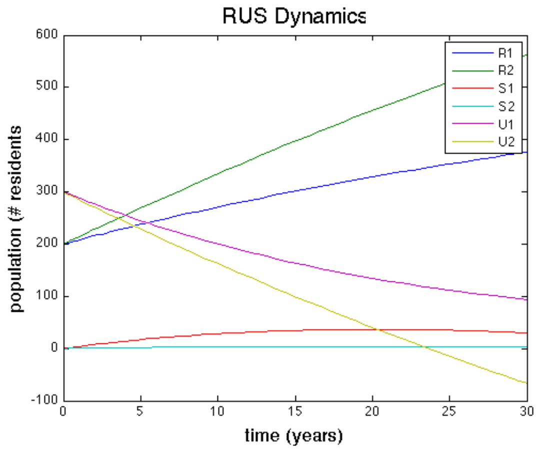
$k = 0$: Resources completely depleted



Interestingly, while the industrial farming population decreases over time, artisan farming increases. Nearly no one migrates to the suburbs (this is prevalent in results). The number of urban wealthy increases over time. Slum dwelling also interestingly increases because $R_1 \rightarrow U_1 \rightarrow S_1$ migration occurs since U_1 income becomes more attractive than R_1 income, which is zero. However, after the R populations transition to U, real estate prices rise, driving some to slums.

$k = 3$: See $h = 100$

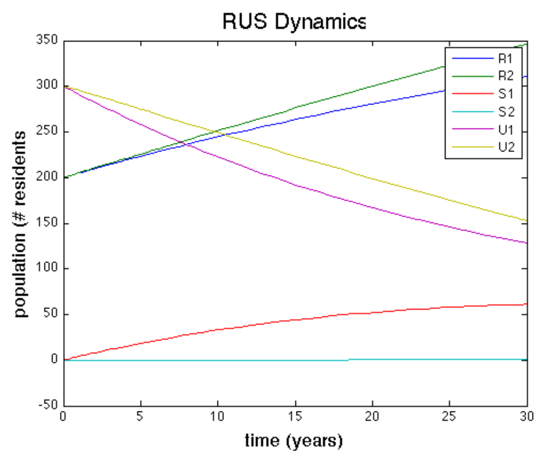
$k = 6$: Greater resources, higher rural income



Due to an increase in resources, rural income increases along with migration from other pools into the rural areas. As expected, R_2 grows at a higher rate than R_1 due to the productivity factor. The slum population is concave and decreases after increasing slightly, suggesting how preservation of natural resources can keep people from crowding into cities and slums, instead allowing a more even spatial distribution.

6.1.5 Assess changes with respect to τ, the communications technology factor

$\tau = 0$: Sets suburban income to 0



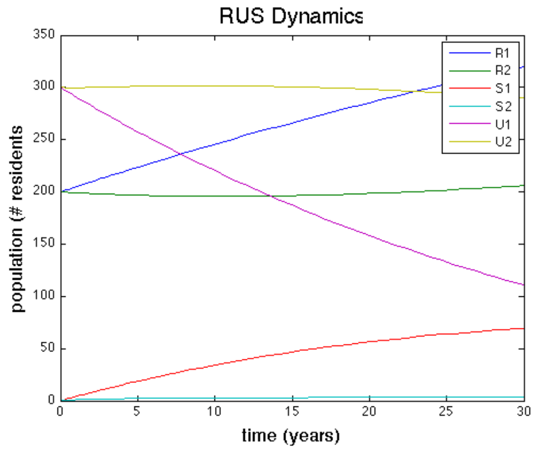
Group S_2 remains nearly flat at 0.

$\tau = 0.4$: See $h = 100$

$\tau = 1$: Sets suburban income to be on par with income in the city for same work
Similar results as $\tau = 0$, which reflects that our other fixed parameter assumptions have made suburbia a financially unattractive option such that technology that allows one to work remotely does not affect the decision significantly. However, this part was not modeled with enough specificity to allow τ and cd, the cost of transportation, change.

6.1.6 Assess changes with respect to γ, the industrial farming productivity factor

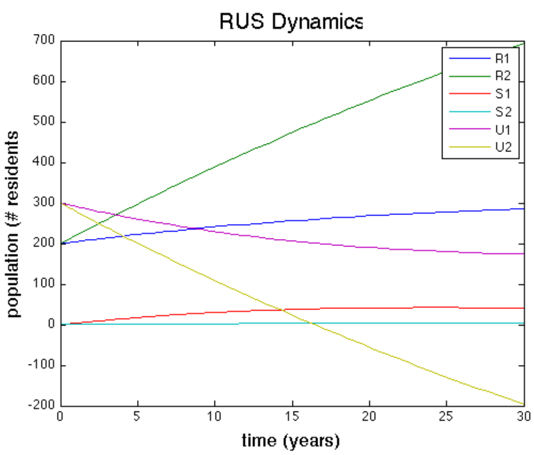
$\gamma = 1$: No difference in productivity- all rural population earns same income



U_1 falls at a faster rate than U_2 and symmetrically, R_1 increases at a faster rate than R_2. This is likely because the U_2 income becomes more attractive as R_2 income is lowered.

$\gamma = 3$: See $h = 100$

$\gamma = 8$: Significantly higher productivity of 8x than not using innovative equipment and farming techniques



Now both U_1 and U_2 fall, but with the latter falling at a faster rate as R_2 rises at a faster rate than R_1. This is because industrial farming becomes quite lucrative relative to urban and suburban wages. This result shows how reverse urbanization might occur, but it does not take into account that productivity or income might drop as R_2 increases to share the same resources.

7 Conclusion and Model Shortcomings

While our matrix ODE does not converge to a stable equilibrium, the model does not become useless since we are assuming a time-step of 1 year because populations do not change very rapidly in reality. As such, in reality we would not have fixed parameters over the span of $\triangle t = 30\ yrs$ so if there were stable equilibria, they would change over time as productivity factors or natural resource levels changed. Therefore it is arguably more meaningful to study the relative changes in flow trends as our parameters of interest are changed. We are able to infer general changes due to increasing ecological degradation, tighter housing supply, higher industrial farming productivity, and the effects of nearby neighborhoods on a given area's real estate prices. Though the factors are simplified and do not mean to be comprehensive, the model aims to provide new insight by interlacing factors that are normally not combined.

The model made simplifying assumptions such as assuming that housing stock/ capacity is the same in all areas and that the areas are treated as points in 1-dimension so that all homes in the suburbs and rural area are respectively the same distance from the city center. In addition, in reality migration to suburbs is more prevalent than our model, which points toward improvements in our assigned parameter values or inadequate representation of the factors that lead to suburban migration.

There is the danger of making too many assumptions that could compound and make this model too unrealistic, but at the same time the alternative would be to consider less factors and have more specificity for the few factors used. For example, instead of having a simplified real estate pricing function and a simplified wage function, we could have focused on one of them more specifically. However, we wanted this model to be able to incorporate as many key components that weigh into an individual's decision to (re)locate in a given location without becoming unnecessarily cumbersome to interpret.

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