A basic evolutionary model with preferential attachment principle and random choice innovation mechanism was developed by Ormerod and Caicado (2017) to analyse the emerging process of different market structures. This model is built on that basic model to further investigate the re-contracting behaviour in consumer choices. We introduce two new parameters, proportion and loyalty. Proportion is the parameter that specifies how many agents will re-enter the market and choose the products again at each period. Loyalty is the probability agents will conform to what they chose last time when they come to the re-contract point. Agents who are not loyal to their previous choices will follow the same decision mechanism as new entrants. We describe the model formally in the following.

The Formal Model

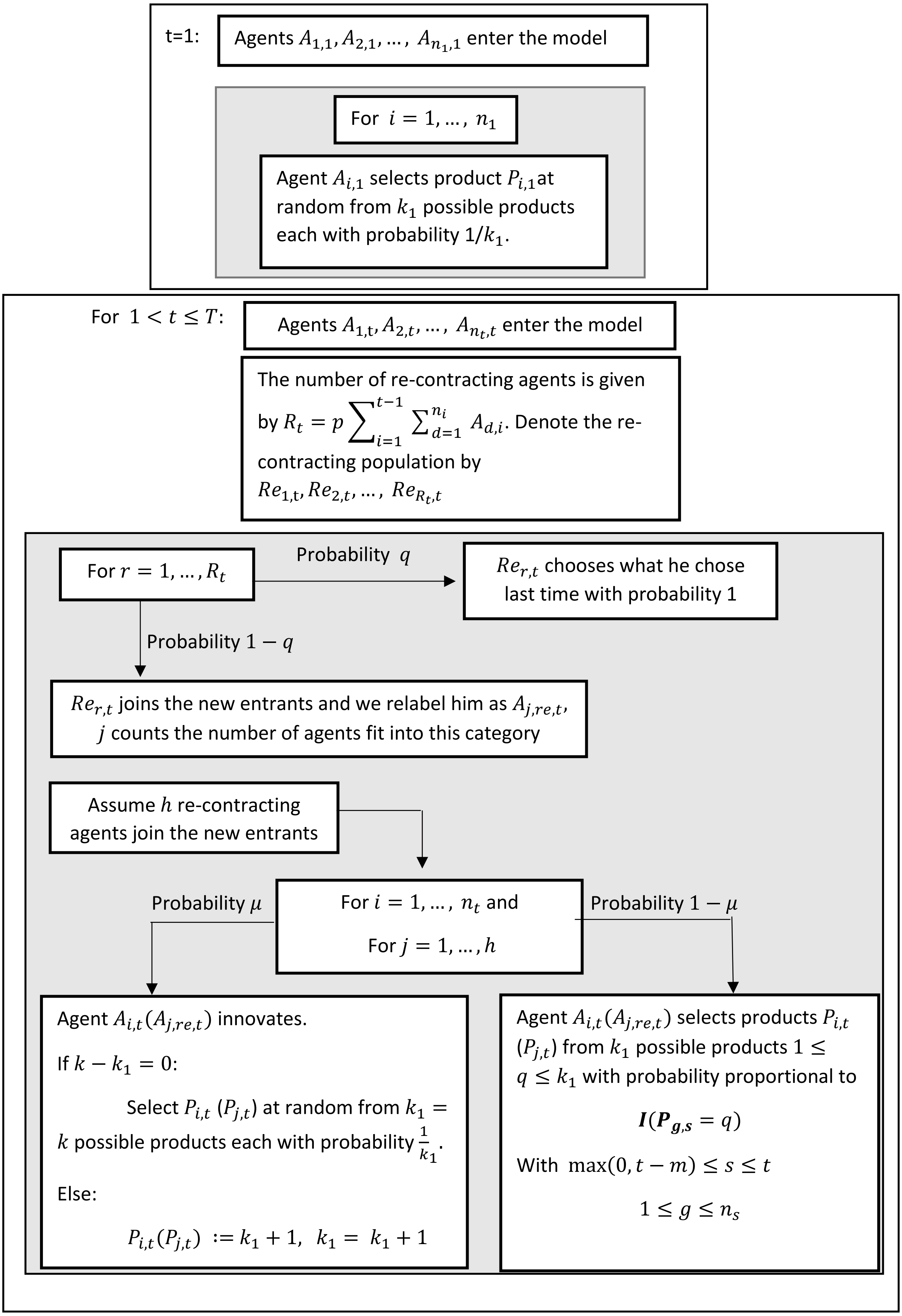
Consider the population growth at each time period is characterized by function N(t). At time period 1, the model is populated by N(1) agents. These agents each select at random one of k_1 products. We assume that each firm produces only one product. From period two onward, both new entrants and re-contracting agents will choose in the market. In each period, N(t) new agents enter the model. Meanwhile, a fixed percentage p of existing agents who have entered in any of the previous period is given the chance to choose again in the market. In our analysis, N(t) was chosen to be the standard logistic population growth, which characterizes the adoption patterns observed in new product markets in general, and we will see how effectively the re-contracting agents disrupt this growth pattern.

Every existing agent has the same probability of being chosen to re-contract at each time step, and any of them may be chosen multiple times to re-enter the market through the time span. For each agent chosen to re-contract, he will either be loyal to his previous choice with probability q, or join the new entrants and choose by the preferential attachment principle and innovation mechanism, with probability $1-q$.

For each of the re-contracting agents who decides not to choose what they chose before and each of the new entrants at each time period, they follow the procedure described in the basic model (Ormerod and Caicado, 2017). With probability ($1-\ \mu$), their choice will be influenced by other agents' choices in the last m periods (memory parameter). Specifically, they will choose a product with probability proportional to how many people chose that product in the last m steps. Or, with probability μ, they may not be happy to just follow others pattern and decide to 'innovate' – choose at random – which is the key force to reduce market concentration and increase competition as shown by Ormerod and Caicado (2017). In particular, if the maximum available products number k is not reached, each of them will add a product to the market until the available products number increases from k_1 to k.

More formally, the simulation algorithm is described by the following flowchart:

Set $k \in \mathbb{N}, k_1 \in \mathbb{N},\ k_1 \leq k,\ m \in \mathbb{N},\ \mu \in\ [0,1], p \in [0,0.01], q \in [0,1]$.
Select $n_1,\ n_2, \dots,\ n_t, \dots, n_T\ \in \mathbb{N}, 1 \leq t \leq T, T \in \mathbb{N}$.



Economics focus and parameter choice

As this paper is developed based on the work done by Ormerod and Caicado (2017), we share the same economics focus with them on market concentration issue. However, the versatility of the model is enormous for potential application in other fields. The Herfindahl-Hirschman Index (HHI) was adopted to measure the market concentration level of our simulation result by summing the squares of the individual firms' market shares. According to US Department of Justice and the Federal Trade Commission, markets can be generally classified into three categories:

• Unconcentrated markets: HHI below 0.15

• Moderately Concentrated markets: HHI between 0.15 and 0.25

• Highly Concentrated markets: HHI above 0.25

In terms of the growth function, we also adhere to the same setting as in Ormerod and Caicado (2017) at first. With an initial population equal to 100, the growth rate is given by the standard logistic population growth:

$$\frac{dP}{dt} = rP(1 - P/c)$$

where P is the population, $C = 1\ million$ is the carrying capacity and $r = 0.001$ is the growth rate. We focus on the market share at the maximum growth point, and it occurs at step 1946, when 250 agents enter the model, given the current setting. Initially, 10 products are available (k1 = 10), and after initial step, agents can choose from a further 10 (k2 = 10), making 20 in total.

Although we make reference to most of the parameter settings in Ormerod and Caicado (2017), our model is considerably more complicated and mysterious than the basic model. The impact of the two new parameters is not clear and there might be several different ways to look into it. With the aim to provide a coherent and clear explanation, our methodology starts by considering the trade-off between innovation and memory, and discover the impact of new parameters by comparing our result with the benchmark result generated by the basic model. We will also provide further insights on their individual behaviour by looking at them separately and investigate in which part of the model they make a difference.

Unless otherwise stated, in the simulation result reported here, the Innovation parameter is chosen from 0 to 0.01 with step of 0.0001, making 101 values in total, and memory parameter is set from 1 to 10 in step of 1. For simulation at each pair of parameters, we obtain 100 separate solutions for analysis. The main proportion value of interest is 0.0008 that means at each time period, 0.0008 of existing agents (with respect to that period) is chosen to re-enter the market. The proportion seems to be small, but in fact, the number of people is not. Under our setting of logistic growth function, there will be 357167 agents have entered the market by time period 1946, which is the time point of our interest.

0.0008 of that gives us 286 agents re-contract at the peak entry, about the same size as the new entrants (the number of re-contracting agents is rounded up at each period). This choice also makes computation more feasible than higher values.

Result

Before we consider the new parameter effect, we first check that when proportion parameter is set to 0, the result should coincide with the result generated in Ormerod and Caicado (2017). Note that when the proportion is set to 0, the loyalty parameter has no effect. The following Figure 1 shows mean of HHI (left panel) and standard deviation (right panel) across 100 samples at each pair of innovation and memory parameters. Both results agree with the data generated by the basic model and these serve as a benchmark for other results generated by our model.

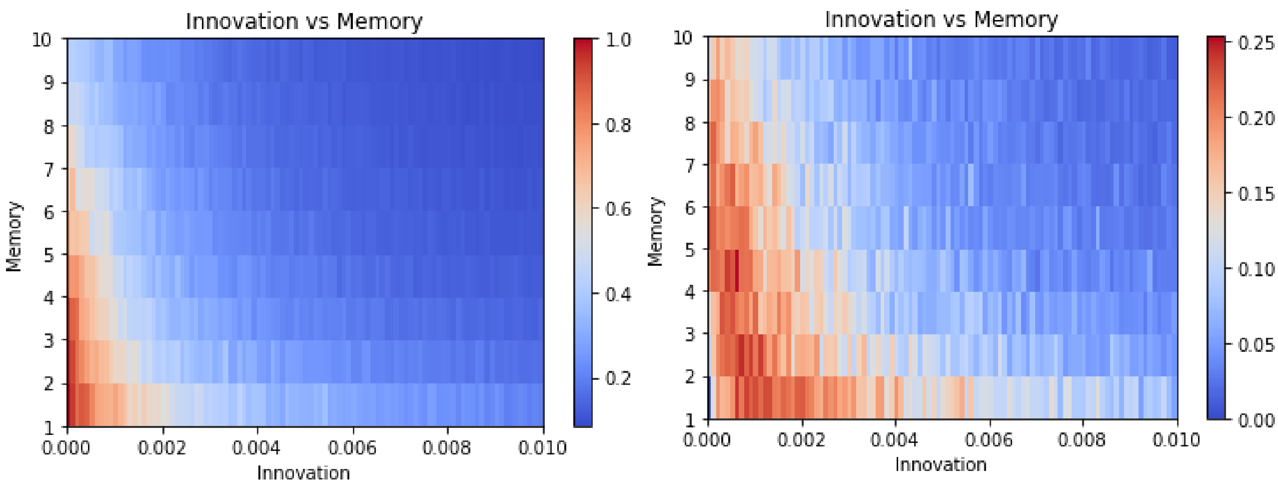


Figure 1

Next, we set proportion parameter to 0.0008 and loyalty to 0, and consider the proportion parameter's impact on the interplay between innovation and memory. The result is shown in Figure 2, again, with mean of HHI (left panel) and standard deviation (right panel).

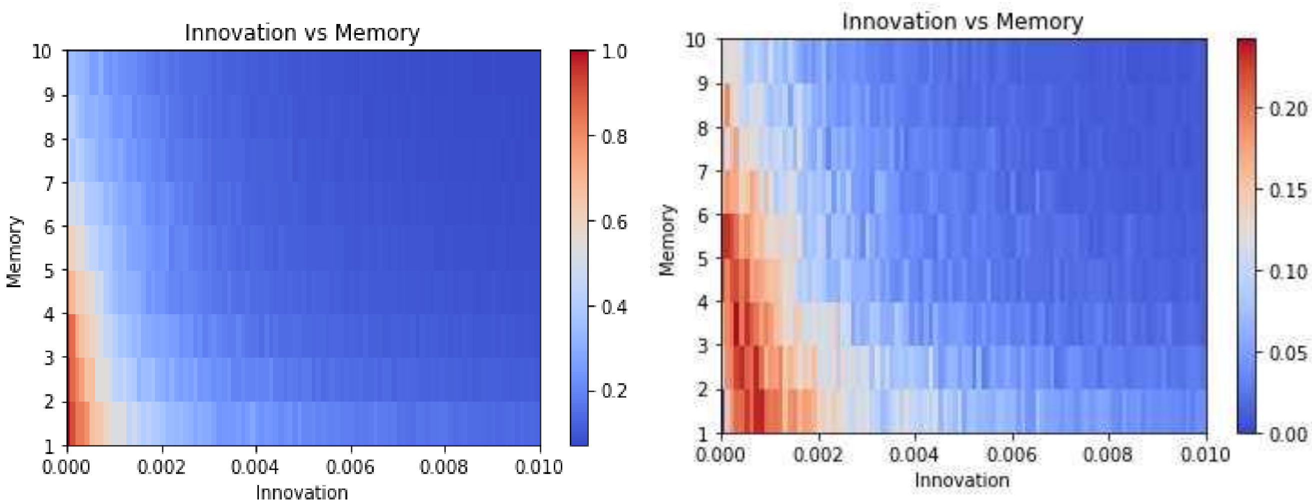


Figure 2

By a direct comparison with Figure 1, we see if agents just choose products following the same rule as new entrants, without any chance of being loyal to what they chose before, its impact on the market concentration level is not obvious. In general, it slightly reduces the average HHI and standard deviation at certain pairs, but it does not change the overall trade-off pattern between innovation and memory. This result is somehow expected as the proportion parameter on itself does nothing but increase the number of agents enter the basic model at each period. In other words, it does not change the mechanism of the system but change the growth function indirectly.

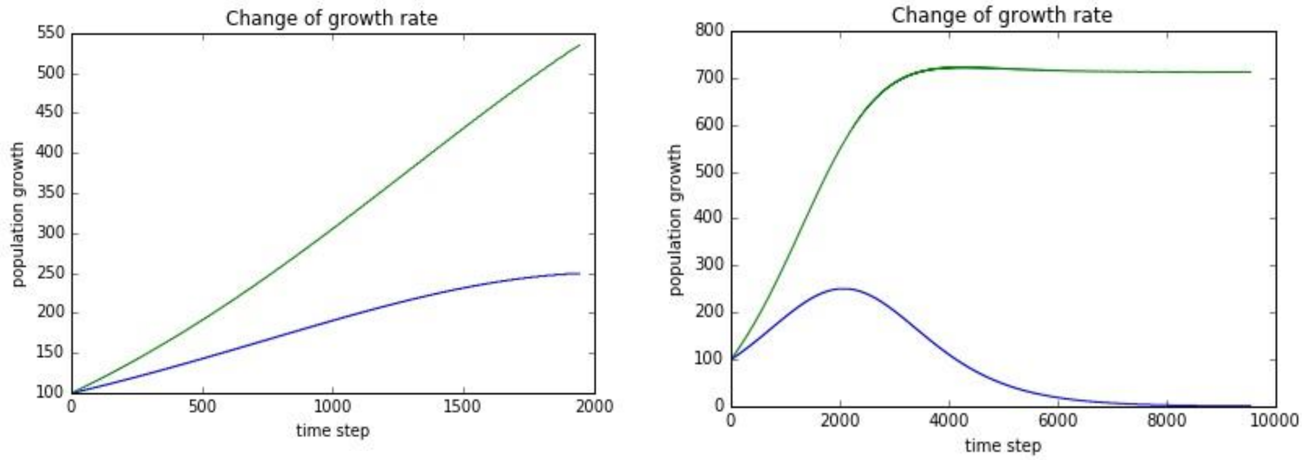


Figure 3

In Figure 3, we show explicitly how the re-contracting agents change the *effective* growth function. The blue line in both panels is the growth rate of our standard logistic growth function, and it takes its peak value at 1946 time point and decreases after that until the growth stops at the 9546 time step. The green line shows that, with re-contracting population added at each time period, the *effective* growth trend is changed from logistic to almost linear in the first part and then reaches a stable constant growth of 712 agents at each period after the logistic growth dies out. We also note this change of *effective* growth rate at each time step poses a significant challenge to the computation and its cumulative effect limited the number of simulations we could perform at each pair of parameters.

From another point of view, since the results generated by the basic model and our model are similar when loyalty is set to zero, it actually unveils the fact that the basic model can also be interpreted in the context of re-contracting in the markets where loyalty is almost 0. In Figure 4, we show an example of this potential interpretation. Consider a new logistic growth function with 100 initial agents, growth rate r = 0.001223 and carrying capacity equal to $10^{6.3}$. The growth rate at each time step is plotted as the blue line in the left panel in Figure 4, up to 1946 time point. The green line in the left panel is our previous *effective* growth rate with 0.0008 proportion parameter value. These two growth functions intersect at the point 1946, the maximum entry point of the previous logistic growth function. By adopting the new logistic growth function, the basic model in Ormerod and Caicado (2017) generates the mean of HHI across 100 samples at each pair as in the right panel of Figure 4. This result is not *statistically significant* from the left panel in Figure 2, which means by choosing appropriate growth function, the basic model could be used to explain some pure re-contracting behaviour (loyalty = 0).

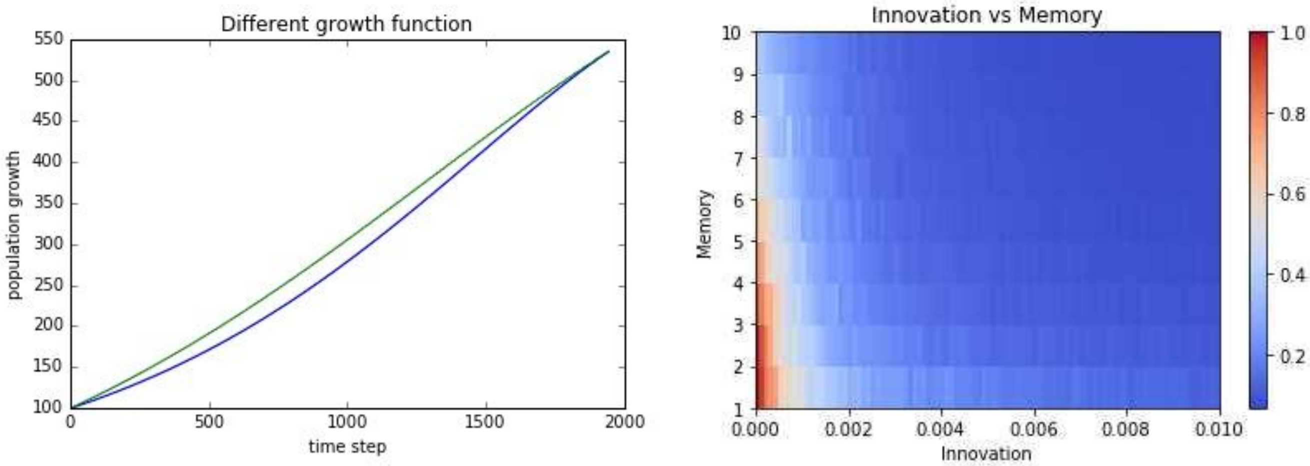


Figure 4

Next, we move our focus on to the loyalty parameter. Figure 5 (a)(b)(c) shows the trade-off between innovation and memory on influencing market concentration level when re-contracting proportion equal to 0.0008, and loyalty set to 0.05,0.5,0.95 respectively. The scale of the results are *different* from previous results and *different* between themselves as noted on the right-hand side colorbar. As the loyalty value is increased from 0, the setting in our previous analysis, the HHI decreases dramatically. From 0 to 0.05, the maximum HHI in the result decreases from 1 to 0.43. With a further increase to 0.5, maximum HHI decreases to 0.32. When loyalty reaches 0.95, the maximum HHI is reduced to 0.26, which still get categorised into highly concentrated market according to the US Department of Justice and the Federal Trade Commission.

Apart from its effect on reducing the market concentration, loyalty also plays a role in changing the trade-off pattern between innovation and memory compared to our benchmark. In Figure 1, the trade-off pattern is that HHI is more sensitive to innovation than memory. An increase of innovation from 0 to 0.002 leads to a significant drop in HHI from 1 to 0.5, whereas it takes the memory increasing from 1 to 7 to have the same effect on its own. However, in Figure 5(b)(c), a clearly different pattern is derived from our model, although the HHI is lower than our benchmark as a whole, it becomes more sensitive to a small increase in memory than the increase in innovation.

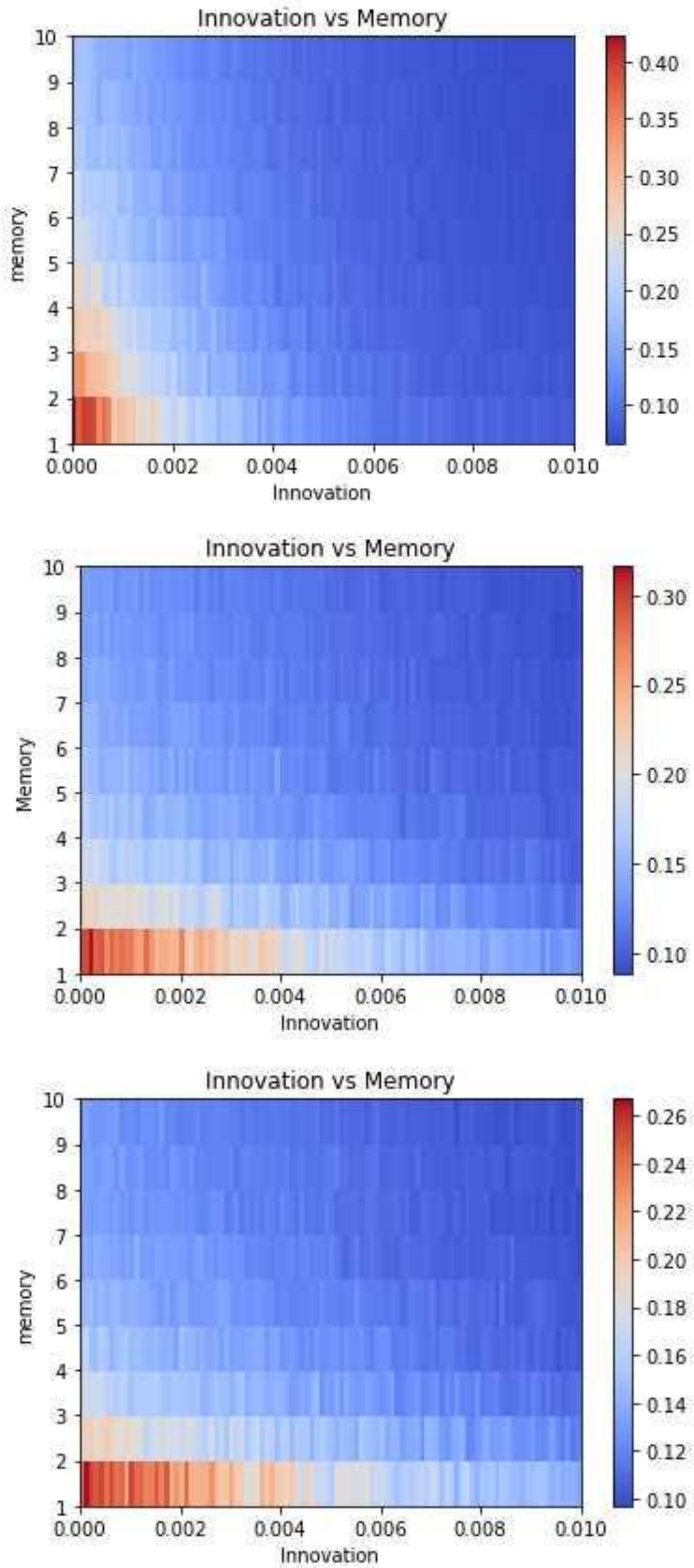

Innovation vs Memory
memory
Innovation
0.000
0.002
0.004
0.006
0.008
0.010
1
2
3
4
5
6
7
8
9
10
0.40
0.35
0.30
0.25
0.20
0.15
0.10
Innovation vs Memory
Memory
Innovation
0.000
0.002
0.004
0.006
0.008
0.010
0.30
0.25
0.20
0.15
0.10
Innovation vs Memory
memory
Innovation
0.000
0.002
0.004
0.006
0.008
0.010
0.26
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10

Figure 5

To understand the behaviour of loyalty and its impact on HHI, we need to consider the re-contracting population in detail. As explained earlier, every agent who has entered the model before time step t has the same chance of being chosen to re-buy products at time step t. However, what matters is not who re-enters the market but the product they chose last time as we need to decide whether they stick to their last choice or follow the other decision rule. Once the focus is shifted from the re-contracting agent to his chosen product, we can make the following approximation: At any time period t, choosing agent to re-contract is *equivalent* to choosing product with respect to the total product distribution across all $(t-1)$ previous time periods. This approximation is valid because every agent has the same probability of being chosen. Furthermore, it shows that the re-contracting agents put a *weight* on total product distribution *across every past time step* in determining the market share at the peak entry level, although this weight will not have any effect unless the loyalty is positive.

When loyalty equals to zero, although the products chosen by re-contracting agents before reflects the whole distribution in the past, they will not stick to their previous choices whatsoever. As a result, they behave just like every new agent who only look back last m steps and make decision based on that m-step product distribution. It is only when loyalty becomes *positive* that the current market concentration level will be influenced by the re-contracting behaviour. The way that loyalty parameter works could be thought as a disruption element in the standard procedure just like the innovation. The products chosen by re-contracting agents before now have a positive probability of being chosen again by the same agent. That 'lucky' product is, indeed, possible to come from innovation or from a relatively less popular choice when it get chose last time. In these two cases, we say the market competition strengthens and hence HHI becomes lower. In the other case when the re-contracting agent is loyal to a product which has high market share, we say the market concentration is not significantly increased, if any, as it is inherently the choice will be made by most of the people. We also note that in terms of the market concentration level, a small increase in a less popular company share has far more impact on decreasing HHI, than the effect on increasing HHI created by the same size increase of a large company share.

Just like the innovation parameter adds noise to standard Preferential Attachment rule, our loyalty parameter adds further randomness by strengthening the innovation effect and create a cumulative effect over time to increase market competition. In conclusion, any positive loyalty parameter would have an impact on lowering market concentration level and from Figure 5, we see that effect is very strong even for small loyalty parameter.