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1. Modeling of carbon cycle

In this problem, we will build a box model to understand the Earth's carbon cycle based on the framework in Tomizuka 2009.

1.1 **[15 points]** Following equation 1-2 (without the buffer effect), build a two-box model to compute the atmospheric CO_2 level in ppm (parts per million) from 1987 to 2004.

思路：我从查阅同济大学版《高等数学上》和冯汇然同学的交谈中得到灵感，知道了求解 two-box model 的原理，并进行了推导。从 Global Fossil-Fuel CO_2 Emissions 下载数据 global.1751_2014.csv，用于计算γ。参考利用 python 的 sympy 库求解常微分方程（组）的方法 - 知乎 (zhihu.com)，python 求初值常微分方程组_mob649e81540090 的技术博客_51CTO 博客用 python 求解常微分方程组。

```
In [169]: #利用sympy库求解解析解
x = sym.symbols("x")
f = sym.Function("f")
f = sym.exp(x)    #只是给出f的形式
#f = sym.tan(x) - sym.exp(x) + x**2 + sym.sqrt(x)
y,z = sym.symbols("y,z",cls = sym.Function)
k12=105/740
k21=102/900
r=2.62   #给定一个初值
eq1=Eq(y(x).diff(x),-k12*y(x)+k21*z(x)+r)   #第一个微分方程
eq2=Eq(z(x).diff(x),k12*y(x)-k21*z(x))    #第二个微分方程
s = sym.dsolve([eq1,eq2])   #求解微分方程组
y= sym.trigsimp(s[0].rhs)   #大气中二氧化碳通解
z= sym.trigsimp(s[1].rhs)  #海洋中二氧化碳通解
y,z

Out[169]: (0.798730158730159*C1 - 1.0*C2*exp(-0.255225225225225*x) + 1.16341687257324*x + 5.70705002274515,
 1.0*C1 + 1.0*C2*exp(-0.255225225225225*x) + 1.45658312742676*x - 5.70705002274515)
```

In [171]:

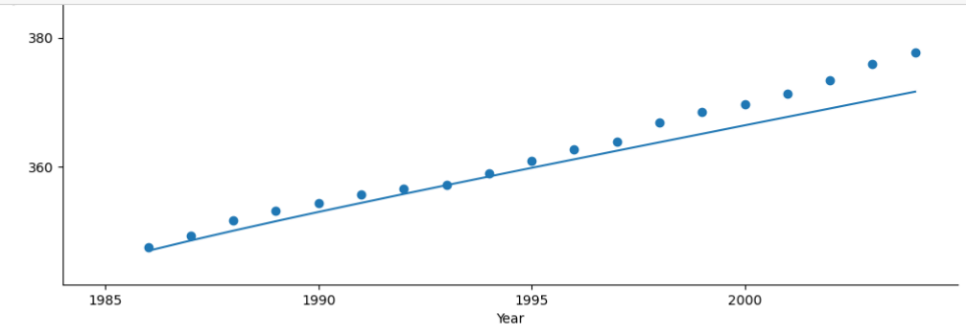
```
#利用scipy库求解数值解
# 定义微分方程组的函数
def equations(u, x):
    y, z = u
    dy_dx = -k12*y+k21*z+r
    dz_dx = k12*y-k21*z
    return [dy_dx,dz_dx]

# 定义初始条件和时间范围
k12=105/740
k21=102/900
r=np.mean(gamma_data['gamma']) ##gamma变化不大，这里选取均值进行替换
initial_conditions = [347, 423]  #给定初值条件   347=740/2.13,423 =900/2.13
time = np.arange(0,19)

# 求解微分方程组
result = odeint(equations, initial_conditions, time) #得到方程的数值解
result
```

Out[171]:

```
array([[347.        , 423.        ],
       [348.58747534, 424.33503516],
       [350.10969121, 425.73532979],
       [351.58134603, 427.18618547],
       [353.01382544, 428.67621657],
       [354.41595828, 430.19659423],
       [355.7945618 , 431.74050121],
       [357.15496783, 433.30260568],
       [358.5012537 , 434.87883031],
       [359.83660777, 436.46598675],
       [361.16349006, 438.06161496],
       [362.48380971, 439.66380581],
       [363.79904441, 441.27108161],
       [365.11034012, 442.8822964 ],
       [366.41858414, 444.49656288],
       [367.72446365, 446.11319387],
       [369.02851137, 447.73165666],
       [370.33113991, 449.35153862],
       [371.63266856, 450.97252047]])
```

In [172]:

```
# 绘制解的图像
times=time+1986
times
plt.figure(figsize=(12,10))
plt.plot(times, result[:, 0], label='N1')
plt.plot(times, result[:, 1], label='N2')
plt.scatter(times, co2_ann_test['mean'], label='Observation')
plt.xlabel('Year')
plt.ylabel('CO_2 Concentration (ppm)')
plt.xlim(1984,2005)
plt.xticks([1985,1990,1995,2000],[1985,1990,1995,2000])
plt.legend()
ax=plt.gca( )#gca:get current axis得到当前轴
#设置图片的右边框和上边框为不显示
ax.spines['right'].set_color('none')
ax.spines['top'].set_color('none')
plt.show()
```



1.2 [20 points] Following equation 3-4 (with the buffer effect), build a two-box model to compute the atmospheric CO_2 level in ppm from 1987 to 2004.

思路：基于 1.1 的基础上，用$N_2^0 + \xi(N2 - N_2^0)$代替 1.1 中的$N2$，用求解 1.1 的方法求解。从 Mauna Loa CO2 annual mean data 下 co2_annmean_mlo.csv，用于ξ。

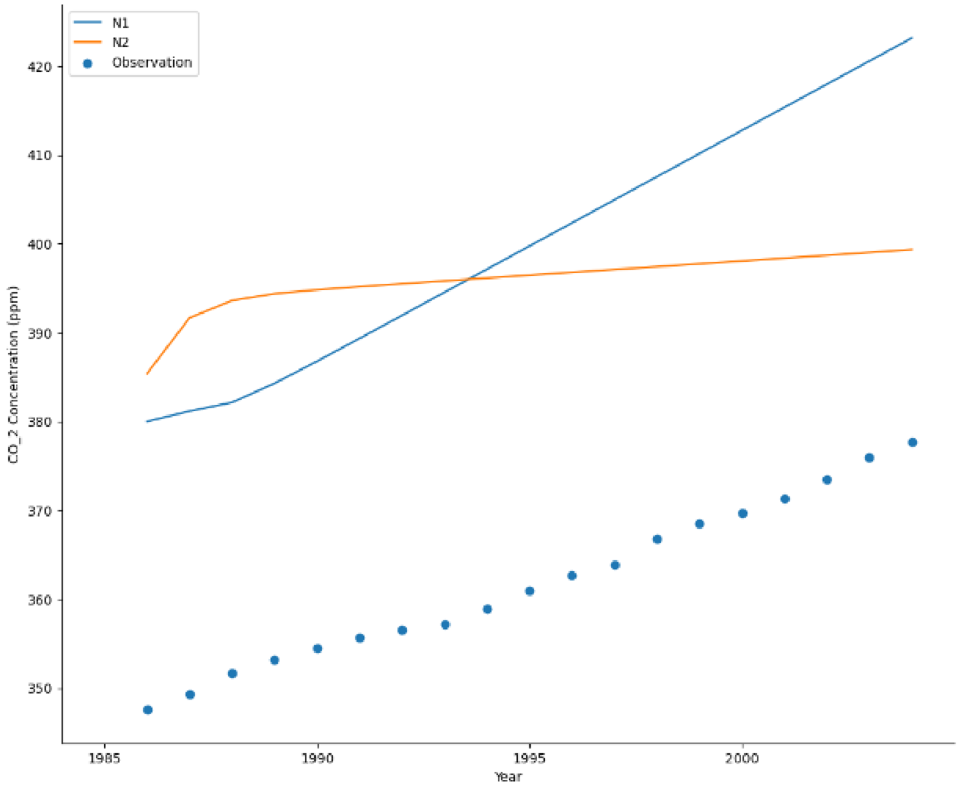
In [173]:

```python
#利用scipy库求解数值解
# 定义微分方程组的函数
def equations1(u, x):
    y, z = u
    #s=(n2_0+a(z-n2_0))
    dy_dx = -k12*y+k21*(n2_0+a*(z-n2_0))+r
    dz_dx = k12*y-k21*(n2_0+a*(z-n2_0))
    return [dy_dx,dz_dx]

# 定义初始条件和时间范围
k12=105/740
k21=102/900
n2_0=821/2.13
initial_conditions1 = [384.5,385.4]  #给定初值条件,384.5=(740+79)/2.13,385.4=900-79/2.13
time = np.arange(0,19)

r=np.mean(gamma_data['gamma']) #gamma变化不大，这里选取均值进行替换
a=np.mean(co2_ann_test['buffer factor'])  #buffer factor变化不大，这里选取均值进行替换
# 求解微分方程组
result1 = odeint(equations1, initial_conditions1, time) #得到方程的数值解
result1
result1[0, 0]=380
result1
```

Out[173]:

```
array([[380.        , 385.4       ],
       [381.15827844, 391.66423206],
       [382.13470552, 393.61031548],
       [384.29243208, 394.37509942],
       [386.77332536, 394.81671665],
       [389.34262614, 395.16992636],
       [391.9361109 , 395.49895211],
       [394.53621206, 395.82136145],
       [397.13812422, 396.14195979],
       [399.74053146, 396.46206305],
       [402.34307264, 396.78203237],
       [404.94565241, 397.10196311],
       [407.54824192, 397.4218841 ],
       [410.15083406, 397.74180246],
       [412.75342695, 398.06172007],
       [415.35601995, 398.38163757],
       [417.95861315, 398.70155487],
       [420.56120642, 399.02147211],
       [423.16379966, 399.34138937]])
```

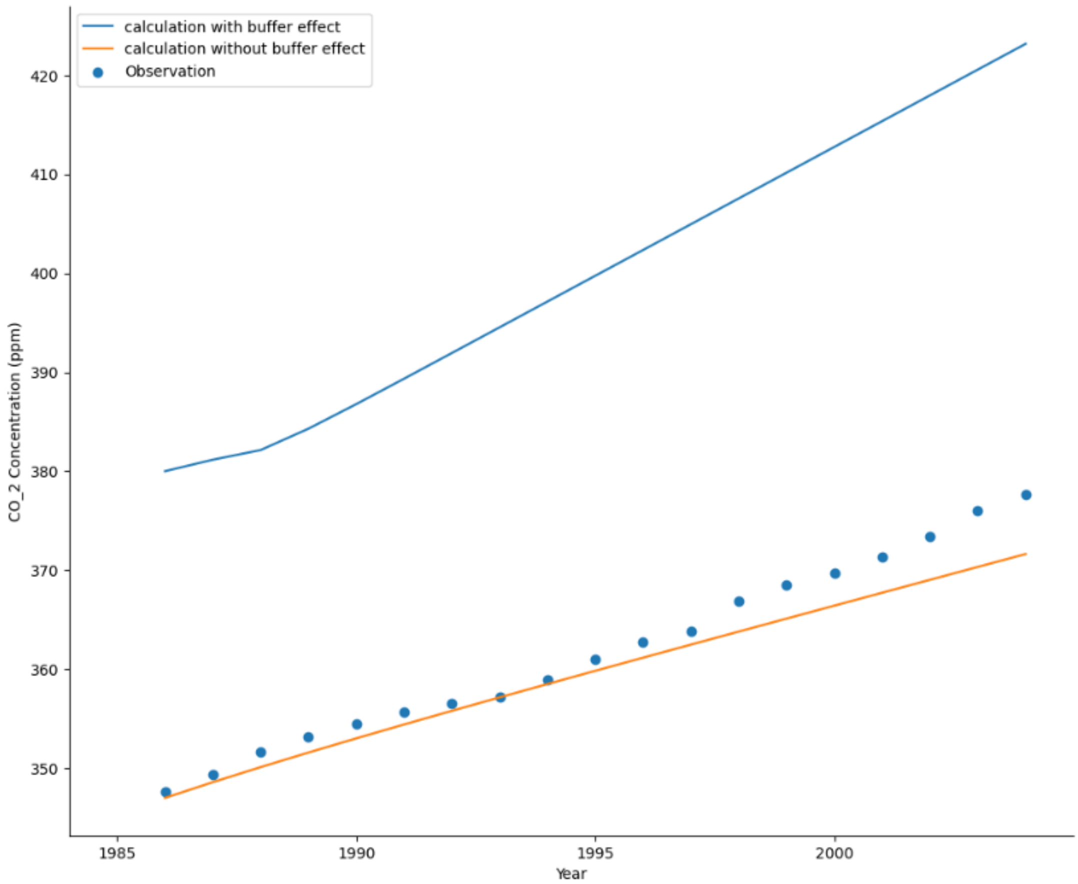
In [174]:
```python
# 绘制解的图像
times=time+1986
times
plt.figure(figsize=(12,10))
plt.plot(times, result1[:, 0], label='N1')
plt.plot(times, result1[:, 1], label='N2')
plt.scatter(times, co2_ann_test['mean'], label='Observation')
plt.xlabel('Year')
plt.ylabel('CO_2 Concentration (ppm)')
plt.xlim(1984,2005)
plt.xticks([1985,1990,1995,2000],[1985,1990,1995,2000])
plt.legend()
ax=plt.gca( )#gca:get current axis得到当前轴
#设置图片的右边框和上边框为不显示
ax.spines['right'].set_color('none')
ax.spines['top'].set_color('none')
plt.show()
```



1.3 [5 points] Based on your results from **1.1** and **1.2**, reproduce Figure 2 in Tomizuka (2009) as much as you can.

思路：基于 1.1 和 1.2 的结果，重现图 2

In [175]:
```python
##第一问和第二问做对比
# 绘制解的图像
times=time+1986
times
plt.figure(figsize=(12,10))
plt.plot(times, result1[:, 0], label='calculation with buffer effect')
plt.plot(times, result[:, 0], label='calculation without buffer effect')
plt.scatter(times, co2_ann_test['mean'], label='Observation')
plt.xlabel('Year')
plt.ylabel('CO_2 Concentration (ppm)')
plt.xlim(1984,2005)
plt.xticks([1985,1990,1995,2000],[1985,1990,1995,2000])
plt.legend()
# 绘制解的图像
ax=plt.gca( )#gca:get current axis得到当前轴
#设置图片的右边框和上边框为不显示
ax.spines['right'].set_color('none')
ax.spines['top'].set_color('none')
plt.show()
```


calculation with buffer effect
calculation without buffer effect
Observation
420
410
400
390
380
370
360
350
CO_2 Concentration (ppm)
1985
1990
1995
2000
Year