ECE 5605: Stochastic Signals and Systems – Fall 2018

Homework #3
Due: Friday, September 21, 2018 (1:45pm)

Reading: Chapters 3 and 4 of the textbook.
Total points: 100

1. (5 pts) Let ξ be a point selected uniformly at random from the unit interval (that is, assume ξ to be uniformly distributed between 0 and 1). Consider the random variable $X = (1-\xi)^{-\frac{1}{2}}$.
   (a) (1 pt) Sketch X as a function of ξ.
   (b) (2 pts) Find the cdf of X.
   (c) (2 pts) Find the probability of the events $\{X > 1\}$ and $\{5 < X < 7\}$.

2. (5 pts) Let X be a Gaussian random variable with mean 0 and standard deviation 1. Define $Y = aX + b$ for $a > 0$. Find the pdf of Y. Is Y a Gaussian random variable?

3. (10 pts) Consider a random variable X with pdf

$$f_X(x) = \begin{cases} \frac{1}{2\sqrt{x}} & 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   A new random variable Y is obtained by subjecting X to the operation $Y = g(X)$ where

$$g(X) = \begin{cases} 0 & X < 0 \\ \sqrt{X} & 0 \leq X \leq 1 \\ 1 & X > 1 \end{cases}$$

   Determine the pdf of random variable Y.

4. (5 pts) Consider a random variable X with pdf

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}.$$

   Compute the pdf $f_Y(y)$ of the random variable Y that is defined as follows:

$$Y = \begin{cases} X & X \geq 0 \\ X^2 & X < 0 \end{cases}$$

5. (10 pts) Suppose that a zero-mean Gaussian random variable X with variance $\sigma^2 = 9$ is passed through the quantizer function $g(x)$ shown in Figure 1. The output is $Y = g(X)$. Express the pmf of Y in terms of $\Phi(\cdot)$, where $\Phi(\cdot)$ is the cdf of Gaussian random variable with zero mean and unit variance.

6. (5 pts) The random variable X is Gaussian with mean m and standard deviation σ. Let $Y = e^X$. Find the pdf of Y. Note that Y is called *lognormal random variable.*

7. (10 pts) Suppose $f_X(x) = 2x/\pi^2, 0 < x < \pi$, and $Y = \sin(X)$. Determine $f_Y(y)$.
   Hint: $\sin x_1 = \sin(\pi - x_1)$.

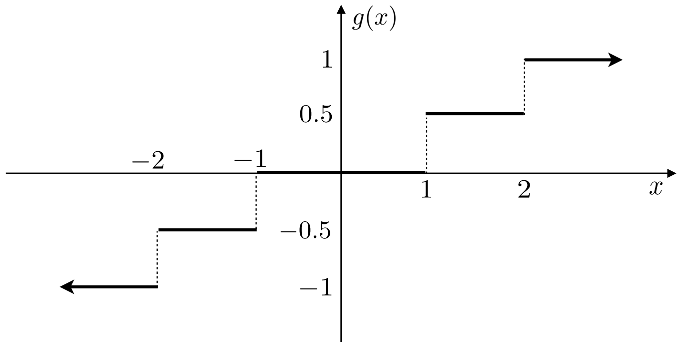

Figure 1: Figure for Problem 5.

8. (10 pts) Consider random variable X that is zero-mean Gaussian with variance equal to b^2. Find the pdf $f_Y(y)$ of random variable $Y = g(X)$ where

$$g(x) = \begin{cases} -b & x < -b \\ x & -b \leq x \leq b \\ b & x > b \end{cases}$$

9. (10 pts) Consider a random variable X with pdf

$$f_X(x) = \begin{cases} \frac{1}{4} + \frac{1}{2}\delta\left(x - \frac{1}{4}\right) & -1 < x < 1 \\ 0 & |x| > 1 \end{cases}$$

Define $Y = X^2$. Determine the pdf and cdf of Y.

10. (10 pts) Find the mean and variance of the Poisson random variable. (Note: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.)

11. (5 pts) Find the n^{th} moment of X if X is uniformly distributed in the interval $[0, 1]$. Repeat for an arbitrary interval $[a, b]$.

12. (5 pts) The time X between customer arrivals at a service station (also called inter-arrival time) has an exponential pdf with parameter λ. Find the mean inter-arrival time (i.e., find $E[X]$).

13. (10 pts) A discrete voltage signal X is uniformly distributed in the set $\{-3, -2, \ldots, 3, 4\}$.

    (a) (4 pts) Find the mean and variance of X.

    (b) (6 pts) Find the mean and variance of $Y = -2X^2 + 3$.