ECE 5605: Stochastic Signals and Systems – Fall 2018

Homework #5
Due: Thursday, October 18, 2018 (1:45pm)

Reading: Chapters 5-6 of the textbook.
Total points: 85

1. (10 pts) X and Y are independent and uniform in the interval $(0, a)$. Find the pdf of $Z = X/Y$.

2. (10 pts) X and Y are independent random variables with pdfs

$$f_X(x) = e^{-x}U(x) \qquad f_Y(y) = e^{-y}U(y)$$

   Find the pdf of the following random variables (a) $X - Y$, (b) XY, and (c) $\min(X, Y)$.

3. (10 pts) The joint pdf of the random variables X and Y is given by

$$f_{X,Y}(x, y) = \begin{cases} 1 & \text{in the shadowed area} \\ 0 & \text{otherwise} \end{cases}$$

   Let $Z = X + Y$. Find $F_Z(z)$ and $f_Z(z)$.

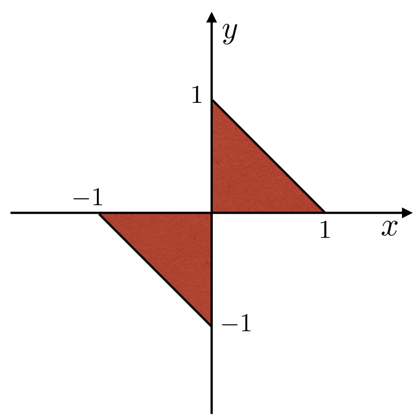

Figure 1: Figure for Problem 3.

4. (a) (5 pts) Determine the pdf of random variable Z in terms of the joint pdf of X and Y, where

$$Z = \frac{X}{X + Y}.$$

   (b) (5 pts) Find the pdf of Z if X and Y are independent exponential random variables with mean 1, i.e., $f_X(x) = e^{-x}U(x)$ and $f_Y(y) = e^{-y}U(y)$, where $U(\cdot)$ is the unit step function.

5. (10 pts) The relation between electric resistance (R), current (I), and power (P) is given by $P = RI^2$. Suppose that R and I are independent random variables with the probability density functions given below. Determine the pdf of P.

$$\begin{aligned} f_R(r) &= 2r, & 0 \le r \le 1 \\ f_I(i) &= 6i(1 - i), & 0 \le i \le 1. \end{aligned}$$

6. (10 pts) X and Y are two independent and exponentially distributed random variables with parameters λ_1 and λ_2, respectively. Define $Z = \max\{X, Y\}$ and $W = \min\{X, Y\}$.

   (a) Find $\mathbb{E}[Z]$.

   (b) Determine $f_W(w)$.

7. (15 pts) An ambulance is continuously traveling back and forth with a constant speed of v on a road of length L, such that at any time t, the location of the ambulance can be assumed to be uniformly distributed on the road. An accident occurs at a uniformly distributed location on the road, which requires the urgent presence of the ambulance. Assume that the locations of the accident and ambulance are independent of each other.

   (a) Find the pdf of the distance between the ambulance and the accident location.

   (b) Find the average time needed for the ambulance to arrive at the accident location (assuming that the ambulance driver has been immediately notified of the accident).

8. (10 pts) X and Y are two independent random variables with the following probability density functions:

$$f_X(x) = \frac{x}{\sigma_X^2} \mathrm{e}^{-\frac{x^2}{2\sigma_X^2}} U(x)$$

$$f_Y(y) = \frac{y}{\sigma_Y^2} \mathrm{e}^{-\frac{y^2}{2\sigma_Y^2}} U(y),$$

   where $U(.)$ is the unit step function. For a given $z > 0$, find $\mathbb{P}[X \leq zY]$.