ECE 5605: Stochastic Signals and Systems – Fall 2018

Homework #1
Due: September 4, 2018 (1:45 pm)

Reading: Chapter 2 of the textbook.
Total points: 90

1. (5 pts) A random experiment has sample space $S = \{a, b, c, d, e\}$. Assuming $P[\{a, b, c\}] = 0.5$ and $P[\{c, d, e\}] = 0.7$, find $P[\{c\}]$.

2. (5 pts) A new family moves in your neighborhood. Assuming that they have two children and one of them is a girl, what is the probability that the other is also a girl?

3. (10 pts) Current students of Hogwarts are studying Astronomy and History of Magic. Denote the event that a student is good in Astronomy by A and is good in History of Magic by M. Denote their complementary events by A^c and M^c, respectively. Let $P[A \cap M] = 0.3$, $P[A \cap M^c] = 0.2$ and $P[A^c \cap M] = 0.2$. We pick up a random student and observe his/her proficiency in these two subjects.

   (a) (1 pts) What is the sample space of this experiment?

   (b) (3 pts) What is the probability that the student is good in Astronomy?

   (c) (3 pts) What is the probability that the student is good in History of Magic?

   (d) (3 pts) What is the probability that the student is good in Astronomy or History of Magic?

4. (5 pts) In World War II, three cryptographers were assigned to independently decode a message. Based on their expertise and skill level, their individual success probability are known to be 0.5, 0.3, and 0.8. What is the probability that the message is finally decoded?

5. (5 pts) In some random experiment, the probability of event A happening is p. This experiment is repeated independently n times. How small must the probability p be so that the probability that A does not occur in any of the n trials is greater than the probability that it occurs at least once during the n trials?

6. (5 pts) Show that if $A \cap C = \varnothing$, then $P[A \cup C|B] = P[A|B] + P[C|B]$.

7. (5 pts) In a trial, let T be the event that testimony is true and G be the event that accused is guilty. Some lawyers have argued on the assumption that $P[G|T] = P[T|G]$. Show that this holds if and only if $P[G] = P[T]$.

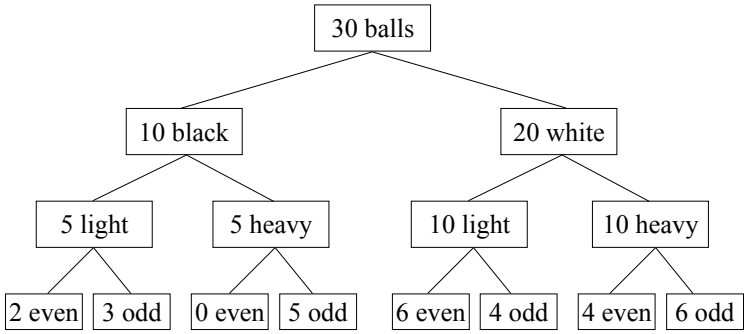
8. (10 pts) Urn A contains 5 balls labeled with numbers $\{1, 3, 5, 7, 9\}$ and urn B contains 4 balls labeled with numbers $\{2, 4, 6, 8\}$. Draw two balls, one at random from each urn, and note down their labels (numbers written on them). What is the probability that the multiplication of the two labels/numbers is greater than 10 but their addition is less than 15?

9. (10 pts) Box A contains 1 white ball and $n > 1$ black balls. Box B contains 1 black ball and $n > 1$ white balls. We pick a ball from a randomly selected box (assume the selection of two boxes to be equally likely). If the ball is black, what is the probability that it came from box A?

10. (5 pts) It is known that if it rains, there is a 50% chance that a sewer will overflow. Also, if the sewer overflows, then there is a 30% chance that the road will flood. If there is a 20% chance that it will rain, what is the probability that the road will flood? Assume that the road floods only if the sewer overflows, and that the sewer overflows only if it rains.

11. (5 pts) An urn contains 10 numbered black balls (some even, some odd) and 20 numbered white balls (some even, some odd). Some of the balls of each color are lighter in weight than the others. The composition of the urn is shown in the diagram below.

    Let A denote the event of picking a black ball, B denote the event of picking one of the lighter balls, and C denote the event of picking an even-numbered ball. Are events A, B, and C independent? Assume that each ball (regardless of color, weight, and number) is equally likely to be picked.



12. (10 pts) Consider that we have two coins, one fair with $P(\text{heads}) = 0.5$ and the other unfair with $P(\text{heads}) = 0.8$. We pick a coin at random and toss it twice.

    (a) (5 pts) Are the two tosses conditionally independent (conditioned on the fact that you know which coin was picked)?

    (b) (5 pts) Are the two tosses independent?

13. (10 pts) A ternary communication channel is shown below. Suppose that the input symbols 0, 1, and 2 are equally likely (i.e., each occurs with probability 1/3).

    (a) (5 pts) Suppose that bit 1 is observed as an output. What is the more probable input symbol?

    (b) (5 pts) Is there a choice of ϵ for which the input to the channel is independent of the output of the channel?