ECE 5605: Stochastic Signals and Systems – Fall 2018

Homework #4
Due: Thursday, October 11, 2018 (1:45pm)

Reading: Chapters 3-5 of the textbook.
Total points: 95

1. (10 pts) The random variable X is Gaussian with mean zero and variance σ^2. Find $E[X|X > 0]$ and VAR$[X|X > 0]$.

2. (5 pts) The random variable X is Gaussian with mean m and standard deviation σ. Using characteristic function approach, find the mean of $Y = \cos(X)$. Use the fact that $\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$.

3. (5 pts) Derive the characteristic function of random variable X that is uniformly distributed in $[-b, b]$. Find $E[X]$ by applying the moment theorem.

4. (10 pts) The random variable X has pdf

$$f_X(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Assuming $g(X) = \frac{\sin(X)}{X}$, find $E[g(X)]$. Hint: $\frac{\sin(x)}{x} = \frac{1}{2}\int_{-1}^{1} e^{jux}du$.

5. (10 pts) Let (X, Y) have the joint pdf

$$f_{X,Y}(x, y) = xe^{-x(1+y)} \qquad x > 0, y > 0.$$

Find the marginal pdf of X and Y.

6. (10 pts) The general form of the joint pdf for two jointly Gaussian random variables is $f_{X,Y}(x, y) =$

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-m_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{x-m_1}{\sigma_1}\right)\left(\frac{y-m_2}{\sigma_2}\right) + \left(\frac{y-m_2}{\sigma_2}\right)^2\right]\right)$$

for $-\infty < x < \infty$ and $-\infty < y < \infty$. Find $P[X^2 + Y^2 < q^2]$ assuming $\rho = 0$, $m_1 = m_2 = 0$ and $\sigma_1 = \sigma_2 = \sigma$. Hint: use polar coordinates to compute the integral.

7. (10 pts) The random variables X and Y have the joint pdf

$$f_{X,Y}(x, y) = c\sin(x + y) \quad 0 \leq x \leq \frac{\pi}{2}, \quad 0 \leq y \leq \frac{\pi}{2}$$

   (a) Find the value of constant c.
   (b) Find the joint cdf of X and Y.
   (c) Find the marginal pdfs of X and Y.

8. (10 pts) Let (X, Y) have the joint pdf

$$f_{X,Y}(x, y) = k(x + y), \quad 0 < x < 1, 0 < y < 1.$$

(a) Are X and Y independent?

(b) Find $f_Y(y|x)$.

9. (10 pts) The random variables X and Y have the joint pdf

$$f_{X,Y}(x,y) = \begin{cases} e^{-y} & 0 \le x \le y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Evaluate the conditional expectations $E[X|y]$ and $E[Y|x]$.

10. (10 pts) (Slide 24 of Handout 4) The joint pdf of (X, Y) is given by

$$f_{X,Y}(x,y) = 6(1 - x - y)$$

for values of x and y for which (x, y) lies within the triangle shown in Figure 1. Find the conditional expected values of X and X^2 given $Y = y$, i.e., find $E[X|y]$ and $E[X^2|y]$.
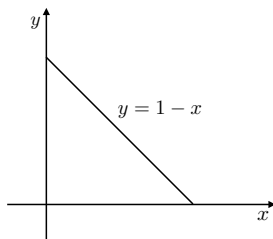

Figure 1: Figure for Problem 10.

11. (5 pts) The random variables X and Y have joint density

$$f_{X,Y}(x,y) = \begin{cases} e^{-x} & 0 \le x < \infty, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Evaluate the probability $P[X \ge Y]$.