Notes on Linear Algebra

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Chapter 1

Preliminaries

1.1 Vector Spaces

Bases

Generating (span)

Independent System

Definition 1.1 (Independent)**.** The system $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \ldots, \mathbf{v_n}\}$ are called "Independent" iff

$$\lambda_1 \mathbf{v_1} + \lambda 2 \mathbf{v_2} + \cdots + \lambda_n \mathbf{v_n} = \mathbf{0} \iff \lambda_i = 0$$

Definition 1.2 (Dependent)**.** The system $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \ldots, \mathbf{v_n}\}$ are called "Dependent" iff there exits some $\lambda_j \neq 0$ such that $j < n$ and

$$\sum_{i=1}^{n} \lambda_i \mathbf{v_i} = \mathbf{0}$$

Proposition 1. *The system $\{v_1, v_2, v_3, \ldots, v_n\}$ are dependent iff there exits j such that $\mathbf{v_j}$ is the linear combination of the other vectors.*

Proof. There exits $j < n$ such that $\lambda_j \neq 0$ and

$$\begin{aligned} & \sum_{i=1}^{n} \lambda_i \mathbf{v_i} = \mathbf{0} \\ \Longrightarrow \quad & \lambda_j \mathbf{v_j} + \sum_{i \neq j} \lambda_i \mathbf{v_i} = \mathbf{0} \\ \Longrightarrow \quad & \mathbf{v_j} = \sum_{i \neq j} \frac{-\lambda_i}{\lambda_j} \mathbf{v_i} \end{aligned}$$

And the converse is simple to prove. □

Proposition 2. *If the system $\{v_1, v_2, v_3, \ldots, v_n\}$ are both*

Proposition 3. *Bases $\iff$ Generating + Independent*