Notes on Abstract Algebra

Sivmeng

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Chapter 1

Benedict Gross

1.1 Preliminaries

- We have group $GL_n(\mathbb{R})$ set of all invertable $n \times n$ matrixes with entries in $\mathbb{R}$
- same thing with $GL_n(\mathbb{C}), GL_n(\mathbb{Q})$

Definition 1.1 (Group)**.** All group G is

- a set with a product structure, that is if $a, b \in G$ then $a \cdot b \in G$
- associative: $a(bc) = (ab)c$
- exits indentiy e such that $ae = ea = a$
- exits inverse a^{-1} such that $aa^{-1} = a^{-1}a = e$

1.2 Ur group

The $\text{sym}(T)$ set of all bijections $a : T \to T$. Deine $a \cdot b(t) = a(b(t))$

- bijection = ormomophism
- $GL_n(\mathbb{R}) \subset \text{sym}(\mathbb{R}^n)$

Definition 1.2. sub group $H \subset G$ closed under $\cdot$ contain indentiy, and closed.

- Let $S_n = \text{sym}\{1, 2, 3, ..., n\}$ permution group of n letter. This is finite group of order n. So $|S_n| = n!$
- $S_1(1) = \{e\}$
- $S_2 = \{e, \tau\}$ where $e\tau = \tau e = \tau$ and $\tau\tau = e$
- $S_3 = \{e, \tau', \tau'', \sigma, \sigma'\}$

Definition 1.3. If $ab = ba$ for all pairs, then G is Abelian group.

Corollary 1. The group S_n is non-abelion for all $n \geq 3$.

Proof. Since $S_3 \subset S_n$. Fixing the letters $\{4, 5, 6, ..., n\}$. But S_3 not non-abelion, then so is S_n. □

Claim 1. For $k \leq n$ then $S_k \subset S_n$.

What is a subgroup of $GL_2(\mathbb{R})$ which stablized the line. The matrix

$$\begin{pmatrix} a & c \\ 0 & d \end{pmatrix}$$

where $ad \neq 0$. $y = 0$.

Proposition 1. *The subgroup of* $(\mathbb{Z}, +)$ *are precisely given by* $(b\mathbb{Z}, +)$.

Proof. First, there are all subgroups. Let $H \subset \mathbb{Z}$. If $H = \{0\}$ then here $b = 0$.

Now suppose that $H \neq \{0\}$. so contains $m \neq 0$. Let $b > 0$ be the smallest positive integer contained in H. Then $H \supset b\mathbb{Z}$. Suppose that $h \in H$ and write $h = mb + r$ with $0 \leq r < b$. We cliam that $r = 0$.

$$h + (-m)b = r$$

somehow $r = 0$ □

For any group G, $g \in G$, Denote $H = \langle g \rangle$ be cyclic subgruop generated by G smallest subgroup contains g. If $g^m = e$ and m is the smallest power, we say that m is the order of $g \in G$. If such m not exits, we say g has infinite order.