Price an arithmetic average call

Price an arithmetic average call using the binomial tree model.

Payoff(t) = max(Save(t) - K; 0)

where Save(t) is the arithmetic average of stock prices from the issue date until the current time point t.

1. Method

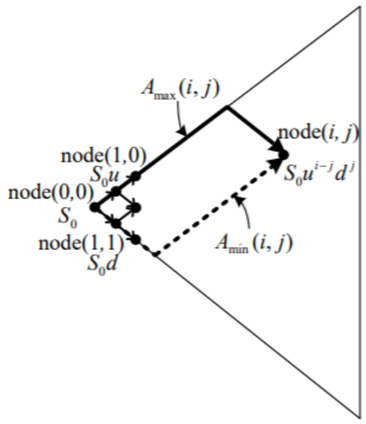
(i) binomial tree model : European and American arithmetic average calls

(ii) Monte Carlo simulation : European arithmetic average calls

2. Algorithm

Inputs: St, K, r, q, sd, T - t, M, n, Save(t), passing time, number of simulations, number of repetitions.

1. build tree and calculate average of max and min price series.



$$A_{\max}(i,j) = S_0(1 + \overbrace{u + u^2 + \cdots + u^{i-j}}^{i-j \text{ up movements}} + \overbrace{u^{i-j}d + u^{i-j}d^2 + \cdots + u^{i-j}d^j}^{j \text{ down movements}})/\,(i+1)$$
$$= (S_0 + S_0u\tfrac{1-u^{i-j}}{1-u} + S_0u^{i-j}d\tfrac{1-d^j}{1-d})/(i+1)$$

$$A_{\min}(i,j) = S_0(1 + \overbrace{d + d^2 + \cdots + d^j}^{j \text{ down movements}} + \overbrace{d^ju + d^ju^2 + \cdots + d^ju^{i-j}}^{i-j \text{ up movements}})/(i+1)$$
$$= (S_0 + S_0d\tfrac{1-d^j}{1-d} + S_0d^ju\tfrac{1-u^{i-j}}{1-u})/(i+1)$$

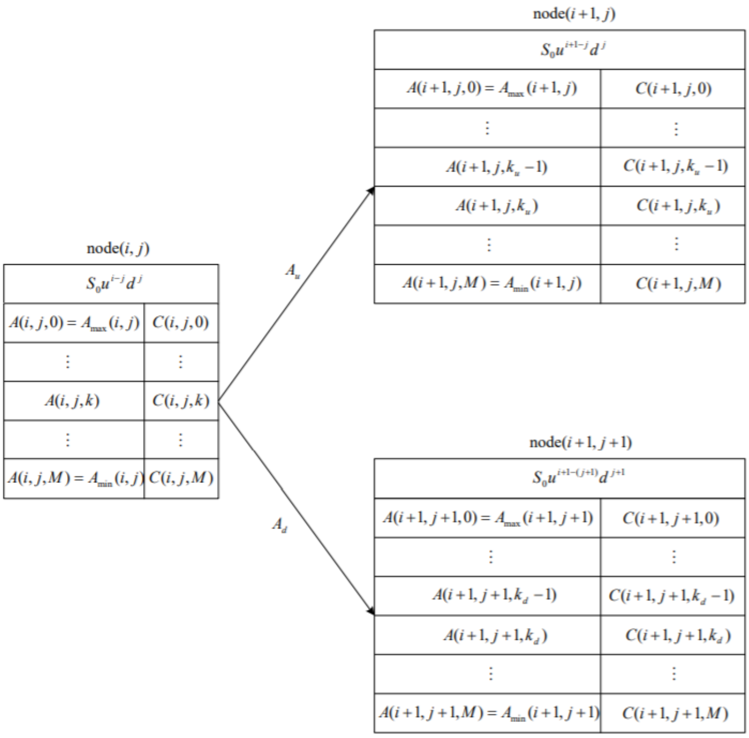
2. Generate M times of average prices for each node between average of max and min price series.

$$\left(A(i,j,k) = \exp\left(\frac{M-k}{M}\ln(A_{\max}(i,j)) + \frac{k}{M}\ln(A_{\min}(i,j))\right),\ \text{for } k = 0, \ldots, M.\right)$$

3. For each terminal node(n, j), decide the payoff for each representative average price A(n, j, k).

4. Backward Induction :

Find corresponding average prices and its option price from terminal



5. Apply Interpolation method to get appropriate average price for A(I,J,K)

For $A(i, j, k)$, $0 \le j \le i \le n$, and $k = 0, 1, \ldots, M$,

$\Rightarrow A_u = \frac{(i+1)A(i,j,k)+S_0u^{i+1-j}d^j}{i+2}$

Suppose A_u is inside the range $[A(i+1, j, k_u), A(i+1, j, k_u - 1)]$. The corresponding option value C_u for A_u can be approximated by the linear interpolation, i.e.,

$$C_u = w_u C(i+1, j, k_u) + (1 - w_u) C(i+1, j, k_u - 1),$$

where

$$w_u = \frac{A(i+1, j, k_u - 1) - A_u}{A(i+1, j, k_u - 1) - A(i+1, j, k_u)}.$$

$\Rightarrow A_d = \frac{(i+1)A(i,j,k)+S_0u^{i+1-(j+1)}d^{(j+1)}}{i+2}$

Similarly, if A_d is inside the range $[A(i+1, j+1, k_d), A(i+1, j+1, k_d - 1)]$. The corresponding option value C_d for A_d can be approximated by the linear interpolation following the same logic as above.

6. backward induction for both euro and American calls

$\Rightarrow C(i,j,k) = [pC_u + (1-p)C_d]e^{-r\Delta t}$

$*$ If American arithmetic average options are considered, the option value $C(i,j,k)$ $= \max(A(i,j,k) - K, [pC_u + (1-p)C_d]e^{-r\Delta t})$.