

# Price American Lookback Put with the Binomial Tree and Monte Carlo

## 1. requirement:

The payoff function of the lookback put is as follows.

Payoff<sub>t</sub> =  $\max(S_{\max,t} - S_t, 0)$ , where  $S_{\max,t} = \max S_u$ , for  $u = 0, \Delta t, 2\Delta t, \dots, t$ .

Implement the binomial tree model to price both European and American lookback puts.

## 2. Binomial Tree Algorithm :

- 1 Build the binomial tree
- 2 Record possible max price( $S_{\max}$ ) for each node.
  - 2.1 Insert max price for monotonic upward and downward prices.
  - 2.2 The forward-tracking method – For a node with the stock price  $S_t$ , inherit  $S_{\max}'$  s from parents :
    1. If  $S_{\max}$  from parents  $\geq S_t \Rightarrow$  Insert this  $S_{\max}$  into its  $S_{\max}$ -list
    2. If  $S_{\max}$  from parents  $< S_t \Rightarrow$  Ignore  $S_{\max}$  and insert  $S_t$  into its  $S_{\max}$ -list
- 3 Two Backward inductions (a/e) – Calculate early exercise at the same time for American put
  - Backward inductions : Try to find  $S_{\max}(t)$  in list of  $S_{\max}(t+1)$ . If found, use the option price of the same  $S_{\max}$ . If not, insert option values of the upward price,

which will be the maximum.

Notice : for monotonic upward price, use the option price of the next upward node if there is no same  $S_{max}$ .

### **3. Monte Carlo Algorithm :**

1. Break a price path into 100/300 prices. Use the new price to calculate next mean.
2. Record the maximum price
3. Create 10000 paths and their value with 20 repetitions.