Problem Set 2

Problems 1-2 correspond to "The generative approach to classification"

Problem 1

1/1 point (graded)

Which of the following accurately describes the generative approach to classification, in the case where there are just two labels?

- ○ Fit a model to the boundary between the two classes.
- ◉ Fit a probability distribution to each class separately. ✔

Submit

Problem 2

1/1 point (graded)

In a generative model with k classes, the class probabilities are $\pi_1, \ldots, \pi_k$ (summing to 1) and the individual class distributions are $P_1(x), \ldots, P_k(x)$. In order to classify a new point x, we should pick the label j that maximizes which of the following quantities?

- ○ π_j
- ○ $P_j(x)$

- ○ $\pi_j + P_j(x)$
- ◉ $\pi_j P_j(x)$ ✔

Submit

Problems 3-8 correspond to "Probability review I: probability spaces, events, conditioning"

Problem 3

3/3 points (graded)

What is the **size** of the **sample space** in each of the following experiments?

a) A fair coin is tossed.

2 ✔

2

b) A fair die is rolled.

6 ✔

6

c) A fair coin is tossed ten times in a row.

1024 ✔

1024

Submit

Problem 4

3/3 points (graded)

Consider a sample space $\Omega = \{a, b, c, d\}$ in which the probability of outcome a is $1/2$, the probability of outcome b is $1/8$, and the probability of outcome c is $1/4$.

a) What is the probability of outcome d?

0.125

0.125

b) Define event $A = \{a, b, c\}$. What is the probability of event A?

0.875

0.875

c) Define event $B = \{a, c, d\}$. What is the probability of event $A \cap B$?

0.75

0.75

Submit

Problem 5

3/3 points (graded)

Two fair dice are rolled. What is the probability that:

a) Their sum is 10, given that the first roll is a 6?

1/6

$\frac{1}{6}$

b) Their sum is 10, given that the first roll is an even number?

1/9

$\frac{1}{9}$

c) They have the same value?

1/6

$\frac{1}{6}$

Submit

Problem 6

1/1 point (graded)

A certain genetic disease occurs in 5% of men but just 1% of women. Let's say there are an equal number of men and women in the world. A person is picked at random and found to possess the disease. What is the probability, given this information, that the person is male?

0.83

0.83

Submit

Problem 7

2/2 points (graded)

The TryMe smartphone company has three factories making its phones. They are all fairly unreliable: 10% of the phones from factory 1 are defective, 20% of the phones from factory 2 are defective, and 24% of the phones from factory 3 are defective. The factories do not produce the same numbers of phones: factory 1 produces $1/2$ of TryMe's phones, while factories 2 and 3 each produce $1/4$.

a) What is the probability that a TryMe phone chosen at random is defective?

0.16

0.16

b) Given that a TryMe phone is defective, what is the probability that it came from factory 1?

0.3125

0.3125

Submit

Problem 8

1/1 point (graded)

Here are some statistics collected by a doctor about patients who walk into her office.

- 25% of the patients have the flu.
- Among patients with the flu, 75% have a fever.
- Among patients who don't have the flu, 50% have a fever.

A new person walks into the doctor's office and turns out to have a fever. What is the probability that he has the flu?

0.33

0.33

Submit

Problems 9-12 correspond to "Generative modeling in one dimension"

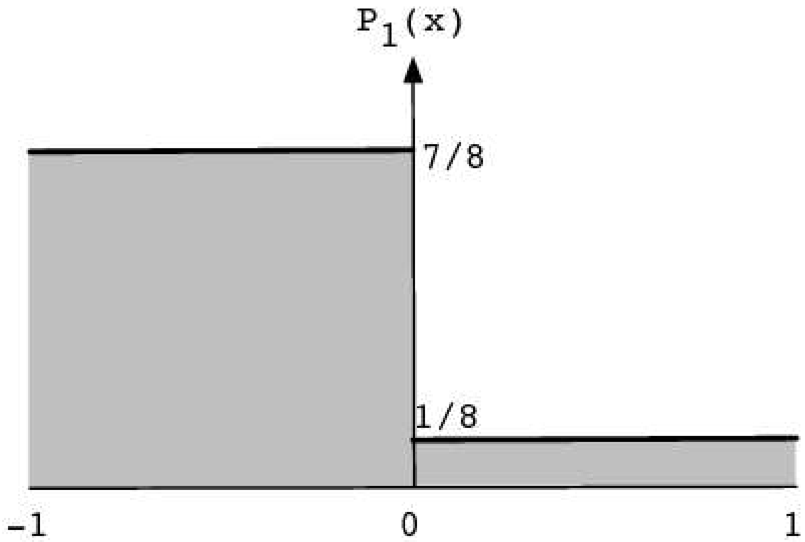
Problem 9

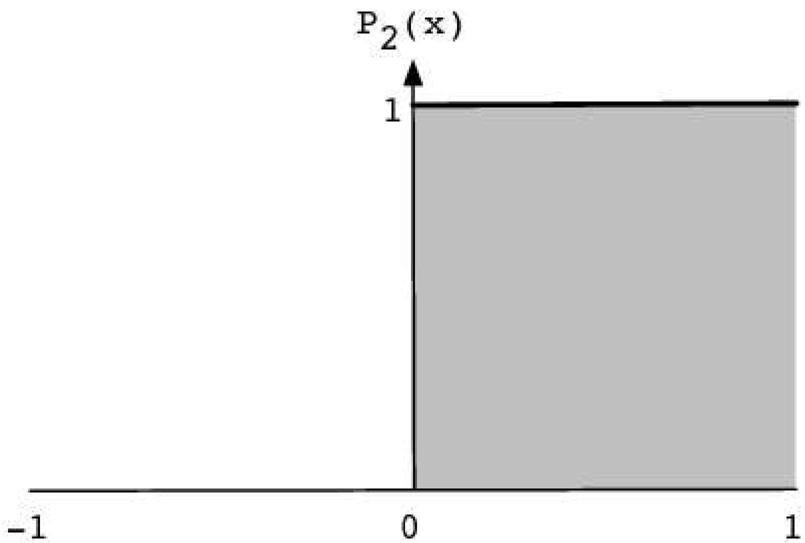
2/2 points (graded)

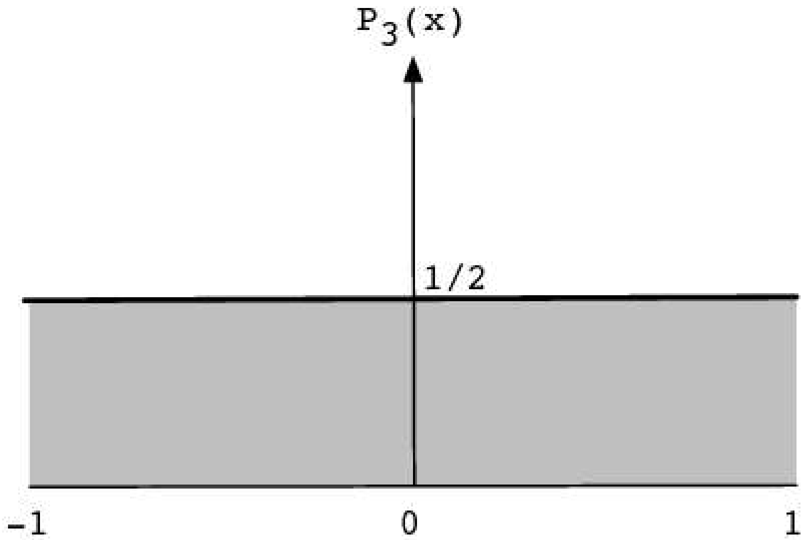
Suppose we have one-dimensional data points lying in $X = [-1, 1]$, that have associated labels in $Y = \{1, 2, 3\}$. The individual classes have weights

$$\pi_1 = \frac{1}{3},\ \ \pi_2 = \frac{1}{6},\ \ \pi_3 = \frac{1}{2}$$

and densities P_1, P_2, P_3 as shown below. (For instance, P_1 is the density of the points whose label is 1; in particular, this means that P_1 integrates to 1.)







Based on this information, what labels should be assigned to the following points?

a) $-1/2$

1

1

b) $1/2$

3

3

Submit

Problem 10

2/2 points (graded)

A set of 100 data points in $\mathbb{R}$ have mean of 20 and standard deviation of 10. We want to fit a Gaussian $N(\mu, \sigma^2)$ to this data. What μ and σ^2 should we pick?

a) μ =

20

20

b) σ^2 =

100 ✔

100

Submit

Problem 11

1/1 point (graded)

A generative approach is used for a binary classification problem and it turns out that the resulting classifier predicts $+$ at **all** points x in the input space. What can we conclude for sure? Check all that apply.

- [ ] There are no $-$ points in the training set.
- [ ] The $+$ points are spread out over the space, while the $-$ points are concentratd in a small region.
- [x] There are fewer $-$ points than $+$ points in the training set.
- [ ] The density of $+$ points is greater than the density of $-$ points everywhere in the space.

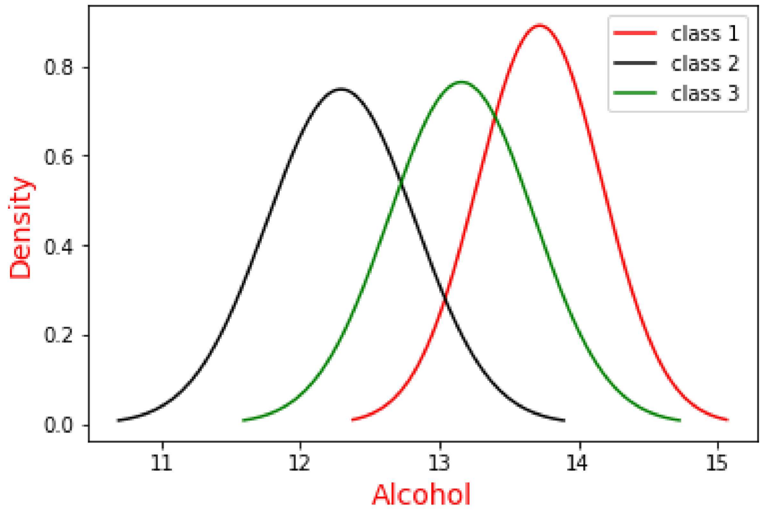
✔

Submit

Problem 12

5/5 points (graded)

For the winery example from lecture, the densities obtained are reproduced here:



The class probabilities are $\pi_1 = 0.33, \pi_2 = 0.39, \pi_3 = 0.28$. What labels would be assigned to the following points?

a) 12.0

2

2

b) 12.5

2

2

c) 13.0

3

3

d) 13.5

1

1

e) 14.0

1

1

Submit

Problems 13-15 correspond to "Probability review II: random variables, expected value, and variance"

Problem 13

4/4 points (graded)

A fair die is rolled twice. Let X_1 and X_2 denote the outcomes, and define random variable X to be the minimum of X_1 and X_2.

a) How many possible values are there for X?

6

6

b) What is the probability that $X = 1$?

11/36

$\frac{11}{36}$

c) What is $E(X)$?

2.52

2.52

d) What is $\mathrm{var}(X)$?

1.96

1.96

Submit

Problem 14

2/2 points (graded)

In a series of ten independent experiments, a random variable X takes on values

$$1, 1, 2, 5, 0, 1, 2, 2, 1, 1.$$

a) Give an estimate of $E(X)$.

1.6

1.6

b) Give an estimate of $\mathbf{var}(X)$.

1.64

1.64

Submit

Problem 15

1/1 point (graded)

Which of the following random variables has $\mathbf{zero\ variance}$? Check all that apply.

- [ ] X takes on values -1 and 1 with equal probability.
- [x] X always takes on value 1.
- [ ] X is always equal to X^2.

- [x] X is always zero.

✔

Submit

Problems 16-18 correspond to "Probability review III: modeling dependence"

Problem 16

4/4 points (graded)

In each of the following cases, say whether X and Y are dependent or independent.

a) Randomly pick a card from a pack of 52 cards. Define X to be 1 if the card is a Jack, and 0 otherwise. Define Y to be 1 if the card is a spade, and 0 otherwise.

- ○ dependent
- ◉ independent ✔

b) Randomly pick two cards from a pack of 52 cards. X is 1 if the first card is a spade, and 0 otherwise. Y is 1 if the second card is a spade, and 0 otherwise.

- ◉ dependent ✔
- ○ independent

c) Toss a coin ten times. X is the number of heads and Y is the number of tails.

- ◉ dependent ✔
- ○ independent

d) Roll a fair die. X is 1 if the outcome is even, and 0 otherwise. Y is 1 if the outcome is ≥ 3, and zero otherwise.

- ( ) dependent
- (•) independent ✔

Submit

Problem 17

2/2 points (graded)

Random variables X, Y take on values in the range $\{-1, 0, 1\}$ and have the following joint distribution.

		Y		
		-1	0	1
	-1	0	0	$1/3$
X	0	0	$1/3$	0
	1	$1/3$	0	0

a) What is the covariance between X and Y?

-2/3 ✔

$-\frac{2}{3}$

b) What is the correlation between X and Y?

-1 ✔

-1

Submit

Problem 18

2/2 points (graded)

Random variables X, Y take on values in the range $\{-1, 0, 1\}$ and have the following joint distribution.

		Y		
		-1	0	1
	-1	$1/6$	0	$1/6$
X	0	0	$1/3$	0
	1	$1/6$	0	$1/6$

a) Are X and Y independent?

- (•) dependent ✔
- ( ) independent

b) Are X and Y uncorrelated?

- ( ) correlated
- (•) uncorrelated ✔

Submit

Problems 19-20 correspond to "Two-dimensional generative modeling with the bivariate Gaussian"

Problem 19

2 points possible (graded)

Each of the following scenarios describes a joint distribution (x, y). In each case, give the parameters of the (unique) bivariate Gaussian that satisfies these properties.

a) x has mean 2 and standard deviation 1, y has mean 2 and standard deviation 0.5, and the correlation between x and y is -0.5.

- $\mu = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$
- $\mu = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 1 & -1 \\ -1 & \frac{1}{2} \end{pmatrix}$
- $\mu = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 1 & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$

b) x has mean 1 and standard deviation 1, and y is equal to x.

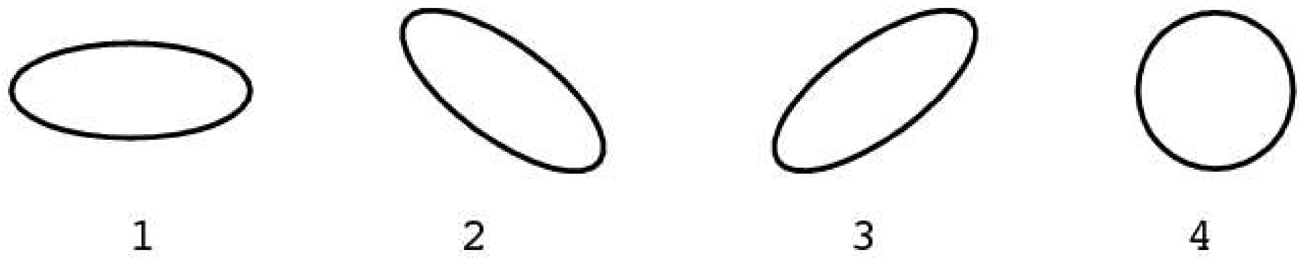
- $\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- $\mu = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
- $\mu = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$
- $\mu = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Submit

Problem 20

3 points possible (graded)

Here are four possible shapes of Gaussian distributions:



For each of the following Gaussians $N(\mu, \Sigma)$, indicate which of these shapes (1,2,3,4) is the best approximation.

a) $\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix}$

b) $\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 9 & 2 \\ 2 & 1 \end{pmatrix}$

c) $\mu = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & -0.75 \\ -0.75 & 1 \end{pmatrix}$

Submit

Course › Week 2... › Compr... › Quiz 2

Quiz 2

Problem 1

1/1 point (graded)
In the generative approach to classification, how do we choose which label to give to a new point x ?

- Choose the label j that occurs most frequently, i.e., $\max(\pi_j)$
- Choose the label j whose distribution P_j assigns the highest probability to x, i.e. $\max(P_j(x))$.
- (selected) Choose the label j that maximizes $\pi_j P_j(x)$. ✔
- Choose the label j for which x is the smallest number of standard deviations away from the mean of P_j

Submit

Problem 2

1/1 point (graded)
Select the correct version of Bayes' Rule from the following:

- (selected) $P(A|B) = P(A) \times P(B|A)/P(B)$ ✔
- $P(A|B) = P(B) \times P(B|A)/P(A)$

- $P(AB) = P(A) \times P(B|A)$
- $P(AB) = P(A) \times P(B)$

Submit

Problem 3

1/1 point (graded)

What is a probability space?

- The set of all possible outcomes for a given experiment
- The probability of an event occurring
- The set of all possible outcomes for an experiment and the probabilities of those outcomes occurring ✔
- The domain of the probability density function

Submit

Problem 4

3/3 points (graded)

Suppose you toss two 6-sided dice and you want to know what the probability is that the difference of their faces is equal to 3.

a) What is the size of the sample space for this problem?

- 6

- 12
- 18
- 36 ✔

b) How many events in the sample space are successful for this experiment?

- 6 ✔
- 8
- 12
- 18

c) What is the probability of this event occurring?

- $\frac{1}{36}$
- $\frac{1}{12}$
- $\frac{1}{6}$ ✔
- $\frac{1}{3}$

Submit

Problem 5

1/1 point (graded)

Suppose there are 3 bags, each containing 3 colored balls. The first two bags contain a red, green, and blue ball. The third bag contains 3 red balls. You choose a bag at random. You then pull out a ball and replace it back into the bag. You then pull out another ball and determine that both of the balls you chose were red. What is the probability you chose the bag with the 3 red balls?

- 55%
- 66%
- 71%
- 82% ✔

Submit

Problem 6

1/1 point (graded)

A particular classification task has three possible labels. Which of the following three features is likely to be the most useful for determining which label should be given to a new data point, based on the Gaussian distributions given?

Feature 1:

Label 1 ~ N(2, 4)

Label 2 ~ N(1, 1)

Label 3 ~ N(3, 6.25)

Feature 2:

Label 1 ~ N(1, 1)

Label 2 ~ N(1, 0.56)

Label 3 ~ N(1, 1.27)

Feature 3:

Label 1 ~ N(4, 0.25)

Label 2 ~ N(2, 1)

Label 3 ~ N(8, 0.25)

- Feature 1
- Feature 2
- Feature 3 ✔

Submit

Problem 7

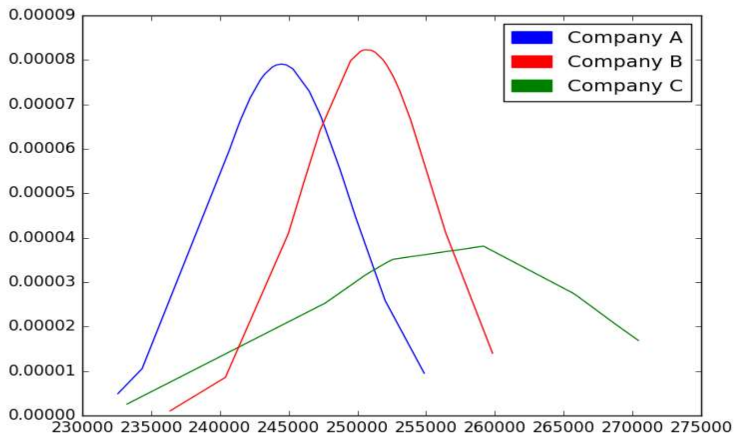
3/3 points (graded)

Let's say you are building a replica of your expensive electric guitar. You wish to use the same electronic hardware as your old guitar uses but none of the parts are labeled with their manufacturer. You know that three companies sell potentiometers to your guitar's maker: company A, company B, and company C. You happen to have a training set of potentiometers from the three companies, consisting of 22 from A, 25 from B and 14 from C.

From your set of potentiometers, what is the probability, π_B, that company B manufactured a particular potentiometer?

- 22%
- 32%
- 41% ✔
- 55%

You decide to analyze the feature of "maximum resistance" to try to determine the manufacturer of your old potentiometer. After analyzing your training set, you find that company A's potentiometers have an average maximum resistance of $244k\Omega$ with a standard deviation of $6.2k\Omega$, company B's are $251k\Omega$ with a standard deviation of $4.1k\Omega$, and company C's are $261k\Omega$ with a standard deviation of $11.1k\Omega$. The distribution functions are shown:



You also find that your old potentiometer has a maximum resistance of $248k\Omega$. Based on this information, which company can you conclude manufactured your old potentiometer?

- Company A
- Company B
- Company C
- Cannot reliably determine ✔

If your old potentiometer instead had a maximum resistance of $269k\Omega$, which company could you reasonably conclude was the manufacturer?

- Company A
- Company B
- Company C ✔

- Cannot reliably determine

Submit

Problem 8

2/2 points (graded)

Suppose you flip 3 fair coins and you let the random variable, X, be the number of tails.

How many different values can X take on?

- 2
- 3
- 4 ✓
- 8

What is the probability that $X = 2$?

- 0.125
- 0.25
- 0.375 ✓
- 0.5

Submit

Problem 9

1/1 point (graded)

Let's say you have an unusual 6-sided die, in which four of the sides show a 1 and the other 2 sides show a 6. If X is the value shown on the die, what is the expected value and variance of X?

- ○ $E[X] = 3.5$, $Var[X] = 2.92$
- ○ $E[X] = 3.5$, $Var[X] = 6.94$
- ◉ $E[X] = 2.67$, $Var[X] = 5.55$ ✔
- ○ $E[X] = 2.67$, $Var[X] = 2.36$

Submit

Problem 10

1/1 point (graded)

True or False:

$$Var[X] = E[(X - \mu)^2]$$

where $\mu = E[X]$

- ◉ True ✔
- ○ False

Submit

Problem 11

4/4 points (graded)

You are given the following joint probability distribution (X values along the left column, Y values along the top row):

X/Y	2	4	6
1	0.2	0.08	0.12
3	0.1	0.04	0.06
5	0.2	0.08	0.12

What is the probability that $X = 1$?

- 0.2
- 0.4 ✔
- 0.5
- 0.6

What is the probability that $Y = 6$?

- 0.12
- 0.18
- 0.3 ✔
- 0.4

What is the probability that $X = 1$ and $Y = 6$?

- (selected) 0.12 ✔
- 0.17
- 0.3
- 0.4

Are X and Y independent?

- (selected) Yes ✔
- No

Submit

Problem 12

1/1 point (graded)

Select the correct version of the correlation coefficient.

- $\rho_{X,Y} = \frac{Cov[X,Y]}{E[X]E[Y]}$
- $\rho_{X,Y} = \frac{Cov[X,Y]}{Var[X]Var[Y]}$
- (selected) $\rho_{X,Y} = \frac{Cov[X,Y]}{Std[X]Std[Y]}$ ✔
- $\rho_{X,Y} = \frac{Cov[X,Y]}{E[XY]}$

Submit

Problem 13

1/1 point (graded)

True or False: If X and Y are independent random variables, then their covariance is 0 and the correlation coefficient is also 0.

- (selected) True ✔
- False

Submit

Problem 14

1/1 point (graded)

Given the following joint probability distribution, find the correlation coefficient.

X/Y	-1	1
1	0.1	0.4
3	0.4	0.1

- $\rho = 0.00$
- $\rho = -0.29$
- (selected) $\rho = -0.60$ ✔

- [ ] $\rho = 0.34$

Submit

Problem 15

1/1 point (graded)

Which of the following statements are true for a bivariate Gaussian distribution?

- [x] The density is highest at the mean
- [ ] The bivariate Gaussian is fully parameterized by the mean and standard deviation for each variable
- [ ] The contour lines of the density are concentric circles
- [x] The contour lines represent specific probability density values

✔

Submit

Problem 16

1/1 point (graded)

What does the covariance matrix, $\Sigma = \begin{pmatrix} 9 & 0 \\ 0 & \frac{1}{4} \end{pmatrix}$ indicate about the contour lines for the density function?

- [ ] They are centered at $(9, \frac{1}{4})$
- [ ] They are centered at $(3, \frac{1}{2})$

- [x] The contour lines are aligned with the coordinate axes
- [x] They are stretched 6 times further in the x_1-direction than in the x_2-direction

✔

Submit

Problem 17

1/1 point (graded)

True or false: If covariance between two variables is positive, then they must be positively correlated.

- (•) True ✔
- ( ) False

Submit

Problem 18

1/1 point (graded)

True or false: the elements $\Sigma_{1,2}$ and $\Sigma_{2,1}$ of the covariance matrix must be equal.

- (•) True ✔
- ( ) False

Submit

Problem Set 2.

PSet2. → Problem 13.

$\Omega = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

$Pr(x_1 = i, x_2 = j) = 1/36$ for $i, j \in \{1,2,3,4,5,6\}$

$X = \min(x_1, x_2)$

$\Omega \ (|\Omega| = 36)$
$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

1 will occur as minimum 11 times

$Pr(X=1) = 11/36$

2 " " 10 times

$Pr(X=2) = 10/36$

$Pr(X=3) = 7/36$

$Pr(X=4) = 5/36$

$Pr(X=5) = 1/12$

$Pr(X=6) = 1/36.$

→ Problem 3.

a) $\Omega = \{H, T\}$

b) $\Omega = \{1, 2, 3, 4, 5, 6\}$

c) $\Omega = \{H,T\} \times \{H,T\} \times \cdots \{H,T\}. = \{H,T\}^{10} = 2^{10}$

→ Problem 4.

$\Omega = \{a, b, c, d\}$ $Pr(a) = \frac{1}{2}$ $Pr(b) = \frac{1}{8}$ $Pr(c) = \frac{1}{4}$

a) $Pr(d) = 1 - Pr(a) - Pr(b) - Pr(c) = 1 - (\frac{1}{2} + \frac{1}{8} + \frac{1}{4})$

b) $A = \{a, b, c\}$ $Pr(A) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

c) $B = \{a, c, d\}$ $A \cap B = \{a, c\}$ $Pr(A \cap B) = \frac{1}{2} + \frac{1}{4}$

→ Problem 5.

a) of the 36 possible outcomes of a roll of 2 dice, 6 corresponds to the $z_1 = 6$.
of those 6, only 1 result is a sum of 10, (6,4).
$\therefore Pr(x) = \frac{1}{6}$

b). $Pr(x) = 2/18$ of 36 possible outcomes, 18 corresponds to the $z_1 =$ an even number
of those 18, only 2 result is a sum of 10. (6,4), (4,6).

→ Problem 6.

$$P(male|sick) = \frac{P(male \cap sick)}{P(sick)} = \frac{P(sick|male) \cdot P(male)}{P(sick|male) \cdot P(male) + P(sick|female) \cdot P(female)}$$

$$= \frac{0.05 \times 0.5}{0.05 \times 0.5 + 0.01 \times 0.5}$$

→ Problem I.

b) $Pr(\text{defective} | \text{factory}1) = \dfrac{Pr(\text{factory}1 | \text{defective}) \cdot Pr(\text{factory}1)}{Pr(\text{factory}1|\text{defective}) \cdot Pr(\text{factory}1) + Pr(\text{factory}2|\text{defective}) \cdot Pr(\text{factory}2) + Pr(\text{factory}3|\text{defective}) \cdot Pr(\text{factory}3)}$

$= (0.05 \times 0.5) / 0.16$

→ Problem II.

Primary Outcomes.

→ Problem II

$X, Y \in \{-1, 0, 1\}$

X	Pr
-1	1/3
0	1/3
1	1/3

Y	Pr
-1	1/3
0	1/3
1	1/3

XY	Pr
-1	2/3
0	1/3
1	0

$E(X) = -1 \cdot 1/3 + 0 + 1 \cdot \frac{1}{3} = 0$

$Var(X) = E(X^2) - E(X)^2 = 1 \cdot 1/3 + 0 + 1 \cdot 1/3 - 0 = 2/3$

$Std(X) = \sqrt{2/3}$

$E(Y) = 0$

$Var(Y) = 2/3$

$Std(Y) = \sqrt{2/3}$

$E(XY) = -1 \cdot 2/3 + 0 + 0 = -2/3$

$Cor(X,Y) = \dfrac{-2/3 - 0 \cdot 0}{\sqrt{2/3} \cdot \sqrt{2/3}} = \dfrac{-\frac{2}{3}}{(\sqrt{2/3})^2} = -1$

→ Problem 18.

$X, Y \in \{-1, 0, 1\}$

X	Pr
-1	1/3
0	1/3
1	1/3

Y	Pr
-1	1/3
0	1/3
1	1/3

XY	Pr
-1	1/3
0	1/3
1	1/3

$Pr(X=-1, Y=-1) = 1/6$ (from table) $\neq 1/3 \cdot 1/3$ Dependent variables

$E(X) = -1 \cdot \frac{1}{3} + 0 + \frac{1}{3} = 0$ $E(XY) = E(X) \cdot E(Y)$ Uncorrelated

$E(Y) = 0$

$E(XY) = -\frac{1}{3} + \frac{1}{3} = 0$

Independent ~ uncorrelated (single-way)

Uncorrelated ⇏ Independent

Quiz 2.

→ Problem 5.

3 bags : 1 with red red red
2 with Green black red

Pick up a bag, pull out a ball (red) pull out another
Probability you pick up the bag with full reds ball.

Event A : Pick up the bag (right bag)
Event B : all red balls.

$P(A|B) = P(\text{right bag} \mid \text{all red balls}) = \frac{P(A) \cdot P(B|A)}{P(B)}$

$P(A) = P(\text{pick up the right bag}) = \frac{1}{3}$
$P(B|A) = P(\text{all red balls} \mid \text{pick up the right bag}) = 1$
$P(B) = P(\text{bag} \mid \text{all red balls}) + P(\text{not the right bag} \mid \text{all red balls})$
$= \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot (\frac{1}{3})^2$
$= \frac{11}{27}$

$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)} = \frac{\frac{1}{3} \cdot 1}{\frac{11}{27}} = 0.82$

→ Problem 9.

1 Die, 4 sides with number 1, 2 sides with number 6
$X \in \{1, 6\}$

$E(X) = 1 \cdot \frac{4}{6} + 6 \cdot \frac{2}{6} = 2.67$
$Var(X) = E(X^2) - \mu^2$
$= 1^2 \cdot \frac{4}{6} + 6^2 \cdot \frac{2}{6} - (2.67)^2$