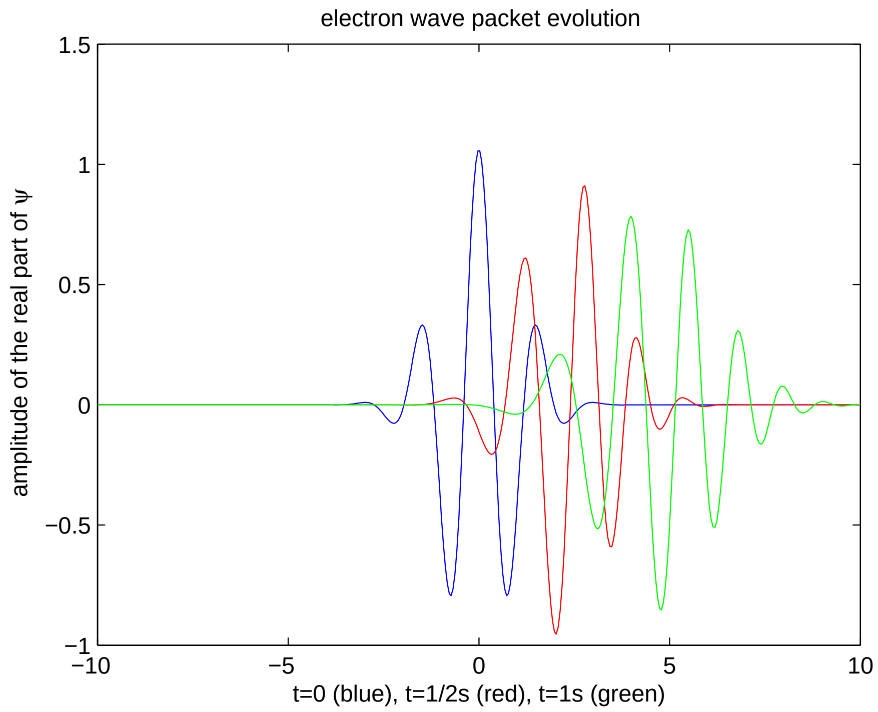
1 ELEC1204 Lab P3 - Wavepackets

This lab is going to get you started with designing and constructing your own classes. By the end of the lab you will be able to numerically evaluate a wavepacket and verify that it it is plausible.



2 Preparation

2.1 Complex Number Class

Design a complex number class yourself (not copied from any other source).
Your class must include :
complex conjugation
modulus and argument functions.
square root of a complex type.
exponentiation of a complex type (e^z)
appropriate Constructors

Outside of the class definition (by overloading + - * /) support
addition, subtraction, multiplication and division of complex types.
multiplication of a double by a complex type.
equality and inequality tests of complex types $==$, $!=$.

2.2 Wave packets

$\psi(x,t)$ is a Gaussian wavefunction .

$$\psi(x,t) = (\frac{\alpha}{2\pi})^{\frac{1}{4}} \exp{(-\alpha\kappa_0^2)} \frac{1}{\sqrt{(\alpha + j\beta t)}} \exp\left(\frac{(2\alpha\kappa_0 + jx)^2}{4(\alpha + j\beta t)}\right) \tag{1}$$

When $\beta \approx 0.5785 cm^2/sec$ it represents an electron with an energy determined by κ_0 which is set to 4 in the MATLAB code below. Its on the web site if you want to play but is not part of the required preparation.

```
alpha = 0.5;
k0 = 4.0;
beta = 0.5785;  %h//2m for electron cm^2/sec
x = linspace(-10,10,200);
t = linspace(0,3,200);
gamma = alpha + i*beta*t;
norm = exp(-k0*k0*alpha)*((alpha)/(2*pi))^0.25;
```

```
[X,T] = meshgrid(x,t);
gamma = alpha + i*beta*T;
phi = (norm./sqrt(gamma)).*exp(((2*alpha*k0+i*X).*(2*alpha*k0+i*X))./(4*gamma));
figure(1);
mesh(X,T,real(phi*phi'));
title('amplitude of the real part of \phi')
ylabel('time');
zlabel('amplitude of the real part of \phi');
xlabel('x');
figure(2)
hold off
t0 = 0.0;
phi0 = PSI(alpha,beta,k0,x,t0);
plot(x,real(phi0));
pause
hold on
t1= 0.5;
phi1 = PSI(alpha,beta,k0,x,t1);
plot(x,real(phi1),'r-');
pause;
t2= 1.0;
phi2 = PSI(alpha,beta,k0,x,t2);
plot(x,real(phi2),'g-');
ylabel('amplitude of the real part of \psi');
xlabel('t=0 (blue), t=1/2s (red), t=1s (green)');
title('electron wave packet evolution');
```

3 Lab work

3.1 Write and test your complex numbers

Implement and test your complex number class.

3.2 Evaulate a wavepacket

Use your complex number class to evaluate an electron wavepacket at $t = 0$, $t = 0.5$ and $t = 1$. These are the values used for the front cover picture.

3.3 Test that it is normalised

Do this at $t = 0$ seconds only, so the only variable is x. When we discretise the x-axis in δx size intervals and write $\psi(x, 0) = \psi(j\delta x, 0) = \psi_j(t)$ where $-N \leq j \leq N$ Verify that the Gaussian wavepacket, equation (2) of the notes is normalised at $t = 0 \quad secs$
I used $N = 4096$, $L = 25$ (length of positive x axis), $\beta = 0.5785$ (an electron), $\alpha = 0.5$, $\kappa_0 = 1$, $x_0 = -L$, $x_N = L$, $\delta x = \frac{-2L}{N}$
You need to show that ;

$$\int_{-L}^{L} \psi^*(x)\psi(x)dx = 1$$

Use the extended Trapezoidal rule

$$\int_{x_0}^{x_m} f(x)dx = \delta x[\frac{f_0}{2} + f_1 + f_2 + \dots f_{m-1} + \frac{f_m}{2}]$$

with $f = \psi^*\psi$

4 Extra work

Output the real part of your wavepacket in a form suitable for display in matlab. Display it in MATLAB for the 3 time values that you have.

Jeff Reeve 10/2/2013