

# Lab #3 Answers

2022-02-16

## Contents

<b>Chapter 4 Exercises</b>	<b>1</b>
Exercise 4.1: Methane . . . . .	1
Exercise 4.2: CO <sub>2</sub> (Graduate students only) . . . . .	14
Exercise 4.3: Water vapor . . . . .	24
<b>Chapter 5 Exercise</b>	<b>29</b>
Exercise 5.2: Skin Height . . . . .	29

## Chapter 4 Exercises

Fill in R code for the exercises (I have put the comment # TODO in all of the code chunks where you need to do this) and then fill in the answers where I have marked **Answer:**. Be sure to write explanations of your answer and don't just put numbers with no text.

### Exercise 4.1: Methane

Methane has a current concentration of 1.7 ppm in the atmosphere and is doubling at a faster rate than CO<sub>2</sub>.

- a) **Would an additional 10 ppm of methane in the atmosphere have a larger or smaller impact on the outgoing IR flux than an additional 10 ppm of CO<sub>2</sub> at current concentrations?**

**Hint:** See the suggestion in the lab-03-instructions document.

```
base_co2 = 400 # parts per million
base_ch4 = 1.7 # parts per million
modtran_baseline = run_modtran(file.path(data_dir,
                                           "ex_4_1_baseline.txt"),
                               co2_ppm = base_co2,
                               ch4_ppm = base_ch4)
modtran_plus_10_co2 = run_modtran(file.path(data_dir,
                                              "ex_4_1_plus_10_co2.txt"),
                                  co2_ppm = base_co2 + 10,
                                  ch4_ppm = base_ch4)
modtran_plus_10_ch4 = run_modtran(file.path(data_dir,
                                              "ex_4_1_plus_10_ch4.txt"),
                                  co2_ppm = base_co2,
                                  ch4_ppm = base_ch4 + 10)
i_out_baseline = modtran_baseline$i_out
```

```
i_out_co2 = modtran_plus_10_co2$i_out
i_out_ch4 = modtran_plus_10_ch4$i_out
delta_i_co2 = i_out_co2 - i_out_baseline
delta_i_ch4 = i_out_ch4 - i_out_baseline
```

**Answer:** I ran MODTRAN three times. One run was a baseline, which used the current concentrations of CO<sub>2</sub> and CH<sub>4</sub>. Then I ran MODTRAN with the CO<sub>2</sub> concentration increased by 10 ppm and I ran it a third time with the baseline value for CO<sub>2</sub>, but with CH<sub>4</sub> increased by 10 ppm.

For the baseline run, the intensity of outgoing longwave light was 298.7. Watts per square meter. Increasing CO<sub>2</sub> by 10 ppm decreased the outgoing longwave light by 0.13 W/m<sup>2</sup>, and increasing CH<sub>4</sub> by 10 ppm decreased the outgoing longwave light by 3.11 W/m<sup>2</sup>, which is around 25. times as much as for CO<sub>2</sub>.

The difference is because absorption for CO<sub>2</sub> is strongly saturated, but the absorption for CH<sub>4</sub> is not saturated. Another way to think about this is that a 10 ppm increase in CO<sub>2</sub> increases the atmospheric concentration by 2.5% and a 10 ppm increase in CH<sub>4</sub> increases the atmospheric concentration by 590.%.

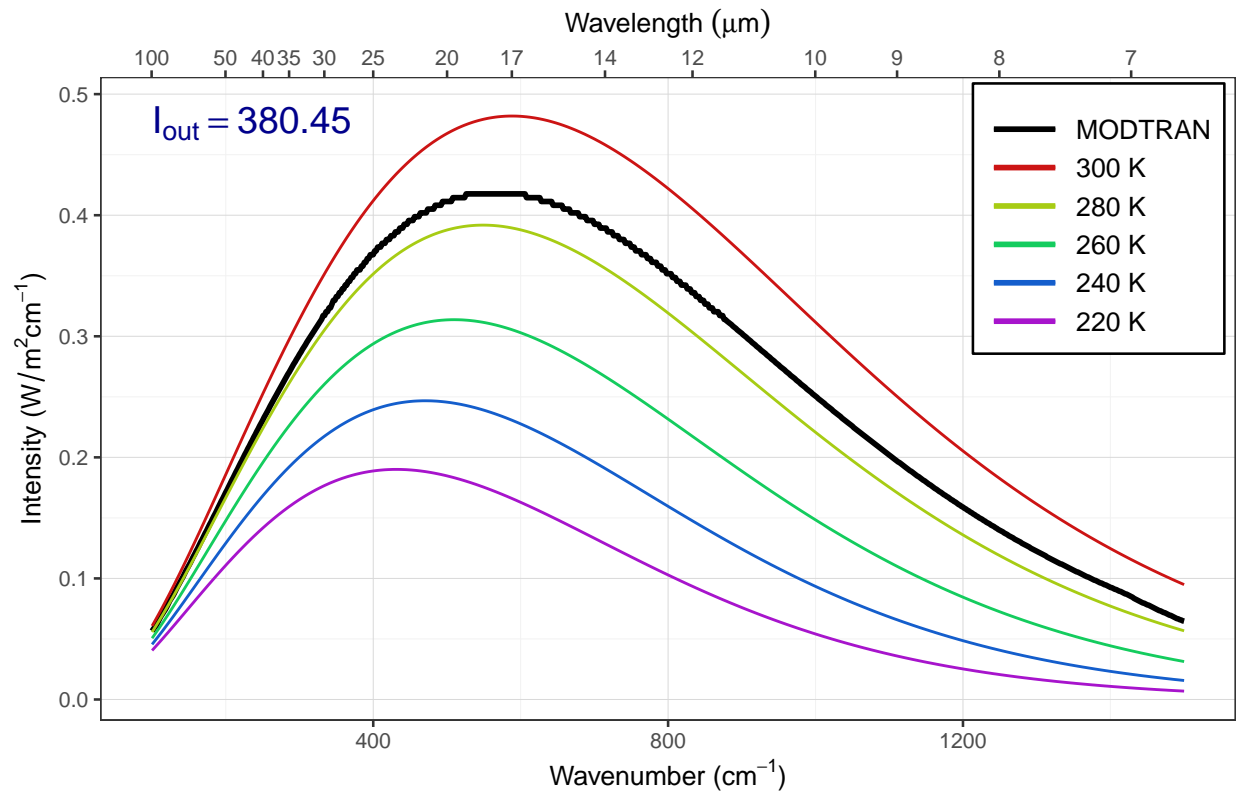
b) **Where in the spectrum does methane absorb? What concentration does it take to begin to saturate the absorption in this band? Explain what you are looking at to judge when the gas is saturated.**

**Hints:**

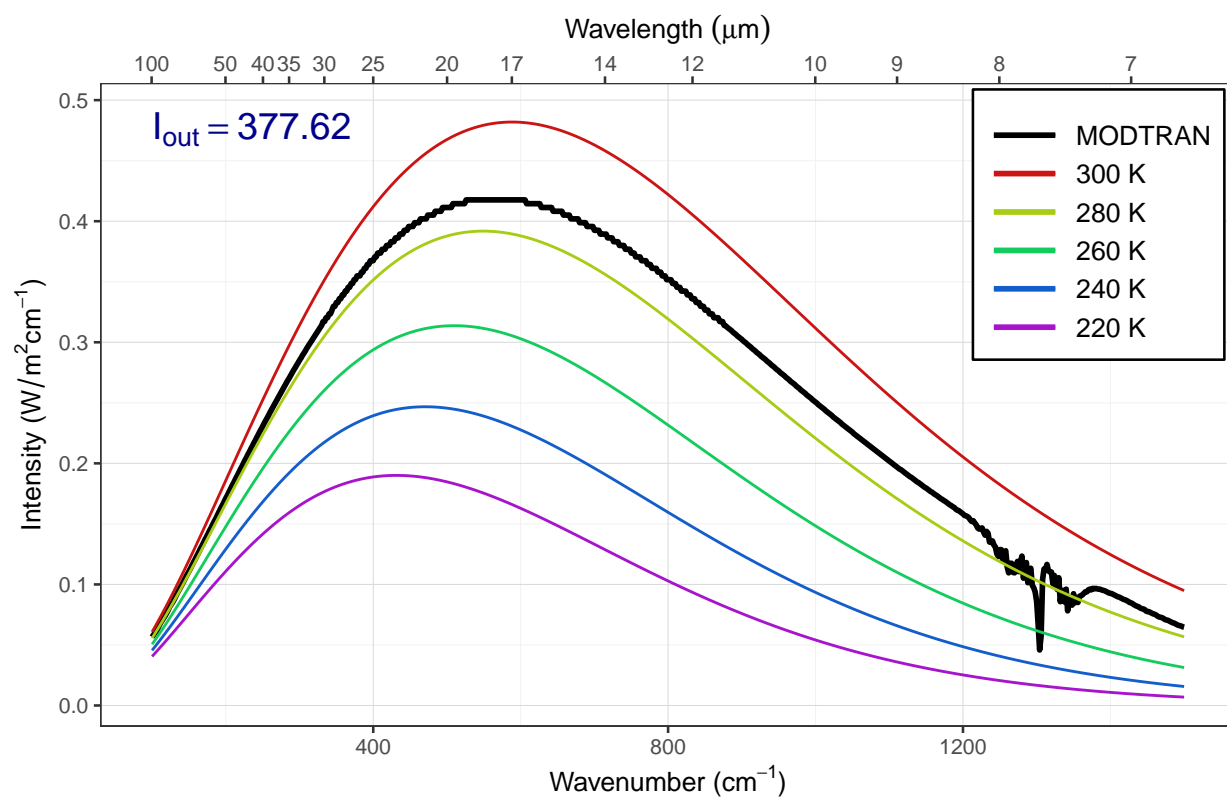
See the hints in the lab-03-instructions document.

```
ch4_values = c(0, 2^seq(0,7))
sat_data = tibble()
for (ch4 in ch4_values) {
  filename = file.path(data_dir, str_c("ex_4_1_ch4_", ch4, ".txt"))
  if (file.exists(filename)) {
    mod_data = read_modtran(filename)
  } else {
    mod_data = run_modtran(filename, co2_ppm = 0, ch4_ppm = ch4,
                           trop_o3_ppb = 0, strat_o3_scale = 0,
                           h2o_scale = 0, freon_scale = 0,
                           atmosphere = "standard", altitude_km = 70)
  }
  p = plot_modtran(mod_data, descr = str_c(ch4, " ppm methane"))
  plot(p)
}
```

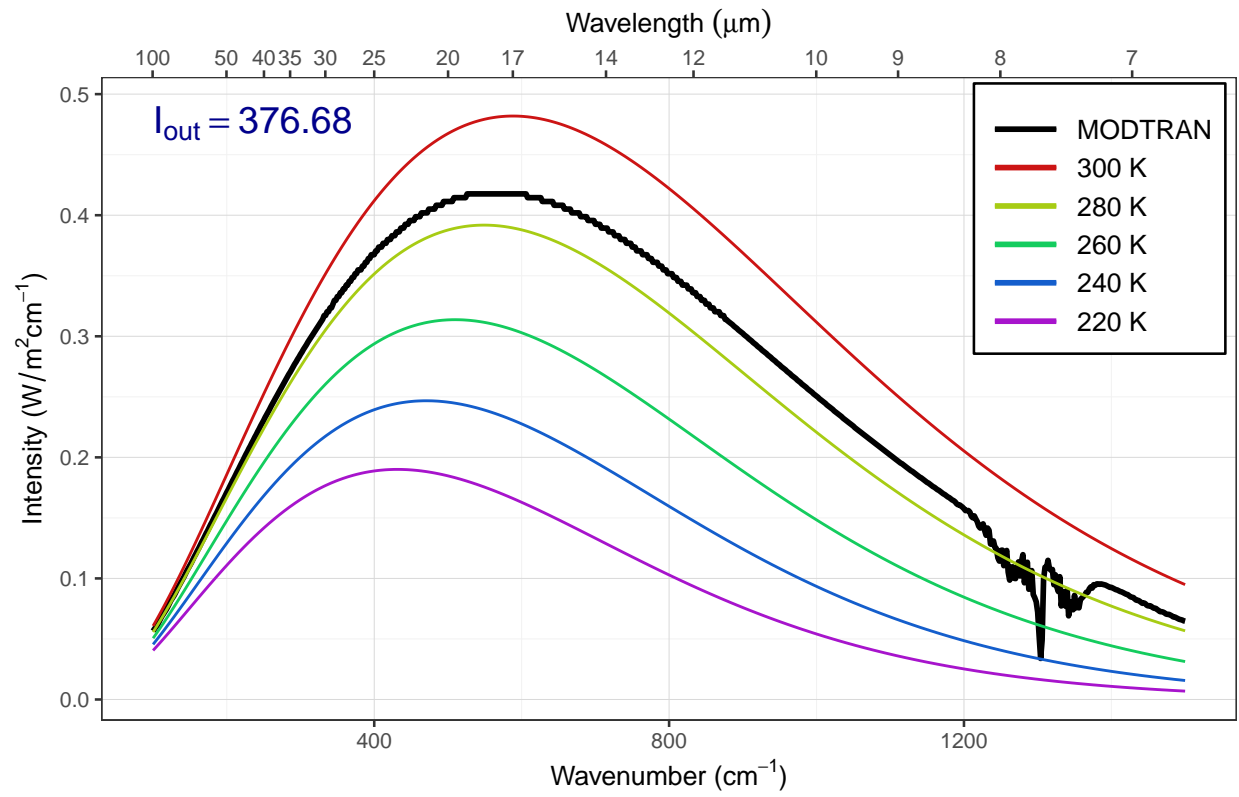
0 ppm methane



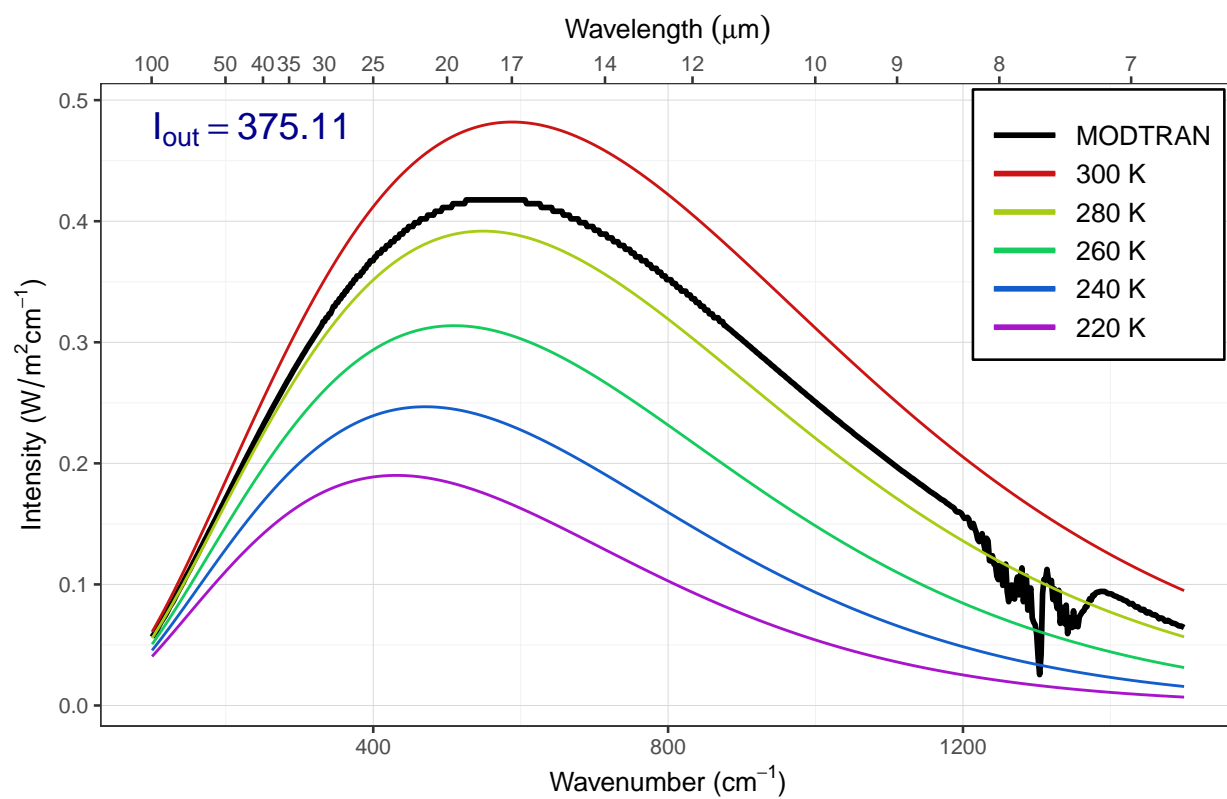
1 ppm methane



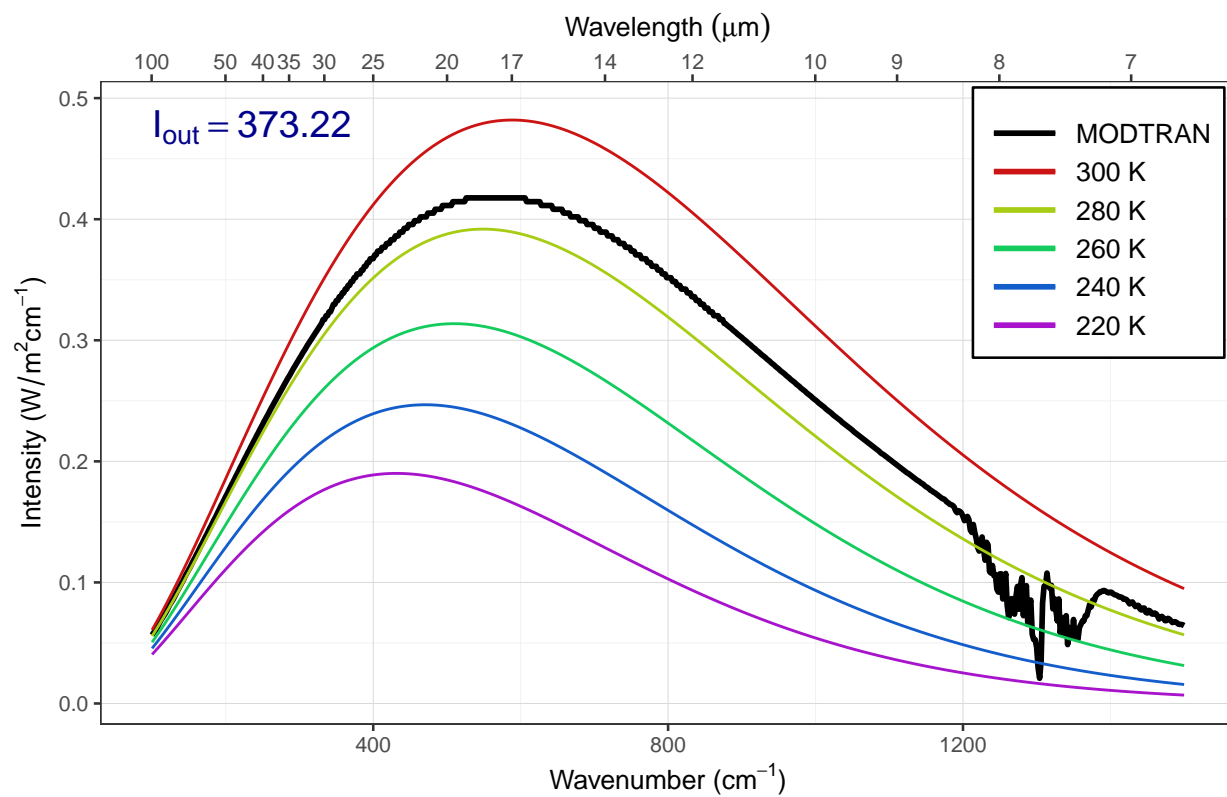
2 ppm methane



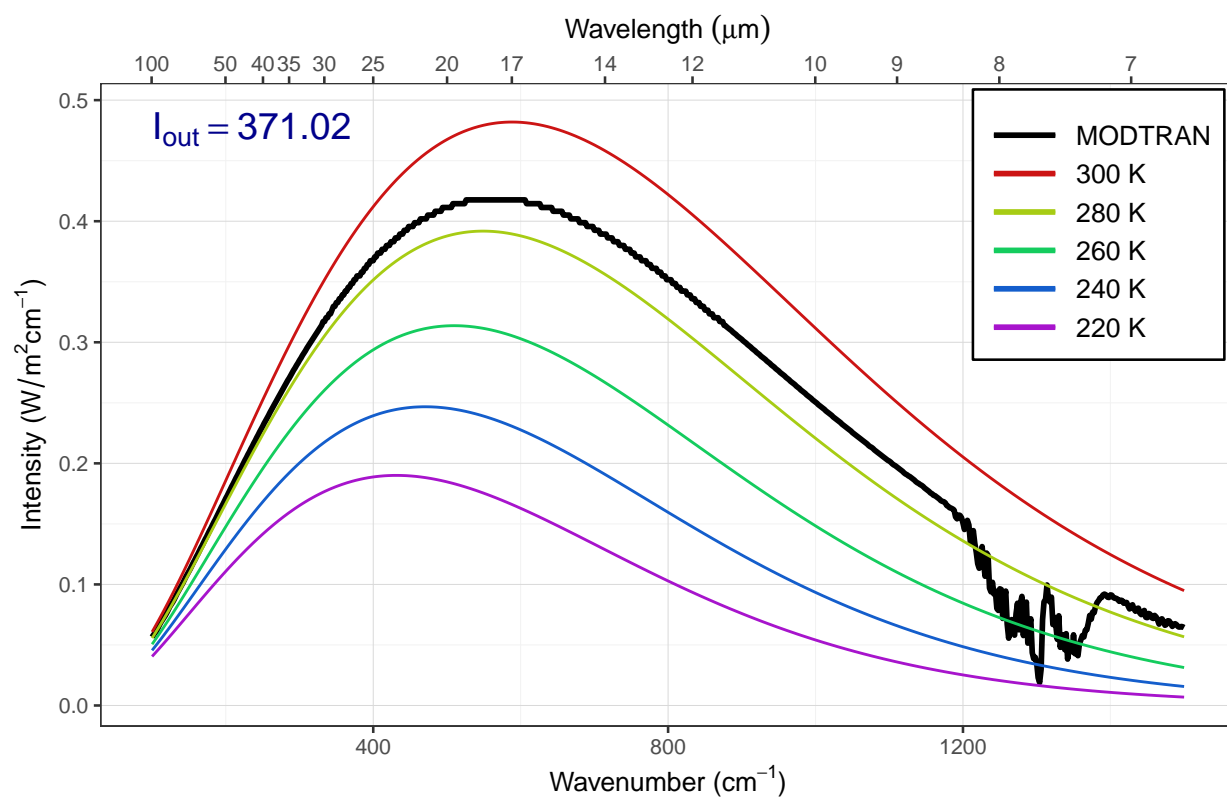
4 ppm methane



8 ppm methane

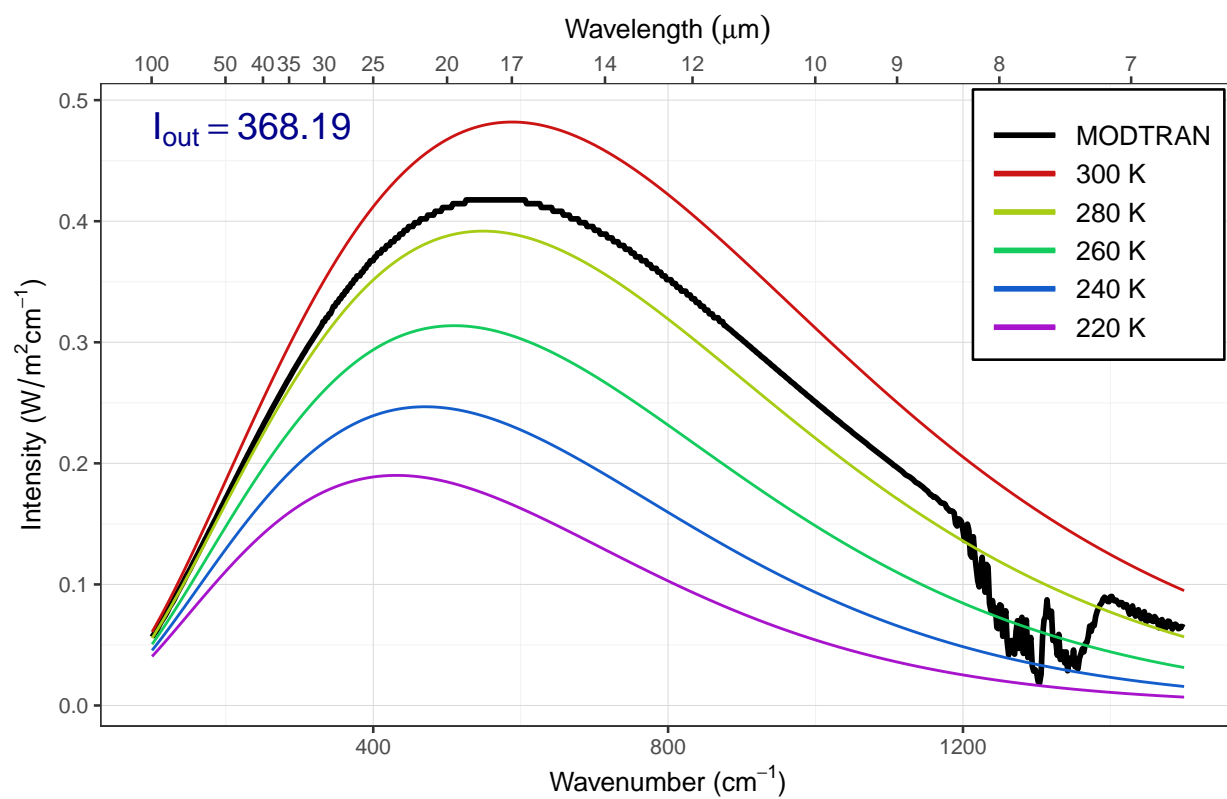


16 ppm methane

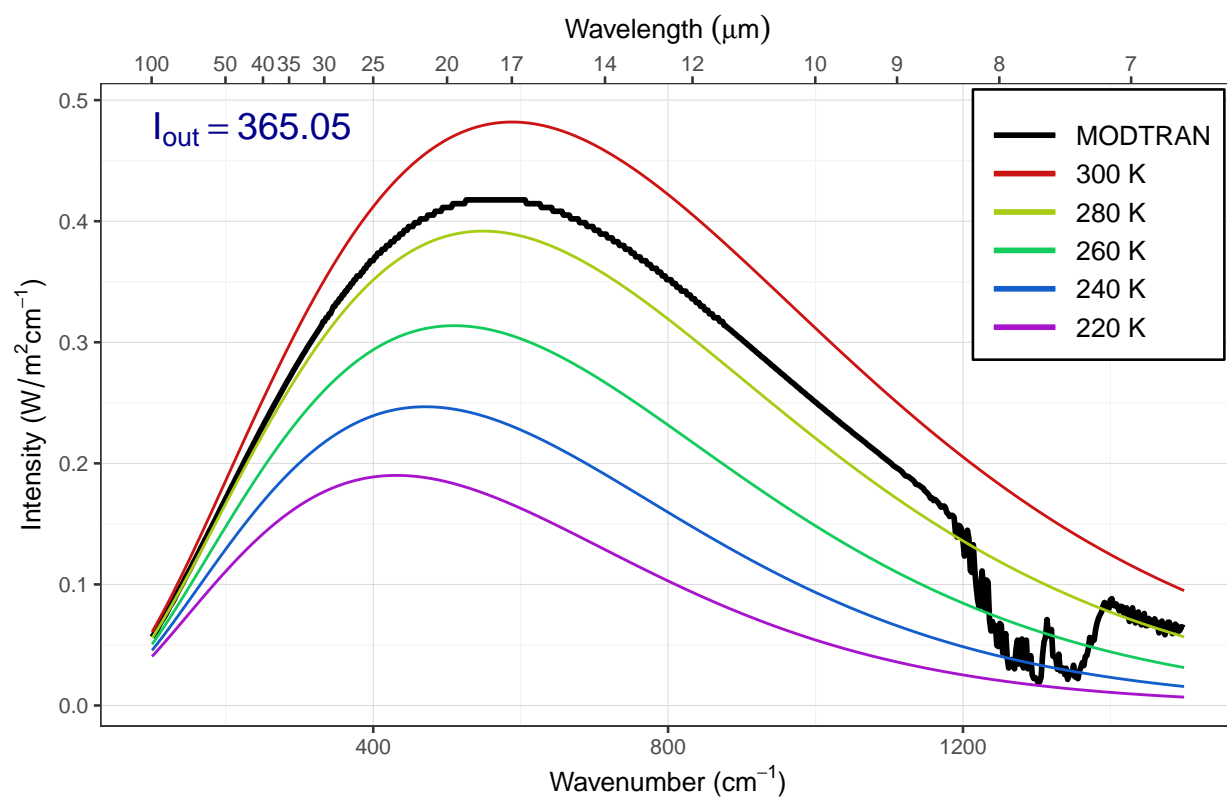




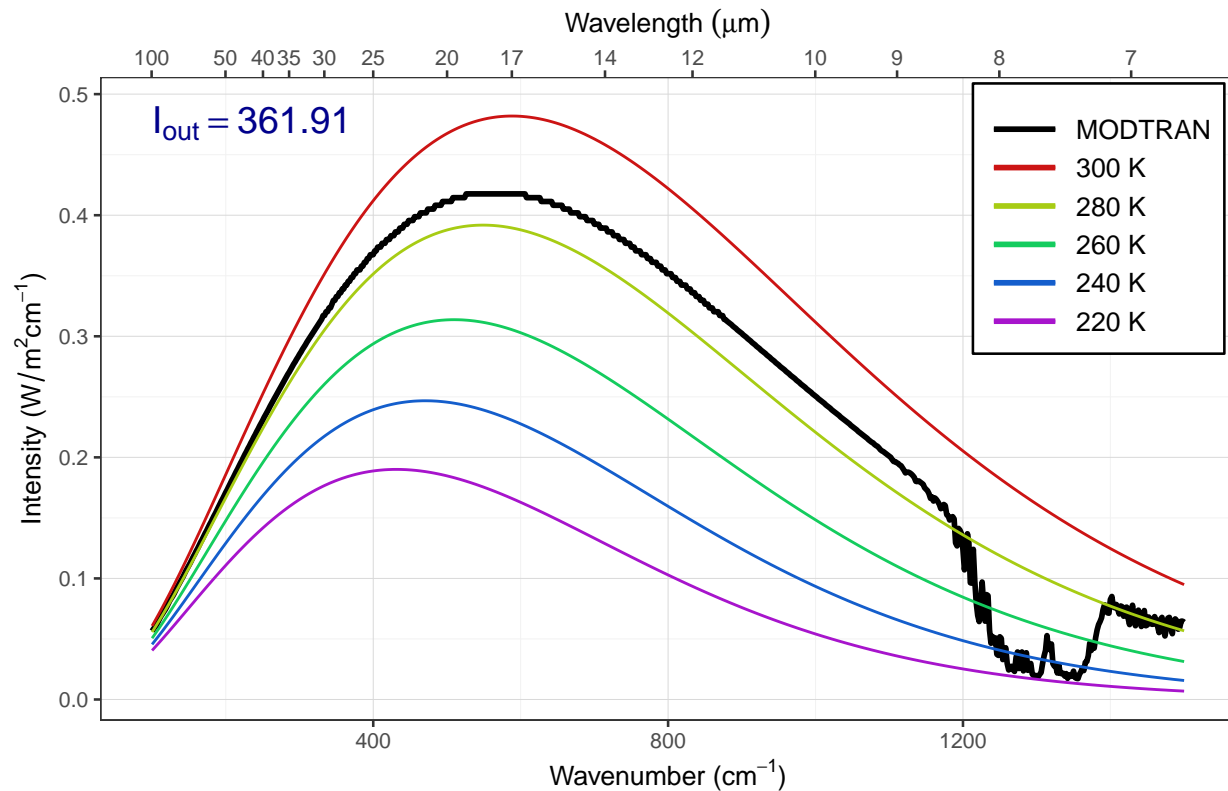
32 ppm methane



64 ppm methane



## 128 ppm methane



**Answer:** The plots above show the MODTRAN spectrum with all gases set to zero except methane. Methane absorbs most strongly around 1300 wavenumbers.

As we increase the methane concentration, the big spike around 1300 wavenumbers gets bigger until it bottoms out on the black line. This happens somewhere around 8, 16, or 32 ppm, so any of those answers would be correct. But the spectrum is complicated and so is its saturation, so other answers are plausible if they are supported by sound reasoning.

c) Would a doubling of methane have as great an impact on the heat balance as a doubling of CO<sub>2</sub>?

**Hint:** See the suggestion in the lab-03-instructions document.

```
base_co2_ppm = 400
base_methane_ppm = 1.7
baseline = read_modtran(file.path(data_dir, "ex_4_1_baseline.txt"))
double_co2 = run_modtran(file.path(data_dir, "ex_4_1_2x_co2.txt"),
                          co2_ppm = 2 * base_co2_ppm,
                          ch4_ppm = base_methane_ppm)
double_ch4 = run_modtran(file.path(data_dir, "ex_4_1_2x_ch4.txt"),
                          co2_ppm = base_co2_ppm,
                          ch4_ppm = 2 * base_methane_ppm)

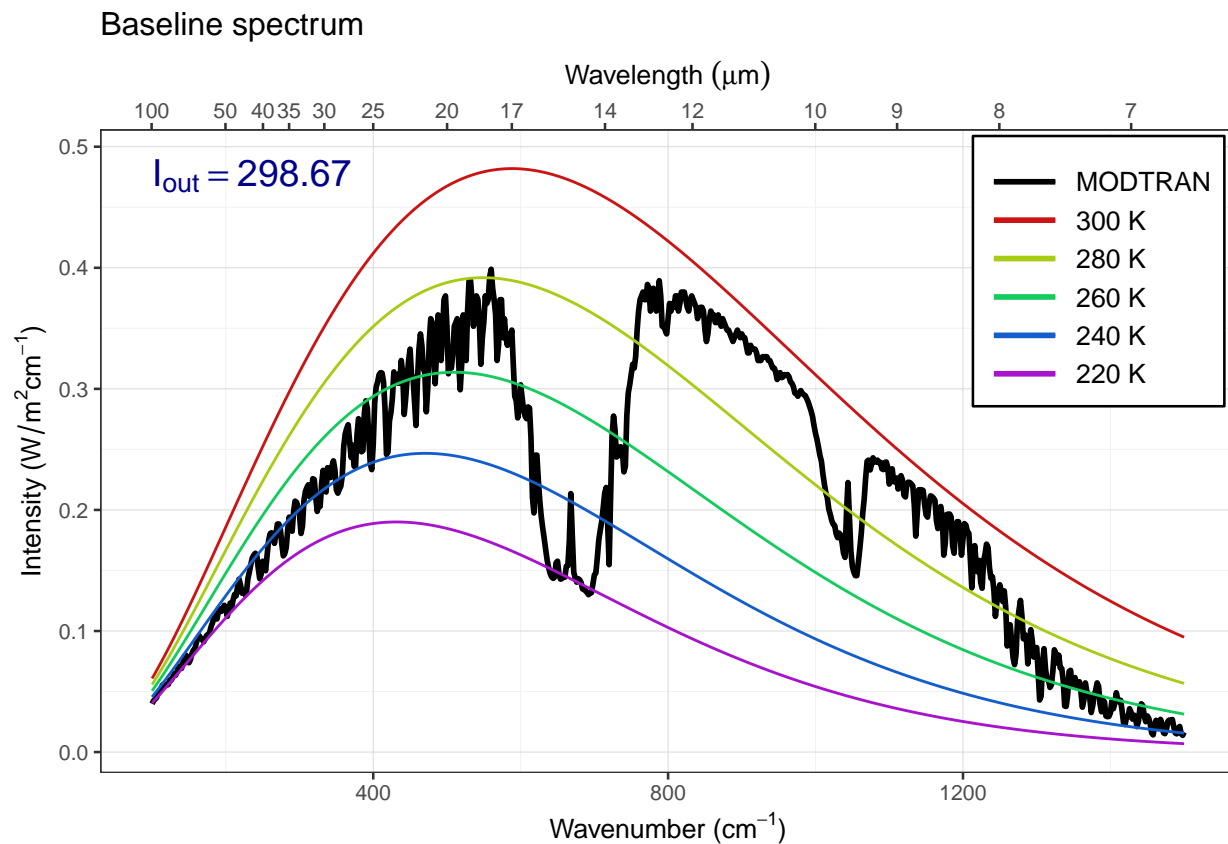
i_out_baseline = baseline$i_out
i_out_co2 = double_co2$i_out
i_out_ch4 = double_ch4$i_out

delta_i_out_co2 = i_out_co2 - i_out_baseline
delta_i_out_ch4 = i_out_ch4 - i_out_baseline
```

**Answer:** The baseline value for  $I_{\text{out}}$  is  $298.7 \text{ W/m}^2$ . If we double  $\text{CO}_2$ , it drops to  $295.3 \text{ W/m}^2$ , a decrease of  $-3.3 \text{ W/m}^2$ , and if we double  $\text{CH}_4$ , it drops to  $297.8 \text{ W/m}^2$ , a decrease of  $-0.88 \text{ W/m}^2$ . Doubling  $\text{CO}_2$  has the larger effect because there is a lot more  $\text{CO}_2$  in the atmosphere and that is more important than how saturated its absorption is.

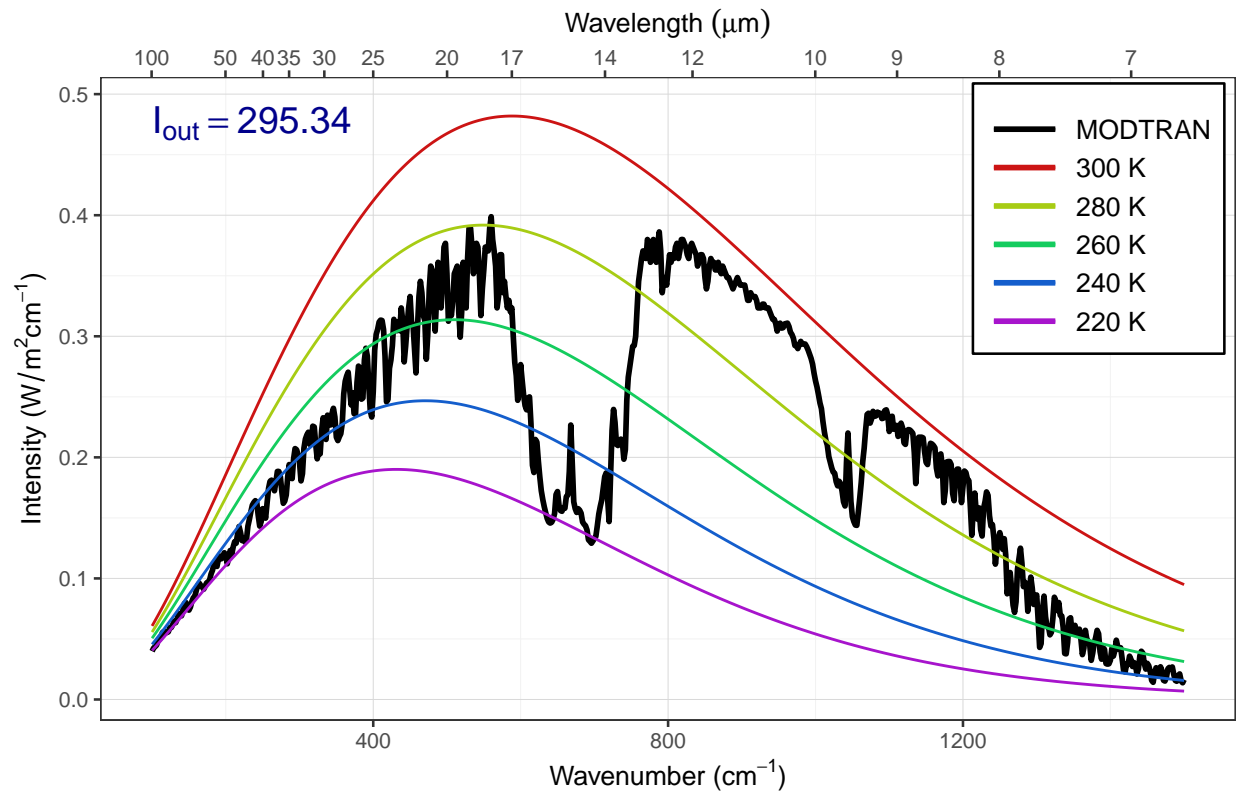
You can see this if you look at the plots below. Notice that the effect of doubling  $\text{CO}_2$  isn't to make the big  $\text{CO}_2$  absorption feature get deeper, but to make it wider. You can see this if you compare the baseline to the doubled  $\text{CO}_2$  spectrum where the purple spectrum crosses the yellow blackbody curve near 600 wavenumbers and around 750 wavenumbers. Compare this to the very small change in the methane spike near 1300 wavenumbers (you have to look very carefully at the doubled methane spectrum to notice this).

```
plot_modtran(baseline, descr = "Baseline spectrum")
```



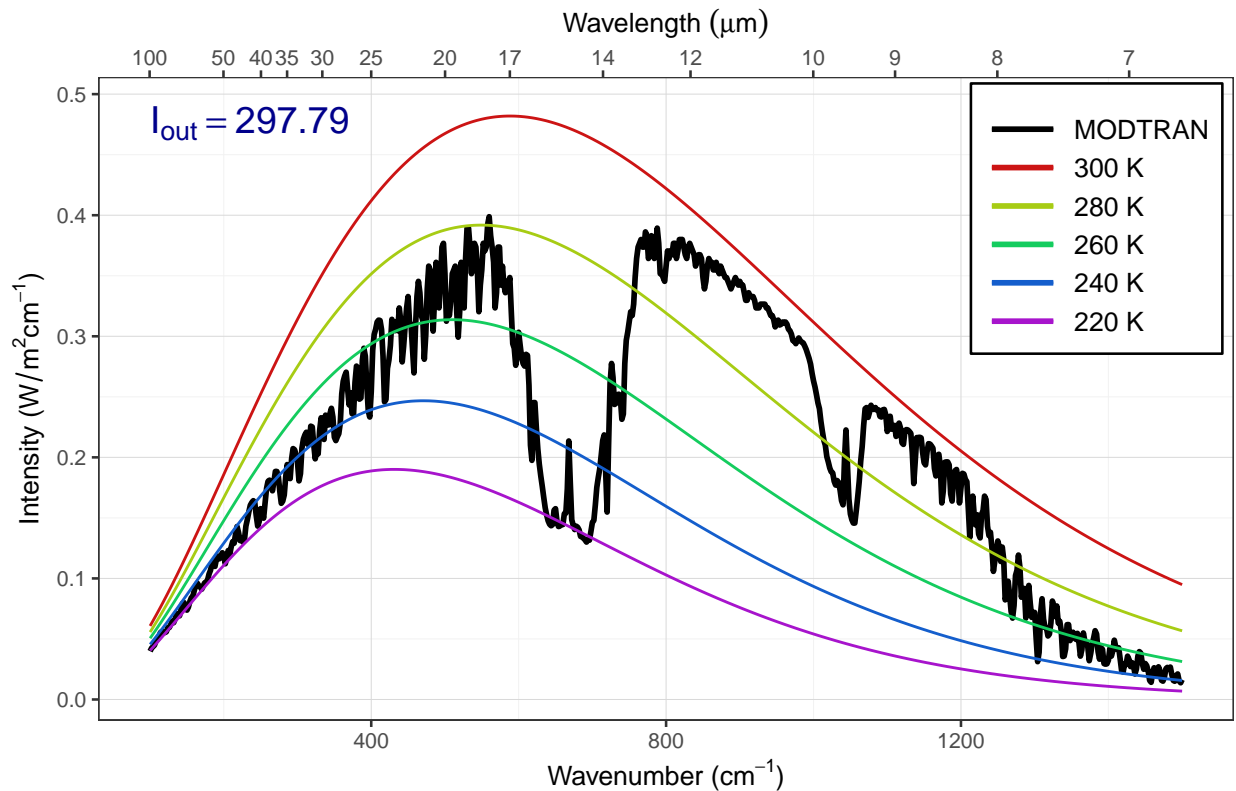
```
plot_modtran(double_co2, descr = "Doubled CO2")
```

## Doubled CO2



```
plot_modtran(double_ch4, descr = "Doubled CH4")
```

## Doubled CH4



- d) What is the “equivalent CO<sub>2</sub>” of doubling atmospheric methane? That is to say, how many ppm of CO<sub>2</sub> would lead to the same change in outgoing IR radiation energy flux as doubling methane? What is the ratio of ppm CO<sub>2</sub> change to ppm methane change?

```
matching_methane = 13
modtran_match_ch4 = run_modtran(file.path(data_dir, "ex_4_1_ch4_match.txt"),
                                co2_ppm = 400, ch4_ppm = matching_methane)
```

**Answer:** When we double CO<sub>2</sub>,  $I_{out}$  is `rformat_md(i_out_co2, digits = 4) * W/m²`. We need to adjust CH<sub>4</sub> to produce the same  $I_{out}$  with the default value of 400 ppm CO<sub>2</sub>. After some trial and error, this turns out to be about 13 ppm, which has  $I_{out} = 295.34$

## Exercise 4.2: CO<sub>2</sub> (Graduate students only)

- a) Is the direct effect of increasing CO<sub>2</sub> on the energy output at the top of the atmosphere larger in high latitudes or in the tropics?

**Hint:** See the hint in the lab-03-instructions document.

```
modtran_lat_df = tibble()
for (atmos in c("tropical", "midlatitude summer", "subarctic summer"
               #, "midlatitude winter", "subarctic winter"
)) {
  for (co2 in c(400, 800)) {
    filename = file.path(data_dir,
```

```

      str_c("ex_4_2_", atmos, "_co2_", co2, ".txt"))
if (file.exists(filename)) {
  mod_data = read_modtran(filename)
} else {
  mod_data = run_modtran(filename, atmosphere = atmos, co2_ppm = co2)
}
modtran_lat_df = bind_rows(modtran_lat_df,
                           tibble(co2 = co2, atmos = atmos,
                                   i_out = mod_data$i_out))
}
}

modtran_lat_df = modtran_lat_df %>%
  mutate(co2 = str_c("co2_", co2)) %>%
  pivot_wider(names_from = "co2", values_from = "i_out") %>%
  mutate(delta_i = co2_400 - co2_800) %>%
  arrange(desc(delta_i))
kable(modtran_lat_df, digits = 1)

```

atmos	co2_400	co2_800	delta_i
tropical	298.7	295.3	3.3
midlatitude summer	289.7	286.7	3.0
subarctic summer	270.9	268.4	2.5

**Answer:** When atmospheric CO<sub>2</sub> doubles, the greatest change in I<sub>out</sub> occurs in the tropics, followed by the midlatitudes, and the smallest change occurs at high latitudes in the subarctic.

- b) Set pCO<sub>2</sub> to an absurdly high value of 10,000 ppm. You will see a spike in the CO<sub>2</sub> absorption band. What temperature is this light coming from? Where in the atmosphere do you think this comes from?

Now turn on clouds and run the model again. Explain what you see. Why are night-time temperatures warmer when there are clouds?

**Hint:** See the hint in the lab-03-instructions document and for the second part of this exercise, try using “altostratus” clouds.

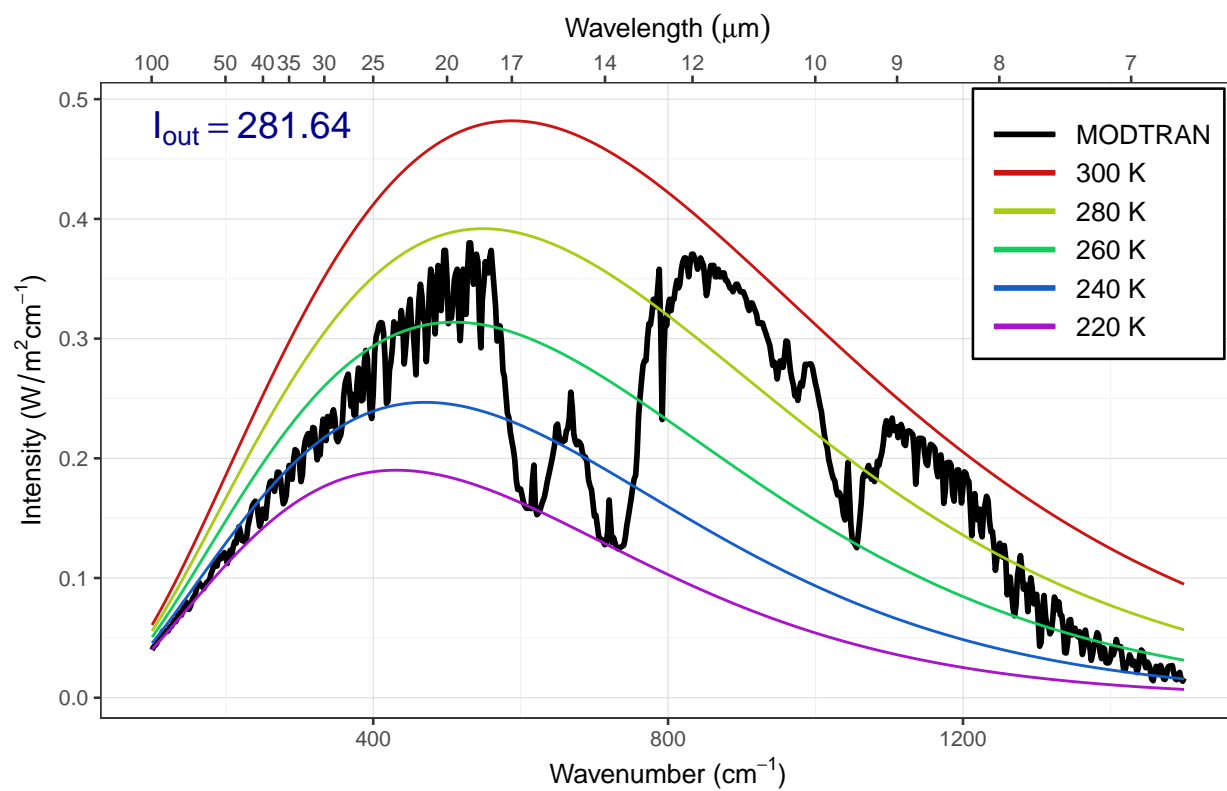
The figure below shows the spectrum with a high concentration of CO<sub>2</sub>.

```

high_co2 = 1E4
for (z in c(70, 60, 50, 40, 30, 20, 10)) {
  filename = file.path(data_dir, str_c("ex_4_2_alt_", z, ".txt"))
  if (file.exists(filename)) {
    modtran_high_co2 = read_modtran(filename)
  } else {
    modtran_high_co2 = run_modtran(filename, co2_ppm = high_co2,
                                   altitude_km = z)
  }
  p = plot_modtran(modtran_high_co2)
  plot(p)
}

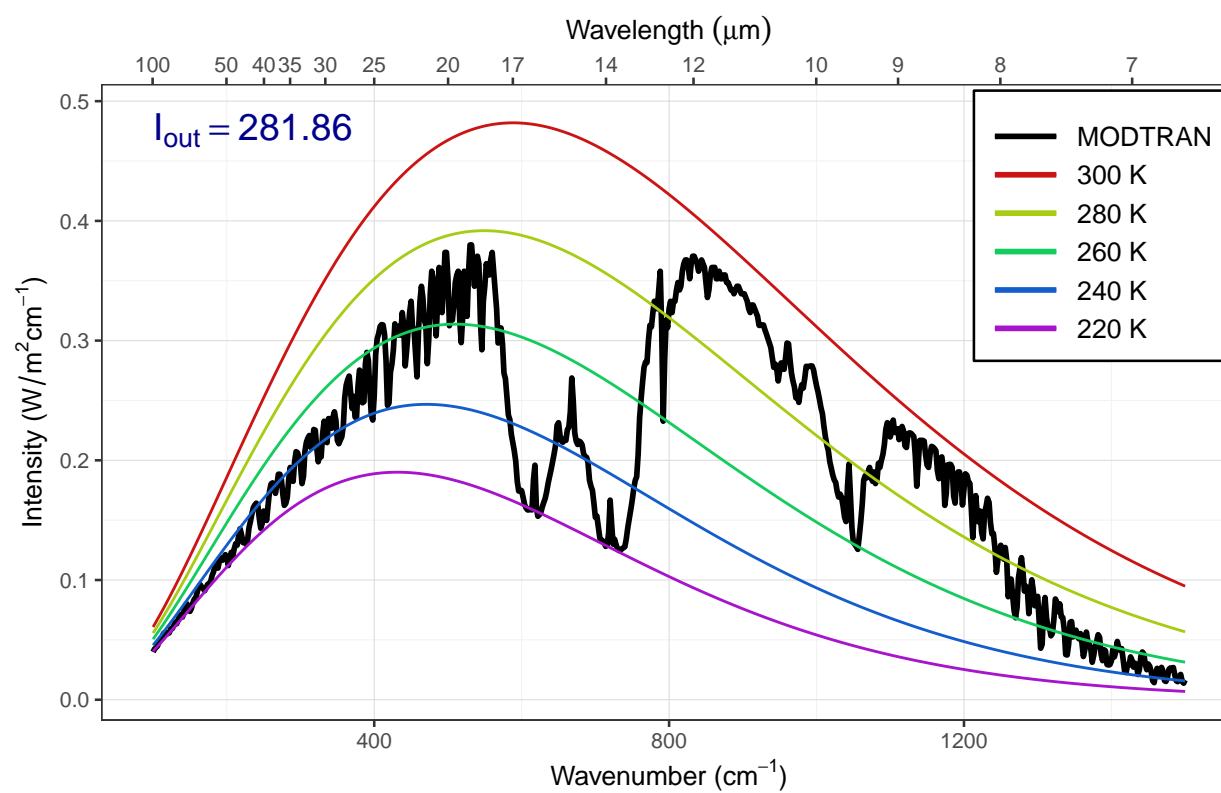
```

MODTRAN: 10000 ppm CO<sub>2</sub>, 70 km altitude

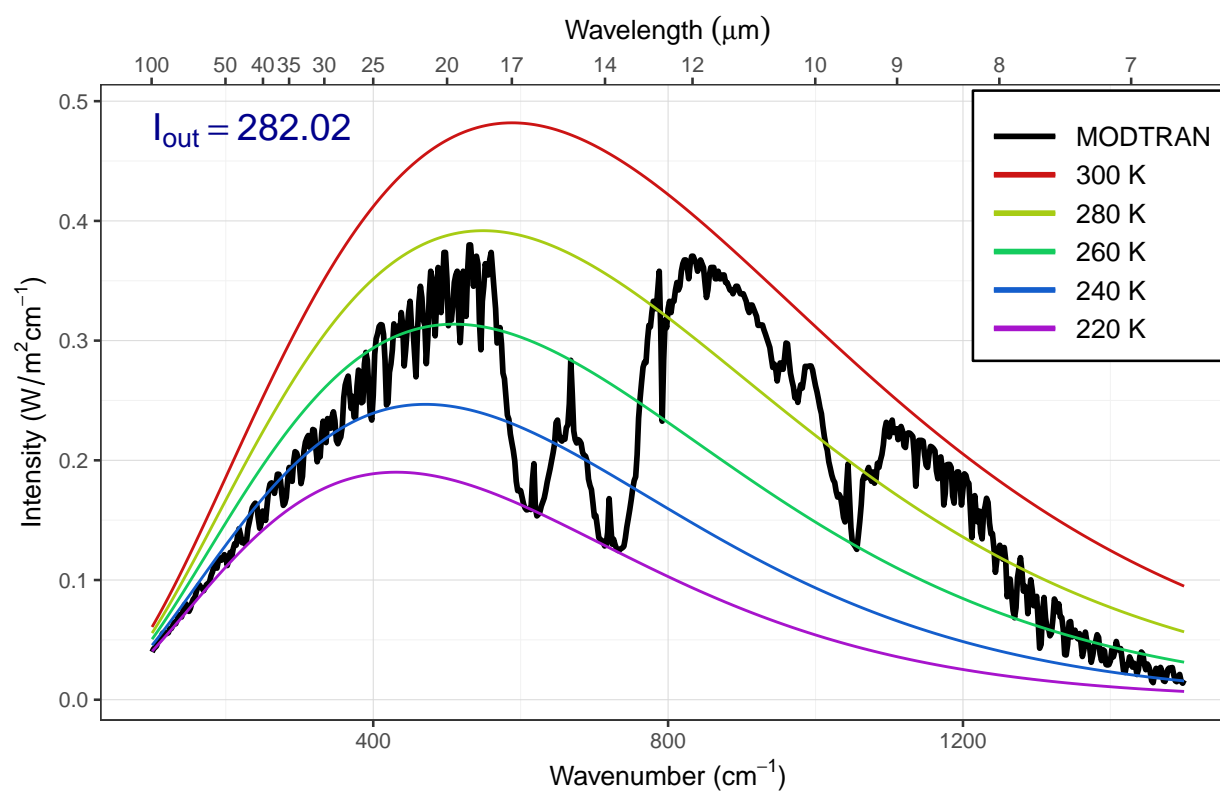




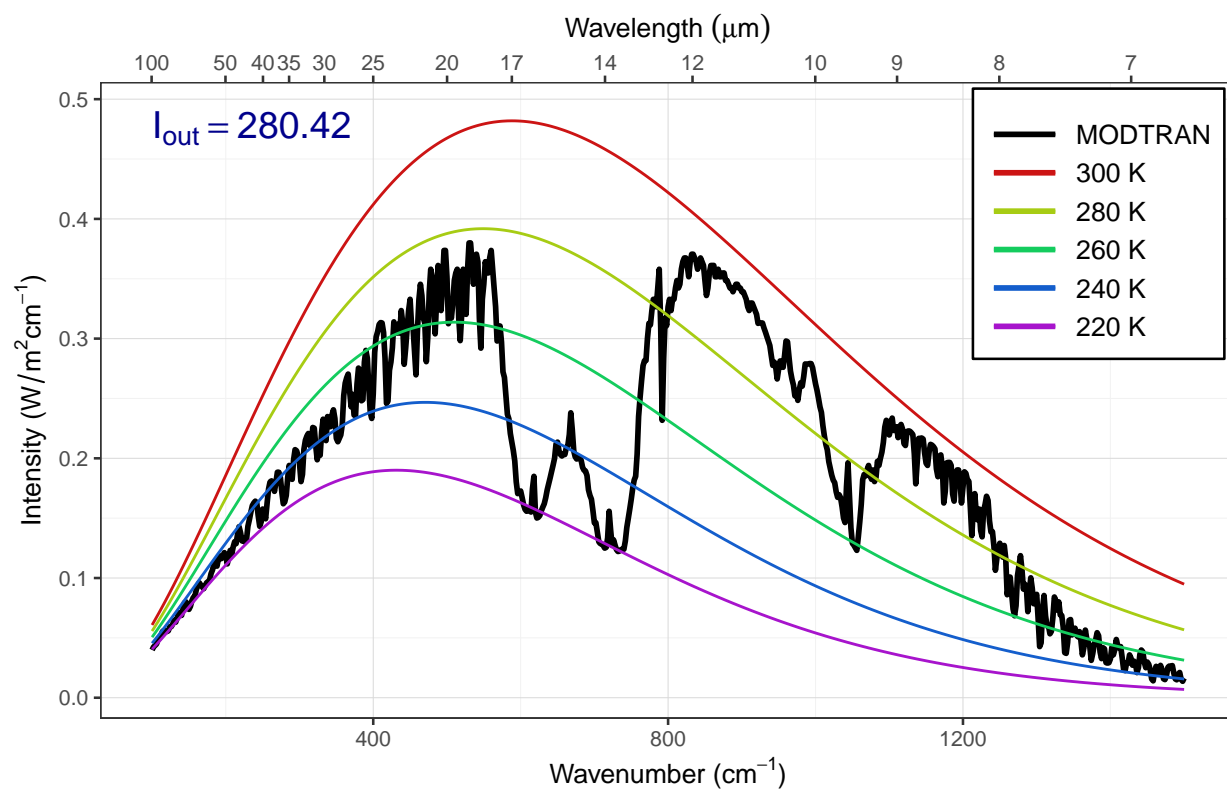
MODTRAN: 10000 ppm CO<sub>2</sub>, 60 km altitude



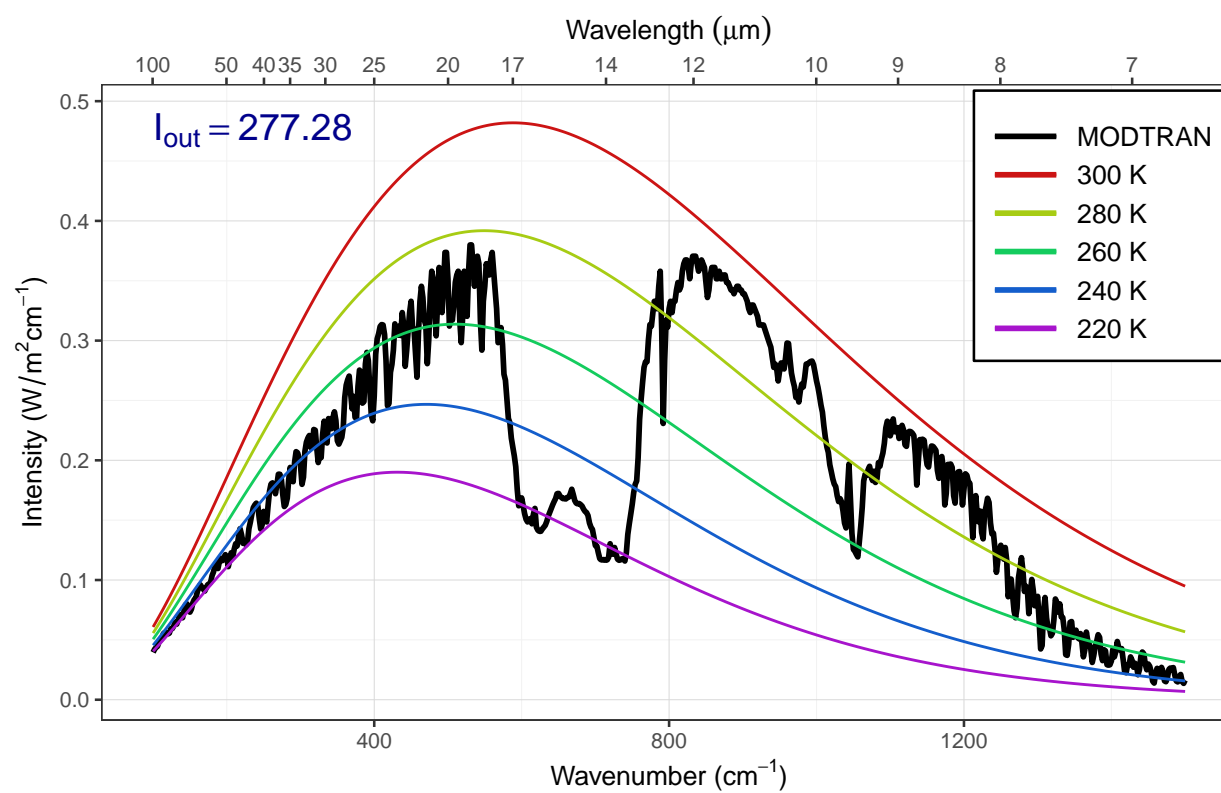
MODTRAN: 10000 ppm CO<sub>2</sub>, 50 km altitude



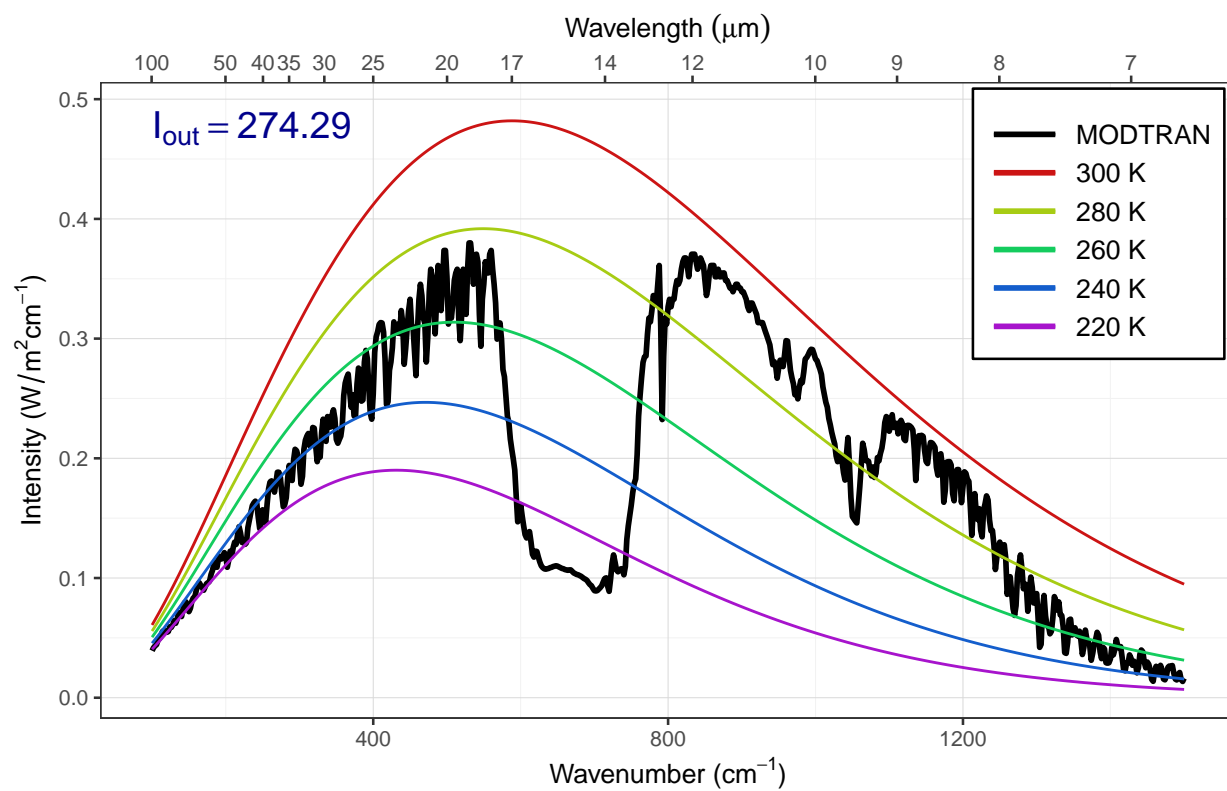
MODTRAN: 10000 ppm CO<sub>2</sub>, 40 km altitude



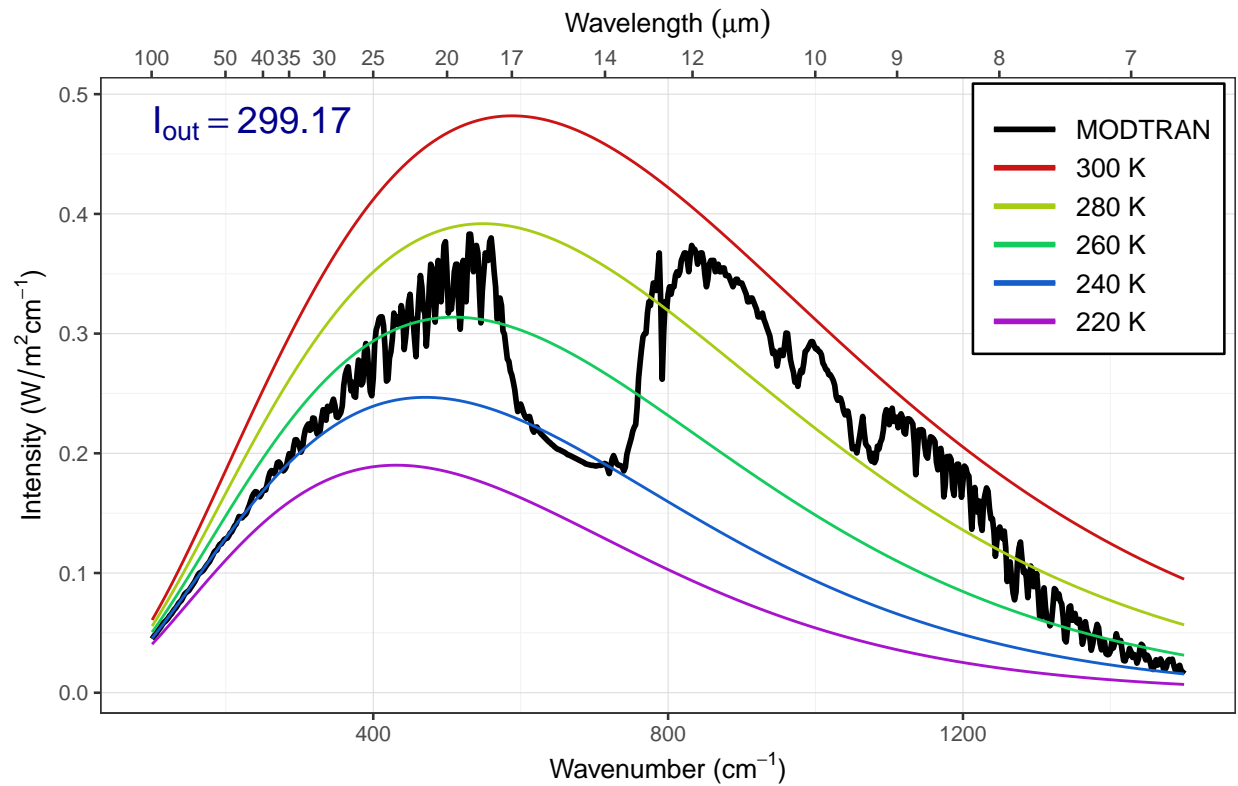
MODTRAN: 10000 ppm CO<sub>2</sub>, 30 km altitude



MODTRAN: 10000 ppm CO<sub>2</sub>, 20 km altitude

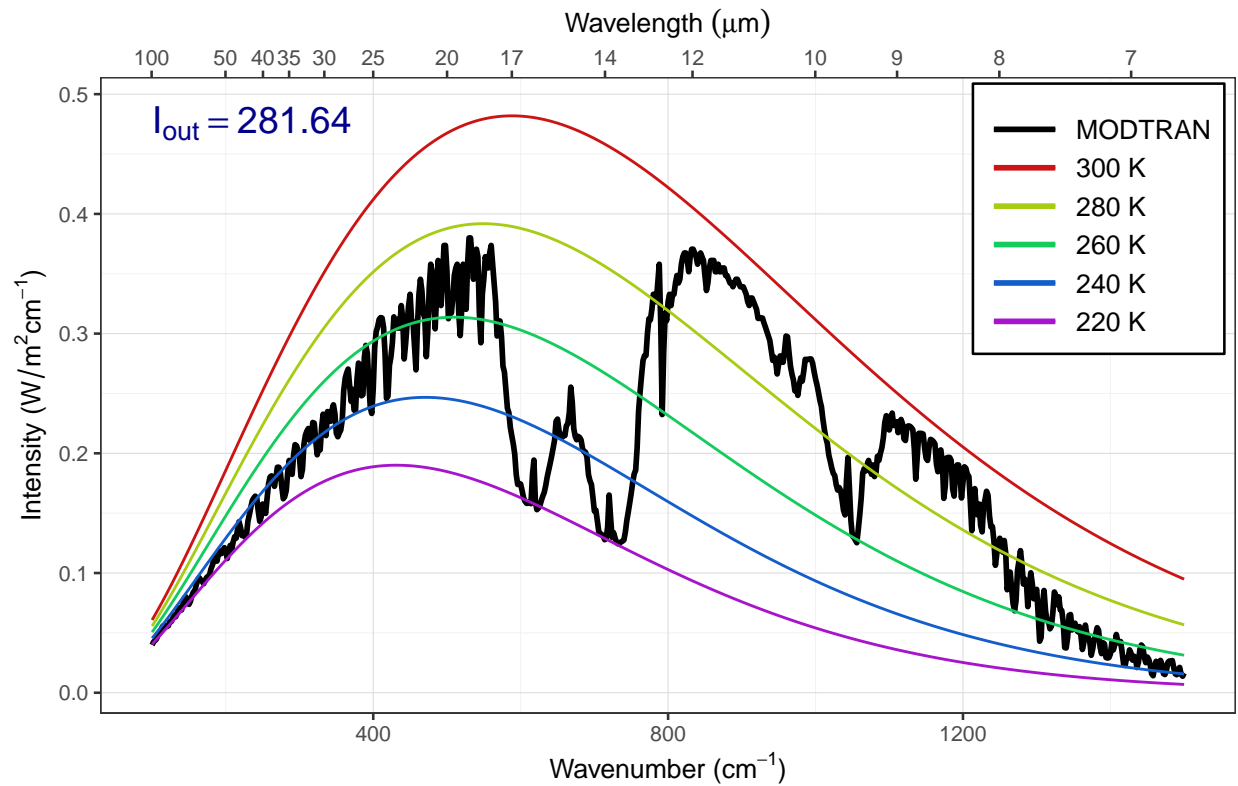


MODTRAN: 10000 ppm CO<sub>2</sub>, 10 km altitude



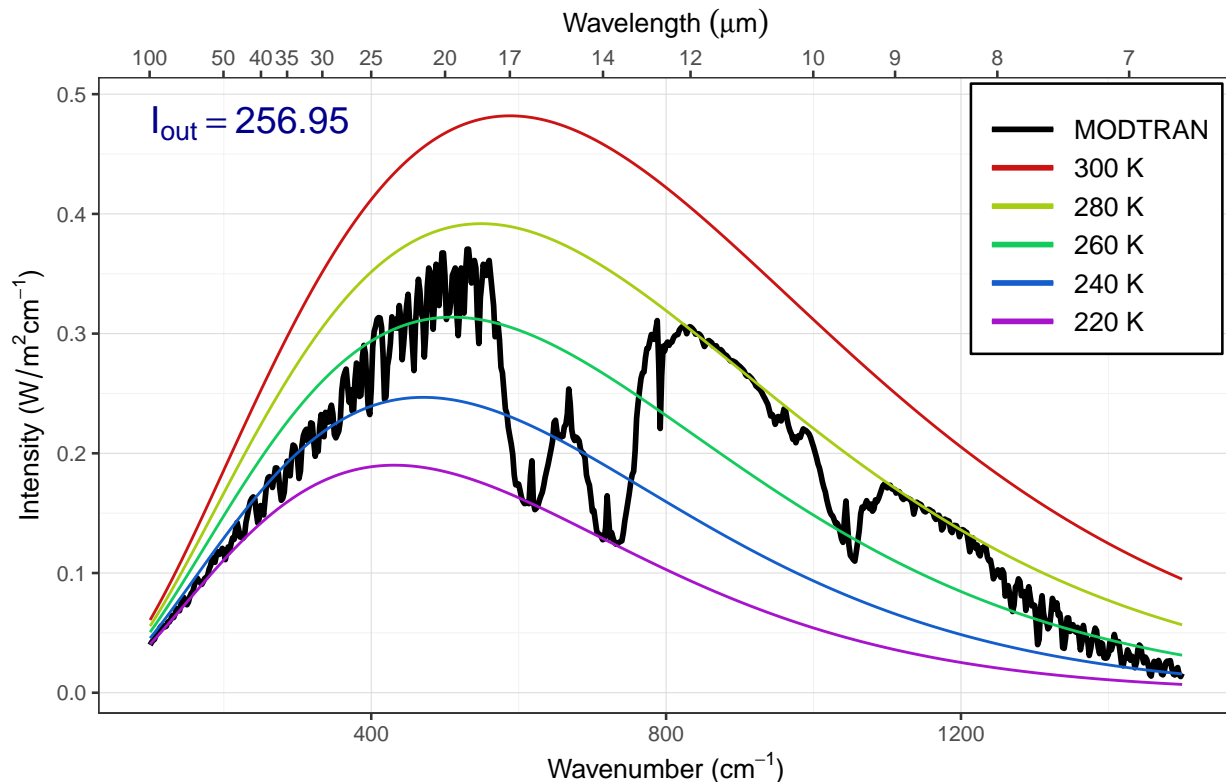
```
high_co2 = 1E4
modtran_high_co2 = run_modtran(file.path(data_dir,
                                           "ex_4_2_hi_co2_clear.txt"),
                               co2_ppm = high_co2)
plot_modtran(modtran_high_co2)
```

MODTRAN: 10000 ppm CO<sub>2</sub>, 70 km altitude



```
high_co2 = 1E4
modtran_high_co2 = run_modtran(file.path(data_dir,
                                           "ex_4_2_hi_co2_cloudy.txt"),
                               co2_ppm = high_co2, clouds = "altostratus")
plot_modtran(modtran_high_co2)
```

MODTRAN: 10000 ppm CO<sub>2</sub>, 70 km altitude



**Answer:** The spike in the CO<sub>2</sub> absorption feature gets smaller as the sensor altitude drops below about 40 km, and disappears entirely as the sensor drops below 30 km. This means that the spike must be coming from the region of the atmosphere between 30 and 40 km.

When you add altostratus clouds to the atmosphere there isn't much change in the emission from the big absorption spikes but the emission from the window region of the spectrum drops considerably. The clouds block longwave emissions in the window region. This traps heat near the surface and is one reason why cloudy nights tend to be warmer than clear nights.

### Exercise 4.3: Water vapor

Our theory of climate presumes that an increase in the temperature at ground level will lead to an increase in the outgoing IR energy flux at the top of the atmosphere.

- a) **How much extra outgoing IR would you get by raising the temperature of the ground by 5°C? What effect does the ground temperature have on the shape of the outgoing IR spectrum and why?**

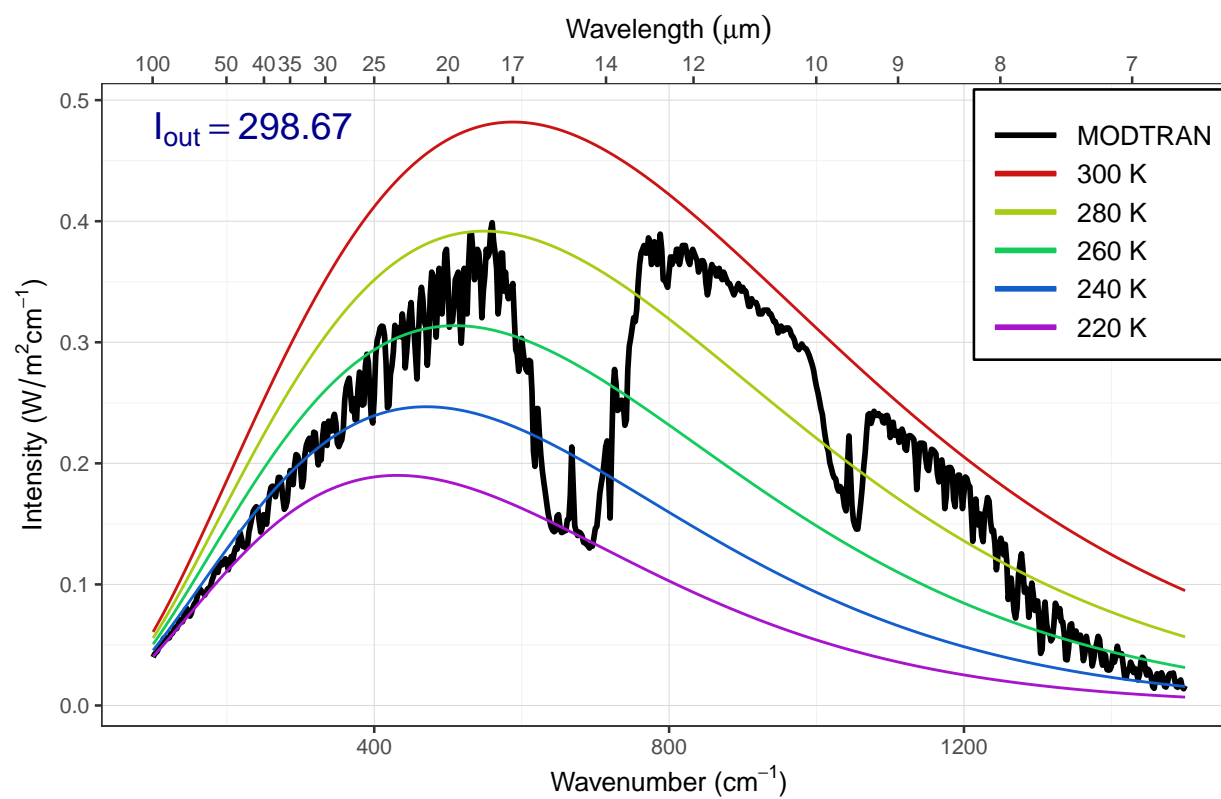
**Hint:** See the hint in the lab-03-instructions document.

```
modtran_baseline = run_modtran(file.path(data_dir, "ex_4_3_baseline.txt"))
modtran_plus_5 = run_modtran(file.path(data_dir, "ex_4_3_t_plus_5"),
                             delta_t = 5)

p_baseline = plot_modtran(modtran_baseline, descr = "Baseline spectrum")
p_plus_5 = plot_modtran(modtran_plus_5,
                       descr = "Ground temperature raised 5K")
plot(p_baseline)
```

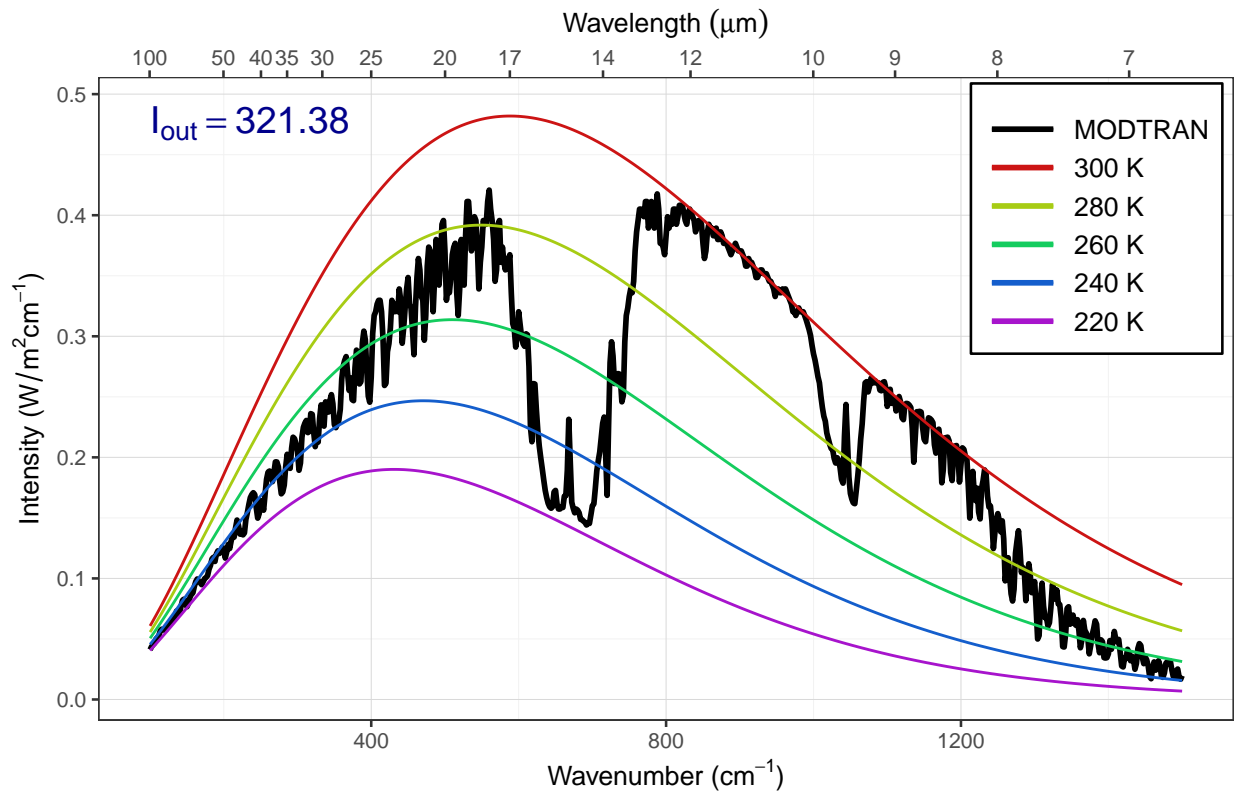


## Baseline spectrum



```
plot(p_plus_5)
```

Ground temperature raised 5K



**Answer:** Raising the ground temperature raises the entire spectrum.

b) More water can evaporate into warm air than into cool air. Change the model settings to hold the water vapor at constant relative humidity rather than constant vapor pressure (the default), calculate the change in outgoing IR energy flux for a 5°C temperature increase. Is it higher or lower? Does water vapor make the Earth more sensitive to CO<sub>2</sub> increases or less sensitive?

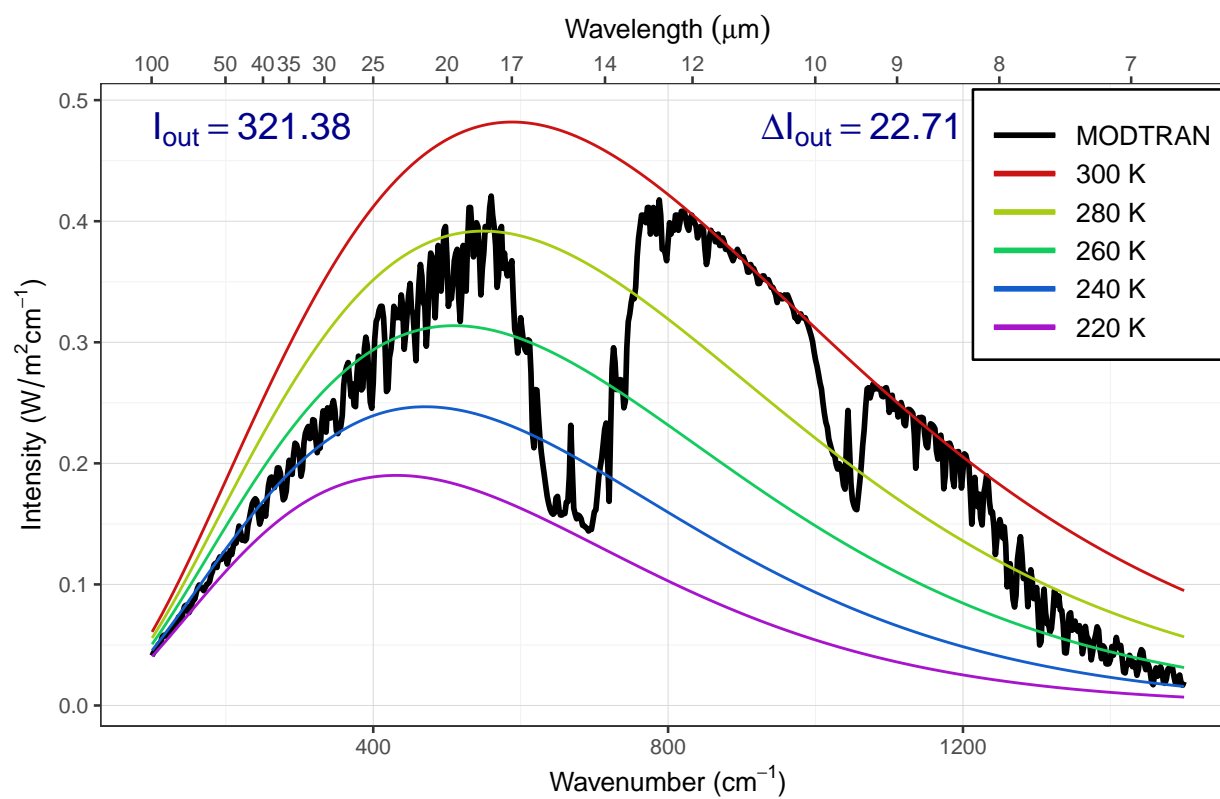
**Note:** By default, the MODTRAN model holds water vapor pressure constant, but you can set it to hold relative humidity constant instead with the option `h2o_fixed = "relative humidity"`, like this:  
`run_modtran(file_name, delta_t = 5, h2o_fixed = "relative humidity")`.

```
modtran_vp = run_modtran(file.path(data_dir, "ex_4_3_vap_press.txt"),
                        delta_t = 5, h2o_fixed = "vapor pressure")
modtran_rh = run_modtran(file.path(data_dir, "ex_4_3_rel_humid.txt"),
                        delta_t = 5, h2o_fixed = "relative humidity")

i_out_baseline = modtran_baseline$i_out
i_out_vp = modtran_vp$i_out
i_out_rh = modtran_rh$i_out

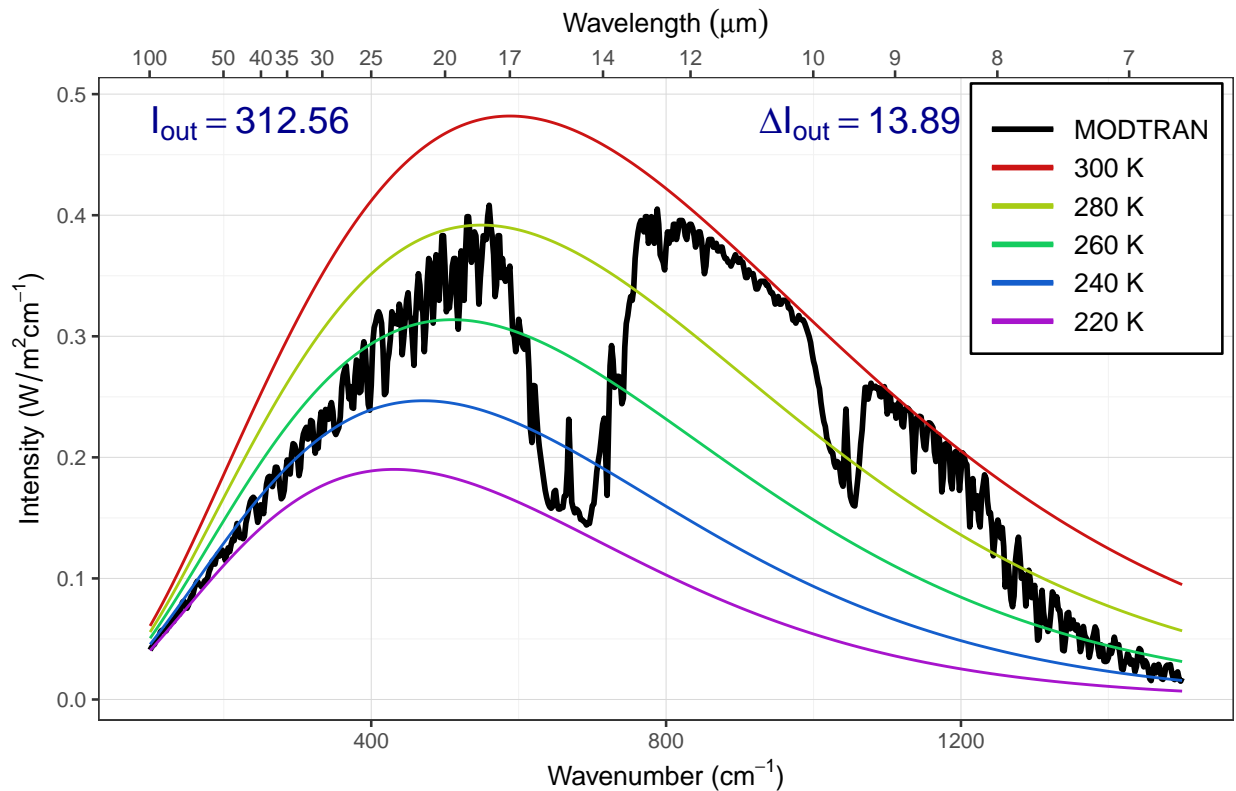
p_vp = plot_modtran(modtran_vp, descr = "Constant vapor pressure",
                    i_out_ref = i_out_baseline)
p_rh = plot_modtran(modtran_rh, descr = "Constant relative humidity",
                    i_out_ref = i_out_baseline)
plot(p_vp)
```

Constant vapor pressure



plot(p\_rh)

### Constant relative humidity



**Answer:** Raising the surface temperature has a bigger effect on  $I_{\text{out}}$  when water vapor pressure is fixed than when relative humidity is fixed. This means that compensating for a change in  $\text{CO}_2$  would require a bigger change in temperature with fixed relative humidity, so the climate is more sensitive to changes in  $\text{CO}_2$  when relative humidity is fixed.

c) Now see this effect in another way.

- Starting from the default base case, record the total outgoing IR flux.
- Now double  $\text{CO}_2$ . The temperature in the model stays the same (that's how the model is written), but the outgoing IR flux goes down.
- Using constant water vapor pressure, adjust the temperature offset until you get the original IR flux back again. Record the change in temperature.
- Now repeat the exercise, but holding the relative humidity fixed instead of the water vapor pressure.
- The ratio of the warming when you hold relative humidity fixed to the warming when you hold water vapor pressure fixed is the feedback factor for water vapor. What is it?

```
modtran_baseline = read_modtran(file.path(data_dir, "ex_4_3_baseline.txt"))
i_baseline = modtran_baseline$i_out

modtran_2xco2 = run_modtran(file.path(data_dir, "ex_4_3_2x_co2_vp.txt"),
                             co2_ppm = 800)
i_2x_vp = modtran_2xco2$i_out

dt_vp = 0.76
modtran_vp_dt = run_modtran(file.path(data_dir, "ex_4_3_vp_dt.txt"),
```

```

co2_ppm = 800, delta_t = dt_vp)
i_vp_dt = modtran_vp_dt$i_out

modtran_2x_rh = run_modtran(file.path(data_dir, "ex_4_3_2x_co2_rh.txt"),
                             co2_ppm = 800, h2o_fixed = "relative humidity")
i_2x_rh = modtran_2x_rh$i_out

dt_rh = 1.21
modtran_rh_dt = run_modtran(file.path(data_dir, "ex_4_3_rh_dt.txt"),
                             co2_ppm = 800, delta_t = dt_rh,
                             h2o_fixed = "relative humidity")
i_rh_dt = modtran_rh_dt$i_out

feedback_factor = dt_rh / dt_vp

```

**Answer:** In the baseline case,  $I_{\text{out}} = 298.67 \text{ W/m}^2$ . When we double  $\text{CO}_2$  with constant water vapor pressure,  $I_{\text{out}}$  drops to  $295.34 \text{ W/m}^2$  and we have to raise the ground temperature by  $0.760 \text{ K}$  to bring  $I_{\text{out}}$  back to  $I_{\text{out}} = 298.67 \text{ W/m}^2$ .

When we double  $\text{CO}_2$  with constant water relative humidity,  $I_{\text{out}}$  drops to  $295.34 \text{ W/m}^2$  and we have to raise the ground temperature by  $1.21 \text{ K}$  to bring  $I_{\text{out}}$  back to  $I_{\text{out}} = 298.67 \text{ W/m}^2$ .

The feedback factor is the ratio of the temperature change with relative humidity fixed to the temperature change with vapor pressure fixed:  $f = 1.59$ .

Notice that there is no difference between holding vapor pressure constant and holding relative humidity constant until the temperature changes.

## Chapter 5 Exercise

### Exercise 5.2: Skin Height

- a) Run the MODTRAN model using the “tropical” atmosphere, without clouds, and with the present-day  $\text{CO}_2$  concentration (400 ppm). Use the ground temperature reported by the model to calculate  $\varepsilon \sigma T_{\text{ground}}^4$ , the heat flux emitted by the ground. Assume  $\varepsilon = 1$ , and I have already provided the value of the Stefan-Boltzmann constant  $\sigma$ , as the R variable `sigma_sb`, which equals  $5.670 \times 10^{-8}$ . (I defined it in the script “utils.R”, which I loaded in the “setup” chunk in the RMarkdown document).

Next, look at the outgoing heat flux at the top of the atmosphere (70 km) ( $I_{\text{out}}$ ) reported by the MODTRAN model. Is it greater or less than the heat flux that you calculated was emitted by the ground?

```

baseline = run_modtran(file.path(data_dir, "ex_5_2_baseline.txt"),
                        co2_ppm = 400, atmosphere = "tropical",
                        clouds = "none")

epsilon = 1.0
t_ground = baseline$t_ground
i_ground = epsilon * sigma_sb * t_ground^4
i_atmos_up = baseline$i_out

```

**Answer:**  $T_{\text{ground}} = 299.7 \text{ K}$ , so  $I_{\text{out}} = \varepsilon \sigma T_{\text{ground}}^4 = 457.5 \text{ W/m}^2$ . The MODTRAN model reports  $I_{\text{out}} = 298.7 \text{ W/m}^2$ .  $I_{\text{ground}}$  is roughly 1.5 times greater than  $I_{\text{out}}$ .

- b) Use the outgoing heat flux at the top of the atmosphere ( $I_{\text{out}}$ ) to calculate the skin temperature (use the equation  $I_{\text{out}} = \varepsilon \sigma T_{\text{skin}}^4$ ). What is the skin temperature, and how does it compare to the ground temperature and the temperature at the tropopause, as reported by the MODTRAN model (`t_tropo`)?

Assuming an environmental lapse rate of 6K/km, and using the skin temperature that you calculated above, and the ground temperature from the model, what altitude would you expect the skin height to be?

```
lapse_rate = 6.0 # Kelvin/km
t_tropo = baseline$t_tropo
t_skin = (i_atmos_up / (epsilon * sigma_sb))^0.25
h_skin = (t_ground - t_skin) / lapse_rate
```

**Answer:** The skin temperature is given by

$$T_{\text{skin}} = \sqrt[4]{\frac{I_{\text{out}}}{\epsilon \sigma}}$$

$T_{\text{skin}} = 269.4$  K, which is 30.3 K less than the ground temperature and greater than the tropopause temperature  $T_{\text{tropo}} = 194.8$ . The skin height is

$$h_{\text{skin}} = \frac{T_{\text{ground}} - T_{\text{skin}}}{\text{lapse rate}}$$

so with a lapse rate of 6 K/km,  $h_{\text{skin}} = 5.1$  km.

- c) **Double the CO<sub>2</sub> concentration and run MODTRAN again. Do not adjust the ground temperature. Repeat the calculations from (b) of the skin temperature and the estimated skin height.**

**What is the new skin temperature? What is the new skin height?**

```
modtran_2x_co2 = run_modtran(file.path(data_dir, "ex_5_2_2x_co2_clear.txt"),
                             co2_ppm = 800, atmosphere = "tropical",
                             clouds = "none")
i_out_2x = modtran_2x_co2$i_out
t_skin_2x = (i_out_2x / (epsilon * sigma_sb))^0.25
h_skin_2x = (t_ground - t_skin_2x) / lapse_rate
```

**Answer:** The new  $I_{\text{out}}$  is 295.3 W/m<sup>2</sup>, so the new  $T_{\text{skin}}$  is 268.6 K, which implies that the new skin height is 5.2 km, 0.13 km higher than for today's CO<sub>2</sub> concentration.

- d) **Put the CO<sub>2</sub> back to today's value, but add cirrus clouds, using the "standard cirrus" value for the clouds. Repeat the calculations from (b) of the skin temperature and the skin height.**

**What is the new skin temperature? What is the new skin height? Did the clouds or the doubled CO<sub>2</sub> have a greater effect on the skin height?**

```
modtran_cirrus = run_modtran(file.path(data_dir, "ex_5_2_cirrus.txt"),
                             co2_ppm = 400, atmosphere = "tropical",
                             clouds = "standard cirrus")
i_out_cirrus = modtran_cirrus$i_out
t_skin_cirrus = (i_out_cirrus / (epsilon * sigma_sb))^0.25
h_skin_cirrus = (t_ground - t_skin_cirrus) / lapse_rate
```

**Answer:** The new skin height is 5.9 km, which is 0.68 km higher than for the doubled CO<sub>2</sub>. To put this in context, doubling CO<sub>2</sub> raises the skin height by 0.13 km and adding cirrus clouds raises the skin height by 0.81, so the cirrus clouds have a much bigger effect on the climate.