COMP9601 Assignment 1 Solution

Problem 1

1) Let $\mathcal{D} = (Q, \Sigma, \delta, q_0, F)$ be the DFA that accepts L, we construct the NFA $\mathcal{N} = (Q', \Sigma, \delta', q_0', F')$ where

1. $Q' = Q_1 \cup Q_2 \cup Q_3$, where $Q_i = \{q^i | q \in Q\}$ for $i = 1, 2, 3$. In other words, each Q_i is a copy of Q with extra superscripts to distinguish from each other.
2. $\delta'(q^1, a) = \delta'(q^3, a) = \delta(q, a)$, for all $q \in Q$ and $a \in \Sigma$.
   $\delta'(q^2, a) = q^2$ for all $q \in Q$ and $a \in \Sigma$.
   $\delta'(q^1, \epsilon) = q^2$, $\delta'(q^2, \epsilon) = q^3$ for all $q \in Q$.
3. $q_0' = q_0^1$.
4. $F' = \{q^3 | q \in F\}$.

2) Let $\mathcal{D} = (Q, \Sigma, \delta, q_0, F)$ be the DFA that accepts L, we construct the NFA $\mathcal{N} = (Q, \Sigma, \delta', q_0, F)$ where

1. $\delta'(q, a) = \delta(q, a)$, for $q \in Q$ and $a \in \Sigma$.
   $\delta'(q, \epsilon) = \{p | p$ is reachable from q in $\mathcal{D}\}$, for $q \in Q$.

Problem 2

A linear grammar is a context-free grammar that has at most one nonterminal in the right hand side of its productions and a right linear grammar can have rules only of the form $A \to Z$ where A is a variable and $Z = aB$ or a, where a is a terminal and B is variable.

A right linear grammar can be denoted as a $5-tuple\,(V,\ \Sigma,\ R,\ S,\ E)$, where,

- V is a finite set called variables,
- Σ is a finite set, disjoint from V, called terminals,
- R is a finite set of rules in the following two forms, (1) $X \to a$, where $X \in V$ and $a \in \Sigma$. (2) $X \to aY$, where $X, Y \in V$ and $a \in \Sigma$
- S is the start variable and $S \in V$
- E is a boolean variable, which indicates whether the grammar accepts empty string ε.

Firstly, it is straightforward to design right linear grammars to generate basic units like Φ, ε and $a \in \Sigma$. Other cases can be generated through union, concatenation and star operations.

Given two regular expressions R_1 and R_2, we assume that they can be derived from $G_1 = (V_1,\ \Sigma,\ R_1,\ S_1,\ E_1)$ and $G_2 = (V_2,\ \Sigma,\ R_2,\ S_2,\ E_2)$ respectively. We also assume that R_1 and R_2 are neither Φ nor ε, otherwise, the right linear grammar R can be derived in a straight forward way.

1. $R = R_1 \cup R_2$, we construct the right linear grammar $G = (V,\ \Sigma,\ R,\ S,\ E)$ as follows,
   - $V = V_1 \cup V_2 \cup S$
   - $R = R_1 \cup R_2 \cup \{S \to \delta | S_1 \to \delta$ is in R_1 or $S_2 \to \delta$ is in $R_2\}$.
   - S is a start variable not in V_1 or V_2.
   - $E = E_1$ OR E_2.
2. $R = R_1 R_2$, we construct the right linear grammar $G = (V,\ \Sigma,\ R,\ S,\ E)$ as follows,
   - $V = V_1 \cup V_2 \cup S$
   - Let $R'' = R_1' \cup R_2 \cup \{S \to \delta | S \to \delta$ is in $R_1'\}$, where R_1' is derived from R_1 by replacing the rules in the form $X \to a$ with $X \to aS_2$; We construct R step by step as follows,
     (a) Set $R = R''$
     (b) if E_1 is TRUE, we set $R = R \cup R_1 \cup \{S \to \delta | S_1 \to \delta$ is in $R_1\}$.
     (c) if E_2 is TRUE, we set $R = R \cup \{S \to \delta | S_2 \to \delta$ is in $R_2\}$.
   - S is a start variable not in V_1 or V_2.
   - $E = E_1$ AND E_2
3. $R = R_1^*$, we construct the right linear grammar $G = (V,\ \Sigma,\ R,\ S,\ E)$ as follows,
   - $V = V_1 \cup \{S\}$
   - $R = R_1 \cup R_1' \cup \{S \to \delta | S_1 \to \delta$ is in $R_1\} \cup \{S \to \delta | S_1 \to \delta$ is in $R_1'\}$, where R_1' is derived from R_1 by replacing the rules in the form $X \to a$ with $X \to aS$;
   - S is a start variable not in V_1 or V_2.
   - E=TRUE.

Problem 3

For each rule in the left linear grammar,

- If it is $S \to a$, add rule $S \to A$.
- If it is $S \to Aa$, add rule $A \to a$.
- If it is $A \to a$, add rule $S \to aA$.
- If it is $A \to Ba$, add rule $B \to aA$.

Problem 4

We denote PDA A as $A\,(Q, \Sigma, \Gamma, \delta, q_0, F)$, and construct another one state PDA $B\,(Q, \Sigma, \Gamma', \delta', q_0, F)$ to simulate A.

Let $\Gamma' = \Gamma \times \Gamma$, and $\Gamma' = \{[z_i, z_j] | z_i \in \Gamma \text{and} z_j \in \Gamma\}$. We denote z as the top element of A and z' as the next top element of A. a is the input symbol.

1. If $\delta\,(q, a, z)$ contains $(q, z_1 z_2, ..., z_k)$, then we let the transition function $\delta'(p, a, [z', z])$ contain $(p, [z', z_1][z_1, z_2]...[z_{k-2}, z_{k-1}][z_{k-1} z_k])$
2. If $\delta(q, a, z)$ contains (q, ϵ), then we let the transition function $\delta'(p, a, [z', z])$ contain $(p, [\epsilon, \epsilon])$
3. If $\delta(q, a, z', z)$ contains $(q, z_1 z_2 ... z_k)$, then we let the transition function $\delta'(p, a, [z', z])$ contain $(p, [z', z_1][z_1, z_2]...[z_{k-2}, z_{k-1}][z_{k-1} z_k])$

4. If $\delta(q, a, z', z)$ contains (q, ϵ), then we let the transition function $\delta'(p, a, [z', z])$ contain the $(p, [\epsilon, \epsilon])$.

Problem 5

Let B be the language of all palindromes over $\{0, 1\}$ which contains an equal number of 0s and 1s. For a contradiction, we assume that B is context free. Therefore, B has a pumping length p. We apply pumping lemma for $s = 0^p1^{2p}0^p \in B$ with $|s| > p$, there exists $uvxyz$ such that (1) $uv^ixy^iz \in B$ for all $i \geq 0$, (2) $|vy| > 0$ and (3) $|vxy| \leq p$. We will now proceed by cases to show that no matter what the values of $uvxyz$ we choose, we will reach a contradiction.

- Case 1: vxy contains only 1s. uv^2xy^2z will not have the same number of 0s and 1s, thus can not be generated by B.
- Case 2: vxy contains at least one 0. From (3), we know that $|vxy| \leq p$. If vxy contains p 0s, then $u = \epsilon$ and $uv^2xy^2z \notin B$, since it will no longer have the same number of 0s and 1s and no longer be a palindrome. If vxy contains less than p 0s, vxy can only contain symbols from the starting 0s or the final 0s, but not both. Thus, after pumping s will either have a different number 0s before and after the 1s or it will no longer be a palindrome, thus $uv^2xy^2z \notin B$.

These are all the cases from (2). In each case we have proof that contradict (1). Therefore, B is not context free.