Theorem: It is undecidable whether or not the languages generated by two given context-free grammars have an empty intersection.

Proof: By a reduction of post correspondence problem (which is known to be undecidable) to the empty intersection problem.
Given a set $d_1,...,d_n$ of dominos where, for i=1,...,n, the top string of d_i is w_i and the bottom string of d_i = x_i. Consider the context-free grammars

$$W \to w_1 W\ d_1 \mid w_2 W\ d_2 \mid ... \mid w_n W\ d_n \mid w_1\ d_1 \mid w_2\ d_2 \mid ... \mid w_n\ d_n \mid$$

and

$$X \to x_1 X\ d_1 \mid x_2 X\ d_2 \mid ... \mid x_n X\ d_n \mid x_1\ d_1 \mid x_2\ d_2 \mid ... \mid x_n\ d_n \mid$$

Now notice that the given instance of PCS has a match exactly when the intersection of the languages generated by the resulting grammars above is nonempty.

Theorem: It is undecidable whether or not a given context-free grammar is ambiguous.

Proof: Given an instance of PCP, create the grammar G with productions S -> W | X and the productions for X and W above. Now the given instance has a match exactly when G is ambiguous.

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