COMP9601 Assignment 2

Problem 1

Let $T = \{Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject}\}$ be the Turning machine that accepts L. We construct a DFA $N = \{Q, \Gamma, f, q_0, F\}$ to accept the same language, where $F = \{q_{accept}\}$. We define a maximal chain of stay-put transitions $\delta(q, a) = (q_1, b_1, S), \delta(q_1, b_1) = (q_2, b_2, S), ..., \delta(q_{k-1}, b_{k-1}) = (q_k, b_k, S), \delta(q_k, b_k) = (q', b', R)$. Then, we define the transition functions:
For all maximal chain of stay-put transitions starting with state q and character a, ending with state q', let $f(q, a) = q'$.
Moreover, let $f(q_{accept}, \Gamma^*) = q_{accept}$ and $f(q_{reject}, \Gamma^*) = q_{reject}$.

Problem 2

We show that $E_{TM} \leq_m EQ_{TM}$.

The mapping function is defined as $f(\langle M \rangle) = \langle M_1, M_2 \rangle$, where $M_1 = M$ and M_2 is a Turing machine that rejects all inputs. It is left to show that $\langle M \rangle \in E_{TM}$ if and only if $f(\langle M \rangle) \in EQ_{TM}$.

Problem 3

We show that $\sim K \leq_m L_\infty$.

The mapping function is defined as $f(\langle M \rangle) = \langle M' \rangle$, the construction is as below: for any input with length n to the Turing machine M', the Turing machine M runs $\langle M \rangle$ for n steps, if M doesn't accept, then M' accepts the input, otherwise reject.

Problem 4

Let $L = \{(LS, LT) | LS$ and LT are context free grammars with nonempty intersection$\}$. We show that $PCP \leq_m L$.

Given a PCP instance $P = \{(s_1, t_1), (s_2, t_2), \cdots (s_n, t_n)\}$. We construct the following context free grammars:

LS: $S \to s_1 S a_1 | s_2 S a_2 | \cdots | s_n S a_n | S | \epsilon$ and LT: $T \to t_1 T a_1 | t_2 T a_2 | \cdots | t_n T a_n | T | \epsilon$. It is left to show that $P \in PCP$ if and only if $(LS, LT) \in L$.

Problem 5

Let $L = \{P | P$ is a Boolean formula with at least two satisfying assignments$\}$. We show that $SAT \leq_p L$.
The mapping function is defined as $f(P) = P \vee (x \wedge \bar{x})$ where x is a variable that doesn't appear in P. (If P has a feasible assignment, $f(P)$ must have at least 2 satisfying assignments.)

Problem 6

We show that $3SAT \leq_p 2COLOR$.

Given a formula P in 3SAT, we construct the following instance (S, C) for 2-Color problem. For each variable x in the formula F, add x and $\bar{x}$ to S and create a corresponding set $\{x, \bar{x}\}$ to C. We further add a special variable named b into S. For each clause in P, add a set containing its variables and the special variable b to C. We are left to show that $P \in 3SAT$ if and only if $(S, C) \in 2COLOR$. (The color of b is considered as false always.)