Sequential data

1. Time series
   - Financial data analysis: stock market, commodities, Forex
   - Healthcare: pulse rate, sugar level (from medical equipment and wearables)
2. Text and speech: speech understanding, text generation
3. Spatiotemporal data
   - Self-driving and object tracking
   - Plate tectonic activity
4. Physics: jet identification
5. etc.

Sequence modelling I

Sequence classification

1. $\boldsymbol{x} = x_1, x_2, \ldots, x_n$, $x_i \in V$ - objects
2. $y \in \{1, \ldots, L\}$ - labels
3. $\{(\boldsymbol{x}^{(1)}, y_1), (\boldsymbol{x}^{(2)}, y_2), \ldots, (\boldsymbol{x}^{(m)}, y_m)\}$ – training data

Classification problem: $\gamma : \boldsymbol{x} \to y$

1. Activity recognition: x – pulse rate, y – activity (walking, running, peace)
2. Opinion mining: x – sentence, y – sentiment (positive, negative)
3. Trading: x – stock market, y – action (sell, buy, do nothing)

Sequence modelling II

Sequence labelling

1. $\boldsymbol{x} = x_1, x_2, \ldots, x_n$, $x_i \in V$ - objects
2. $\boldsymbol{y} = y_1, y_2, \ldots, y_n$, $y_i \in \{1, \ldots, L\}$ - labels
3. $\{(\boldsymbol{x}^{(1)}, \boldsymbol{y}^{(1)}), (\boldsymbol{x}^{(2)}, \boldsymbol{y}^{(2)}), \ldots, (\boldsymbol{x}^{(m)}, \boldsymbol{y}^{(m)})\}$ – training data
4. exponential number of possible solutions : if $\texttt{length}(x) = n$, there are L^n possible solutions

Classification problem: $\gamma : \boldsymbol{x} \to \boldsymbol{y}$

1. Part of speech tagging: x – word, y – part of speech (verb, noun, etc.)
2. Genome annotation: x – DNA, y – genes
3. HEP tracking: x - a set of hits with backgrounds, y – hit classification

Sequence labelling tasks

POS tagging and Named Entity Recognition

X (words)	the	cat	sat	on	a	mat
Y (tags)	DET	NOUN	VERB	PREP	DET	NOUN

Table: POS tagging

Alex	is	going	to	Los	Angeles
B-PER	O	O	O	B-LOC	I-LOC

Table: NER (IOB2)

Alex	travels	with	Marty	A.	Rick	to	NY	city
S-PER	O	O	B-PER	I-PER	E-PER	O	B-LOC	E-LOC

Table: NER (IOBES)

Sequence modelling III

Sequence transduction / transformation

1. $\boldsymbol{x} = x_1, x_2, \ldots, x_n$, $x_i \in V_{source}$ - objects
2. $\boldsymbol{y} = y_1, y_2, \ldots, y_n$, $y_i \in V_{target}$ - objects
3. $\{(\boldsymbol{x}^{(1)}, \boldsymbol{y}^{(1)}), (\boldsymbol{x}^{(2)}, \boldsymbol{y}^{(2)}), \ldots, (\boldsymbol{x}^{(m)}, \boldsymbol{y}^{(m)})\}$ – training data
4. $\boldsymbol{x}^{(1)}$, $\boldsymbol{y}^{(1)}$ are of different length

Transduction problem: $\boldsymbol{x}_{source} \rightarrow \boldsymbol{y}_{target}$

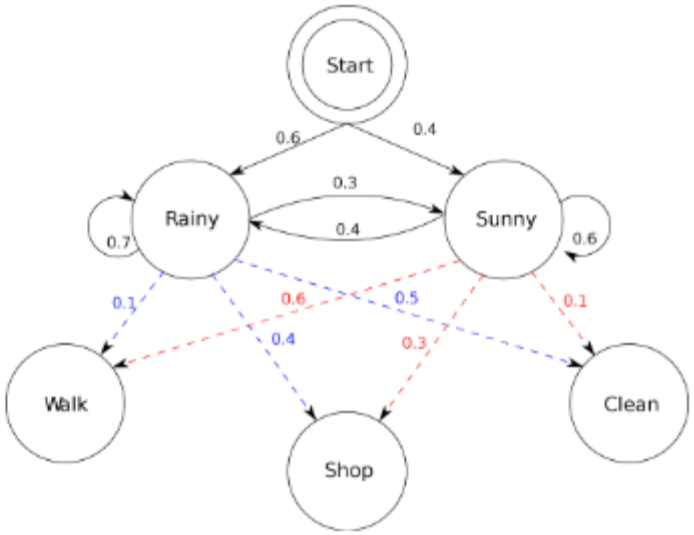
1. Machine translation: x – sentence in German, y – sentence in English
2. Speech recognition: x – spoken language, y – text
3. Chat bots: x – question, y – answer

Traditional ML approaches to sequence modeling

- Hidden Markov Models (HMM)
- Conditional Random Fields (CRF)
- Local classifier: for each x define features, based on x_{-1}, x_{+1}, etc, and perform classification n times

Problems:

1. Markov assumption: fixed length history
2. Computation complexity

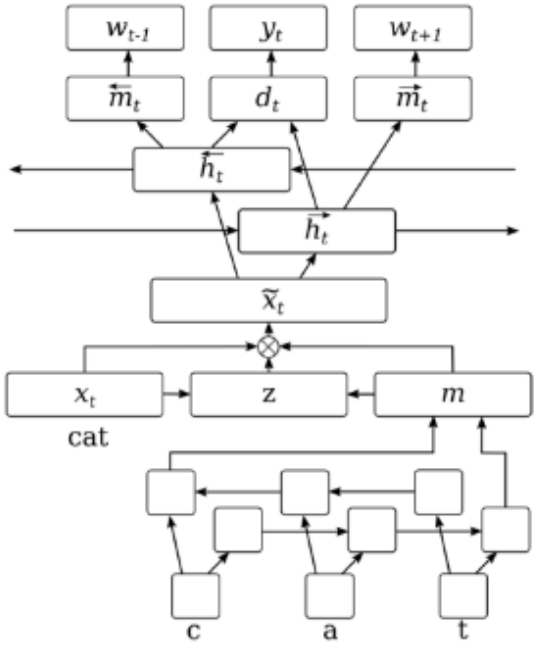


DL approaches to sequence modeling

- Neural networks
- Recurrent neural network and its modifications: LSTM, GRU, Highway
- 2D Convolutional Neural Network
- Transformer
- Pointer network

Problems:

1. Training time
2. Amount of training data



Neural language model

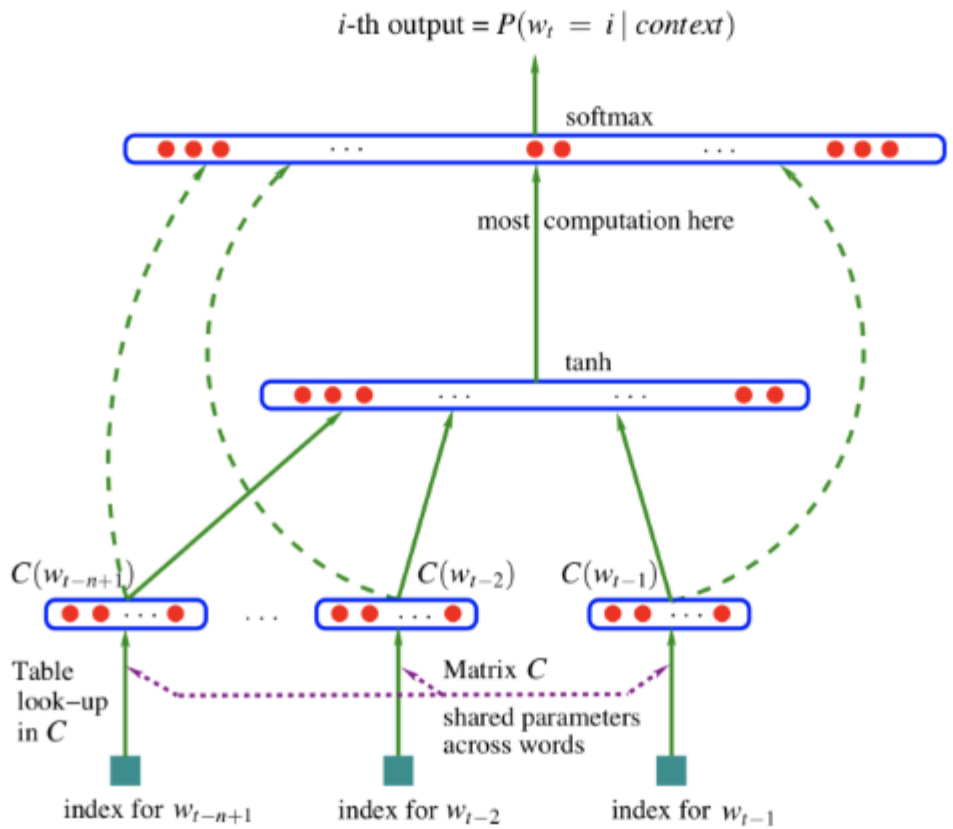
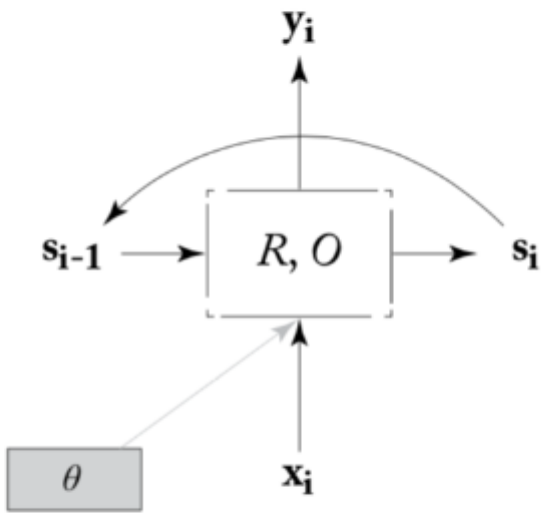


Figure: Neural language model

Recurrent neural network

- Input: sequence of vectors
- $x_{1:n} = x_1, x_2, \ldots, x_n,\ x_i \in \mathbb{R}^{d_{in}}$
- Output: a single vector
  $y_n = RNN(x_{1:n}),\ y_n \in \mathbb{R}^{d_{out}}$
- For each prefix $x_{i:j}$ define an output vector y_i:
  $y_i = RNN(x_{1:i})$
- RNN^* is a function returning this sequence for input sequence $x_{1:n}$:
  $y_{1:n} = RNN^*(x_{1:n}),\ y_i \in \mathbb{R}^{d_{out}}$



Sequence modelling with RNN

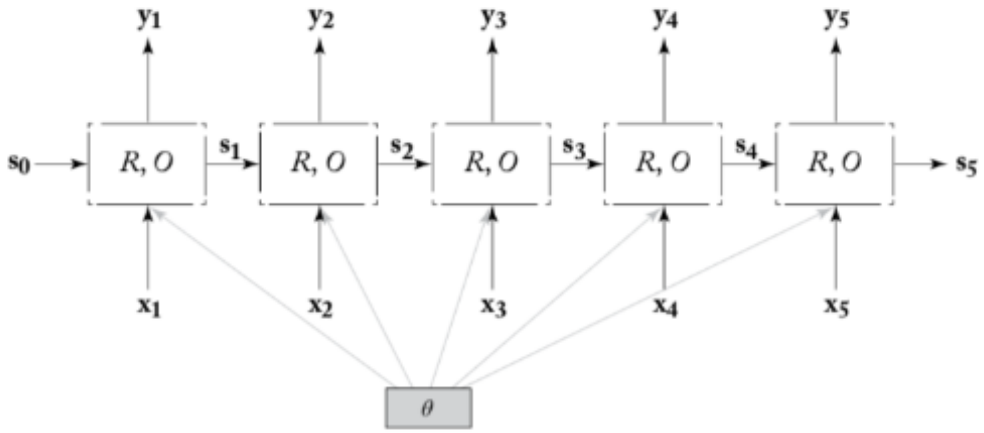
1. Sequence classification
   Put a dense layer on top of RNN to predict the desired class of the sequence after the whole sequence is processed

$$p(l_j|\boldsymbol{x}_{1:n}) = \texttt{softmax}(RNN(\boldsymbol{x}_{1:n}) \times W + b)_{[j]}$$

2. Sequence labelling
   Produce an output y_i for each input RNN reads in. Put a dense layer on top of each output to predict the desired class of the input

$$p(l_j|\boldsymbol{x}_j) = \texttt{softmax}(RNN(\boldsymbol{x}_{1:j}) \times W + b)_{[j]}$$

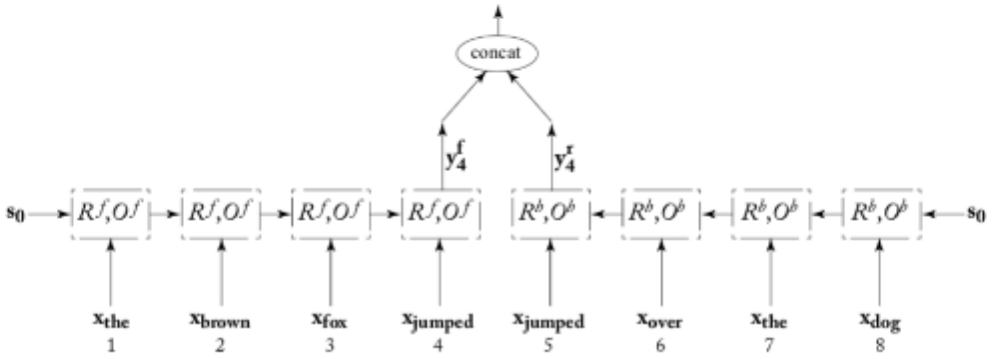
RNN unrolled



$$s_4 = R(s_3, x_4) = R(R(s_2, x_3), x_4) = R(R(R(s_1, x_2), x_3), x_4) =$$
$$= R(R(R(R(s_0, x1), x_2), x_3), x_4)$$

Bidirectional RNN (Bi-RNN)

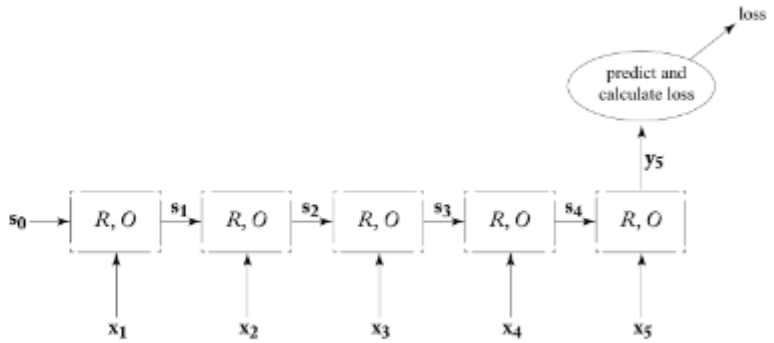
The input sequence can be read from left to right and from right to left.
Which direction is better?



$$biRNN(x_{1:n}, i) = y_i = [RNN^f(x_{1:i}); RNN^r(x_{n:i})]$$

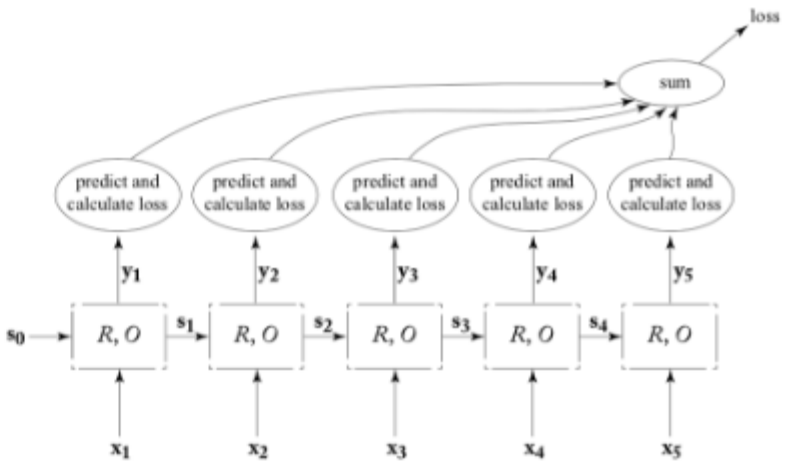
Sequence classification

- $\hat{y}_n = O(s_n)$
- $\texttt{prediction} = MLP(\hat{y}_n)$
- Loss: $L(\hat{y}_n, y_n)$
- L can take any form: cross entropy, hinge, margin, etc.

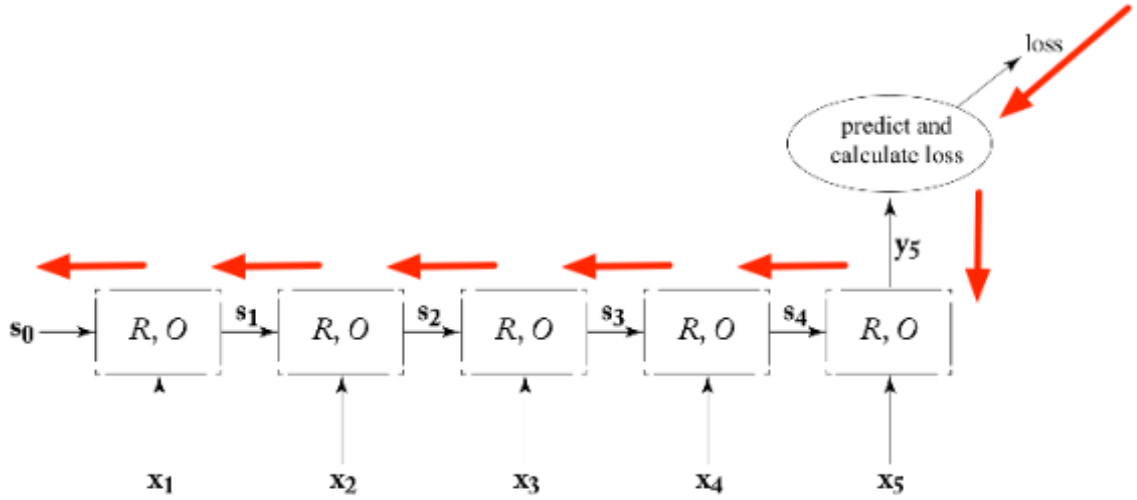


Sequence labelling

- Output $\hat{t}_i$ for each input $x_{1,i}$
- Local loss: $L_{local}(\hat{t}_i, t_i)$
- Global loss: $L(\hat{t}_n, t_n) = \sum_i L_{local}(\hat{t}_i, t_i)$
- L can take any form: cross entropy, hinge, margin, etc.



Backpropagation through time



$$s_i = R(x_i, s_{i-1}) = g(s_{i-1}W^s + x_i W^x + b)$$

Chain rule: $\frac{\partial L}{\partial w} = \frac{\partial L}{\partial p(\hat{y}_5)} \frac{\partial p(\hat{y}_5)}{\partial s_4} \left(\frac{\partial s_4}{\partial w} + \frac{\partial s_4}{\partial s_3} \frac{\partial s_3}{\partial w} + \frac{\partial s_4}{\partial s_3} \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial s_w} + \ldots \right)$

Vanishing gradient problem

Chain rule: $\frac{\partial L}{\partial w} = \frac{\partial L}{\partial p(\hat{y}_5)} \frac{\partial p(\hat{y}_5)}{\partial s_4} \left(\frac{\partial s_4}{\partial w} + \frac{\partial s_4}{\partial s_3} \frac{\partial s_3}{\partial w} + \frac{\partial s_4}{\partial s_3} \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial s_w} + \ldots \right)$
g – sigmoid

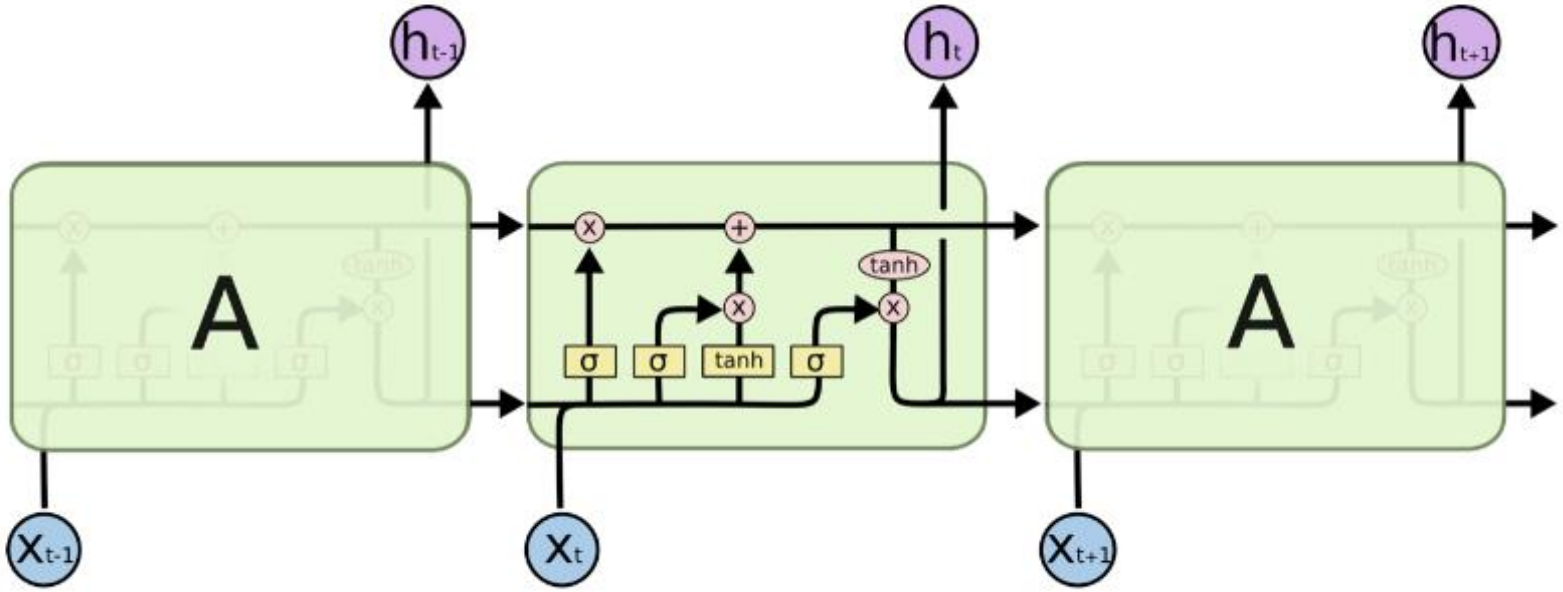
1. Many sigmoids near 0 and 1
   - Gradients $\rightarrow 0$
   - Not training for long term dependencies
2. Many sigmoids > 1
   - Gradients $\rightarrow +\text{inf}$
   - Not training again

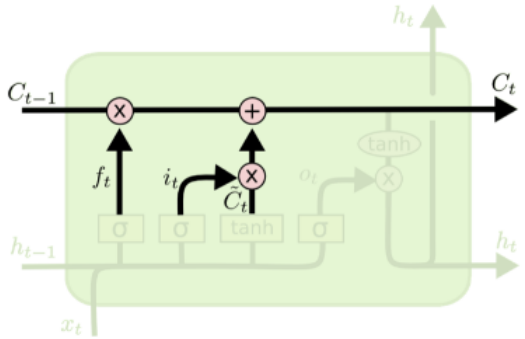
Solution: gated architectures (LSTM and GRU)

Controlled memory access

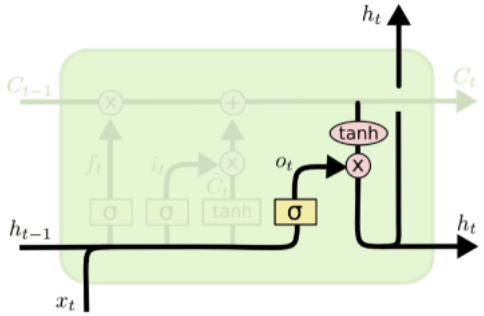
- Entire memory vector is changed: $s_{i+1} = R(x_i, s_i)$
- Controlled memory access: $s_{i+1} = g \odot R(x_i, s_i) + (1 - g)s_i$
  $g \in [0, 1]^d, s, x \in \mathbb{R}^d$
- Differential gates: $\sigma(g), g' \in \mathbb{R}^d$
- This controllable gating mechanism is the basis of the LSTM and the GRU architectures

Long short term memory

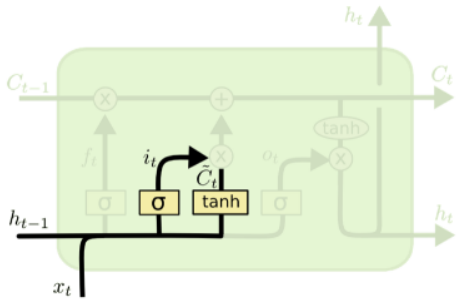




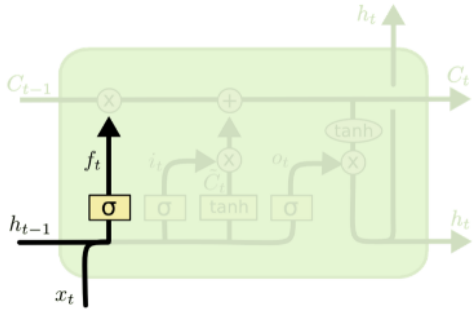
$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$



$$o_t = \sigma\left(W_o\left[h_{t-1}, x_t\right] + b_o\right)$$
$$h_t = o_t * \tanh\left(C_t\right)$$

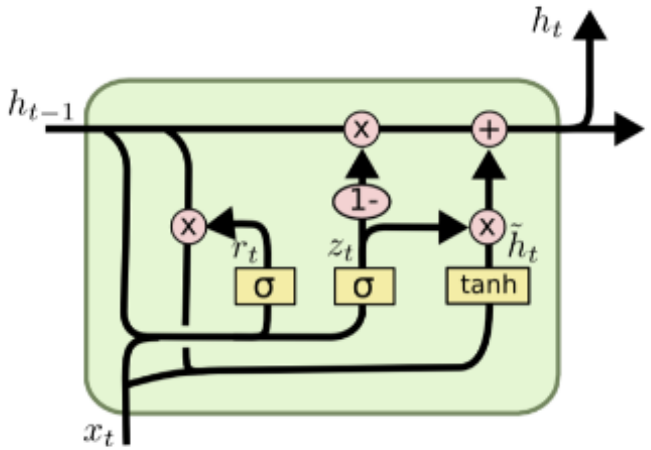


$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] + b_i\right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] + b_f\right)$$

Gated recurrent unit



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

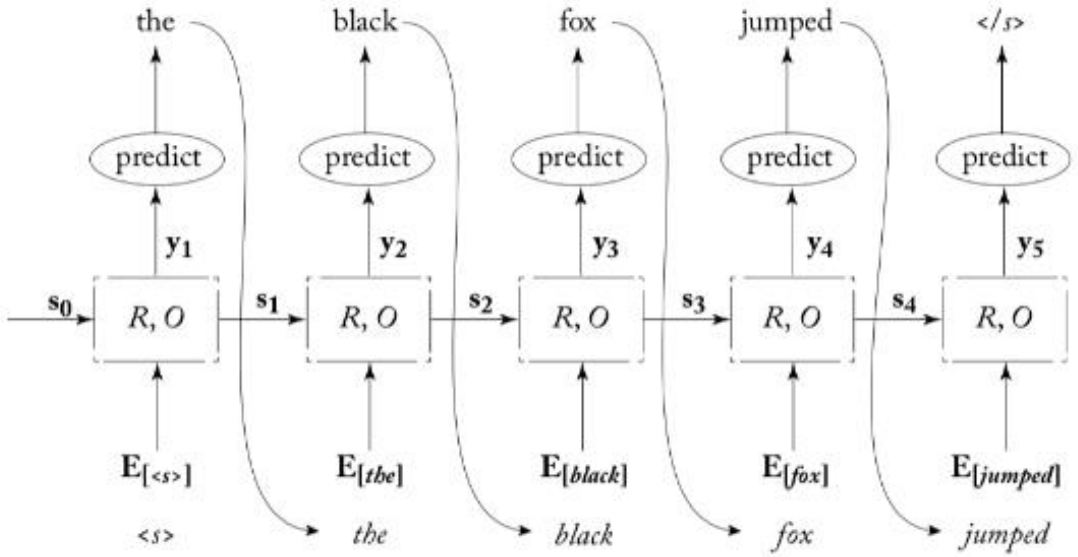
$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

Sequence generation

Teacher forcing: $x := < s > x, y := x < /s >$

$x : < s > x_1 x_2 \ldots x_n$

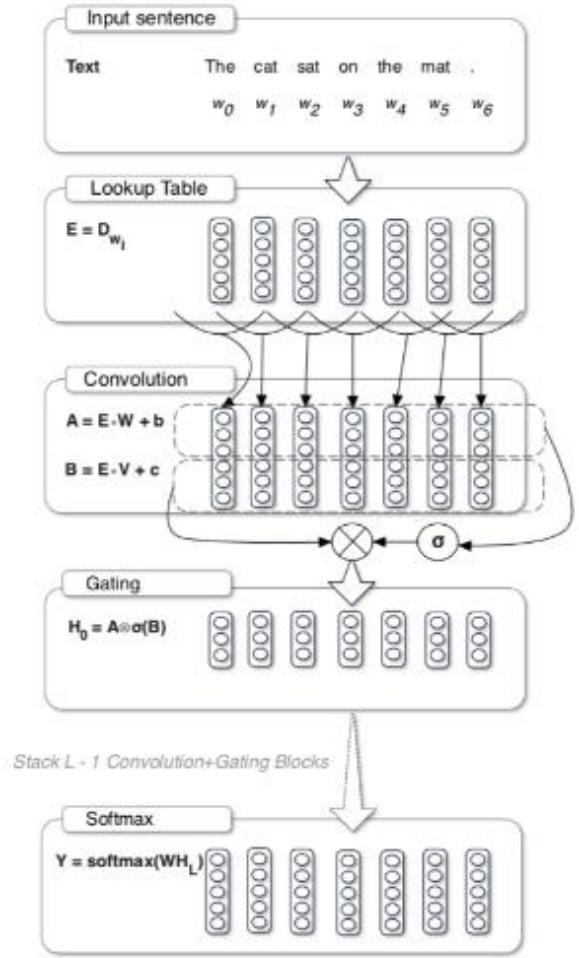
$y : x_1 x_2 \ldots x_n < /s >$



Pros and cons of RNNs

1. Advantages:
   - RNNs are popular and successful for variable-length sequences
   - The gating models such as LSTM are suited for long-range error propagation
2. Problems:
   - The sequentiality prohibits parallelization within instances
   - Long-range dependencies still tricky, despite gating

Language Modeling with Gated Convolutional Networks



- **Embeddings** $\in D^{|V| \times e}$
- **Input:** $w_0, \ldots, w_n \rightarrow E = [D_{w_0}, \ldots, D_{w_n}]$
- **Hidden layers**: $h_0, \ldots, h_n$:

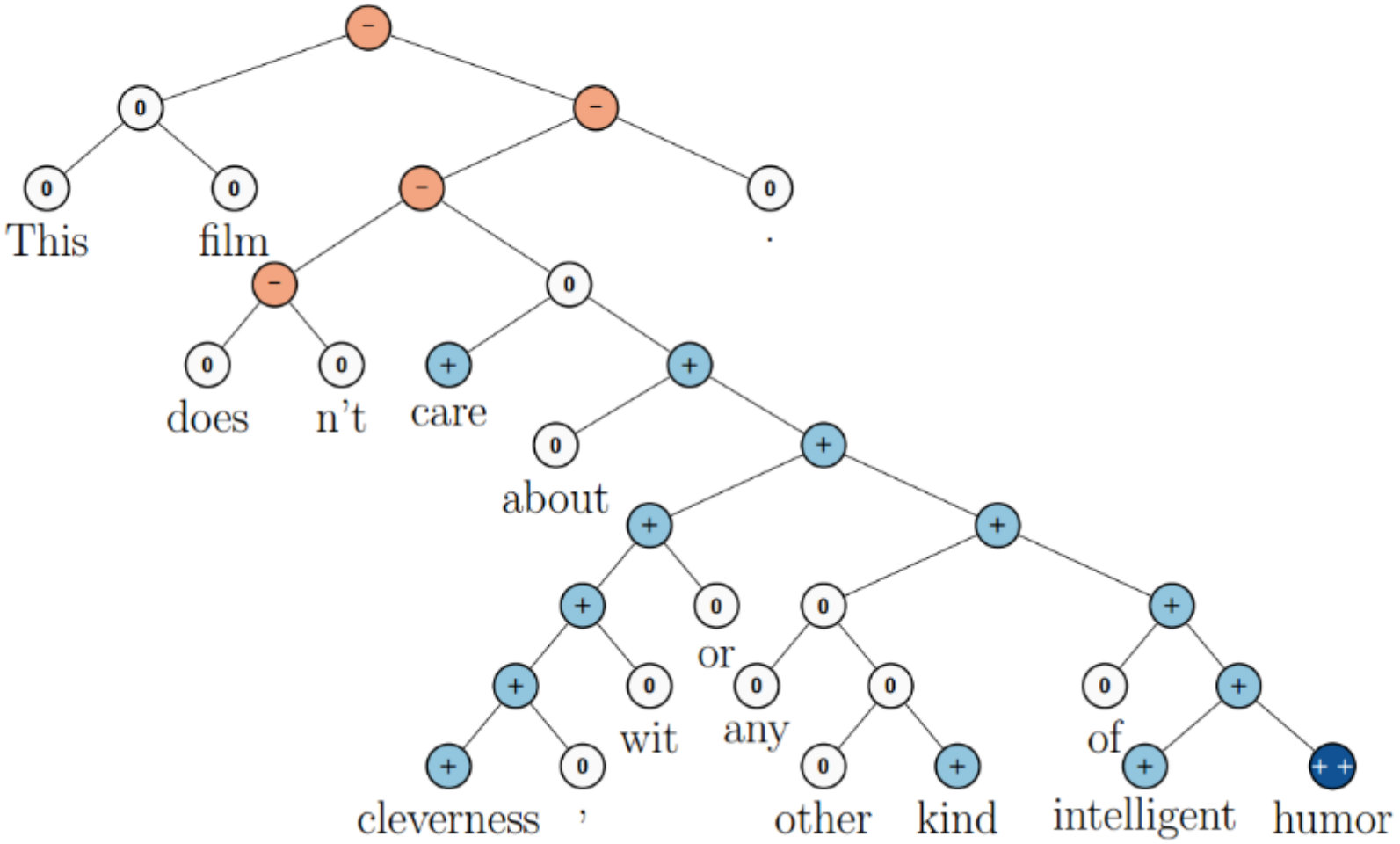
$$h_l(X) = (X \times W + b) \circ \sigma(X \times V + c)$$

- **Gated linear unit**: $X \circ \sigma(X)$
- **Output**: $Y = \text{softmax}(WH_L)$

Modeling trees with Recursive NN

- Input: $x_1, x_2, \ldots, x_n$
- A binary tree T can be represented as a unique set of triplets (i, k, j), s.t. $i < k < j$, $x_{i:j}$ is parent of $x_{i:k}, i_{k+1,j}$
- RecNN takes as an input a binary tree and returns as output a corresponding set of inside state vectors $\boldsymbol{s}_{\boldsymbol{i:j}}^{\boldsymbol{A}} \in \mathbb{R}^d$
- Each state vector $\boldsymbol{s}_{\boldsymbol{i:j}}^{\boldsymbol{A}}$ represents the corresponding tree node $q_{i:j}^{A}$ and encodes the entire structure rooted at that node

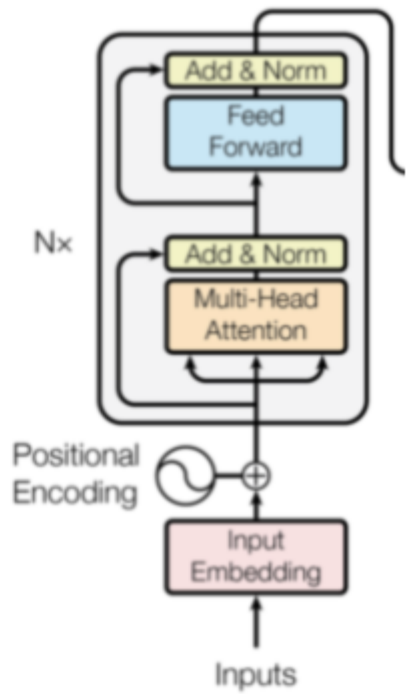
RecNN


This
film
does
n't
care
about
or
any
other
kind
of
intelligent
humor
cleverness
,
wit
.

The Transformer

An alternative architecture to RNN which allows of parallel and faster training

- Several layers of identical modules
- Each module consists of Multi-Head Attention and Feed Forward layers
- Input: embeddings. To get embeddings for numerical input, apply any dense layer
- Positional embeddings to make use of the order of the sequence

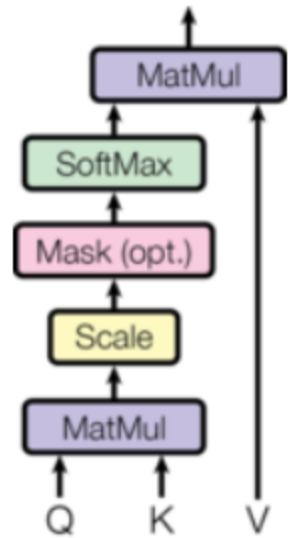


Scaled Dot-Product Attention

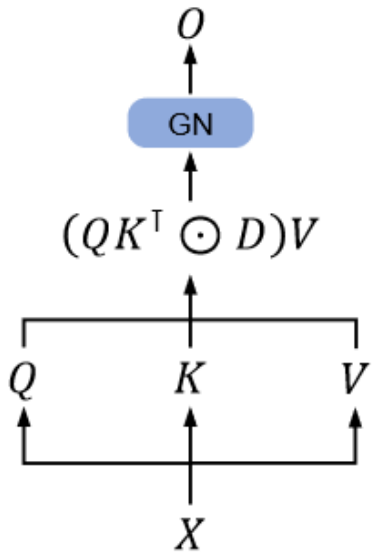
An attention function can be described as mapping a query and a set of key-value pairs to an output, where the query, keys, values, and output are all vectors:

$$Attention(Q, K, V) = \texttt{softmax}(\frac{QK^T}{\sqrt{d_k}})V,$$

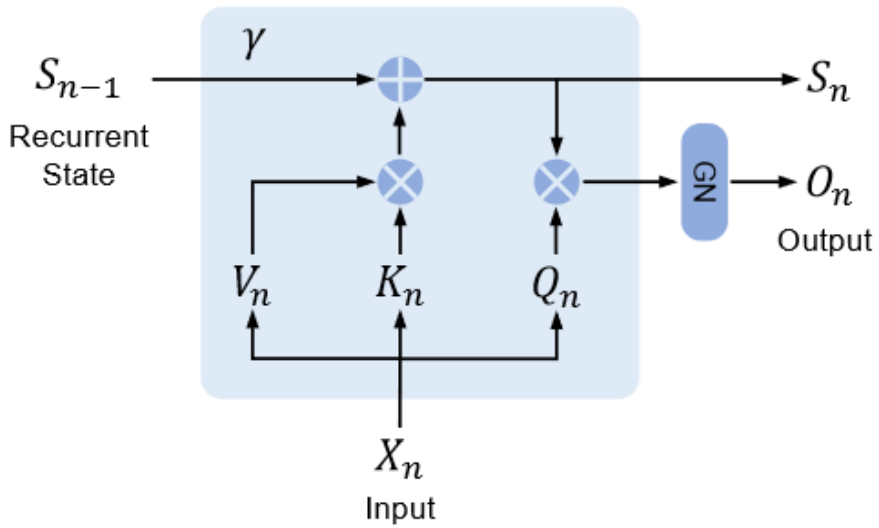
where the input consists of queries Q and keys K of dimension d_k and values V of dimension d_v



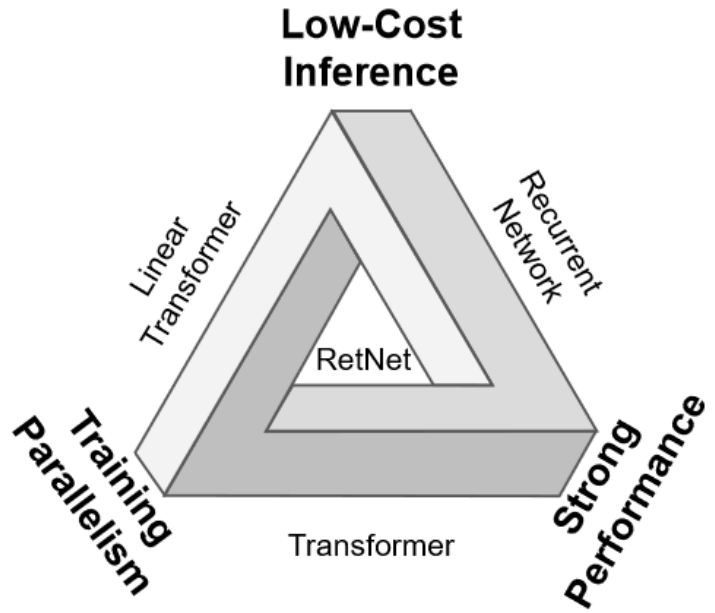
Modern RNN: RetNet



(a) Parallel representation.



(b) Recurrent representation.



In this work, we propose retentive networks (RetNet), achieving low-cost inference, efficient long-sequence modeling, Transformer-comparable performance, and parallel model training simultaneously. Specifically, we introduce a multi-scale retention mechanism to substitute multi-head attention, which has three computation paradigms, i.e., parallel, recurrent, and chunkwise recurrent representations. First, the parallel representation empowers training parallelism to utilize GPU devices fully. Second, the recurrent representation enables efficient $O(1)$ inference in terms of memory and computation. The deployment cost and latency can be significantly reduced. Moreover, the implementation is greatly simplified without key-value cache tricks. Third, the chunkwise recurrent representation can perform efficient long-sequence modeling. We parallelly encode each local block for computation speed while recurrently encoding the global blocks to save GPU memory.

Modern RNN: Mamba

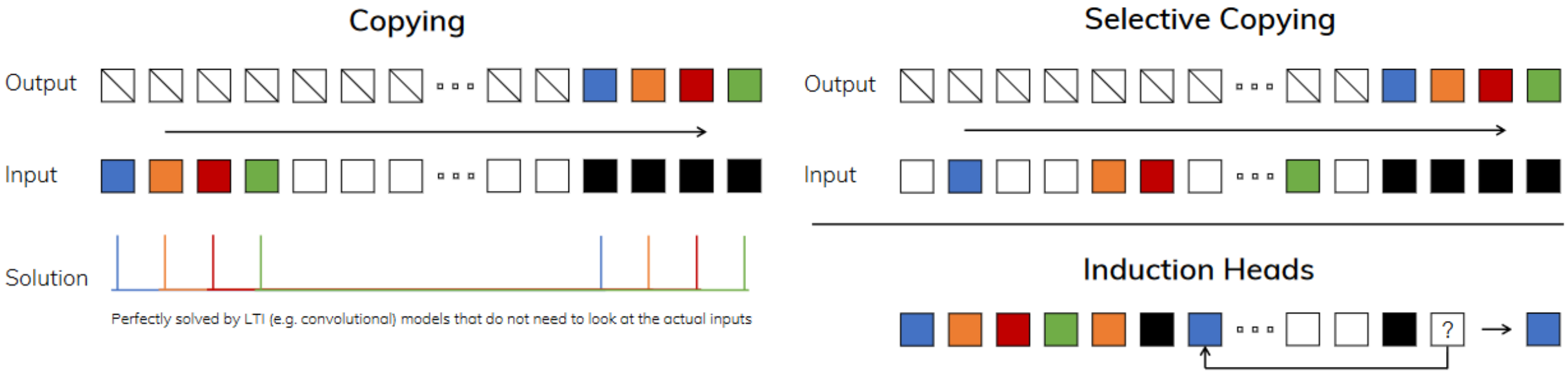
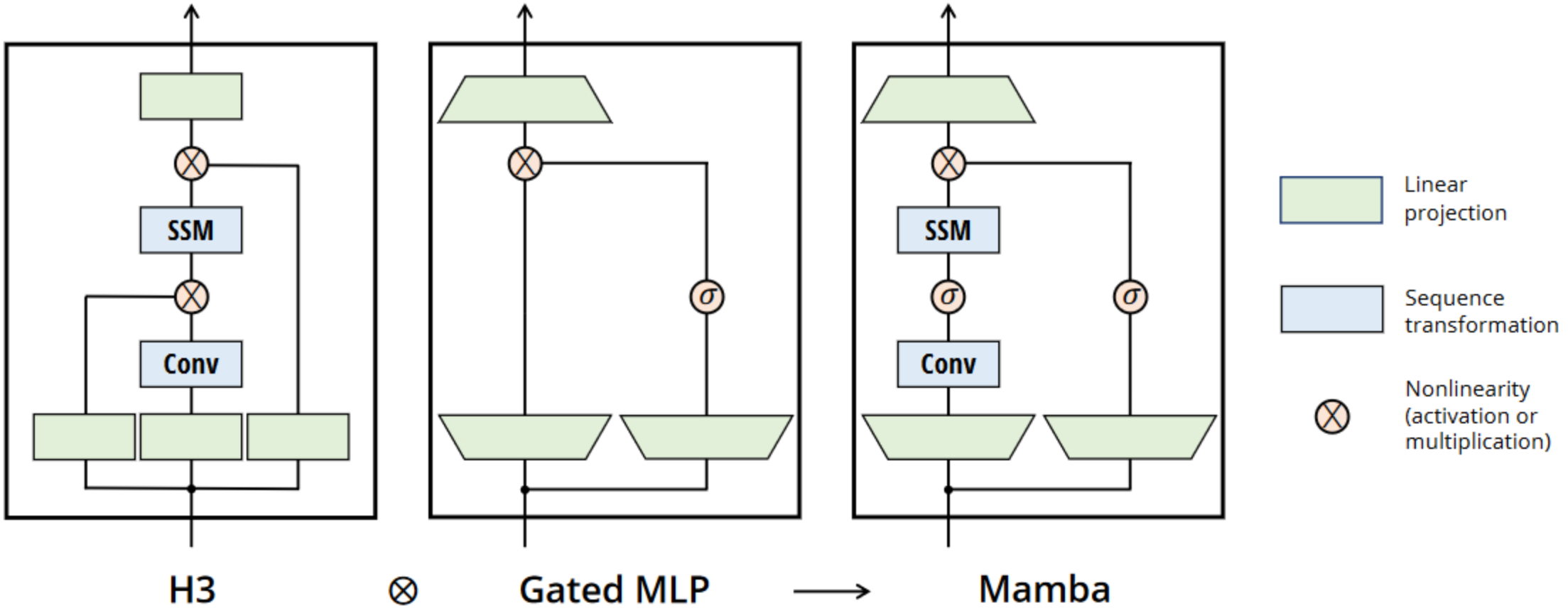
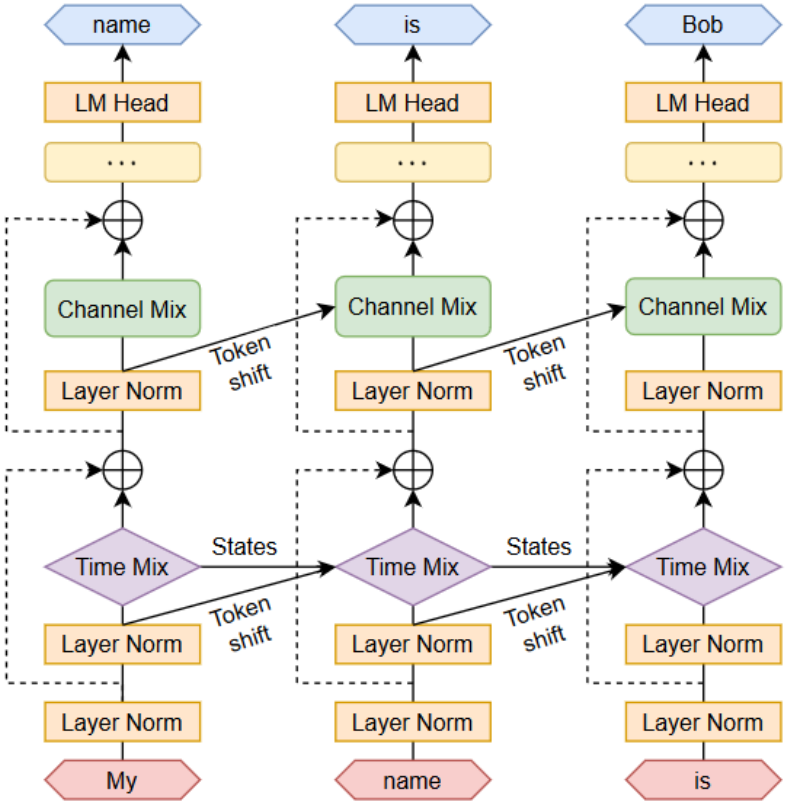


Figure 2: (*Left*) The standard version of the Copying task involves constant spacing between input and output elements and is easily solved by time-invariant models such as linear recurrences and global convolutions. (*Right Top*) The Selective Copying task has random spacing in between inputs and requires time-varying models that can *selectively* remember or ignore inputs depending on their content. (*Right Bottom*) The Induction Heads task is an example of associative recall that requires retrieving an answer based on context, a key ability for LLMs.

Modern RNN: Mamba



Modern RNN: RWKV



- R: The **Receptance** vector acts as the receiver of past information.
- W: The **Weight** signifies the positional weight decay vector, a trainable parameter within the model.
- K: The **Key** vector performs a role analogous to K in traditional attention mechanisms.
- V: The **Value** vector functions similarly to V in conventional attention processes.

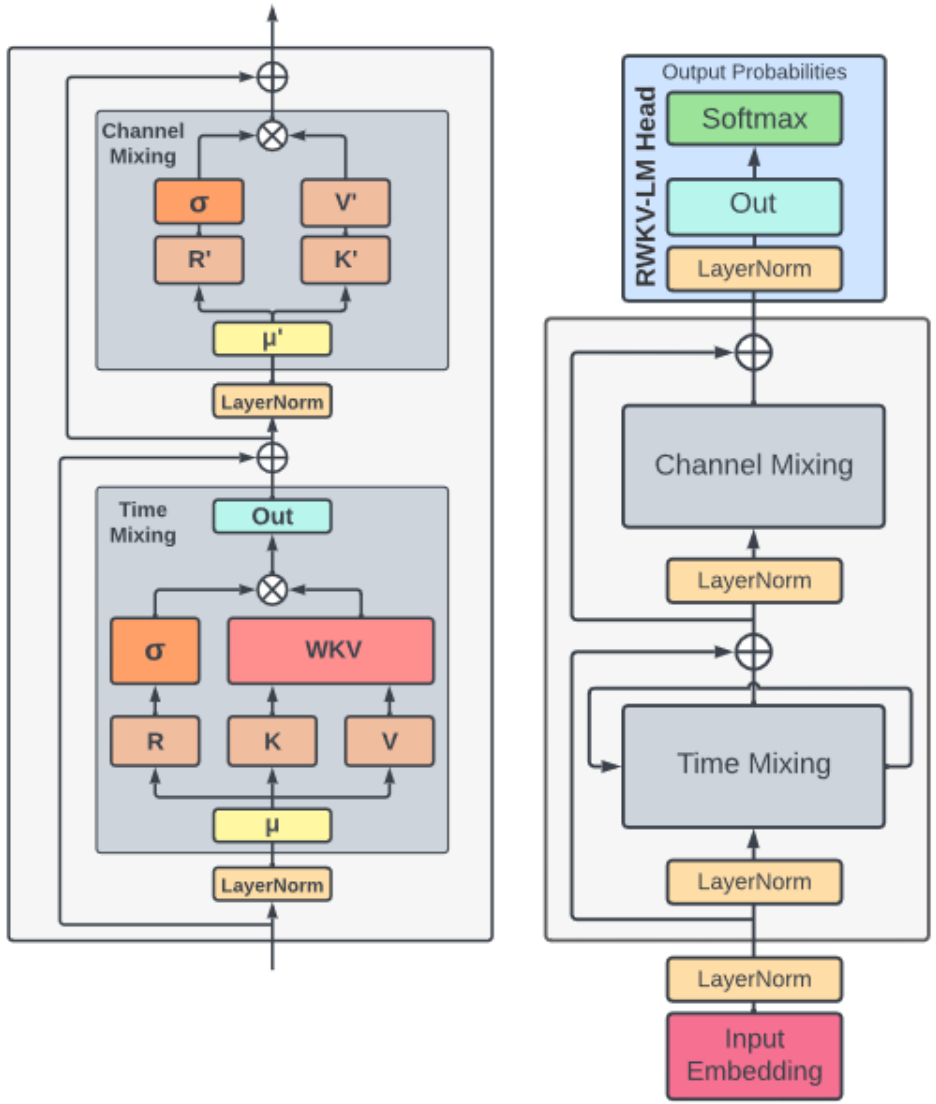


Figure 2: Elements within an RWKV block (left) and the complete RWKV residual block, equipped with a final head for language modeling (right).