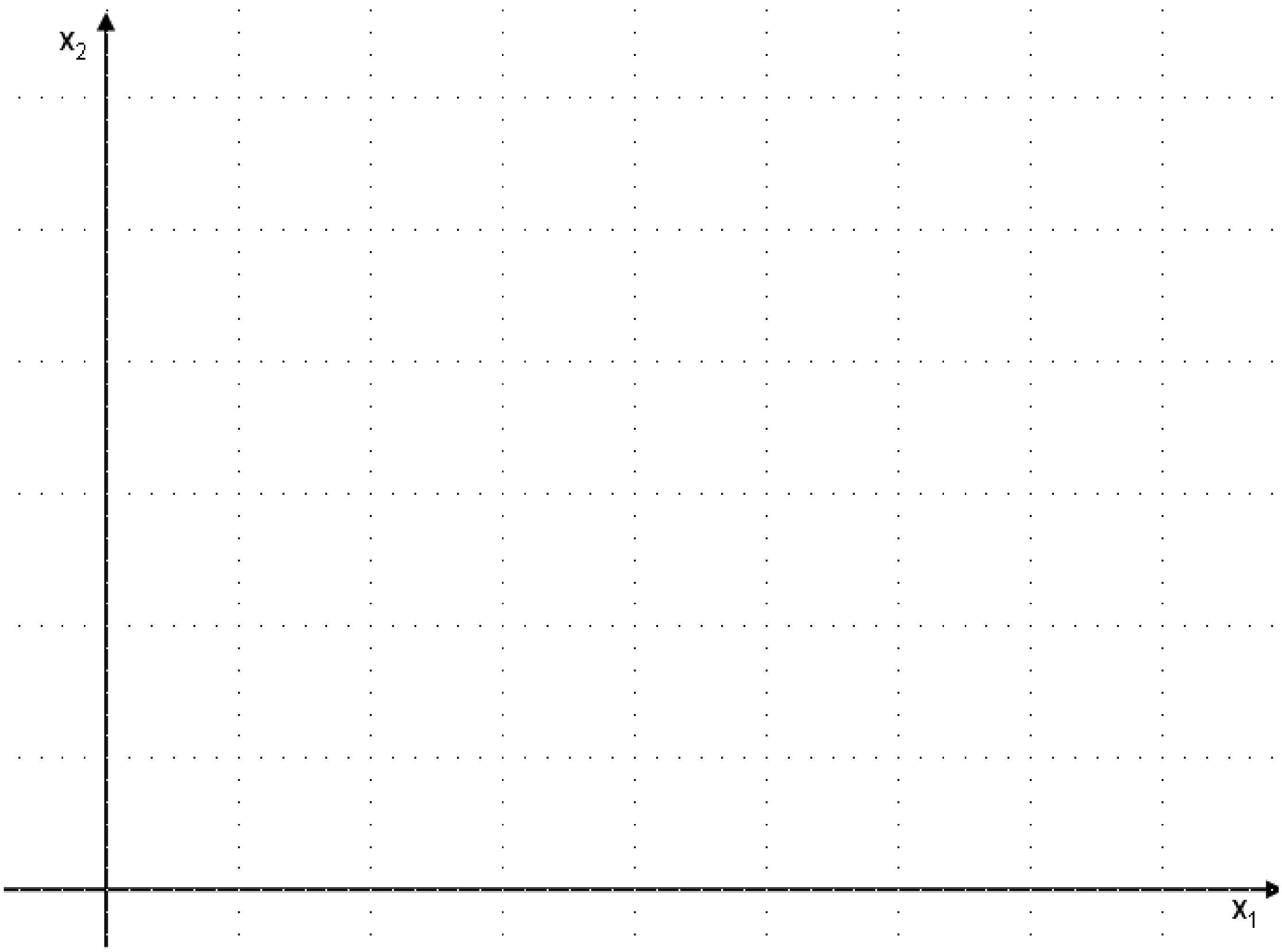
Goal Programming Example Problems & Solutions

Problem 1:

Given the following Goal Program in Goal Programming Form:

$$
\begin{aligned}
4x_1 + \; & 5x_2 && + \eta_1 - \rho_1 = 20 \\
2x_1 + \; & x_2 && + \eta_2 - \rho_2 = 4 \\
& x_2 && + \eta_3 - \rho_3 = 4 \\
x_1 + \; & x_2 && + \eta_4 - \rho_4 = 5
\end{aligned}
$$

Lexicographically minimize the following achievement vector:
$\mathbf{A} = \{(\rho_1), (\eta_2), (\eta_3 + \rho_3), (\eta_4), (\rho_2)\}$



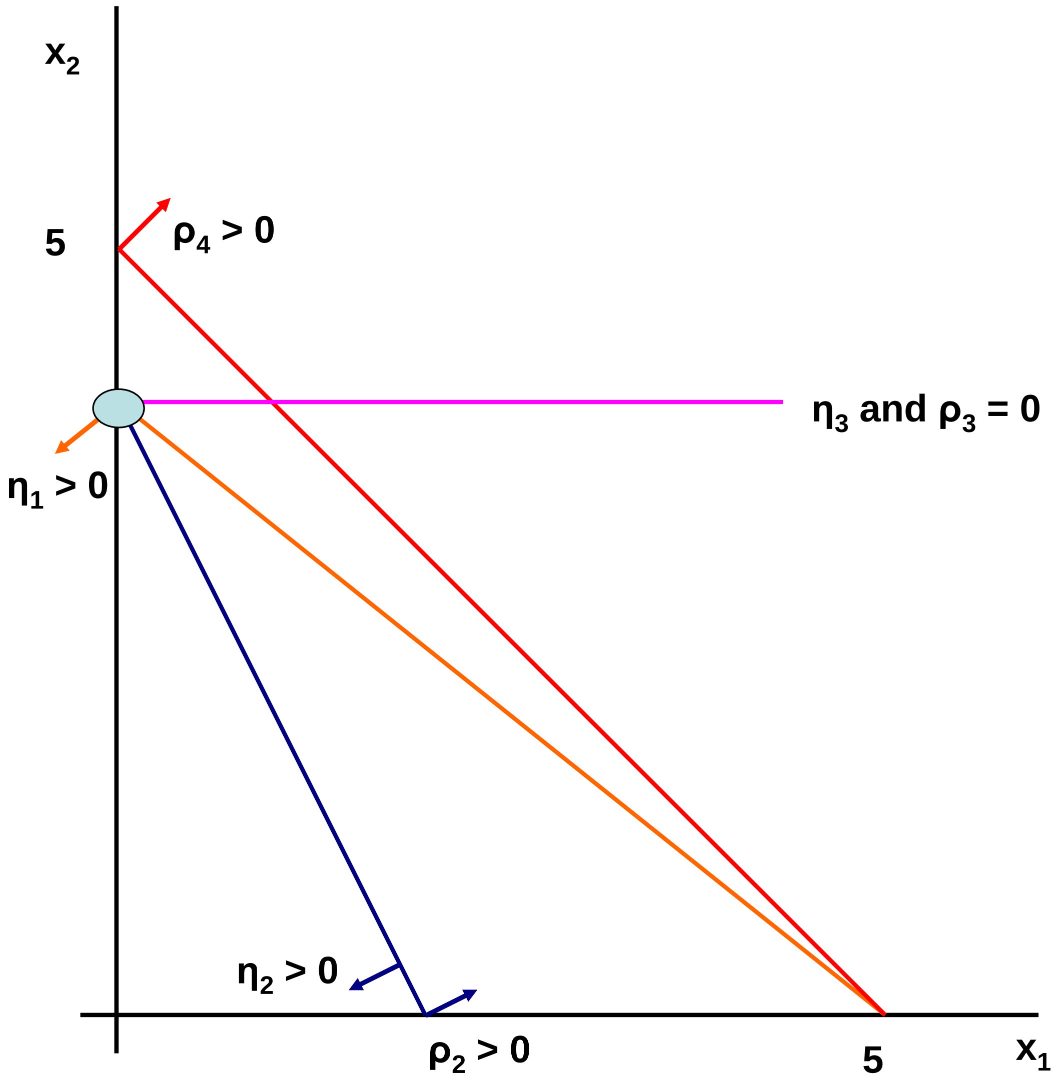
Below, identify the Optimal Solution $x^* = (x_1, x_2)$ and the Achievement Vector Values at the Optimal Solution.

Problem 1 Solution:

Given the following Goal Program in Goal Programming Form:

$$4x_1 + 5x_2 + \eta_1 - \rho_1 = 20$$
$$2x_1 + x_2 + \eta_2 - \rho_2 = 4$$
$$x_2 + \eta_3 - \rho_3 = 4$$
$$x_1 + x_2 + \eta_4 - \rho_4 = 5$$

Lexicographically minimize the following achievement vector:
A = $\{(\rho_1), (\eta_2), (\eta_3 + \rho_3), (\eta_4), (\rho_2)\}$



Below, identify the Optimal Solution x* = (x_1, x_2) and the Achievement Vector Values at the Optimal Solution.
x*: $x_1 = 0$, $x_2 = 4$

$$\mathbf{x_1 + x_2 + \eta_4 - \rho_4 = 5}$$
$$\mathbf{0 + 4 + \eta_4 - 0 = 5 \qquad \eta_4 = 1}$$

A = {0, 0, 0, (η_4), 0}
A = {0, 0, 0, 1, 0}

Problem 2:
Formulate the lexicographic linear goal programming model for the following problem. Energistics, Inc., produces a single type of moped (i.e., a small, gasoline-powered motorbike) called the "Spirit of America," or "Spirit" for short. It also imports an Italian moped, called the "Lirasaver," which it simply assembles and checks out. The Spirit sells for $650, ant the Lirasaver sells for $725. Demand is such that essentially all the mopeds that the firm could make or import could easily be sold. The cost, to Energistics, of the imported, unassembled Lirasaver is $185 each. Data with regard to production time, assembly time, test time, and labor costs are given in the accompanying table.

Moped	Hours per Unit		
	Manufacture	Assembly	Test
Spirit	20	5	3
Lirasaver	0	7	6
Labor Cost per Hour	$12 regular	$8 regular	$10 regular

After discussing the problem in more detail with Luigi Smith, the company president, we conclude that:
1. A profit of at least $3,000 per week is desired.
2. There are 120, 80, and 40 hours of regular time available per week for manufacture, assembly, and testing, respectively.
3. The firm believes that it would be politically wise to sell as many Spirits as possible.
4. The president wishes to minimize worker idle time without resorting, in any case, to overtime.

First formulate the baseline model and then, using your own assumptions as to the ranking of the various objectives and goals, convert to both a lexicographic linear goal programming model and a single-objective linear programming model.

Solution:
Variables:
x_1 = # of spirits x_2 = # of lirasavers

Profit:	$(650 - 12(20) - 8(5) - 10(3))x_1 + (725 - 185 - 8(7) - 10(6))x_2 \geq 3000$	(Objective)
Manufacturing:	$20x_1 \leq 120$	(Constraint)
Assembly:	$5x_1 + 7x_2 \leq 80$	(Constraint)
Test:	$3x_1 + 6x_2 \leq 40$	(Constraint)
Maximize spirit:	$x_1 \geq 8$ (or some other number)	(Goal 1)
Minimize idle time:	Goal 2 (But taken care of by constraints)	
Minimize OT:	Goal 3 (But taken care of by constraints)	

$x_1, x_2 \geq 0$

$(650 - 12(20) - 8(5) - 10(3))x_1 + (725 - 185 - 8(7) - 10(6))x_2 + \eta_1 - \rho_1 = 3000$
$20x_1 + \eta_2 - \rho_2 = 120$
$5x_1 + 7x_2 + \eta_3 - \rho_3 = 80$
$3x_1 + 6x_2 + \eta_4 - \rho_4 = 40$
$x_1 + \eta_5 - \rho_5 = 8$
$x_1, x_2 \geq 0$ and $\eta_i, \rho_i \geq 0$ for all i

$$\text{Lex Min} \begin{pmatrix} \rho_2 + \rho_3 + \rho_4 \\ \eta_1 \\ \eta_2 + \eta_3 + \eta_4 \\ \eta_5 \end{pmatrix}$$

Problem 3:

Highland Appliance must determine how many DVD players and Blue Ray players should be stocked. It costs Highland $300 to purchase a DVD player and $200 to purchase a Blue Ray player. A DVD player requires 3 sq yd of storage space, and a Blue Ray player requires 1 sq yd of storage space. The sale of a DVD players earns Highland a profit of $150, and the sale of a Blue Ray player earns Highland a profit of $100. Highland has set the following goals (listed in order of importance):

Goal 1: A maximum of $20,000 can be spent on purchasing DVD players and Blue Ray players.
Goal 2: Highland should earn at least $11,000 in profits from the sale of DVD players and Blue Ray players.
Goal 3: DVD players and Blue Ray players should not use up more than 200 sq yd of storage space.

Formulate a pre-emptive goal programming model that Highland could use to determine how many DVD players and Blue Ray players to order. How would you modify the pre-emptive goal formulation if Highland's goal were to have a profit of exactly $11,000?

Solution:

Variables:
x_1 = # of DVD players
x_2 = # of Blue Ray players

Goal 1: $300x_1 + 200x_2 \leq 20{,}000$
Goal 2: $150x_1 + 100x_2 \geq 11{,}000$
Goal 3: $3x_1 + x_2 \leq 200$
$x_1, x_2 \geq 0$

$300x_1 + 200x_2 + \eta_1 - \rho_1 = 20{,}000$
$150x_1 + 100x_2 + \eta_2 - \rho_2 = 11{,}000$
$3x_1 + x_2 + \eta_3 - \rho_3 = 200$
$x_1, x_2 \geq 0$ and $\eta_i, \rho_i \geq 0$ for all i

$$\text{Lex Min} \begin{pmatrix} \rho_1 \\ \eta_2 \\ \rho_3 \end{pmatrix}$$

Problem 4:

A company has two machines that can manufacture a product. Machine 1 makes 2 units per hour, while Machine 2 makes 3 per hour. The company has an order for 160 units. Each machine can be run for 40 hours on regular time; however, overtime is available. The company's goals, in order of importance, are to:

1. Avoid falling short of meeting the demand.
2. Avoid any overtime of Machine 2 beyond 10 hours.
3. Minimize the sum of overtime (assign differential weights according to the relative cost of overtime hours, assuming that the operating cost of the two machines are the same).
4. Avoid under-utilization of regular working hours (assign weights according to the productivity of the machines).

Solution:

Variables:
x_1 = # of hours for Machine 1
x_2 = # of hours for Machine 2

Demand:	$2x_1 + 3x_2 \geq 160$
Regular Time M1:	$x_1 \leq 40$
Regular Time M2:	$x_2 \leq 40$
M2 OT:	$x_2 \leq 50$

$x_1, x_2 \geq 0$

$2x_1 + 3x_2 + \eta_1 - \rho_1 = 160$
$x_1 + \eta_2 - \rho_2 = 40$
$x_2 + \eta_3 - \rho_3 = 40$
$x_2 + \eta_4 - \rho_4 = 50$
$x_1, x_2 \geq 0$ and $\eta_i, \rho_i \geq 0$ for all i

$$\text{Lex Min} \begin{pmatrix} \eta_1 \\ \rho_4 \\ 3\rho_2 + 2\rho_3 \\ 2\eta_2 + 3\eta_3 \end{pmatrix}$$