EXERCISES: CHAPTER 2

DISCUSSION

Chapter 2 of the text focuses on the basic assumptions of linear programming. In particular, various linear programming modeling techniques are developed.

ANSWERS TO EXERCISE SET

2.1 Let: x_1 = # of Comp386 produced per week

x_2 = # of Comp486 produced per week

maximize profit = $300x_1 + 450x_2$

subject to

$4x_1 + 6x_2 \leq 150$	(Assembly)
$3x_1 + 3.5x_2 \leq 70$	(Testing)
$x_1, x_2 \geq 0$	(Nonnegativity)

2.2 Let: x_1 = # of luxury cars produced

x_2 = # of mid-sized cars produced

x_3 = # of compact cars produced

maximize profit = $600x_1 + 460x_2 + 320x_3$

subject to

$\dfrac{18x_1 + 29x_2 + 38x_3}{x_1 + x_2 + x_3} \geq 30$ (or $-12x_1 - x_2 + 8x_3 \geq 0$)	(Average mpg)
$x_1 \leq 600{,}000$	(Demand for luxury cars)
$x_2 \leq 800{,}000$	(Demand for mid-sized cars)

$x_3 \leq 700{,}000$	(Demand for compact cars)
$x_1, x_2, x_3 \geq 0$	(Nonnegativity)

2.3 Let: x_1 = # of acres used to grow corn

x_2 = # of acres used to grow wheat

x_3 = # of acres used to grow soybeans

x_4 = # of acres used to grow oats

maximize profit = $(0.36)(110)x_1 + (0.90)(35)x_2 + (0.82)(32)x_3 + (0.98)(55)x_4$

subject to

$x_1 + x_2 + x_3 + x_4 = 500$	(Total acreage)
$x_3 \leq 120$	(Soybean acreage limit)
$110x_1 \geq 10{,}000$	(Bushels of corn requirement)
$x_2 \geq (x_3 + x_4)$	(Wheat restriction)
$x_1, x_2, x_3, x_4 \geq 0$	(Nonnegativity)

2.4 Let: A_i = # of units of Item A produced in month i (i = 1,...,6)

B_i = # of units of Item B produced in month i (i = 1,...,6)

IA_i = # of units of inventory of Item A carried over from month i to month i+1 (i = 1,...,5)

IB_i = # of units of inventory of Item B carried over from month i to month i+1 (i = 1,...,5)

minimize cost = $15(A_1 + A_2) + 16(A_3 + A_4) + 18(A_5 + A_6) + 8(B_1 + B_2 + B_3) + 10(B_4 + B_5 + B_6) + 2(IA_1 + IA_2 + IA_3 + IA_4 + IA_5 + IB_1 + IB_2 + IB_3 + IB_4 + IB_5)$

subject to

$A_1 - IA_1 = 200$	(Demand for A in Month 1)
$A_2 + IA_1 - IA_2 = 250$	(Demand for A in Month 2)
$A_3 + IA_2 - IA_3 = 400$	(Demand for A in Month 3)

$A_4 + IA_3 - IA_4 = 650$ (Demand for A in Month 4)
$A_5 + IA_4 - IA_5 = 700$ (Demand for A in Month 5)
$A_6 + IA_5 = 450$ (Demand for A in Month 6)
$B_1 - IB_1 = 160$ (Demand for B in Month 1)
$B_2 + IB_1 - IB_2 = 180$ (Demand for B in Month 2)
$B_3 + IB_2 - IB_3 = 370$ (Demand for B in Month 3)
$B_4 + IB_3 - IB_4 = 500$ (Demand for B in Month 4)
$B_5 + IB_4 - IB_5 = 420$ (Demand for B in Month 5)
$B_6 + IB_5 = 350$ (Demand for B in Month 6)
$A_i + B_i \le 800$ for $i = 1,\dots,6$ (Production limits)
$IA_i + IB_i \le 200$ for $1 = 1,\dots,5$ (Inventory limits)
$A_i, B_i, IA_i, IB_i \ge 0$ for all i (Nonnegativity)

2.5 Let: $x_{i,j}$ = gallons of crude oil type i (i = 1, 2, 3) used to produce gasoline of grade j (j = A, B, C)

maximize profit = sales - costs

$$= 1.39(x_{1,A} + x_{2,A} + x_{3,A}) + 1.24(x_{1,B} + x_{2,B} + x_{3,B}) + 1.18(x_{1,C} + x_{2,C} + x_{3,C}) - 1.10(x_{1,A} + x_{1,B} + x_{1,C}) - 0.84(x_{2,A} + x_{2,B} + x_{2,C}) - 0.90(x_{3,A} + x_{3,B} + x_{3,C})$$

subject to

$x_{1,A} + x_{1,B} + x_{1,C} \le 10{,}000$ (Availability of crude I)
$x_{2,A} + x_{2,B} + x_{2,C} \le 9{,}000$ (Availability of crude II)
$x_{3,A} + x_{3,B} + x_{3,C} \le 3{,}000$ (Availability of crude III)

$\dfrac{x_{1,A}}{x_{1,A} + x_{2,A} + x_{3,A}} \ge 0.5$ (Blending specification, Grade A)

$\dfrac{x_{2,A}}{x_{1,A} + x_{2,A} + x_{3,A}} \le 0.3$ (Blending specification, Grade A)

$\dfrac{x_{1,B}}{x_{1,B} + x_{2,B} + x_{3,B}} \ge 0.35$ (Blending specification, Grade B)

$\dfrac{x_{2,B}}{x_{1,B} + x_{2,B} + x_{3,B}} \le 0.45$ (Blending specification, Grade B)

$\dfrac{x_{3,C}}{x_{1,C} + x_{2,C} + x_{3,C}} \le 0.20$ (Blending specification, Grade C)

$x_{i,j} \ge 0$ for all i,j (Nonnegativity)

2.6 Let: w_j = amount invested in the one-year investment at the beginning of year j (j = 1 ,...,6)
x_j = amount invested in the three-year investment at the beginning of year j (j = 1, 2, 3, 4)
y_j = amount invested in the five-year investment at the beginning of year j (j = 1, 2)

maximize $1.051w_6 + 1.162x_4 + 1.285y_2$
subject to

$w_1 + x_1 + y_1 = 10{,}000$ (Initial investment, year 1)
$w_2 + x_2 + y_2 = 1.051w_1$ (Beginning of year 2)
$w_3 + x_3 = 1.051w_2$ (Beginning of year 3)
$w_4 + x_4 = 1.051w_3 + 1.162x_1$ (Beginning of year 4)
$w_5 = 1.051w_4 + 1.162x_2$ (Beginning of year 5)
$w_6 = 1.051w_5 + 1.162x_3 + 1.285y_1$ (Beginning of year 6)
$w_i, x_i, y_i \ge 0$ for all i (Nonnegativity)

2.7 Let: w_1 = # of widgets produced per week using method i (i = 1, 2, 3)
g_1 = # of gadgets produced per week using method i (i = 1, 2, 3, 4)

minimize cost = $1.35w_1 + 1.28w_2 + 1.47w_3 + 1.14g_1 + 1.19g_2 + 1.26g_3 + 1.16g_4$
subject to

$w_1 + w_2 + w_3 \ge 320$ (Weekly demand for widgets)
$g_1 + g_2 + g_3 \ge 250$ (Weekly demand for gadgets)
$0.25w_1 + 0.25w_3 + 0.26g_2 + 0.20g_3 \le 80$ (Production time, Center 1)

$0.13w_1 + 0.34w_2 + 0.18g_1 + 0.12g_4 \le 80$	(Production time, Center 2)
$0.15w_2 + 0.42w_3 + 0.22g_2 + 0.18g_4 \le 80$	(Production time, Center 3)
$0.20w_1 + 0.30g_3 + 0.20g_4 \le 80$	(Production time, Center 4)
$0.28w_2 + 0.32g_1 + 0.14g_2 \le 80$	(Production time, Center 5)
$w_i, g_i \ge 0$ for all i	(Nonnegativity)

2.8 Let: x_i = kilograms of ingredient i in one kilogram of feed (i = 1, 2, 3)

minimize cost = $0.55x_1 + 0.42x_2 + 0.38x_3$

subject to

$x_1 + x_2 + x_3 = 1$	(Components sum to one kilogram)
$25x_1 + 45x_2 + 32x_3 \ge 35$	(Protein lower limit)
$11x_1 + 10x_2 + 7x_3 \ge 8$	(Fat lower limit)
$11x_1 + 10x_2 + 7x_3 \le 10$	(Fat upper limit)
$235x_1 + 160x_2 + 190x_3 \ge 200$	(Vitamin s lower limit)
$12x_1 + 6x_2 + 10x_3 \ge 10$	(Mineral t lower limit)
$x_1, x_2, x_3 \ge 0$	(Nonnegativity)

2.9 Let: x_i = amount invested in Investment Opportunity i (i = 1,...,7)

maximize return = $0.09x_1 + 0.10x_2 + 0.08x_3 + 0.07x_4 + 0.07x_5 + 0.06x_6 + 0.11x_7$

subject to

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 40{,}000{,}000$	(Total amount invested)
$x_1 + x_4 + x_6 + x_7 \ge x_2 + x_3 + x_5$	(USA restriction)
$x_3 \le 0.20(x_1 + x_3 + x_4 + x_6 + x_7)$	(Canadian restriction)
$x_1 \le 2{,}000{,}000$	(Investment 1 upper limit)
$x_2 \le 12{,}000{,}000$	(Investment 2 upper limit)
$x_3 \le 8{,}000{,}000$	(Investment 3 upper limit)
$x_4 \le 6{,}000{,}000$	(Investment 4 upper limit)
$x_5 \le 10{,}000{,}000$	(Investment 5 upper limit)
$x_6 \le 4{,}000{,}000$	(Investment 6 upper limit)
$x_7 \le 9{,}000{,}000$	(Investment 7 upper limit)
$x_i \ge 0$ for all i	(Nonnegativity)

Chapter 2

2.10 Let: A = # of units of Casting A produced per week
B = # of units of Casting B produced per week
C = # of units of Casting C produced per week
D = # of units of Casting D produced per week

maximize profit = 18A + 15B + 13C + 14D

subject to

$3A + B + 2C + D \le 40(60)$	(Available minutes/week of Pouring)
$8A + 12B + 6C + 7D \le 80(60)$	(Available minutes/week of Cleaning)
$10A + 6B + 9C + 7D \le 80(60)$	(Available minutes/week of Grinding)
$A + B + C + D \le 20(60)$	(Available minutes/week of Inspection)
$3A + 5B + 3C + 2D \le 40(60)$	(Available minutes/week of Packing)
$A \ge 200$	(Lower limit for production of A)
$D \ge 300$	(Lower limit for production of D)
$A, B, C, D \ge 0$	(Nonnegativity)