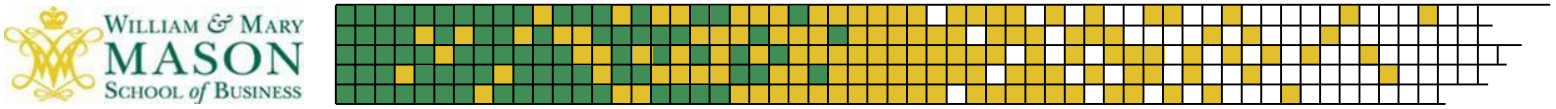


Sorting & Running Time

BUAD 5042

Topics

- Big-Oh running time
- Broad perspective:
  - Getting code to run faster
    - Running time and micro-optimization
  - Optimizing programs
- Sorting methods and running time
  - **Bubble sort**
  - **Selection sort**
  - **Insertion sort**
  - **Merge sort**
  - Heap sort
  - Quick sort

Big-Oh Running Time

- We are concerned with computing how long programs will take to run
  - … particularly for large problems
  - Interesting and important problems are large

Big-Oh Examples

- Compute the sum of an `n` element array

```
 9 n = 3
10 array = list(range(n))
11 array_sum = 0
12 for element in array:
13     array_sum += element
14 print('array_sum = ' + str(array_sum))
```

Big-Oh Examples

- How many computations are required to find the closest DC for each store?

```
 8 """
 9 The j-th element in the i-th sub list of dist represents the distance
10 from store i to distribution center j
11 """
12 dist = [[1,5,11,9],[3,7,14,21],[14,9,7,6],[23,15,9,17]]
13 answer = []
14
15 """ Find the closest DC for each store """
16 for store in dist:
17     dist_min = 99999999999999999
18     for j in range(len(store)):
19         if store[j] < dist_min:
20             dist_min = store[j]
21             dc_num = j
22     answer.append((dist_min,dc_num))
23 print('The minimum distances are as follow: ' + str(answer))
```

Big-Oh Examples

- How many computations are required for this matrix multiplication program?

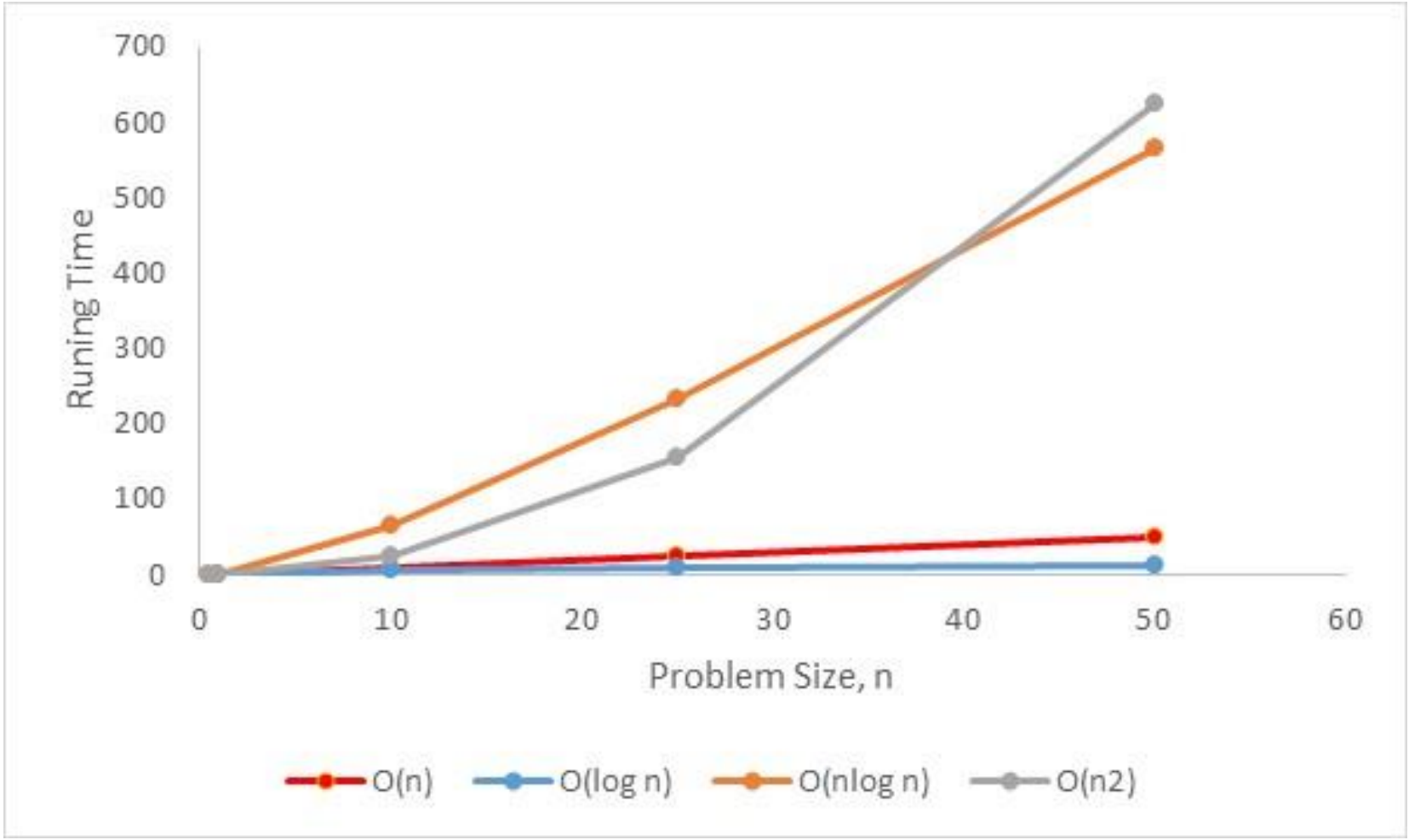
```
 8 A = [[2,5,9],[1,5,3],[7,1,4]]
 9 I = [[1,0,0],[0,1,0],[0,0,1]]
10 AI = [[0,0,0],[0,0,0],[0,0,0]]
11
12 for i in range(len(A)):
13     for k in range(len(AI)):
14         for j in range(len(A[i])):
15             AI[i][k] +=A[i][j] * I[j][k]
16 print('AI = ' + str(AI))
```

Big-Oh Examples

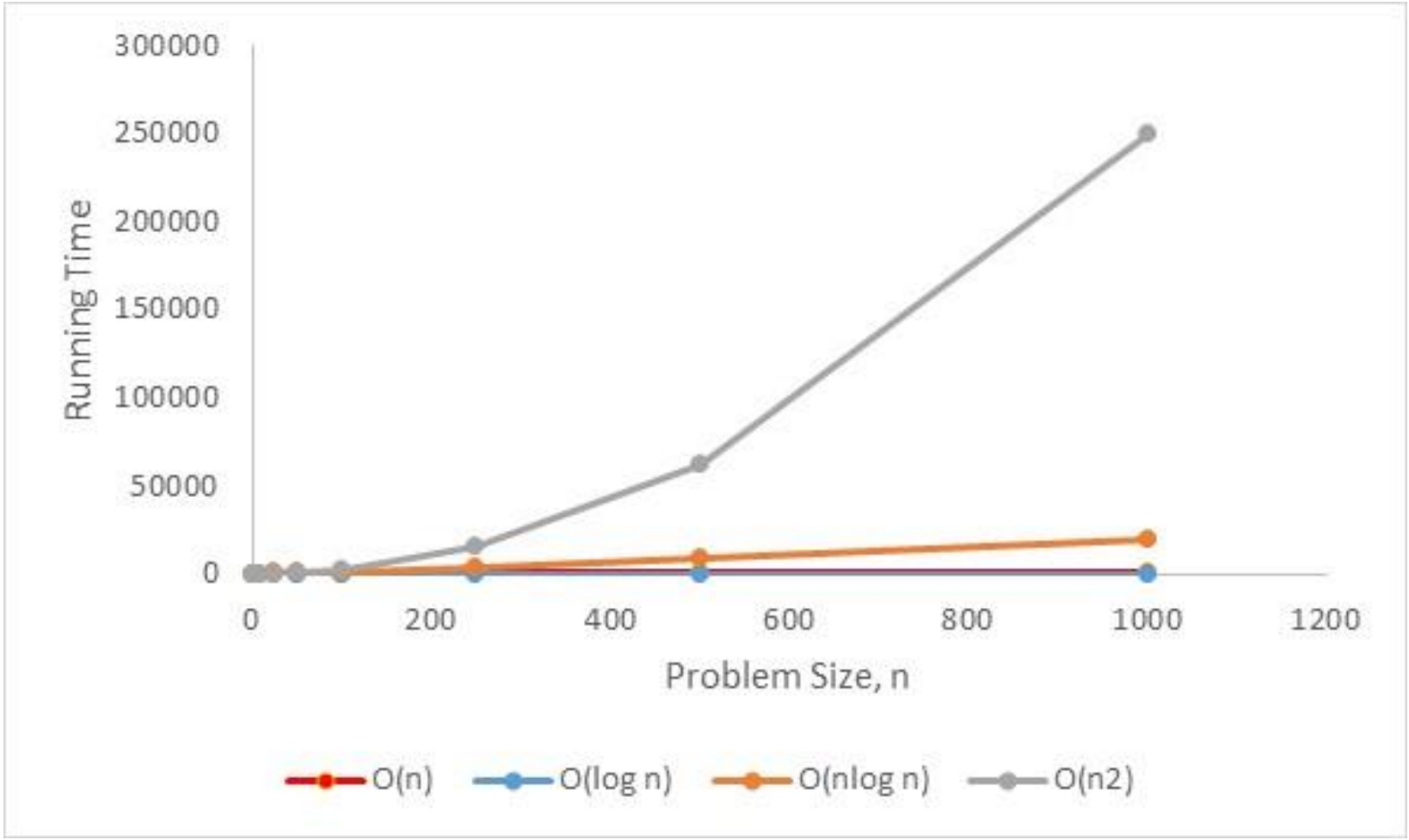
- How many calculations for distances between each pair of `n` locations?

```
7  import math
8
9  R = 6371.0 * 0.621371
10 def hav_dist(lat1, lon1, lat2, lon2):
11     """ latitude and longitude inputs are in degrees """
12
13     """ convert latitude and longitude to radians """
14     lat1 = lat1 * math.pi /180.0
15     lon1 = lon1 * math.pi /180.0
16     lat2 = lat2 * math.pi /180.0
17     lon2 = lon2 * math.pi /180.0
18
19     a = math.sin((lat2-lat1)/2)**2 + math.cos(lat1) * math.cos(lat2) * (math.sin((lon2-lon1)/2))**2
20     return R * 2 * math.atan2(math.sqrt(a),math.sqrt(1-a))
21
22 """ Store location data: [id, latitude, longitude] """
23 stores = [[0, 36.8176, -76.26684],[1, 36.8368239286, -76.3111960714],[2, 36.784765, -76.25588],[3, 36.80
24 R = 6371.0 * 0.621371
25
26 for i in range(len(stores)):
27     for j in range(i+1,len(stores)):
28         thisDist = hav_dist(stores[i][1],stores[i][2],stores[j][1],stores[j][2])
29         print('Distance Store ' + str(i) + ' to Store ' + str(j) + ': ' + str(thisDist))
```

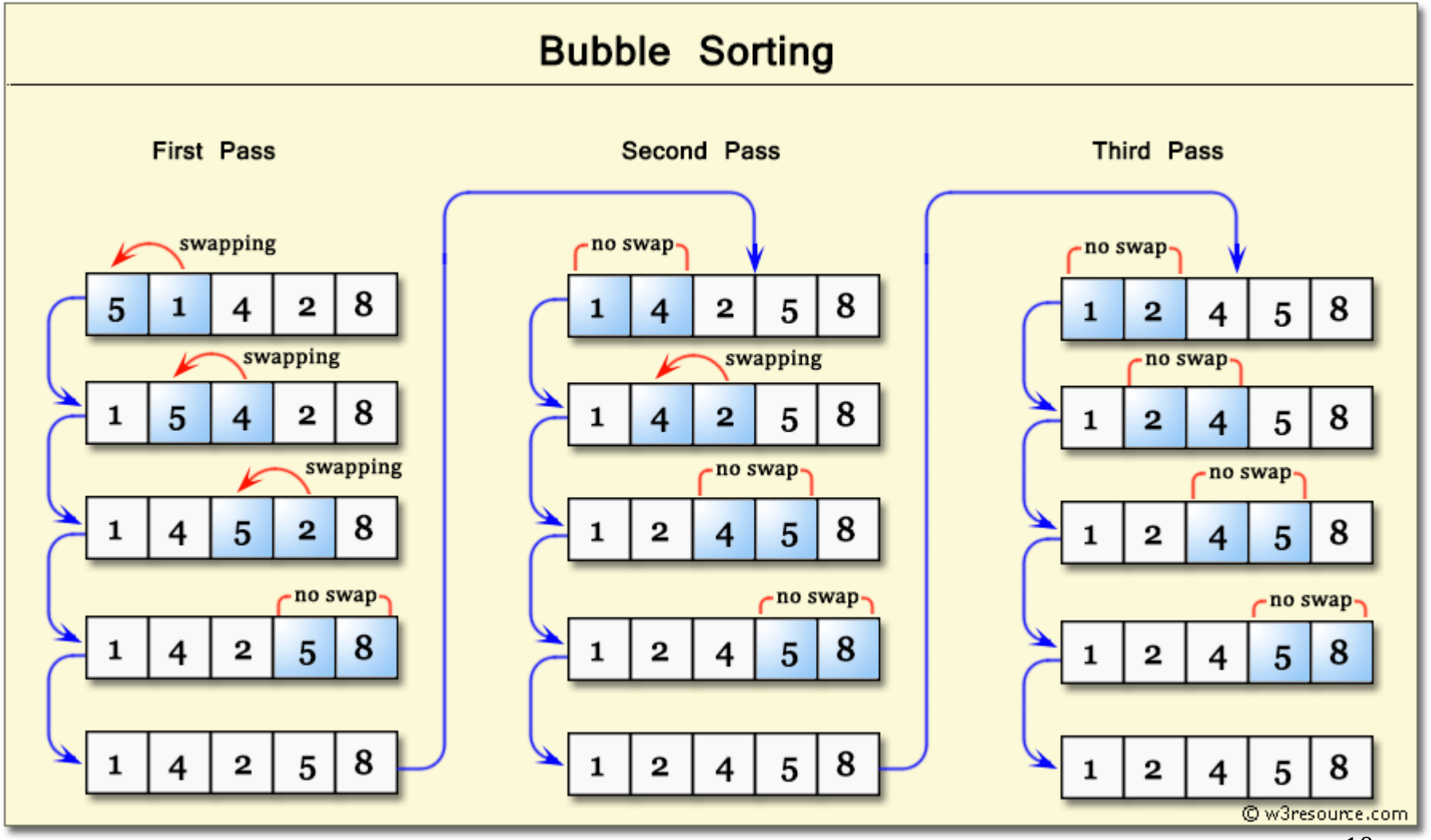
Why is Big-Oh Important?



Why is Big-Oh Important?



Bubble Sort


Bubble Sorting
First Pass
Second Pass
Third Pass
swapping
no swap
5 1 4 2 8
1 5 4 2 8
1 4 5 2 8
1 4 2 5 8
1 4 2 5 8
1 4 2 5 8
1 2 4 5 8
1 2 4 5 8
1 2 4 5 8
1 2 4 5 8
1 2 4 5 8
1 2 4 5 8
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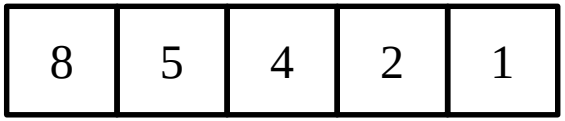
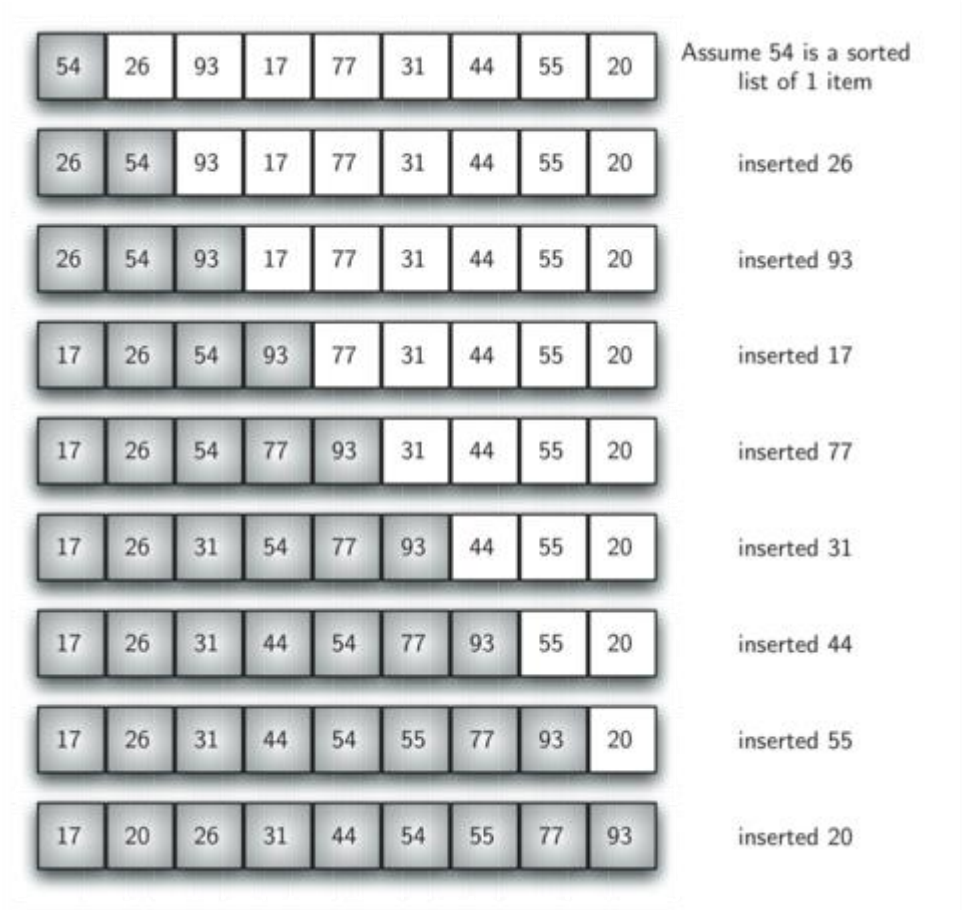
Bubble Sort

- Calculate running time = O(?)
- What is worst case?
  - Putting an list in descending order into ascending order



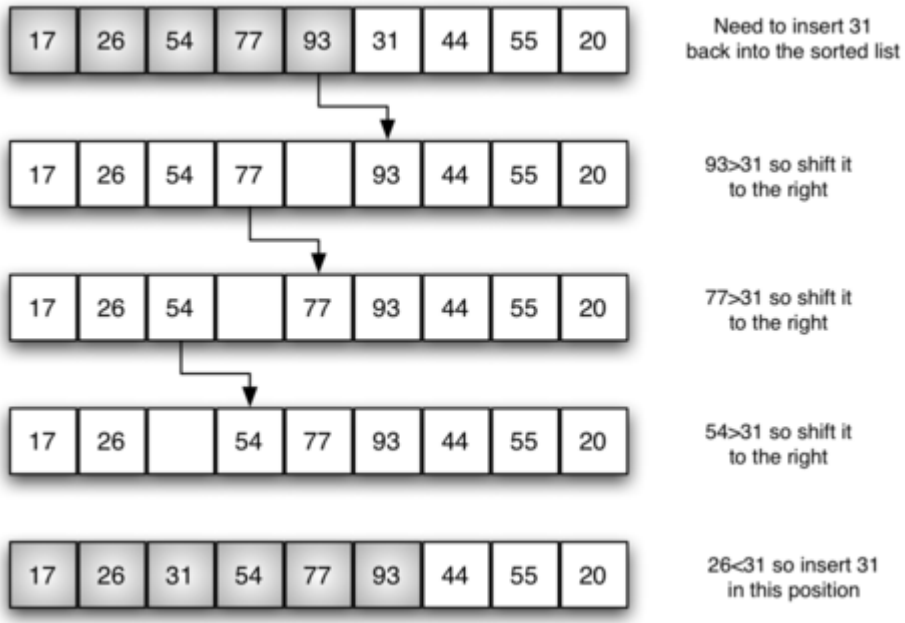
- How many passes are required to sort?
- How many comparisons/swaps are made in each pass?

Insertion Sort



http://interactivepython.org/XikcZ/courselib/static/pythonds/SortSearch/TheInsertionSort.html

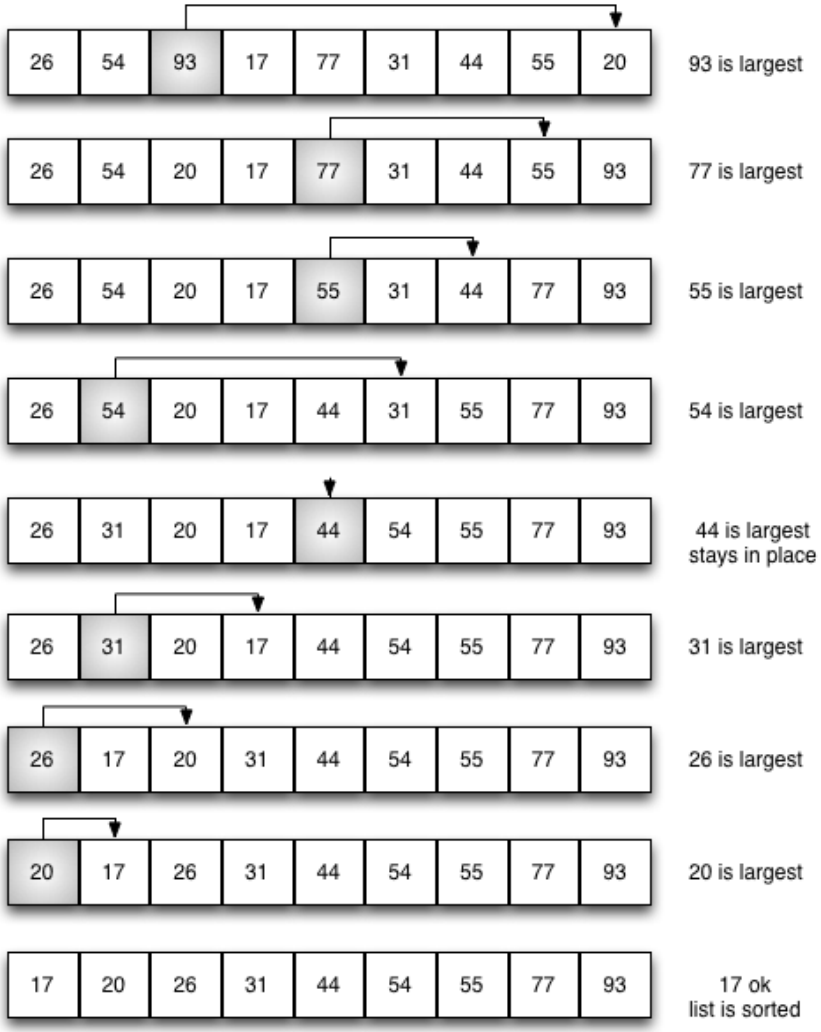
Insertion Sort



Insertion Sort

- Let's calculate the running time, $O(?)$
- How many insertions?
- Worst case number of comparisons/shifts for each insertion?

Selection Sort

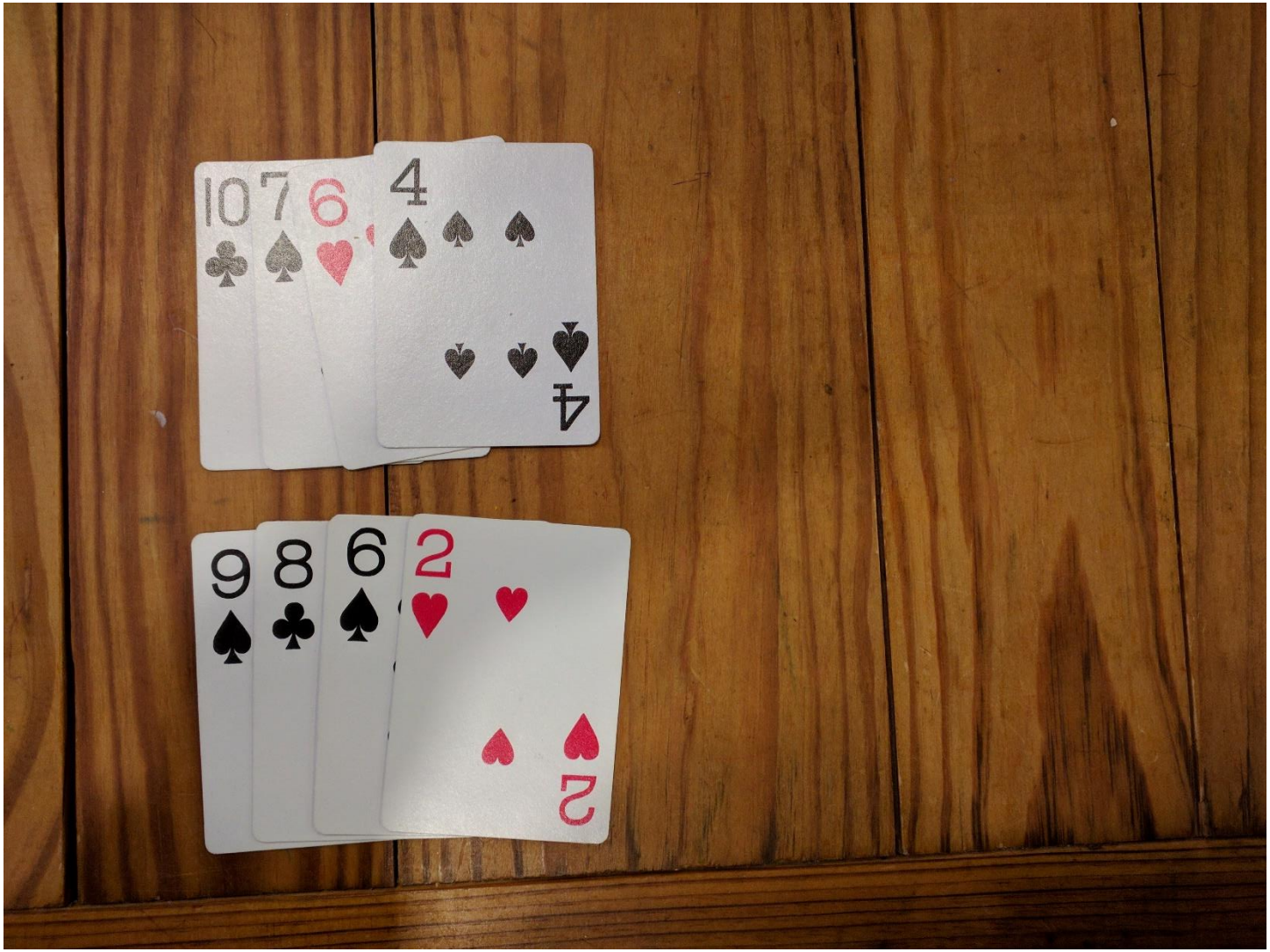


http://interactivepython.org/XikcZ/courselib/static/pythonds/SortSearch/TheSelectionSort.html

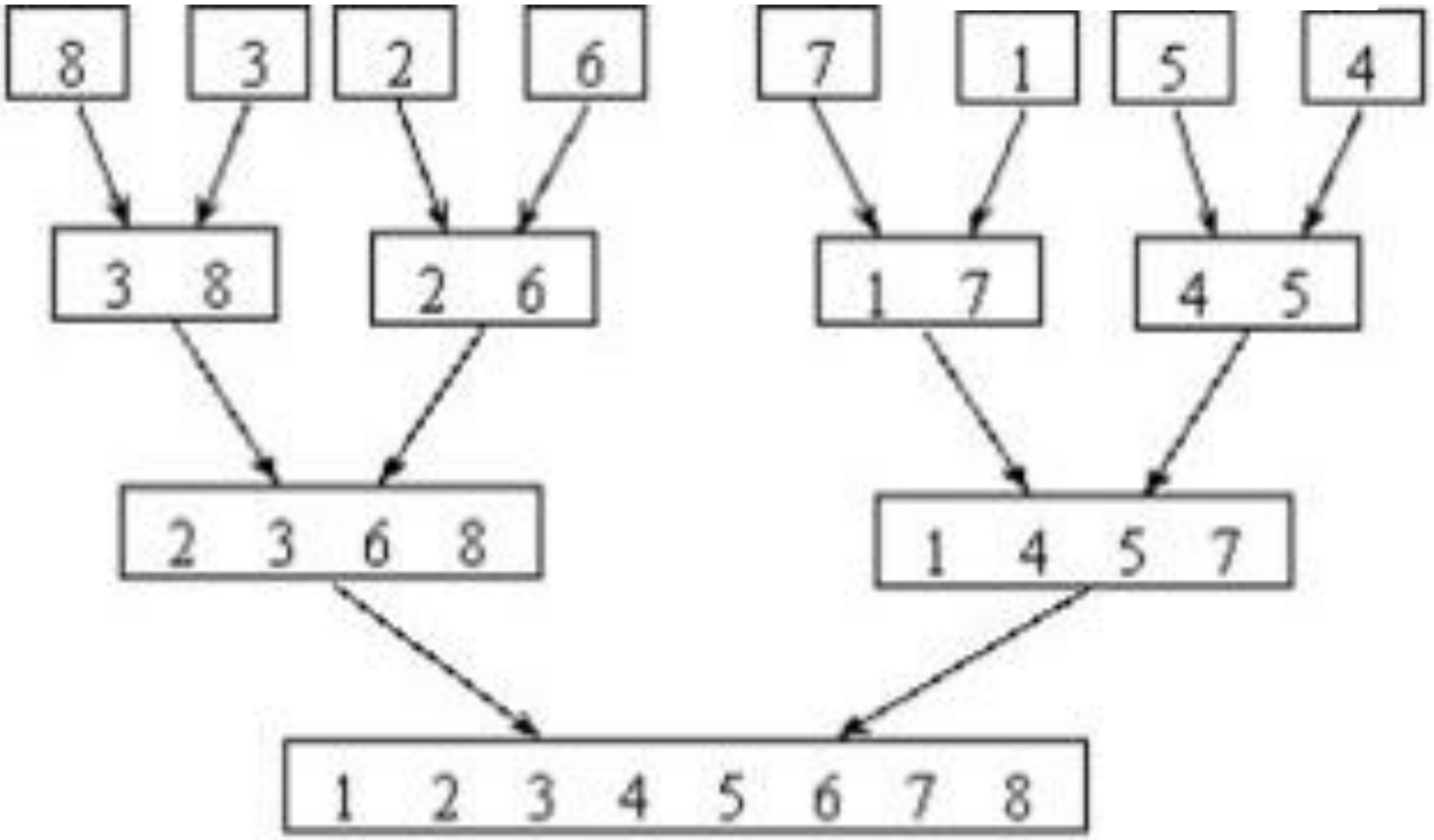
Selection Sort

- Let's calculate the running time, *O*(?)
- How many maximum computations?
  - How many comparisons for each one?
- How many memory changes to move the maximum to its position toward the right of the array?

Merge Sort

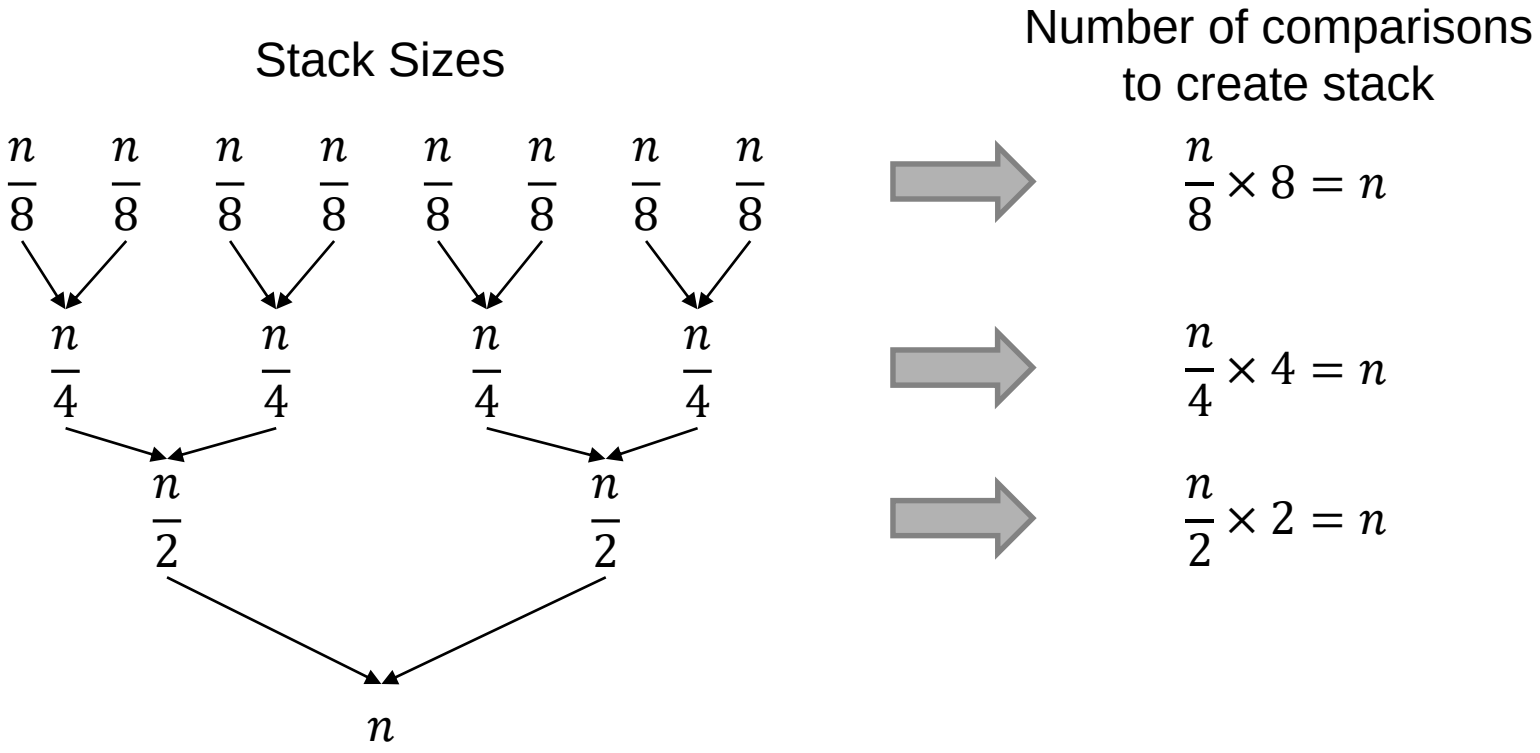


Merge Sort



http://lifexplorer.me/leetcode-sort-list/

Merge Sort



- Let's calculate the running time
  - How many comparisons in each step?
  - How many merge steps?

Heap Sort

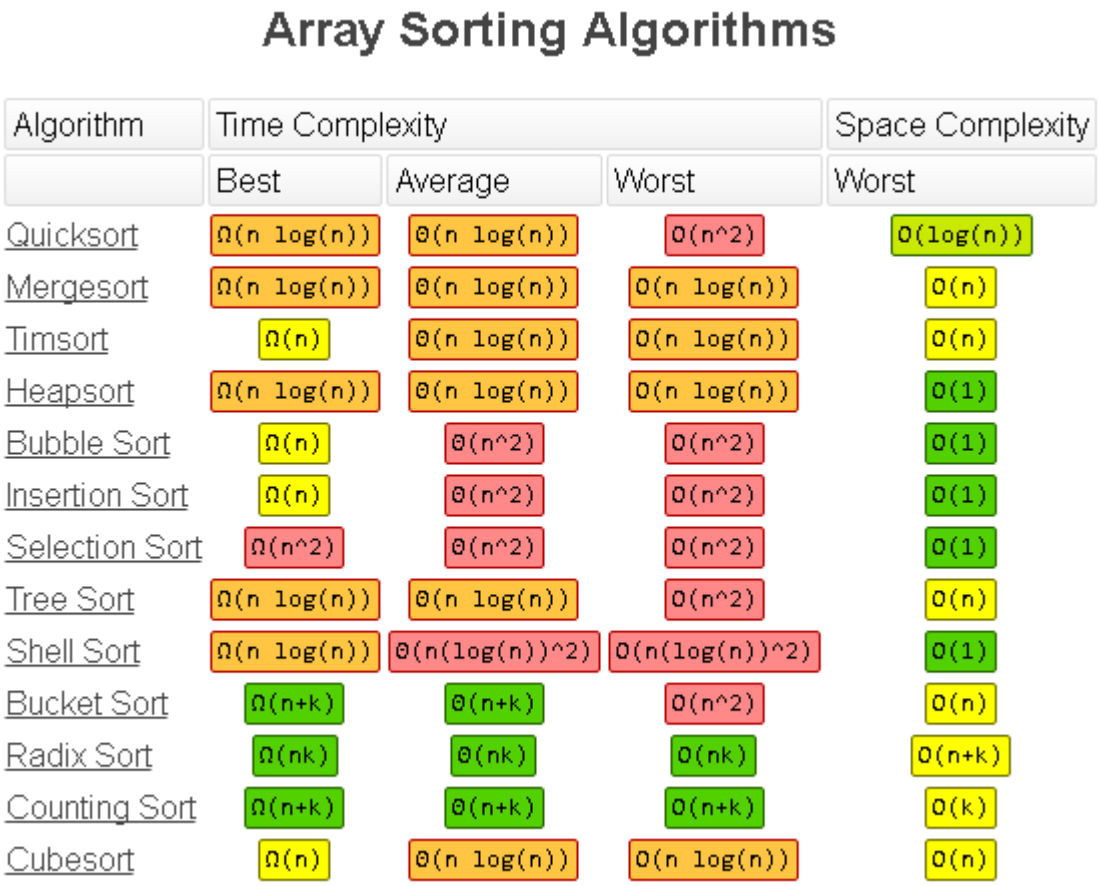
- “[…] improved selection sort: like that algorithm, it divides its input into a sorted and an unsorted region, and it iteratively shrinks the unsorted region by extracting the largest element and moving that to the sorted region.”
- https://en.wikipedia.org/wiki/Heapsort

Quicksort

- “[…] it can be about two or three times faster than its main competitors, merge sort and heapsort”
- https://en.wikipedia.org/wiki/Quicksort

Sorting Algorithm Running Times

- http://bigocheatsheet.com/



Array Sorting Algorithms

Algorithm	Time Complexity			Space Complexity
	Best	Average	Worst	Worst
Quicksort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(log(n))
Mergesort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(n)
Timsort	Ω(n)	Θ(n log(n))	O(n log(n))	O(n)
Heapsort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(1)
Bubble Sort	Ω(n)	Θ(n^2)	O(n^2)	O(1)
Insertion Sort	Ω(n)	Θ(n^2)	O(n^2)	O(1)
Selection Sort	Ω(n^2)	Θ(n^2)	O(n^2)	O(1)
Tree Sort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(n)
Shell Sort	Ω(n log(n))	Θ(n(log(n))^2)	O(n(log(n))^2)	O(1)
Bucket Sort	Ω(n+k)	Θ(n+k)	O(n^2)	O(n)
Radix Sort	Ω(nk)	Θ(nk)	O(nk)	O(n+k)
Counting Sort	Ω(n+k)	Θ(n+k)	O(n+k)	O(k)
Cubesort	Ω(n)	Θ(n log(n))	O(n log(n))	O(n)

Takeaways

- You should be computing the running times for your algorithms
- You should be aware of the running times of the built-in functions you use
  - Test them to see how fast they are

Python Sorting

- What are the alternative Python methods?
- Native Python `list.sort()`
  - timsort
  - Hybrid merge sort & insertion sort, takes advantage of existing order in list
- Numpy
  - Offers alternative sorting methods
  - https://docs.scipy.org/doc/numpy/reference/generated/numpy.sort.html
- Research is required to find the best/fastest methods

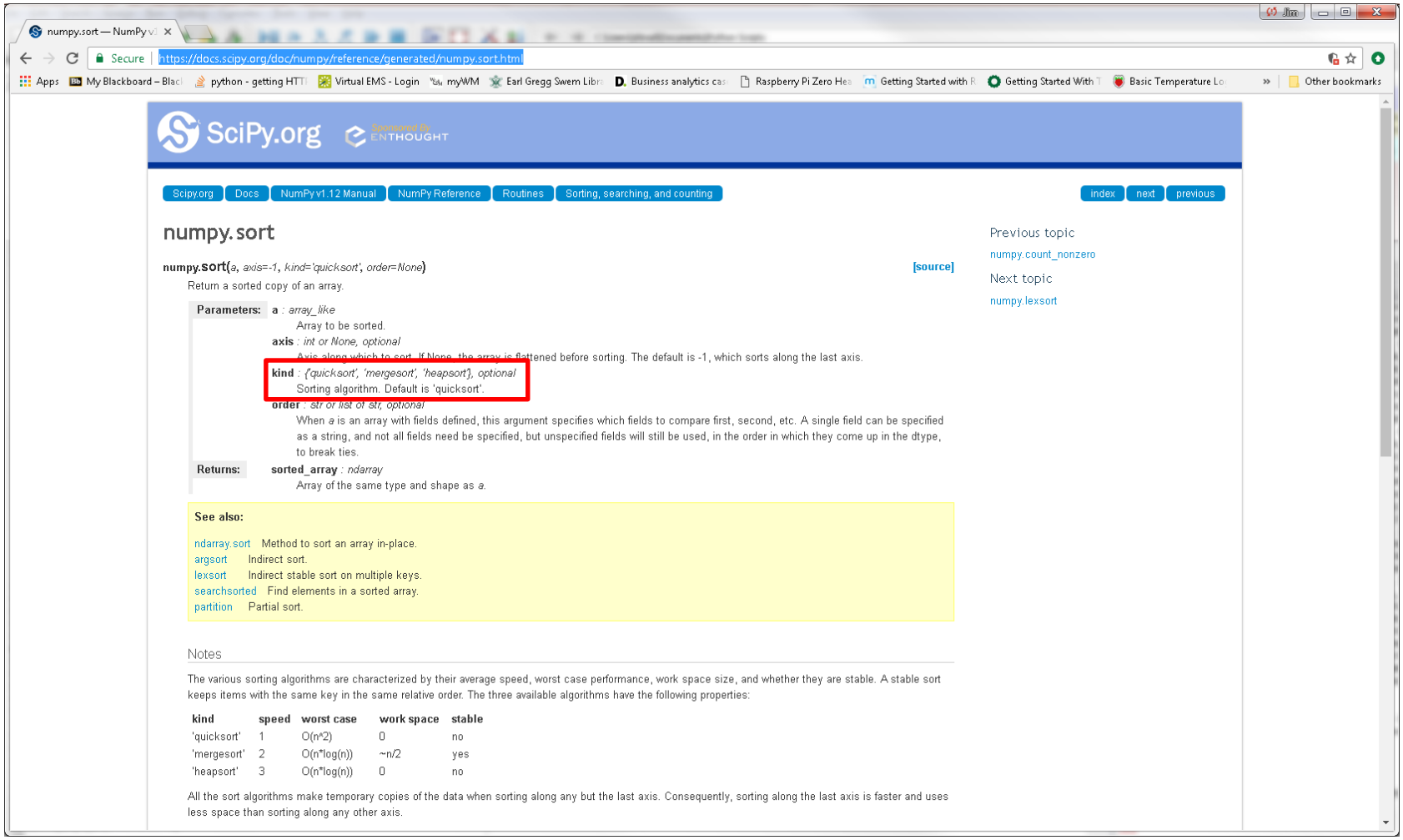
Optimizing Code

- Most of this course is…
  - Getting an algorithm that runs
- Once your algorithm is running…
  - Try to reduce running time
  - Use methods that are fastest
    - Choose among the alternatives
    - It sometime takes research
- This includes simplifying your code

Backup Slides

Python Sorting

- `numpy` offers alternatives



Python Sorting

- Reference for other Python sorting utilities
  - https://docs.python.org/3.6/howto/sorting.html
  - https://en.wikipedia.org/wiki/Timsort
  - http://stackoverflow.com/questions/10948920/what-algorithm-does-pythons-sorted-use

Python Sorting Test

Note, using Python 2.7 back in the day.

```
 7
 8 import time
 9 import random
10 import numpy as np
11
12 x = range(1000000)
13 random.shuffle(x)
14 print x
15 start_time = time.time()
16 x.sort()
17 print "Time to sort random x.sort():",time.time() - start_time
18
19 xtemp = x[500000]
20 x[500000] = x[500001]
21 x[500001] = xtemp
22 start_time = time.time()
23 x.sort()
24 print "Time to sort x.sort() when it is almost sorted:",time.time() - start_time
25
26 x.sort(reverse=True)
27 start_time = time.time()
28 x.sort()
29 print "Time to sort x.sort() when list is sorted in reverse:",time.time() - start_time
30
31 random.shuffle(x)
32 x_numpy = np.array(x)
33 start_time = time.time()
34 x_numpy.sort()
35 print "Time for numpy quicksort of random data:",time.time() - start_time
36
37 x_numpy = np.array(x)
38 start_time = time.time()
39 x_numpy.sort(kind='mergesort')
40 print "Time for numpy mergesort of random data:",time.time() - start_time
41
42 x_numpy = np.array(x)
43 start_time = time.time()
44 x_numpy.sort(kind='heapsort')
45 print "Time for numpy heapsort of random data:",time.time() - start_time
46
47 random.shuffle(x)
48 start_time = time.time()
49 y = sorted(x)
50 print "Time to sort sorted(x):",time.time() - start_time
51
52 random.shuffle(x)
53 start_time = time.time()
54 x.sort(key = lambda x:x)
55 print "Time to sort x.sort() with lambda key:",time.time() - start_time
56
57 z = [[random.random(),random.random()] for aa in range(1000000)]
58
59 z1 = [(w[1],w) for w in z]
60
61 start_time = time.time()
62 z1.sort()
```

Python Sorting Takeaways

- Lessons
  - Sorting methods take advantage of existing order in a list
  - Use `numpy` quicksort

```
Time to sort random list: 0.517000198364
Time to sort list that is almost sorted: 0.0140001773834
Time to sort list in reverse order: 0.0160000324249
Time for numpy quicksort of random list: 0.0599999427795
Time for numpy quicksort of almost in order list: 0.00600004196167
Time for numpy mergesort of random list: 0.0769999027252
Time for numpy mergesort of almost in order list: 0.010999917984
Time for numpy heapsort of random list: 0.0989999771118
Time for numpy heapsort of almost in order list: 0.0420000553131
Time to execute sorted(x): 0.742000102997
Time to execute sorted(x) on almost sorted list: 0.0379998683929
Time to sort x.sort() with lambda key: 1.6369998455
Time to sort with decorator sort: 1.882999897
Time to sort tuples using lambda function: 1.42999982834
```