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A fast algorithm for local minimum and maximum filters on rectangular and octagonal kernels

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Abstract

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A new algorithm is presented for local maximum and minimum filters requiring only 6 comparisons per pixel independent of kernel size. The algorithm is based on separability and a combination of block recursive series which are evaluated forwards and backwards.

Keywords. Mathematical morphology, recursive filters, local maximum and minimum, separability.

1. Introduction

In our institution we are developing methods to analyse high energy x-ray images obtained with a newly developed x-ray camera during radiotherapeutical treatment of cancer patients. These so called *portal images* are intended for the verification of the patient setup during their treatment (e.g., Van Herk et al. (1988)). Because of the inherent poor image quality (low contrast and poor sharpness), we implemented a number of image enhancement algorithms such as least squares inverse filtering (Meertens et al. (1988)). More recently, work was started on the fully automatic analysis of portal images. Bijhold et al. (1991a) describe an algorithm for automatic portal edge detection. Gilhuijs et al. (1991) and Van Herk et al. (1990) describe a method for automatic image matching for the purpose of on-line patient setup error evaluation. The latter method is a fully automatic implementation of interactive procedures for portal image evaluation as described by Bijhold et al. (1991b) and Meertens et al. (1990).

Following the widespread use of mathematical morphology (e.g. Haralick et al. (1987), Serra (1982) and Sternberg (1986)), we applied morphological filters for some of our image processing problems which were difficult to solve using linear filters. Gilhuijs et al. (1991) indicate that grey tone morphological filters (e.g., local minimum and maximum) can be efficient for extracting anatomical details (such as bones) from our type of (very low contrast) x-ray images. In general, however,

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the computation time of morphological filters for large kernel sizes becomes prohibitively long for on-line applications.

The aim of this study was to find fast algorithms for local minimum and maximum filtering. For this purpose, speed improvements obtained by utilizing separability and recursive algorithms have been investigated.

2. Separable morphological operations

The following notations are used: f, g and h represent discrete images or filter kernels (structuring elements), x and y are image indices and i and j kernel indices. The image indices run from 1 to N, where N is the image size. The kernel indices run from $-(k-1)/2$ to $(k-1)/2$, where k is the kernel size (assumed odd for maintaining symmetry). Edge effects are taken into account in the following way: image values outside the admissible index range, e.g. f_0, f_{-1} or f_{N+1}, are taken as $-\infty$ for local maximum filters and as $+\infty$ for local minimum filters.

Serra (1982) defines a grey tone morphological dilation (Minkowski addition) with structuring element g on image f as:

$$\max_{i,j} [f_{x-i,y-j} + g_{i,j}] \tag{1}$$

which we denote as $(g \oplus f)$. We limit ourselves in this paper to dilations. The relations for erosions (Minkowski subtraction) result, when using the minimum instead of the maximum and negating the structuring element.

Separation of (1) in row and column operations is possible when the structuring element g can be written as a sum of x and y terms.

Well known separable morphological filters are the streetblock distance transform as described, e.g., by Borgefors (1986), Gonzales and Wintz (1977) and Sternberg (1986). Also local minimum and maximum filters for a rectangular kernel fall in this category (e.g., Groen et al. (1988)). The advantage of separation for a kernel of $k*k$ pixels lies in the reduction of the number of operations from k^2 operations per pixel for direct evaluation to $2k$ operations when separated (k for the rows and k for the columns). A different approach for fast filtering with large kernel size is repeated application of a small, e.g. 3×3, kernel size filter. For a $k*k$ kernel (k odd) the 3×3 filter must be applied $(k-1)/2$ times. Such an approach can be implemented with $3(k-1)$ operations when the 3×3 filter is separable.

3. Recursive morphological filters

After separation, the remaining one-dimensional filter can sometimes be evaluated recursively. A good example is the distance transform. In one dimension the basic recursive series for a distance transform is:

$$g_x = \max[g_{x-1} + a, f_x] \quad \text{for } x = 1, \dots, M \tag{2}$$

where a defines the distance scale ($a<0$). The structuring element that is formed in this way, has the shape of a single saw-tooth, resulting in a one-sided distance transform. To obtain a full distance transform, which has a triangular structuring element, the same recursive operation must be performed backwards through the result of equation (2), i.e.,

$$h_x = \max[h_{x+1} + a, g_x] \quad \text{for } x = M, \dots, 1. \tag{3}$$

This operation also makes the filter symmetrical. Borgefors (1986) gives an extensive description of recursive distance transforms.

It is not possible to obtain a flat structuring element with this algorithm, as this requires the value of a to be 0. In that case the size of the kernel is not restricted. For the distance transform this means that every image pixel is eventually set to the maximum of the *whole* image.

We describe a modification of the previous recursive algorithm so that local minimum and maximum on one-dimensional images can be found with only three operations per pixel, independent of the structuring element size. This algorithm can be utilized to create local optimum filters on rectangular and octagonal kernels in arbitrary dimensions.

4. The new algorithm

With linear filters subtraction can be used to limit the range of a recursive series, e.g., to obtain high speed uniform filters (e.g., Groen et al. (1988)). Because the uniform filter is also separable, it can be computed with only 4 operations per pixel. Unfortunately this technique is not applicable to morphological filters, as there is no reverse operation of taking a maximum. Once the output variable has been replaced by the maximum of that variable and an input value, there is no simple and fast way to undo that operation. We therefore had to find an alternative way to limit the propagation of the recursive morphological filter.

We solved the problem as follows. The input array is divided into sub-arrays (groups) of size k, with k the structuring size. If necessary, the array is extended to a multiple of k by padding the array on the right side with values $-\infty$. The recursive procedure (2) with $a=0$ is applied to every sub-array. In the resulting sub-arrays, stored in array g, the ith pixel of each group thus contains the maximum of the first i pixels of the same group of input array f. The same recursive operation is performed again on the original input array, but now backwards. The latter result is stored in array h. Here the $(k+1-i)$th pixel of each group contains the maximum of the last i pixels of the same group of the input array. In Figure 1 these two arrays are depicted for a structuring element of $k=5$. From this figure it can be seen that it is possible to obtain the local maximum around every array element by combining the array g shifted left by $k/2$ and the array h shifted right by $k/2$. Some examples of this operation are shown by the arrows in the figure.

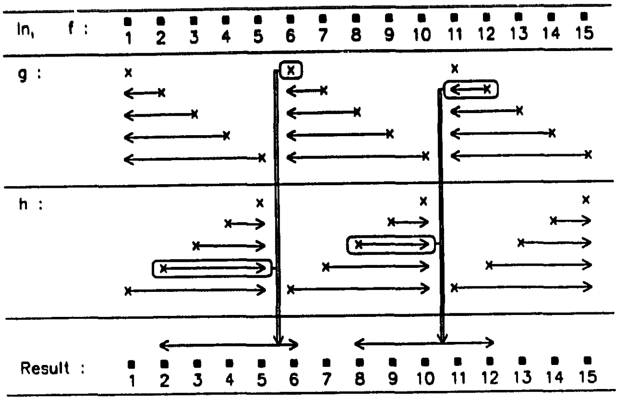

Figure 1. Recursive method for performing local maximum and minimum filters. In this example the kernel size, k, is 5. The arrays g and h are temporary results obtained with a recursive morphological filter. A symbol '×⟶' denotes that the value stored in location '×' is the maximum of all values covered by that arrow. The figure demonstrates that the end result is obtained by combining the arrays g and h shifted left and right, respectively, by half the kernel size.

The recursive procedures (4a) and (4b) and the operation (4c) describe the complete algorithm:

$$g_x = \begin{cases} f_x & \text{for } x=1,k+1,2k+1,\ldots \\ \max[g_{x-1},f_x] & \text{for } x=2,\ldots,k;k+2,\ldots,2k; \\ & \qquad 2k+2,\ldots,3k;\ldots \end{cases} \tag{4a}$$

$$h_x = \begin{cases} f_x & \text{for } x=N,N-k-1, \\ & \qquad N-2k-1,\ldots \\ \max[h_{x+1},f_x] & \text{for } x=N-1,\ldots,N-k; \\ & \qquad N-k-2,\ldots,N-2k-1;\ldots \end{cases} \tag{4b}$$

$$\text{Result}_x = \max[g_{x+(k-1)/2}, h_{x-(k-1)/2}] \quad \text{for } x=1,\ldots,N. \tag{4c}$$

The advantage of this method is that the operations (4a), (4b) and (4c) each require only one single comparison per array element. The entire operation therefore requires only 3 operations per pixel, *independent of the structuring element size.* In this way local maximum and minimum operations with large structuring elements can be performed rapidly. In principle this approach is faster than a non-recursive approach for structuring elements larger than 3 pixels. We have found, however, that the additional time required for setting up the arrays results in a break-even point occurring at values somewhat larger than 3.

In the two-dimensional case the filter is implemented by applying operation (4) consecutively to rows and columns of the image. A local maximum or minimum operation in 2D for a rectangular kernel therefore requires only 6 operations per pixel, independent of the size of the kernel. Extension to higher dimensions is straightforward.

A disadvantage of this technique is the need for two temporary buffers. As this technique is applied to only one row (or column) of the image at the time, the extra memory requirements are modest. Another disadvantage is that the image or buffer size has to be a multiple of the structuring element size. We achieved this by padding the end of the buffers with large negative values. In practice we pad the two buffers further on both sides with $k/2$ elements. In this way the implementation can ignore edge effects completely.

For making the kernel of the local optimum filters more circular symmetric the filters are run additionally in the two diagonal directions across the image matrix. This doubles the execution time, but allows the determination of, for example, octagonal dilations or eight-connected distance transforms.

5. Results and discussion

The above mentioned filters were implemented on IBM PC-AT computers. In Table 1, the computation times are listed. These values hold for a 12 MHz, 1 wait state 80286 system, being a somewhat slow standard (approximately 1 MIPS). The routines were coded in assembly language for optimal performance and applied to images of 256×256 pixels, 16 bits per pixel.

For comparison we list computation times for a classical implementation of local minimum and maximum filters and for recursive and non-recursive distance transforms. The local maximum is evaluated by a 3×3 local maximum repeated 1, 4 and 13 times. Its 3×3 structuring element is obtained by applying a 3 point horizontal maximum, followed by a 3 point vertical maximum filter. The classical distance transform is a 3×3 cone dilation, again applied 1, 4 and 13 times to obtain the required structuring element sizes. The new morphological filter was implemented recursively as described in Section 4. The computation time of the flat morphological filter has a small dependence on the structuring element size, because of an increase in length of the arrays by the padding operations.

The table shows that, with the new recursive algorithms, large kernel size local maximum and minimum can be performed in 1 to 2 seconds even on a small size 80286 computer. The filters are an order of magnitude faster than corresponding non-recursive operations.

Even though the local minimum and maximum operations are grey tone morphological operations, the presented algorithms are faster than many existing algorithms for binary erosion and dilation (e.g., Van Vliet and Verwer (1987)). Assuming grey tone image values of 0 and 1 described algorithms can directly be used for these applications.

Because the algorithm is recursive, it cannot easily be implemented on parallel computers. Specialized hardware for such an algorithm should not be hard to develop, however, because of the simplicity of the algorithm. For the binary morphological operations where local maximum is equivalent to the bitwise *or* operation, implementation in hardware should be specifically simple.

6. Conclusions

We present a highly efficient recursive implementation of local minimum and maximum filter types. The algorithm is suitable both for rectangular and octagonal kernels, requiring respec-

Table 1

Computation times of recursive and classical implementation of local minimum and the distance transform The computation times are for a 256×256 image with 16 bites/pixel

kernel size filter type	3×3	9×9	27×27	equation(s)
Classical minimum or maximum	1.0 s	4.0 s	12.9 s	1
Recursive minimum or maximum	1.7 s	1.7 s	1.8 s	4
Classical distance transform	1.6 s	6.6 s	21.4 s	1
Recursive distance transform		1.2 s		2,3

tively 6 or 12 operations per pixel, independent of kernel size. For large structuring element size, say 27×27 pixels, the new recursive algorithm runs an order of magnitude faster than classical non-recursive implementations. With this algorithm local minimum and maximum filters with very large structuring elements can now also be applied for time-critical applications.

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