Belief Space Partitioning for Symbolic Motion Planning

Mengxue Hou[1], Tony X. Lin[1], Haomin Zhou[2], Wei Zhang[3], Catherine R. Edwards[4], Fumin Zhang[1]

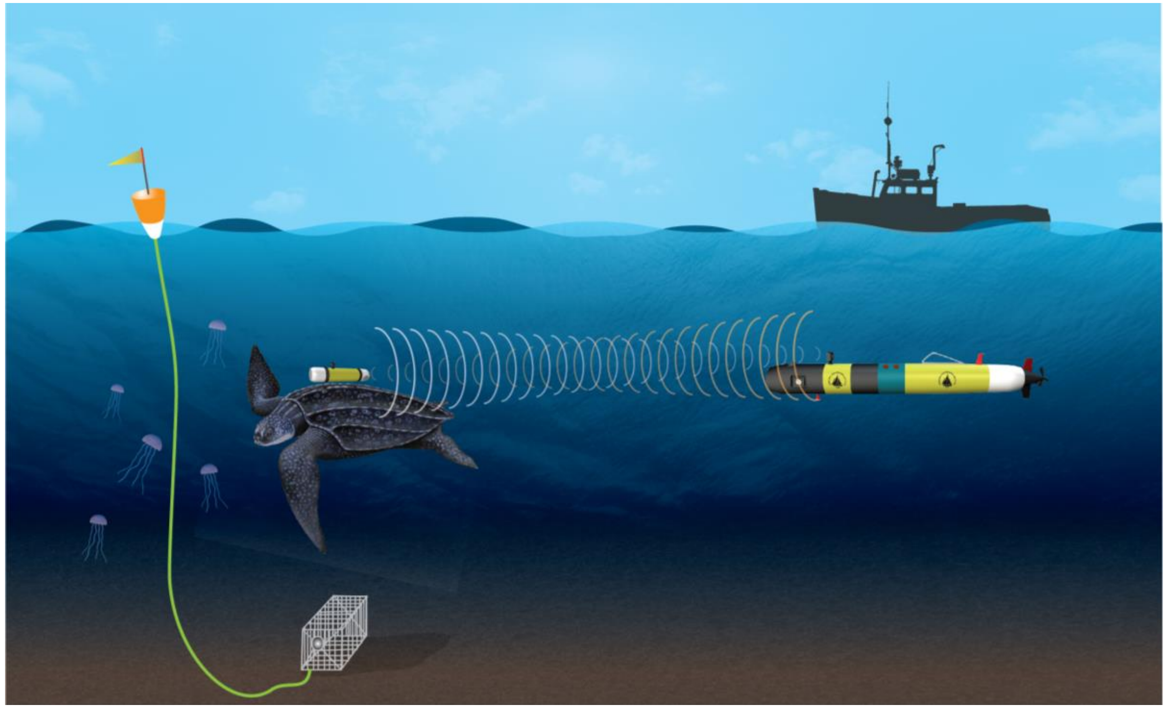
[1] Electrical & Computer Engineering, Georgia Institute of Technology, Atlanta, Georgia, USA

[2] Mathematics, Georgia Institute of Technology, Atlanta, Georgia, USA

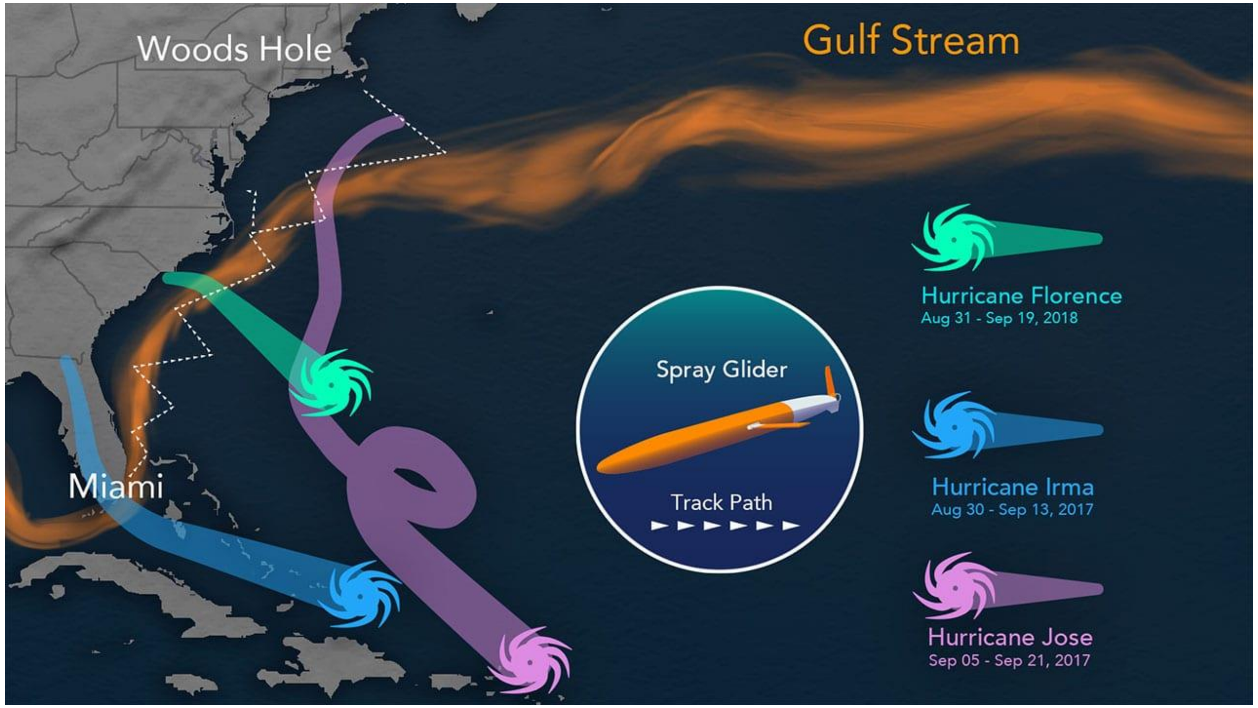
[3] Mechanical & Energy Engineering, Southern University of Science and Technology, Shenzhen, Guangdong, China

[4] Skidaway Institute of Oceanography, University of Georgia, Savannah, Georgia, USA

AUV marine autonomy

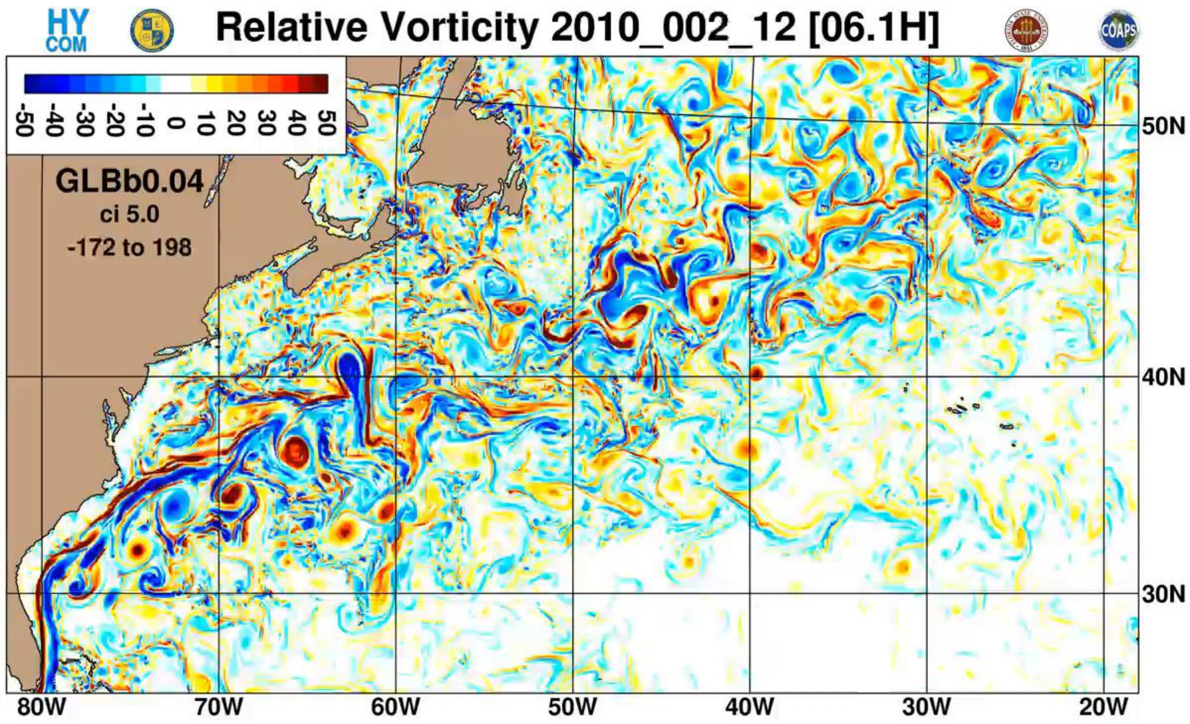


Localize & track hotspots (courtesy Woods Hole)

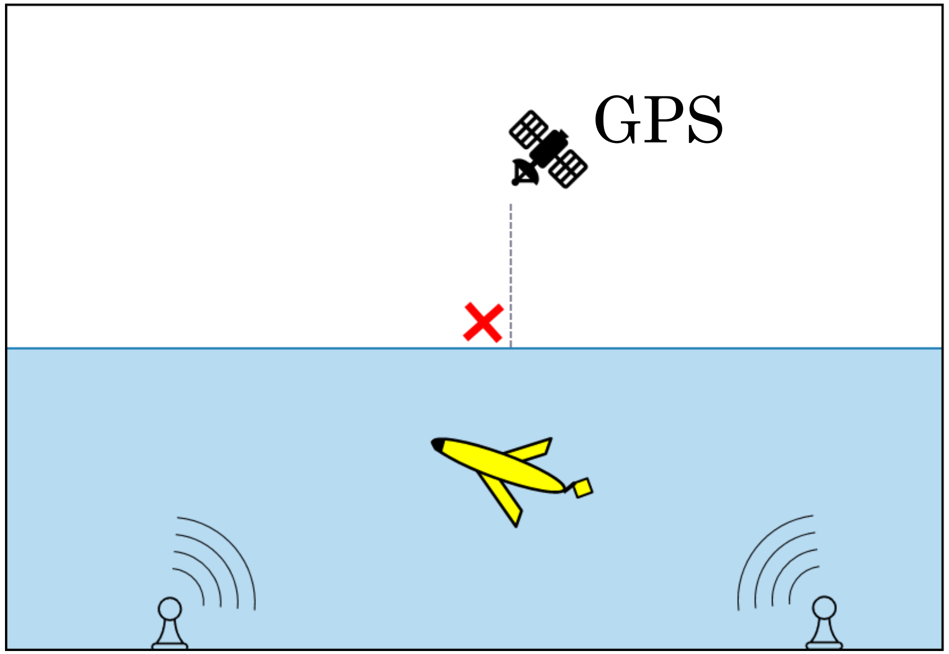


Ocean sampling (courtesy Woods Hole)

Challenge in marine autonomy



Flow forecasts can contain high uncertainty



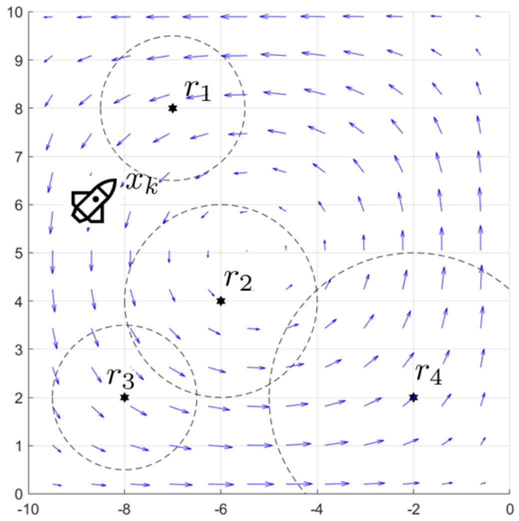
Acoustic localization contains high uncertainty due to multi-path effects and doppler spread

Problem formulation

Vehicle dynamics:

$$x_{k+1} = x_k + F(x_k, t_k) + \begin{bmatrix} V(\cos(u_k^{(1)}) - \frac{\sqrt{2}}{2}|u_k^{(2)}|) \\ V(\sin(u_k^{(1)}) - \frac{\sqrt{2}}{2}|u_k^{(2)}|) \\ u_k^{(2)} \end{bmatrix} + w_k$$

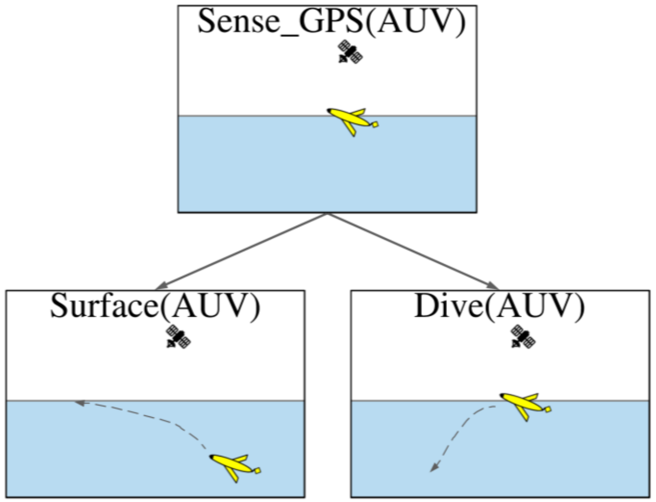
Predicted flow velocity   Vehicle forward speed   Noise



Control input:

$$u_k \in \mathcal{U}_{xy} \cup \mathcal{U}_z = \{[\tfrac{j\pi}{4} \quad 0]^\intercal\}_{j=0}^{7} \cup \{[\tfrac{\pi}{4} \quad 1]^\intercal, [\tfrac{\pi}{4} \quad -1]^\intercal\}$$

Move   Active sensing: diving & surfacing



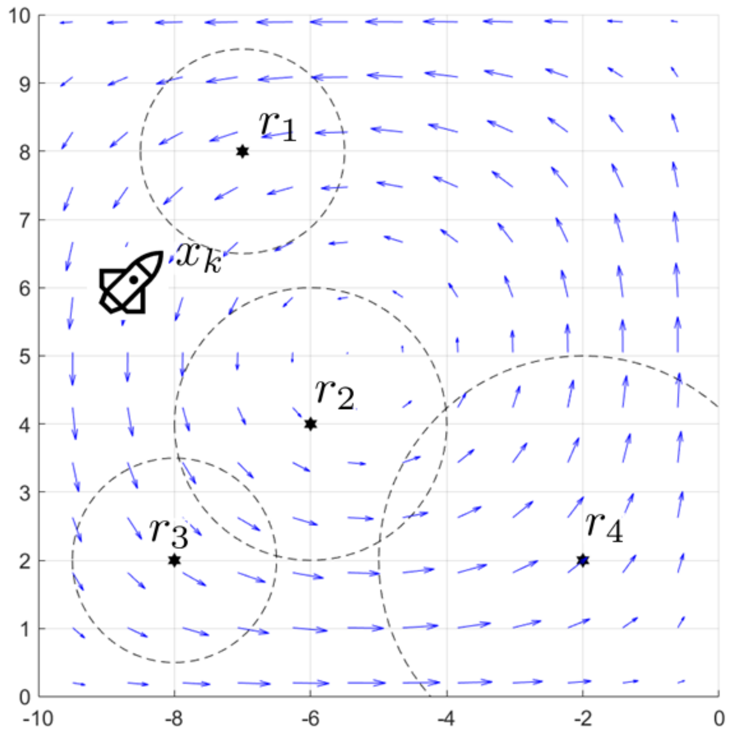
Problem formulation

Localization signal for vehicle: $y_k \in \mathbb{R}^M, y_k^{(j)} \in \{0, 1\}$

Observation model:

$$\Pr(y_k^{(j)} = 1) = p\mathbb{1}\{\|x_k - r_j\| \leq R_j\} + (1-p)\mathbb{1}\{\|x_k - r_j\| > R_j\}$$

$$\Pr(y_k^{(j)} = 0) = (1-p)\mathbb{1}\{\|x_k - r_j\| \leq R_j\} + p\mathbb{1}\{\|x_k - r_j\| > R_j\}$$



Problem formulation

Georgia Tech Systems Research GTSR

Belief state: $b_k = \Pr(x_k | u_{k-1}, u_{k-2}, \ldots, u_1, y_k, y_{k-1}, \ldots, y_1)$

Push forward operator (Bayes' law)

Belief dynamics: $b_{k+1} = \mathcal{L}(y_{k+1}, u_k) b_k$

Problem 1: find an approximation model of the belief dynamics

Problem 2: solve the planning problem

$$\min_{[u_0, \ldots, u_N]} \sum_{k=1}^{N} l(b_k, u_k)$$

s.t. approx. belief dynamics,

$$b_0 = \rho_0, g(b_N) \leq 0$$

Difficulty in modeling the belief dynamics

Difficulty:

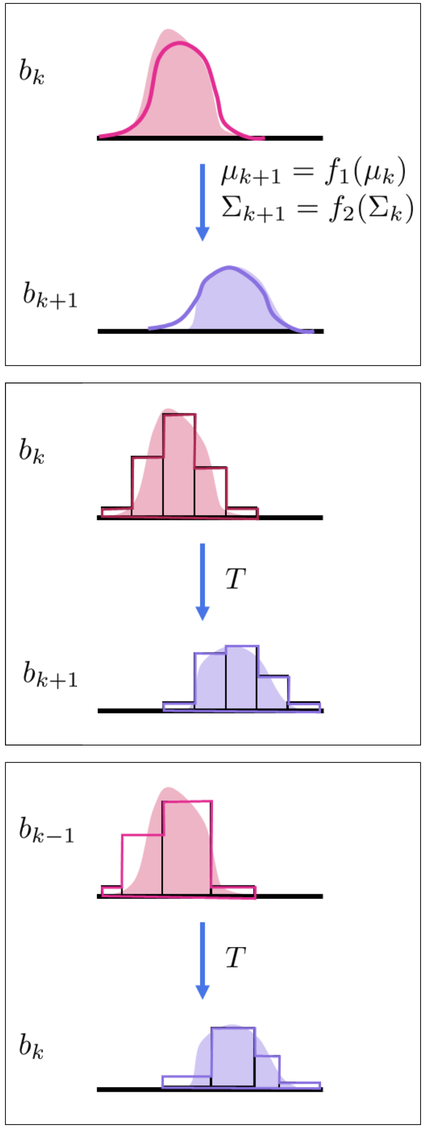
-Nonlinear dynamics, non-Gaussian noise

-Curse of dimensionality

Our contribution:

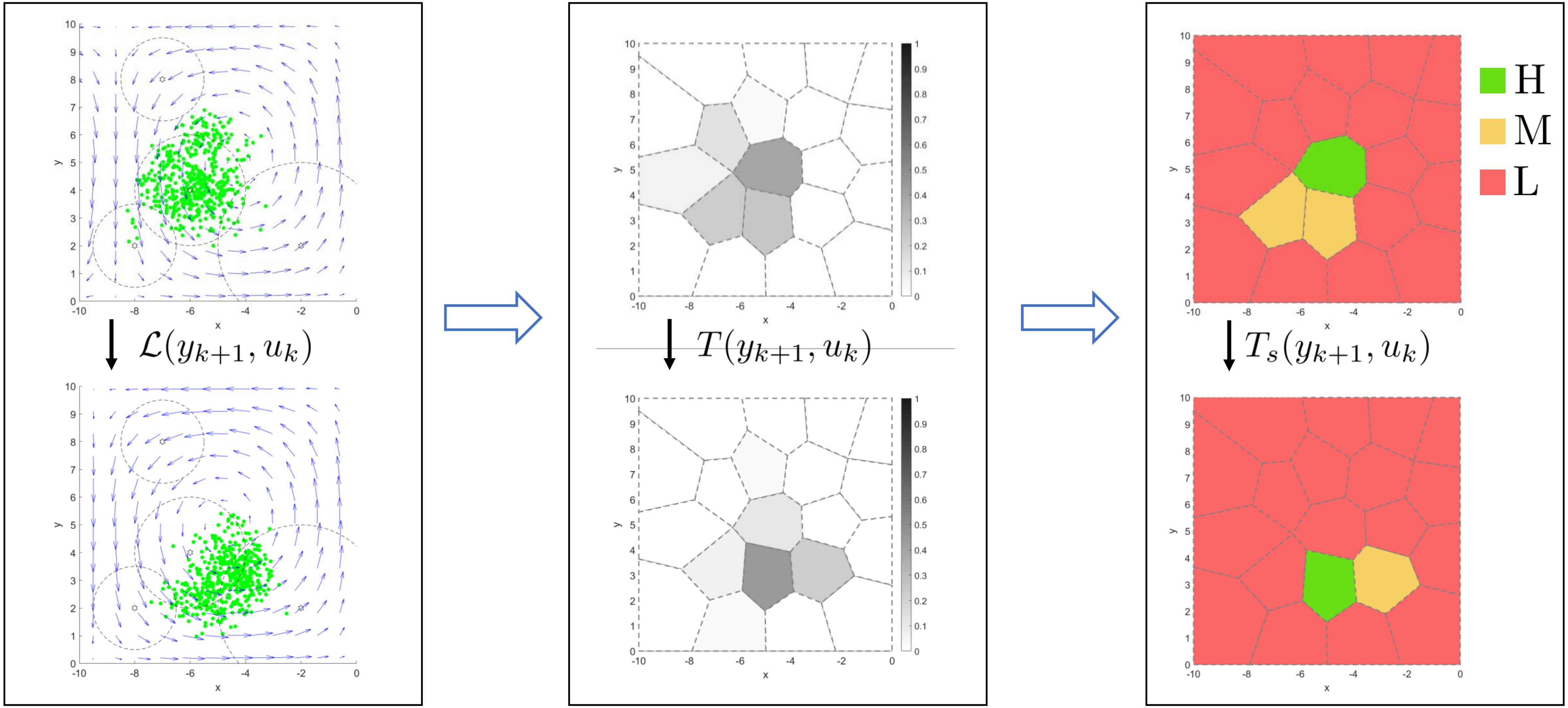
A novel belief partition method that allocates a fixed number of partitions over regions that the AUV most likely will visit

The partition method enables computationally efficient solutions to the continuous-state POMDP problem



Proposed belief abstraction method

Unparameterized dyn. | Belief dyn. in parameter space | Belief dyn. in symbolic space



Focused finite partition

Intuition: in many robotic applications, vehicle has zero probability to be in most of the state space.

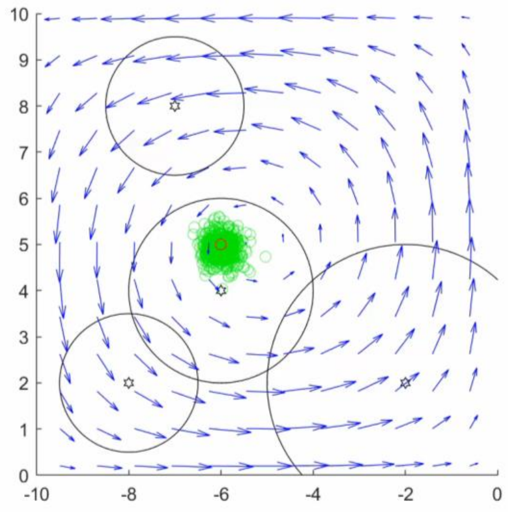
We allocate a fixed number of partitions over regions that the system state most likely will visit

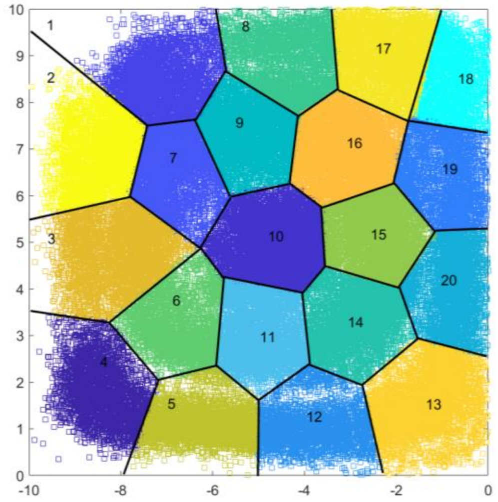
$$\text{Simulated particle: } z_k^{(m)} = [(\overbrace{r_k^{(m)}}^{\text{Particle position}})^\intercal, \underbrace{w_k^{(m)}}_{\text{Weight}}]^\intercal$$

$$\min_{\mathcal{R}} \sum_{k\in[1,n]} \sum_{i\in[1,K]} \sum_{r_k^m\in\mathcal{R}_i} \mathrm{dist}(\overbrace{z_k^{(m)}}^{\text{Simulated particle}}, \overbrace{\mu^{(i)}}^{\text{Centroid of partitioned cells}}) \qquad (1)$$

$$\text{s.t.} \quad \sum_{i\in[1,K]} I\mu^{(i)} = 1, I\mu^{(i)} > 0, \forall i \in [1, K]$$

(1) can be solved using the K-means method



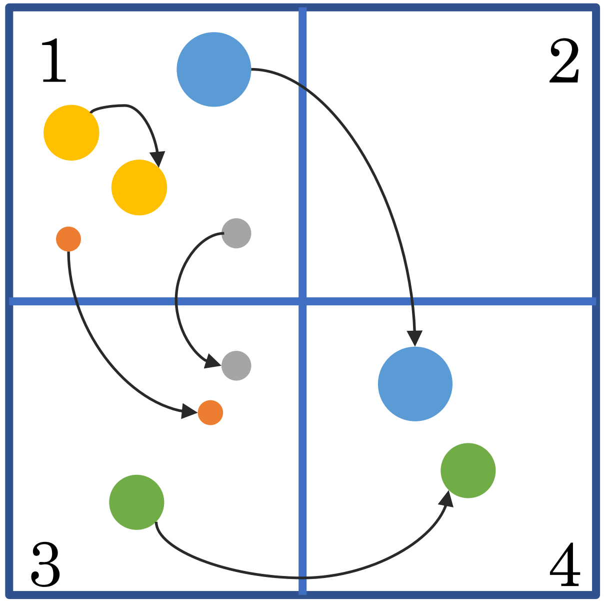


Identify the parameter space belief dynamics

Simulate particle trajectory given a fixed control input u,

Numerically estimate the partitioned belief dynamics by:

$$T^{ij}(u) = \frac{\text{mass transported from } j \text{ to } i}{\text{total mass in } j}$$



Belief dynamics in parameter space:

$$\Theta_k = T(u_{k-1})\Theta_{k-1},$$

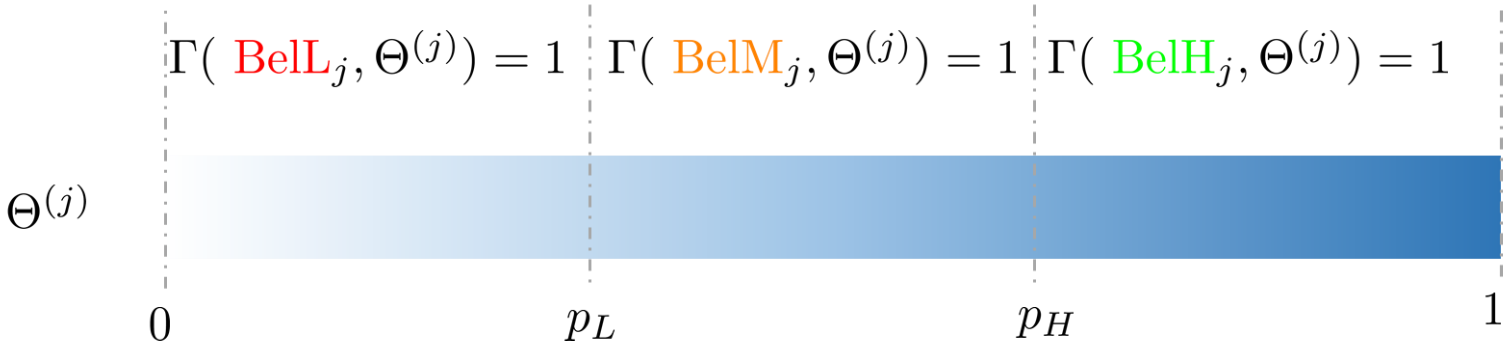
$$\Theta_0 = \arg\min_{\Theta} \operatorname{dist}(b_0 - \Theta^T \Phi)$$

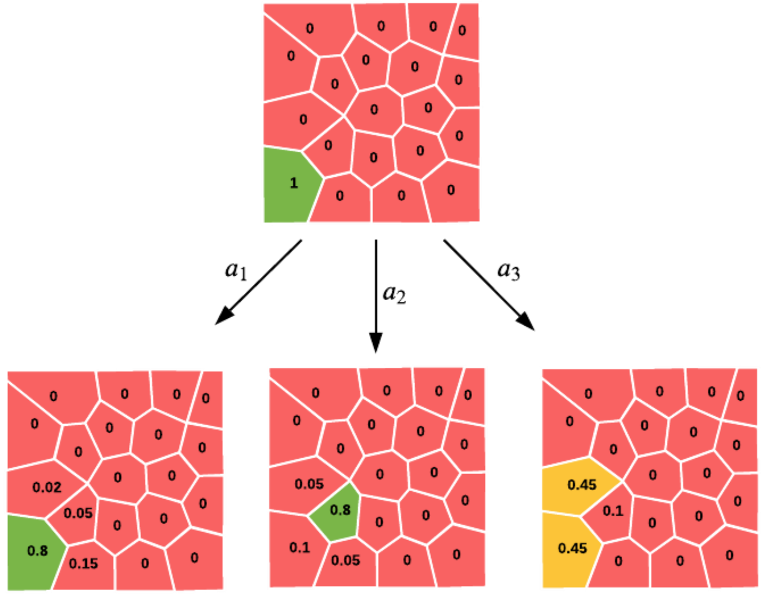
Identify the symbolic space belief dynamics

Predicates: $\mathbb{S} = \{\text{at\_surf}, \text{at\_depth}\} \cup \{\text{BelH}_j, \text{BelM}_j, \text{BelL}_j\}_{j=1}^{K}$

Active predicates: $s_k = \{s \in \mathbb{S} : \Gamma(s, \Theta_k) = 1\}$

Γ defines a mapping from belief values to symbols:

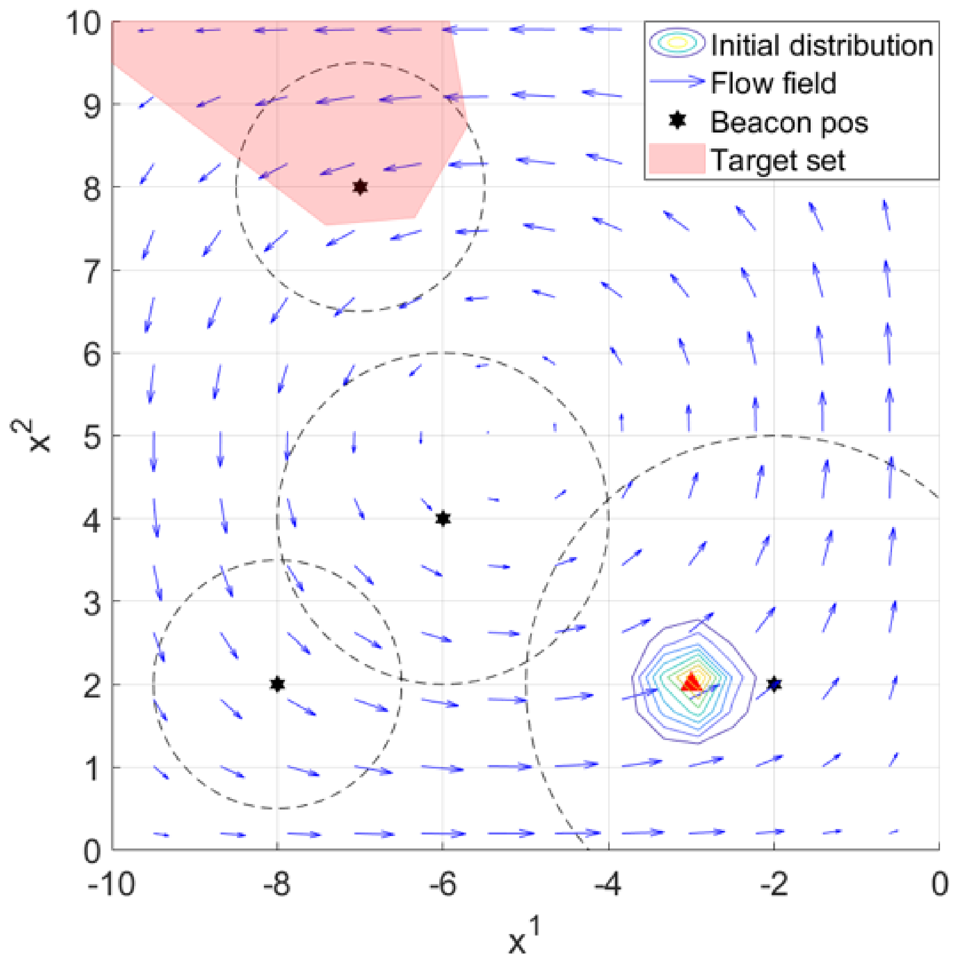




Simulation

We derive the approximated belief dynamics using both the proposed method and the generalized cell mapping method

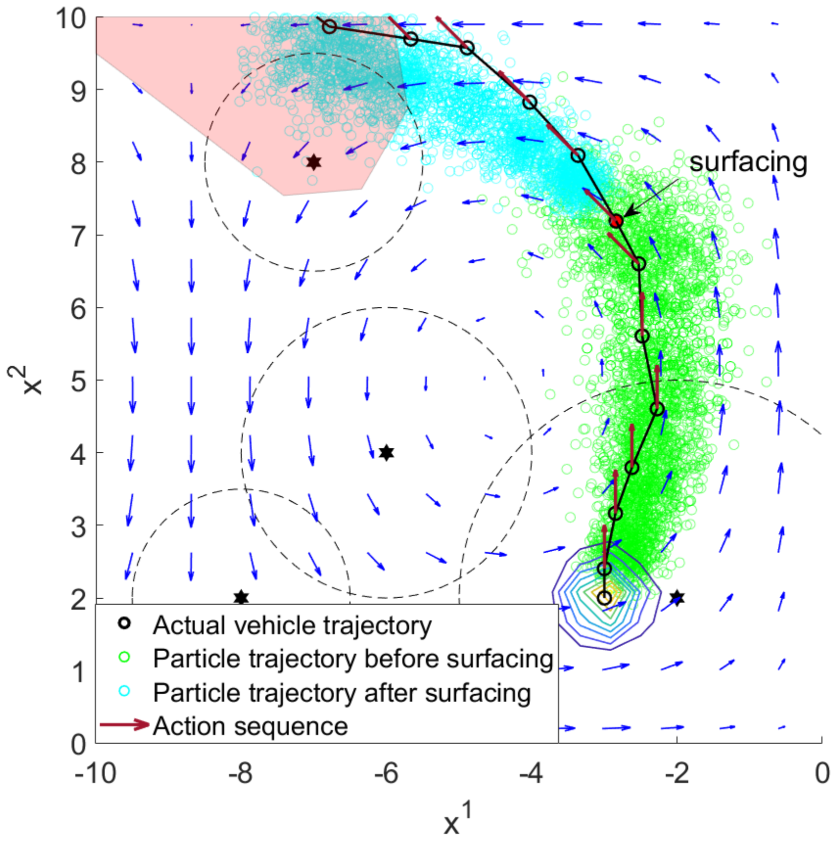
The symbolic planning problem is solved by the A* algorithm



Sun, Jian-Qiao, et al. *Cell Mapping Methods*. Springer, New York, 2018.

Simulation

Grid size	# of Pre.	Mod. time	Plan time
Generalized cell mapping			
$10 \times 10 \times 2$	600	14.2845 s	**5.9873** s
$100 \times 100 \times 2$	$6e4$	15.5396	**54.3957** s
$150 \times 150 \times 2$	$1.35e5$	16.4289 s	**154.368** s
Proposed solution			
$10 \times 10 \times 2$	120	18.3886	**3.9702** s
$100 \times 100 \times 2$	120	19.845 s	**4.7825** s
$150 \times 150 \times 2$	120	20.5480 s	**4.9796** s



Belief state partition reduces the computational cost of symbolic planning

Conclusion

- We propose a novel belief partition method that allocate a fixed number of partitions over regions that the system state most likely will visit

- The partition method enables computationally efficient solutions to the continuous-state POMDP problem