Flow Field Partition for Underwater Vehicle Path Planning

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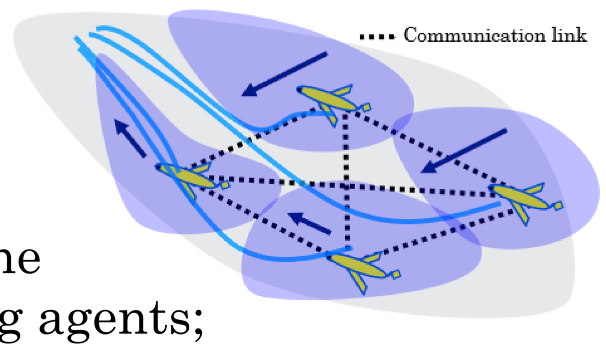
Objective

To develop a compressed representation of the spatial and temporal variation of the ocean flow field that facilitate fast path planning.

Motivation

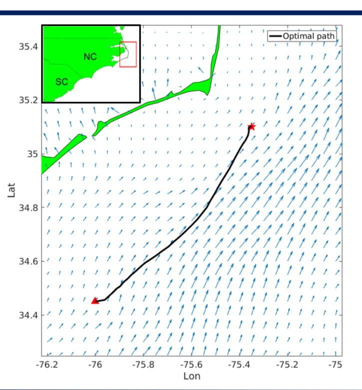
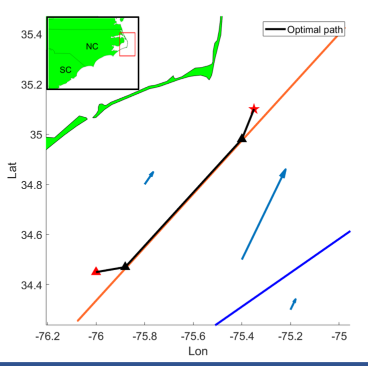
Sharing field estimation data among agents will facilitate planning performance of the fleet.



- Constrained communication capacity limits the amount of information that can be shared among agents;
- Computation cost of AUV path planning increases in the case of complicated flow map.

Novelty and Contribution

Grid-based flow map	Feature map
Spatial variation	Partitions of uniform flow speed
Temporal variation	Temporal variation of the partitioned flow

	Map size	Planning cost
Grid-based map Grid based path planning, e.g. Level Set Method 	Large	High
Feature map Method of Evolving Junctions 	Small	Low

Relating flow partition to Koopman Operator Theory

Evolution of state $\mathbf{z}_{k+1} = \mathbf{f}(\mathbf{z}_k)$ → Observable $\mathbf{u}(\mathbf{z}, \mathbf{x}, t)$ → Evolution of observables $\mathcal{K}\mathbf{u}(\mathbf{z}_k, \mathbf{x}) = \mathbf{u}(\mathbf{z}_{k+1}, \mathbf{x})$

Finite dimension nonlinear dynamics

Infinite dimension linear dynamics

Define $\phi_j(\mathbf{x}) = \mathbb{I}_{\mathbf{x} \in R_j}$,

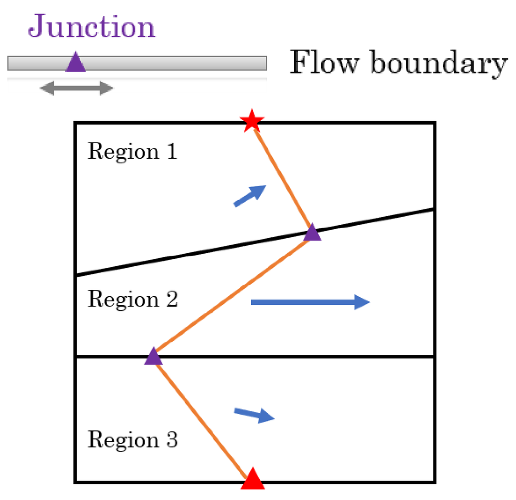
$$\mathbf{u}(\mathbf{z}_k, \mathbf{x}) = \sum_{j=1}^{\infty} \bar{\mathbf{u}}_j(\mathbf{z}_k)\phi_j(\mathbf{x})$$

$$\mathcal{K}\mathbf{u}(\mathbf{z}_k, \mathbf{x}) = \sum_{j=1}^{\infty} \bar{\mathbf{u}}_j(\mathbf{z}_{k+1})\phi_j(\mathbf{x})$$

Flow partition can be a novel data driven method to compute the Koopman modes and Koopman eigenfunctions of the flow field.

Method of Evolving Junctions

Infinite dimensional path planning → Finite dimension optimization on junction positions



Features:

- Low computation cost
- Guaranteed global optimality
- Applicable to various cost functions

Representing Spatial Variation of the Flow Field

$$\mathbf{y}(t) = [\mathbf{x}; \mathbf{F}(\mathbf{x}, t)]$$

$\mathbf{x}$: Data point position; $\mathbf{F}(\mathbf{x}, t)$: Flow speed

$$\text{dist}^2(\mathbf{y}, \mathbf{y}') = (\mathbf{y} - \mathbf{y}')^T \mathbf{Q}(\mathbf{y} - \mathbf{y}')$$

$$\min J = \sum_{\alpha=1}^{k} \sum_{\mathbf{y} \in R_\alpha} \sum_{t \in T} \text{dist}^2(\mathbf{y}(t), \mu_\alpha)$$

Partitioned regions (R_α); Centroid of data points in α^{th} region (μ_α)

$$\sum_{t \in T} \text{dist}^2(\mathbf{y}(t), \mu_\alpha) = T\text{dist}^2(\bar{\mathbf{y}}, \mu_\alpha) + \sum_{t \in T} \text{dist}^2(\mathbf{y}(t), \bar{\mathbf{y}})$$

$$\min \sum_{\alpha=1}^{k} \sum_{\mathbf{y} \in R_\alpha} \text{dist}^2(\bar{\mathbf{y}}, \mu_\alpha)$$

Difference between time-averaged flow obs. and centroid of data points in α^{th} region

[1] Haoyan Zhai, Mengxue Hou, Fumin Zhang, and Haomin Zhou, "Method of evolving junction on optimal path planning in flow fields," in preparation for submission. Preprint available at http://arxiv.org/abs/1904.11554

Representing Temporal Variation of the Flow Field

$$\min_{\Theta_\alpha} J_\alpha = \sum_{\mathbf{x} \in R_\alpha} \sum_{t \in T} ||\mathbf{f}_\alpha(\Theta_\alpha, t) - \mathbf{F}(\mathbf{x}, t)||^2$$

Time series model containing a set of unknown parameters Θ_α

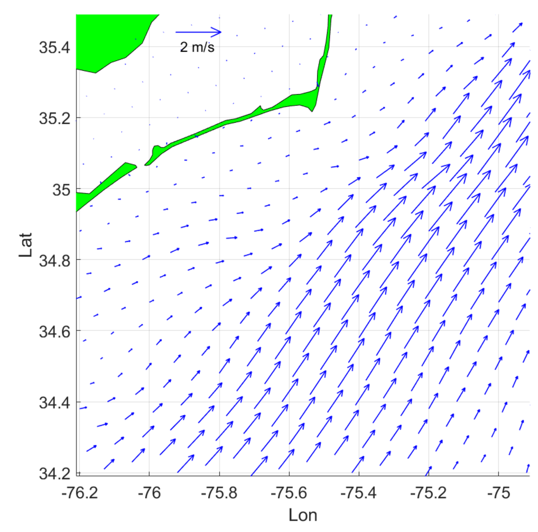
$$\sum_{\mathbf{x} \in R_\alpha} ||\mathbf{f}_\alpha(\Theta_\alpha, t) - \mathbf{F}(\mathbf{x}, t)||^2$$

$$= n(R_\alpha)||\mathbf{f}_\alpha(\Theta_\alpha, t) - \phi_\alpha(t)||^2 + \sum ||\mathbf{F}(\mathbf{x}, t) - \phi_\alpha(t)||^2$$

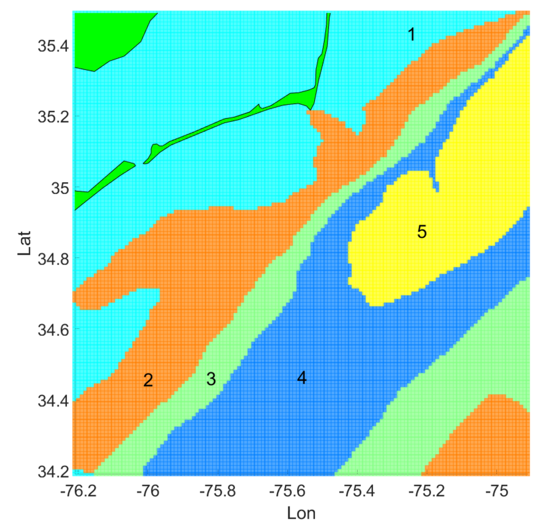
$$\min_{\Theta_\alpha} \sum_{t \in T} ||\mathbf{f}_\alpha(\Theta_\alpha, t) - \phi_\alpha(t)||^2$$

Difference between spatial-averaged flow and time-series model in α^{th} region

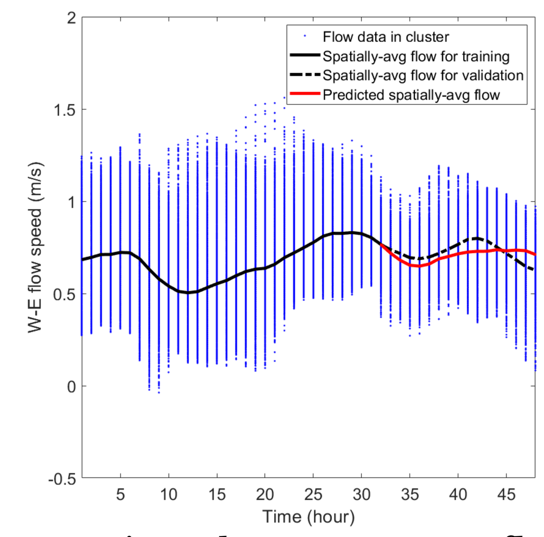
Partition of Ocean Surface Flow Field

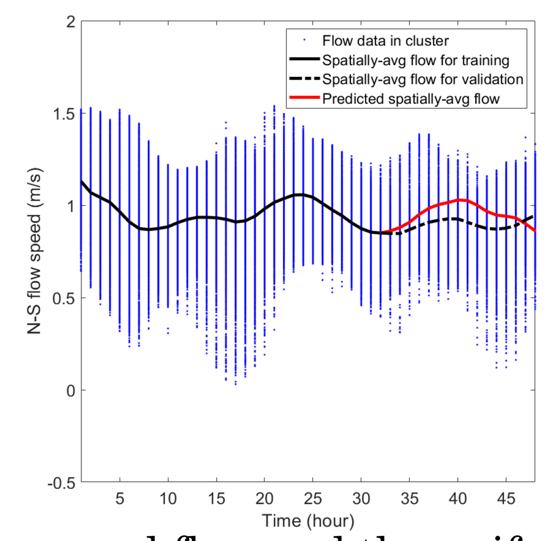


48 hrs time-averaged flow 5/27, 00:00– 5/29, 00:00 UTC, 2017 at Cape Hatteras, NC



Partitioned flow field

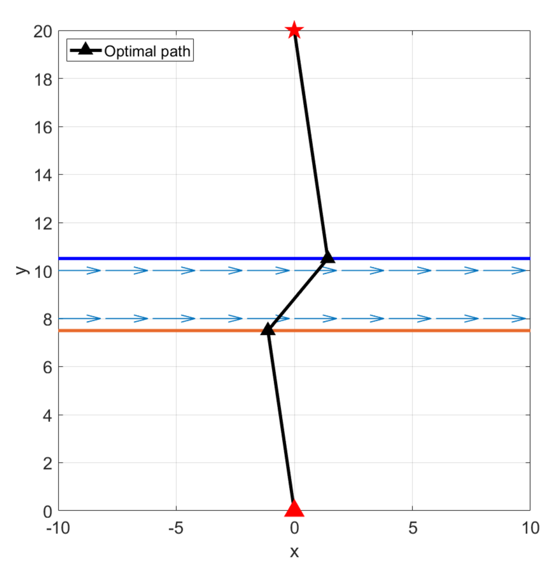




Comparison between true flow, spatially averaged flow and the uniform flow predicted by ARIMA model in region 4. Left: W-E flow comparison. Right: N-S flow comparison.

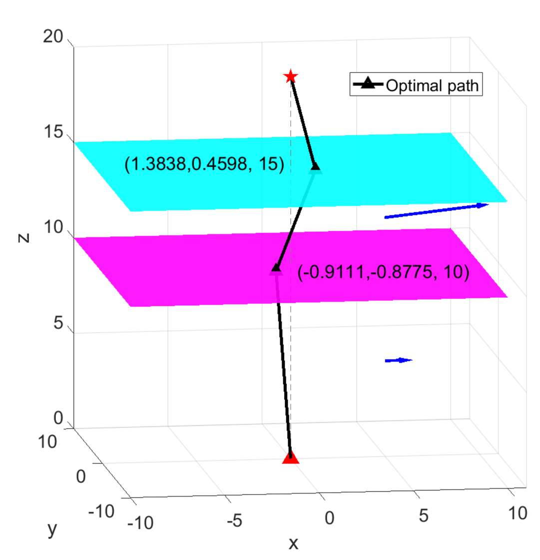
Original flow field ~ 5×10^5 data points	Partitioned flow field ~ 10^3 data points

Comparison between MEJ and LSM



Comparison between time-optimal path planned by MEJ and LSM in 2D jet flow

	MEJ	LSM
θ_1	8.4785	8.4113
θ_2	-40.1878	-39.7923
θ_3	8.4783	8.2147
Time	0.110 secs	10.328 secs



Comparison between time-optimal path planned by MEJ and LSM in 3D jet flow

	MEJ	LSM
θ_1	82.7924	83.5659
θ_2	62.0255	63.3118
θ_3	73.7397	73.8027
γ_1	-136.0775	-135.6592
γ_2	30.2293	30.2407
γ_3	-161.6199	-161.2246
Time	0.57 secs	10125.2 secs

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