Transitions in (1+1) Light Front ϕ^4 Theory using Quantum Computing Method

Mengyao Huang[1], James P. Vary[1]

[1]Department of Physics & Astronomy, Iowa State University

Abstract

We study the phase transitions in (1+1) ϕ^4 theory in the light-front frame using both classical computing and quantum computing methods. The transition at the ground state from a single-particle dominant state to a three-particle dominant state can be revealed by the parton distribution function and indicated by crossing of mass square eigenvalues at a strong coupling. We discuss quantum computing as a method of exploring quantum phase transition in the light-front (1+1) ϕ^4 theory.

Motivation

- Classical computing hits the memory bound in non-perturbative field theory calculations with increasing resolution. Quantum computing is promising in reducing the memory consumption, as N configurations can be encoded by only $log_2 N$ qubits in compact encoding [1].
- Demonstrate that transition in the (1+1) light front ϕ^4 theory in the strong coupling region can be observed by solving the ground state using the VQE quantum algorithm.

Introduction

Topological phase transition [2] in quantum field theory is an emerging collective phenomenon that can be explored by non-perturbative methods.

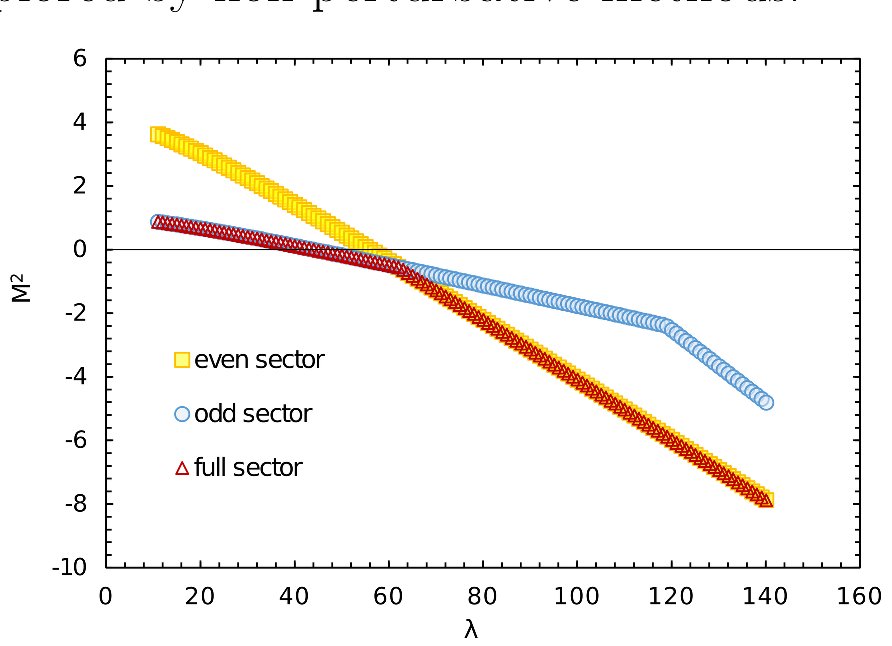

Figure 1: Example of a transition implied by the crossing of mass-squared eigenvalues between the even sector ground state and the odd sector ground state of the (1+1) ϕ^4 theory at $K = 16$.

The (1+1) ϕ^4 theory in discretized light-front quantization (DLCQ)

1. Starting from the (1+1) light front ϕ^4 theory Lagrangian (in symmetric phase for example) [3, 4]

$$\mathcal{L} = \frac{1}{2}\partial^+\phi\partial^-\phi - \frac{\mu^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4,$$

discretizing the field in the domain of $-L < x^- < L$ ($n = 1, 2, \cdots$ for periodic boundary condition neglecting the zero mode)

$$\phi = \frac{1}{\sqrt{4\pi}}\sum_n \frac{1}{\sqrt{n}}\left(a_n e^{-i\frac{n\pi}{L}x^-} + a_n^\dagger e^{i\frac{n\pi}{L}x^-}\right),$$

the resulting Hamiltonian is

$$H = \mu^2\sum_n^K \frac{1}{n}a_n^\dagger a_n + \frac{\lambda}{4\pi}\left(\sum_{k\le l, m\le n}^K \frac{1}{N_{kl}}\frac{1}{N_{mn}}\frac{a_k^\dagger a_l^\dagger a_m a_n}{\sqrt{klmn}}\delta_{m+n,k+l} + \sum_{k,l\le m\le n}^K \frac{1}{N_{lmn}}\frac{a_k^\dagger a_l a_m a_n + a_n^\dagger a_m^\dagger a_l^\dagger a_k}{\sqrt{klmn}}\delta_{k,m+n+l}\right)$$

where $N_{kl} = 1, k \neq l$; $N_{kl} = 2!, k = l$. And $N_{lmn} = 1, l \neq m \neq n$; $N_{lmn} = 2!, l = m \neq n$ or $l \neq m = n$; $N_{lmn} = 3!, l = m = n$. K is the total momentum that controls the resolution of the discretization.

2. Solving the lowest eigenvalue of $M^2 = KH$ using the Variational Quantum Eigensolver (VQE) quantum algorithm in Qiskit [5] developed by IBM. Classical optimizers are used for optimizing the parameters of the ansatze in each iteration. Physical observables can be evaluated with the final optimal ansatze.

Choosing Optimizers

We prepare a circuit "TwoLocal" with 16 parameters as an ansatze for $K = 9$ odd sector (Hamiltonian matrix of 16×16 dimension encoded by 4 qubits). It is crucial of choosing a suitable optimization method which gives a satisfactory performance.

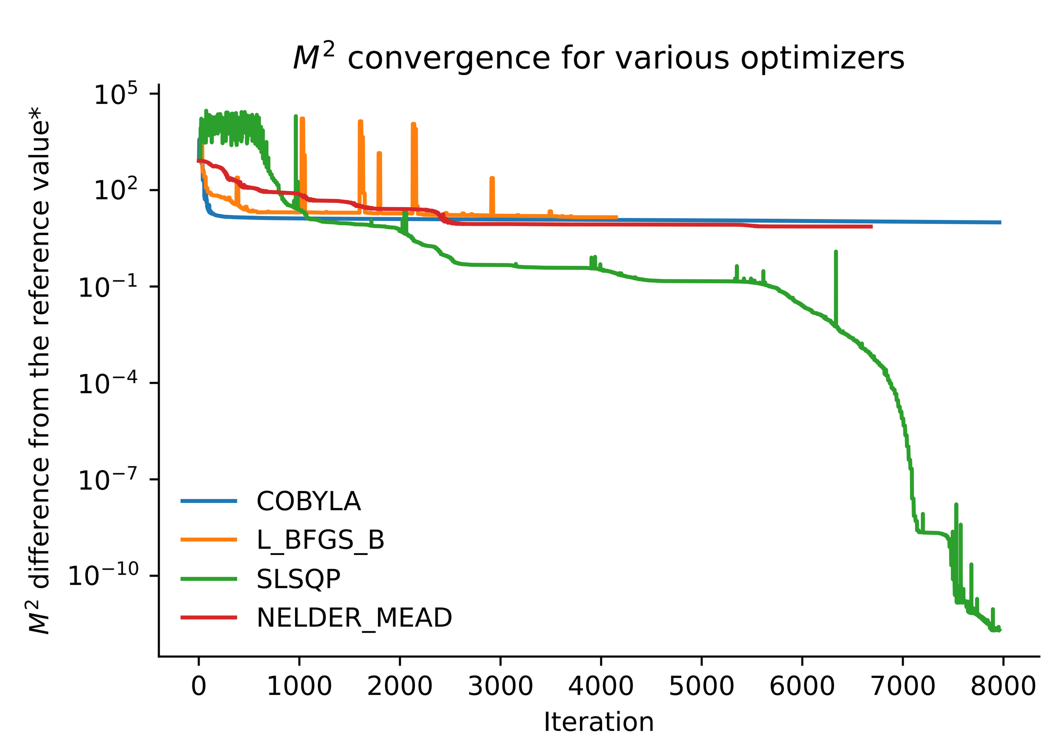

Figure 2: M^2 convergence for various optimizers. COBYLA and Nelder-Mead are derivative-free optimization methods. L-BFGS-B uses the solutions and gradients from the most recent iterations to estimate the Hessian matrix. SLSQP is a local gradient-based optimization method using sequential least squares programming. (*The reference value is calculated by classical computing.)

The Parton Distribution Function

Parton distribution function (PDF) $s(x)$ represents the number density of a particle carrying momentum fraction x. In discretized version,

$$s_i(x_k) = \sum_j |\alpha_{ji}|^2 \frac{\langle\phi_j|a_k^+ a_k|\phi_j\rangle}{N_j},$$

where N_j is the number of particles in j-th configuration $|\phi_j\rangle$ and $x_k = k/K$. One can check that $\sum_k s_i(x_k) = 1$.

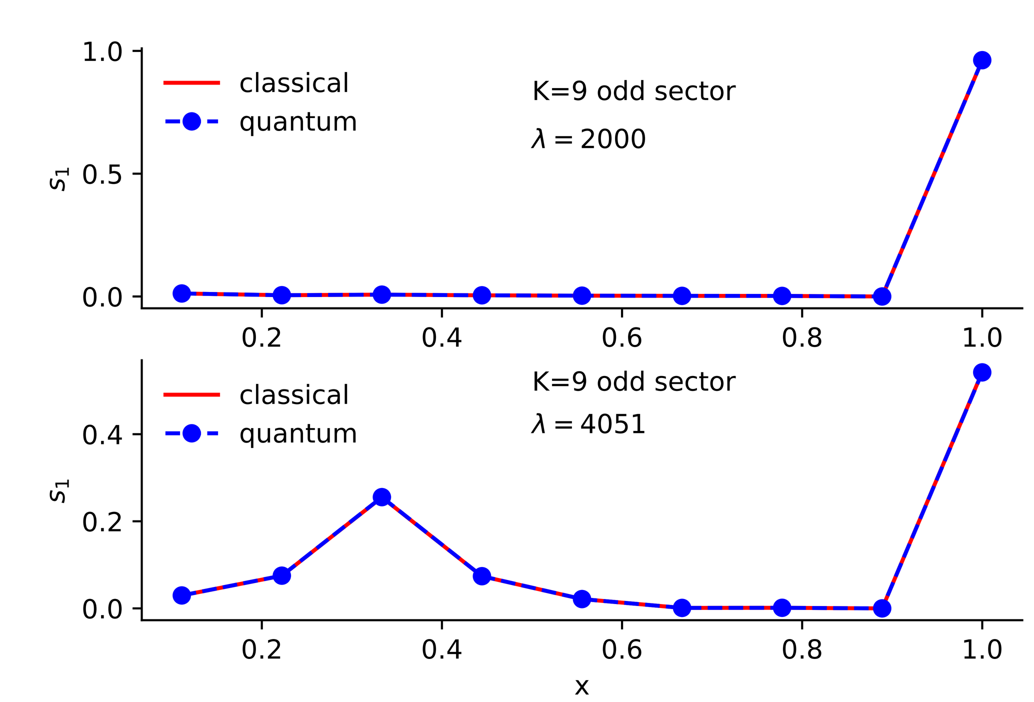

Figure 3: PDF of the $K = 9$ odd sector ground state of (1+1) light front ϕ^4 theory. We can see a second peak at $x \sim 0.33$ at the stronger coupling, indicating a significant portion of comoving three particles at the odd sector ground state.

Discussions and outlook

We show that the VQE quantum algorithm can be used to explore the transition properties of the (1+1) light front ϕ^4 theory. In the future, we hope to solve the theory for a larger Hamiltonian matrix with a finer resolution, to see if the supposed advantages of quantum computing can be realized. We will also perform the noisy simulation which better represents the current stage of quantum devices, and apply the error mitigation technique for improving the performance.

Acknowledgments

This work is supported by US DOE (DE-FG02-87ER40371, DE-SC0018223). Computational resources provided by NERSC (US DOE Contract No. DE-AC02-05CH11231).

References

[1] M. Kreshchuk, S. Jia, W. M. Kirby, G. Goldstein, J. P. Vary, and P. J. Love, "Simulating hadronic physics on noisy intermediate-scale quantum devices using basis light-front quantization," *Physical Review A*, vol. 103, p. 062601, 2021.

[2] D. Chakrabarti, A. Harindranath, and J. P. Vary, "Transition in the spectrum of the topological sector of ϕ_2^4 theory at strong coupling," *Physical Review D*, vol. 71, p. 125012, 2005.

[3] A. Harindranath and J. P. Vary, "Solving two-dimensional φ^4 theory by discretized light-front quantization," *Physical Review D*, vol. 36, pp. 1141–1147, 1987.

[4] J. P. Vary, M. Huang, S. Jawadekar, M. Sharaf, A. Harindranath, and D. Chakrabarti, "Critical coupling for two-dimensional ϕ^4 theory in discretized light-cone quantization," *Physical Review D*, vol. 105, p. 016020, 2022.

[5] M. S. A. et al., "Qiskit: An open-source framework for quantum computing," 2021.

Iowa State University