Appendix: Pricing and Market Analysis for Used BMW Cars

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Contents

Load in packages

```
mylibrary <- c("tidyverse", "cowplot", "GGally", "MASS", "ggplot2",
    "glmnet", "data.table", "car", "MLmetrics", "patchwork")
invisible(lapply(mylibrary, library, character.only = TRUE))
```

Function for drawing diagnostic plots

```
diagPlot <- function(model) {
    p_resid <- ggplot(model, aes(.fitted, .resid)) + geom_point() +
        stat_smooth(method = "loess") + geom_hline(yintercept = 0,
        col = "red", linetype = "dashed") + xlab("Fitted values") +
        ylab("Residuals") + ggtitle("Residual vs Fitted Plot") +
        theme_bw()

    p_QQ <- ggplot(model, aes(sample = .stdresid)) + stat_qq() +
        stat_qq_line() + xlab("Theoretical Quantiles") + ylab("Standardized Residuals")
         ↪  +
        ggtitle("Normal Q-Q") + theme_bw()

    p_SL <- ggplot(model, aes(.fitted, sqrt(abs(.stdresid)))) +
        geom_point(na.rm = TRUE) + stat_smooth(method = "loess",
        na.rm = TRUE) + xlab("Fitted Value") + ylab(expression(sqrt("|Standardized
         ↪  residuals|"))) +
        ggtitle("Scale-Location") + theme_bw()

    p_lev <- ggplot(model, aes(.hat, .stdresid)) + geom_point(na.rm = TRUE) +
        stat_smooth(method = "loess", na.rm = TRUE) + xlab("Leverage") +
        ylab("Standardized Residuals") + ggtitle("Residual vs Leverage Plot") +
        scale_size_continuous("Cook's Distance", range = c(1,
            5)) + theme_bw() + theme(legend.position = "bottom")

    ## combine plots
    plot_grid(p_resid, p_QQ, p_SL, p_lev, align = "h")
}
```

read in dataset

```
dat_bmw <- read.csv("BMWpricing_updated.csv")
```

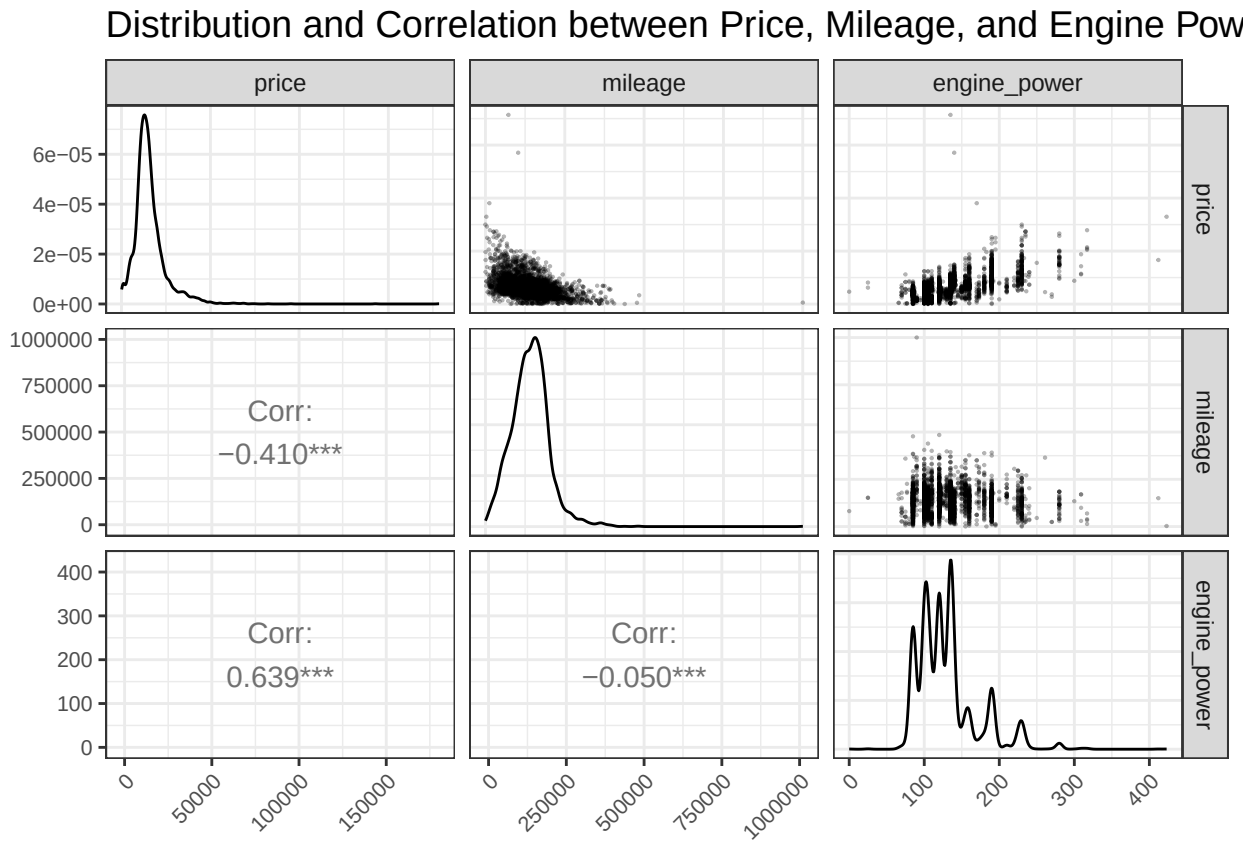
Data

Data Overview

We first drew the scatterplot matrix between price, mileage, and engine power.

```
### Create a data frame only has the variable of interests
dat_bmw2 <- dat_bmw[, c("price", "mileage", "engine_power")]
### Drop the data point with NA
dat_bmw2 <- dat_bmw2[complete.cases(dat_bmw2), ]

ggpairs(dat_bmw2, upper = list(continuous = wrap("points", alpha = 0.3,
    size = 0.1)), lower = list(continuous = wrap("cor", size = 4))) +
    labs(title = "Distribution and Correlation between Price, Mileage, and Engine
     ↪  Power") +
```

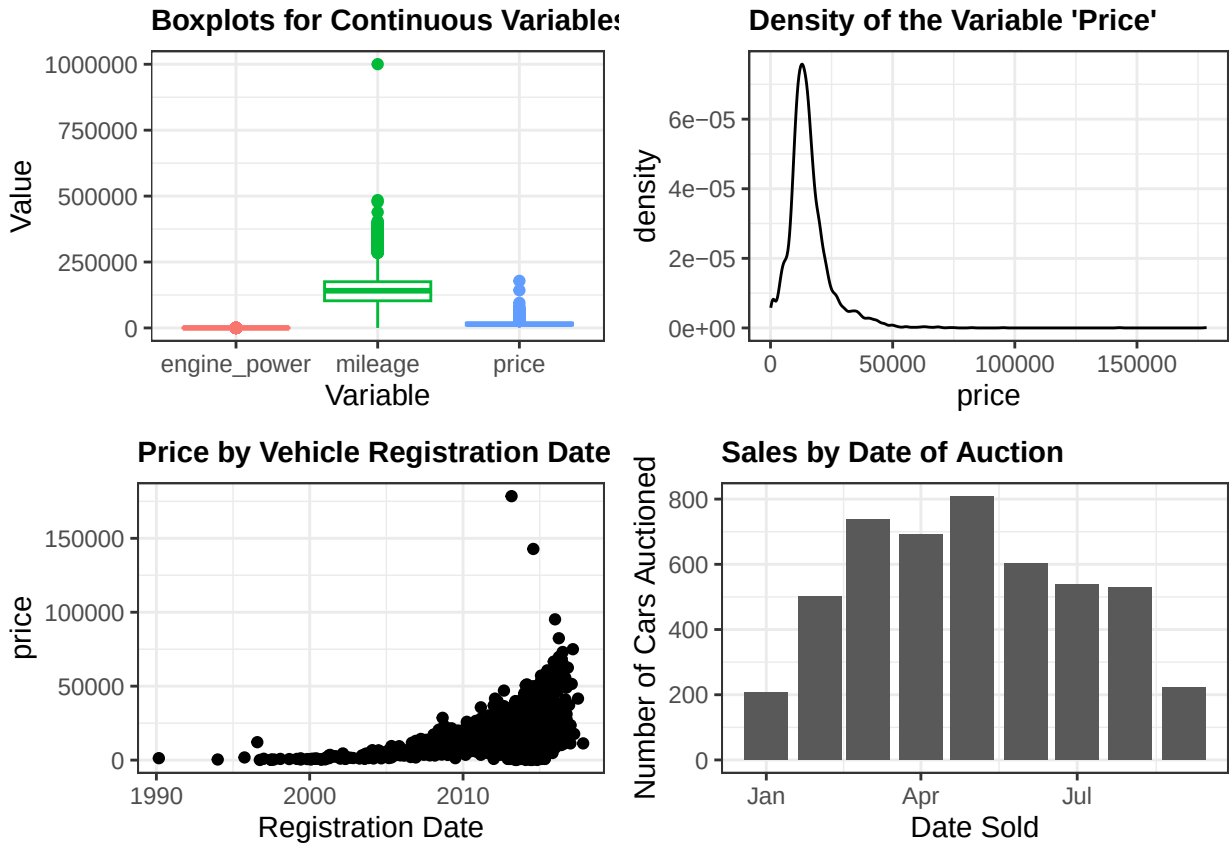
```
    theme_bw() + theme(axis.text.x = element_text(angle = 45,
    hjust = 1, size = 8), axis.text.y = element_text(size = 8),
    title = element_text(size = 12))
```



Then we investigated the distribution of price, the relationship between price and registribution date, and the sales by action date.

```
p_box_conti <- dat_bmw[, c("price", "mileage", "engine_power")] |>
    pivot_longer(everything(), values_to = "Value", names_to = "Variable") |>
    ggplot() + geom_boxplot(aes(x = Variable, y = Value, color = Variable)) +
    labs(title = "Boxplots for Continuous Variables") + theme_bw() +
    theme(plot.title = element_text(size = 11, face = "bold"),
        legend.position = "none")

p_price <- dat_bmw |>
    ggplot() + geom_density(aes(x = price)) + labs(title = "Density of the Variable
     ↪  'Price'") +
    theme_bw() + theme(plot.title = element_text(size = 11, face = "bold"))

p_price_date <- dat_bmw |>
    ggplot(aes(x = as.Date(registration_date, format = "%m/%d/%Y"),
        y = price)) + geom_point() + labs(title = "Price by Vehicle Registration Date")
         ↪  +
    xlab("Registration Date") + theme_bw() + theme(plot.title = element_text(size = 11,
    face = "bold"))
```

```
p_sale_Date <- dat_bmw |>
    ggplot(aes(x = as.Date(sold_at, format = "%m/%d/%Y"))) +
    geom_bar() + labs(title = "Sales by Date of Auction") + xlab("Date Sold") +
    ylab("Number of Cars Auctioned") + theme_bw() + theme(plot.title =
     ↪  element_text(size = 11,
    face = "bold"))

plot_grid(p_box_conti, p_price, p_price_date, p_sale_Date, ncol = 2)
```

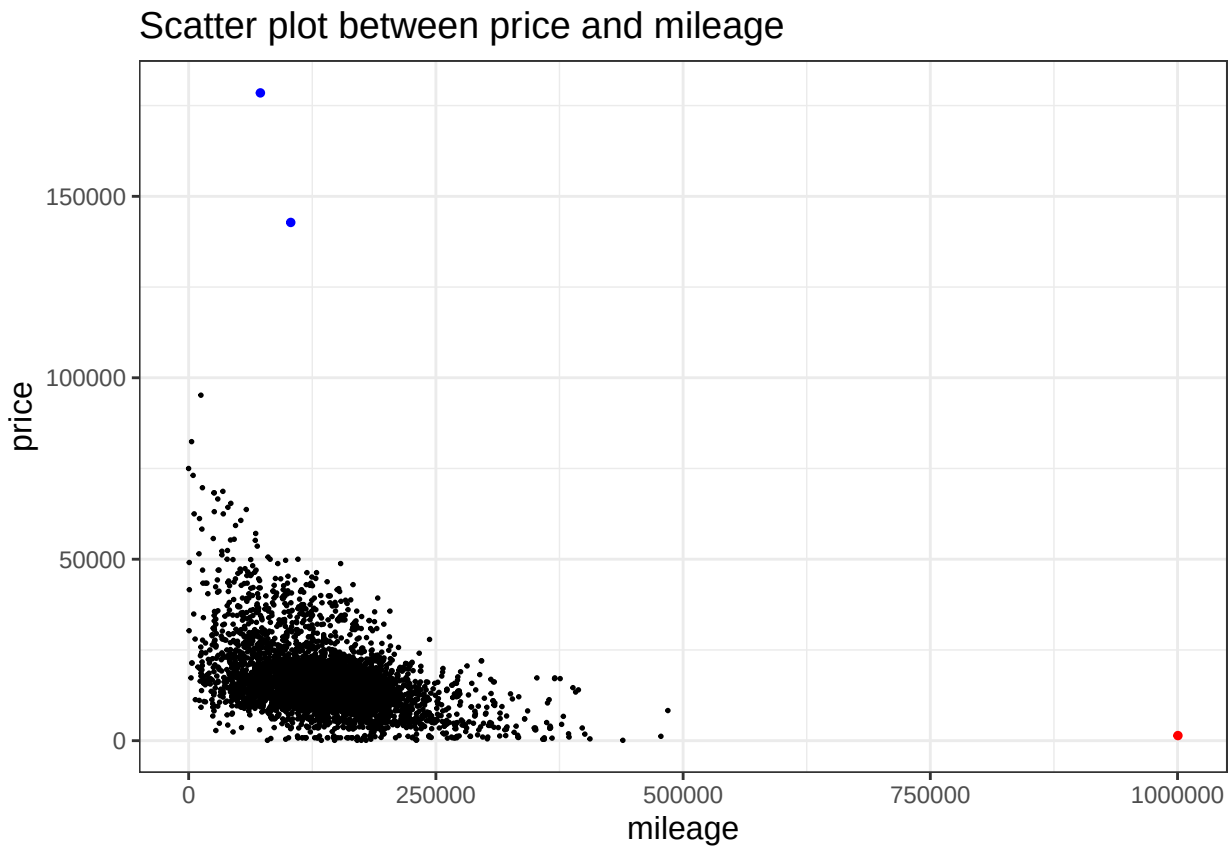


Missing data and implausible value handling

We drew the scatter plot between price and mileage to explore potential outliers. We highlighted these potential outliers.

```
scatterPlotPriceMileage <- ggplot(data = dat_bmw2, mapping = aes(x = mileage, y =
 ↪  price)) +
  geom_point(pch=19, cex=0.3) +  # Default points
  geom_point(data = subset(dat_bmw2, mileage > 500000), mapping = aes(x = mileage, y =
   ↪  price), pch=19, cex=1, color = "red") +  # Highlight points with mileage >
   ↪  500000 in red
  geom_point(data = subset(dat_bmw2, price > 100000), mapping = aes(x = mileage, y =
   ↪  price), pch=19, cex=1, color = "blue") + # Highlight points with price > 100000
   ↪  in blue
  labs(title = "Scatter plot between price and mileage") +
```

```
  theme_bw() + theme(title = element_text(size=12))

scatterPlotPriceMileage
```



Before, we fit the model, we removed the outliers we observed from the scatter plot (price > 100000 and mileage > 500000). We reasonably conclude that these points are errors in data entry that skew the model too heavily.

```
dat_bmw_clean <- dat_bmw |>
    filter(mileage < 5e+05 & mileage > 0) |>
    filter(price < 1e+05) |>
    filter(engine_power != 0)

head(dat_bmw_clean)
```

```
##   maker_key model_key mileage engine_power registration_date   fuel paint_color
## 1       BMW       118  140411          100          2/1/2012 diesel       black
## 2       BMW        M4   13929          317          4/1/2016 petrol        grey
## 3       BMW       320  183297          120          4/1/2012 diesel       white
## 4       BMW       420  128035          135          7/1/2014 diesel         red
## 5       BMW       425   97097          160         12/1/2014 diesel      silver
## 6       BMW       335  152352          225          5/1/2011 petrol       black
##      car_type feature_1 feature_2 feature_3 feature_4 feature_5 feature_6
## 1 convertible      TRUE      TRUE     FALSE     FALSE      TRUE      TRUE
## 2 convertible      TRUE      TRUE     FALSE     FALSE     FALSE      TRUE
## 3 convertible     FALSE     FALSE     FALSE     FALSE      TRUE     FALSE
```

```
## 4 convertible      TRUE      TRUE     FALSE     FALSE      TRUE      TRUE
## 5 convertible      TRUE      TRUE     FALSE     FALSE     FALSE      TRUE
## 6 convertible      TRUE      TRUE     FALSE     FALSE      TRUE      TRUE
##   feature_7 feature_8 price  sold_at obs_type
## 1      TRUE     FALSE 11300 1/1/2018 Training
## 2      TRUE      TRUE 69700 2/1/2018 Training
## 3      TRUE     FALSE 10200 2/1/2018 Training
## 4      TRUE      TRUE 25100 2/1/2018 Training
## 5      TRUE      TRUE 33400 4/1/2018 Training
## 6      TRUE      TRUE 17100 2/1/2018 Training
```

We regrouped the variable "model_key" based on cars' series.

```
### Create Model series variable
dat_bmw_clean <- dat_bmw_clean %>%
    mutate(model_series = case_when(grepl("^1", model_key) ~
        "1_Series", grepl("^2", model_key) ~ "2_Series", grepl("^3",
        model_key) ~ "3_Series", grepl("^4", model_key) ~ "4_Series",
        grepl("^5", model_key) ~ "5_Series", grepl("^7", model_key) ~
            "7_Series", grepl("^M|M$", model_key) ~ "M_Power",
        model_key %in% c("X1") ~ "X1", model_key %in% c("X3") ~
            "X3", model_key %in% c("X5") ~ "X5", model_key %in%
            c("X6") ~ "X6", TRUE ~ "Other"))
```

Modelling and Analysis for the Full Data Set

Question 1

To explore which factor between mileage and engine power impacts the price of the car the most, we fit the baseline model: $price = \beta_0 + \beta_1 mileage + \beta_2 engine\ power + \epsilon$.

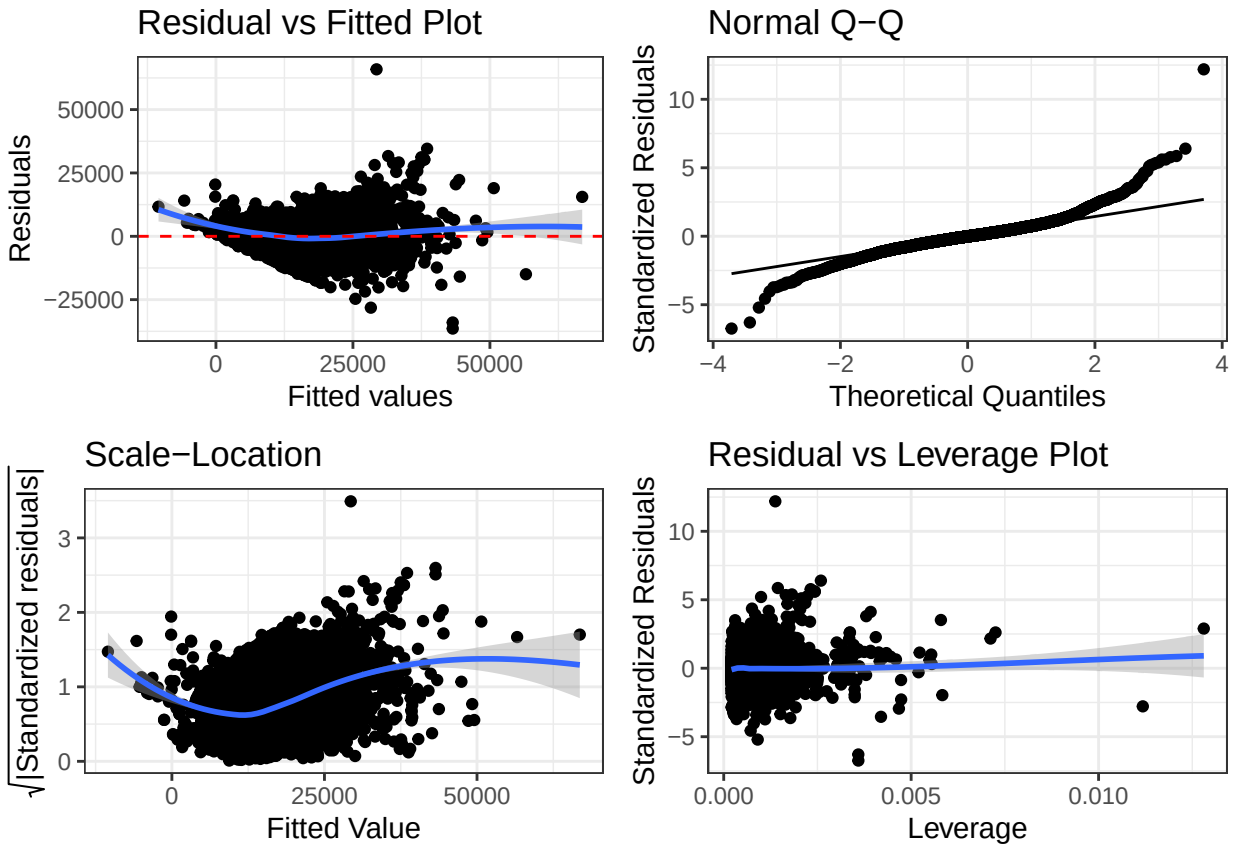
```
baseline_model <- lm(price ~ mileage + engine_power, data = dat_bmw_clean)
summary(baseline_model)
```

```
##
## Call:
## lm(formula = price ~ mileage + engine_power, data = dat_bmw_clean)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -36415  -2785    -73   2546  65891
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   5.193e+03  3.345e+02   15.53   <2e-16 ***
## mileage      -5.883e-02  1.322e-03  -44.49   <2e-16 ***
## engine_power  1.462e+02  2.000e+00   73.08   <2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5410 on 4835 degrees of freedom
## Multiple R-squared:  0.6127, Adjusted R-squared:  0.6125
## F-statistic:  3824 on 2 and 4835 DF,  p-value: < 2.2e-16
```

```
diagPlot(baseline_model)
```

```
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
```

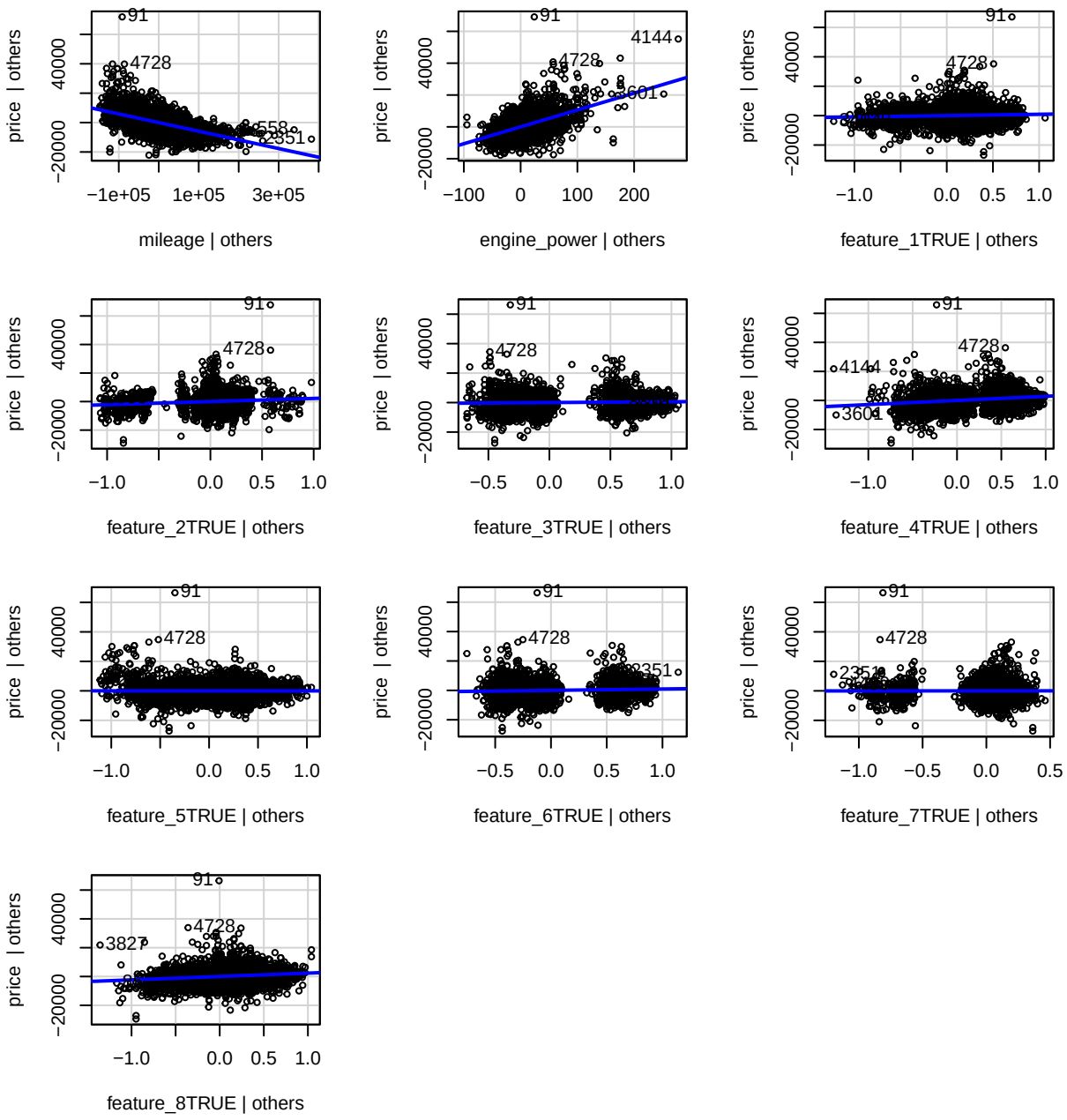


Question 2

We utilized added variable plots to solve our second question: "Which categorical predictors explain a significant amount of variance in auction price, after controlling for mileage, engine power?".

We first investigated if any feature variables could explain a significant amount of variance in auction price.

```
feature_model <- lm(price ~ mileage + engine_power + feature_1 +
    feature_2 + feature_3 + feature_4 + feature_5 + feature_6 +
    feature_7 + feature_8, data = dat_bmw_clean)

avPlots(feature_model, layout = c(4, 3))
```



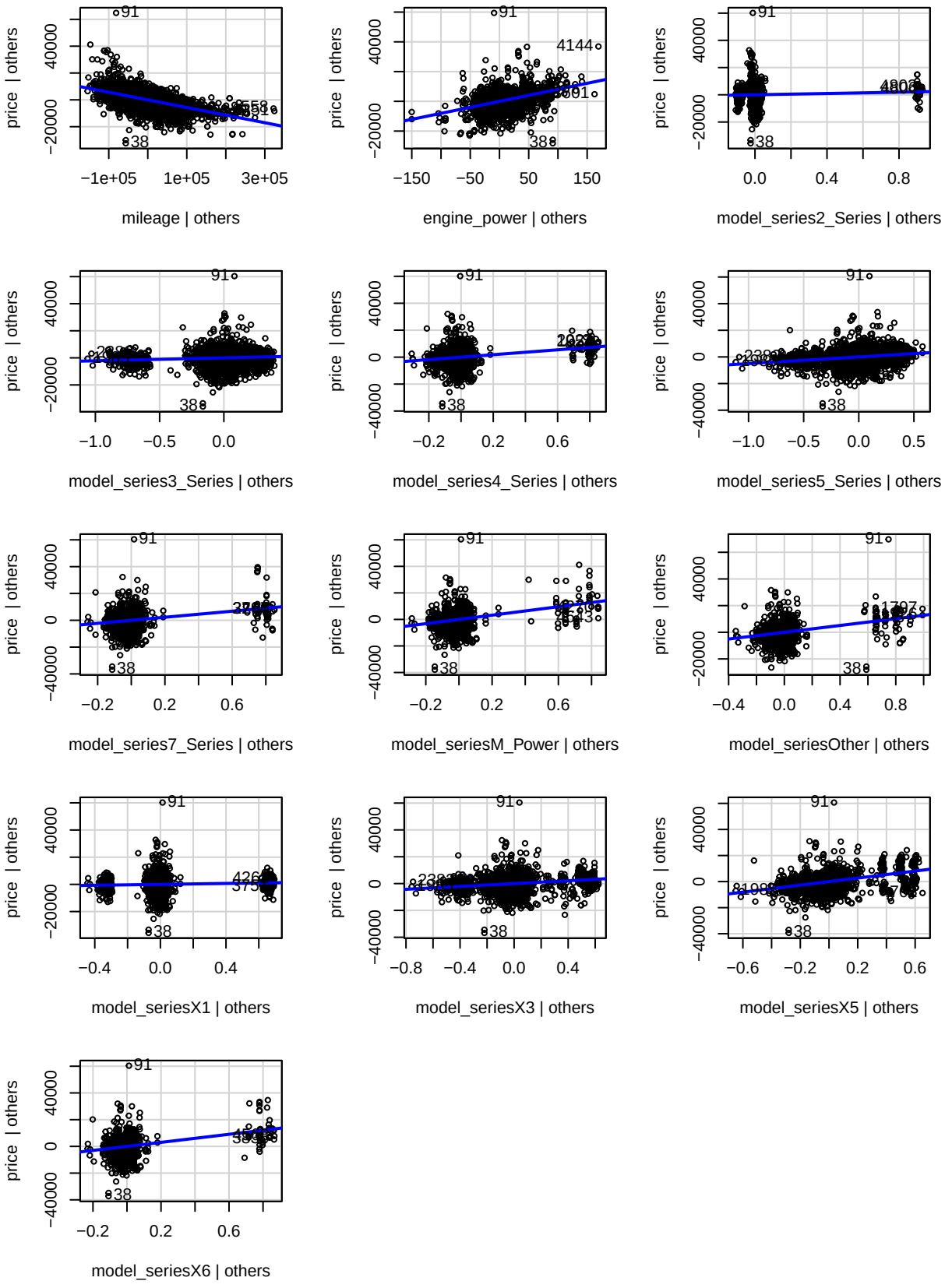
We then investigated if model series could explain a significant amount of variance in auction price with model: $price = \beta_0 + \beta_1 mileage + \beta_2 engine\ power + \beta_3 model\ series + \epsilon$.

```
key_model <- lm(price ~ mileage + engine_power + model_series,
    data = dat_bmw_clean)
summary(key_model)
```

```
##
## Call:
## lm(formula = price ~ mileage + engine_power + model_series, data = dat_bmw_clean)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -35607  -2007    266   2432  60188
```

```
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)           9.240e+03  3.521e+02  26.243  < 2e-16 ***
## mileage              -5.681e-02  1.225e-03 -46.380  < 2e-16 ***
## engine_power          8.080e+01  2.543e+00  31.770  < 2e-16 ***
## model_series2_Series  2.389e+03  6.975e+02   3.426 0.000618 ***
## model_series3_Series  2.169e+03  2.377e+02   9.126  < 2e-16 ***
## model_series4_Series  9.068e+03  5.249e+02  17.275  < 2e-16 ***
## model_series5_Series  4.992e+03  2.798e+02  17.841  < 2e-16 ***
## model_series7_Series  1.128e+04  7.279e+02  15.495  < 2e-16 ***
## model_seriesM_Power   1.596e+04  7.991e+02  19.967  < 2e-16 ***
## model_seriesOther     1.274e+04  5.736e+02  22.213  < 2e-16 ***
## model_seriesX1        1.742e+03  3.520e+02   4.949 7.73e-07 ***
## model_seriesX3        5.334e+03  3.201e+02  16.661  < 2e-16 ***
## model_seriesX5        1.349e+04  4.427e+02  30.482  < 2e-16 ***
## model_seriesX6        1.508e+04  8.018e+02  18.804  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4744 on 4824 degrees of freedom
## Multiple R-squared:  0.7028, Adjusted R-squared:  0.702
## F-statistic: 877.4 on 13 and 4824 DF,  p-value: < 2.2e-16
```

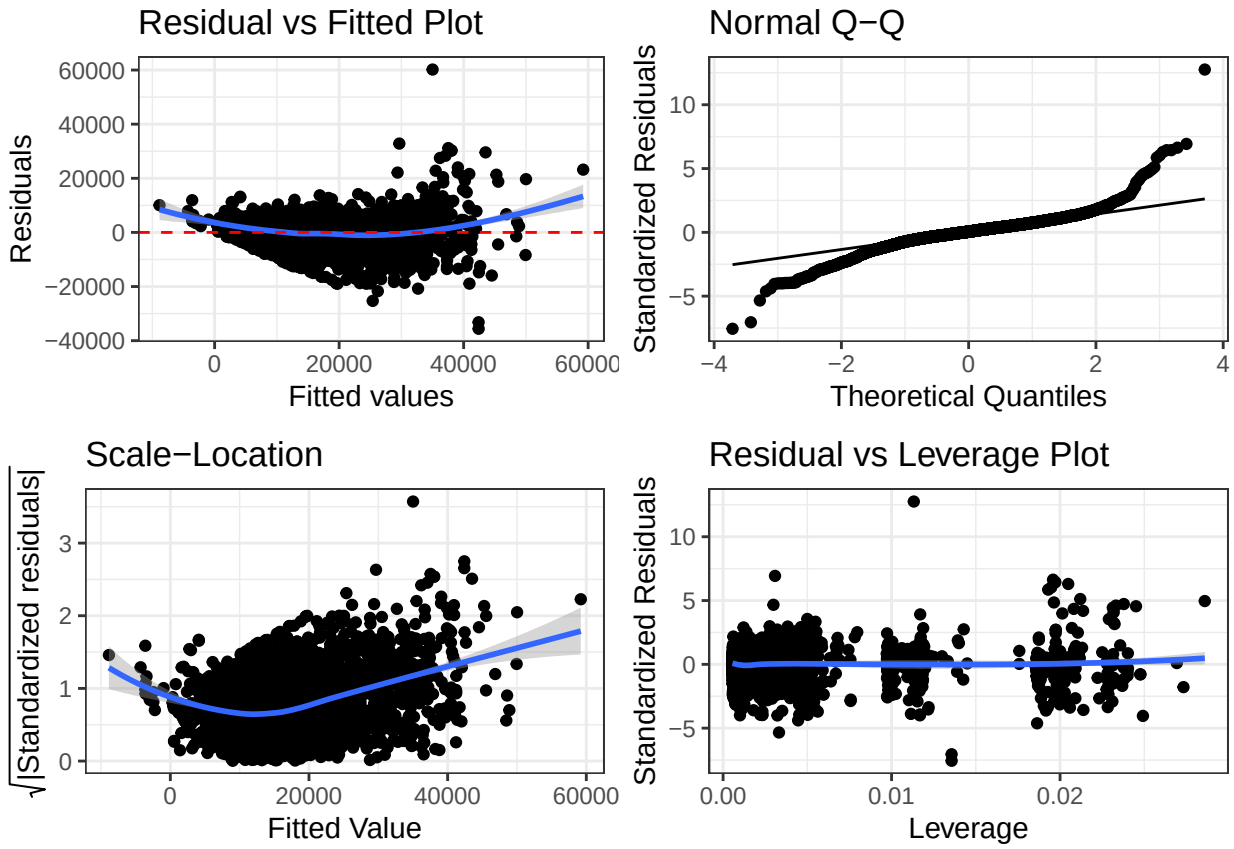
```r
avPlots(key_model, layout = c(5, 3))
```




```
diagPlot(key_model)
```

```
## `geom_smooth()` using formula = 'y ~ x'
```

```
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
```



Model Refinement

Based on the curved line observed in the "Residual vs Fitted" Plot in the "key_model" and the quadratic relationship between price and mileage, we add $\sqrt{mileage}$ to the model: $price = \beta_0 + \beta_1 mileage + \beta_2 engine\ power + \beta_3 model\ series + \beta 4\sqrt{mileage} + \epsilon$. The fitted line in the "Residual vs Fitted" Plot is not as quadratic as it in the "key_model". The deviated tail may be caused by potential outliers.

```
refine_model <- lm(price ~ mileage + engine_power + model_series +
    I(mileage^(1/2)), data = dat_bmw_clean)
summary(refine_model)
```

```
##
## Call:
## lm(formula = price ~ mileage + engine_power + model_series +
##     I(mileage^(1/2)), data = dat_bmw_clean)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -37193  -2065    357   2464  57586
##
## Coefficients:
```

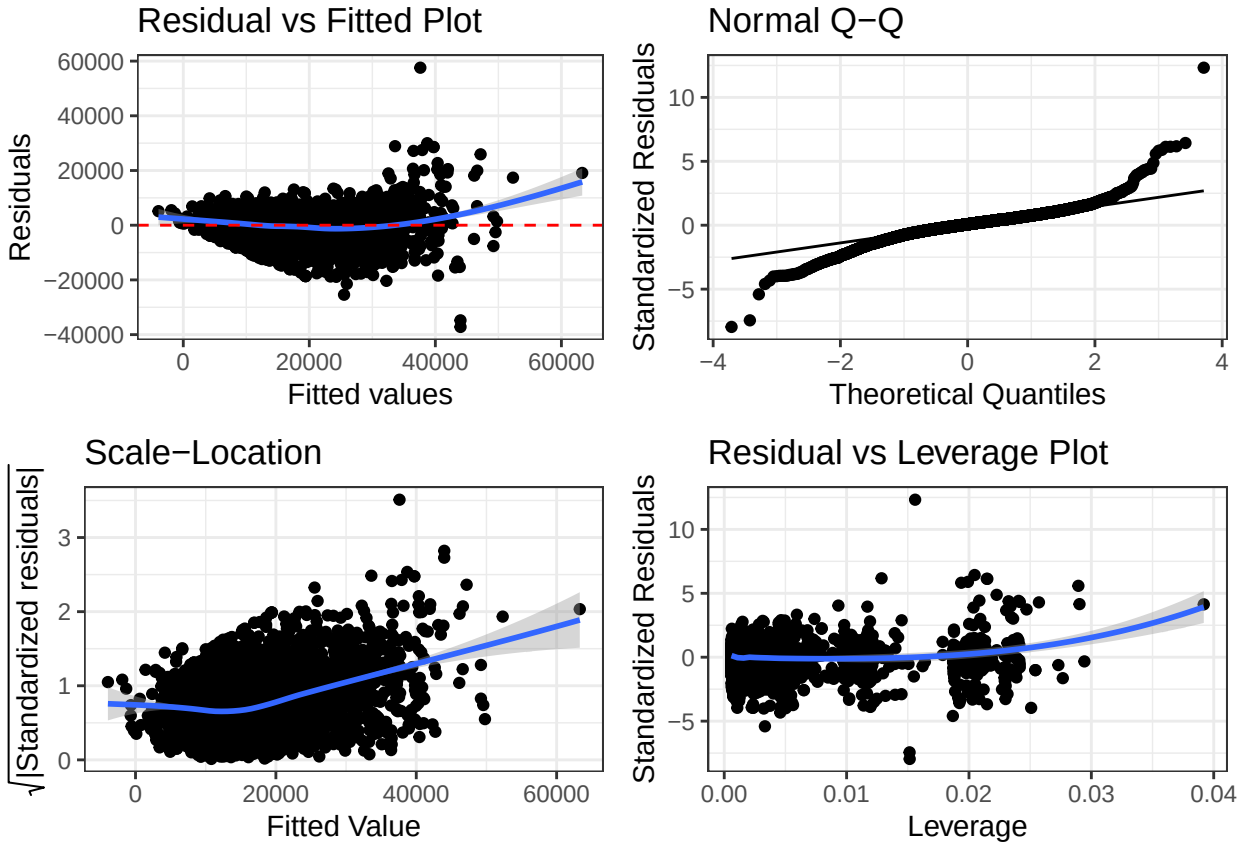
```
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)             1.550e+04  8.191e+02  18.918  < 2e-16 ***
## mileage                -7.259e-03  5.992e-03  -1.211   0.2258
## engine_power            8.105e+01  2.525e+00  32.098  < 2e-16 ***
## model_series2_Series    1.602e+03  6.987e+02   2.292   0.0219 *
## model_series3_Series    2.305e+03  2.366e+02   9.746  < 2e-16 ***
## model_series4_Series    8.727e+03  5.227e+02  16.698  < 2e-16 ***
## model_series5_Series    5.062e+03  2.779e+02  18.213  < 2e-16 ***
## model_series7_Series    1.120e+04  7.227e+02  15.496  < 2e-16 ***
## model_seriesM_Power     1.552e+04  7.950e+02  19.528  < 2e-16 ***
## model_seriesOther       1.248e+04  5.702e+02  21.890  < 2e-16 ***
## model_seriesX1          1.894e+03  3.499e+02   5.411 6.57e-08 ***
## model_seriesX3          5.472e+03  3.182e+02  17.194  < 2e-16 ***
## model_seriesX5          1.342e+04  4.396e+02  30.528  < 2e-16 ***
## model_seriesX6          1.492e+04  7.962e+02  18.740  < 2e-16 ***
## I(mileage^(1/2))       -3.640e+01  4.310e+00  -8.446  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4710 on 4823 degrees of freedom
## Multiple R-squared:  0.7071, Adjusted R-squared:  0.7063
## F-statistic: 831.7 on 14 and 4823 DF,  p-value: < 2.2e-16
```

```
anova(key_model, refine_model)  #partial F-test between key_model and refine_model
```

```
## Analysis of Variance Table
##
## Model 1: price ~ mileage + engine_power + model_series
## Model 2: price ~ mileage + engine_power + model_series + I(mileage^(1/2))
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1   4824 1.0857e+11
## 2   4823 1.0699e+11  1 1582302175 71.328 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
diagPlot(refine_model)
```

```
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
```



We further added the interaction term between mileage and engine_power to see how engine_power influence the association between price and mileage. The model was $price = \beta_0 + \beta_1 mileage + \beta_2 engine\ power + \beta_3 model\ series + \beta 4\sqrt{mileage} + \beta_5 mileage \times engine\ power + \epsilon$

```
interact_model <- lm(price ~ mileage + engine_power + model_series +
    I(mileage^(1/2)) + mileage:engine_power, data = dat_bmw_clean)
summary(interact_model)
```

```
##
## Call:
## lm(formula = price ~ mileage + engine_power + model_series +
##     I(mileage^(1/2)) + mileage:engine_power, data = dat_bmw_clean)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -43422  -1962    368   2378  56422
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)             6.568e+03  9.513e+02   6.904 5.70e-12 ***
## mileage                 4.731e-02  6.635e-03   7.129 1.16e-12 ***
## engine_power            1.440e+02  4.419e+00  32.575  < 2e-16 ***
## model_series2_Series    2.289e+03  6.797e+02   3.367 0.000765 ***
## model_series3_Series    1.885e+03  2.310e+02   8.159 4.26e-16 ***
## model_series4_Series    8.160e+03  5.086e+02  16.044  < 2e-16 ***
## model_series5_Series    4.917e+03  2.700e+02  18.213  < 2e-16 ***
```

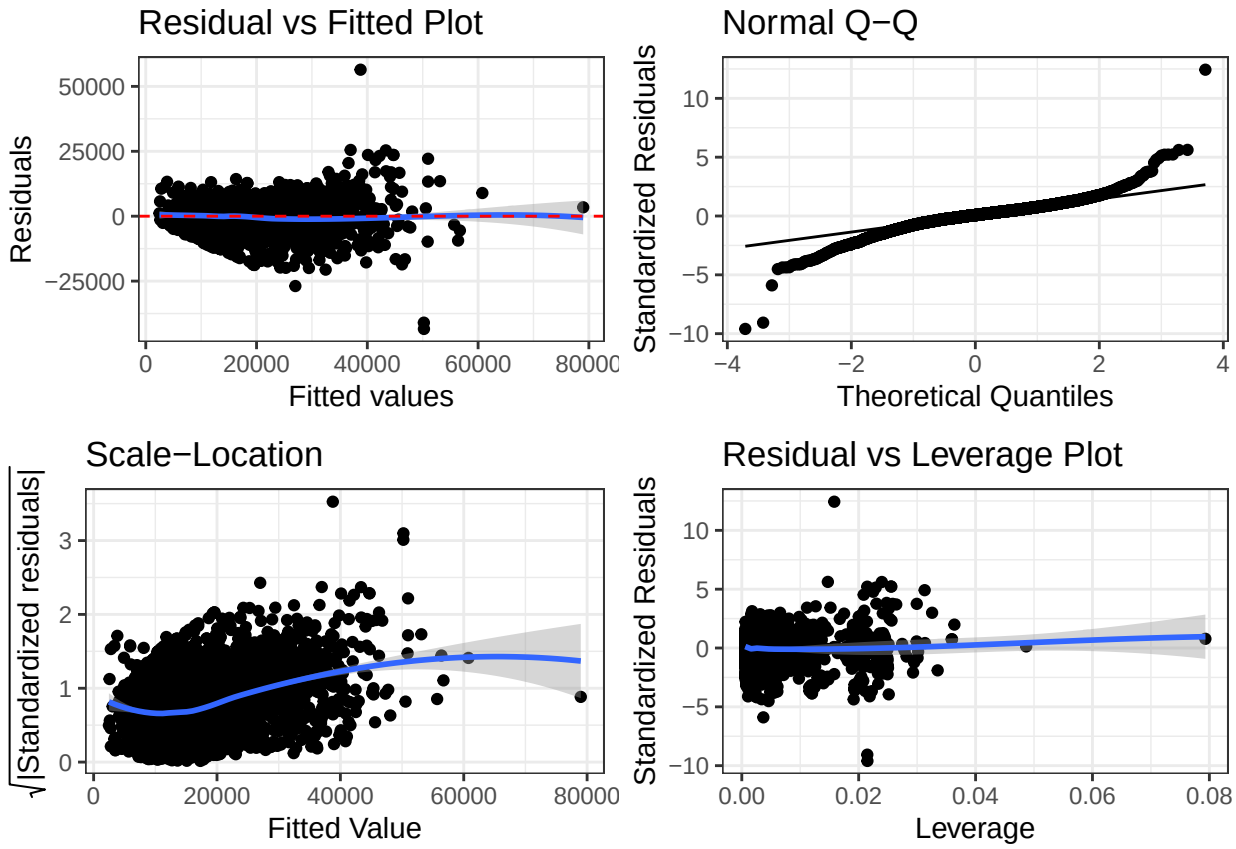
```
## model_series7_Series  1.094e+04  7.020e+02  15.582  < 2e-16 ***
## model_seriesM_Power   1.366e+04  7.796e+02  17.526  < 2e-16 ***
## model_seriesOther     1.161e+04  5.561e+02  20.875  < 2e-16 ***
## model_seriesX1        1.750e+03  3.399e+02   5.149 2.72e-07 ***
## model_seriesX3        5.241e+03  3.093e+02  16.942  < 2e-16 ***
## model_seriesX5        1.302e+04  4.275e+02  30.465  < 2e-16 ***
## model_seriesX6        1.409e+04  7.747e+02  18.182  < 2e-16 ***
## I(mileage^(1/2))     -3.108e+01  4.197e+00  -7.405 1.54e-13 ***
## mileage:engine_power -4.723e-04  2.761e-05 -17.109  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4574 on 4822 degrees of freedom
## Multiple R-squared:  0.7239, Adjusted R-squared:  0.723
## F-statistic: 842.8 on 15 and 4822 DF,  p-value: < 2.2e-16
```

```
anova(refine_model, interact_model)
```

```
## Analysis of Variance Table
##
## Model 1: price ~ mileage + engine_power + model_series + I(mileage^(1/2))
## Model 2: price ~ mileage + engine_power + model_series + I(mileage^(1/2)) +
##     mileage:engine_power
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1   4823 1.0699e+11
## 2   4822 1.0087e+11  1 6123355871 292.73 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
diagPlot(interact_model)
```

```
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
```



Question 3

We calculated the variable "age" to solve the third question: "How does the annual depreciation rate (price decrease per year of age) differ across the top five model keys?"

```
dat_age <- dat_bmw_clean |>
    mutate(age = 2018 - year(as.Date(registration_date, format = "%m/%d/%Y"))) |>
    mutate(is_116 = ifelse(model_key == "116", 1, 0)) |>
    mutate(is_318 = ifelse(model_key == "318", 1, 0)) |>
    mutate(is_X1 = ifelse(model_key == "X1", 1, 0)) |>
    mutate(is_X3 = ifelse(model_key == "X3", 1, 0))

# determine models with sufficient data (n > 100)
model_candidates <- dat_bmw_clean |>
    group_by(model_key) |>
    summarize(n = n()) |>
    filter(n > 100)
model_candidates
```

```
## # A tibble: 11 x 2
##    model_key     n
##    <chr>     <int>
##  1 116         358
##  2 118         142
##  3 316         235
```

```
##  4 318         569
##  5 320         752
##  6 520         633
##  7 525         184
##  8 530         157
##  9 X1          274
## 10 X3          437
## 11 X5          231
```

```
# model_candidates$model_key

# narrow down number of models considered
coefs = c()
confintlbs = c()

for (i in seq(1:length(model_candidates$model_key))) {
    temp.df <- dat_age |>
        filter(model_key == model_candidates$model_key[i])
    temp.lm <- lm(log(price) ~ age, data = temp.df)
    coefs = c(coefs, coef(temp.lm)[2])
    confintlbs = c(confintlbs, confint(temp.lm)[2])
}

results <- cbind(coefs, confintlbs, model_candidates$model_key)
results
```

```
##     coefs                 confintlbs
## age "-0.12278957659823"   "-0.145500785077517"  "116"
## age "-0.143433541715236"  "-0.162697652058148"  "118"
## age "-0.149038443543364"  "-0.171057626029924"  "316"
## age "-0.136161010233223"  "-0.149777693349068"  "318"
## age "-0.16563587460071"   "-0.178490627890781"  "320"
## age "-0.183404655126876"  "-0.191612478867974"  "520"
## age "-0.206231690635819"  "-0.224201035913407"  "525"
## age "-0.185604933819817"  "-0.198075674413492"  "530"
## age "-0.0182544527786139" "-0.0676029110385667" "X1"
## age "-0.11571932710388"   "-0.13260525273682"   "X3"
## age "-0.171213304919039"  "-0.177936709330697"  "X5"
```

```
age_model <- lm(formula = price ~ mileage + engine_power + model_series +
    I(mileage^(1/2)) + mileage:engine_power + age + age:is_318 +
    age:is_X1 + age:is_X3 + age:is_116, data = dat_age)
summary(age_model)
```

```
##
## Call:
## lm(formula = price ~ mileage + engine_power + model_series +
##     I(mileage^(1/2)) + mileage:engine_power + age + age:is_318 +
```

```
##     age:is_X1 + age:is_X3 + age:is_116, data = dat_age)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -35980  -1533    110   1807  53514
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)            1.327e+04  8.165e+02  16.258  < 2e-16 ***
## mileage                1.126e-01  5.654e-03  19.918  < 2e-16 ***
## engine_power           1.445e+02  3.645e+00  39.643  < 2e-16 ***
## model_series2_Series   4.833e+02  5.894e+02   0.820  0.41221
## model_series3_Series   1.496e+03  2.724e+02   5.492 4.19e-08 ***
## model_series4_Series   7.114e+03  4.545e+02  15.654  < 2e-16 ***
## model_series5_Series   4.903e+03  2.852e+02  17.192  < 2e-16 ***
## model_series7_Series   1.240e+04  6.010e+02  20.631  < 2e-16 ***
## model_seriesM_Power    1.415e+04  6.591e+02  21.473  < 2e-16 ***
## model_seriesOther      1.192e+04  4.873e+02  24.472  < 2e-16 ***
## model_seriesX1        -1.629e+03  1.106e+03  -1.473  0.14094
## model_seriesX3         5.742e+03  5.442e+02  10.552  < 2e-16 ***
## model_seriesX5         1.393e+04  3.895e+02  35.759  < 2e-16 ***
## model_seriesX6         1.539e+04  6.577e+02  23.396  < 2e-16 ***
## I(mileage^(1/2))      -5.259e+01  3.501e+00 -15.020  < 2e-16 ***
## age                   -1.239e+03  2.712e+01 -45.710  < 2e-16 ***
## mileage:engine_power -5.414e-04  2.286e-05 -23.680  < 2e-16 ***
## age:is_318             7.269e+01  3.008e+01   2.417  0.01569 *
## age:is_X1              6.289e+02  2.028e+02   3.102  0.00193 **
## age:is_X3             -4.629e+01  8.148e+01  -0.568  0.56998
## age:is_116             1.145e+02  5.594e+01   2.047  0.04073 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3761 on 4817 degrees of freedom
## Multiple R-squared:  0.8134, Adjusted R-squared:  0.8127
## F-statistic:  1050 on 20 and 4817 DF,  p-value: < 2.2e-16
```

```
vif(age_model)
```

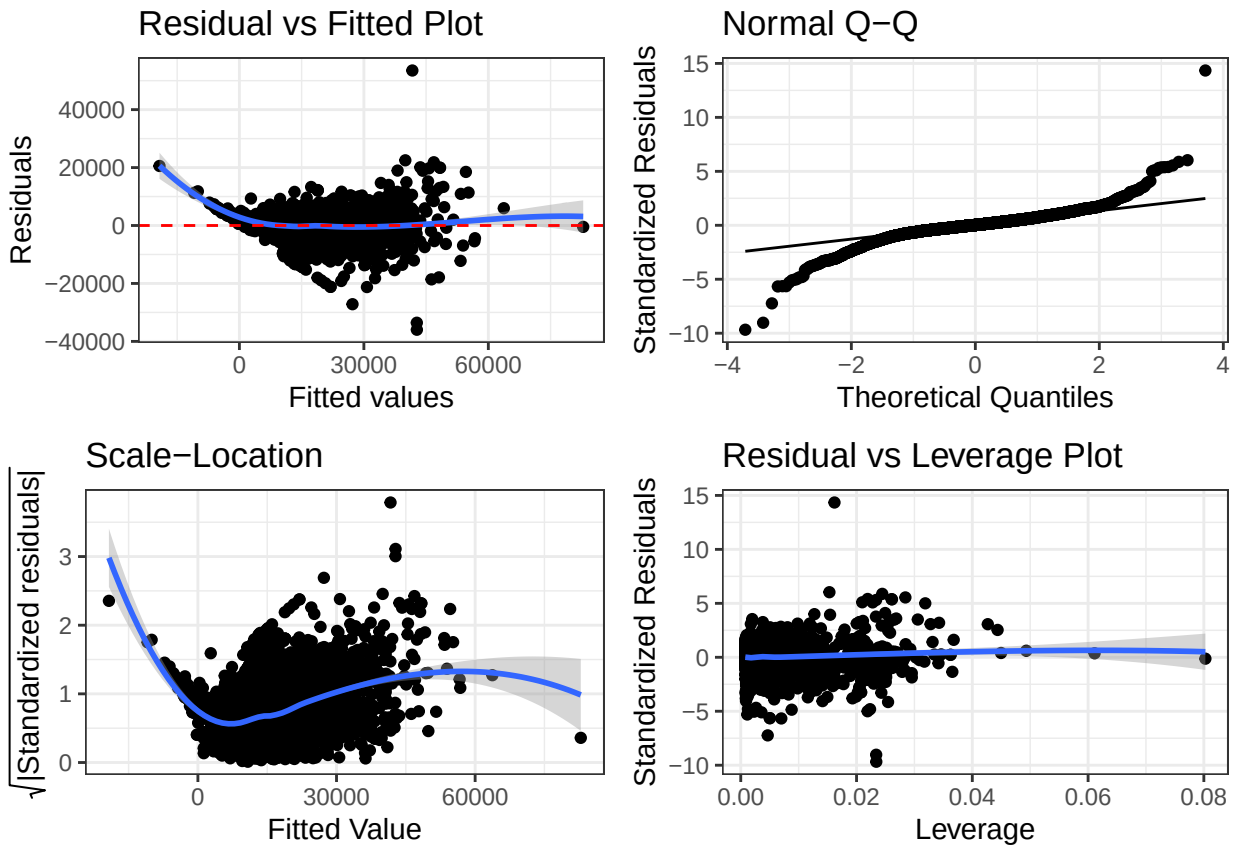
```
## there are higher-order terms (interactions) in this model
## consider setting type = 'predictor'; see ?vif

##                            GVIF Df GVIF^(1/(2*Df))
## mileage                37.904179  1        6.156637
## engine_power            6.888626  1        2.624619
## model_series          814.487760 11        1.356166
## I(mileage^(1/2))       28.659254  1        5.353434
## age                     1.602662  1        1.265963
## mileage:engine_power   15.477916  1        3.934198
## age:is_318              1.333149  1        1.154621
```

```
## age:is_X1              21.331302  1        4.618582
## age:is_X3               6.746347  1        2.597373
## age:is_116              2.314989  1        1.521509
```

```
diagPlot(age_model)
```

```
## `geom_smooth()` using formula = 'y ~ x'
```

```
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
```



Final Model

As we found before, age significantly associated with price, therefore we included it into our final model: $price = \beta_0 + \beta_1 mileage + \beta_2 engine\ power + \beta_3 model\ series + \beta 4\sqrt{mileage} + \beta_5 mileage \times engine\ power + \beta_6 age + \epsilon$.

```
dat_bmw_clean$sold_at <- as.Date(dat_bmw_clean$sold_at, format = "%m/%d/%Y")
dat_bmw_clean$registration_date <- as.Date(dat_bmw_clean$registration_date,
    format = "%m/%d/%Y")
dat_bmw_clean$age <- as.numeric((dat_bmw_clean$sold_at -
 ↪  dat_bmw_clean$registration_date)/365.25)
```

```r
final_model <- lm(formula = price ~ mileage + engine_power +
    model_series + I(mileage^(1/2)) + mileage:engine_power +
    age, data = dat_bmw_clean)
summary(final_model)
```

```
##
## Call:
## lm(formula = price ~ mileage + engine_power + model_series +
##     I(mileage^(1/2)) + mileage:engine_power + age, data = dat_bmw_clean)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -35270  -1546     91   1850  54055
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)             1.327e+04  7.997e+02  16.594  < 2e-16 ***
## mileage                 1.109e-01  5.652e-03  19.616  < 2e-16 ***
## engine_power            1.429e+02  3.656e+00  39.100  < 2e-16 ***
## model_series2_Series    2.472e+02  5.639e+02   0.438    0.661
## model_series3_Series    1.308e+03  1.915e+02   6.830 9.52e-12 ***
## model_series4_Series    6.900e+03  4.216e+02  16.365  < 2e-16 ***
## model_series5_Series    4.659e+03  2.234e+02  20.854  < 2e-16 ***
## model_series7_Series    1.221e+04  5.813e+02  21.006  < 2e-16 ***
## model_seriesM_Power     1.420e+04  6.450e+02  22.018  < 2e-16 ***
## model_seriesOther       1.177e+04  4.600e+02  25.581  < 2e-16 ***
## model_seriesX1          1.369e+03  2.813e+02   4.868 1.16e-06 ***
## model_seriesX3          5.270e+03  2.559e+02  20.596  < 2e-16 ***
## model_seriesX5          1.383e+04  3.541e+02  39.055  < 2e-16 ***
## model_seriesX6          1.536e+04  6.415e+02  23.939  < 2e-16 ***
## I(mileage^(1/2))       -5.126e+01  3.498e+00 -14.653  < 2e-16 ***
## age                    -1.206e+03  2.556e+01 -47.171  < 2e-16 ***
## mileage:engine_power   -5.453e-04  2.289e-05 -23.825  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3784 on 4821 degrees of freedom
## Multiple R-squared:  0.8111, Adjusted R-squared:  0.8105
## F-statistic:  1294 on 16 and 4821 DF,  p-value: < 2.2e-16
```

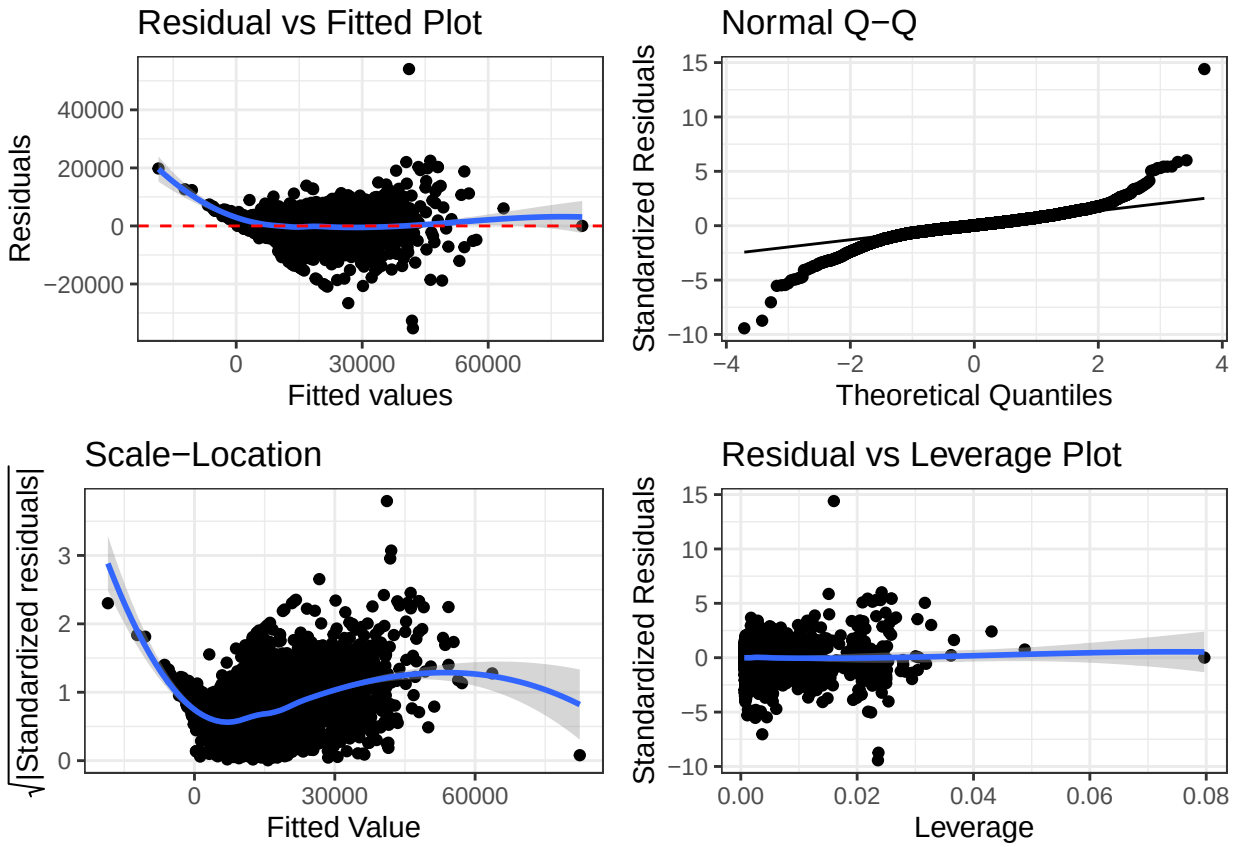
```r
anova(interact_model, final_model)
```

```
## Analysis of Variance Table
##
## Model 1: price ~ mileage + engine_power + model_series + I(mileage^(1/2)) +
##     mileage:engine_power
## Model 2: price ~ mileage + engine_power + model_series + I(mileage^(1/2)) +
##     mileage:engine_power + age
```

```
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1   4822 1.0087e+11
## 2   4821 6.9014e+10  1 3.1853e+10 2225.1 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
diagPlot(final_model)
```

```
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
```



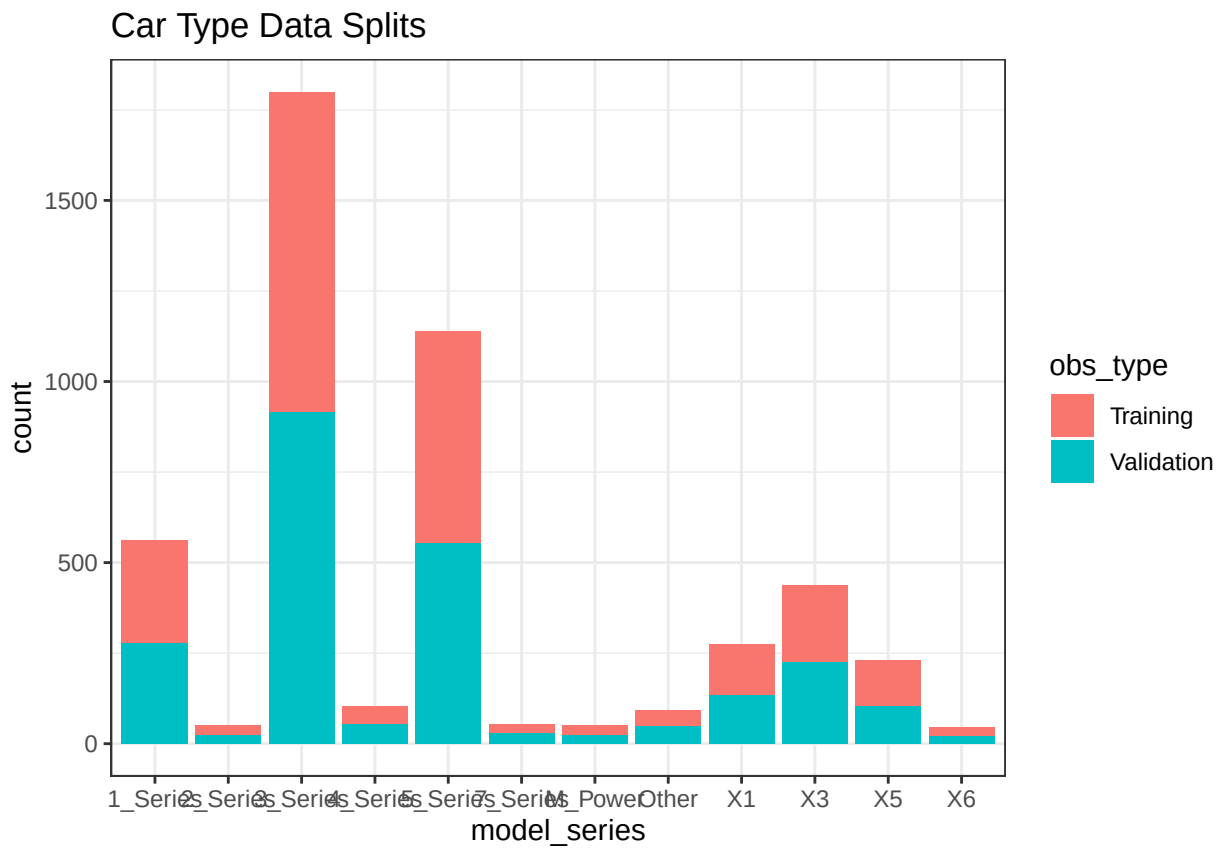
Modelling and Analysis for the Training Data Set

Split Training and Testing Data

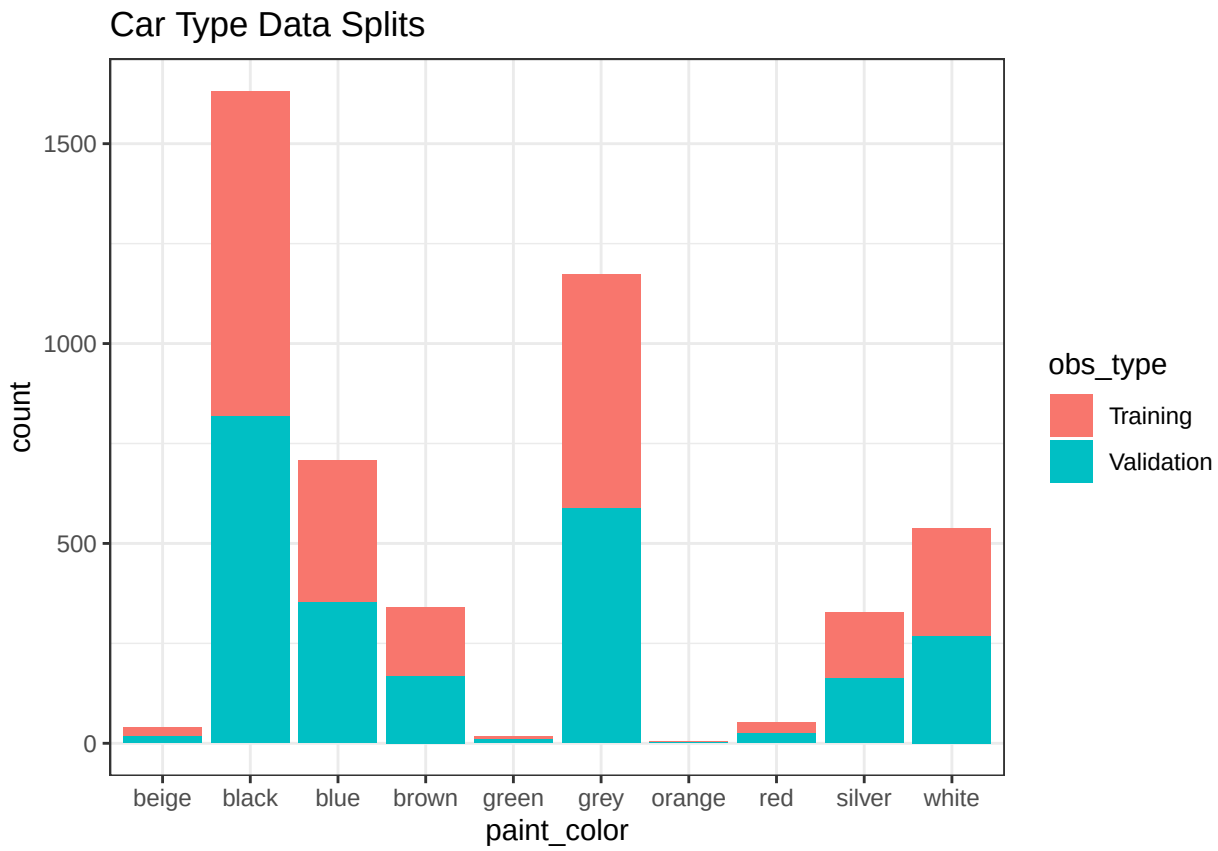
```
train_index <- which(dat_bmw_clean$obs_type == "Training")
test_index <- which(dat_bmw_clean$obs_type == "Validation")
dat_bmw_train <- dat_bmw_clean[train_index, ]
dat_bmw_test <- dat_bmw_clean[test_index, ]
```

Data Overview of the Training and Testing Data

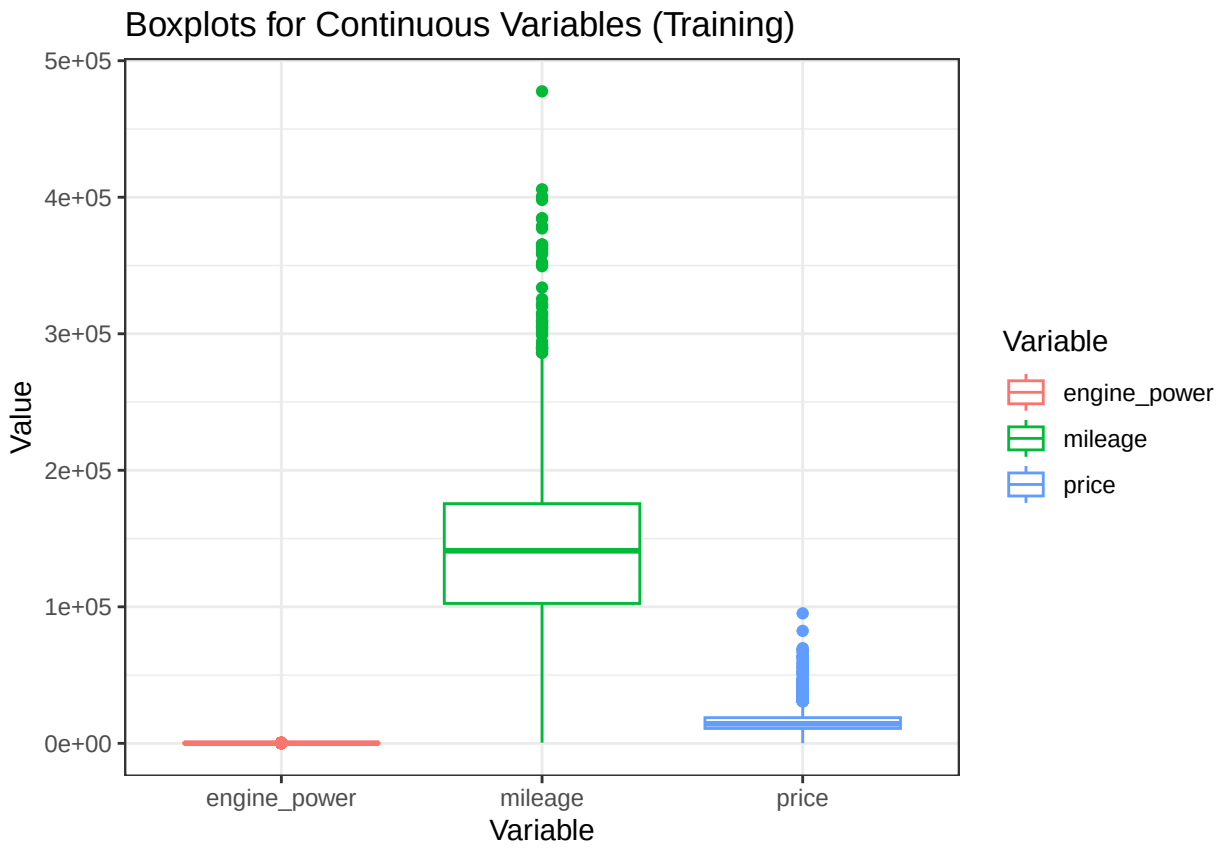
```
dat_bmw_clean |>
    ggplot(aes(x = model_series, fill = obs_type)) + geom_bar() +
    labs(title = "Car Type Data Splits") + theme(axis.text.x = element_text(angle =
     ↪  30)) +
    theme_bw()
```




```
dat_bmw_clean |>
    ggplot(aes(x = paint_color, fill = obs_type)) + geom_bar() +
    labs(title = "Car Type Data Splits") + theme(axis.text.x = element_text(angle =
     ↪  30)) +
    theme_bw()
```




```
dat_bmw_clean[, c("price", "mileage", "engine_power", "obs_type")] |>
    filter(obs_type == "Training") |>
    dplyr::select(price, mileage, engine_power) |>
    pivot_longer(everything(), values_to = "Value", names_to = "Variable") |>
    ggplot() + geom_boxplot(aes(x = Variable, y = Value, color = Variable)) +
    labs(title = "Boxplots for Continuous Variables (Training)") +
    theme_bw()
```



Final Model Fit on Training Data

The final model we fit is $price = \beta_0 + \beta_1 mileage + \beta_2 engine\,power + \beta_3 model\,series + \beta_4 age + \beta 5\sqrt{mileage} + \beta_6 mileage \times engine\,power + \epsilon$.

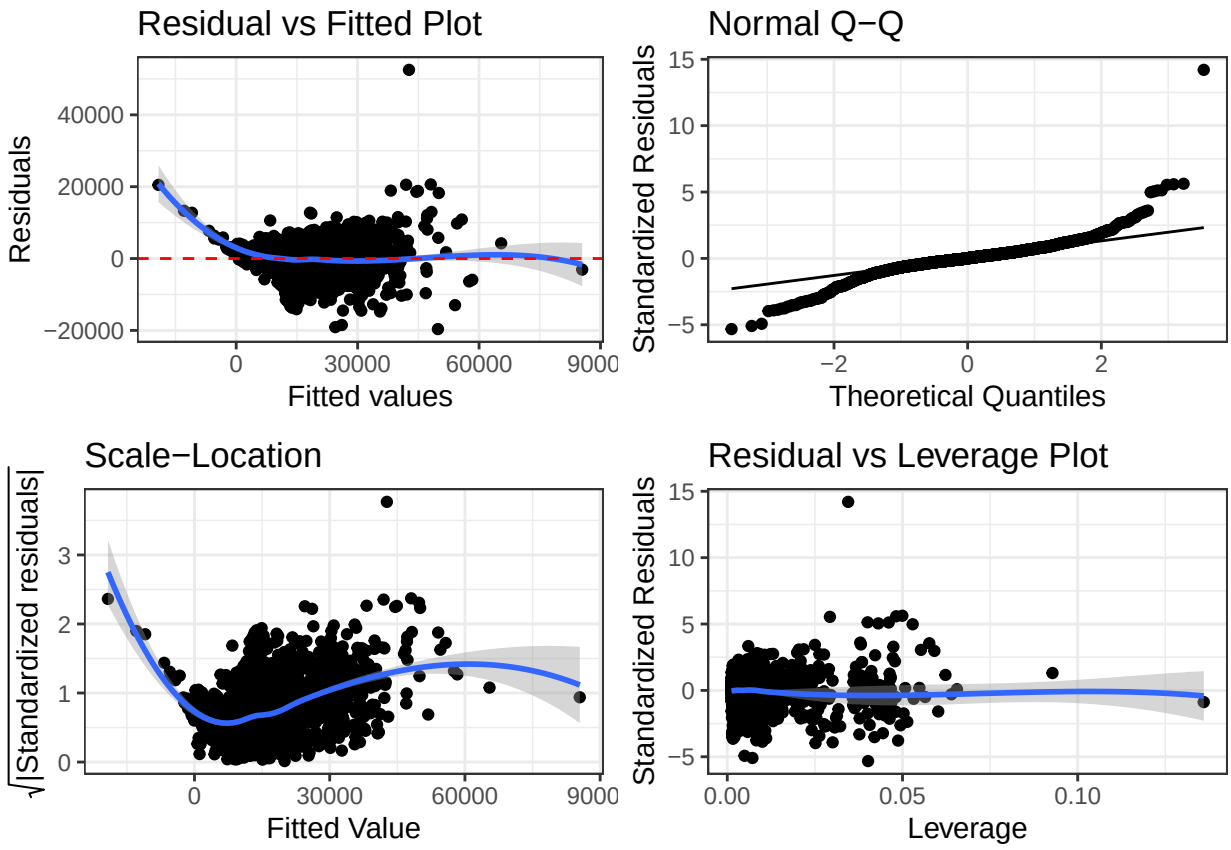
```
final_model_train <- lm(price ~ mileage + engine_power + model_series +
    age + I(mileage^(1/2)) + mileage:engine_power, data = dat_bmw_train)
summary(final_model_train)
```

```
##
## Call:
## lm(formula = price ~ mileage + engine_power + model_series +
##     age + I(mileage^(1/2)) + mileage:engine_power, data = dat_bmw_train)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -19635  -1583     57   1716  52516
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)             1.423e+04  1.173e+03  12.128  < 2e-16 ***
## mileage                 1.289e-01  7.851e-03  16.419  < 2e-16 ***
## engine_power            1.504e+02  5.023e+00  29.945  < 2e-16 ***
## model_series2_Series -3.297e+02  7.598e+02  -0.434 0.664418
## model_series3_Series  1.097e+03  2.676e+02   4.098 4.31e-05 ***
```

```
## model_series4_Series  6.372e+03  6.026e+02  10.574  < 2e-16 ***
## model_series5_Series  4.348e+03  3.117e+02  13.952  < 2e-16 ***
## model_series7_Series  1.281e+04  8.726e+02  14.677  < 2e-16 ***
## model_seriesM_Power   1.372e+04  9.143e+02  15.008  < 2e-16 ***
## model_seriesOther     1.198e+04  6.530e+02  18.343  < 2e-16 ***
## model_seriesX1        1.391e+03  3.909e+02   3.559 0.000379 ***
## model_seriesX3        5.376e+03  3.598e+02  14.942  < 2e-16 ***
## model_seriesX5        1.302e+04  4.848e+02  26.852  < 2e-16 ***
## model_seriesX6        1.409e+04  8.929e+02  15.775  < 2e-16 ***
## age                  -1.235e+03  3.590e+01 -34.399  < 2e-16 ***
## I(mileage^(1/2))     -5.973e+01  5.061e+00 -11.803  < 2e-16 ***
## mileage:engine_power -5.994e-04  3.097e-05 -19.353  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3763 on 2414 degrees of freedom
## Multiple R-squared:  0.8231, Adjusted R-squared:  0.8219
## F-statistic:   702 on 16 and 2414 DF,  p-value: < 2.2e-16
```

```
diagPlot(final_model_train)
```

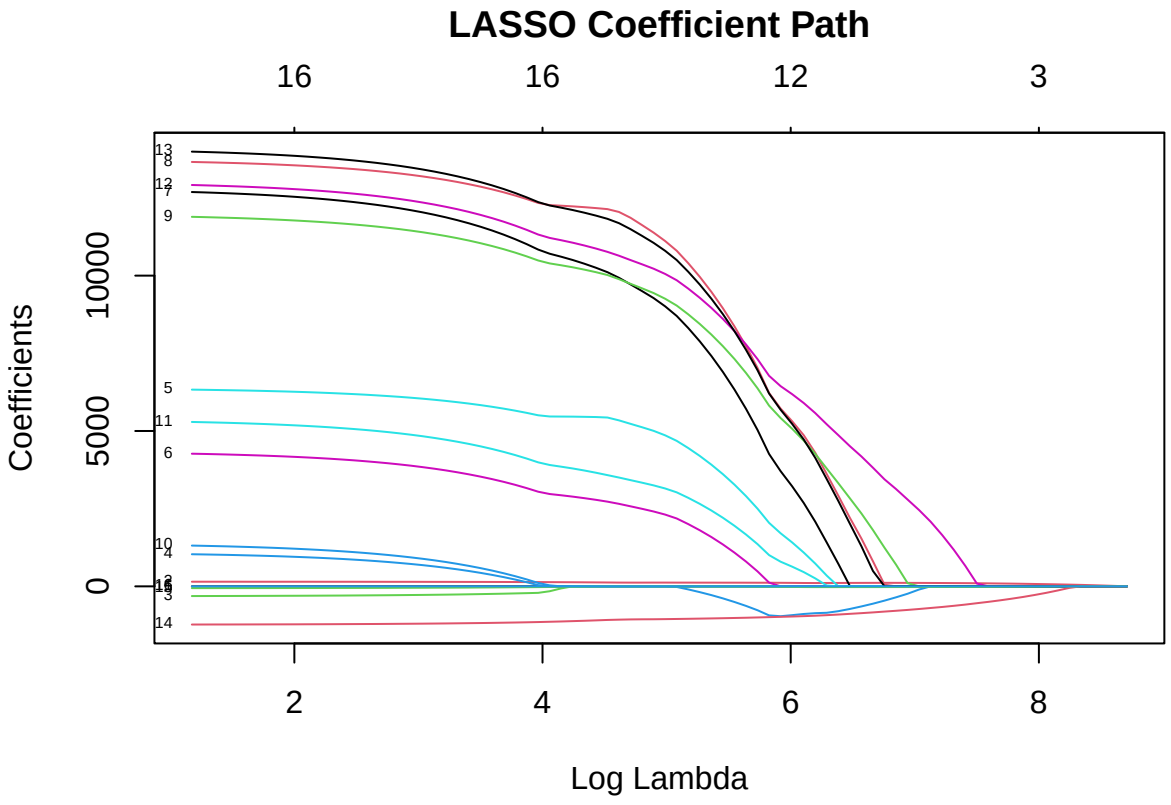
```
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
## `geom_smooth()` using formula = 'y ~ x'
```



Lasso Regression

We performed Lasso regression for the variable selection.

```
# Create a model matrix
X <- model.matrix(price ~ mileage + engine_power + model_series +
    age + I(mileage^(1/2)) + mileage:engine_power, data = dat_bmw_train)[,
    -1]

# Create the response variable
Y <- dat_bmw_train$price

# set seed for reproductivity
set.seed(20250408)

# Fit the LASSO regression model using glmnet with alpha =
# 1
lasso_model <- glmnet(x = X, y = Y, alpha = 1)

# The x-axis shows log(lambda) and each line corresponds to
# a coefficient.
plot(lasso_model, xvar = "lambda", label = TRUE)
title("LASSO Coefficient Path", line = 2.5)
```



We used cross-validation to determine the lambda that minimizes the mean squared error.

```r
cv_model <- cv.glmnet(x = X, y = Y, alpha = 1)
best_lambda <- cv_model$lambda.min

cat("Best Lambda - LASSO:", best_lambda)
```

```
## Best Lambda - LASSO: 3.234687
```

The coefficient for the interaction was close to 0 (i.e., $\beta_{mileage \times engine\ power} = -5.87 \times 10^{-4}$). Therefore, we used partial F-test to investigate if we can remove this interaction term.

```r
lasso_coef <- coef(lasso_model, s = best_lambda)
print(lasso_coef)
```

```
## 17 x 1 sparse Matrix of class "dgCMatrix"
##                                    s1
## (Intercept)            1.397773e+04
## mileage                1.233594e-01
## engine_power           1.491882e+02
## model_series2_Series -3.124235e+02
## model_series3_Series  1.033387e+03
## model_series4_Series  6.331205e+03
## model_series5_Series  4.269069e+03
## model_series7_Series  1.269119e+04
## model_seriesM_Power   1.365871e+04
## model_seriesOther     1.189258e+04
## model_seriesX1        1.311942e+03
## model_seriesX3        5.290716e+03
## model_seriesX5        1.291696e+04
## model_seriesX6        1.399076e+04
## age                  -1.228902e+03
## I(mileage^(1/2))     -5.700250e+01
## mileage:engine_power -5.870496e-04
```

```r
reduced_model <- lm(price ~ mileage + engine_power + model_series +
    age + I(mileage^(1/2)), data = dat_bmw_train)
anova(final_model_train, reduced_model)
```

```
## Analysis of Variance Table
##
## Model 1: price ~ mileage + engine_power + model_series + age + I(mileage^(1/2)) +
##     mileage:engine_power
## Model 2: price ~ mileage + engine_power + model_series + age + I(mileage^(1/2))
##   Res.Df        RSS Df   Sum of Sq      F    Pr(>F)
## 1   2414 3.4175e+10
## 2   2415 3.9477e+10 -1 -5302067949 374.52 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Prediction

With the testing data, We used the coefficients from Lasso regression to evaluate the prediction performance of our final model: $price = \beta_0 + \beta_1 mileage + \beta_2 engine\ power + \beta_3 model\ series + \beta_4 age + \beta 5\sqrt{mileage} + \beta_6 mileage \times engine\ power + \epsilon$.

```
newX <- model.matrix(price ~ mileage + engine_power + model_series +
    age + I(mileage^(1/2)) + mileage:engine_power, data = dat_bmw_test)[,
    -1]

best_model <- glmnet(x = X, y = Y, alpha = 1, lambda = best_lambda)

pred_values <- predict(best_model, newx = newX)
obs_values <- dat_bmw_test$price

RMSE <- RMSE(pred_values, obs_values)
MAE <- MAE(pred_values, obs_values)
MAPE <- MAPE(pred_values, obs_values)

rbind(RMSE, MAE, MAPE)
```

```
##              [,1]
## RMSE 3826.6398506
## MAE  2538.6284046
## MAPE    0.6997168
```

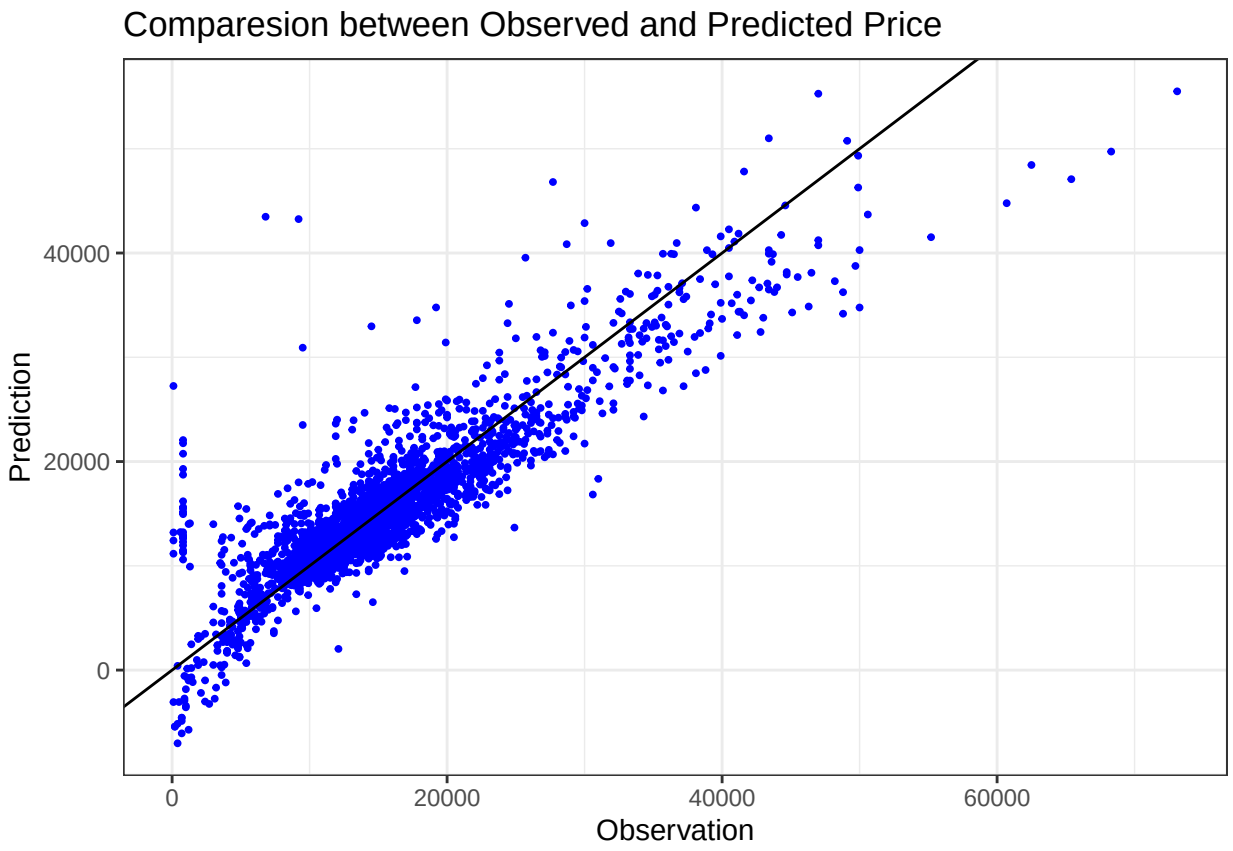
```
summary(dat_bmw_test$price)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     100   10800   14000   15601   18600   73100
```

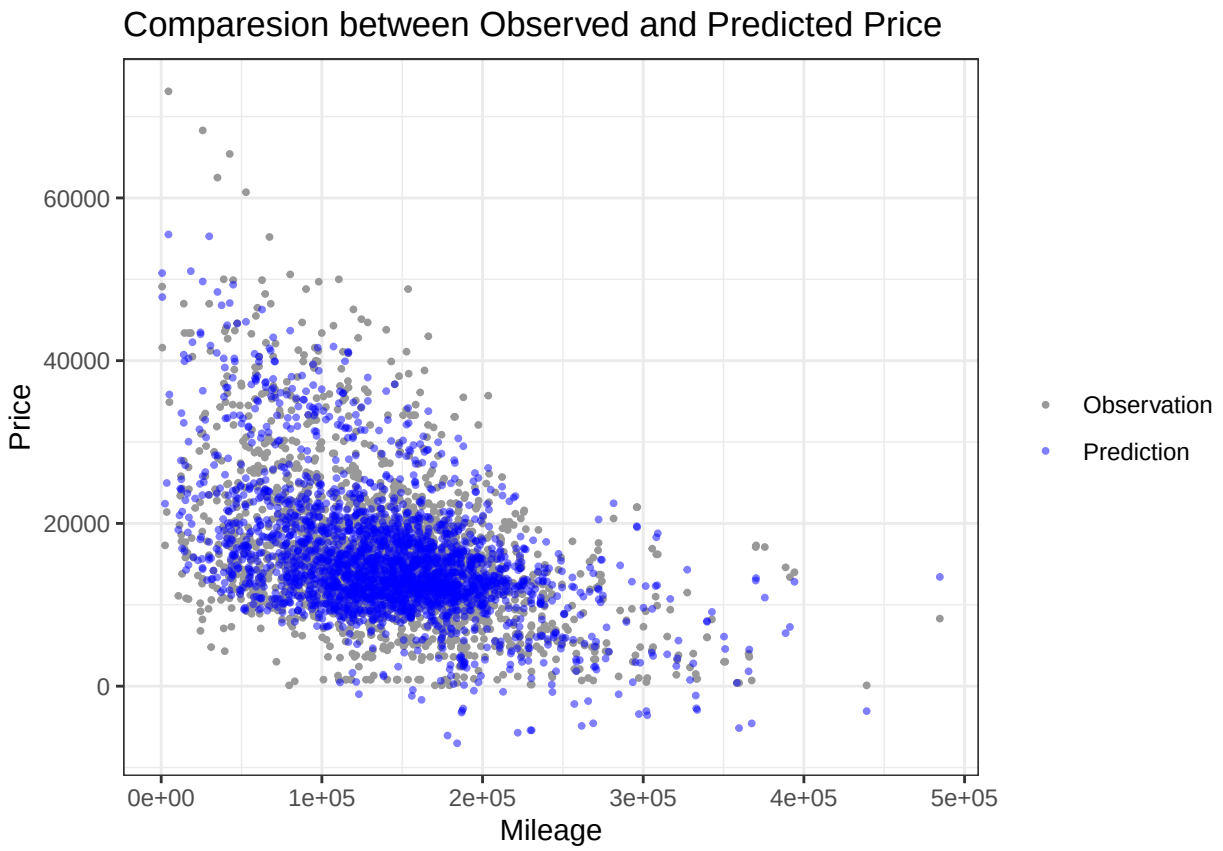
```
summary(dat_bmw_clean$price)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     100   10800   14200   15760   18600   95200
```

```
dat_bmw_test$pred <- pred_values

ggplot(data = dat_bmw_test, mapping = aes(x = price, y = pred)) +
    geom_point(size = 0.7, color = "blue") + labs(title = "Comparesion between Observed
     ↪  and Predicted Price") +
    xlab("Observation") + ylab("Prediction") + geom_abline(slope = 1,
    intercept = 0) + theme_bw()
```




```
ggplot() + geom_point(data = dat_bmw_test, aes(x = mileage, y = price,
    color = "Observation"), size = 0.7) + geom_point(data = dat_bmw_test,
    aes(x = mileage, y = pred, color = "Prediction"), alpha = 0.5,
    size = 0.7) + labs(title = "Comparesion between Observed and Predicted Price") +
    xlab("Mileage") + ylab("Price") + scale_color_manual(name = element_blank(),
    values = c(Observation = "grey60", Prediction = "blue")) +
    theme_bw()
```



Data and Codes Availability

All data and codes could be found in our GitHub repository: "https://github.com/mengyaoww/MA575_Lab2_GroupA.git".