

Homework 1

Yuqiao Meng

1. Array: [1, 2, -1, 4, 10]
 - a. Mean = $(1+2-1+4+10)/5=3.2$
 - b. Median = 2
 - c. Variance = $((1-3.2)^2+(1-3.2)^2+(1-3.2)^2+(1-3.2)^2+(1-3.2)^2)/(5-1) = 17.7$
2. $E[Z] = E[(X^2)Y] = E[X^2]E[Y] = (\text{Var}[X] + E[X]^2)*E[Y] = (1+2^2)*3 = 15$
3. $P[Y|X] = P[X,Y]/P[X] = 0.5$
4. $\log 45 = \log(5*3*3) = \log 5 + \log 3 + \log 3 = 2.32+1.58+1.58 = 5.48$
5. The money he is likely to get is: $E(\text{money}) = 90*0.05 + 10*0.1 + (-10)*(1-0.05-0.1) = 4.5+1-8.5 = -3$. So I think he should not enter the lottery.
6. $P = 10!/5!(0.4)^5(0.6)^5 = 0.2006581248$
7. The probability B win one round: $P = 5/12$, thus the probability of B win the game:
 $P(B) = C_{10}^6(P)^6(1-P)^4 + C_{10}^7(P)^7(1-P)^3 + C_{10}^8(P)^8(1-P)^2 + C_{10}^9(P)^9(1-P) + P^{10}$
8. 5 red, 5 green, 4 yellow, 6 white
 - a. $P(\text{white}) = 6 / (5+5+6+4) = 0.3$
 - b. $P(\text{Green}) = 3 / (5+3+6+4) = 1/6$
 - c. $P(\text{white}) = 6 / (5+2+6+4) = 6/17$
 - d. $P(\text{white}) = P(\text{the fourth ball is white})P(\text{white} | \text{the fourth ball is white}) + P(\text{the fourth ball is not white})P(\text{white} | \text{the fourth ball is not white}) = 6/17*5/16 + 11/17*6/16 = 96/272$
9. $P(\text{win}) = 1/4*(20\%+50\%+50\%+90\%) = 0.525$
10. Expected gain = $n*(0.2*10+0.8*(-5)) = -2n$