Machine Learning: Programming Exercise 1

Linear Regression

In this exercise, you will implement linear regression and get to see it work on data.

Files needed for this exercise

- `ex1.mlx` - MATLAB Live Script that steps you through the exercise
- ex1data1.txt - Dataset for linear regression with one variable
- ex1data2.txt - Dataset for linear regression with multiple variables
- `submit.m` - Submission script that sends your solutions to our servers
- *`warmUpExercise.m` - Simple example function in MATLAB
- *`plotData.m` - Function to display the dataset
- *`computeCost.m` - Function to compute the cost of linear regression
- *`gradientDescent.m` - Function to run gradient descent
- **`computeCostMulti.m` - Cost function for multiple variables
- **`gradientDescentMulti.m` - Gradient descent for multiple variables
- **`featureNormalize.m` - Function to normalize features
- **`normalEqn.m` - Function to compute the normal equations

indicates files you will need to complete

****indicates optional exercises***

Clear existing variables and confirm that your Current Folder is set correctly

Click into this section, then click the 'Run Section' button above. This will execute the `clear` command to clear existing variables and the `dir` command to list the files in your Current Folder. The output should contain all of the files listed above and the 'lib' folder. If it does not, right-click the 'ex1' folder and select 'Open' before proceding or see the instructions in `README.mlx` for more details.

```
clear
dir
```

```
.                    computeCost.m         ex1data1.txt          gradientDescentMulti.m  token.mat
..                   computeCostMulti.m    ex1data2.txt          normalEqn.m             warmUpExer
Copy_of_submit.m     ex1.mlx               featureNormalize.m    plotData.m
computeCost.asv      ex1_companion.mlx     gradientDescent.m     submit.m
```

Before you begin

The workflow for completing and submitting the programming exercises in MATLAB Online differs from the original course instructions. Before beginning this exercise, make sure you have read through the instructions in `README.mlx` which is included with the programming exercise files. `README.mlx` also contains solutions to the many common issues you may encounter while completing and submitting the exercises in MATLAB Online. Make sure you are following instructions in `README.mlx` and have checked for an existing solution before seeking help on the discussion forums.

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1. A simple MATLAB function

The first part of this script gives you practice with MATLAB syntax and the homework submission process. In the file `warmUpExercise.m`, you will find the outline of a MATLAB function. Modify it to return a 5 x 5 identity matrix by filling in the following code:

```
A = eye(5);
```

When you are finished, save `warmUpExercise.m`, then run the code contained in this section to call `warmUpExercise()`.

5x5 Identity Matrix:

```
warmUpExercise()
```

```
ans = 5×5
     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
```

You should see output similar to the following:

```
ans =

     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
```

You can toggle between right-hand-side output and in-line output for printing results and displaying figures inside a Live Script by selecting the appropriate box in the upper right of the Live Editor window.

1.1 Submitting Solutions

After completing a part of the exercise, you can submit your solutions for that section by running the section of code below, which calls the `submit.m` script. Your score for each section will then be displayed as output. **Enter your login and your unique submission token *inside the command window* when prompted. For future submissions of this exercise, you will only be asked to confirm your credentials.** Your submission token for each exercise is found in the corresponding homework assignment course page. New tokens can be generated if you are experiencing issues with your current token. You are allowed to submit your solutions multiple times, and we will take only the highest score into consideration.

You should now submit your solutions. Enter **`submit`** *at the command prompt, then enter your login and token when prompted.*

2. Linear regression with one variable

In this part of this exercise, you will implement linear regression with one variable to predict profits for a food truck. Suppose you are the CEO of a restaurant franchise and are considering different cities for opening a new outlet. The chain already has trucks in various cities and you have data for profits and populations from the cities. You would like to use this data to help you select which city to expand to next.

The file `ex1data1.txt` contains the dataset for our linear regression problem. The first column is the population of a city and the second column is the profit of a food truck in that city. A negative value for profit indicates a loss. This script has already been set up to load this data for you.

2.1 Plotting the data

Before starting on any task, it is often useful to understand the data by visualizing it. For this dataset, you can use a scatter plot to visualize the data, since it has only two properties to plot (profit and population). Many other problems that you will encounter in real life are multi-dimensional and can't be plotted on a 2-d plot.

Run the code below to load the dataset from the data file into the variables X and y:

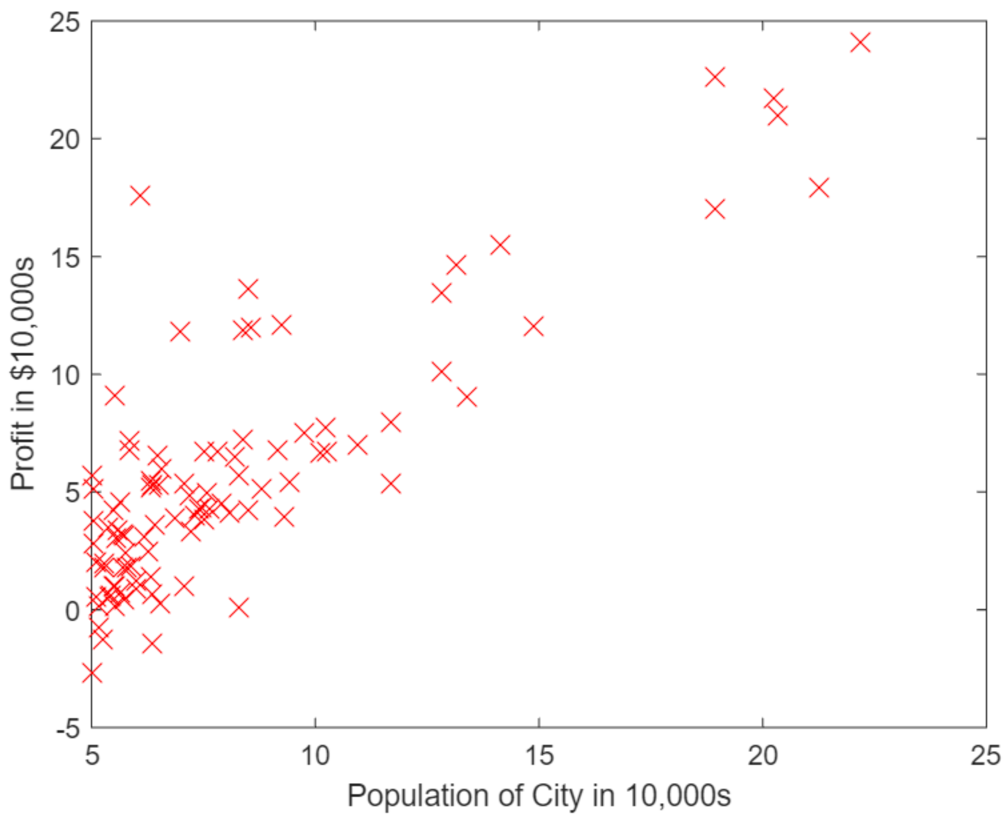
```
data = load('ex1data1.txt'); % read comma separated data
X = data(:, 1); y = data(:, 2);
```

Your job is to complete **`plotData.m`** to draw the plot; modify the file and fill in the following code:

```
plot(x, y, 'rx', 'MarkerSize', 10); % Plot the data
ylabel('Profit in $10,000s'); % Set the y-axis label
xlabel('Population of City in 10,000s'); % Set the x-axis label
```

Once you are finished, save `plotData.m`, and execute the code in this section which will call `plotData`.

```
plotData(X,y)
```



The resulting plot should appear as in Figure 1 below:

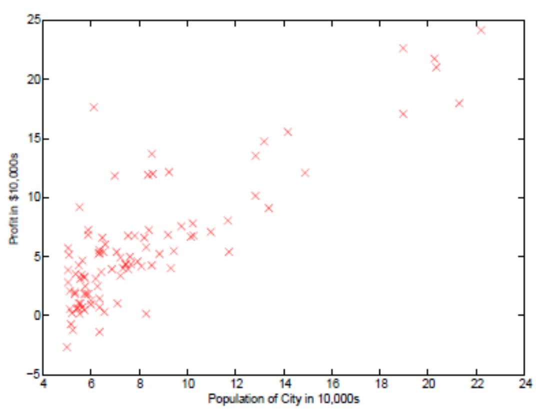


Figure 1: Scatter plot of training data

To learn more about the `plot` command, you can run the command **`help plot`** at the command prompt, type `plot()` inside the MATLAB Live Editor and click on the "(?)" tooltip, or you can search the MATLAB documentation for "plot". Note that to change the markers to red x's in the plot, we used the option: **`'rx'`** together with the **`plot`** command, i.e.,

```
plot(..,[your options here],..,'rx');
```

2.2 Gradient Descent

In this section, you will fit the linear regression parameters to our dataset using gradient descent.

2.2.1 Update Equations

The objective of linear regression is to minimize the cost function

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

where the hypothesis $h_\theta(x)$ is given by the linear model

$$h_\theta(x) = \theta^T x = \theta_0 + \theta_1 x_1$$

Recall that the parameters of your model are the θ values. These are the values you will adjust to minimize cost $J(\theta)$. One way to do this is to use the batch gradient descent algorithm. In batch gradient descent, each iteration performs the update

$$\theta_j := \theta_j - \alpha\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)x_j^{(i)} \qquad \left(\text{simultaneously update } \theta_j \text{ for all } j\right)$$

With each step of gradient descent, your parameters j come closer to the optimal values that will achieve the lowest cost $J(\theta)$.

Implementation Note: We store each example as a row in the the X matrix in MATLAB. To take into account the intercept term (θ_0), we add an additional first column to **X** and set it to all ones. This allows us to treat θ_0 as simply another 'feature'.

2.2.2 Implementation

In this script, we have already set up the data for linear regression. In the following lines, we add another dimension to our data to accommodate the θ_0 intercept term. Run the code below to initialize the parameters to 0 and the learning rate `alpha` to 0.01.

```
m = length(X); % number of training examples
X = [ones(m,1),data(:,1)]; % Add a column of ones to x
theta = zeros(2, 1); % initialize fitting parameters
iterations = 1500;
alpha = 0.01;
```

2.2.3 Computing the cost $J(\theta)$

As you perform gradient descent to minimize the cost function $J(\theta)$, it is helpful to monitor the convergence by computing the cost. In this section, you will implement a function to calculate $J(\theta)$ so you can check the convergence of your gradient descent implementation.

Your next task is to complete the code in the file `computeCost.m`, which is a function that computes $J(\theta)$. As you are doing this, remember that the variables `X` and `y` are not scalar values, but matrices whose rows represent the examples from the training set.

Once you have completed the function definition, run this section. The code below will call `computeCost` once using θ initialized to zeros, and you will see the cost printed to the screen. You should expect to see a cost of 32.07 for the first output below:

```
% Compute and display initial cost with theta all zeros
computeCost(X, y, theta)
```

```
ans = 32.0727
```

Next we call `computeCost` again, this time with non-zero `theta` values as an additional test. You should expect to see an output of 54.24 below:

```
% Compute and display initial cost with non-zero theta
computeCost(X, y,[-1; 2])
```

```
ans = 54.2425
```

If the outputs above match the expected values, you can submit your solution for assessment. If the outputs do not match or you receive an error, check your cost function for mistakes, then rerun this section once you have addressed them.

You should now submit your solutions. Enter **`submit`** *at the command prompt, then enter or confirm your login and token when prompted.*

2.2.4 Gradient descent

Next, you will implement gradient descent in the file `gradientDescent.m`. The loop structure has been written for you, and you only need to supply the updates to θ within each iteration.

As you program, make sure you understand what you are trying to optimize and what is being updated. Keep in mind that the cost $J(\theta)$ is parameterized by the vector θ, not X and y. That is, we minimize the value of $J(\theta)$ by changing the values of the vector θ, not by changing X or y. Refer to the equations given earlier and to the video lectures if you are uncertain.

A good way to verify that gradient descent is working correctly is to look at the value of `J` and check that it is decreasing with each step. The starter code for `gradientDescent.m` calls `computeCost` on every iteration and prints the cost. Assuming you have implemented gradient descent and `computeCost` correctly, your value of $J(\theta)$ should never increase, and should converge to a steady value by the end of the algorithm.

After you are finished, run this execute this section. The code below will use your final parameters to plot the linear fit. The result should look something like Figure 2 below:

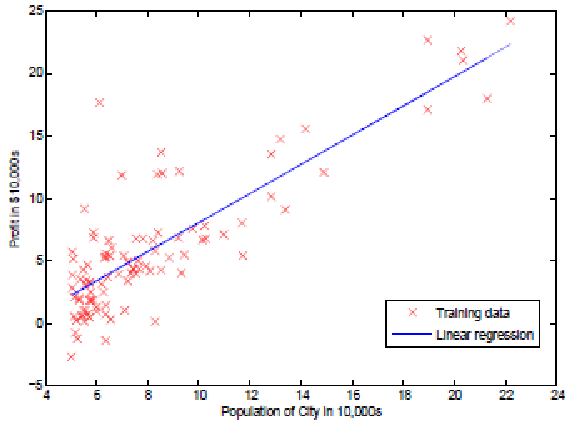


Figure 2: Training data with linear regression fit

Your final values for θ will also be used to make predictions on profits in areas of 35,000 and 70,000 people.

```
% Run gradient descent:
% Compute theta
theta = gradientDescent(X, y, theta, alpha, iterations);
```

```
theta = 2×1
    0.0584
    0.6533
theta = 2×1
    0.0629
    0.7700
theta = 2×1
    0.0578
    0.7913
theta = 2×1
    0.0511
    0.7957
theta = 2×1
    0.0440
    0.7971
theta = 2×1
    0.0369
    0.7979
theta = 2×1
    0.0298
    0.7987
theta = 2×1
    0.0228
    0.7994
theta = 2×1
    0.0157
    0.8001
theta = 2×1
    0.0086
    0.8008
theta = 2×1
    0.0016
    0.8015
```

```
theta = 2×1
   -0.0054
    0.8022
theta = 2×1
   -0.0124
    0.8029
theta = 2×1
   -0.0194
    0.8036
theta = 2×1
   -0.0264
    0.8043
theta = 2×1
   -0.0334
    0.8050
theta = 2×1
   -0.0404
    0.8057
theta = 2×1
   -0.0473
    0.8064
theta = 2×1
   -0.0542
    0.8071
theta = 2×1
   -0.0612
    0.8078
theta = 2×1
   -0.0681
    0.8085
theta = 2×1
   -0.0750
    0.8092
theta = 2×1
   -0.0819
    0.8099
theta = 2×1
   -0.0887
    0.8106
theta = 2×1
   -0.0956
    0.8113
theta = 2×1
   -0.1025
    0.8120
theta = 2×1
   -0.1093
    0.8126
theta = 2×1
   -0.1161
    0.8133
theta = 2×1
   -0.1229
    0.8140
theta = 2×1
   -0.1297
    0.8147
theta = 2×1
   -0.1365
    0.8154
theta = 2×1
   -0.1433
```

```
    0.8161
theta = 2×1
   -0.1501
    0.8167
theta = 2×1
   -0.1568
    0.8174
theta = 2×1
   -0.1636
    0.8181
theta = 2×1
   -0.1703
    0.8188
theta = 2×1
   -0.1770
    0.8194
theta = 2×1
   -0.1837
    0.8201
theta = 2×1
   -0.1904
    0.8208
theta = 2×1
   -0.1971
    0.8215
theta = 2×1
   -0.2037
    0.8221
theta = 2×1
   -0.2104
    0.8228
theta = 2×1
   -0.2170
    0.8235
theta = 2×1
   -0.2237
    0.8241
theta = 2×1
   -0.2303
    0.8248
theta = 2×1
   -0.2369
    0.8255
theta = 2×1
   -0.2435
    0.8261
theta = 2×1
   -0.2501
    0.8268
theta = 2×1
   -0.2566
    0.8274
theta = 2×1
   -0.2632
    0.8281
theta = 2×1
   -0.2698
    0.8288
theta = 2×1
   -0.2763
    0.8294
theta = 2×1
   -0.2828
```

```
    0.8301
theta = 2×1
   -0.2893
    0.8307
theta = 2×1
   -0.2958
    0.8314
theta = 2×1
   -0.3023
    0.8320
theta = 2×1
   -0.3088
    0.8327
theta = 2×1
   -0.3153
    0.8333
theta = 2×1
   -0.3217
    0.8340
theta = 2×1
   -0.3282
    0.8346
theta = 2×1
   -0.3346
    0.8353
theta = 2×1
   -0.3410
    0.8359
theta = 2×1
   -0.3474
    0.8366
theta = 2×1
   -0.3538
    0.8372
theta = 2×1
   -0.3602
    0.8378
theta = 2×1
   -0.3666
    0.8385
theta = 2×1
   -0.3729
    0.8391
theta = 2×1
   -0.3793
    0.8398
theta = 2×1
   -0.3856
    0.8404
theta = 2×1
   -0.3919
    0.8410
theta = 2×1
   -0.3983
    0.8417
theta = 2×1
   -0.4046
    0.8423
theta = 2×1
   -0.4109
    0.8429
theta = 2×1
   -0.4171
    0.8436
```

```
theta = 2×1
   -0.4234
    0.8442
theta = 2×1
   -0.4297
    0.8448
theta = 2×1
   -0.4359
    0.8455
theta = 2×1
   -0.4422
    0.8461
theta = 2×1
   -0.4484
    0.8467
theta = 2×1
   -0.4546
    0.8473
theta = 2×1
   -0.4608
    0.8480
theta = 2×1
   -0.4670
    0.8486
theta = 2×1
   -0.4732
    0.8492
theta = 2×1
   -0.4793
    0.8498
theta = 2×1
   -0.4855
    0.8504
theta = 2×1
   -0.4916
    0.8511
theta = 2×1
   -0.4978
    0.8517
theta = 2×1
   -0.5039
    0.8523
theta = 2×1
   -0.5100
    0.8529
theta = 2×1
   -0.5161
    0.8535
theta = 2×1
   -0.5222
    0.8541
theta = 2×1
   -0.5283
    0.8547
theta = 2×1
   -0.5344
    0.8553
theta = 2×1
   -0.5404
    0.8560
theta = 2×1
   -0.5465
    0.8566
theta = 2×1
```

```
   -0.5525
    0.8572
theta = 2×1
   -0.5585
    0.8578
theta = 2×1
   -0.5646
    0.8584
theta = 2×1
   -0.5706
    0.8590
theta = 2×1
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    0.8596
theta = 2×1
   -0.5825
    0.8602
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    0.8608
theta = 2×1
   -0.5945
    0.8614
theta = 2×1
   -0.6004
    0.8620
theta = 2×1
   -0.6064
    0.8626
theta = 2×1
   -0.6123
    0.8632
theta = 2×1
   -0.6182
    0.8638
theta = 2×1
   -0.6241
    0.8644
theta = 2×1
   -0.6300
    0.8650
theta = 2×1
   -0.6359
    0.8655
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   -0.6418
    0.8661
theta = 2×1
   -0.6476
    0.8667
theta = 2×1
   -0.6535
    0.8673
theta = 2×1
   -0.6593
    0.8679
theta = 2×1
   -0.6652
    0.8685
theta = 2×1
   -0.6710
    0.8691
theta = 2×1
   -0.6768
```

```
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    0.8702
theta = 2×1
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theta = 2×1
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    0.8714
theta = 2×1
   -0.7000
    0.8720
theta = 2×1
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    0.8726
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    0.8731
theta = 2×1
   -0.7172
    0.8737
theta = 2×1
   -0.7229
    0.8743
theta = 2×1
   -0.7287
    0.8749
theta = 2×1
   -0.7344
    0.8754
theta = 2×1
   -0.7401
    0.8760
theta = 2×1
   -0.7458
    0.8766
theta = 2×1
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    0.8772
theta = 2×1
   -0.7571
    0.8777
theta = 2×1
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    0.8783
theta = 2×1
   -0.7684
    0.8789
theta = 2×1
   -0.7740
    0.8794
theta = 2×1
   -0.7797
    0.8800
theta = 2×1
   -0.7853
    0.8806
theta = 2×1
   -0.7909
    0.8811
theta = 2×1
   -0.7965
    0.8817
```

```
theta = 2×1
   -0.8021
    0.8822
theta = 2×1
   -0.8077
    0.8828
theta = 2×1
   -0.8132
    0.8834
theta = 2×1
   -0.8188
    0.8839
theta = 2×1
   -0.8243
    0.8845
theta = 2×1
   -0.8299
    0.8850
theta = 2×1
   -0.8354
    0.8856
theta = 2×1
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    0.8861
theta = 2×1
   -0.8464
    0.8867
theta = 2×1
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    0.8872
theta = 2×1
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    0.8878
theta = 2×1
   -0.8629
    0.8883
theta = 2×1
   -0.8683
    0.8889
theta = 2×1
   -0.8738
    0.8894
theta = 2×1
   -0.8793
    0.8900
theta = 2×1
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    0.8905
theta = 2×1
   -0.8901
    0.8911
theta = 2×1
   -0.8955
    0.8916
theta = 2×1
   -0.9009
    0.8922
theta = 2×1
   -0.9063
    0.8927
theta = 2×1
   -0.9117
    0.8933
theta = 2×1
```

```
   -0.9171
    0.8938
theta = 2×1
   -0.9225
    0.8943
theta = 2×1
   -0.9278
    0.8949
theta = 2×1
   -0.9332
    0.8954
theta = 2×1
   -0.9385
    0.8959
theta = 2×1
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    0.8965
theta = 2×1
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    0.8970
theta = 2×1
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    0.8975
theta = 2×1
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    0.8981
theta = 2×1
   -0.9651
    0.8986
theta = 2×1
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    0.8991
theta = 2×1
   -0.9756
    0.8997
theta = 2×1
   -0.9809
    0.9002
theta = 2×1
   -0.9862
    0.9007
theta = 2×1
   -0.9914
    0.9013
theta = 2×1
   -0.9966
    0.9018
theta = 2×1
   -1.0019
    0.9023
theta = 2×1
   -1.0071
    0.9028
theta = 2×1
   -1.0123
    0.9034
theta = 2×1
   -1.0175
    0.9039
theta = 2×1
   -1.0227
    0.9044
theta = 2×1
   -1.0279
```

```
    0.9049
theta = 2×1
   -1.0330
    0.9054
theta = 2×1
   -1.0382
    0.9060
theta = 2×1
   -1.0433
    0.9065
theta = 2×1
   -1.0485
    0.9070
theta = 2×1
   -1.0536
    0.9075
theta = 2×1
   -1.0587
    0.9080
theta = 2×1
   -1.0638
    0.9085
theta = 2×1
   -1.0690
    0.9090
theta = 2×1
   -1.0740
    0.9096
theta = 2×1
   -1.0791
    0.9101
theta = 2×1
   -1.0842
    0.9106
theta = 2×1
   -1.0893
    0.9111
theta = 2×1
   -1.0943
    0.9116
theta = 2×1
   -1.0994
    0.9121
theta = 2×1
   -1.1044
    0.9126
theta = 2×1
   -1.1095
    0.9131
theta = 2×1
   -1.1145
    0.9136
theta = 2×1
   -1.1195
    0.9141
theta = 2×1
   -1.1245
    0.9146
theta = 2×1
   -1.1295
    0.9151
theta = 2×1
   -1.1345
    0.9156
```

```
theta = 2×1
   -1.1395
    0.9161
theta = 2×1
   -1.1444
    0.9166
theta = 2×1
   -1.1494
    0.9171
theta = 2×1
   -1.1543
    0.9176
theta = 2×1
   -1.1593
    0.9181
theta = 2×1
   -1.1642
    0.9186
theta = 2×1
   -1.1691
    0.9191
theta = 2×1
   -1.1741
    0.9196
theta = 2×1
   -1.1790
    0.9201
theta = 2×1
   -1.1839
    0.9206
theta = 2×1
   -1.1887
    0.9211
theta = 2×1
   -1.1936
    0.9216
theta = 2×1
   -1.1985
    0.9221
theta = 2×1
   -1.2034
    0.9226
theta = 2×1
   -1.2082
    0.9230
theta = 2×1
   -1.2131
    0.9235
theta = 2×1
   -1.2179
    0.9240
theta = 2×1
   -1.2227
    0.9245
theta = 2×1
   -1.2275
    0.9250
theta = 2×1
   -1.2323
    0.9255
theta = 2×1
   -1.2371
    0.9259
theta = 2×1
```

```
   -1.2419
    0.9264
theta = 2×1
   -1.2467
    0.9269
theta = 2×1
   -1.2515
    0.9274
theta = 2×1
   -1.2563
    0.9279
theta = 2×1
   -1.2610
    0.9283
theta = 2×1
   -1.2658
    0.9288
theta = 2×1
   -1.2705
    0.9293
theta = 2×1
   -1.2752
    0.9298
theta = 2×1
   -1.2800
    0.9302
theta = 2×1
   -1.2847
    0.9307
theta = 2×1
   -1.2894
    0.9312
theta = 2×1
   -1.2941
    0.9317
theta = 2×1
   -1.2988
    0.9321
theta = 2×1
   -1.3035
    0.9326
theta = 2×1
   -1.3081
    0.9331
theta = 2×1
   -1.3128
    0.9335
theta = 2×1
   -1.3175
    0.9340
theta = 2×1
   -1.3221
    0.9345
theta = 2×1
   -1.3267
    0.9349
theta = 2×1
   -1.3314
    0.9354
theta = 2×1
   -1.3360
    0.9359
theta = 2×1
   -1.3406
```

```
    0.9363
theta = 2×1
   -1.3452
    0.9368
theta = 2×1
   -1.3498
    0.9373
theta = 2×1
   -1.3544
    0.9377
theta = 2×1
   -1.3590
    0.9382
theta = 2×1
   -1.3636
    0.9386
theta = 2×1
   -1.3681
    0.9391
theta = 2×1
   -1.3727
    0.9396
theta = 2×1
   -1.3772
    0.9400
theta = 2×1
   -1.3818
    0.9405
theta = 2×1
   -1.3863
    0.9409
theta = 2×1
   -1.3908
    0.9414
theta = 2×1
   -1.3953
    0.9418
theta = 2×1
   -1.3998
    0.9423
theta = 2×1
   -1.4043
    0.9427
theta = 2×1
   -1.4088
    0.9432
theta = 2×1
   -1.4133
    0.9436
theta = 2×1
   -1.4178
    0.9441
theta = 2×1
   -1.4223
    0.9445
theta = 2×1
   -1.4267
    0.9450
theta = 2×1
   -1.4312
    0.9454
theta = 2×1
   -1.4356
    0.9459
```

```
theta = 2×1
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    0.9463
theta = 2×1
   -1.4445
    0.9468
theta = 2×1
   -1.4489
    0.9472
theta = 2×1
   -1.4533
    0.9477
theta = 2×1
   -1.4577
    0.9481
theta = 2×1
   -1.4621
    0.9485
theta = 2×1
   -1.4665
    0.9490
theta = 2×1
   -1.4709
    0.9494
theta = 2×1
   -1.4752
    0.9499
theta = 2×1
   -1.4796
    0.9503
theta = 2×1
   -1.4840
    0.9507
theta = 2×1
   -1.4883
    0.9512
theta = 2×1
   -1.4926
    0.9516
theta = 2×1
   -1.4970
    0.9520
theta = 2×1
   -1.5013
    0.9525
theta = 2×1
   -1.5056
    0.9529
theta = 2×1
   -1.5099
    0.9533
theta = 2×1
   -1.5142
    0.9538
theta = 2×1
   -1.5185
    0.9542
theta = 2×1
   -1.5228
    0.9546
theta = 2×1
   -1.5271
    0.9551
theta = 2×1
```

```
   -1.5313
    0.9555
theta = 2×1
   -1.5356
    0.9559
theta = 2×1
   -1.5399
    0.9564
theta = 2×1
   -1.5441
    0.9568
theta = 2×1
   -1.5484
    0.9572
theta = 2×1
   -1.5526
    0.9576
theta = 2×1
   -1.5568
    0.9581
theta = 2×1
   -1.5610
    0.9585
theta = 2×1
   -1.5652
    0.9589
theta = 2×1
   -1.5694
    0.9593
theta = 2×1
   -1.5736
    0.9597
theta = 2×1
   -1.5778
    0.9602
theta = 2×1
   -1.5820
    0.9606
theta = 2×1
   -1.5862
    0.9610
theta = 2×1
   -1.5903
    0.9614
theta = 2×1
   -1.5945
    0.9618
theta = 2×1
   -1.5986
    0.9623
theta = 2×1
   -1.6028
    0.9627
theta = 2×1
   -1.6069
    0.9631
theta = 2×1
   -1.6110
    0.9635
theta = 2×1
   -1.6151
    0.9639
theta = 2×1
   -1.6193
```

```
    0.9643
theta = 2×1
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    0.9647
theta = 2×1
   -1.6275
    0.9652
theta = 2×1
   -1.6315
    0.9656
theta = 2×1
   -1.6356
    0.9660
theta = 2×1
   -1.6397
    0.9664
theta = 2×1
   -1.6438
    0.9668
theta = 2×1
   -1.6478
    0.9672
theta = 2×1
   -1.6519
    0.9676
theta = 2×1
   -1.6559
    0.9680
theta = 2×1
   -1.6600
    0.9684
theta = 2×1
   -1.6640
    0.9688
theta = 2×1
   -1.6680
    0.9692
theta = 2×1
   -1.6720
    0.9696
theta = 2×1
   -1.6760
    0.9700
theta = 2×1
   -1.6800
    0.9704
theta = 2×1
   -1.6840
    0.9708
theta = 2×1
   -1.6880
    0.9712
theta = 2×1
   -1.6920
    0.9716
theta = 2×1
   -1.6960
    0.9720
theta = 2×1
   -1.6999
    0.9724
theta = 2×1
   -1.7039
    0.9728
```

```
theta = 2×1
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    0.9732
theta = 2×1
   -1.7118
    0.9736
theta = 2×1
   -1.7157
    0.9740
theta = 2×1
   -1.7197
    0.9744
theta = 2×1
   -1.7236
    0.9748
theta = 2×1
   -1.7275
    0.9752
theta = 2×1
   -1.7314
    0.9756
theta = 2×1
   -1.7353
    0.9760
theta = 2×1
   -1.7392
    0.9764
theta = 2×1
   -1.7431
    0.9768
theta = 2×1
   -1.7470
    0.9772
theta = 2×1
   -1.7508
    0.9776
theta = 2×1
   -1.7547
    0.9779
theta = 2×1
   -1.7586
    0.9783
theta = 2×1
   -1.7624
    0.9787
theta = 2×1
   -1.7663
    0.9791
theta = 2×1
   -1.7701
    0.9795
theta = 2×1
   -1.7739
    0.9799
theta = 2×1
   -1.7778
    0.9803
theta = 2×1
   -1.7816
    0.9806
theta = 2×1
   -1.7854
    0.9810
theta = 2×1
```

```
   -1.7892
    0.9814
theta = 2×1
   -1.7930
    0.9818
theta = 2×1
   -1.7968
    0.9822
theta = 2×1
   -1.8006
    0.9825
theta = 2×1
   -1.8043
    0.9829
theta = 2×1
   -1.8081
    0.9833
theta = 2×1
   -1.8119
    0.9837
theta = 2×1
   -1.8156
    0.9841
theta = 2×1
   -1.8194
    0.9844
theta = 2×1
   -1.8231
    0.9848
theta = 2×1
   -1.8269
    0.9852
theta = 2×1
   -1.8306
    0.9856
theta = 2×1
   -1.8343
    0.9859
theta = 2×1
   -1.8380
    0.9863
theta = 2×1
   -1.8417
    0.9867
theta = 2×1
   -1.8454
    0.9871
theta = 2×1
   -1.8491
    0.9874
theta = 2×1
   -1.8528
    0.9878
theta = 2×1
   -1.8565
    0.9882
theta = 2×1
   -1.8602
    0.9885
theta = 2×1
   -1.8639
    0.9889
theta = 2×1
   -1.8675
```

```
    0.9893
theta = 2×1
   -1.8712
    0.9896
theta = 2×1
   -1.8748
    0.9900
theta = 2×1
   -1.8785
    0.9904
theta = 2×1
   -1.8821
    0.9907
theta = 2×1
   -1.8857
    0.9911
theta = 2×1
   -1.8894
    0.9915
theta = 2×1
   -1.8930
    0.9918
theta = 2×1
   -1.8966
    0.9922
theta = 2×1
   -1.9002
    0.9926
theta = 2×1
   -1.9038
    0.9929
theta = 2×1
   -1.9074
    0.9933
theta = 2×1
   -1.9110
    0.9936
theta = 2×1
   -1.9145
    0.9940
theta = 2×1
   -1.9181
    0.9944
theta = 2×1
   -1.9217
    0.9947
theta = 2×1
   -1.9252
    0.9951
theta = 2×1
   -1.9288
    0.9954
theta = 2×1
   -1.9323
    0.9958
theta = 2×1
   -1.9359
    0.9961
theta = 2×1
   -1.9394
    0.9965
theta = 2×1
   -1.9429
    0.9968
```

```
theta = 2×1
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    0.9972
theta = 2×1
   -1.9500
    0.9976
theta = 2×1
   -1.9535
    0.9979
theta = 2×1
   -1.9570
    0.9983
theta = 2×1
   -1.9605
    0.9986
theta = 2×1
   -1.9640
    0.9990
theta = 2×1
   -1.9674
    0.9993
theta = 2×1
   -1.9709
    0.9997
theta = 2×1
   -1.9744
    1.0000
theta = 2×1
   -1.9778
    1.0004
theta = 2×1
   -1.9813
    1.0007
theta = 2×1
   -1.9848
    1.0011
theta = 2×1
   -1.9882
    1.0014
theta = 2×1
   -1.9916
    1.0017
theta = 2×1
   -1.9951
    1.0021
theta = 2×1
   -1.9985
    1.0024
theta = 2×1
   -2.0019
    1.0028
theta = 2×1
   -2.0053
    1.0031
theta = 2×1
   -2.0087
    1.0035
theta = 2×1
   -2.0121
    1.0038
theta = 2×1
   -2.0155
    1.0041
theta = 2×1
```

```
   -2.0189
    1.0045
theta = 2×1
   -2.0223
    1.0048
theta = 2×1
   -2.0257
    1.0052
theta = 2×1
   -2.0291
    1.0055
theta = 2×1
   -2.0324
    1.0058
theta = 2×1
   -2.0358
    1.0062
theta = 2×1
   -2.0391
    1.0065
theta = 2×1
   -2.0425
    1.0068
theta = 2×1
   -2.0458
    1.0072
theta = 2×1
   -2.0492
    1.0075
theta = 2×1
   -2.0525
    1.0079
theta = 2×1
   -2.0558
    1.0082
theta = 2×1
   -2.0591
    1.0085
theta = 2×1
   -2.0624
    1.0089
theta = 2×1
   -2.0657
    1.0092
theta = 2×1
   -2.0690
    1.0095
theta = 2×1
   -2.0723
    1.0098
theta = 2×1
   -2.0756
    1.0102
theta = 2×1
   -2.0789
    1.0105
theta = 2×1
   -2.0822
    1.0108
theta = 2×1
   -2.0854
    1.0112
theta = 2×1
   -2.0887
```

```
    1.0115
theta = 2×1
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    1.0118
theta = 2×1
   -2.0952
    1.0121
theta = 2×1
   -2.0985
    1.0125
theta = 2×1
   -2.1017
    1.0128
theta = 2×1
   -2.1049
    1.0131
theta = 2×1
   -2.1082
    1.0134
theta = 2×1
   -2.1114
    1.0138
theta = 2×1
   -2.1146
    1.0141
theta = 2×1
   -2.1178
    1.0144
theta = 2×1
   -2.1210
    1.0147
theta = 2×1
   -2.1242
    1.0151
theta = 2×1
   -2.1274
    1.0154
theta = 2×1
   -2.1306
    1.0157
theta = 2×1
   -2.1338
    1.0160
theta = 2×1
   -2.1370
    1.0163
theta = 2×1
   -2.1401
    1.0167
theta = 2×1
   -2.1433
    1.0170
theta = 2×1
   -2.1465
    1.0173
theta = 2×1
   -2.1496
    1.0176
theta = 2×1
   -2.1528
    1.0179
theta = 2×1
   -2.1559
    1.0182
```

```
theta = 2×1
   -2.1590
    1.0186
theta = 2×1
   -2.1622
    1.0189
theta = 2×1
   -2.1653
    1.0192
theta = 2×1
   -2.1684
    1.0195
theta = 2×1
   -2.1715
    1.0198
theta = 2×1
   -2.1746
    1.0201
theta = 2×1
   -2.1777
    1.0204
theta = 2×1
   -2.1808
    1.0207
theta = 2×1
   -2.1839
    1.0211
theta = 2×1
   -2.1870
    1.0214
theta = 2×1
   -2.1901
    1.0217
theta = 2×1
   -2.1932
    1.0220
theta = 2×1
   -2.1962
    1.0223
theta = 2×1
   -2.1993
    1.0226
theta = 2×1
   -2.2024
    1.0229
theta = 2×1
   -2.2054
    1.0232
theta = 2×1
   -2.2085
    1.0235
theta = 2×1
   -2.2115
    1.0238
theta = 2×1
   -2.2145
    1.0241
theta = 2×1
   -2.2176
    1.0244
theta = 2×1
   -2.2206
    1.0247
theta = 2×1
```

```
   -2.2236
    1.0250
theta = 2×1
   -2.2266
    1.0253
theta = 2×1
   -2.2296
    1.0257
theta = 2×1
   -2.2326
    1.0260
theta = 2×1
   -2.2356
    1.0263
theta = 2×1
   -2.2386
    1.0266
theta = 2×1
   -2.2416
    1.0269
theta = 2×1
   -2.2446
    1.0272
theta = 2×1
   -2.2476
    1.0275
theta = 2×1
   -2.2505
    1.0278
theta = 2×1
   -2.2535
    1.0280
theta = 2×1
   -2.2565
    1.0283
theta = 2×1
   -2.2594
    1.0286
theta = 2×1
   -2.2624
    1.0289
theta = 2×1
   -2.2653
    1.0292
theta = 2×1
   -2.2683
    1.0295
theta = 2×1
   -2.2712
    1.0298
theta = 2×1
   -2.2741
    1.0301
theta = 2×1
   -2.2770
    1.0304
theta = 2×1
   -2.2800
    1.0307
theta = 2×1
   -2.2829
    1.0310
theta = 2×1
   -2.2858
```

```
    1.0313
theta = 2×1
   -2.2887
    1.0316
theta = 2×1
   -2.2916
    1.0319
theta = 2×1
   -2.2945
    1.0322
theta = 2×1
   -2.2974
    1.0325
theta = 2×1
   -2.3002
    1.0327
theta = 2×1
   -2.3031
    1.0330
theta = 2×1
   -2.3060
    1.0333
theta = 2×1
   -2.3088
    1.0336
theta = 2×1
   -2.3117
    1.0339
theta = 2×1
   -2.3146
    1.0342
theta = 2×1
   -2.3174
    1.0345
theta = 2×1
   -2.3203
    1.0348
theta = 2×1
   -2.3231
    1.0350
theta = 2×1
   -2.3259
    1.0353
theta = 2×1
   -2.3288
    1.0356
theta = 2×1
   -2.3316
    1.0359
theta = 2×1
   -2.3344
    1.0362
theta = 2×1
   -2.3372
    1.0365
theta = 2×1
   -2.3400
    1.0367
theta = 2×1
   -2.3428
    1.0370
theta = 2×1
   -2.3456
    1.0373
```

```
theta = 2×1
   -2.3484
    1.0376
theta = 2×1
   -2.3512
    1.0379
theta = 2×1
   -2.3540
    1.0381
theta = 2×1
   -2.3568
    1.0384
theta = 2×1
   -2.3596
    1.0387
theta = 2×1
   -2.3623
    1.0390
theta = 2×1
   -2.3651
    1.0393
theta = 2×1
   -2.3678
    1.0395
theta = 2×1
   -2.3706
    1.0398
theta = 2×1
   -2.3734
    1.0401
theta = 2×1
   -2.3761
    1.0404
theta = 2×1
   -2.3788
    1.0406
theta = 2×1
   -2.3816
    1.0409
theta = 2×1
   -2.3843
    1.0412
theta = 2×1
   -2.3870
    1.0415
theta = 2×1
   -2.3897
    1.0417
theta = 2×1
   -2.3925
    1.0420
theta = 2×1
   -2.3952
    1.0423
theta = 2×1
   -2.3979
    1.0426
theta = 2×1
   -2.4006
    1.0428
theta = 2×1
   -2.4033
    1.0431
theta = 2×1
```

```
   -2.4060
    1.0434
theta = 2×1
   -2.4086
    1.0436
theta = 2×1
   -2.4113
    1.0439
theta = 2×1
   -2.4140
    1.0442
theta = 2×1
   -2.4167
    1.0444
theta = 2×1
   -2.4193
    1.0447
theta = 2×1
   -2.4220
    1.0450
theta = 2×1
   -2.4247
    1.0452
theta = 2×1
   -2.4273
    1.0455
theta = 2×1
   -2.4300
    1.0458
theta = 2×1
   -2.4326
    1.0460
theta = 2×1
   -2.4352
    1.0463
theta = 2×1
   -2.4379
    1.0466
theta = 2×1
   -2.4405
    1.0468
theta = 2×1
   -2.4431
    1.0471
theta = 2×1
   -2.4457
    1.0474
theta = 2×1
   -2.4484
    1.0476
theta = 2×1
   -2.4510
    1.0479
theta = 2×1
   -2.4536
    1.0481
theta = 2×1
   -2.4562
    1.0484
theta = 2×1
   -2.4588
    1.0487
theta = 2×1
   -2.4614
```

```
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theta = 2×1
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    1.0492
theta = 2×1
   -2.4665
    1.0494
theta = 2×1
   -2.4691
    1.0497
theta = 2×1
   -2.4717
    1.0500
theta = 2×1
   -2.4742
    1.0502
theta = 2×1
   -2.4768
    1.0505
theta = 2×1
   -2.4794
    1.0507
theta = 2×1
   -2.4819
    1.0510
theta = 2×1
   -2.4845
    1.0513
theta = 2×1
   -2.4870
    1.0515
theta = 2×1
   -2.4895
    1.0518
theta = 2×1
   -2.4921
    1.0520
theta = 2×1
   -2.4946
    1.0523
theta = 2×1
   -2.4971
    1.0525
theta = 2×1
   -2.4997
    1.0528
theta = 2×1
   -2.5022
    1.0530
theta = 2×1
   -2.5047
    1.0533
theta = 2×1
   -2.5072
    1.0535
theta = 2×1
   -2.5097
    1.0538
theta = 2×1
   -2.5122
    1.0540
theta = 2×1
   -2.5147
    1.0543
```

```
theta = 2×1
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    1.0545
theta = 2×1
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    1.0548
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    1.0550
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    1.0553
theta = 2×1
   -2.5271
    1.0555
theta = 2×1
   -2.5296
    1.0558
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   -2.5320
    1.0560
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   -2.5345
    1.0563
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   -2.5369
    1.0565
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    1.0568
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    1.0570
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   -2.5443
    1.0573
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   -2.5467
    1.0575
theta = 2×1
   -2.5491
    1.0577
theta = 2×1
   -2.5516
    1.0580
theta = 2×1
   -2.5540
    1.0582
theta = 2×1
   -2.5564
    1.0585
theta = 2×1
   -2.5588
    1.0587
theta = 2×1
   -2.5612
    1.0590
theta = 2×1
   -2.5636
    1.0592
theta = 2×1
   -2.5660
    1.0594
theta = 2×1
```

```
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theta = 2×1
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    1.0599
theta = 2×1
   -2.5732
    1.0602
theta = 2×1
   -2.5756
    1.0604
theta = 2×1
   -2.5780
    1.0606
theta = 2×1
   -2.5803
    1.0609
theta = 2×1
   -2.5827
    1.0611
theta = 2×1
   -2.5851
    1.0614
theta = 2×1
   -2.5874
    1.0616
theta = 2×1
   -2.5898
    1.0618
theta = 2×1
   -2.5922
    1.0621
theta = 2×1
   -2.5945
    1.0623
theta = 2×1
   -2.5969
    1.0625
theta = 2×1
   -2.5992
    1.0628
theta = 2×1
   -2.6015
    1.0630
theta = 2×1
   -2.6039
    1.0632
theta = 2×1
   -2.6062
    1.0635
theta = 2×1
   -2.6085
    1.0637
theta = 2×1
   -2.6108
    1.0639
theta = 2×1
   -2.6132
    1.0642
theta = 2×1
   -2.6155
    1.0644
theta = 2×1
   -2.6178
```

```
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    1.0649
theta = 2×1
   -2.6224
    1.0651
theta = 2×1
   -2.6247
    1.0653
theta = 2×1
   -2.6270
    1.0656
theta = 2×1
   -2.6293
    1.0658
theta = 2×1
   -2.6315
    1.0660
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   -2.6338
    1.0663
theta = 2×1
   -2.6361
    1.0665
theta = 2×1
   -2.6384
    1.0667
theta = 2×1
   -2.6406
    1.0669
theta = 2×1
   -2.6429
    1.0672
theta = 2×1
   -2.6452
    1.0674
theta = 2×1
   -2.6474
    1.0676
theta = 2×1
   -2.6497
    1.0678
theta = 2×1
   -2.6519
    1.0681
theta = 2×1
   -2.6541
    1.0683
theta = 2×1
   -2.6564
    1.0685
theta = 2×1
   -2.6586
    1.0687
theta = 2×1
   -2.6608
    1.0690
theta = 2×1
   -2.6631
    1.0692
theta = 2×1
   -2.6653
    1.0694
```

```
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theta = 2×1
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    1.0699
theta = 2×1
   -2.6719
    1.0701
theta = 2×1
   -2.6741
    1.0703
theta = 2×1
   -2.6763
    1.0705
theta = 2×1
   -2.6785
    1.0707
theta = 2×1
   -2.6807
    1.0710
theta = 2×1
   -2.6829
    1.0712
theta = 2×1
   -2.6851
    1.0714
theta = 2×1
   -2.6873
    1.0716
theta = 2×1
   -2.6895
    1.0718
theta = 2×1
   -2.6916
    1.0721
theta = 2×1
   -2.6938
    1.0723
theta = 2×1
   -2.6960
    1.0725
theta = 2×1
   -2.6981
    1.0727
theta = 2×1
   -2.7003
    1.0729
theta = 2×1
   -2.7025
    1.0732
theta = 2×1
   -2.7046
    1.0734
theta = 2×1
   -2.7068
    1.0736
theta = 2×1
   -2.7089
    1.0738
theta = 2×1
   -2.7110
    1.0740
theta = 2×1
```

```
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    1.0742
theta = 2×1
   -2.7153
    1.0744
theta = 2×1
   -2.7174
    1.0747
theta = 2×1
   -2.7196
    1.0749
theta = 2×1
   -2.7217
    1.0751
theta = 2×1
   -2.7238
    1.0753
theta = 2×1
   -2.7259
    1.0755
theta = 2×1
   -2.7280
    1.0757
theta = 2×1
   -2.7301
    1.0759
theta = 2×1
   -2.7322
    1.0761
theta = 2×1
   -2.7343
    1.0764
theta = 2×1
   -2.7364
    1.0766
theta = 2×1
   -2.7385
    1.0768
theta = 2×1
   -2.7406
    1.0770
theta = 2×1
   -2.7427
    1.0772
theta = 2×1
   -2.7448
    1.0774
theta = 2×1
   -2.7468
    1.0776
theta = 2×1
   -2.7489
    1.0778
theta = 2×1
   -2.7510
    1.0780
theta = 2×1
   -2.7530
    1.0782
theta = 2×1
   -2.7551
    1.0784
theta = 2×1
   -2.7571
```

```
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theta = 2×1
   -2.7592
    1.0789
theta = 2×1
   -2.7613
    1.0791
theta = 2×1
   -2.7633
    1.0793
theta = 2×1
   -2.7653
    1.0795
theta = 2×1
   -2.7674
    1.0797
theta = 2×1
   -2.7694
    1.0799
theta = 2×1
   -2.7714
    1.0801
theta = 2×1
   -2.7735
    1.0803
theta = 2×1
   -2.7755
    1.0805
theta = 2×1
   -2.7775
    1.0807
theta = 2×1
   -2.7795
    1.0809
theta = 2×1
   -2.7815
    1.0811
theta = 2×1
   -2.7835
    1.0813
theta = 2×1
   -2.7855
    1.0815
theta = 2×1
   -2.7876
    1.0817
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theta = 2×1
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    1.0821
theta = 2×1
   -2.7935
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   -2.7955
    1.0825
theta = 2×1
   -2.7975
    1.0827
theta = 2×1
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    1.0829
```

```
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theta = 2×1
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    1.0833
theta = 2×1
   -2.8054
    1.0835
theta = 2×1
   -2.8074
    1.0837
theta = 2×1
   -2.8093
    1.0839
theta = 2×1
   -2.8113
    1.0841
theta = 2×1
   -2.8132
    1.0843
theta = 2×1
   -2.8152
    1.0845
theta = 2×1
   -2.8171
    1.0847
theta = 2×1
   -2.8191
    1.0849
theta = 2×1
   -2.8210
    1.0851
theta = 2×1
   -2.8230
    1.0853
theta = 2×1
   -2.8249
    1.0855
theta = 2×1
   -2.8268
    1.0856
theta = 2×1
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    1.0858
theta = 2×1
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    1.0860
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   -2.8326
    1.0862
theta = 2×1
   -2.8345
    1.0864
theta = 2×1
   -2.8364
    1.0866
theta = 2×1
   -2.8383
    1.0868
theta = 2×1
   -2.8402
    1.0870
theta = 2×1
```

```
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    1.0872
theta = 2×1
   -2.8440
    1.0874
theta = 2×1
   -2.8459
    1.0876
theta = 2×1
   -2.8478
    1.0878
theta = 2×1
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    1.0879
theta = 2×1
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    1.0881
theta = 2×1
   -2.8535
    1.0883
theta = 2×1
   -2.8554
    1.0885
theta = 2×1
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    1.0893
theta = 2×1
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    1.0895
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    1.0896
theta = 2×1
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    1.0898
theta = 2×1
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    1.0900
theta = 2×1
   -2.8721
    1.0902
theta = 2×1
   -2.8740
    1.0904
theta = 2×1
   -2.8758
    1.0906
theta = 2×1
   -2.8777
    1.0908
theta = 2×1
   -2.8795
    1.0909
theta = 2×1
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```

```
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theta = 2×1
   -2.8831
    1.0913
theta = 2×1
   -2.8850
    1.0915
theta = 2×1
   -2.8868
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theta = 2×1
   -2.8886
    1.0919
theta = 2×1
   -2.8904
    1.0920
theta = 2×1
   -2.8922
    1.0922
theta = 2×1
   -2.8941
    1.0924
theta = 2×1
   -2.8959
    1.0926
theta = 2×1
   -2.8977
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theta = 2×1
   -2.9030
    1.0933
theta = 2×1
   -2.9048
    1.0935
theta = 2×1
   -2.9066
    1.0937
theta = 2×1
   -2.9084
    1.0938
theta = 2×1
   -2.9102
    1.0940
theta = 2×1
   -2.9120
    1.0942
theta = 2×1
   -2.9137
    1.0944
theta = 2×1
   -2.9155
    1.0946
theta = 2×1
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    1.0947
theta = 2×1
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    1.0949
```

```
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theta = 2×1
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    1.0953
theta = 2×1
   -2.9243
    1.0954
theta = 2×1
   -2.9261
    1.0956
theta = 2×1
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    1.0958
theta = 2×1
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    1.0960
theta = 2×1
   -2.9313
    1.0961
theta = 2×1
   -2.9330
    1.0963
theta = 2×1
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    1.0972
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    1.0974
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    1.0975
theta = 2×1
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    1.0979
theta = 2×1
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    1.0980
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    1.0982
theta = 2×1
   -2.9537
    1.0984
theta = 2×1
   -2.9554
    1.0986
theta = 2×1
```

```
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    1.0987
theta = 2×1
   -2.9587
    1.0989
theta = 2×1
   -2.9604
    1.0991
theta = 2×1
   -2.9621
    1.0992
theta = 2×1
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    1.0994
theta = 2×1
   -2.9655
    1.0996
theta = 2×1
   -2.9672
    1.0997
theta = 2×1
   -2.9688
    1.0999
theta = 2×1
   -2.9705
    1.1001
theta = 2×1
   -2.9722
    1.1002
theta = 2×1
   -2.9738
    1.1004
theta = 2×1
   -2.9755
    1.1006
theta = 2×1
   -2.9772
    1.1007
theta = 2×1
   -2.9788
    1.1009
theta = 2×1
   -2.9805
    1.1011
theta = 2×1
   -2.9821
    1.1012
theta = 2×1
   -2.9838
    1.1014
theta = 2×1
   -2.9854
    1.1016
theta = 2×1
   -2.9870
    1.1017
theta = 2×1
   -2.9887
    1.1019
theta = 2×1
   -2.9903
    1.1021
theta = 2×1
   -2.9920
```

```
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theta = 2×1
   -2.9936
    1.1024
theta = 2×1
   -2.9952
    1.1026
theta = 2×1
   -2.9968
    1.1027
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   -2.9984
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   -3.0001
    1.1030
theta = 2×1
   -3.0017
    1.1032
theta = 2×1
   -3.0033
    1.1034
theta = 2×1
   -3.0049
    1.1035
theta = 2×1
   -3.0065
    1.1037
theta = 2×1
   -3.0081
    1.1039
theta = 2×1
   -3.0097
    1.1040
theta = 2×1
   -3.0113
    1.1042
theta = 2×1
   -3.0129
    1.1043
theta = 2×1
   -3.0145
    1.1045
theta = 2×1
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   -3.0177
    1.1048
theta = 2×1
   -3.0193
    1.1050
theta = 2×1
   -3.0208
    1.1051
theta = 2×1
   -3.0224
    1.1053
theta = 2×1
   -3.0240
    1.1055
theta = 2×1
   -3.0256
    1.1056
```

```
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    1.1058
theta = 2×1
   -3.0287
    1.1059
theta = 2×1
   -3.0303
    1.1061
theta = 2×1
   -3.0318
    1.1062
theta = 2×1
   -3.0334
    1.1064
theta = 2×1
   -3.0349
    1.1066
theta = 2×1
   -3.0365
    1.1067
theta = 2×1
   -3.0380
    1.1069
theta = 2×1
   -3.0396
    1.1070
theta = 2×1
   -3.0411
    1.1072
theta = 2×1
   -3.0427
    1.1073
theta = 2×1
   -3.0442
    1.1075
theta = 2×1
   -3.0457
    1.1076
theta = 2×1
   -3.0473
    1.1078
theta = 2×1
   -3.0488
    1.1079
theta = 2×1
   -3.0503
    1.1081
theta = 2×1
   -3.0518
    1.1083
theta = 2×1
   -3.0534
    1.1084
theta = 2×1
   -3.0549
    1.1086
theta = 2×1
   -3.0564
    1.1087
theta = 2×1
   -3.0579
    1.1089
theta = 2×1
```

```
   -3.0594
    1.1090
theta = 2×1
   -3.0609
    1.1092
theta = 2×1
   -3.0624
    1.1093
theta = 2×1
   -3.0639
    1.1095
theta = 2×1
   -3.0654
    1.1096
theta = 2×1
   -3.0669
    1.1098
theta = 2×1
   -3.0684
    1.1099
theta = 2×1
   -3.0699
    1.1101
theta = 2×1
   -3.0714
    1.1102
theta = 2×1
   -3.0729
    1.1104
theta = 2×1
   -3.0744
    1.1105
theta = 2×1
   -3.0759
    1.1107
theta = 2×1
   -3.0773
    1.1108
theta = 2×1
   -3.0788
    1.1110
theta = 2×1
   -3.0803
    1.1111
theta = 2×1
   -3.0818
    1.1113
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   -3.0832
    1.1114
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theta = 2×1
   -3.0876
    1.1118
theta = 2×1
   -3.0891
    1.1120
theta = 2×1
   -3.0905
```

```
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theta = 2×1
   -3.0920
    1.1123
theta = 2×1
   -3.0934
    1.1124
theta = 2×1
   -3.0949
    1.1126
theta = 2×1
   -3.0963
    1.1127
theta = 2×1
   -3.0977
    1.1129
theta = 2×1
   -3.0992
    1.1130
theta = 2×1
   -3.1006
    1.1132
theta = 2×1
   -3.1021
    1.1133
theta = 2×1
   -3.1035
    1.1134
theta = 2×1
   -3.1049
    1.1136
theta = 2×1
   -3.1063
    1.1137
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   -3.1078
    1.1139
theta = 2×1
   -3.1092
    1.1140
theta = 2×1
   -3.1106
    1.1142
theta = 2×1
   -3.1120
    1.1143
theta = 2×1
   -3.1134
    1.1144
theta = 2×1
   -3.1148
    1.1146
theta = 2×1
   -3.1162
    1.1147
theta = 2×1
   -3.1177
    1.1149
theta = 2×1
   -3.1191
    1.1150
theta = 2×1
   -3.1205
    1.1151
```

```
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   -3.1219
    1.1153
theta = 2×1
   -3.1232
    1.1154
theta = 2×1
   -3.1246
    1.1156
theta = 2×1
   -3.1260
    1.1157
theta = 2×1
   -3.1274
    1.1158
theta = 2×1
   -3.1288
    1.1160
theta = 2×1
   -3.1302
    1.1161
theta = 2×1
   -3.1316
    1.1163
theta = 2×1
   -3.1329
    1.1164
theta = 2×1
   -3.1343
    1.1165
theta = 2×1
   -3.1357
    1.1167
theta = 2×1
   -3.1371
    1.1168
theta = 2×1
   -3.1384
    1.1169
theta = 2×1
   -3.1398
    1.1171
theta = 2×1
   -3.1412
    1.1172
theta = 2×1
   -3.1425
    1.1174
theta = 2×1
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    1.1175
theta = 2×1
   -3.1452
    1.1176
theta = 2×1
   -3.1466
    1.1178
theta = 2×1
   -3.1479
    1.1179
theta = 2×1
   -3.1493
    1.1180
theta = 2×1
```

```
   -3.1506
    1.1182
theta = 2×1
   -3.1520
    1.1183
theta = 2×1
   -3.1533
    1.1184
theta = 2×1
   -3.1546
    1.1186
theta = 2×1
   -3.1560
    1.1187
theta = 2×1
   -3.1573
    1.1188
theta = 2×1
   -3.1587
    1.1190
theta = 2×1
   -3.1600
    1.1191
theta = 2×1
   -3.1613
    1.1192
theta = 2×1
   -3.1626
    1.1194
theta = 2×1
   -3.1640
    1.1195
theta = 2×1
   -3.1653
    1.1196
theta = 2×1
   -3.1666
    1.1198
theta = 2×1
   -3.1679
    1.1199
theta = 2×1
   -3.1692
    1.1200
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   -3.1705
    1.1202
theta = 2×1
   -3.1718
    1.1203
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   -3.1731
    1.1204
theta = 2×1
   -3.1744
    1.1206
theta = 2×1
   -3.1757
    1.1207
theta = 2×1
   -3.1770
    1.1208
theta = 2×1
   -3.1783
```

```
    1.1210
theta = 2×1
   -3.1796
    1.1211
theta = 2×1
   -3.1809
    1.1212
theta = 2×1
   -3.1822
    1.1213
theta = 2×1
   -3.1835
    1.1215
theta = 2×1
   -3.1848
    1.1216
theta = 2×1
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    1.1563
theta = 2×1
   -3.5304
    1.1563
theta = 2×1
   -3.5311
    1.1564
theta = 2×1
   -3.5317
    1.1565
theta = 2×1
   -3.5324
    1.1565
theta = 2×1
   -3.5330
    1.1566
theta = 2×1
   -3.5337
    1.1567
theta = 2×1
   -3.5343
    1.1567
theta = 2×1
   -3.5350
    1.1568
theta = 2×1
   -3.5356
    1.1569
theta = 2×1
   -3.5363
    1.1569
theta = 2×1
   -3.5369
```

```
    1.1570
theta = 2×1
   -3.5376
    1.1570
theta = 2×1
   -3.5382
    1.1571
theta = 2×1
   -3.5389
    1.1572
theta = 2×1
   -3.5395
    1.1572
theta = 2×1
   -3.5402
    1.1573
theta = 2×1
   -3.5408
    1.1574
theta = 2×1
   -3.5414
    1.1574
theta = 2×1
   -3.5421
    1.1575
theta = 2×1
   -3.5427
    1.1576
theta = 2×1
   -3.5434
    1.1576
theta = 2×1
   -3.5440
    1.1577
theta = 2×1
   -3.5446
    1.1578
theta = 2×1
   -3.5453
    1.1578
theta = 2×1
   -3.5459
    1.1579
theta = 2×1
   -3.5465
    1.1579
theta = 2×1
   -3.5472
    1.1580
theta = 2×1
   -3.5478
    1.1581
theta = 2×1
   -3.5484
    1.1581
theta = 2×1
   -3.5490
    1.1582
theta = 2×1
   -3.5497
    1.1583
theta = 2×1
   -3.5503
    1.1583
```

```
theta = 2×1
   -3.5509
    1.1584
theta = 2×1
   -3.5515
    1.1584
theta = 2×1
   -3.5521
    1.1585
theta = 2×1
   -3.5528
    1.1586
theta = 2×1
   -3.5534
    1.1586
theta = 2×1
   -3.5540
    1.1587
theta = 2×1
   -3.5546
    1.1588
theta = 2×1
   -3.5552
    1.1588
theta = 2×1
   -3.5558
    1.1589
theta = 2×1
   -3.5565
    1.1589
theta = 2×1
   -3.5571
    1.1590
theta = 2×1
   -3.5577
    1.1591
theta = 2×1
   -3.5583
    1.1591
theta = 2×1
   -3.5589
    1.1592
theta = 2×1
   -3.5595
    1.1593
theta = 2×1
   -3.5601
    1.1593
theta = 2×1
   -3.5607
    1.1594
theta = 2×1
   -3.5613
    1.1594
theta = 2×1
   -3.5619
    1.1595
theta = 2×1
   -3.5625
    1.1596
theta = 2×1
   -3.5631
    1.1596
theta = 2×1
```

```
   -3.5637
    1.1597
theta = 2×1
   -3.5643
    1.1597
theta = 2×1
   -3.5649
    1.1598
theta = 2×1
   -3.5655
    1.1599
theta = 2×1
   -3.5661
    1.1599
theta = 2×1
   -3.5667
    1.1600
theta = 2×1
   -3.5673
    1.1600
theta = 2×1
   -3.5679
    1.1601
theta = 2×1
   -3.5685
    1.1602
theta = 2×1
   -3.5691
    1.1602
theta = 2×1
   -3.5697
    1.1603
theta = 2×1
   -3.5703
    1.1603
theta = 2×1
   -3.5708
    1.1604
theta = 2×1
   -3.5714
    1.1604
theta = 2×1
   -3.5720
    1.1605
theta = 2×1
   -3.5726
    1.1606
theta = 2×1
   -3.5732
    1.1606
theta = 2×1
   -3.5738
    1.1607
theta = 2×1
   -3.5743
    1.1607
theta = 2×1
   -3.5749
    1.1608
theta = 2×1
   -3.5755
    1.1609
theta = 2×1
   -3.5761
```

```
    1.1609
theta = 2×1
   -3.5766
    1.1610
theta = 2×1
   -3.5772
    1.1610
theta = 2×1
   -3.5778
    1.1611
theta = 2×1
   -3.5784
    1.1611
theta = 2×1
   -3.5789
    1.1612
theta = 2×1
   -3.5795
    1.1613
theta = 2×1
   -3.5801
    1.1613
theta = 2×1
   -3.5807
    1.1614
theta = 2×1
   -3.5812
    1.1614
theta = 2×1
   -3.5818
    1.1615
theta = 2×1
   -3.5824
    1.1615
theta = 2×1
   -3.5829
    1.1616
theta = 2×1
   -3.5835
    1.1617
theta = 2×1
   -3.5840
    1.1617
theta = 2×1
   -3.5846
    1.1618
theta = 2×1
   -3.5852
    1.1618
theta = 2×1
   -3.5857
    1.1619
theta = 2×1
   -3.5863
    1.1619
theta = 2×1
   -3.5868
    1.1620
theta = 2×1
   -3.5874
    1.1621
theta = 2×1
   -3.5880
    1.1621
```

```
theta = 2×1
   -3.5885
    1.1622
theta = 2×1
   -3.5891
    1.1622
theta = 2×1
   -3.5896
    1.1623
theta = 2×1
   -3.5902
    1.1623
theta = 2×1
   -3.5907
    1.1624
theta = 2×1
   -3.5913
    1.1624
theta = 2×1
   -3.5918
    1.1625
theta = 2×1
   -3.5924
    1.1626
theta = 2×1
   -3.5929
    1.1626
theta = 2×1
   -3.5935
    1.1627
theta = 2×1
   -3.5940
    1.1627
theta = 2×1
   -3.5946
    1.1628
theta = 2×1
   -3.5951
    1.1628
theta = 2×1
   -3.5956
    1.1629
theta = 2×1
   -3.5962
    1.1629
theta = 2×1
   -3.5967
    1.1630
theta = 2×1
   -3.5973
    1.1630
theta = 2×1
   -3.5978
    1.1631
theta = 2×1
   -3.5983
    1.1632
theta = 2×1
   -3.5989
    1.1632
theta = 2×1
   -3.5994
    1.1633
theta = 2×1
```

```
   -3.5999
    1.1633
theta = 2×1
   -3.6005
    1.1634
theta = 2×1
   -3.6010
    1.1634
theta = 2×1
   -3.6015
    1.1635
theta = 2×1
   -3.6021
    1.1635
theta = 2×1
   -3.6026
    1.1636
theta = 2×1
   -3.6031
    1.1636
theta = 2×1
   -3.6037
    1.1637
theta = 2×1
   -3.6042
    1.1637
theta = 2×1
   -3.6047
    1.1638
theta = 2×1
   -3.6052
    1.1638
theta = 2×1
   -3.6058
    1.1639
theta = 2×1
   -3.6063
    1.1639
theta = 2×1
   -3.6068
    1.1640
theta = 2×1
   -3.6073
    1.1641
theta = 2×1
   -3.6078
    1.1641
theta = 2×1
   -3.6084
    1.1642
theta = 2×1
   -3.6089
    1.1642
theta = 2×1
   -3.6094
    1.1643
theta = 2×1
   -3.6099
    1.1643
theta = 2×1
   -3.6104
    1.1644
theta = 2×1
   -3.6109
```

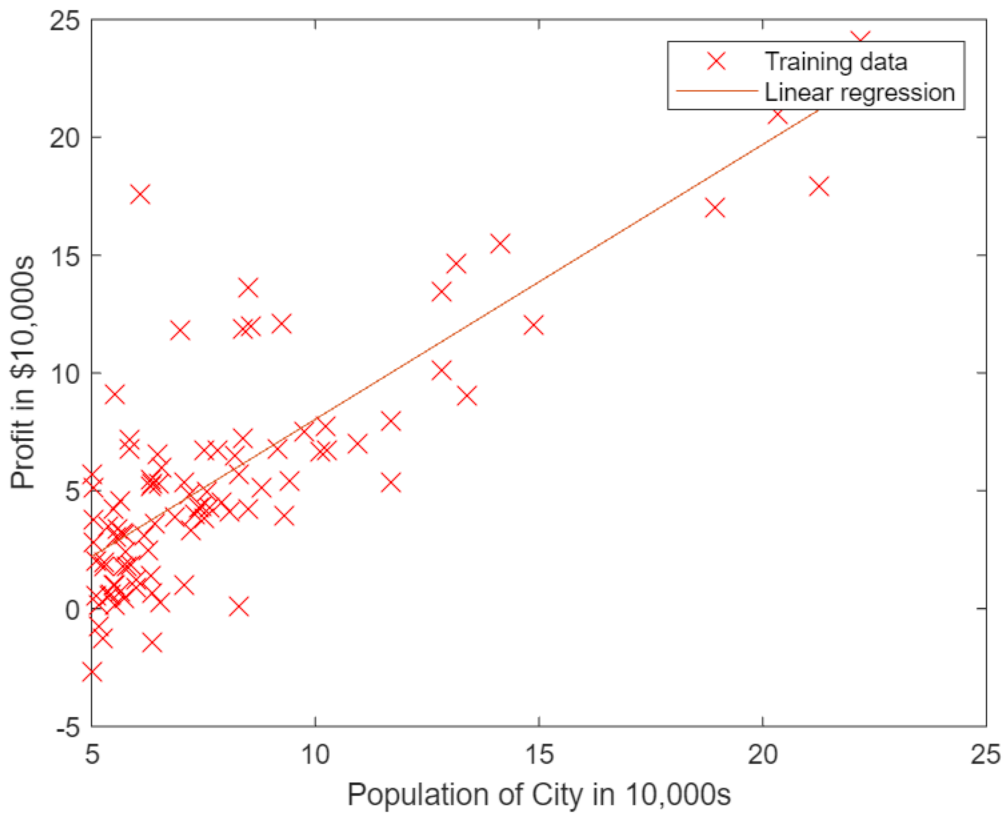
```
    1.1644
theta = 2×1
   -3.6115
    1.1645
theta = 2×1
   -3.6120
    1.1645
theta = 2×1
   -3.6125
    1.1646
theta = 2×1
   -3.6130
    1.1646
theta = 2×1
   -3.6135
    1.1647
theta = 2×1
   -3.6140
    1.1647
theta = 2×1
   -3.6145
    1.1648
theta = 2×1
   -3.6150
    1.1648
theta = 2×1
   -3.6155
    1.1649
theta = 2×1
   -3.6160
    1.1649
theta = 2×1
   -3.6165
    1.1650
theta = 2×1
   -3.6170
    1.1650
theta = 2×1
   -3.6175
    1.1651
theta = 2×1
   -3.6180
    1.1651
theta = 2×1
   -3.6185
    1.1652
theta = 2×1
   -3.6190
    1.1652
theta = 2×1
   -3.6195
    1.1653
theta = 2×1
   -3.6200
    1.1653
theta = 2×1
   -3.6205
    1.1654
theta = 2×1
   -3.6210
    1.1654
theta = 2×1
   -3.6215
    1.1655
```

```
theta = 2×1
   -3.6220
    1.1655
theta = 2×1
   -3.6225
    1.1656
theta = 2×1
   -3.6230
    1.1656
theta = 2×1
   -3.6235
    1.1657
theta = 2×1
   -3.6240
    1.1657
theta = 2×1
   -3.6245
    1.1658
theta = 2×1
   -3.6250
    1.1658
theta = 2×1
   -3.6255
    1.1659
theta = 2×1
   -3.6259
    1.1659
theta = 2×1
   -3.6264
    1.1660
theta = 2×1
   -3.6269
    1.1660
theta = 2×1
   -3.6274
    1.1661
theta = 2×1
   -3.6279
    1.1661
theta = 2×1
   -3.6284
    1.1662
theta = 2×1
   -3.6289
    1.1662
theta = 2×1
   -3.6293
    1.1663
theta = 2×1
   -3.6298
    1.1663
theta = 2×1
   -3.6303
    1.1664
```

```
% Print theta to screen
% Display gradient descent's result
fprintf('Theta computed from gradient descent:\n%f,\n%f',theta(1),theta(2))
```

```
Theta computed from gradient descent:
-3.630291,
1.166362
```

```
% Plot the linear fit
hold on; % keep previous plot visible
plot(X(:,2), X*theta, '-')
legend('Training data', 'Linear regression')
hold off % don't overlay any more plots on this figure
```




```
% Predict values for population sizes of 35,000 and 70,000
predict1 = [1, 3.5] *theta;
fprintf('For population = 35,000, we predict a profit of %f\n', predict1*10000);
```

```
For population = 35,000, we predict a profit of 4519.767868
```

```
predict2 = [1, 7] * theta;
fprintf('For population = 70,000, we predict a profit of %f\n', predict2*10000);
```

```
For population = 70,000, we predict a profit of 45342.450129
```

Note the way that the lines above use matrix multiplication, rather than explicit summation or looping, to calculate the predictions. This is an example of *code vectorization* in MATLAB.

You should now submit your solutions. Enter **`submit`** *at the command prompt, then enter or confirm your login and token when prompted.*

2.3 Debugging

Here are some things to keep in mind as you implement gradient descent:

- MATLAB array indices start from one, not zero. If you're storing θ_0 and θ_1 in a vector called **`theta`**, the values will be **`theta(1)`** and **`theta(2)`**.
- If you are seeing many errors at runtime, inspect your matrix operations to make sure that you're adding and multiplying matrices of compatible dimensions. Printing the dimensions of variables with the `size` command will help you debug.
- By default, MATLAB interprets math operators to be matrix operators. This is a common source of size incompatibility errors. If you don't want matrix multiplication, you need to add the "dot" notation to specify this to MATLAB. For example, `A*B` does a matrix multiply, while `A.*B` does an element-wise multiplication.

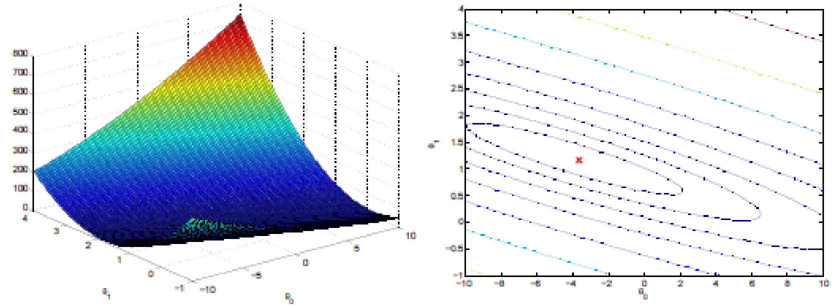
2.4 Visualizing $J(\theta)$

To understand the cost function $J(\theta)$ better, you will now plot the cost over a 2-dimensional grid of θ_0 and θ_1 values. You will not need to code anything new for this part, but you should understand how the code you have written already is creating these images.

In the next step, there is code set up to calculate $J(\theta)$ over a grid of values using the `computeCost` function that you wrote.

```
% Visualizing J(theta_0, theta_1):
% Grid over which we will calculate J
theta0_vals = linspace(-10, 10, 100);
theta1_vals = linspace(-1, 4, 100);

% initialize J_vals to a matrix of 0's
J_vals = zeros(length(theta0_vals), length(theta1_vals));

% Fill out J_vals
for i = 1:length(theta0_vals)
    for j = 1:length(theta1_vals)
      t = [theta0_vals(i); theta1_vals(j)];
      J_vals(i,j) = computeCost(X, y, t);
    end
end
```

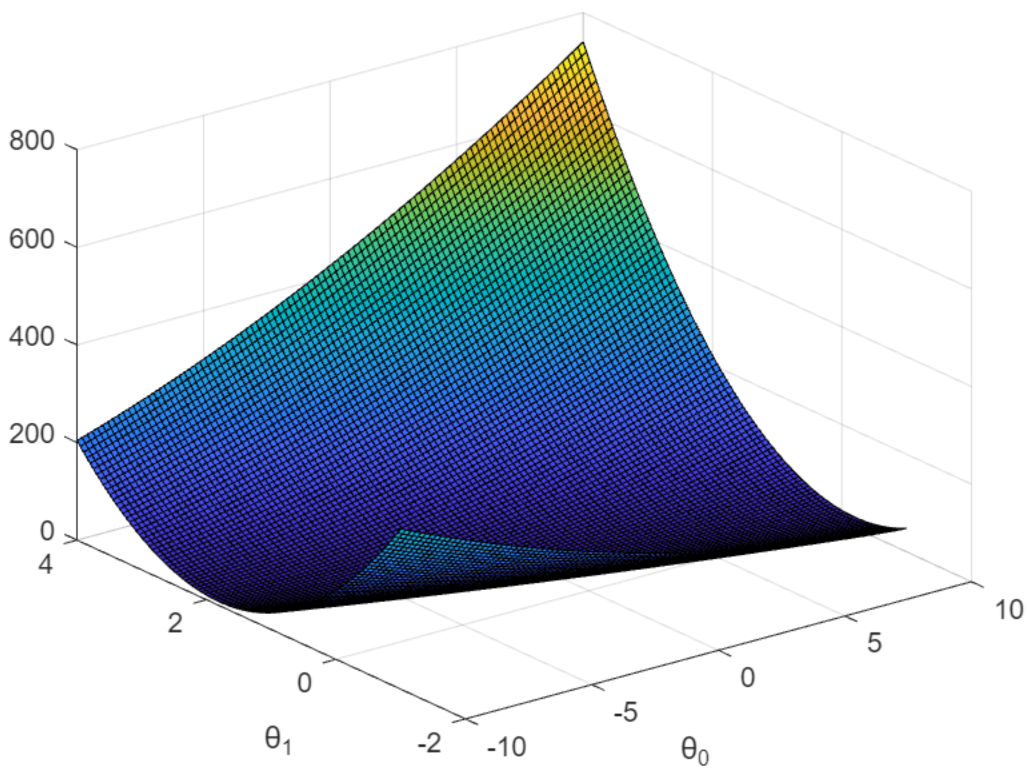
After the code above is executed, you will have a 2-D array of $J(\theta)$ values. The code below will then use these values to produce surface and contour plots of $J(\theta)$ using the **`surf`** and **`contour`** commands. Run the code in this section now. The resulting plots should look something like the figure below.



(a) Surface (b) Contour, showing minimum

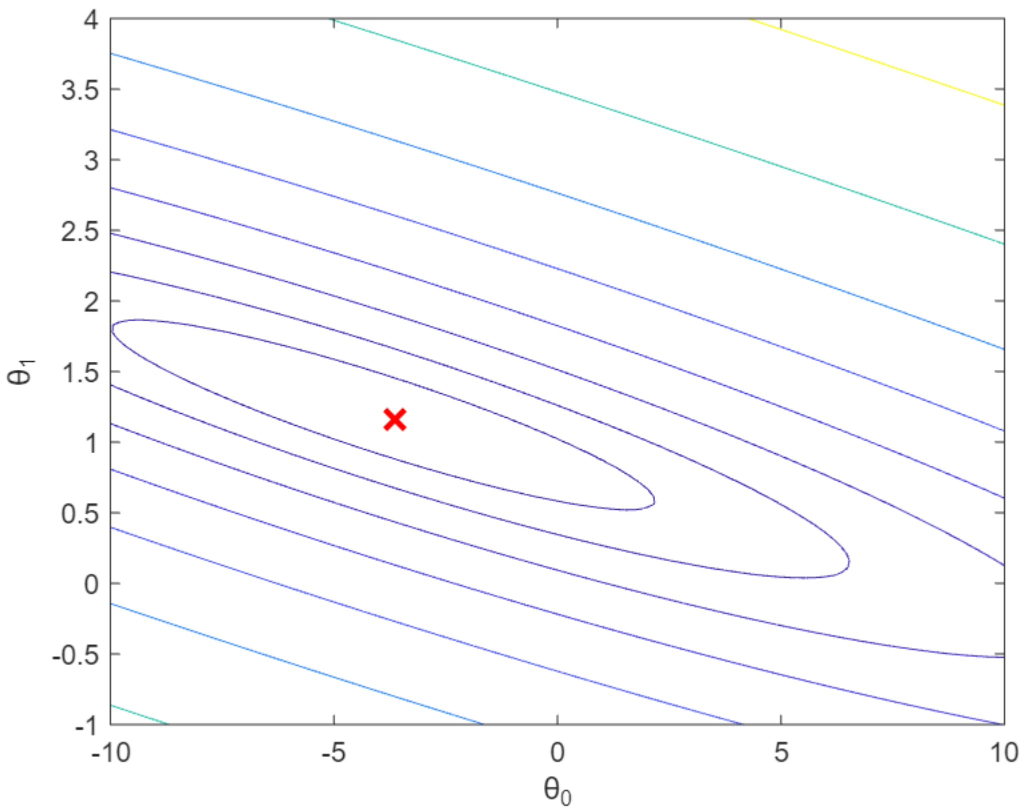
Figure 3: Cost function $J(\theta)$

```
% Because of the way meshgrids work in the surf command, we need to
% transpose J_vals before calling surf, or else the axes will be flipped
J_vals = J_vals';

% Surface plot
figure;
surf(theta0_vals, theta1_vals, J_vals)
xlabel('\theta_0'); ylabel('\theta_1');
```




```

% Contour plot
figure;
```

```
% Plot J_vals as 15 contours spaced logarithmically between 0.01 and 100
contour(theta0_vals, theta1_vals, J_vals, logspace(-2, 3, 20))
xlabel('\theta_0'); ylabel('\theta_1');
hold on;
plot(theta(1), theta(2), 'rx', 'MarkerSize', 10, 'LineWidth', 2);
hold off;
```



The purpose of these graphs is to show you that how $J(\theta)$ varies with changes in θ_0 and θ_1. The cost function $J(\theta)$ is bowl-shaped and has a global mininum. (This is easier to see in the contour plot than in the 3D surface plot). This minimum is the optimal point for θ_0 and θ_1, and each step of gradient descent moves closer to this point.

Optional Exercises:

If you have successfully completed the material above, congratulations! You now understand linear regression and should able to start using it on your own datasets. For the rest of this programming exercise, we have included the following optional exercises. These exercises will help you gain a deeper understanding of the material, and if you are able to do so, we encourage you to complete them as well.

3. Linear regression with multiple variables

In this part, you will implement linear regression with multiple variables to predict the prices of houses. Suppose you are selling your house and you want to know what a good market price would be. One way to do this is to first collect information on recent houses sold and make a model of housing prices.

The file `ex1data2.txt` contains a training set of housing prices in Portland, Oregon. The first column is the size of the house (in square feet), the second column is the number of bedrooms, and the third column is the price of the house. Run this section now to preview the data.

```
% Load Data
data = load('ex1data2.txt');
X = data(:, 1:2);
y = data(:, 3);
m = length(y);

% Print out some data points
% First 10 examples from the dataset
fprintf(' x = [%.0f %.0f], y = %.0f \n', [X(1:10,:) y(1:10,:)]');
```

```
x = [2104 3], y = 399900
x = [1600 3], y = 329900
x = [2400 3], y = 369000
x = [1416 2], y = 232000
x = [3000 4], y = 539900
x = [1985 4], y = 299900
x = [1534 3], y = 314900
x = [1427 3], y = 198999
x = [1380 3], y = 212000
x = [1494 3], y = 242500
```

The remainder of this script has been set up to help you step through this exercise.

3.1 Feature Normalization

This section of the script will start by loading and displaying some values from this dataset. By looking at the values, note that house sizes are about 1000 times the number of bedrooms. When features differ by orders of magnitude, first performing feature scaling can make gradient descent converge much more quickly.

Your task here is to complete the code in `featureNormalize.m` to:

- Subtract the mean value of each feature from the dataset.
- After subtracting the mean, additionally scale (divide) the feature values by their respective "standard deviations".

The standard deviation is a way of measuring how much variation there is in the range of values of a particular feature (most data points will lie within ± 2 standard deviations of the mean); this is an alternative to taking the range of values (*max - min*). In MATLAB, you can use the `std` function to compute the standard deviation. For example, inside `featureNormalize.m`, the quantity `X(:,1)` contains all the values of x_1 (house sizes) in the training set, so `std(X(:,1))` computes the standard deviation of the house sizes. At the time that `featureNormalize.m` is called, the extra column of 1's corresponding to $x_0 = 1$ has not yet been added to `X` (see the code below for details).

You will do this for all the features and your code should work with datasets of all sizes (any number of features / examples). Note that each column of the matrix **X** corresponds to one feature. When you are finished with `featureNormailize.m`, run this section to normailze the features of the housing dataset.

```
% Scale features and set them to zero mean
[X, mu, sigma] = featureNormalize(X);
```

Implementation Note: When normalizing the features, it is important to store the values used for normalization - the mean value and the standard deviation used for the computations. After learning the parameters from the model, we often want to predict the prices of houses we have not seen before. Given a new `x` value (living room area and number of bedrooms), we must first normalize `x` using the mean and standard deviation that we had previously computed from the training set.

Add the bias term

Now that we have normailzed the features, we again add a column of ones corresponding to θ_0 to the data matrix `X`.

```
% Add intercept term to X
X = [ones(m, 1) X];
```

3.2 Gradient Descent

Previously, you implemented gradient descent on a univariate regression problem. The only difference now is that there is one more feature in the matrix `X`. The hypothesis function and the batch gradient descent update rule remain unchanged.

You should complete the code in `computeCostMulti.m` and `gradientDescentMulti.m` to implement the cost function and gradient descent for linear regression *with multiple variables*. If your code in the previous part (single variable) already supports multiple variables, you can use it here too.

Make sure your code supports any number of features and is well-vectorized. You can use the command `size(X,2)` to find out how many features are present in the dataset.

We have provided you with the following starter code below that runs gradient descent with a particular learning rate (`alpha`). Your task is to first make sure that your functions `computeCost` and `gradientDescent` already work with this starter code and support multiple variables.

Implementation Note: In the multivariate case, the cost function can also be written in the following vectorized form:

$$J(\theta) = \frac{1}{2m}\left(X\theta - \vec{y}\right)^T\left(X\theta - \vec{y}\right)$$

where

$$X = \begin{bmatrix} - \left(x^{(1)}\right)^T - \\ - \left(x^{(2)}\right)^T - \\ \vdots \\ - \left(x^{(m)}\right)^T - \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}$$

The vectorized version is efficient when you're working with numerical computing tools like MATLAB. If you are an expert with matrix operations, you can prove to yourself that the two forms are equivalent.

```
% Run gradient descent
% Choose some alpha value
alpha = 0.1;
num_iters = 400;

% Init Theta and Run Gradient Descent
theta = zeros(3, 1);
[theta, ~] = gradientDescentMulti(X, y, theta, alpha, num_iters);

% Display gradient descent's result
fprintf('Theta computed from gradient descent:\n%f\n%f\n%f',theta(1),theta(2),theta(3))
```

```
Theta computed from gradient descent:
340412.659574
110631.048958
-6649.472950
```

Finally, you should complete and run the code below to predict the price of a 1650 sq-ft, 3 br house using the value of `theta` obtained above.

Hint: At prediction, make sure you do the same feature normalization. Recall that the first column of X is all ones. Thus, it does not need to be normalized.

```
% Estimate the price of a 1650 sq-ft, 3 br house
% ====================== YOUR CODE HERE ======================

house = featureNormalize([1650, 3]);
house = [ones(1) house];
price = house * theta; % Enter your price formula here

% ============================================================

fprintf('Predicted price of a 1650 sq-ft, 3 br house (using gradient descent):\n $%f',
```

```
Predicted price of a 1650 sq-ft, 3 br house (using gradient descent):
 $423342.511917
```

3.2.1 Optional (ungraded) exercise: Selecting learning rates

In this part of the exercise, you will get to try out dierent learning rates for the dataset and find a learning rate that converges quickly. You can change the learning rate by modifying the code below and changing the part of the code that sets the learning rate.

The code below will call your `gradientDescent` function and run gradient descent for about 50 iterations at the chosen learning rate. The function should also return the history of $J(\theta)$ values in a vector `J`. After the last iteration, the code plots the **J** values against the number of the iterations. If you picked a learning rate within a good range, your plot should look similar Figure 4 below.

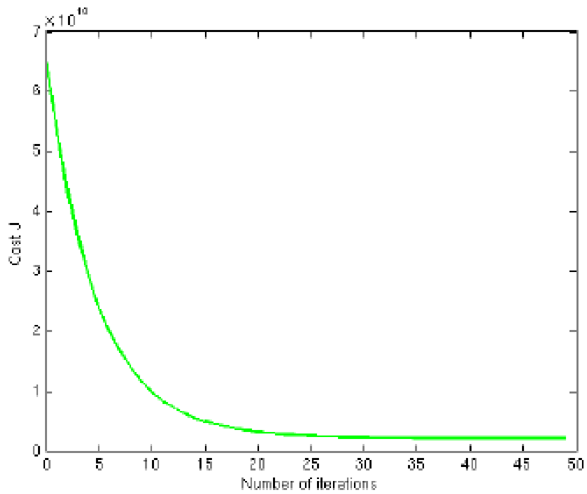


Figure 4: Convergence of gradient descent with an appropriate learning rate

If your graph looks very different, especially if your value of $J(\theta)$ increases or even blows up, use the control to adjust your learning rate and try again. We recommend trying values of the learning rate on a log-scale, at multiplicative steps of about 3 times the previous value (i.e., 0.3, 0.1, 0.03, 0.01 and so on). You may also want to adjust the number of iterations you are running if that will help you see the overall trend in the curve.

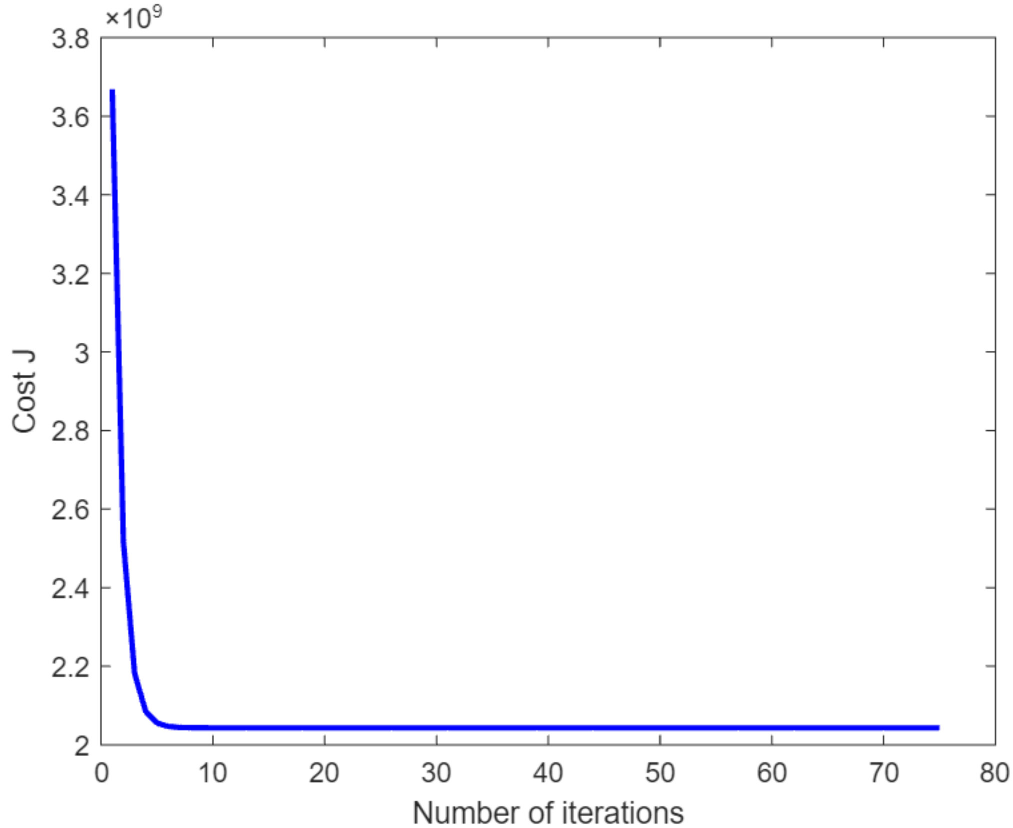
Implementation Note: If your learning rate is too large, $J(\theta)$ can diverge and 'blow up', resulting in values which are too large for computer calculations. In these situations, MATLAB will tend to return `NaNs`. `NaN` stands for 'not a number' and is often caused by undefined operations that involve $\pm\infty$.

MATLAB Tip: To compare how different learning learning rates affect convergence, it's helpful to plot `J` for several learning rates on the same figure. In MATLAB, this can be done by performing gradient descent multiple times with a `hold on` command between plots. Make sure to use the `hold off` command when you are done plotting in that figure. Concretely, if you've tried three different values of `alpha` (you should probably try more values than this) and stored the costs in `J1`, `J2` and `J3`, you can use the following commands to plot them on the same figure:

```
plot(1:50, J1(1:50), 'b');
hold on
plot(1:50, J2(1:50), 'r');
plot(1:50, J3(1:50), 'k');
hold off
```

The final arguments 'b','r', and 'k' specify different colors for the plots. If desired, you can use this technique and adapt the code below to plot multiple convergence histories in the same plot.

```
% Run gradient descent:
% Choose some alpha value
alpha = 1;
num_iters = 75;

% Init Theta and Run Gradient Descent
theta = zeros(3, 1);
[~, J_history] = gradientDescentMulti(X, y, theta, alpha, num_iters);

% Plot the convergence graph
plot(1:num_iters, J_history, '-b', 'LineWidth', 2);
xlabel('Number of iterations');
ylabel('Cost J');
```



Notice the changes in the convergence curves as the learning rate changes. With a small learning rate, you should find that gradient descent takes a very long time to converge to the optimal value. Conversely, with a large learning rate, gradient descent might not converge or might even diverge!

Using the best learning rate that you found, run the section of code below, which will run gradient descent until convergence to find the final values of θ. Next, use this value of θ to predict the price of a house with 1650 square feet and 3 bedrooms. You will use value later to check your implementation of the normal equations. Don't forget to normalize your features when you make this prediction!

```
% Run gradient descent
% Replace the value of alpha below best alpha value you found above
alpha = 1;
num_iters = 400;

% Init Theta and Run Gradient Descent
theta = zeros(3, 1);
[theta, ~] = gradientDescentMulti(X, y, theta, alpha, num_iters);

% Display gradient descent's result
fprintf('Theta computed from gradient descent:\n%f\n%f\n%f',theta(1),theta(2),theta(3))
```

```
Theta computed from gradient descent:
340412.659574
110631.050279
-6649.474271
```

```
% Estimate the price of a 1650 sq-ft, 3 br house. You can use the same
% code you entered ealier to predict the price
% ====================== YOUR CODE HERE ======================

house = featureNormalize([1650, 3]);
house = [ones(1) house];
price = house * theta; % Enter your price formula here

% ============================================================

fprintf('Predicted price of a 1650 sq-ft, 3 br house (using gradient descent):\n $%f',
```

```
Predicted price of a 1650 sq-ft, 3 br house (using gradient descent):
 $423342.513785
```

3.3 Normal Equations

In the lecture videos, you learned that the closed-form solution to linear regression is

$$\theta = \left(X^T X\right)^{-1} X^T \vec{y}$$

Using this formula does not require any feature scaling, and you will get an exact solution in one calculation: there is no "loop until convergence" like in gradient descent.

Complete the code in `normalEqn.m` to use the formula above to calculate θ, then run the code in this section. Remember that while you don't need to scale your features, we still need to add a column of 1's to the X matrix to have an intercept term (θ_0) . Note that the code below will add the column of 1's to X for you.

```
% Solve with normal equations:
% Load Data
data = csvread('ex1data2.txt');
X = data(:, 1:2);
y = data(:, 3);
m = length(y);

% Add intercept term to X
X = [ones(m, 1) X];

% Calculate the parameters from the normal equation
theta = normalEqn(X, y);
```

```
theta = 3×1
10^4 ×
    8.9598
    0.0139
   -0.8738
```

```


% Display normal equation's result
fprintf('Theta computed from the normal equations:\n%f\n%f\n%f', theta(1),theta(2),thet
```

```
Theta computed from the normal equations:
89597.909544
139.210674
-8738.019113
```

Optional (ungraded) exercise: Now, once you have found θ using this method, use it to make a price prediction for a 1650-square-foot house with 3 bedrooms. You should find that gives the same predicted price as the value you obtained using the model fit with gradient descent (in Section 3.2.1).

```
% Estimate the price of a 1650 sq-ft, 3 br house.
% ====================== YOUR CODE HERE ======================

price = [1, 1650, 3] * theta; % Enter your price forumla here

% ============================================================

fprintf('Predicted price of a 1650 sq-ft, 3 br house (using normal equations):\n $%f',
```

```
Predicted price of a 1650 sq-ft, 3 br house (using normal equations):
 $293081.464335
```

Submission and Grading

After completing various parts of the assignment, be sure to use the submit function system to submit your solutions to our servers. The following is a breakdown of how each part of this exercise is scored.

Part	Submitted File	Points
Warm up exercise	`warmUpExercise.m`	10 points
Compute cost for one variable	`computeCost.m`	40 points
Gradient descent for one variable	`gradientDescent.m`	50 points
Total Points		100 points

Optional Exercises

Part	Submitted File	Points
Feature normalization	`featureNormalize.m`	0 points
Compute cost for multiple variables	`computeCostMulti.m`	0 points
Gradient descent for multiple variables	`gradientDescentMulti.m`	0 points
Normal Equations	`normalEqn.m`	0 points

You are allowed to submit your solutions multiple times, and we will take only the highest score into consideration.