Assignment2

September 30, 2018

1 Assignment 2

In [1]:
```
#install.packages("rjags")
```

```
Updating HTML index of packages in '.Library'
Making 'packages.html' ... done
```

1.1 problem 1

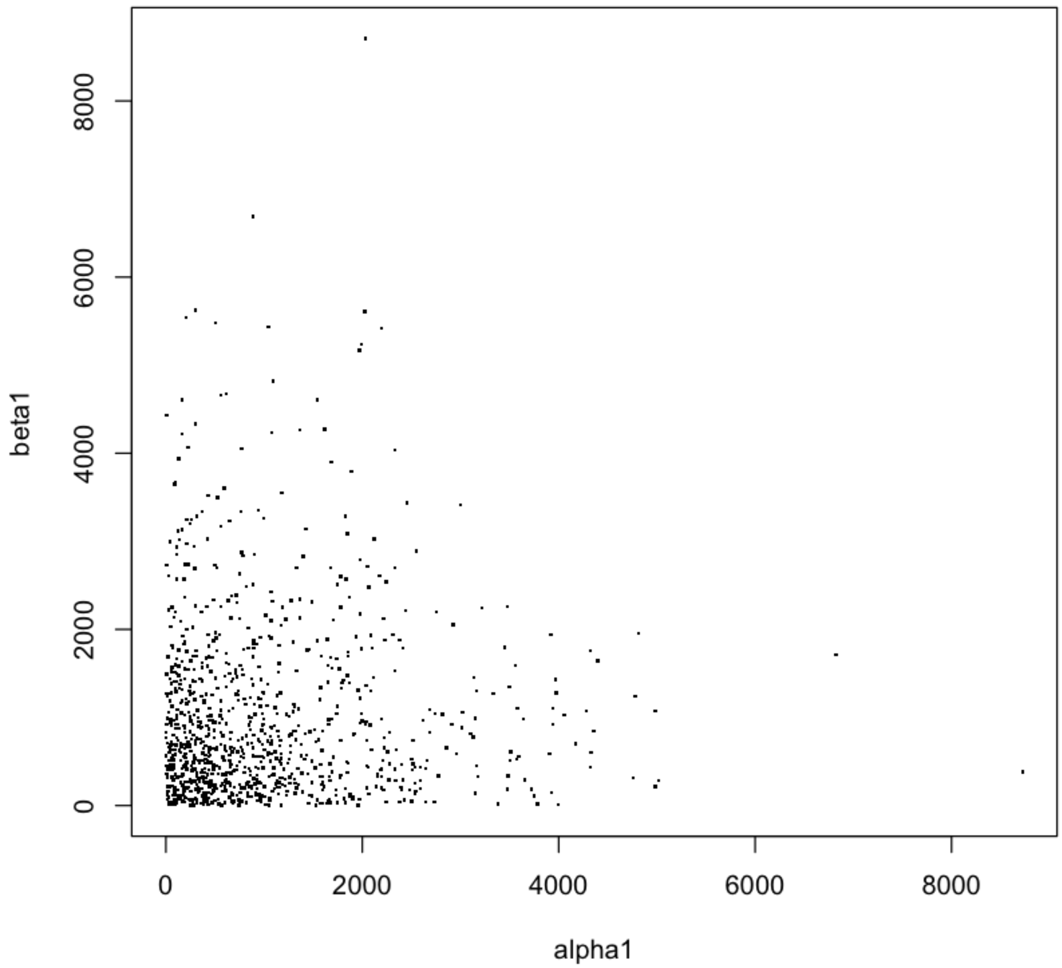
Compare two different hyperprior formulations for the binomial hierarchical model of Lesson 3.2: Hierarchical Modeling Fundamentals.

(a) first prior formulation was

$$\theta_j \mid \alpha, \beta \sim \text{Beta}(\alpha, \beta)$$
$$\alpha, \beta \sim \text{iid}\,\text{Expon}(0.001)$$

(a)(i) Simulate 1000 pairs (α, β) from the hyperprior, and produce a scatterplot of β versus α. Now produce a scatterplot of β versus α.

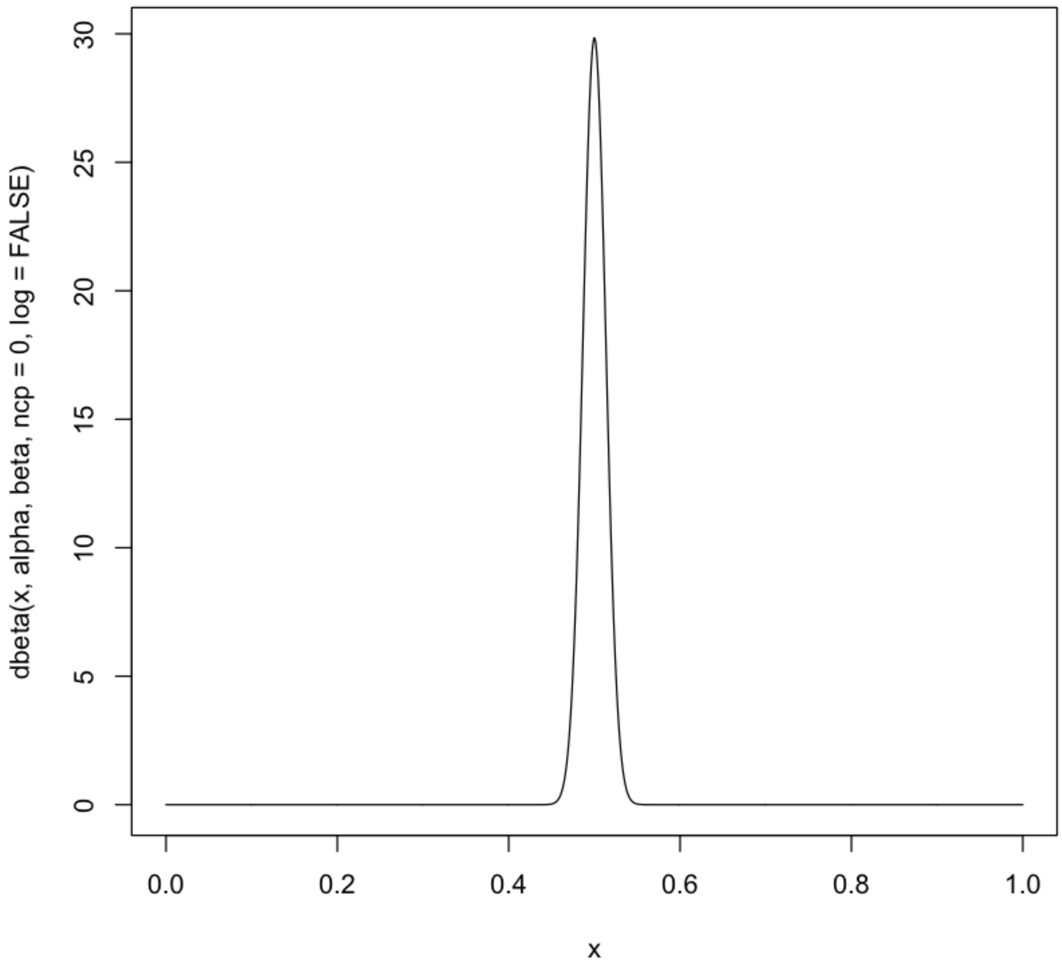
In [17]:
```
alpha1 <- rexp(n = 1000, rate =0.001)
beta1 <- rexp(n = 1000, rate =0.001)
plot(alpha1, beta1, pch=".", cex=2)
```



(a)(ii) Now let $\alpha = \beta = 700$, plot the (conditional prior) density of θ_j.
In this case,

$$p(\theta_j \mid \alpha, \beta) \sim \text{Beta}(\alpha = 700, \beta = 700)$$

In [14]:
```
alpha<-700
beta<-700
curve(dbeta(x, alpha, beta, ncp = 0, log = FALSE),0, 1, n=1000)
```

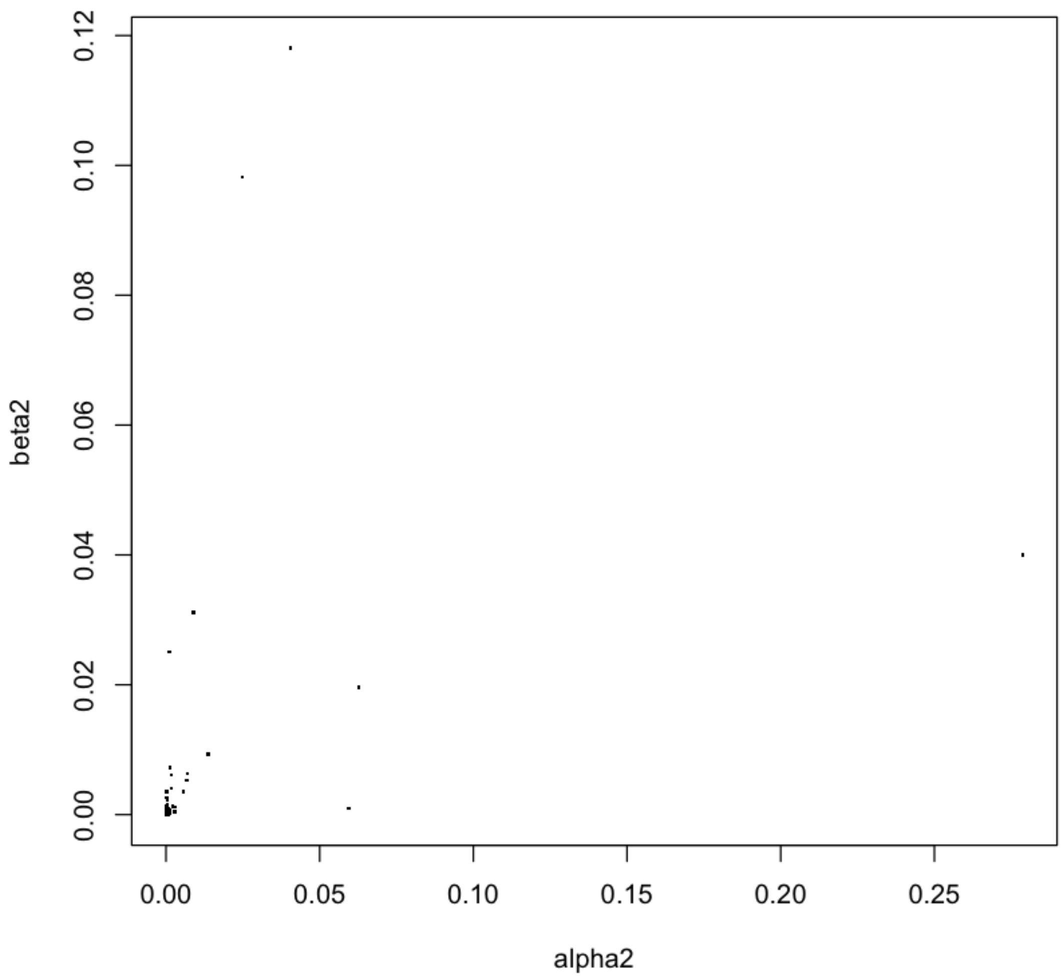


(b) second prior formulation was

$$\theta_j \mid \alpha, \beta \sim \text{Beta}(\alpha, \beta)$$
$$\alpha = \frac{\phi_1}{\phi_2^2} \quad \beta = \frac{1-\phi_1}{\phi_2^2}$$
$$\phi_1 \sim \text{U}(0,1) \quad \phi_2 \sim \text{U}(0,1000)$$

(b)(i) Now produce a scatterplot of β versus α.

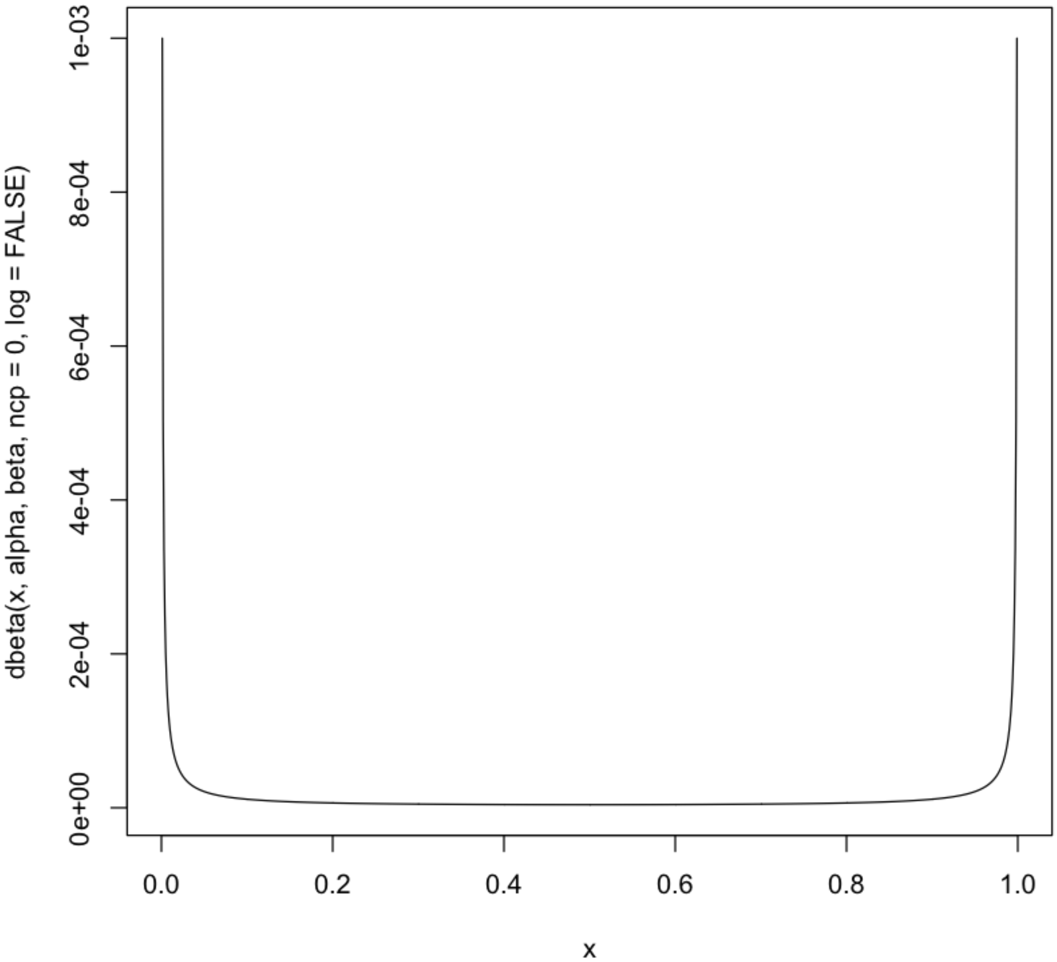
```
In [19]: phi1<-runif(1000, min = 0, max = 1)
         phi2<-runif(1000, min = 0, max = 1000)
         alpha2=phi1/phi2^2
         beta2=(1-phi1)/phi2^2
         plot(alpha2, beta2, pch=".", cex=2)
```



(b)(ii) Now let $\alpha = \beta = 0.000002$, plot the (conditional prior) density of θ_j.
In this case,

$$p(\theta_j \mid \alpha, \beta) \sim \text{Beta}(\alpha = 0.000002, \beta = 0.000002)$$

In [20]:
```
alpha<-0.000002
beta<-0.000002
curve(dbeta(x, alpha, beta, ncp = 0, log = FALSE),0, 1, n=1000)
```



1.2 problem2

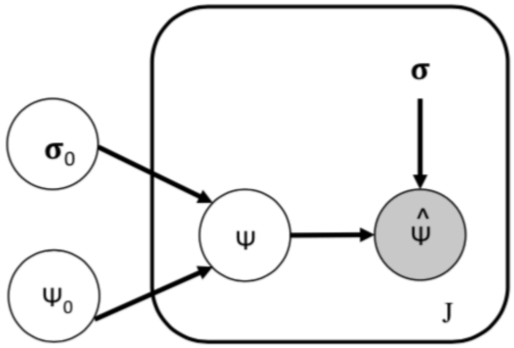
Bayesian hierarchical model in this case:

$$\begin{aligned}\hat{\psi}_j \mid \psi_j &\sim \text{indep.}\,\mathrm{N}(\psi_j, \sigma_j^2) \quad j = 1, ..., 12\\ \psi_j \mid \psi_0, \sigma_0^2 &\sim \text{iid}\,\mathrm{N}(\psi_0, \sigma_0^2) \quad j = 1, ..., 12\\ \psi_0 &\sim \mathrm{N}(0, 1000^2)\\ \sigma_0 &\sim \mathrm{U}(0, 1000)\end{aligned}$$

with ψ_0 and σ_0 independent, and the values $\sigma_j^2, j = 1, ..., 12$, regarded as fixed and known.

(a) The hyperparameters are:

$$\begin{aligned}\psi_0 &\sim \mathrm{N}(0, 1000^2)\\ \sigma_0 &\sim \mathrm{U}(0, 1000)\end{aligned}$$



title

(b) directed acyclic graph (DAG) for this model:
(c) completed asgn2template.bug and set up any R (rjags) statements appropriate for creating a JAGS model.

```
In [2]: 'model {
          for (j in 1:12) {
            psihat[j] ~ dnorm(psi[j], 1/sigma[j]^2)
            psi[j] ~ dnorm(psi0, 1/sigma0^2)
          }

          psi0 ~ dnorm(0,1/1000^2)
          sigma0 ~ dunif(0,1000)

          sigmasq0 <- sigma0^2
        }

        '
        library(rjags)
```

'model {\n for (j in 1:12) {\n psihat[j] ~ dnorm(psi[j], 1/sigma[j]^2)\n psi[j] ~ dnorm(psi0, 1/sigma0^2)\n }\n\n psi0 ~ dnorm(0,1/1000^2)\n sigma0 ~ dunif(0,1000)\n\n sigmasq0 <- sigma0^2\n}\n\n'

```
Loading required package: coda
Linked to JAGS 4.3.0
```

```
Loaded modules: basemod,bugs
```

```
In [4]: #load data
        d <- read.table("data.txt", header=FALSE)
        colnames(d) <- c("psi", "sigma")
```

```
In [5]: d
```

psi	sigma
1.055	0.373
-0.097	0.116
0.626	0.229
0.017	0.117
1.068	0.471
-0.025	0.120
-0.117	0.220
-0.381	0.239
0.507	0.186
0.000	0.328
0.385	0.206
0.405	0.254

```
In [51]: #install.packages("xtable")
```

```
Updating HTML index of packages in '.Library'
Making 'packages.html' ... done
```

(d) Using the JAGS model to produce posterior numerical summary and also graphical estimates of the posterior densities.
the approximations of the posterior expected values, posterior standard deviations, and 95% central posterior intervals are showing below code and output in the end of this section:

```
In [31]: m1 <- jags.model("asgn2template.bug", d)
```

```
Compiling model graph
   Resolving undeclared variables
   Allocating nodes
Graph information:
   Observed stochastic nodes: 12
   Unobserved stochastic nodes: 14
   Total graph size: 70

Initializing model
```

```
In [32]: update(m1, 10000) #Run at least 10,000 iterations of burn-in
```

In [34]:
```
#100,000 iterations to use for inference
x1 <- coda.samples(m1, c("psi0","sigmasq0","psi"), n.iter=100000)
```

In [36]:
```
psi0<- as.matrix(x1)[,"psi0"]
sigmasq0<- as.matrix(x1)[,"sigmasq0"]
```

In [44]:
```
summary(psi0)
summary(sigmasq0)
psi0_mean=mean(psi0)
sigmasq0_mean=mean(sigmasq0)
psi0_sd=sd(psi0)
sigmasq0_sd=sd(sigmasq0)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.7974  0.1887  0.2879  0.2879  0.3872  1.6583


   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.05912 0.19107 0.25560 0.29809 0.35450 3.91334
```

In [50]:
```
#the approximations of the posterior expected values
psi0_mean
sigmasq0_mean

#posterior standard deviations
psi0_sd
sigmasq0_sd
```

0.158496143696757
0.170284481570614

In [87]:
```
#Calculating a 95% Confidence Interval From a Normal Distribution

ci<-function(n,s,mean){

    error <- qnorm(0.975)*s/sqrt(n)
    left <- mean-error
    right <- mean+error
    ci<-list(left,right)
    return (ci)
}

psi0_ci=ci(100000,psi0_sd,psi0_mean)
sigmasq0_ci=ci(100000,sigmasq0_sd,sigmasq0_mean)
```

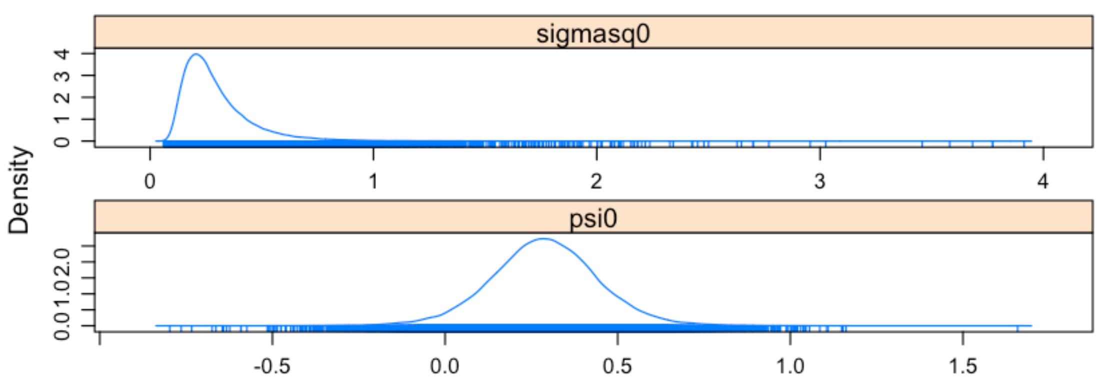
In [88]:
```
#posterior 95% central posterior intervals
psi0_ci
sigmasq0_ci
```

1. 0.286886984337488
2. 0.288851686787541

1. 0.297037259934381
2. 0.299148089449502

In [38]:
```
#graphical estimates of the posterior densities.
require(lattice)
densityplot(x1[,c("psi0","sigmasq0")])
```

```
Loading required package: lattice
```



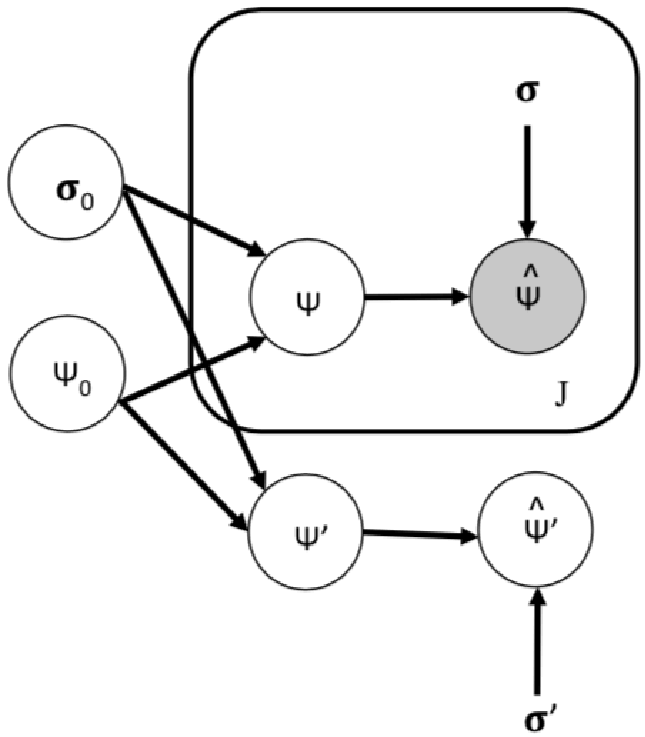
```
In [96]: posterior.expected.values<-c(psi0_mean,sigmasq0_mean)
         posterior.standard.deviations<-c(psi0_sd,sigmasq0_sd)
         CI<-rbind(psi0_ci,sigmasq0_ci)
```

```
In [104]: summary = data.frame(posterior.expected.values, posterior.standard.deviations,CI)
          row.names(summary) <- c('psi0','sigmasq0')
          colnames(summary)<-c('posterior.expected.values','posterior.standard.deviations','95%
```

```
In [105]: #the approximations of the posterior expected values,
          #posterior standard deviations, and 95% central posterior intervals.
          summary
```

	posterior.expected.values	posterior.standard.deviations	95%CI.left	95%CI.right
psi0	0.2878693	0.1584961	0.286887	0.2888517
sigmasq0	0.2980927	0.1702845	0.2970373	0.2991481

(e)(i) directed acyclic graph (DAG) for the new model:



(e)(ii) Estimate the posterior mean and posterior standard deviation, and form a 95% central posterior predictive interval for the estimated log-odds ratio that the new study will obtain.
As the result showing below, **posterior mean value of new $\hat{\psi}$ is 0.28719, posterior standard deviation is 0.603, the 95% central posterior predictive interval for the estimated log-odds ratio is (0.283,0.291)**

```
In [55]: #JAGS model for the new study case asgn2template_new2.bug
         '
```

```
model {
  for (j in 1:12) {
    psihat[j] ~ dnorm(psi[j], 1/sigma[j]^2)
    psi[j] ~ dnorm(psi0, 1/sigma0^2)
  }

  psi_new ~ dnorm(psi0, 1/sigma0^2)
  psihat_new ~ dnorm(psi_new,1/sigma_new^2)
  sigma_new <- 0.2
  sigma_newsq <- sigma_new^2
  psi0 ~ dnorm(0,1/1000^2)
  sigma0 ~ dunif(0,1000)

  sigmasq0 <- sigma0^2
}
'
```

'\nmodel {\n for (j in 1:12) {\n psihat[j] ~ dnorm(psi[j], 1/sigma[j]^2)\n psi[j] ~ dnorm(psi0, 1/sigma0^2)\n }\n\n psi_new ~ dnorm(psi0, 1/sigma0^2)\n psihat_new ~ dnorm(psi_new,1/sigma_new^2)\n sigma_new <- 0.2\n sigma_newsq <- sigma_new^2\n psi0 ~ dnorm(0,1/1000^2)\n sigma0 ~ dunif(0,1000)\n\n sigmasq0 <- sigma0^2\n}\n'

```
In [9]: m2 <- jags.model("asgn2template_new2.bug", d)
```

```
Compiling model graph
   Resolving undeclared variables
   Allocating nodes
Graph information:
   Observed stochastic nodes: 12
   Unobserved stochastic nodes: 16
   Total graph size: 75

Initializing model
```

```
In [10]: update(m2, 10000)
```

```
In [11]: #100,000 iterations to use for inference
         x2 <- coda.samples(m2, c("psi_new","psihat_new"), n.iter=100000)
```

```
In [15]: psi_new<-as.matrix(x2)[,"psi_new"]
         psihat_new<-as.matrix(x2)[,"psihat_new"]
```

```
In [16]: summary(psi_new)
         summary(psihat_new)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-3.9058 -0.0714  0.2862  0.2880  0.6462  4.3642
```

```
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-3.78554 -0.09837  0.28690  0.28719  0.67068  4.60852
```

```
In [53]: psihat_new_mean<-mean(psihat_new)
         psihat_new_sd<-sd(psihat_new)
         psihat_new_mean #posterior mean
         psihat_new_sd #posterior standard deviation
```

0.287190869984071
0.603062542503832

```
In [19]: ci<-function(n,s,mean){

             error <- qnorm(0.975)*s/sqrt(n)
             left <- mean-error
             right <- mean+error
             ci<-list(left,right)
             return (ci)
         }
```

```
In [49]: psihat_new_ci=ci(100000,psihat_new_sd,psihat_new_mean)
         #95% central posterior predictive interval for the estimated log-odds ratio that the 
         psihat_new_ci
```

1. 0.283453118303942

2. 0.290928621664199

(e)(iii) Estimate the posterior predictive probability that the new estimated log-odds ratio will be at least twice its standard error.
As the result showing below, the probability is 0.04776.

```
In [50]: two_sigma_pos<-psihat_new_mean+psihat_new_sd*2
         two_sigma_neg<-psihat_new_mean-psihat_new_sd*2
```

```
In [51]: count_sig<-0
         x <- c(1:100000)
         for (i in x){
             if((psihat_new[i]>two_sigma_pos) | (psihat_new[i]<two_sigma_neg)){
                 count_sig=count_sig+1
                 }
         }
```

```
In [52]: #predictive probability that the new estimated log-odds ratio will be at least twice
         probability<-count_sig/100000
         probability
```

0.04776