Assignment1-MengyuXie

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1 Problem 1

(a) In the case of movie review, let

θ = proportion of positive review
n = size of review for a random movie
y = number of positive review for a random movie

Therefore sampling distribution:

$$y \mid \theta \sim \text{Bin}(n, \theta)$$

Sampling density:

$$p(y \mid \theta) = \binom{n}{y} \cdot \theta^y (1-\theta)^{n-y}, \quad y = 0, ..., n$$

For Movie_1:
y = 150 from n = 200
So the likelihood:

$$\begin{aligned} p(y = 150 \mid \theta) &= \binom{200}{150} \cdot \theta^{150} (1-\theta)^{200-150} \\ &\propto \theta^{150}(1-\theta)^{50} \end{aligned}$$

Now assume a uniform prior on p:

$$\theta \sim \text{U}(0, 1)$$

so that

$$p(\theta) = 1, \quad 0 < \theta < 1$$

Apply Bayes' rule, now the posterior density:

$$\begin{aligned} p(\theta \mid y = 150) &\propto 1 \cdot \theta^{150}(1-\theta)^{50} \\ &\propto \theta^{150}(1-\theta)^{50}, \quad 0 < \theta < 1 \end{aligned}$$

therefore the posterior distribution of Movie_1 is:

$$\theta \mid y = 150 \sim \text{Beta}(\alpha = 151, \beta = 51)$$

Similarily, **For Movie_2:**
y = 4 from n = 5

the posterior distribution of Movie_2 is:

$$\theta \mid y = 4 \sim \text{Beta}(\alpha = 5, \beta = 2)$$

(b) The posterior mean for Movie_1:

$$\text{E}(\theta \mid y = 150) = \frac{\alpha}{\alpha + \beta} = \frac{151}{151 + 51} \approx 0.748$$

The posterior mean for Movie_2:

$$\text{E}(\theta \mid y = 4) = \frac{\alpha}{\alpha + \beta} = \frac{5}{5 + 2} \approx 0.714$$

The posterior mode for Movie_1:

$$\text{E}(\theta \mid y = 150) = \frac{\alpha - 1}{\alpha + \beta - 2} = \frac{151 - 1}{151 + 51 - 2} = 0.75$$

The posterior mode for Movie_2:

$$\text{E}(\theta \mid y = 4) = \frac{\alpha - 1}{\alpha + \beta - 2} = \frac{5 - 1}{5 + 2 - 2} = 0.8$$

Now use R function `qbeta` to simulate the distribution and find posterior median for
Movie_1: 0.748
Movie_2: 0.736

```
In [2]: posterior.samples <- qbeta(c(0.5), 151, 51)
        median(posterior.samples)
```

0.748342963724847

```
In [5]: posterior.samples <- qbeta(c(0.5), 5, 2)
        median(posterior.samples)
```

0.73555001670434

Conclusion: Movie_1 ranks higher according to posterior mean, Movie_2 ranks higher according to posterior mode, and Movie_1 ranks higher according to posterior median.

2 Problem 2

(a) load the randomwikipedia data:

```
In [2]: wikidata <- read.table("randomwikipedia.txt", header=TRUE)
```

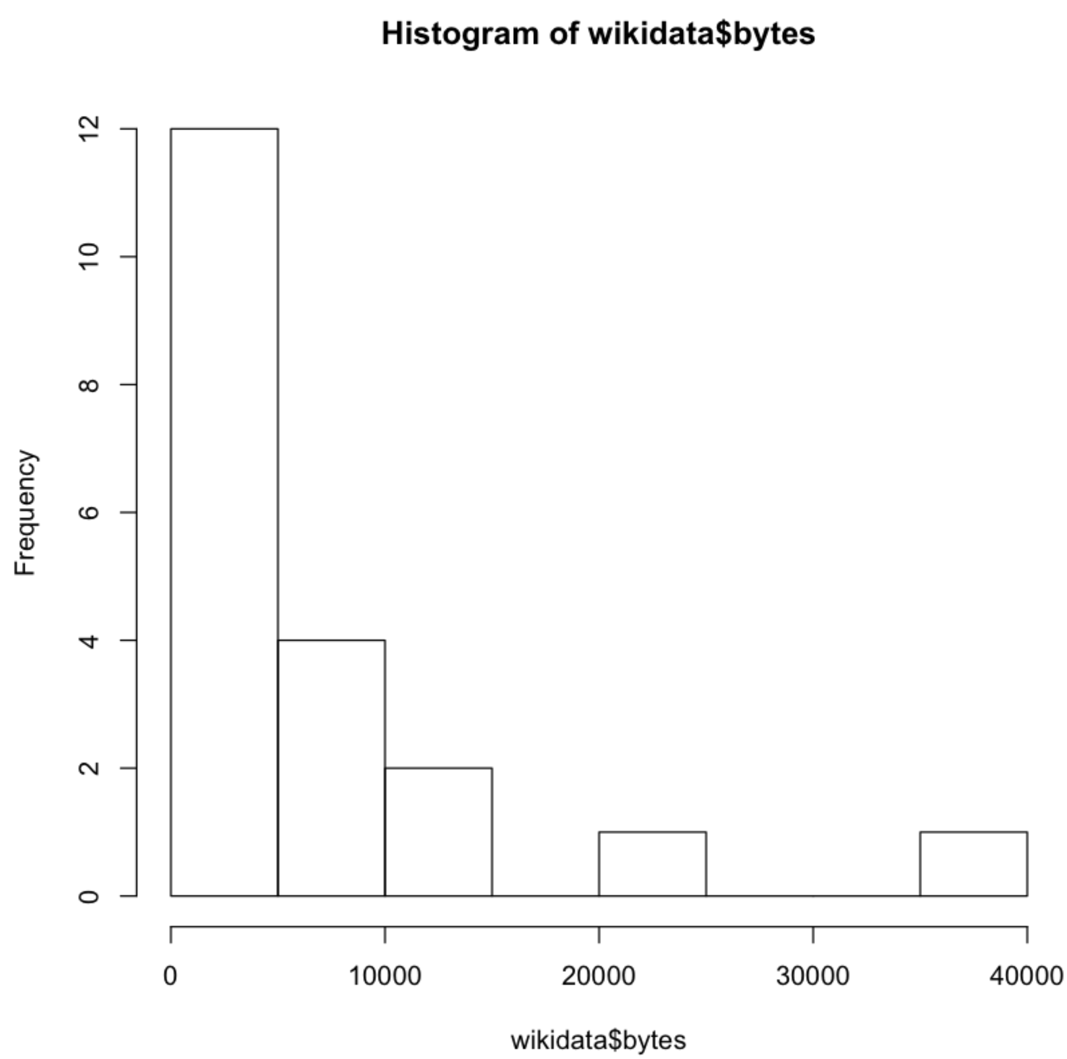
```
In [3]: head(wikidata)
```

pageID	bytes
13143885	628
3042586	11433
3193309	4156
2807448	1713
39456694	7880
3096195	3398

(a)(i) a histogram of article length:

This histogram is right skewed. As shown in the summary code block, the data has a median of 3912 bytes, mean of 6756 bytes.

```
In [4]: hist(wikidata$bytes)
```

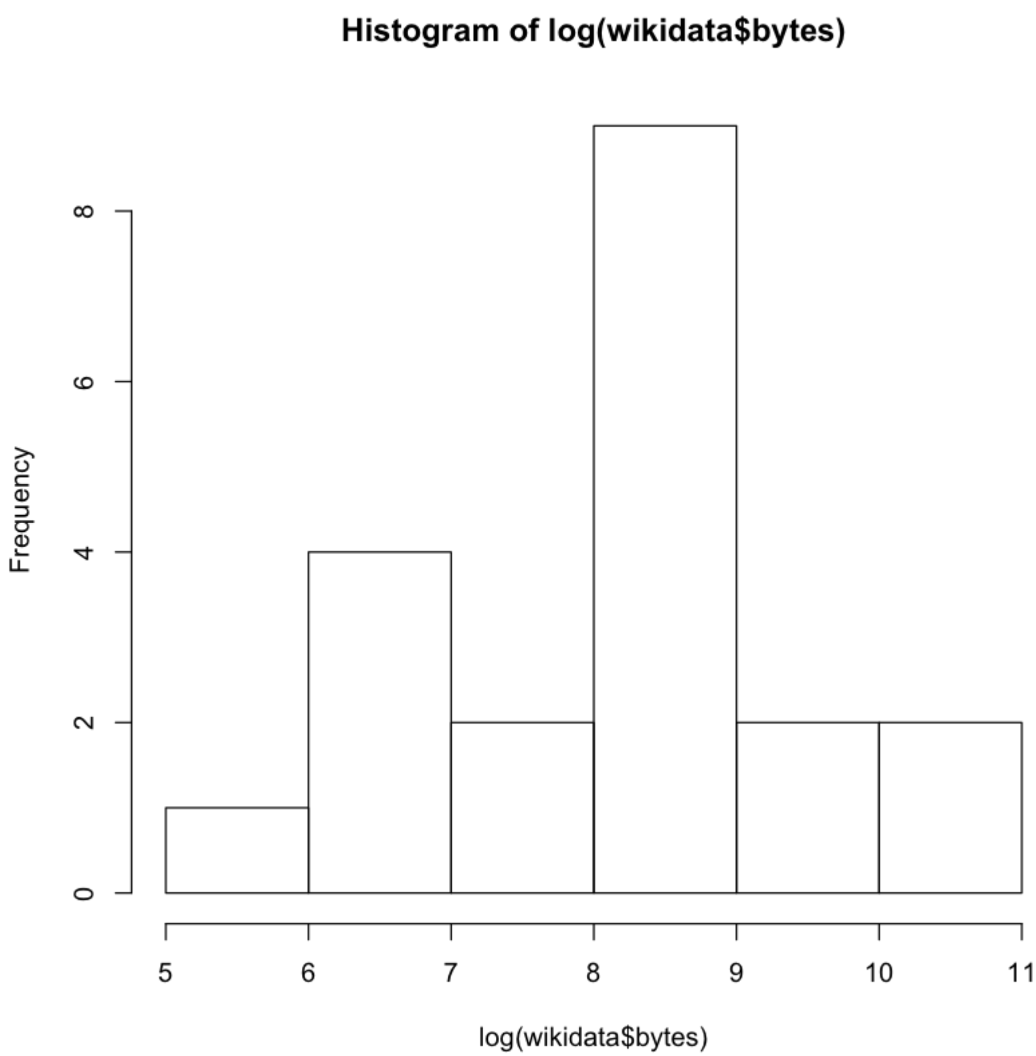



```
In [5]: summary(wikidata$bytes)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
    265    1322    3912    6756    7908   35151 
```

(a)(ii) a histogram of log transformed article length: It is now less skewed and more looks like a normal distribution.

```
In [6]: hist(log(wikidata$bytes))
```




```
In [7]: summary(log(wikidata$bytes))
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  5.580   7.181   8.270   8.160   8.976  10.467 
```

(a)(iii) I think log scale is better to use for the remainder of the analysis, because in part (b) it assumes each y_i is normally distributed.

(b) Let y_i be length of article i on the log scale, sample mean:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = 8.160$$

sample standard deviation:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2} = 1.242$$

```
In [8]: mean(log(wikidata$bytes)) #sample mean deviation_log scale
        sd(log(wikidata$bytes)) #sample standard deviation_log scale
```

8.16040014466563
1.24242675959823

(c) Now assume y_i is normally distributed with unknown mean μ, variance equal to the sample variance.

(c)(i) a conjugate prior for μ is also normally distributed:

$$\mu \sim \mathrm{N}(\mu_0, \tau_0)$$

$$p(\mu) \propto \exp(-\frac{1}{2\tau_0^2}(\mu - \mu_0)^2)$$

so the posterior density of μ given y is:

$$\begin{aligned} p(\mu \mid y) &\propto p(\mu)p(y \mid \mu) \\ &\propto \exp(-\frac{1}{2\tau_n^2}(\mu - \mu_n)^2) \end{aligned}$$

where

$$\mu_n = \frac{\frac{1}{\tau_0^2}\mu_0 + \frac{n}{\sigma^2}\bar{y}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}} \qquad \frac{1}{\tau_n^2} = \frac{1}{\tau_0^2} + \frac{n}{\sigma^2}$$

known that
$\bar{y} = 8.160$
$\mu_0 = 8.270$
$\tau_0^2 = s^2 = 1.544$
therefore:
posterior mean: $\mu_n \approx 8.165$
posterior variance: $\tau_n^2 \approx 0.074$
posterior precision: $\frac{1}{\tau_n^2} \approx 13.601$

```
In [9]: var(log(wikidata$bytes)) #sample variance_log scale
```

1.54362425296577

```
In [10]: mu_0 = 8.270
         tau_0_sqrd = 1.544
         n = nrow(wikidata)
         y_bar = 8.160

         mu_n = ((mu_0/tau_0_sqrd)+(y_bar*n/tau_0_sqrd))/((1/tau_0_sqrd)+n/tau_0_sqrd)
         mu_n #posterior mean
```

8.1652380952381

```
In [11]: tau_n_sqrd = 1/((1/tau_0_sqrd)+(n/tau_0_sqrd))
         tau_n_sqrd #posterior variance
```

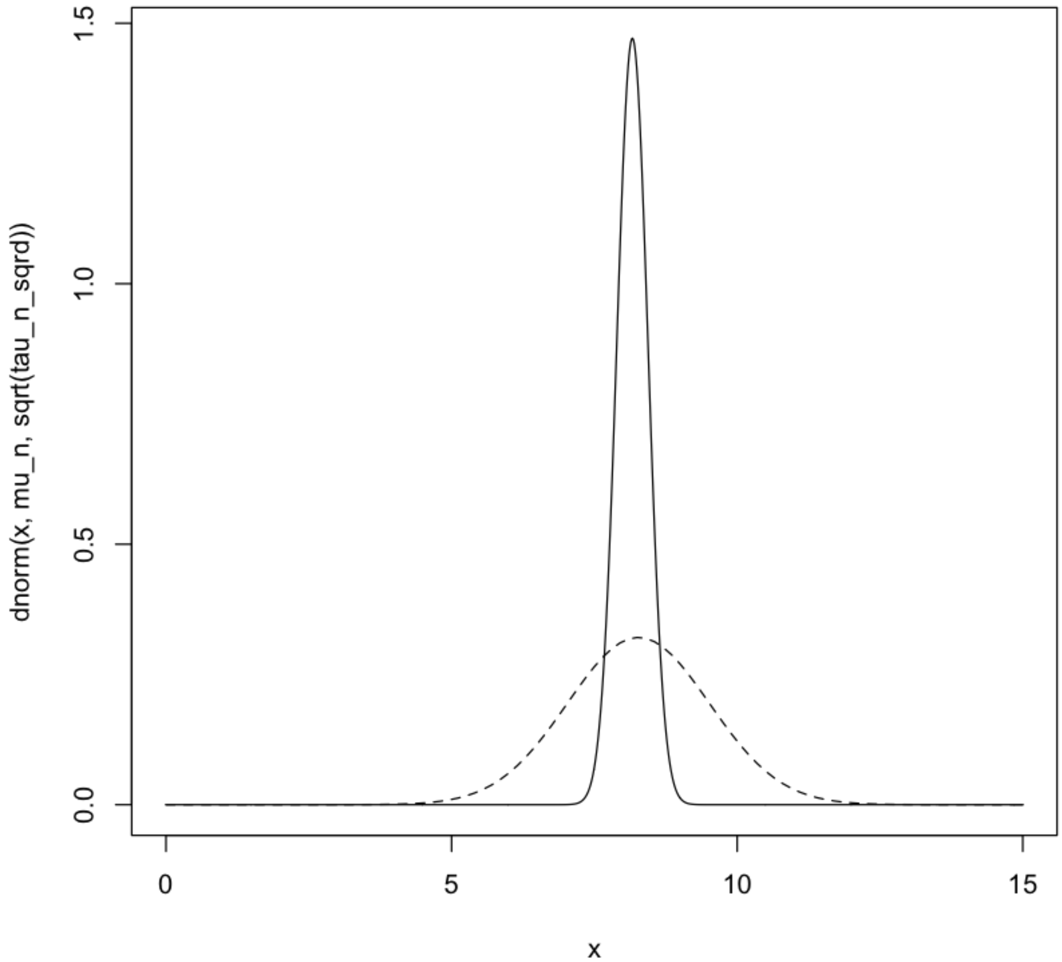
0.0735238095238095

```
In [12]: posterior.precision = (1/tau_0_sqrd)+(n/tau_0_sqrd)
         posterior.precision
```

13.6010362694301

(c)(ii) Plot the prior and posterior densities in a single plot.

```
In [13]: curve(dnorm(x,mu_n,sqrt(tau_n_sqrd)), 0, 15, n=1000)  # posterior
         curve(dnorm(x,mu_0,sqrt(tau_0_sqrd)), 0, 15, add=TRUE, lty=2)  # conjugate prior
```



(c)(iii)a 90% central posterior interval for $\mu \approx (7.719, 8.611)$

In [14]:
```
qnorm(.05)
qnorm(.95)
```

-1.64485362695147
1.64485362695147

In [15]:
```
#margin of error is
me <- 1.645*(sqrt(tau_n_sqrd))
mu_n - me
mu_n + me
```

7.71919183455803

8.61128435591816
(d) Consider a flat prior for μ, so the $\tau_0^2 \to \infty$, therefore

$$\mu \mid y \longrightarrow \mathrm{N}(\overline{y}, \frac{\sigma^2}{n})$$

consider

$$p(\mu) \propto 1 \qquad -\infty < \mu < \infty$$

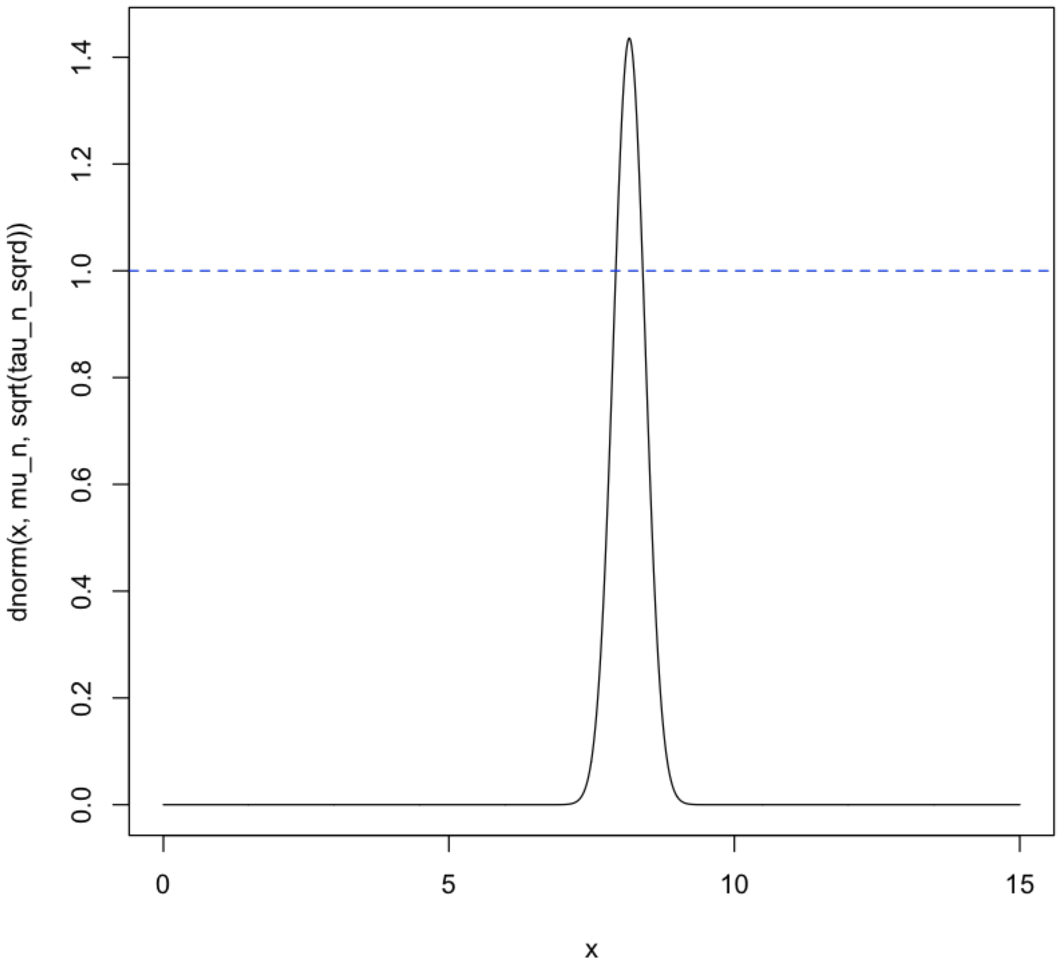
so that posterior density of μ

$$\begin{aligned} p(\mu \mid y) &\propto p(\mu)p(y \mid \mu) \\ &\propto \exp(-\frac{n}{2\sigma^2}(\mu - \overline{y})^2) \qquad -\infty < \mu < \infty \end{aligned}$$

$n = 20$ therefore: posterior mean: $\mu_n = \overline{y} \approx 8.160$
posterior variance: $\tau_n^2 = \sigma^2/n \approx 0.0772$
posterior precision: $\frac{1}{\tau_n^2} \approx 12.953$
(d)(ii) Plot the prior and posterior densities in a single plot.

In [16]:
```
n=20
mu_n=y_bar
tau_n_sqrd=var(log(wikidata$bytes))/n
```

In [17]:
```
curve(dnorm(x,mu_n,sqrt(tau_n_sqrd)), 0, 15, n=1000)  # posterior
abline(h=1,col="blue", lty=2)  # flat prior
```



(d)(iii) A 90% central posterior interval for $\mu \approx (7.703, 8.617)$

In [18]:
```
qnorm(.05)
qnorm(.95)
```

-1.64485362695147
1.64485362695147

In [19]:
```
#margin of error is
me <- 1.645*(sqrt(tau_n_sqrd))

#90% CI
mu_n - me
mu_n + me
```

7.7029942112439
8.6170057887561

(e)(i) simulate 1000 samples from the posterior predictive distribution of article length (in bytes).
The simulated sample mean is 3591.135.
The simulated sample variance is 953417.724

```
In [20]: post.mu.sim <- exp(rnorm(1000, mu_n, sqrt(tau_n_sqrd)))
```

```
In [21]: mean(post.mu.sim)
```

3591.13499540045

```
In [22]: var(post.mu.sim)
```

953417.723756613

(e)(ii) Given that there are about 5.7 million articles on the English Wikipedia, estimate the total number of bytes that represents, based on the posterior predictive distribution: 43166141085.8314

```
In [65]: sum(exp(rnorm(5700000, mu_n, sqrt(tau_n_sqrd))))
```

43166141085.8314