PHY 982 Homework 3

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1 Choice of beam energies and potentials

The ^{12}C(d, p)^{13}C reaction is discussed in this work. Two deuteron beam energies are used, one is 2.84 MeV near the Coulomb barrier around 2 MeV and another one is 4.51 MeV.

At both of the experimental energies chosen, the reaction is more accurately modeled as a compound reaction, as mentioned in [1]. Therefore our calculations using FRESCO may not yield satisfying agreement with experiment.

Optical potentials are needed that described the incoming and outgoing distorted waves. These are interactions between: ^{12}C, d [2]; ^{13}C, p [3]; and ^{12}C, p[4]. For the deuteron wavefunction, the binding for the proton and neutron was described by a simple gaussian potential

$$V_{np}(r) = -72.15e^{-(r/1.484)^2}. \tag{1}$$

scaled to reproduce a bound state at 2.2 MeV.

The neutron that is transferred in the reaction is expected to occupy a $1p_{\frac{1}{2}}$ orbit with an experimental single-particle binding energy of 4.946 MeV. The FRESCO calculation dynamically adjusts the Woods-Saxon depth for ^{13}C to reproduce this energy.

2 Results of DWBA post-form calculations

In a transfer reaction A(d,p)B showed in Fig. 1, by introduction the auxiliary potential $U_f(R_2)$, the transfer T-matrix has a formula [5]

$$T_{post} = \langle \phi_{nA}\chi_{pB}^{(-)} \left| V_{np}(r_1) + U_{pA}(r_p) - U_f(R_2) \right| \Psi_1^{(+)}(\vec{r}_1, \vec{R}_1) \rangle, \tag{2}$$

where ϕ_{nA} and χ_{pB} are bound states wave-functions. Under first-order DWBA, it becomes

$$T_{post}^{DWBA} = \langle \phi_{nA}\chi_{pB}^{(-)} \left| V_{np}(r_1) + U_{pA}(r_p) - U_f(R_2) \right| \phi_{np}\chi_{dA} \rangle. \tag{3}$$

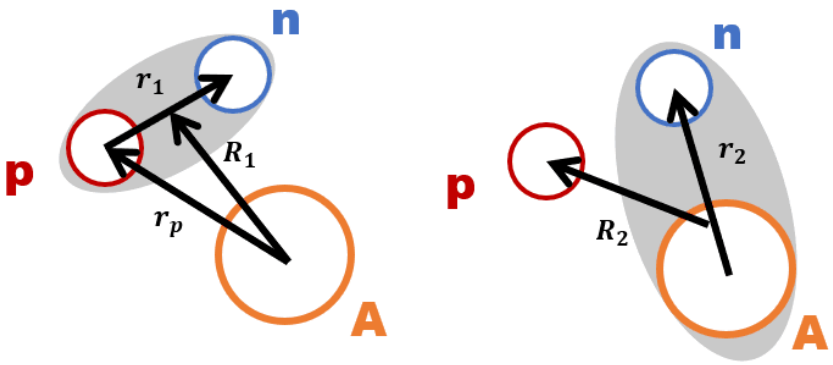

Figure 1: Coordinates used in one neutron transfer reaction.

Besides that, we still need information for the auxiliary potential $U_f(R_2)$. It's usually chosen as $U_{pB}(R_2)$ fitted from elastic scattering. We name $V_{np}(r_1)$ as binding potential and the rest two remnants.

Here are three different handling methods we used in our calculations.

1. Zero range approximation (ZRA): Remnants are neglected; $V_{np}(r_1)$ is considered as a local interaction with strength D_0. Correspondingly, the T-matrix becomes

$$T_{post}^{ZR-DWBA} = D_0 < \phi_{nA}(R_1)\chi_{pB}^{(-)}|\chi_{dA}(R_1) > \tag{4}$$

   It now relies on R_1 only, which simplifies calculation.

2. First-order DWBA, finite-range interactions, without or with full complex remnant: The former abandons the remnants but the latter keeps, as well as the nonlocality of $V_{np}(r_1)$ is preserved in both.

The results together with experimental data are presented in figures (need to be supplemented). We can see ZRA gives result deviates most from experiment because it applies the roughest approximation. DWBA with or without remnants yield close results. This makes sense because U_{pA} and U_{pB} are so similar that they almost cancel each other in Eq. 3. But looking closer, we find the one with remnants is more contiguous to experiment.

3 Results of prior-form DWBA calculations

In a transfer reaction A(d,p)B, the transfer T-matrix has a prior-form formula [5]

$$T_{prior} = \langle \Psi_2^{(-)}(\vec{r}_2, \vec{R}_2)\,|V_{nA}(r_n) + U_{pA}(r_p) - U_{dA}(R_1)|\,\phi_{np}\chi_{dA}\rangle, \tag{5}$$

where ϕ_{np} and χ_{dA} are bound states wave-functions, $\vec{r}_n = \vec{r}_p + \vec{r}_1$, and $U_{dA}(R_1)$ is the auxiliary potential we choose. Under first-order DWBA, it becomes

$$T_{prior} = \langle \phi_{nA} \chi_{pB}^{(-)} | V_{nA}(r_n) + U_{pA}(r_p) - U_{dA}(R_1) | \phi_{np} \chi_{dA} \rangle, \tag{6}$$

The differential cross sections of ^{12}C(d, p)^{13}C calculated in both post and prior forms are given in Fig. ??. First-order DWBA with finite-range interactions and full complex remnant is used in both types of calculations. The convergence of calculation in prior form is checked in the same way as we discussed in Sec. 2. Variables *rnl* and *center* are chosen based on FRESCO's recommendations. As shown in Fig. ??, the results from post- and prior-form calculations have similar trends and are not far from each other. If we calculate without any approximation (Eq. 2 and 5), the same results should be obtained from both forms. But if only first-order DWBA is considered, we expect that small differences between the two results appear, which can be seen in Fig. ??.

It is worth mentioning that the recommended *rnl*, which represents the non-local range, is larger in prior form (??) than that in post form (??). In post form (Eq. 3) $U_{pA}(r_p)$ and $U_{pB}(R_2)$ are close to each other as nuclei A and B are very similar. Thus, the operator in Eq. 3 is approximately $V_{np}(r_1)$, which has a very short range. However, in prior form (Eq. 6) $U_{pA}(r_p)$ and $U_{dA}(R_1)$ cannot cancel each other as the elastic scatterings of deuteron on A and proton on A are very different. Thus, the operator in Eq. 6 has a longer range which comes from optical potentials $U_{pA}(r_p)$ and $U_{dA}(R_1)$.

4 Extraction of spectroscopic factor

As is normal, the spectroscopic factor is extracted by compare the theory to the data at the first peak in the angular distribution [6], as we expect that the reaction is mostly direct at the forward angle. The spectroscopic factors at beam energies 2.84 MeV and 4.51 MeV are given in Table 1. The angle of first peak θ_p is given by the calculations in Sec. 2, and σ^{exp} is obtained by interpolation of experimental data. The spectroscopic factors we extract are energy-dependent, which is expected because Ref. [1] shows that ^{12}C(d, p)^{13}C is more accurately modeled as a compound reaction at the beam energies we choose.

References

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Table 1: Spectroscopic factors S extracted from ^{12}C(d, p)^{13}C. θ_p is the angle of the first peak, and σ^{exp} and σ^{DWBA} are corresponding differential cross sections obtained from experimental data and post-form DWBA calculation, respectively.

Beam energy (MeV)	2.84	4.51
θ_p (degree)		
σ^{exp} (mb/sr)		
σ^{DWBA} (mb/sr)		
$S = \sigma^{\mathrm{exp}}/\sigma^{\mathrm{DWBA}}$		

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