1 Mathematical Model

The notation used for this mathematical formulation is provided in Table 1.

Table 1: Sets, Indices, Parameters, and Decision Variables

Sets and indices	**Description**	**Units**
P	Set of products	-
S	Set of suppliers	-
T	Set of months	-
Parameters	**Description**	**Units**
$d_{p,t}$	Demand for product p in month t	[kg]
h_p	Holding cost for product p	[€/kg]
c_s	Cost of raw material from supplier s	[€/kg]
m_s	Maximum supply from supplier s	[kg/month]
Cr_s	Chromium content in raw material from supplier s	[%]
Ni_s	Nickel content in raw material from supplier s	[%]
$CrReq_p$	Required chromium content for product p	[%]
$NiReq_p$	Required nickel content for product p	[%]
C^t	Monthly production capacity	[kg/month]
Decision Variables	**Description**	**Units**
$x_{p,t}$	Quantity of product p produced in month t	[kg]
$I_{p,t}$	Inventory of product p at the end of month t	[kg]
$z_{p,s,t}$	Quantity of raw material purchased from supplier s in month t for product p	[kg]

The mathematical formulation then follows as:

1.1 Objective Function

$$\min \sum_{t \in T} \left(\sum_{p \in P} h_p \cdot I_{p,t} + \sum_{s \in S} \sum_{p \in P} c_s \cdot z_{p,s,t} \right)$$

1.2 Constraints

$$x_{p,t} + I_{p,t-1} - I_{p,t} = d_{p,t}, \quad \forall p \in P, \forall t \in T \tag{1}$$

$$\sum_{p \in P} x_{p,t} \leq C^t, \quad \forall t \in T \tag{2}$$

$$\sum_{p \in P} z_{p,s,t} \leq m_s, \quad \forall s \in S, \forall t \in T \tag{3}$$

$$\sum_{s \in S} Cr_s \cdot z_{p,s,t} = CrReq_p \cdot x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{4}$$

$$\sum_{s \in S} Ni_s \cdot z_{p,s,t} = NiReq_p \cdot x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{5}$$

$$\sum_{s \in S} z_{p,s,t} = x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{6}$$

$$x_{p,t}, I_{p,t}, z_{p,s,t} \geq 0, \quad \forall p \in P, \forall s \in S, \forall t \in T \tag{7}$$

1.3 Description of the Constraints

- The first constraint (1) ensures that the production and inventory satisfy the demand for each product in each month.
- The second constraint (2) limits the total monthly production to a fixed capacity C^t.
- The third constraint (3) ensures that the quantity of raw material purchased from each supplier does not exceed their maximum supply limit.

- The fourth constraint (4) requires that the purchased raw materials contain sufficient chromium to meet the product's composition.
- The fifth constraint (5) ensures the same for nickel content.
- The sixth constraint (6) ensures that the total raw materials purchased match the quantity of products produced.
- The final constraint (7) enforces the non-negativity of the decision variables.