Assignment Q1

QUANTITATIVE METHODS FOR LOGISTICS

ME44206

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Contents

1 Mathematical Model

The notation used for this mathematical formulation is provided in Table 1.

Table 1: Sets, Indices, Parameters, and Decision Variables

Sets and indices	Description	Units
P	Set of products	-
S	Set of suppliers	-
T	Set of months	-
Parameters	**Description**	**Units**
$d_{p,t}$	Demand for product p in month t	[kg]
h_p	Holding cost for product p	[€/kg]
c_s	Cost of raw material from supplier s	[€/kg]
m_s	Maximum supply from supplier s	[kg/month]
Cr_s	Chromium content in raw material from supplier s	[%]
Ni_s	Nickel content in raw material from supplier s	[%]
$CrReq_p$	Required chromium content for product p	[%]
$NiReq_p$	Required nickel content for product p	[%]
C^t	Monthly production capacity	[kg/month]
Decision Variables	**Description**	**Units**
$x_{p,t}$	Quantity of product p produced in month t	[kg]
$I_{p,t}$	Inventory of product p at the end of month t	[kg]
$z_{p,s,t}$	Quantity of raw material purchased from supplier s in month t for product p	[kg]

The mathematical formulation then follows as:

1.1 Objective Function

$$\min \sum_{t\in T} \left(\sum_{p\in P} h_p \cdot I_{p,t} + \sum_{s\in S} \sum_{p\in P} c_s \cdot z_{p,s,t} \right)$$

1.2 Constraints

$$x_{p,t} + I_{p,t-1} - I_{p,t} = d_{p,t}, \quad \forall p \in P, \forall t \in T \tag{1}$$

$$\sum_{p\in P} x_{p,t} \le C^t, \quad \forall t \in T \tag{2}$$

$$\sum_{p\in P} z_{p,s,t} \le m_s, \quad \forall s \in S, \forall t \in T \tag{3}$$

$$\sum_{s\in S} Cr_s \cdot z_{p,s,t} = CrReq_p \cdot x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{4}$$

$$\sum_{s\in S} Ni_s \cdot z_{p,s,t} = NiReq_p \cdot x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{5}$$

$$\sum_{s\in S} z_{p,s,t} = x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{6}$$

$$x_{p,t}, I_{p,t}, z_{p,s,t} \ge 0, I_{p,0} = 0 \quad \forall p \in P, \forall s \in S, \forall t \in T \tag{7}$$

1.3 Description of the Constraints

- The first constraint (1) ensures that the production and inventory satisfy the demand for each product in each month.
- The second constraint (2) limits the total monthly production to a fixed capacity C^t.
- The third constraint (3) ensures that the quantity of raw material purchased from each supplier does not exceed their maximum supply limit.

- The fourth constraint (4) requires that the purchased raw materials contain sufficient chromium to meet the product's composition.
- The fifth constraint (5) ensures the same for nickel content.
- The sixth constraint (6) ensures that the total raw materials purchased match the quantity of products produced.
- The final constraint (7) enforces the non-negativity of the decision variables.

2 b: Implementation of the Model

Question

Implement the mathematical model in part (a) in python and solve with Gurobi. Provide the optimal solution. By the optimal solution it is meant that you need to report the optimal cost value and the production schedule. Your python model needs to also output the values of the decision variables but only those that you discuss need to be in the report.

2.1 Implementation in Python

Python code of the model and the data is in the Appendix part B.

2.2 Output of the model (optimal solution)

Following are the output of the code and provide the optimal solution (in table format with explanation).

Optimal cost Output

Optimal cost: 9646.78 EURO

Production Plan Schedule Table

Table 2 shows the monthly breakdown of the production quantities (in kilograms) for each of the three products: 18/10, 18/8, and 18/0 stainless steel.

- **Rows**: Each row corresponds to one of the three products.
- **Columns**: Each column represents a month (from Month 1 to Month 12).

Table 2: Production Plan (kg)

Product	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
18/10	25.0	25.0	0.0	0.0	0.0	50.0
18/8	10.0	10.0	10.0	14.375	5.625	10.0
18/0	8.086957	65.0	86.956522	86.956522	85.625	44.375

Product	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
18/10	12.0	0.0	10.0	10.0	45.0	99.0
18/8	10.0	10.0	10.0	19.0	1.0	0.0
18/0	78.0	80.0	62.0	80.0	36.0	0.0

Inventory Levels Schedule Table

Table 3 displays the monthly inventory levels for each product at the end of the month (in kilograms). The inventory level represents how much stock is leftover after meeting the demand.

- **Rows**: Each row corresponds to one of the three products.
- **Columns**: Each column shows the inventory at the end of each month (Month 1 to Month 12).

Table 3: Inventory Levels (kg)

Product	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
18/10	0.0	0.0	0.0	0.0	0.0	0.0
18/8	0.0	0.0	0.0	0.0	4.375	0.0
18/0	3.086957	48.086957	55.043478	117.0	152.625	72.0

Product	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
18/10	0.0	0.0	0.0	0.0	0.0	0.0
18/8	0.0	0.0	0.0	0.0	9.0	0.0
18/0	0.0	0.0	22.0	0.0	100.0	0.0

Purchase Plan Schedule Table

Table 4 5 6 shows the quantities of raw materials (in kilograms) purchased from each supplier (A, B, C, D, E) across 12 months.

- **Rows**: Each row corresponds to a supplier.
- **Columns**: Each column represents the raw material purchased from a supplier for each month (Month 1 to Month 12).

Table 4: Procurement Plan for Product 1 (kg)

Supplier	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Supplier 1	9.011628	9.011628	0.0	0.0	0.0	18.023256
Supplier 2	5.813953	5.813953	0.0	0.0	0.0	11.627907
Supplier 3	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 4	10.174419	10.174419	0.0	0.0	0.0	20.348837
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

Supplier	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Supplier 1	4.325581	0.0	3.604651	16.220930	0.0	35.686047
Supplier 2	2.790698	0.0	2.325581	10.465116	0.0	23.023256
Supplier 3	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 4	4.883721	0.0	4.069767	18.313953	0.0	40.290698
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

Table 5: Procurement Plan for Product 2 (kg)

Supplier	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Supplier 1	4.883721	4.883721	3.043478	3.043478	4.375000	2.747093
Supplier 2	1.860465	1.860465	2.086957	2.086957	3.000000	1.046512
Supplier 3	0.0	0.0	4.869565	4.869565	7.0	0.0
Supplier 4	3.255814	3.255814	0.0	0.0	0.0	1.831395
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

Supplier	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Supplier 1	4.883721	4.883721	4.883721	9.279070	0.488372	0.0
Supplier 2	1.860465	1.860465	1.860465	3.534884	0.186047	0.0
Supplier 3	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 4	3.255814	3.255814	3.255814	6.186047	0.325581	0.0
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

Table 6: Procurement Plan for Product 3 (kg)

Supplier	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Supplier 1	8.086957	65.000000	86.956522	86.956522	85.625000	44.375000
Supplier 2	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 3	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 4	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

Supplier	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Supplier 1	78.000000	80.000000	62.000000	36.000000	0.0	0.0
Supplier 2	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 3	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 4	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

2.3 Discussion

The procurement, production, and inventory strategies outline resource management over 12 months. Product 3's production plan shows variability, with peaks in months 2 and 3, likely reflecting seasonal demand or strategic stockpiling. Product 1 has a balanced but intermittent production pattern, with increased output in the final months, aligning with year-end demand.

The inventory plan indicates Product 3's accumulation in months 4 and 5 before depleting, suggesting a push strategy to buffer against demand variability. Products 1 and 2 maintain minimal inventory, reflecting a lean production approach focused on cost efficiency.

The procurement plan shows heavy reliance on Supplier 1, especially for Product 3, with peaks in months 2, 3, and 10, posing a potential risk if disruptions occur. Suppliers 2 and 4 contribute less frequently. Product 2's procurement relies on Supplier 2 during months 4 to 6 and Supplier 1 in months 7 and 8, suggesting an adaptive strategy.

Product 3's inventory buildup between months 4 and 6 supports future production or mitigates supply chain disruptions. Lean inventory for Products 1 and 2 suggests predictable demand or a just-in-time (JIT) model to minimize carrying costs.

Overall, the organization seeks efficiency with minimal inventory while maintaining contingencies. Diversifying suppliers or increasing inventory buffers could enhance supply chain resilience.

3 c: Verification of the Implementation

Question

Verify the implementation, to convince that the code for (b) matches the mathematical formulation of (a) with a number of verification tests that cover different types of parameters used in the model. Provide a discussion on the verification experiments and their results with your justifications.

3.1 Verification Strategy

The following tests were performed by adjusting key parameters:

- **Objective Function Parameters**: Objective function verification ensures that the objective function correctly represents the actual optimization goal, such as cost minimization or profit maximization. By adjusting parameters or variables (e.g., increasing the storage cost for a specific alloy), we observe the resulting changes in the objective function. The method typically involves running small-scale experiments and manually calculating the total cost to compare it with the model's output. If the behavior matches the expected outcome (e.g., reducing storage while increasing costs), it confirms that the objective function is correctly formulated.
- **Function Parameters**: Function parameters verification aims to ensure that the model's parameters (e.g., material content, supply limits) are correctly applied in the model. By modifying specific parameters (such as increasing Supplier A's chromium content), we expect the model to adjust the material procurement to optimize production. If the model responds as expected by reducing costs, it confirms that the function parameters are appropriately influencing the model's behavior. It is also important to verify if the "unmixing" constraint (i.e., preventing the supply from being split into different products in a way that changes the composition) was correctly implemented
- **RHS parameters**: RHS (right-hand side) parameters verification checks if changes in the constraint parameters (such as demand levels or supply limits) are properly reflected in the model's decisions. By significantly altering demand or supply constraints, we observe whether the model appropriately adjusts its production or procurement plan. For example, reducing demand to zero in most months should result in a zero production plan, while reducing the availability of a key supplier should increase costs. If the model reacts as expected, it confirms that the RHS parameters are correctly applied.

3.2 Key Tests for Verification

The Table 7 below summarizes the key tests that were conducted to verify the implementation of the model. For each test, the objective is described, followed by the setup, expected results, and justifications. These tests ensure that the model behaves correctly under different scenarios and satisfies the defined constraints.

Table 7: Summary of Test Experiments for Model Verification

Test Category	Experiment Description	Expected Results	Remarks	Test Result	Passed?
Objective Function Verification	Increase storage cost of the third alloy (18/0) by 2x.	The plan will reduce monthly storage for 18/0, and overall cost will increase (cost > 9646.78).	Verifies model sensitivity to storage cost changes.	Cost=12712.02, 18/0 alloy storage reduced	Yes
Function Parameters Verification	Set chromium content requirement for first alloy to 100% and the demand for it in first month as 1, all other alloys in other months demand to 0.	Model becomes infeasible.	Suppliers can't meet demand without "unmixing" materials, which should be disallowed.	Model infeasible	Yes
RHS Parameters Verification	Demand for the first product (18/10) is 30 in month 1, with 0 for all other months.	Minimum cost will be 185.58 EUR, with 0 production/inventory in other months.	Tests how the model adjusts production with significant demand changes.	Cost=185.58 EUR, 0 production/inventory after January	Yes
RHS Parameters Verification	Reduce Supplier D's max purchase limit (high nickel content).	Overall cost increases due to reduced high-quality nickel availability.	Verifies impact of reducing key material supply on total costs.	Cost=9665.97	Yes

3.3 Conclusion

These tests demonstrate that the code correctly implements the mathematical model, with all constraints (demand, production capacity, supplier limits, and material composition) and the objective function properly adhered to. The model behaves as expected under different scenarios, verifying that the code aligns with the mathematical formulation.

4 d: Experimenting

Question

Experiment with different values of the maximum production capacity and the holding costs in order to get insights for the trade-off between these in the production and inventory holding decisions for products. Justify the ranges of values you use for the experiments. Provide your interpretation of the results by discussing the impact on the objective function, different costs involved and the decisions.

4.1 Experimenting

To gain insights into the trade-offs between maximum production capacity and holding costs, a series of experiments were conducted by varying these parameters. Specifically, production capacities of 50, 100 and 150 units were analyzed under different holding cost scenarios to understand their impact on the overall objective function, procurement, holding, and total costs. The ranges for production capacity were selected based on feasibility constraints, while holding costs were varied to capture the effects of cost sensitivity across alloys.

4.1.1 Steps in Each Experiment

1. Define the production capacities to be tested: 50, 100, and 150 units.
2. Vary the holding costs for each product: 5, 10, and 20 units for each of the 18/10, 18/8, and 18/0 alloys.
3. For each combination of production capacity and holding costs, run the optimization model to minimize the total cost.
4. Ensure that demand is met for each product and that inventory levels are maintained across the 12-month planning horizon.
5. Extract and save the results for total cost, production plan, inventory plan, and purchase plan for each experiment.
6. Compare the results to identify the trade-off between production capacity and holding costs and their impact on the overall system cost.

The total number of experiments is determined by the combination of different parameter values. Specifically, there are:

- 3 levels of production capacity (50, 100, 150 units),
- 3 levels of holding cost for the 18/10 alloy (5, 10, 20 units),
- 3 levels of holding cost for the 18/8 alloy (5, 10, 20 units),
- 3 levels of holding cost for the 18/0 alloy (5, 10, 20 units).

Thus, the total number of experiments is calculated as:

$$3 \times 3 \times 3 \times 3 = 81$$

4.1.2 Data Outputs

Each experiment outputs:

- Different kind of cost for the system (total, inventory, and purchase costs).
- Production plan: the quantity of each product to produce in each month.
- Inventory plan: the stock levels for each product at the end of each month.
- Purchase plan: the quantity of each product to be purchased from each supplier.

The full output are saved in an Excel file, with each experiment occupying separate sheets and a summary sheet shows all the cost.

4.2 Results Analysis

The following section provides an in-depth analysis of the experimental results, drawing on three sets of heatmaps: one representing holding costs, another depicting procurement costs, and the third illustrating total costs. The primary objective is to identify optimal configurations by evaluating different alloy types (18/10, 18/8, and 18/0) under varying production capacities and holding cost combinations. This analysis employs visual heatmaps to elucidate trends and interdependencies among holding, procurement, and total costs under different experimental conditions. Furthermore, the comprehensive evaluation aims to provide detailed insights into the effects of these variables on overall cost efficiency, thereby informing strategic decision-making in inventory management and procurement processes.

4.2.1 Heatmap of Holding Cost

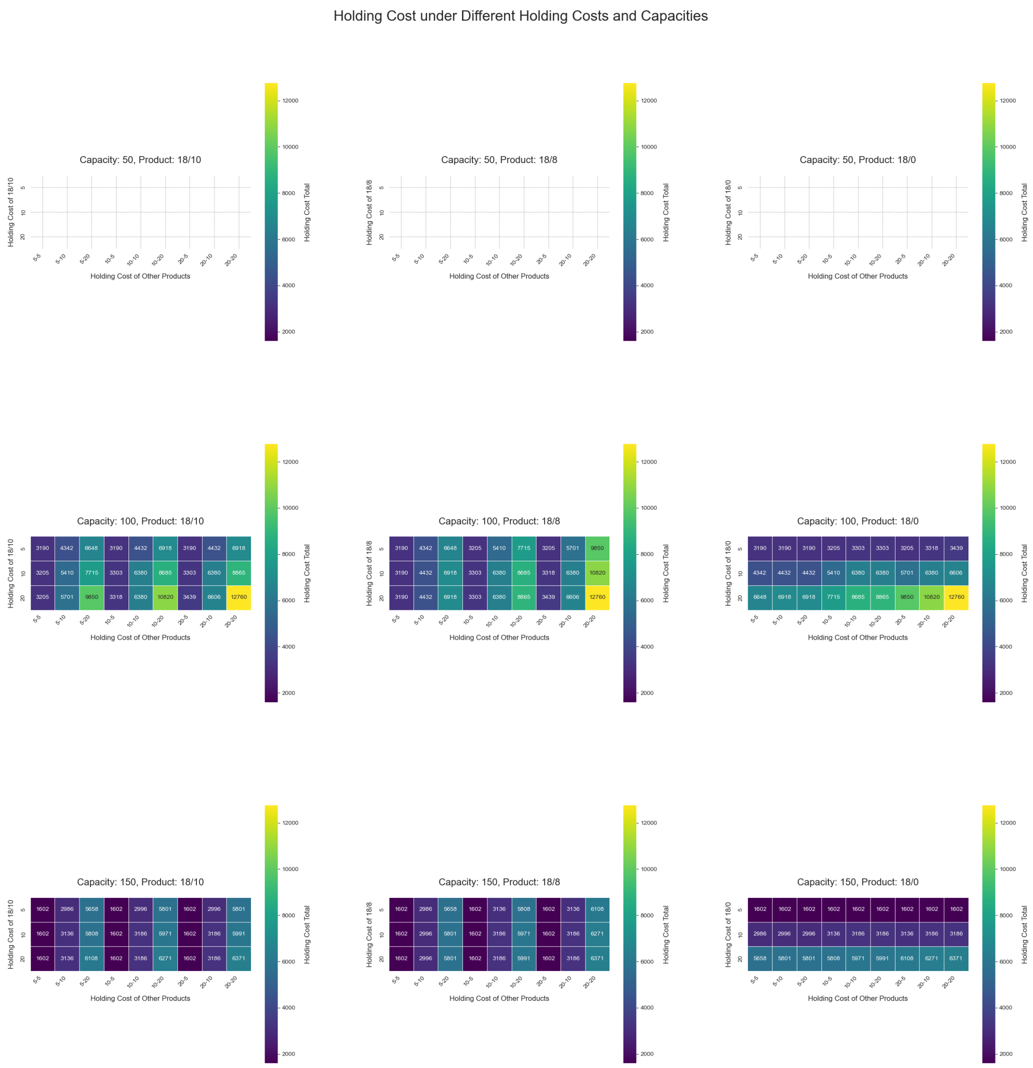

Figure 1: Heatmap showing holding cost across all experiments.

Figure 1 illustrates the heatmap of holding costs across different production capacities and holding cost configurations for alloys 18/10, 18/8, and 18/0. The tested production capacities are 50, 100, and 150 units. Each heatmap's color gradient signifies the holding cost, where lighter colors indicate higher costs and darker colors represent lower costs.

Effect of Production Capacity

At a capacity of 50 units, the model yields no feasible solution, indicating that the production constraints are too restrictive to meet the demands effectively. As a result, no data is available for this capacity level. With an increase to 100 units, holding costs are reduced, particularly for alloys 18/10 and 18/8, indicating enhanced inventory management flexibility. At 150 units, holding costs are effectively minimized, especially for scenarios with reduced holding costs for other products. The substantial reduction in holding costs at higher capacities can be attributed to economies of scale, which allow for more efficient allocation of storage resources and better responsiveness to demand fluctuations.

Impact of Alloy Combinations

The holding cost for alloy 18/10 exhibits significant sensitivity to changes in the holding cost of other products. This sensitivity is also observed for alloy 18/8, though to a lesser extent, while alloy 18/0 demonstrates reduced responsiveness to such changes. These results imply that alloy 18/10 necessitates more sophisticated inventory control mechanisms for cost optimization. The differential impact across alloys suggests that intrinsic material properties and demand profiles may influence holding cost sensitivity, necessitating a more tailored approach to inventory management for each alloy type.

General Observations

The heatmaps indicate that higher production capacities are generally associated with lower procurement costs, particularly when holding costs for other products are effectively controlled. The cost optimization effects are more substantial for alloys 18/10 and 18/8 compared to alloy 18/0. This suggests that procurement cost efficiency can be enhanced through strategic planning of production capacities and by closely managing the holding costs of other products.

4.2.2 Heatmap of Procurement Cost

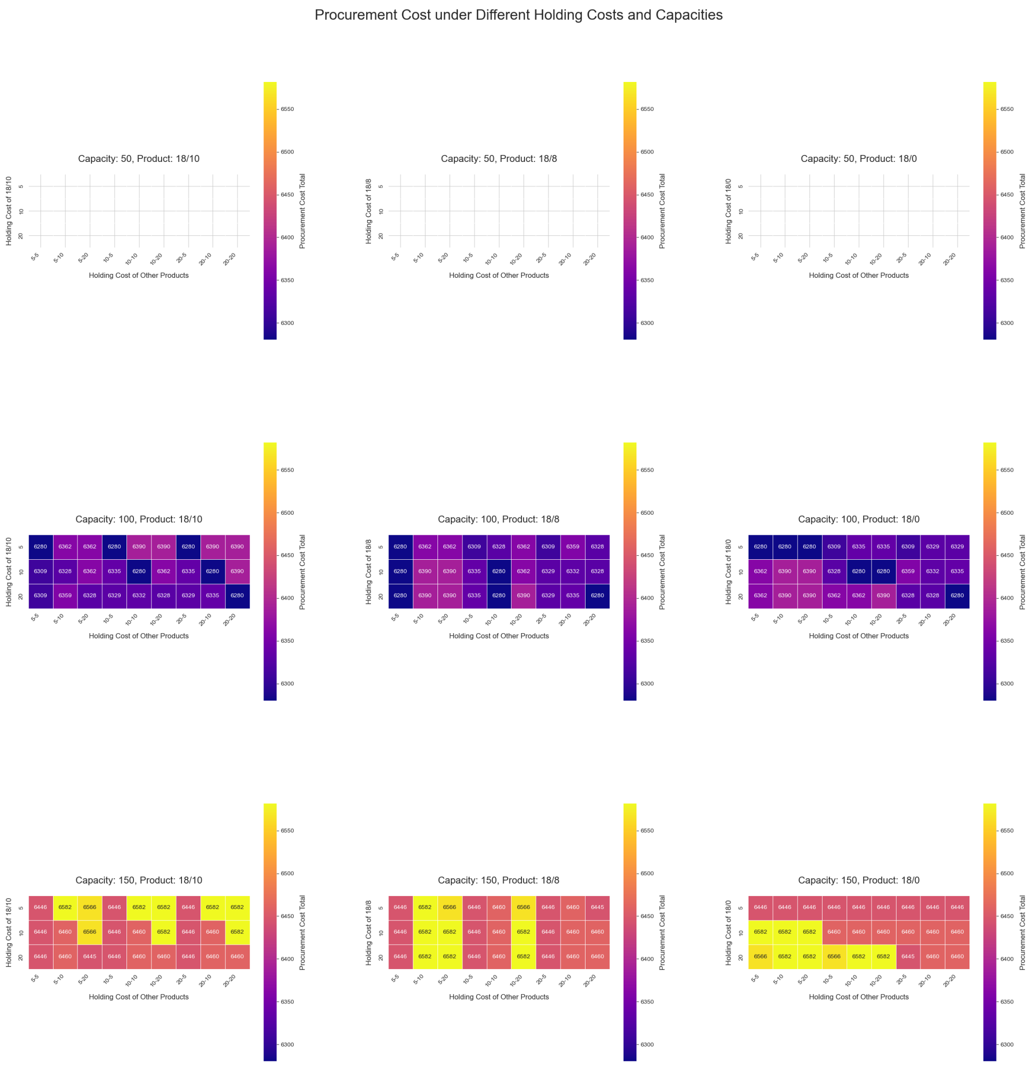

Figure 2: Heatmap showing minimum procurement cost across all experiments.

Figure 2 presents the heatmap of procurement costs as a function of varying holding costs and production capacities for the three alloys (18/10, 18/8, 18/0).

Impact of Production Capacity

At a production capacity of 50 units, the model yields no feasible solution due to the inability to meet demand within the given constraints. Thus, no procurement cost data is available for this capacity level. Conversely, at 100 units, procurement costs begin to exhibit variability, especially for alloys 18/8 and 18/10. With a capacity of 150 units, procurement costs show significant sensitivity to holding cost levels, with optimal values generally occurring when holding costs for other products are maintained at lower levels. This observation underscores the importance of a balanced approach to inventory and procurement management, wherein increasing production capacity not only reduces procurement costs but also enhances the overall system's adaptability to changes in cost parameters.

Alloy-Specific Observations

Procurement cost variability is most pronounced for the 18/10 alloy in response to changes in holding costs. In contrast, alloys 18/8 and 18/0 exhibit more stability, with procurement costs showing less pronounced changes under varying holding costs. This behavior can be explained by the differing characteristics inherent to each alloy type and their respective roles in the production process. The 18/10 alloy, for example, requires precise alignment between procurement and inventory strategies to mitigate cost fluctuations effectively.

Insights Derived from Heatmaps

The heatmaps indicate that higher production capacities are generally associated with lower procurement costs, particularly when holding costs for other products are effectively controlled. The cost optimization effects are more substantial for alloys 18/10 and 18/8 compared to alloy 18/0. This suggests that procurement cost efficiency can be enhanced through strategic planning of production capacities and by closely managing the holding costs of other products.

4.2.3 Heatmap of Total Cost

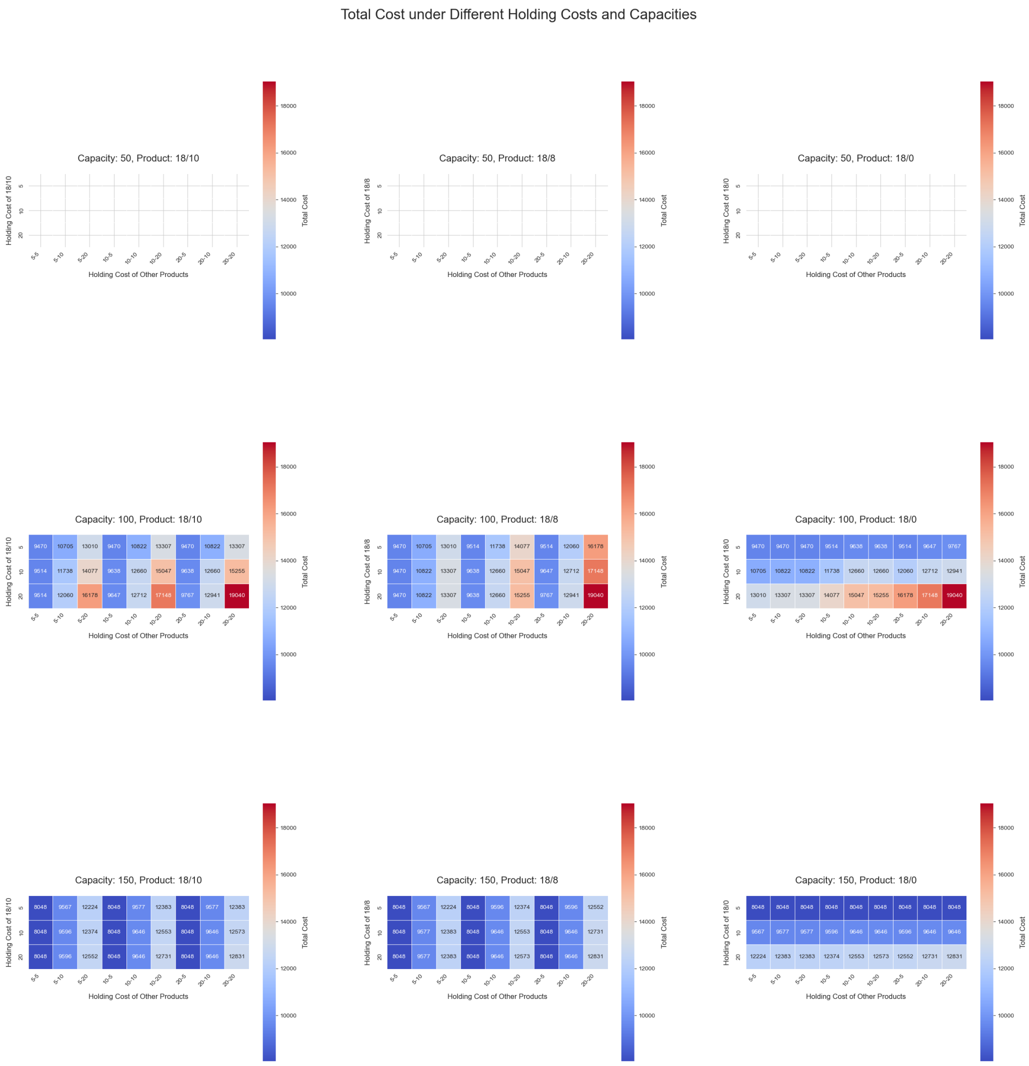

Figure 3: Heatmap showing minimum procurement cost across all experiments.

Figure 3 depicts the heatmap of total costs, which include holding, procurement, and production costs, across different production capacities and holding cost configurations for alloys 18/10, 18/8, and 18/0. The production capacities considered are 50, 100, and 150 units, with the color gradient representing the total cost, where lighter shades indicate higher costs and darker shades represent lower costs.

Effect of Production Capacity

At a capacity of 50 units, the model yields no feasible solution, suggesting that the production constraints are insufficient to meet the required production and inventory needs. Consequently, no total cost data is available for this capacity level. As production capacity increases to 100 units, a noticeable decrease in total cost occurs, particularly for alloys 18/8 and 18/10, due to improved optimization opportunities in production and inventory management. At 150 units, total cost is minimized, especially when the holding costs for other products are kept at lower levels. The reduction in total cost at higher capacities can be linked to better coordination between procurement, production, and inventory management, leading to enhanced operational synergies and reduced waste.

Alloy-Specific Cost Sensitivity

The total cost for alloy 18/10 demonstrates considerable sensitivity to variations in the holding cost of other products. For alloy 18/8, a similar trend is noted, though less prominently, while alloy 18/0 shows relatively reduced sensitivity. This indicates that alloy 18/10 requires more precise management practices to achieve cost efficiency. The analysis of the cost sensitivity suggests that each alloy requires a customized approach to effectively minimize the overall costs in the production process.

General Observations

The lowest total costs are generally achieved at higher production capacities (150 units), particularly when the holding costs of other products are minimized. These findings suggest that increasing production capacity fosters enhanced flexibility and consequently yields more cost-effective solutions. Additionally, the synergistic benefits observed at higher capacities imply that production planning must be aligned with inventory and procurement strategies to achieve optimal results.

4.3 General Findings

The experimental results consistently indicate that increasing production capacity results in reductions in holding, procurement, and total costs. Specifically:

- **Production Capacity vs. Holding Costs**: The experimental results indicate that increasing production capacity from 100 to 150 units consistently reduces total costs, particularly when holding costs are minimized. The experiments highlight that higher production capacities allow for more efficient management of inventory, effectively reducing the holding cost burden. At 150 units, both holding and procurement costs were observed to decrease, emphasizing the benefits of increased capacity in providing operational flexibility.

- **Holding Cost Variability**: The holding cost variability plays a crucial role in determining the overall cost efficiency of the system. For alloys such as 18/10, which are highly sensitive to holding cost changes, reducing holding costs leads to significant savings in total costs. On the other hand, alloys 18/0 showed less sensitivity, suggesting that targeted cost control measures could be alloy-specific to optimize the cost structure more effectively.

- **Objective Function Impact**: The objective function, which aims to minimize the total cost, responded favorably to higher production capacities under low holding cost conditions. The experiments demonstrate that with increased capacity, the system gains the ability to respond to fluctuating demand without incurring high holding costs. This adaptability is crucial for maintaining cost efficiency while meeting production requirements.

- **Trade-off Interpretation**: The trade-off between production capacity and holding costs is evident in the experimentation results. While increasing capacity involves higher operational throughput, it enables more effective inventory management and reduces reliance on high-cost procurement strategies. Conversely, minimizing holding costs without adequate production capacity can lead to infeasibility in meeting demand. Thus, a balanced approach that optimizes both parameters is necessary to achieve cost-effective production.

4.4 Conclusion

Increasing production capacity effectively contributes to reductions in holding, procurement, and total costs. The sensitivity of total cost to holding cost variations is particularly notable for specific alloys, especially 18/10, which requires meticulous inventory management strategies. In contrast, alloys 18/8 and 18/0 exhibit greater cost stability under changing holding costs, implying a more consistent cost structure irrespective of holding cost variations. These findings advocate for a differentiated approach to inventory and procurement strategies to optimize total cost efficiency across varying alloy types and production capacities. Furthermore, the results suggest that maximizing production capacity, while simultaneously managing holding costs strategically, yields significant cost advantages. To this end, a nuanced understanding of each alloy's cost behavior is critical to developing effective supply chain strategies that enhance overall system robustness and economic viability. The differential cost sensitivities observed across alloys necessitate a flexible and alloy-specific approach to both production and inventory management, ensuring that the benefits of increased production capacity are fully realized while mitigating potential cost-related risks.

5 e: CopperLimit

Introduction

The company wants the amount of copper in its products to be below a certain value, called the CopperLimit. In order to remove copper from a mix, electrolysis can be used. Each month it is decided whether electrolysis will be used; if so, fixed costs for electrolysis must be paid (100 euro) in that month and the production of that month for each product can be treated. The weight reduction caused by electrolysis is equal to the weight of the copper that is removed. The variable costs for electrolysis depend on the amount of copper that is removed this way and is 5 euro per kg.

Question

Extend the mathematical model so that the use of electrolysis is included. Use binary decision variables and keep it as a mixed integer linear programming problem (no multiplication of variables with variables). Provide the changes in your mathematical formulation. Use the model to determine the lowest CopperLimit that can be used without extra costs, compared to the solution for (b). Experiment with other values of the CopperLimit and show the effect it has on electrolysis costs and holding costs.

5.1 Extended Model

5.1.1 Notation (sets and indices, parameters, variables)

The notation used for this mathematical formulation is provided in Table 8:

Table 8: Notation

Sets and indices	Description	Indices/Units
P	Set of products	$p \in P$
S	Set of suppliers	$s \in S$
T	Set of months	$t \in T$
Parameter		**Units**
$d_{p,t}$	Demand for product p in month t	[kg]
h_p	Holding cost for product p	[€/kg]
c_s	Cost of raw material from supplier s	[€/kg]
m_s	Maximum supply from supplier s	[kg/month]
Cr_s	Chromium content in raw material from supplier s	[%]
Ni_s	Nickel content in raw material from supplier s	[%]
Cu_s	Copper content in raw material from supplier s	[%]
$CrReq_p$	Required chromium content for product p	[%]
$NiReq_p$	Required nickel content for product p	[%]
$CuLimit_t$	Maximum allowed copper content in month t	[%]
e_c	Cost per kg for copper electrolysis	[€/kg]
C'	Monthly production capacity	[kg/month]
E_c	fixed cost for copper electrolysis	[€]
M	a very huge number	[1]
Variable		**Units**
$x_{p,t}$	Quantity of product p produced in month t	[kg]
$I_{p,t}$	Inventory of product p at the end of month t	[kg]
$z_{p,s,t}$	Quantity of raw material purchased from supplier s for product p in month t	[kg]
$CuRemoved_{p,t}$	Quantity of electrolyzed copper for product p in month t	[kg]
y_t	Binary variable indicating whether electrolysis is used in month t	[binary]

5.1.2 Objective Function

$$\min \sum_{t \in T} \left(\sum_{p \in P} h_p \cdot I_{p,t} + \sum_{s \in S} \sum_{p \in P} c_s \cdot z_{p,s,t} + \sum_{p \in P} e_c \cdot CuRemoved_{p,t} + y_t \cdot E_c \right)$$

5.1.3 Constraints

$$(x_{p,t} - CuRemoved_{p,t}) + I_{p,t-1} - I_{p,t} \geq d_{p,t}, \quad \forall p \in P, \forall t \in T \tag{1}$$

$$\sum_{p \in P} x_{p,t} \leq C', \quad \forall t \in T \tag{2}$$

$$\sum_{p \in P} z_{p,s,t} \leq m_s, \quad \forall s \in S, \forall t \in T \tag{3}$$

$$\sum_{s \in S} Cr_s \cdot z_{p,s,t} = CrReq_p \cdot (x_{p,t} - CuRemoved_{p,t}), \quad \forall p \in P, \forall t \in T \tag{4}$$

$$\sum_{s \in S} Ni_s \cdot z_{p,s,t} = NiReq_p \cdot (x_{p,t} - CuRemoved_{p,t}), \quad \forall p \in P, \forall t \in T \tag{5}$$

$$\sum_{s \in S} z_{p,s,t} = x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{6}$$

$$\sum_{s \in S} Cu_s \cdot z_{p,s,t} - CuRemoved_{p,t} \leq CuLimit_t \cdot (x_{p,t} - CuRemoved_{p,t}), \quad \forall p \in P, \forall t \in T \tag{7}$$

$$CuRemoved_{p,t} \leq \sum_{s \in S} Cu_s \cdot z_{p,s,t}, \quad \forall p \in P, \forall t \in T \tag{8}$$

$$CuRemoved_{p,t} \leq y_t \cdot M, \quad \forall p \in P, \forall t \in T \tag{8}$$

$$x_{p,t}, I_{p,t}, z_{p,s,t}, CuRemoved_{p,t} \geq 0, I_{p,0} = 0, \quad \forall p \in P, \forall s \in S, \forall t \in T \tag{9}$$

5.1.4 Description of the Constraints

The first constraint ensures that the production and inventory satisfy the demand for each product in each month. The second constraint limits the total monthly production to a fixed capacity C'. The third constraint ensures that the quantity of raw material purchased from each supplier does not exceed their maximum supply limit. The fourth and fifth constraints require that the purchased raw materials contain sufficient chromium and nickel to meet the required product composition for each alloy. The sixth constraint manages the copper content by ensuring that, after electrolysis, the total copper content does not exceed the allowed copper limit in a given month. The seventh constraint ensures that the copper removed via electrolysis for each product from each supplier is less than or equal to the copper content in the purchased raw material. The eighth constraint ties electrolysis activation to the binary variable y_t, ensuring copper electrolysis occurs only when needed. Finally, the ninth constraint guarantees that all decision variables (production, inventory, purchases, and copper removed) remain non-negative throughout the planning horizon.

5.2 Changes

The model that includes copper introduces significant changes compared to the original non-copper model. The primary differences and modifications are summarized below:

5.2.1 New Decision Variables

The copper model introduces two new decision variables:

- $CuRemoved_{p,s,t}$: The quantity of copper removed by electrolysis for product p from supplier s in month t. This variable did not exist in the original model.
- y_t: A binary variable that indicates whether electrolysis is activated in month t (1 if electrolysis is used, 0 otherwise). This variable controls the activation of copper electrolysis in the model.

5.2.2 Changes to the Objective Function

The objective function in the non-copper model only minimized two types of costs: inventory holding costs and raw material procurement costs. In the copper model, a third component—the copper electrolysis cost—is added to the objective function:

$$\min \sum_{t \in T} \left(\sum_{p \in P} h_p \cdot I_{p,t} + \sum_{s \in S} \sum_{p \in P} c_s \cdot z_{p,s,t} + \sum_{p \in P} e_c \cdot CuRemoved_{p,t} + y_t \cdot E_c \right)$$

In this version, the electrolysis cost per kilogram of removed copper e_c is multiplied by the amount of copper removed $CuRemoved_{p,s,t}$, which was absent in the non-copper model.

5.2.3 New Copper-Related Constraints

Several new constraints have been added to manage copper content and electrolysis. These constraints do not exist in the non-copper model and serve to control how copper is treated during production.

Copper Content Limit

The total copper content in the purchased raw materials, after copper removal through electrolysis, must not exceed the allowed copper limit for each month $CuLimit_t$:

$$\sum_{s \in S} Cu_s \cdot z_{p,s,t} - CuRemoved_{p,t} \leq CuLimit_t \cdot (x_{p,t} - CuRemoved_{p,t}), \quad \forall p \in P, \forall t \in T$$

Electrolysis Activation

Copper removal can only occur if electrolysis is activated. This is controlled by the binary variable y_t, which ensures that electrolysis is performed only when needed:

$$CuRemoved_{p,t} \leq y_t \cdot M, \quad \forall p \in P, \forall t \in T \tag{8}$$

These constraints are entirely new and have been added to manage the copper content and its removal in the production process, something the non-copper model did not need to consider.

5.2.4 Changes in Constraint Structure

In the non-copper model, constraints primarily focused on production and inventory management, raw material procurement, and ensuring that the chromium and nickel content requirements for the alloys were met. In the copper model, the structure has been modified by adding copper-related constraints, while still retaining the original production, inventory, and procurement constraints.

5.2.5 Increased Model Complexity

The copper model increases the complexity of the problem, primarily due to the introduction of:

- The binary variable y_t, which controls whether electrolysis is activated, adding a discrete decision-making component to the model.
- The need to manage copper content, with constraints ensuring that copper removal occurs correctly and does not exceed the available copper from suppliers.

This makes the copper model a mixed-integer linear programming (MILP) problem, whereas the original non-copper model was a linear programming (LP) problem. This increase in complexity can make the copper model more difficult and time-consuming to solve.

5.3 Extended Model in Python and the Lowest CopperLimit

5.3.1 the Lowest CopperLimit

To find the lowest possible copper limit without increasing the production cost, first I set a Python file which uses a binary search method to adjust the copper limit incrementally until the smallest feasible value is found. Initially, a range for the copper limit is defined, and the midpoint of this range is tested each time. If using this limit keeps the production plan feasible and does not increase costs, the search continues to lower the limit. If the limit makes the plan infeasible or increases costs, the search adjusts to a higher value.
This process continues until the difference between the highest and lowest limits is very small, effectively finding the minimum copper limit that maintains the original cost. This approach is efficient for balancing resource use with production goals.
The code get a result of the lowest possible copper limit of 2.96%

5.3.2 Experiment

The file named modelb&e.py include both the model without considering copper and the model includes copper, when running th file users can input b or e to choose which model to use.
By changing the CopperLimit multiple times (from 2.96% to 0.1%), the effect it has on electrolysis costs and holding costs, total cost can be shown. The diagram 4 below shows the result.

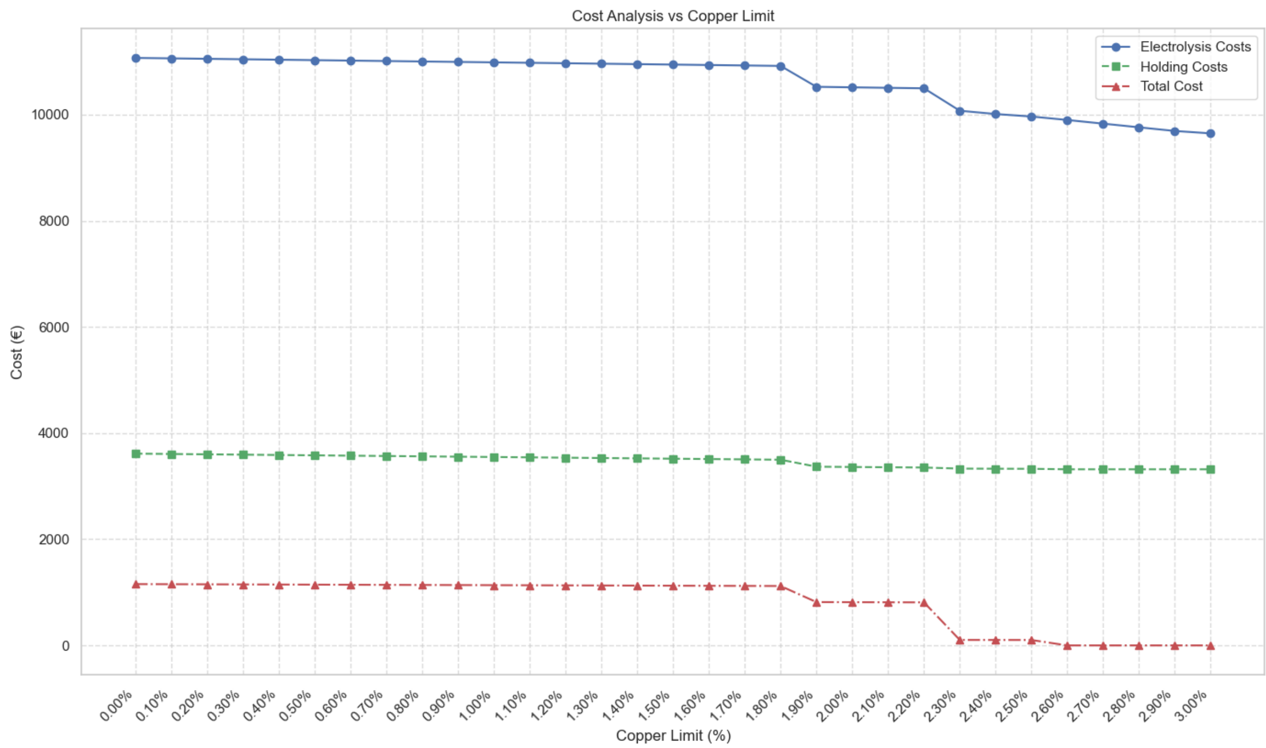

Figure 4: The effect of copperlimit on Electrolysis Costs, Holding cost and Total cost.

5.3.3 Analysis

Electrolysis Costs

Trend: The electrolysis costs (blue line) remain relatively stable up to approximately a 1.4% copper limit, after which there is a noticeable decline, continuing gradually until the copper limit reaches around 2.3%, where the decline becomes steeper.
Analysis: The initial flat trend suggests that the copper limit has little to no impact on electrolysis costs up to a certain point. Beyond 1.4%, as the copper content allowed increases, there seems to be a significant decrease in these costs, indicating that the process may become more efficient or less resource-intensive as higher copper levels are accepted.
Key Point: Reducing copper content complicates the electrolysis process, raising costs.

Holding Costs

Trend: The holding costs (green line) remain constant throughout the range of copper limits, showing no significant fluctuation.
Analysis: The stability of holding costs suggests that varying the copper limit does not influence the cost of holding materials. This implies that the cost of holding may be fixed and independent of the operational or input changes related to the copper content.
Key Point: Holding costs are not sensitive to copper content changes, being influenced by logistical factors.

Total Cost

Trend: The total cost (red line) remains relatively constant up to around a 1.9% copper limit, where it starts to decrease more significantly, mirroring the trend seen in the electrolysis costs.
Analysis: The total cost is heavily influenced by the electrolysis costs, given that holding costs are stable. The decline in total cost after 1.9% shows that an increase in the copper limit is beneficial to the overall cost-efficiency of the process, likely due to reduced electrolysis expenses.
Key Point: Keeping copper content above 2.96% helps maintain lower total costs.

6 Statements

6.1 AI tools Statements

I use chatGPT mainly to write the assignment in proper Latex form, I asked it questions like: help me change this table into Latex form or Tell me how to refer a picture in the paragraph. It gave me the Latex code without changing my original context so that I can write the assignment in a more efficient way.
I also use GitHub copilot to help me write the doc of a function in Python language and let it check my grammar as well.
You can see my complete files including all codes, full report and excel files in my Github: https://github.com/mengzi667/QML-assignment1.git