Assignment Q1

QUANTITATIVE METHODS FOR LOGISTICS

ME44206

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Contents

1 Mathematical Model

The notation used for this mathematical formulation is provided in Table 1.

Table 1: Sets, Indices, Parameters, and Decision Variables

Sets and indices	Description	Units
P	Set of products	-
S	Set of suppliers	-
T	Set of months	-
Parameters	**Description**	**Units**
$d_{p,t}$	Demand for product p in month t	[kg]
h_p	Holding cost for product p	[€/kg]
c_s	Cost of raw material from supplier s	[€/kg]
m_s	Maximum supply from supplier s	[kg/month]
Cr_s	Chromium content in raw material from supplier s	[%]
Ni_s	Nickel content in raw material from supplier s	[%]
$CrReq_p$	Required chromium content for product p	[%]
$NiReq_p$	Required nickel content for product p	[%]
C^t	Monthly production capacity	[kg/month]
Decision Variables	**Description**	**Units**
$x_{p,t}$	Quantity of product p produced in month t	[kg]
$I_{p,t}$	Inventory of product p at the end of month t	[kg]
$z_{p,s,t}$	Quantity of raw material purchased from supplier s in month t for product p	[kg]

The mathematical formulation then follows as:

1.1 Objective Function

$$\min \sum_{t\in T}\left(\sum_{p\in P} h_p \cdot I_{p,t} + \sum_{s\in S}\sum_{p\in P} c_s \cdot z_{p,s,t}\right)$$

1.2 Constraints

$$x_{p,t} + I_{p,t-1} - I_{p,t} = d_{p,t}, \quad \forall p \in P, \forall t \in T \tag{1}$$

$$\sum_{p\in P} x_{p,t} \leq C^t, \quad \forall t \in T \tag{2}$$

$$\sum_{p\in P} z_{p,s,t} \leq m_s, \quad \forall s \in S, \forall t \in T \tag{3}$$

$$\sum_{s\in S} Cr_s \cdot z_{p,s,t} = CrReq_p \cdot x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{4}$$

$$\sum_{s\in S} Ni_s \cdot z_{p,s,t} = NiReq_p \cdot x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{5}$$

$$\sum_{s\in S} z_{p,s,t} = x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{6}$$

$$x_{p,t}, I_{p,t}, z_{p,s,t} \geq 0, \quad \forall p \in P, \forall s \in S, \forall t \in T \tag{7}$$

1.3 Description of the Constraints

- The first constraint (1) ensures that the production and inventory satisfy the demand for each product in each month.
- The second constraint (2) limits the total monthly production to a fixed capacity C^t.
- The third constraint (3) ensures that the quantity of raw material purchased from each supplier does not exceed their maximum supply limit.

- The fourth constraint (4) requires that the purchased raw materials contain sufficient chromium to meet the product's composition.
- The fifth constraint (5) ensures the same for nickel content.
- The sixth constraint (6) ensures that the total raw materials purchased match the quantity of products produced.
- The final constraint (7) enforces the non-negativity of the decision variables.

2 b: Implementation of the Model

Question

Implement the mathematical model in part (a) in python and solve with Gurobi. Provide the optimal solution. By the optimal solution it is meant that you need to report the optimal cost value and the production schedule. Your python model needs to also output the values of the decision variables but only those that you discuss need to be in the report.

2.1 Implementation in Python

Python code of the model and the data is in the Appendix part B.

2.2 Output of the model (optimal solution)

Following are the output of the code and provide the optimal solution (in table format with explanation).

Optimal cost Output

Optimal cost: 9646.78 EURO

Production Plan Schedule Table

Table 2 shows the monthly breakdown of the production quantities (in kilograms) for each of the three products: 18/10, 18/8, and 18/0 stainless steel.

- **Rows**: Each row corresponds to one of the three products.
- **Columns**: Each column represents a month (from Month 1 to Month 12).

Table 2: Production Plan (kg)

Product	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
18/10	25.0	25.0	0.0	0.0	0.0	50.0
18/8	10.0	10.0	10.0	14.375	5.625	10.0
18/0	8.086957	65.0	86.956522	86.956522	85.625	44.375

Product	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
18/10	12.0	0.0	10.0	10.0	45.0	99.0
18/8	10.0	10.0	10.0	19.0	1.0	0.0
18/0	78.0	80.0	62.0	80.0	36.0	0.0

Inventory Levels Schedule Table

Table 3 displays the monthly inventory levels for each product at the end of the month (in kilograms). The inventory level represents how much stock is leftover after meeting the demand.

- **Rows**: Each row corresponds to one of the three products.
- **Columns**: Each column shows the inventory at the end of each month (Month 1 to Month 12).

Table 3: Inventory Levels (kg)

Product	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
18/10	0.0	0.0	0.0	0.0	0.0	0.0
18/8	0.0	0.0	0.0	0.0	4.375	0.0
18/0	3.086957	48.086957	55.043478	117.0	152.625	72.0

Product	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
18/10	0.0	0.0	0.0	0.0	0.0	0.0
18/8	0.0	0.0	0.0	0.0	9.0	0.0
18/0	0.0	0.0	22.0	0.0	100.0	0.0

Purchase Plan Schedule Table

Table 4 5 6 shows the quantities of raw materials (in kilograms) purchased from each supplier (A, B, C, D, E) across 12 months.

- **Rows**: Each row corresponds to a supplier.
- **Columns**: Each column represents the raw material purchased from a supplier for each month (Month 1 to Month 12).

Table 4: Procurement Plan for Product 1 (kg)

Supplier	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Supplier 1	9.011628	9.011628	0.0	0.0	0.0	18.023256
Supplier 2	5.813953	5.813953	0.0	0.0	0.0	11.627907
Supplier 3	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 4	10.174419	10.174419	0.0	0.0	0.0	20.348837
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

Supplier	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Supplier 1	4.325581	0.0	3.604651	16.220930	0.0	35.686047
Supplier 2	2.790698	0.0	2.325581	10.465116	0.0	23.023256
Supplier 3	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 4	4.883721	0.0	4.069767	18.313953	0.0	40.290698
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

Table 5: Procurement Plan for Product 2 (kg)

Supplier	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Supplier 1	4.883721	4.883721	3.043478	3.043478	4.375000	2.747093
Supplier 2	1.860465	1.860465	2.086957	2.086957	3.000000	1.046512
Supplier 3	0.0	0.0	4.869565	4.869565	7.0	0.0
Supplier 4	3.255814	3.255814	0.0	0.0	0.0	1.831395
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

Supplier	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Supplier 1	4.883721	4.883721	4.883721	9.279070	0.488372	0.0
Supplier 2	1.860465	1.860465	1.860465	3.534884	0.186047	0.0
Supplier 3	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 4	3.255814	3.255814	3.255814	6.186047	0.325581	0.0
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

Table 6: Procurement Plan for Product 3 (kg)

Supplier	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Supplier 1	8.086957	65.000000	86.956522	86.956522	85.625000	44.375000
Supplier 2	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 3	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 4	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

Supplier	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
Supplier 1	78.000000	80.000000	62.000000	36.000000	0.0	0.0
Supplier 2	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 3	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 4	0.0	0.0	0.0	0.0	0.0	0.0
Supplier 5	0.0	0.0	0.0	0.0	0.0	0.0

2.3 Discussion

The procurement, production, and inventory strategies outline resource management over 12 months. Product 3's production plan shows variability, with peaks in months 2 and 3, likely reflecting seasonal demand or strategic stockpiling. Product 1 has a balanced but intermittent production pattern, with increased output in the final months, aligning with year-end demand.
The inventory plan indicates Product 3's accumulation in months 4 and 5 before depleting, suggesting a push strategy to buffer against demand variability. Products 1 and 2 maintain minimal inventory, reflecting a lean production approach focused on cost efficiency.
The procurement plan shows heavy reliance on Supplier 1, especially for Product 3, with peaks in months 2, 3, and 10, posing a potential risk if disruptions occur. Suppliers 2 and 4 contribute less frequently. Product 2's procurement relies on Supplier 2 during months 4 to 6 and Supplier 1 in months 7 and 8, suggesting an adaptive strategy.
Product 3's inventory buildup between months 4 and 6 supports future production or mitigates supply chain disruptions. Lean inventory for Products 1 and 2 suggests predictable demand or a just-in-time (JIT) model to minimize carrying costs.
Overall, the organization seeks efficiency with minimal inventory while maintaining contingencies. Diversifying suppliers or increasing inventory buffers could enhance supply chain resilience.

3 c: Verification of the Implementation

Question

Verify the implementation, to convince that the code for (b) matches the mathematical formulation of (a) with a number of verification tests that cover different types of parameters used in the model. Provide a discussion on the verification experiments and their results with your justifications.

3.1 Verification Strategy

The following tests were performed by adjusting key parameters:

- **Objective Function Parameters**: Objective function verification ensures that the objective function correctly represents the actual optimization goal, such as cost minimization or profit maximization. By adjusting parameters or variables (e.g., increasing the storage cost for a specific alloy), we observe the resulting changes in the objective function. The method typically involves running small-scale experiments and manually calculating the total cost to compare it with the model's output. If the behavior matches the expected outcome (e.g., reducing storage while increasing costs), it confirms that the objective function is correctly formulated.
- **Function Parameters**: Function parameters verification aims to ensure that the model's parameters (e.g., material content, supply limits) are correctly applied in the model. By modifying specific parameters (such as increasing Supplier A's chromium content), we expect the model to adjust the material procurement to optimize production. If the model responds as expected by reducing costs, it confirms that the function parameters are appropriately influencing the model's behavior. It is also important to verify if the "unmixing" constraint (i.e., preventing the supply from being split into different products in a way that changes the composition) was correctly implemented
- **RHS parameters**: RHS (right-hand side) parameters verification checks if changes in the constraint parameters (such as demand levels or supply limits) are properly reflected in the model's decisions. By significantly altering demand or supply constraints, we observe whether the model appropriately adjusts its production or procurement plan. For example, reducing demand to zero in most months should result in a zero production plan, while reducing the availability of a key supplier should increase costs. If the model reacts as expected, it confirms that the RHS parameters are correctly applied.

3.2 Key Tests for Verification

The Table 7 below summarizes the key tests that were conducted to verify the implementation of the model. For each test, the objective is described, followed by the setup, expected results, and justifications. These tests ensure that the model behaves correctly under different scenarios and satisfies the defined constraints.

Table 7: Summary of Test Experiments for Model Verification

Test Category	Experiment Description	Expected Results	Remarks	Test Result	Passed?
Objective Function Verification	Increase storage cost of the third alloy (18/0) by 2x.	The plan will reduce monthly storage for 18/0, and overall cost will increase (cost > 9646.78).	Verifies model sensitivity to storage cost changes.	Cost=12712.02, 18/0 alloy storage reduced	Yes
Function Parameters Verification	Set chromium content requirement for first alloy to 100% and the demand for it in first month as 1, all other alloys in other months demand to 0.	Model becomes infeasible.	Suppliers can't meet demand without "unmixing" materials, which should be disallowed.	Model infeasible	Yes
RHS Parameters Verification	Demand for the first product (18/10) is 30 in month 1, with 0 for all other months.	Minimum cost will be 185.58 EUR, with 0 production/inventory in other months.	Tests how the model adjusts production with significant demand changes.	Cost=185.58 EUR, 0 production/inventory after January	Yes
RHS Parameters Verification	Reduce Supplier D's max purchase limit (high nickel content).	Overall cost increases due to reduced high-quality nickel availability.	Verifies impact of reducing key material supply on total costs.	Cost=9665.97	Yes

3.3 Conclusion

These tests demonstrate that the code correctly implements the mathematical model, with all constraints (demand, production capacity, supplier limits, and material composition) and the objective function properly adhered to. The model behaves as expected under different scenarios, verifying that the code aligns with the mathematical formulation.

4 d: Experimenting

Question

Experiment with different values of the maximum production capacity and the holding costs in order to get insights for the trade-off between these in the production and inventory holding decisions for products. Justify the ranges of values you use for the experiments. Provide your interpretation of the results by discussing the impact on the objective function, different costs involved and the decisions.

4.1 Experimenting

The purpose of the experiments is to evaluate the performance of the production and inventory system under various configurations of production capacity and holding costs. Each experiment modifies the production capacity and the holding costs of three different products (18/10, 18/8, 18/0 alloys) across several suppliers. The goal is to minimize the total cost, which includes production, inventory, and purchase costs, while satisfying demand and maintaining inventory balance.

4.1.1 Steps in Each Experiment

1. Define the production capacities to be tested: 50, 100, and 150 units.
2. Vary the holding costs for each product: 5, 10, and 20 units for each of the 18/10, 18/8, and 18/0 alloys.
3. For each combination of production capacity and holding costs, run the optimization model to minimize the total cost.
4. Ensure that demand is met for each product and that inventory levels are maintained across the 12-month planning horizon.
5. Extract and save the results for total cost, production plan, inventory plan, and purchase plan for each experiment.
6. Compare the results to identify the trade-off between production capacity and holding costs and their impact on the overall system cost.

The total number of experiments is determined by the combination of different parameter values. Specifically, there are:

- 3 levels of production capacity (50, 100, 150 units),
- 3 levels of holding cost for the 18/10 alloy (5, 10, 20 units),
- 3 levels of holding cost for the 18/8 alloy (5, 10, 20 units),
- 3 levels of holding cost for the 18/0 alloy (5, 10, 20 units).

Thus, the total number of experiments is calculated as:

$$3 \times 3 \times 3 \times 3 = 81$$

4.1.2 Data Outputs

Each experiment outputs:

- Total cost for the system (production, inventory, and purchase costs).
- Production plan: the quantity of each product to produce in each month.
- Inventory plan: the stock levels for each product at the end of each month.
- Purchase plan: the quantity of each product to be purchased from each supplier.

The full output are saved in an Excel file named which you can find in the additional deliverables, with each experiment occupying separate sheets.

4.2 Results Analysis

The following section presents the analysis of the experimental results. The purpose is to identify the optimal configurations by comparing various production capacities and holding costs. The analysis is visualized using heatmaps and statistical charts, followed by a discussion of the trends observed in production, inventory, and purchase plans. Detailed results are included in the appendix.

4.2.1 Heatmap of Minimum Total Cost

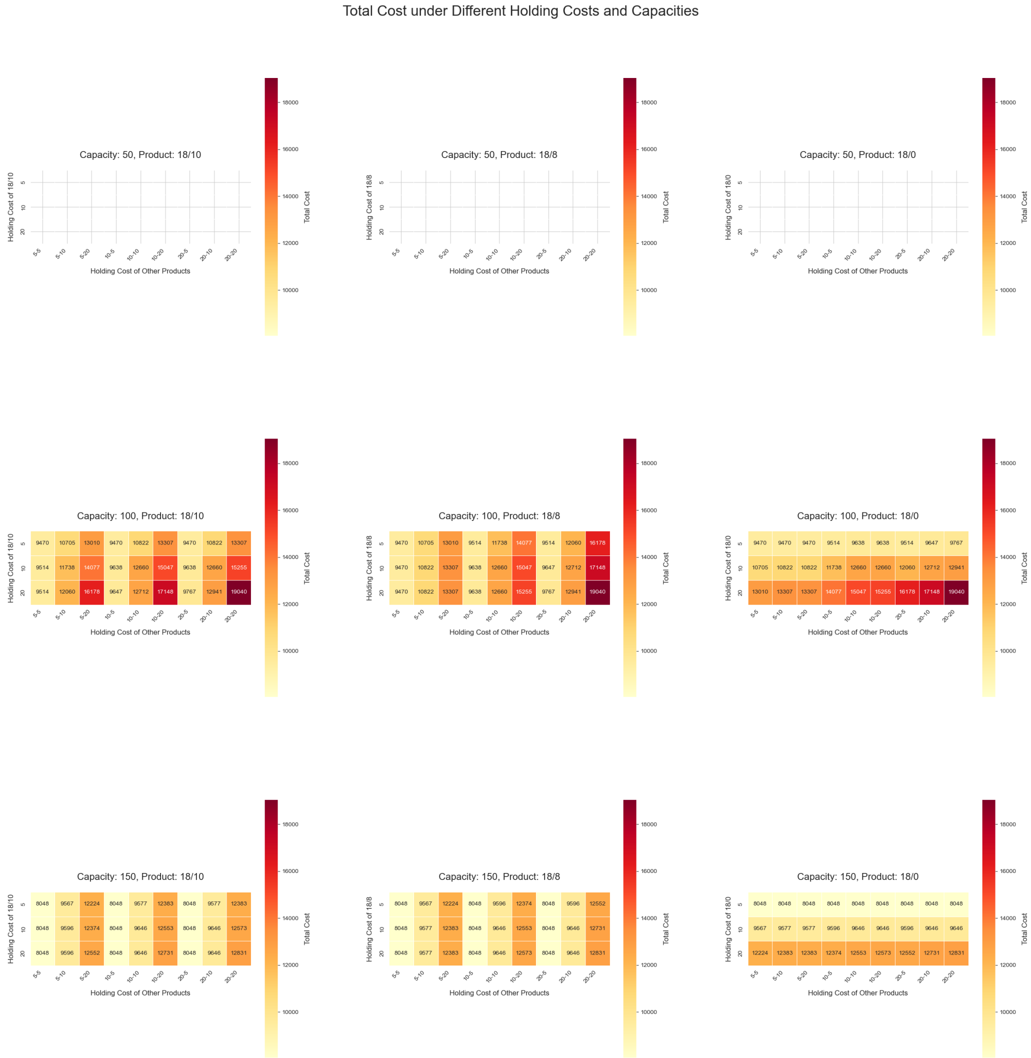

Figure 1: Heatmap showing minimum total cost across all experiments.

The heatmap (Figure 1, also can be found in additional deliverables) shows how total cost varies across different configurations of production capacity and holding costs. Specifically, the heatmaps depict three production capacities (50, 100, 150) and three alloy combinations (18/10, 18/8, 18/0), with the color gradient representing total cost (from yellow for lower costs to red for higher costs). Lower costs are concentrated in areas where holding costs are minimized for specific production capacities. For instance, production capacities of 100 or 150 units with lower holding costs for the 18/10 alloy yield the most cost-efficient results. The detailed analysis is as below.

4.2.2 Effect of Production Capacity on Cost

As production capacity increases from 50 to 150, the total cost tends to decrease for the same storage cost combination. This can be expressed as:

$$\text{Total cost decreases as production capacity increases.}$$

For example, in all product combinations, the minimum total cost with a production capacity of 150 is lower than with a capacity of 50. This suggests that increasing production capacity helps reduce total cost. This could be due to the ability of higher capacities to better manage demand fluctuations and optimize the allocation of production and storage resources.

4.2.3 Effect of Different Alloy Combinations on Cost

Each alloy combination (18/10, 18/8, 18/0) exhibits different impacts on total cost:
- For alloy 18/10, storage cost has a more significant effect on total cost. As the holding cost increases, the total cost varies more distinctly across the heatmap.
- On the other hand, for alloy 18/8 and 18/0, the total cost is more concentrated, and the effect of storage cost on total cost is less pronounced.

This suggests that different alloy combinations require different storage strategies. For example, the 18/10 alloy may require more precise storage strategies to optimize costs, while for alloys 18/8 and 18/0, changes in storage cost will not substantially affect total cost.

4.2.4 Effect of Storage Cost on Minimum Cost

The influence of storage cost on total cost becomes more evident as production capacity increases:
- In the heatmaps for a production capacity of 150, total cost significantly decreases in the regions with lower storage costs, especially for the 18/10 alloy combination.
- Overall, the optimal point for storage cost is not the same across different production capacities and alloy combinations, indicating that optimal storage strategies need to be tailored for each alloy combination.

4.3 Conclusion

Increasing production capacity can help reduce total cost, but the sensitivity of total cost to storage costs varies across different alloy combinations. For some combinations, like 18/10, optimizing storage cost is crucial to minimizing total cost, while for others, such as 18/8 and 18/0, storage costs have a less significant impact on total cost.

5 e: CopperLimit

Introduction

The company wants the amount of copper in its products to be below a certain value, called the CopperLimit. In order to remove copper from a mix, electrolysis can be used. Each month it is decided whether electrolysis will be used; if so, fixed costs for electrolysis must be paid (100 euro) in that month and the production of that month for each product can be treated. The weight reduction caused by electrolysis is equal to the weight of the copper that is removed. The variable costs for electrolysis depend on the amount of copper that is removed this way and is 5 euro per kg.

Question

Extend the mathematical model so that the use of electrolysis is included. Use binary decision variables and keep it as a mixed integer linear programming problem (no multiplication of variables with variables). Provide the changes in your mathematical formulation. Use the model to determine the lowest CopperLimit that can be used without extra costs, compared to the solution for (b). Experiment with other values of the CopperLimit and show the effect it has on electrolysis costs and holding costs.

5.1 Extended Model

5.1.1 Notation (sets and indices, parameters, variables)

The notation used for this mathematical formulation is provided in Table 8:

Table 8: Notation

Sets and indices	Description	Indices/Units
P	Set of products	$p \in P$
S	Set of suppliers	$s \in S$
T	Set of months	$t \in T$
Parameter		**Units**
$d_{p,t}$	Demand for product p in month t	[kg]
h_p	Holding cost for product p	[€/kg]
c_s	Cost of raw material from supplier s	[€/kg]
m_s	Maximum supply from supplier s	[kg/month]
Cr_s	Chromium content in raw material from supplier s	[%]
Ni_s	Nickel content in raw material from supplier s	[%]
Cu_s	Copper content in raw material from supplier s	[%]
$CrReq_p$	Required chromium content for product p	[%]
$NiReq_p$	Required nickel content for product p	[%]
$CuLimit_t$	Maximum allowed copper content in month t	[kg]
e_c	Cost per kg for copper electrolysis	[€/kg]
C'	Monthly production capacity	[kg/month]
Variable		**Units**
$x_{p,t}$	Quantity of product p produced in month t	[kg]
$I_{p,t}$	Inventory of product p at the end of month t	[kg]
$z_{p,s,t}$	Quantity of raw material purchased from supplier s for product p in month t	[kg]
$CuRemoved_{p,s,t}$	Quantity of electrolyzed copper for product p from supplier s in month t	[kg]
y_t	Binary variable indicating whether electrolysis is used in month t	[binary]

5.1.2 Objective Function

$$\min \sum_{t \in T} \left(\sum_{p \in P} h_p \cdot I_{p,t} + \sum_{s \in S} \sum_{p \in P} c_s \cdot z_{p,s,t} + \sum_{p \in P} \sum_{s \in S} e_c \cdot CuRemoved_{p,s,t} \right)$$

5.1.3 Constraints

$$x_{p,t} + I_{p,t-1} - I_{p,t} \geq d_{p,t}, \quad \forall p \in P, \forall t \in T \tag{1}$$

$$\sum_{p \in P} x_{p,t} \le C', \quad \forall t \in T \tag{2}$$

$$\sum_{p \in P} z_{p,s,t} \le m_s, \quad \forall s \in S, \forall t \in T \tag{3}$$

$$\sum_{s \in S} Cr_s \cdot z_{p,s,t} = CrReq_p \cdot x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{4}$$

$$\sum_{s \in S} Ni_s \cdot z_{p,s,t} = NiReq_p \cdot x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{5}$$

$$\sum_{s \in S} z_{p,s,t} = x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{6}$$

$$\sum_{s \in S} Cu_s \cdot z_{p,s,t} - \sum_{s \in S} CuRemoved_{p,s,t} \le CuLimit_t, \quad \forall p \in P, \forall t \in T \tag{7}$$

$$CuRemoved_{p,s,t} \le Cu_s \cdot z_{p,s,t}, \quad \forall p \in P, \forall s \in S, \forall t \in T \tag{8}$$

$$CuRemoved_{p,s,t} \le y_t \cdot Cu_s \cdot z_{p,s,t}, \quad \forall p \in P, \forall s \in S, \forall t \in T \tag{9}$$

$$x_{p,t}, I_{p,t}, z_{p,s,t}, CuRemoved_{p,s,t} \ge 0, \quad \forall p \in P, \forall s \in S, \forall t \in T \tag{10}$$

5.1.4 Description of the Constraints

The first constraint ensures that the production and inventory satisfy the demand for each product in each month. The second constraint limits the total monthly production to a fixed capacity C'. The third constraint ensures that the quantity of raw material purchased from each supplier does not exceed their maximum supply limit. The fourth and fifth constraints require that the purchased raw materials contain sufficient chromium and nickel to meet the required product composition for each alloy. The sixth constraint manages the copper content by ensuring that, after electrolysis, the total copper content does not exceed the allowed copper limit in a given month. The seventh constraint ensures that the copper removed via electrolysis for each product from each supplier is less than or equal to the copper content in the purchased raw material. The eighth constraint ties electrolysis activation to the binary variable y_t, ensuring copper electrolysis occurs only when needed. Finally, the ninth constraint guarantees that all decision variables (production, inventory, purchases, and copper removed) remain non-negative throughout the planning horizon.

5.2 Changes

The model that includes copper introduces significant changes compared to the original non-copper model. The primary differences and modifications are summarized below:

5.2.1 New Decision Variables

The copper model introduces two new decision variables:

- $CuRemoved_{p,s,t}$: The quantity of copper removed by electrolysis for product p from supplier s in month t. This variable did not exist in the original model.
- y_t: A binary variable that indicates whether electrolysis is activated in month t (1 if electrolysis is used, 0 otherwise). This variable controls the activation of copper electrolysis in the model.

5.2.2 Changes to the Objective Function

The objective function in the non-copper model only minimized two types of costs: inventory holding costs and raw material procurement costs. In the copper model, a third component—the copper electrolysis cost—is added to the objective function:

$$\min \sum_{t \in T} \left(\sum_{p \in P} h_p \cdot I_{p,t} + \sum_{s \in S} \sum_{p \in P} c_s \cdot z_{p,s,t} + \sum_{p \in P} \sum_{s \in S} e_c \cdot CuRemoved_{p,s,t} \right)$$

In this version, the electrolysis cost per kilogram of removed copper e_c is multiplied by the amount of copper removed $CuRemoved_{p,s,t}$, which was absent in the non-copper model.

5.2.3 New Copper-Related Constraints

Several new constraints have been added to manage copper content and electrolysis. These constraints do not exist in the non-copper model and serve to control how copper is treated during production.

Copper Content Limit

The total copper content in the purchased raw materials, after copper removal through electrolysis, must not exceed the allowed copper limit for each month $CuLimit_t$:

$$\sum_{s \in S} Cu_s \cdot z_{p,s,t} - \sum_{s \in S} CuRemoved_{p,s,t} \leq CuLimit_t, \quad \forall p \in P, \forall t \in T$$

Electrolysis Quantity Limit

The quantity of copper removed by electrolysis for each product must be less than or equal to the copper content in the purchased raw material from each supplier:

$$CuRemoved_{p,s,t} \leq Cu_s \cdot z_{p,s,t}, \quad \forall p \in P, \forall s \in S, \forall t \in T$$

Electrolysis Activation

Copper removal can only occur if electrolysis is activated. This is controlled by the binary variable y_t, which ensures that electrolysis is performed only when needed:

$$CuRemoved_{p,s,t} \leq y_t \cdot Cu_s \cdot z_{p,s,t}, \quad \forall p \in P, \forall s \in S, \forall t \in T$$

These constraints are entirely new and have been added to manage the copper content and its removal in the production process, something the non-copper model did not need to consider.

5.2.4 Changes in Constraint Structure

In the non-copper model, constraints primarily focused on production and inventory management, raw material procurement, and ensuring that the chromium and nickel content requirements for the alloys were met. In the copper model, the structure has been modified by adding copper-related constraints, while still retaining the original production, inventory, and procurement constraints.

5.2.5 Increased Model Complexity

The copper model increases the complexity of the problem, primarily due to the introduction of:

- The binary variable y_t, which controls whether electrolysis is activated, adding a discrete decision-making component to the model.
- The need to manage copper content, with constraints ensuring that copper removal occurs correctly and does not exceed the available copper from suppliers.

This makes the copper model a mixed-integer linear programming (MILP) problem, whereas the original non-copper model was a linear programming (LP) problem. This increase in complexity can make the copper model more difficult and time-consuming to solve.

5.3 Extended Model in Python and the Lowest CopperLimit

5.3.1 the Lowest CopperLimit

To find the lowest possible copper limit without increasing the production cost, first I set a Python file (e_exp.py) which uses a binary search method to adjust the copper limit incrementally until the smallest feasible value is found.
Initially, a range for the copper limit is defined, and the midpoint of this range is tested each time. If using this limit keeps the production plan feasible and does not increase costs, the search continues to lower the limit. If the limit makes the plan infeasible or increases costs, the search adjusts to a higher value.

This process continues until the difference between the highest and lowest limits is very small, effectively finding the minimum copper limit that maintains the original cost. This approach is efficient for balancing resource use with production goals.
The code get a result of the lowest possible copper limit of 2.96%

5.3.2 Experiment

Set a new Python file which can change the CopperLimit manually (e_data.py) and import data from it. The file named e_switchmodel.py include both the model without considering copper and the model includes copper by switching a bool variable (when set to True, it runs the copper model).
By changing the CopperLimit multiple times (from 2.96% to 0.1%), the effect it has on electrolysis costs and holding costs, total cost can be shown. The diagram 2 below shows the result.

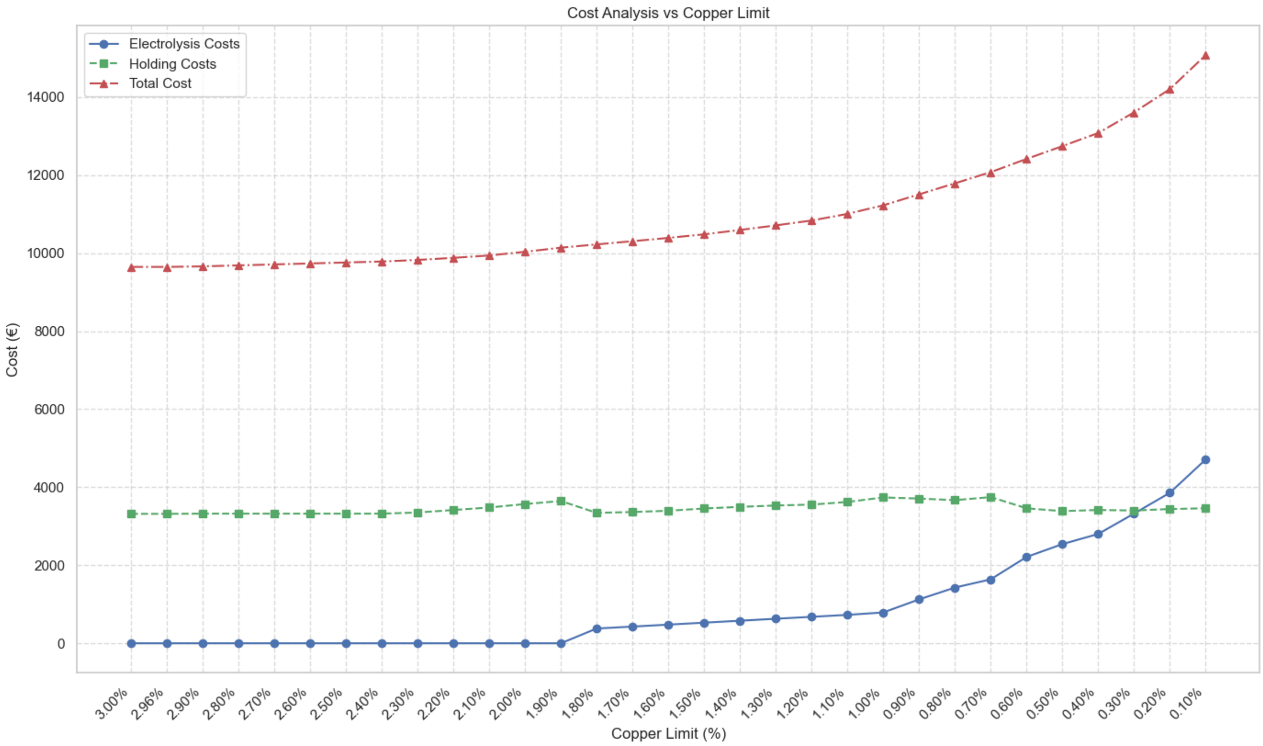

Figure 2: The effect of copperlimit on Electrolysis Costs, Holding cost and Total cost.

5.3.3 Analysis

Electrolysis Costs

Trend: Electrolysis costs increase as copper content decreases, especially below 1.00%.
Analysis: Lower copper content requires more complex electrolysis or higher energy use, increasing costs. Electrolysis costs are stable between 3.00% and 1.00% but rise sharply below 1.00%.
Key Point: Reducing copper content complicates the electrolysis process, raising costs.

Holding Costs

Trend: Holding costs remain stable across different copper content levels.
Analysis: Holding costs are minimally affected by copper content changes and depend more on factors like storage duration. Using alternative suppliers also helps stabilize these costs.
Key Point: Holding costs are not sensitive to copper content changes, being influenced by logistical factors.

Total Cost

Trend: Total costs are stable above 1.50% copper content but rise below this level, especially under 1.00%.
Analysis: Total costs are driven by electrolysis costs. When copper content falls below 1.50%, electrolysis costs rise sharply, increasing total costs.
Key Point: Keeping copper content above 1.50% helps maintain lower total costs.

6 Statements

6.1 AI tools Statements

I use chatGPT mainly to write the assignment in proper Latex form, I asked it questions like: help me change this table into Latex form or Tell me how to refer a picture in the paragraph. It gave me the Latex code without changing my original context so that I can write the assignment in a more efficient way.
I also use GitHub copilot to help me write the doc of a function in Python language and let it check my grammar as well.