5 e: CopperLimit

Introduction

The company wants the amount of copper in its products to be below a certain value, called the CopperLimit. In order to remove copper from a mix, electrolysis can be used. Each month it is decided whether electrolysis will be used; if so, fixed costs for electrolysis must be paid (100 euro) in that month and the production of that month for each product can be treated. The weight reduction caused by electrolysis is equal to the weight of the copper that is removed. The variable costs for electrolysis depend on the amount of copper that is removed this way and is 5 euro per kg.

Question

Extend the mathematical model so that the use of electrolysis is included. Use binary decision variables and keep it as a mixed integer linear programming problem (no multiplication of variables with variables). Provide the changes in your mathematical formulation. Use the model to determine the lowest CopperLimit that can be used without extra costs, compared to the solution for (b). Experiment with other values of the CopperLimit and show the effect it has on electrolysis costs and holding costs.

5.1 Extended Model

5.1.1 Notation (sets and indices, parameters, variables)

The notation used for this mathematical formulation is provided in Table 8:

Table 8: Notation

Sets and indices	Description	Indices/Units
P	Set of products	$p \in P$
S	Set of suppliers	$s \in S$
T	Set of months	$t \in T$
Parameter		**Units**
$d_{p,t}$	Demand for product p in month t	[kg]
h_p	Holding cost for product p	[€/kg]
c_s	Cost of raw material from supplier s	[€/kg]
m_s	Maximum supply from supplier s	[kg/month]
Cr_s	Chromium content in raw material from supplier s	[%]
Ni_s	Nickel content in raw material from supplier s	[%]
Cu_s	Copper content in raw material from supplier s	[%]
$CrReq_p$	Required chromium content for product p	[%]
$NiReq_p$	Required nickel content for product p	[%]
$CuLimit_t$	Maximum allowed copper content in month t	[%]
e_c	Cost per kg for copper electrolysis	[€/kg]
C'	Monthly production capacity	[kg/month]
E_c	fixed cost for copper electrolysis	[€]
M	a very huge number	[1]
Variable		**Units**
$x_{p,t}$	Quantity of product p produced in month t	[kg]
$I_{p,t}$	Inventory of product p at the end of month t	[kg]
$z_{p,s,t}$	Quantity of raw material purchased from supplier s for product p in month t	[kg]
$CuRemoved_{p,t}$	Quantity of electrolyzed copper for product p in month t	[kg]
y_t	Binary variable indicating whether electrolysis is used in month t	[binary]

5.1.2 Objective Function

$$\min \sum_{t \in T} \left(\sum_{p \in P} h_p \cdot I_{p,t} + \sum_{s \in S} \sum_{p \in P} c_s \cdot z_{p,s,t} + \sum_{p \in P} e_c \cdot CuRemoved_{p,t} + y_t \cdot E_c \right)$$

5.1.3 Constraints

$$(x_{p,t} - CuRemoved_{p,t}) + I_{p,t-1} - I_{p,t} \geq d_{p,t}, \quad \forall p \in P, \forall t \in T \tag{1}$$

$$\sum_{p \in P} x_{p,t} \leq C', \quad \forall t \in T \tag{2}$$

$$\sum_{p \in P} z_{p,s,t} \leq m_s, \quad \forall s \in S, \forall t \in T \tag{3}$$

$$\sum_{s \in S} Cr_s \cdot z_{p,s,t} = CrReq_p \cdot (x_{p,t} - CuRemoved_{p,t}), \quad \forall p \in P, \forall t \in T \tag{4}$$

$$\sum_{s \in S} Ni_s \cdot z_{p,s,t} = NiReq_p \cdot (x_{p,t} - CuRemoved_{p,t}), \quad \forall p \in P, \forall t \in T \tag{5}$$

$$\sum_{s \in S} z_{p,s,t} = x_{p,t}, \quad \forall p \in P, \forall t \in T \tag{6}$$

$$\sum_{s \in S} Cu_s \cdot z_{p,s,t} - CuRemoved_{p,t} \leq CuLimit_t \cdot (x_{p,t} - CuRemoved_{p,t}), \quad \forall p \in P, \forall t \in T \tag{7}$$

$$CuRemoved_{p,t} \leq \sum_{s \in S} Cu_s \cdot z_{p,s,t}, \quad \forall p \in P, \forall t \in T \tag{8}$$

$$CuRemoved_{p,t} \leq y_t \cdot M, \quad \forall p \in P, \forall t \in T \tag{8}$$

$$x_{p,t}, I_{p,t}, z_{p,s,t}, CuRemoved_{p,t} \geq 0, I_{p,0} = 0, \quad \forall p \in P, \forall s \in S, \forall t \in T \tag{9}$$

5.1.4 Description of the Constraints

The first constraint ensures that the production and inventory satisfy the demand for each product in each month. The second constraint limits the total monthly production to a fixed capacity C'. The third constraint ensures that the quantity of raw material purchased from each supplier does not exceed their maximum supply limit. The fourth and fifth constraints require that the purchased raw materials contain sufficient chromium and nickel to meet the required product composition for each alloy. The sixth constraint manages the copper content by ensuring that, after electrolysis, the total copper content does not exceed the allowed copper limit in a given month. The seventh constraint ensures that the copper removed via electrolysis for each product from each supplier is less than or equal to the copper content in the purchased raw material. The eighth constraint ties electrolysis activation to the binary variable y_t, ensuring copper electrolysis occurs only when needed. Finally, the ninth constraint guarantees that all decision variables (production, inventory, purchases, and copper removed) remain non-negative throughout the planning horizon.

5.2 Changes

The model that includes copper introduces significant changes compared to the original non-copper model. The primary differences and modifications are summarized below:

5.2.1 New Decision Variables

The copper model introduces two new decision variables:

- $CuRemoved_{p,s,t}$: The quantity of copper removed by electrolysis for product p from supplier s in month t. This variable did not exist in the original model.
- y_t: A binary variable that indicates whether electrolysis is activated in month t (1 if electrolysis is used, 0 otherwise). This variable controls the activation of copper electrolysis in the model.

5.2.2 Changes to the Objective Function

The objective function in the non-copper model only minimized two types of costs: inventory holding costs and raw material procurement costs. In the copper model, a third component—the copper electrolysis cost—is added to the objective function:

$$\min \sum_{t \in T} \left(\sum_{p \in P} h_p \cdot I_{p,t} + \sum_{s \in S} \sum_{p \in P} c_s \cdot z_{p,s,t} + \sum_{p \in P} e_c \cdot CuRemoved_{p,t} + y_t \cdot E_c \right)$$

In this version, the electrolysis cost per kilogram of removed copper e_c is multiplied by the amount of copper removed $CuRemoved_{p,s,t}$, which was absent in the non-copper model.

5.2.3 New Copper-Related Constraints

Several new constraints have been added to manage copper content and electrolysis. These constraints do not exist in the non-copper model and serve to control how copper is treated during production.

Copper Content Limit

The total copper content in the purchased raw materials, after copper removal through electrolysis, must not exceed the allowed copper limit for each month $CuLimit_t$:

$$\sum_{s \in S} Cu_s \cdot z_{p,s,t} - CuRemoved_{p,t} \leq CuLimit_t \cdot (x_{p,t} - CuRemoved_{p,t}), \quad \forall p \in P, \forall t \in T$$

Electrolysis Activation

Copper removal can only occur if electrolysis is activated. This is controlled by the binary variable y_t, which ensures that electrolysis is performed only when needed:

$$CuRemoved_{p,t} \leq y_t \cdot M, \quad \forall p \in P, \forall t \in T \tag{8}$$

These constraints are entirely new and have been added to manage the copper content and its removal in the production process, something the non-copper model did not need to consider.

5.2.4 Changes in Constraint Structure

In the non-copper model, constraints primarily focused on production and inventory management, raw material procurement, and ensuring that the chromium and nickel content requirements for the alloys were met. In the copper model, the structure has been modified by adding copper-related constraints, while still retaining the original production, inventory, and procurement constraints.

5.2.5 Increased Model Complexity

The copper model increases the complexity of the problem, primarily due to the introduction of:

- The binary variable y_t, which controls whether electrolysis is activated, adding a discrete decision-making component to the model.
- The need to manage copper content, with constraints ensuring that copper removal occurs correctly and does not exceed the available copper from suppliers.

This makes the copper model a mixed-integer linear programming (MILP) problem, whereas the original non-copper model was a linear programming (LP) problem. This increase in complexity can make the copper model more difficult and time-consuming to solve.

5.3 Extended Model in Python and the Lowest CopperLimit

5.3.1 the Lowest CopperLimit

To find the lowest possible copper limit without increasing the production cost, first I set a Python file which uses a binary search method to adjust the copper limit incrementally until the smallest feasible value is found. Initially, a range for the copper limit is defined, and the midpoint of this range is tested each time. If using this limit keeps the production plan feasible and does not increase costs, the search continues to lower the limit. If the limit makes the plan infeasible or increases costs, the search adjusts to a higher value.
This process continues until the difference between the highest and lowest limits is very small, effectively finding the minimum copper limit that maintains the original cost. This approach is efficient for balancing resource use with production goals.
The code get a result of the lowest possible copper limit of 2.965116500854492%

5.3.2 Experiment

The file named modelb&e.py include both the model without considering copper and the model includes copper, when running th file users can input b or e to choose which model to use.
By changing the CopperLimit multiple times (from 3.0% to 0%), the effect it has on electrolysis costs and holding costs, total cost can be shown. The diagram 4 below shows the result.

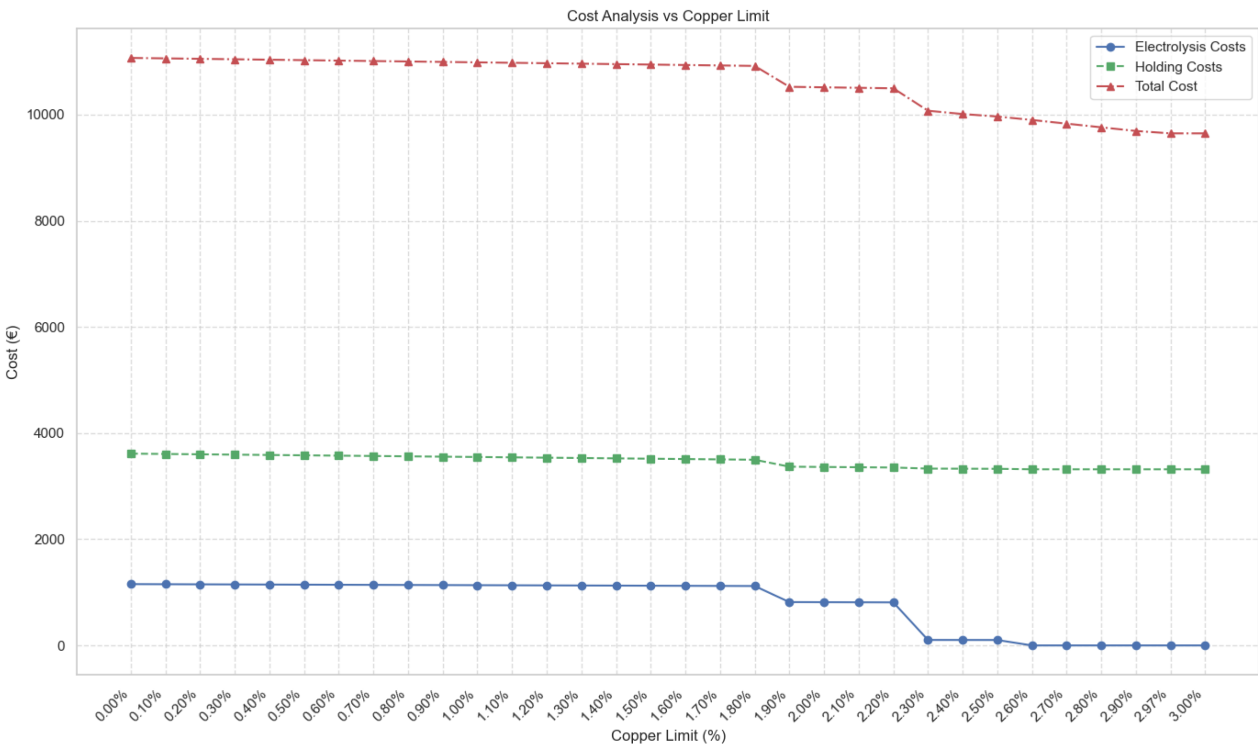

Figure 4: The effect of copperlimit on Electrolysis Costs, Holding cost and Total cost.

5.3.3 Analysis

Electrolysis Costs

Trend: The blue line represents electrolysis costs, which remain relatively stable up to a copper limit of 2.0%, after which there is a noticeable decrease, especially after 2.3%.
Analysis: This trend indicates that when the copper limit is low, electrolysis costs are higher and stable. As the copper limit increases, the process likely becomes more efficient, resulting in significantly reduced electrolysis costs.
Key Point: Reducing copper content complicates the electrolysis process, raising costs.

Holding Costs

Trend: The green line represents holding costs, which remain almost constant throughout the range of copper limits, with a slight decrease around a 1.9% copper limit.
Analysis: This suggests that holding costs are largely unaffected by changes in the copper limit, likely due to fixed storage costs, which are independent of material fluctuations.
Key Point: Holding costs are not sensitive to copper content changes, being influenced by logistical factors.

Total Cost

Trend: The red line represents total costs, which remain stable until about 1.9%, then gradually decrease, with a more significant drop after 2.0%.
Analysis: The trend in total costs closely follows the electrolysis costs, indicating that electrolysis is a major factor in total costs. Increasing the copper limit leads to a notable reduction in total costs, especially after 2.3%.
Key Point: Keeping copper content above 2.96% helps maintain minimum total costs.