Theorem in algorithm RecConcave[1] for privacy parameters ϵ, δ ,promise r database X, domain R and sensetivity-1 utility function u :
if we use $Exponential - Mechanism$ instead of A_{dist} (at step 9) it will return the highest scored value $OPT_u(x)$, in probability at least $1 - \beta$ if $r > \frac{16}{3\epsilon\alpha} ln\left(\frac{|R|}{\beta}\right)$

proof by The Algorithmic Foundations of Differential Privacy[2] Theorem 3.11

$$Pr\left[u(M_E(x,u,R)) \leq OPT_u(x) - \frac{2\Delta u}{\epsilon}\left(ln\left(\frac{|R|}{|R_{OPT}|}\right) + t\right)\right] \leq e^{-t}$$

if we set $e^{-t} = \beta \Rightarrow t = ln\left(\frac{1}{\beta}\right)$, $\Delta u = 1$ and $|R_{OPT}| = 1$we get:

$$Pr\left[u(M_E(x,u,R)) \leq OPT_u(x) - \tfrac{2}{\epsilon}\left(ln\left(|R|\right) + ln\left(\tfrac{1}{\beta}\right)\right)\right] \quad =$$

$$Pr\left[u(M_E(x,u,R)) \leq OPT_u(x) - \frac{2}{\epsilon} ln\left(\frac{|R|}{\beta}\right)\right] \leq \beta$$

we want the gap to be at least $\frac{2}{\epsilon} ln\left(\frac{|R|}{\beta}\right)$
from the other hand in RecConcave we know that the gap is at least $\frac{3\alpha}{8}r$
if we combine the two we get

$$\frac{2}{\epsilon} ln\left(\frac{|R|}{\beta}\right) < \frac{3\alpha}{8} r \Rightarrow r > \frac{16}{3\epsilon\alpha} ln\left(\frac{|R|}{\beta}\right)$$

and in the case of median $r = \frac{|s|}{2}$ so the bound on the sample size is

$$|s| > \frac{32}{3\epsilon\alpha} ln\left(\frac{|R|}{\beta}\right)$$

alternatively if we want to bound |R|

$$\frac{3\epsilon\alpha r}{16} > ln\left(\frac{|R|}{\beta}\right) \Rightarrow e^{\frac{3\epsilon\alpha r}{16}} > \frac{|R|}{\beta} \Rightarrow |R| < \beta e^{\frac{3\epsilon\alpha r}{16}} = \beta e^{\frac{3\epsilon\alpha|s|}{32}}$$

[1] A. Beimel, K. Nissim, and U. Stemmer. Private learning and sanitization- Pure vs. Approximate Differential Privacy

[2] C.Dwork , A.roth