

**Theorem** in algorithm RecConcave<sup>1</sup> for privacy parameters  $\epsilon, \delta$ , promise  $r$  database  $X$ , domain  $R$  and sensitivity-1 utility function  $u$  :

$A_{dist}$  (step 9) will return a value, in probability at least  $1 - \beta$  if  $r > \frac{8 \ln(\frac{r}{\beta})}{3\epsilon\alpha}$

**proof** by defenition

$$q(S, j) \geq (1 - 1/4)R \Rightarrow r - L(j+1) \geq (1 - 1/4)R = \frac{3\alpha}{8}r \Rightarrow L(j+1) \leq r - \frac{3\alpha}{8}r = (1 - \frac{3\alpha}{8})r$$

and by the promise there is  $p$  such  $Q(p) \geq r$

so the gap is at least

$$r - (1 - \frac{3\alpha}{8})r = \frac{3\alpha}{8}r$$

in other words we need that

$$\frac{3\alpha}{8}r > \frac{\ln\left(\frac{1}{\beta\delta}\right)}{\epsilon} \Rightarrow r > \frac{8 \ln\left(\frac{1}{\beta\delta}\right)}{3\epsilon\alpha}$$

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<sup>1</sup>A. Beimel, K. Nissim, and U. Stemmer. Private learning and sanitization- Pure vs. Approximate Differential Privacy