

$$Pr \left[u(M_E(x, u, R)) \leq OPT_u(x) - \frac{2\Delta u}{\epsilon} \left(\ln \left(\frac{|R|}{|R_{OPT}|} \right) + t \right) \right] \leq e^{-t}$$

if we set $e^{-t} = \beta \Rightarrow t = \ln \left(\frac{1}{\beta} \right)$, $\Delta u = 1$ and $|R_{OPT}| = 1$ we get:

$$Pr \left[u(M_E(x, u, R)) \leq OPT_u(x) - \frac{2}{\epsilon} \left(\ln(|R|) + \ln \left(\frac{1}{\beta} \right) \right) \right] =$$

$$Pr \left[u(M_E(x, u, R)) \leq OPT_u(x) - \frac{2}{\epsilon} \ln \left(\frac{|R|}{\beta} \right) \right] \leq \beta$$

we want the gap to be at least $\frac{2}{\epsilon} \ln \left(\frac{|R|}{\beta} \right)$

from the other hand in the reconcave we know that the gap is at least $\frac{3\alpha}{8}r$
if we combine the two we get

$$\frac{2}{\epsilon} \ln \left(\frac{|R|}{\beta} \right) < \frac{3\alpha}{8}r \Rightarrow r > \frac{16}{3\epsilon\alpha} \ln \left(\frac{|R|}{\beta} \right)$$

and in the case of median $r = \frac{|s|}{2}$ so the bound on the sample size is

$$|s| > \frac{32}{3\epsilon\alpha} \ln \left(\frac{|R|}{\beta} \right)$$

alternatively if we want to bound $|R|$

$$\frac{3\epsilon\alpha r}{16} > \ln \left(\frac{|R|}{\beta} \right) \Rightarrow e^{\frac{3\epsilon\alpha r}{16}} > \frac{|R|}{\beta} \Rightarrow |R| < \beta e^{\frac{3\epsilon\alpha r}{16}} = \beta e^{\frac{3\epsilon\alpha |s|}{32}}$$