by defenition

$$q(S,j) \geq (1-{}^1\!/_4)R \Rightarrow r - L(j+1) \geq (1-{}^1\!/_4)R = \frac{3\alpha}{8}r \Rightarrow L(j+1) \leq r - \frac{3\alpha}{8}r = (1-\frac{3\alpha}{8})r$$

and by the promise there is p such $Q(p) \geq r$
so the gap is at least

$$r - (1 - \frac{3\alpha}{8})r = \frac{3\alpha}{8}r$$

in other words we need that

$$\frac{3\alpha}{8}r > \frac{ln\left(\frac{1}{\beta\delta}\right)}{\epsilon} \Rightarrow r > \frac{8ln\left(\frac{1}{\beta\delta}\right)}{3\epsilon\alpha}$$

recall that from privacy we must have $\delta < \frac{1}{T}$ so

$$r > \frac{8ln\left(\frac{T}{\beta}\right)}{3\epsilon\alpha}$$

in the case of median quality $r = \frac{|S|}{2}$ so we get that the sample complecsity bound:

$$|S| > \frac{16ln\left(\frac{T}{\beta}\right)}{3\epsilon\alpha}$$