Fixing a database $x \in X^*$, a quality function $u : X^* \times \mathcal{R} \to \mathbb{R}$, approximation and probability parameters α, β.

denote: $OPT_u(x) = \max_{r \in \mathcal{R}} (u(x, r))$

if $|R_{OPT}| = 1$, Δu=1 and the there is a "gap" $OPT_u(x) - max(R \backslash OPT_u(x)) \geq \alpha$

we get that if

$$OPT_u(x) \geq \frac{2}{\epsilon \alpha} \cdot ln\left(\frac{|R|}{\beta}\right)$$

it holds that

$$Pr\left[u(\mathcal{M}(x, u, \mathcal{R})) \neq OPT_u(x)\right] \leq \beta$$

now denote r as the promise parameter so by definition

$$OPT_u(x) > r$$

if we deal with $u(\cdot)$ s.t $\forall x: \ u(x) \geq 0$ then

$$r \geq \alpha > 0$$

so if

$$\alpha \geq \frac{2}{\epsilon \alpha} \cdot ln\left(\frac{|R|}{\beta}\right) \Rightarrow Pr\left[u(\mathcal{M}(x, u, \mathcal{R})) \neq OPT_u(x)\right] \leq \beta$$

and that happens when

$$\frac{\alpha^2 \epsilon}{2} \geq ln\left(\frac{|R|}{\beta}\right) \Leftrightarrow \beta \cdot exp\left(\frac{\alpha^2 \epsilon}{2}\right) \geq |R|$$

recall that $Adist$ returs a value iff

$$a \geq \frac{1}{\epsilon} \cdot ln\left(\frac{1}{\beta \delta}\right)$$

and if that is the case we can "plug in" α and get a bound on $|R|$

$$\beta \cdot exp\left(\frac{\alpha^2 \epsilon}{2}\right) \geq \beta \cdot exp\left(\frac{\frac{1}{\epsilon} \cdot \left(ln\left(\frac{1}{\beta\delta}\right)\right)^2 \epsilon}{2}\right) = \beta \cdot \left(\frac{1}{\beta \delta}\right)^{\frac{1}{2} ln\left(\frac{1}{\beta\delta}\right)} \geq |R|$$

note that for not-so-extreme paramteres value we get extreme high bound on domain size.

for example

$$\beta = 0.01, \ \delta = 2^{-20} \Rightarrow |R| \leq 2^{485}$$