input: $S,T,Q,r,\alpha,\epsilon,\delta$

we also get as input two function 1

$$1.\ L(S,T',j) \text{ - from step 3 - returns } \max \max_{\substack{[a,b]\subseteq[0,T']\\b-a+1=2^j}} \left(\min_{i\in[a,b]} \left(Q\left(S,i\right)\right)\right)$$

2.
$$u(S,I)$$
 - from step 8 -returns $\max_{i \in I} \left(Q(S,i)\right)$

algorithm implementation:

- 1. $l \leftarrow \lceil log_2(T) \rceil$
- 2. $T' \leftarrow 2^l$
- 3. define an extended quality function:
 - Q(S,j):

$$-$$
 if $T < j \le T'$:

* return
$$min(0, Q(S, j))$$

- else:

* return
$$Q(S, j)$$

- 4. define q(S,j):
 - return $min(L(S, T', j) (1 \alpha)r, r L(S, T', j + 1))$
- 5. $k \leftarrow Exponential Mechanism(S, range(l+1), q, \epsilon)$
- 6. $K \leftarrow 8 \cdot 2^k$
- 7. $As \leftarrow [(i, i + K) \mid i \text{ from } 0 \text{ to } T' \text{ in steps of } K] \# \text{ that's a list of tuples}$
- 8. $Bs \leftarrow [(i, i+K) \mid i \text{ from } K/2 \text{ to } T' \text{ in steps of } K] \# \text{ that's a list of tuples}$
- 9. $A \leftarrow A_{dist}(S, As, u, \epsilon, \delta)$
- 10. $B \leftarrow A_{dist}(S, Bs, u, \epsilon, \delta)$
- 11. if A = bottom' or B = bottom' raise Error
- 12. convert the tuples A and B to lists A' and B' # i.e $(x,y) \Rightarrow [x,...,y]$
- 13. $answer \leftarrow Exponential Mechanism(S, A' \cup B', Q, \epsilon)$
- 14. return answer

¹the implementation of them is in a different file