

denote the *FlatConcave* result as m

define a series of n random variables X_i where $X_i = 1$ if $q(m) > (1 - \alpha)r$ and $X_i = 0$ otherwise. or in other words $X_1, \dots, X_n \sim \text{bernoulli}(p)$ when p is unknown.

denote $\hat{p}_n = \frac{1}{n} \sum_{i=1}^n X_i$ as an estimator for p

say we want to make sure $p > A$ for some constant A

and that we run the mechanism n times and got \hat{p}_n success rate and $\hat{p}_n = A + 0.05$.

by the large-number law we get that a 95% percent confidence interval for bernoulli distribution can be computed as

$$(\hat{p}_n - \epsilon_n, \hat{p}_n + \epsilon_n)$$

when

$$\epsilon_n = z_{1-\alpha/2} \sqrt{\frac{\hat{p}_n (1 - \hat{p}_n)}{n}}$$

since we are only interested in the lower bound of the interval we can take one-sided z-score so $z_{1-\alpha/2} \approx 1.64$

say we want to make sure $p > A$ for some constant A

in our case $\hat{p}_n - A = \frac{1}{20}$ and we want that the following inequality will hold

$$\hat{p}_n - 1.64 \sqrt{\frac{\hat{p}_n (1 - \hat{p}_n)}{n}} > A \Rightarrow n > 400 \cdot 1.64^2 \cdot \hat{p}_n (1 - \hat{p}_n) \approx 1076 \cdot \hat{p}_n (1 - \hat{p}_n)$$

and if $\hat{p}_n = 0.95$

$$n \gtrsim 52$$

note that if we take two-sided z-score we get $n \gtrsim 73$