

Numerical Report

Analyzing the behavior of numerical methods

Contents:

- 1-Methods code
- 2-sample run , Analysis and Observation
- 3-pitfalls
- 4-problematic functions
- 5-Data Structure used
- 6-User manual

TEAM

Name: Mayar Ahmed Ramadan **ID:**6055

Name: Menna Abd El Meguid **ID:**6538

❖ BISECTION METHOD:

A) The code of the bisection method:

```
from sympy import* #library for symbolic mathematics to be full featured computer algebra
import os.path
def func(expr, value, x):
    return expr.subs(x, value)
# Prints root of func(x)
# with error of EPSILON
def bisection(a, b, expr, maxIteration, Epsilon, x):
    print("In bisection")
    file = open("../View/values.txt", "w")
    if os.path.isfile('../Viewss/values.txt'):
        print("File exist")
    else:
        print("File not exist")
    if (func(expr,a, x) * func(expr,b, x) >= 0):
        print("You have not assumed right a and b\n")
        return
    c = a
    step = 1
    while (step < maxIteration):
        # Find middle point
        print('Iteration(%d): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-
mid)' % step)
        print("\t\t\t\t\t %.6f \t %.6f \t %.6f \t %.6f \t %.6f \t %.6f \n" % (
a, func(expr, a, x), b, func(expr, b, x), c, func(expr, c, x)))
        file.write("%.6f %.6f %.6f %.6f %.6f %.6f \n" % (
a, func(expr, a, x), b, func(expr, b, x), c, func(expr, c, x)))
        c_old=c
        c = (a+b)/2
        if(abs((c-c_old)/c)<Epsilon):
            break
        # Check if middle point is root
        if (func(expr,c, x) == 0.0):
            break
        # Decide the side to repeat the steps
        if (func(expr,c, x)*func(expr,a, x) < 0):
            b = c
        else:
            a = c
        step +=1
    file.close()
    finalIteration = step
    return "%d ): %.6f"%(finalIteration,c)

# Main code
def mainFunc(function, maxIteration, epsilon, a, b):
    # the possible variable names must be known beforehand...
    expr = sympify(function)
    x = var('x')

    return bisection(a, b, expr, maxIteration, epsilon, x)
```

B)Sample runs &Analysis

Now we will choose some examples to test the function:

1) first function:

$$E^{-x-x}$$

-lower bound: 0

-upper bound:2

-number of iterations:9

-Epsilon: 10^{-2}

The screenshot shows a software window titled "NUMERICAL PROJECT". The main area has a pink background with the title "Numerical Project" in a stylized font. Below the title, there are input fields for "Function:" (containing E^{-x-x}), "Max Iteration:" (9), and "Epsilon:" (0.01). There is a checkbox labeled "Select from File" and a "Method:" label. Below these, there are tabs for "Bisection", "Fixed point", "False-position", "Newton-Raphson", and "Secant". The "Bisection" tab is selected, and it shows the "Lower Bound:" (0) and "Upper Bound:" (2). The "Root Result:" is displayed as $x(9): 0.570312$. Below this, it shows "Exact Root X: 0" and "Execution Time: 0.005012 s". At the bottom, there are three buttons: "Calculate", "Show Iterations", and "Show graph".

Iterations:

```
In bisection
File not exist
Iteration(1): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               0.000000   1.000000   2.000000   -1.864665   0.000000   1.000000

Iteration(2): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               0.000000   1.000000   1.000000   -0.632121   1.000000   -0.632121

Iteration(3): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               0.500000   0.106531   1.000000   -0.632121   0.500000   0.106531

Iteration(4): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               0.500000   0.106531   0.750000   -0.277633   0.750000   -0.277633

Iteration(5): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               0.500000   0.106531   0.625000   -0.089739   0.625000   -0.089739

Iteration(6): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               0.562500   0.007283   0.625000   -0.089739   0.562500   0.007283

Iteration(7): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               0.562500   0.007283   0.593750   -0.041498   0.593750   -0.041498

Iteration(8): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               0.562500   0.007283   0.578125   -0.017176   0.578125   -0.017176
```

C)Observation :

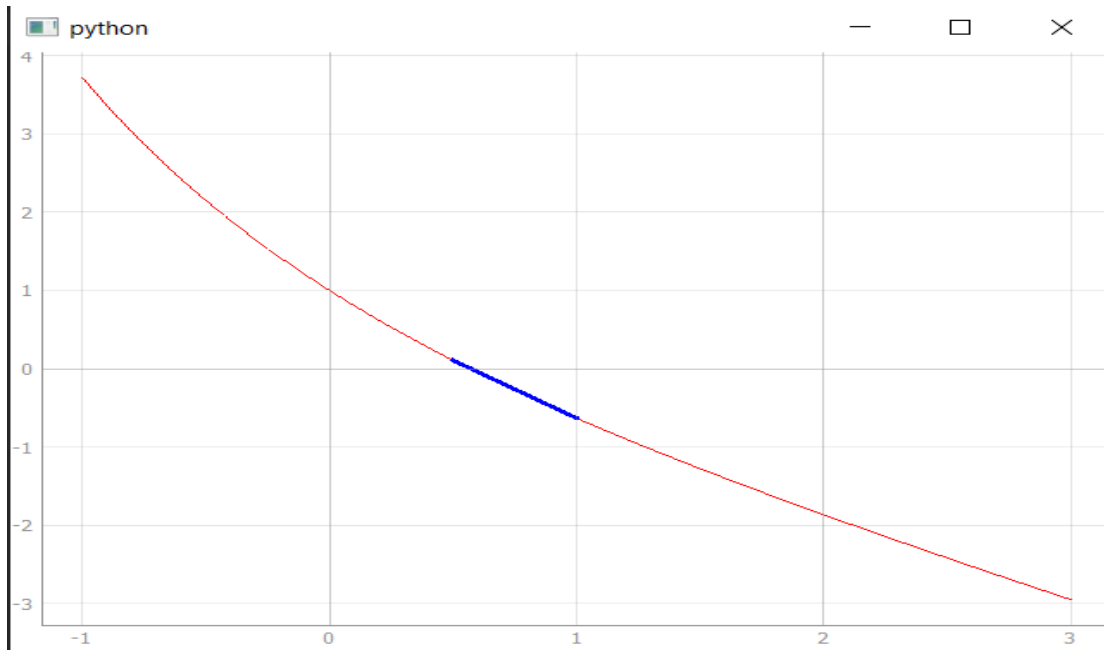
As the number of iterations increase we reaches value near the exact solution.

Bisection method can be done here on this function as the :

$$f(x\text{-lower}) * f(x\text{-upper}) < 0$$

and here we can notice that we have stopped at the 8th iteration despite we have entered 9 iterations , as the relative error became less than the tolerance.

the graph of the bisection method of the previous function:



2)second equation:

NUMERICAL PROJECT

Numerical Project

Function: Max Iteration:
☐ Select from File Epsilon:

Method:

Bisection Fixed point False-position Newton-Raphson Secant

Lower Bound: Upper Bound:

Root Result: $x(9)$: 1.332031

Exact Root X: 0 Execution Time: 0.125113 s

Iterations:

```
Iteration(1): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               -1.000000  -1.000000   5.000000  119.000000   -1.000000  -1.000000

Iteration(2): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               -1.000000  -1.000000   2.000000   5.000000   2.000000   5.000000

Iteration(3): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               0.500000  -1.375000   2.000000   5.000000   0.500000  -1.375000

Iteration(4): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               1.250000  -0.296875   2.000000   5.000000   1.250000  -0.296875

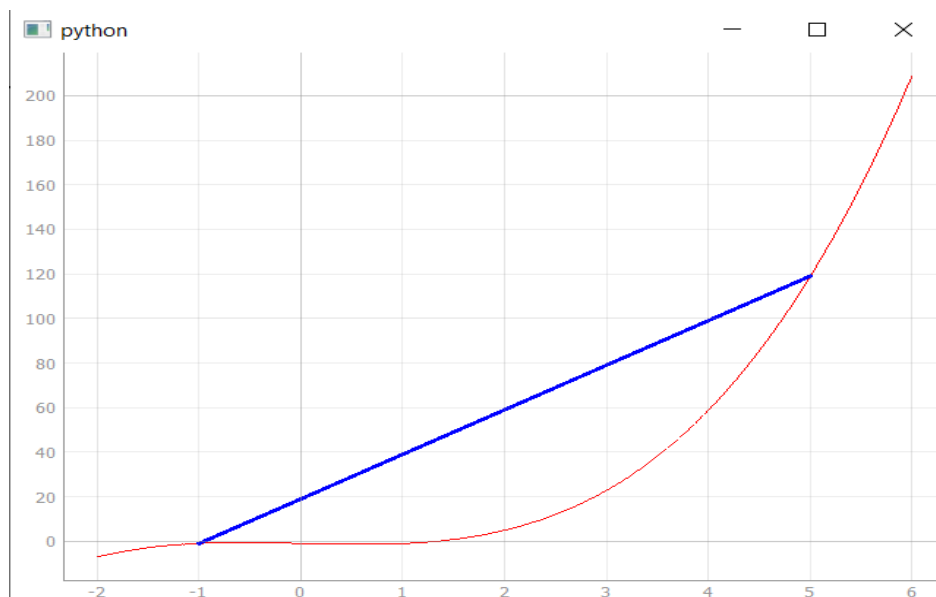
Iteration(5): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               1.250000  -0.296875   1.625000   1.666016   1.625000   1.666016

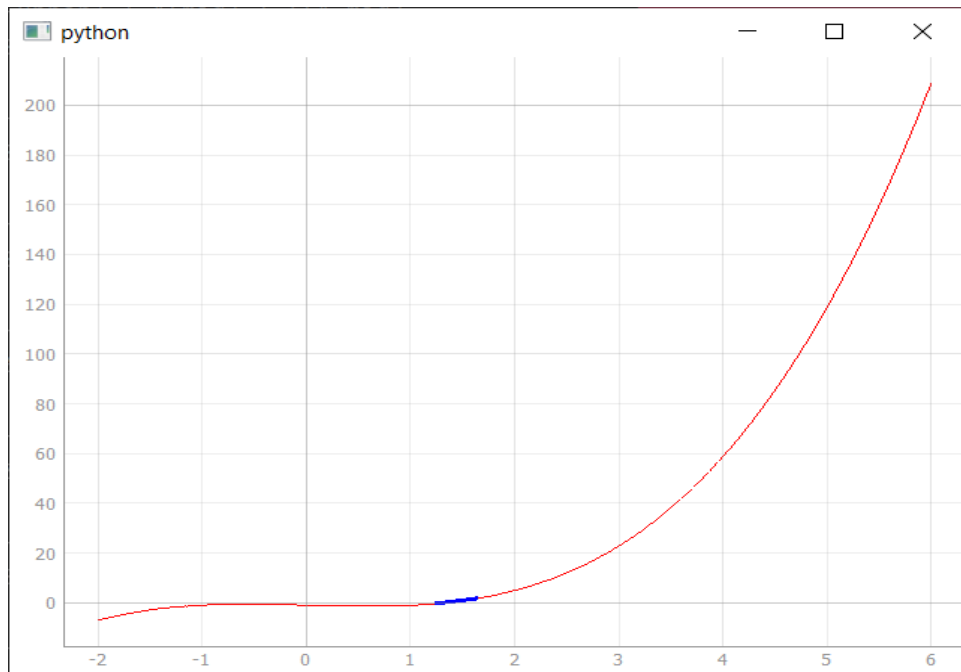
Iteration(6): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               1.250000  -0.296875   1.437500   0.532959   1.437500   0.532959

Iteration(7): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
               1.250000  -0.296875   1.343750   0.082611   1.343750   0.082611

Iteration(8): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
```

GRAPH FOR THIS FUNCTION.





3)Third function:

NUMERICAL PROJECT

Numerical Project

Function:

☐ Select from File

Max Iteration:

Epsilon:

Method:

Bisection

Fixed point

False-position

Newton-Raphson

Secant

Lower Bound:

Upper Bound:

Root Result: X(6): 1.609375

Excact Root X: 0

Execution Time: 0.007976 s

Calculate

Show Iterations

Show graph

Iterations:

```
Iteration(1): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
                1.000000   -5.000000   2.000000   7.000000   1.000000   -5.000000

Iteration(2): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
                1.500000   -1.250000   2.000000   7.000000   1.500000   -1.250000

Iteration(3): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
                1.500000   -1.250000   1.750000   2.218750   1.750000   2.218750

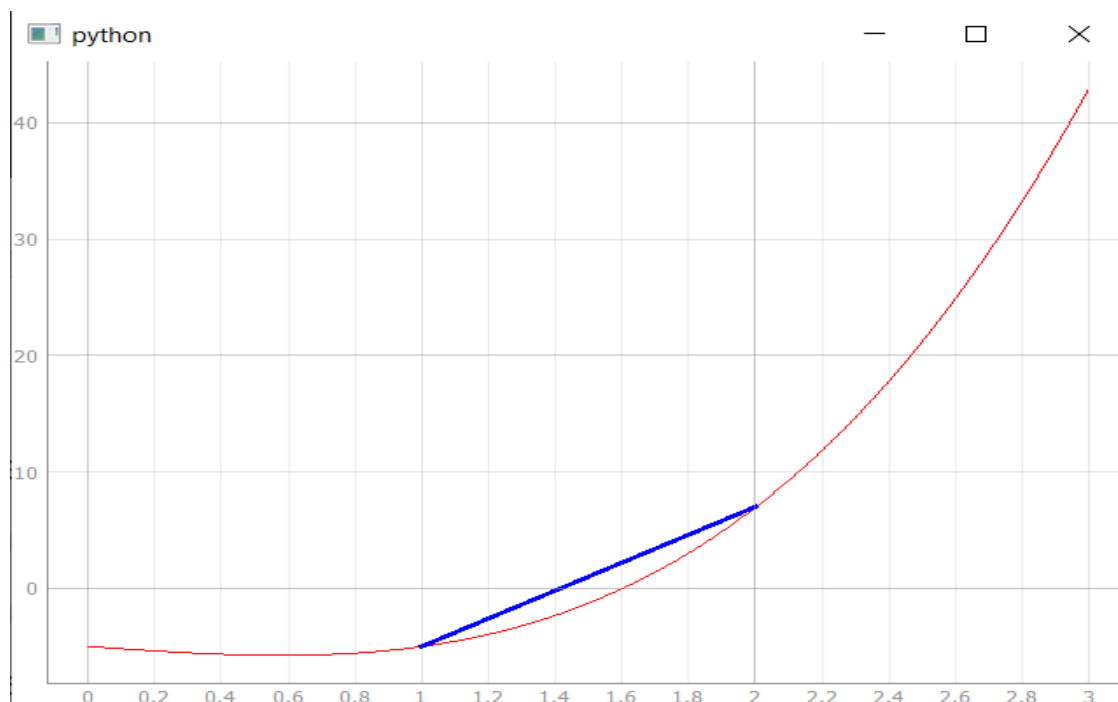
Iteration(4): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
                1.500000   -1.250000   1.625000   0.332031   1.625000   0.332031

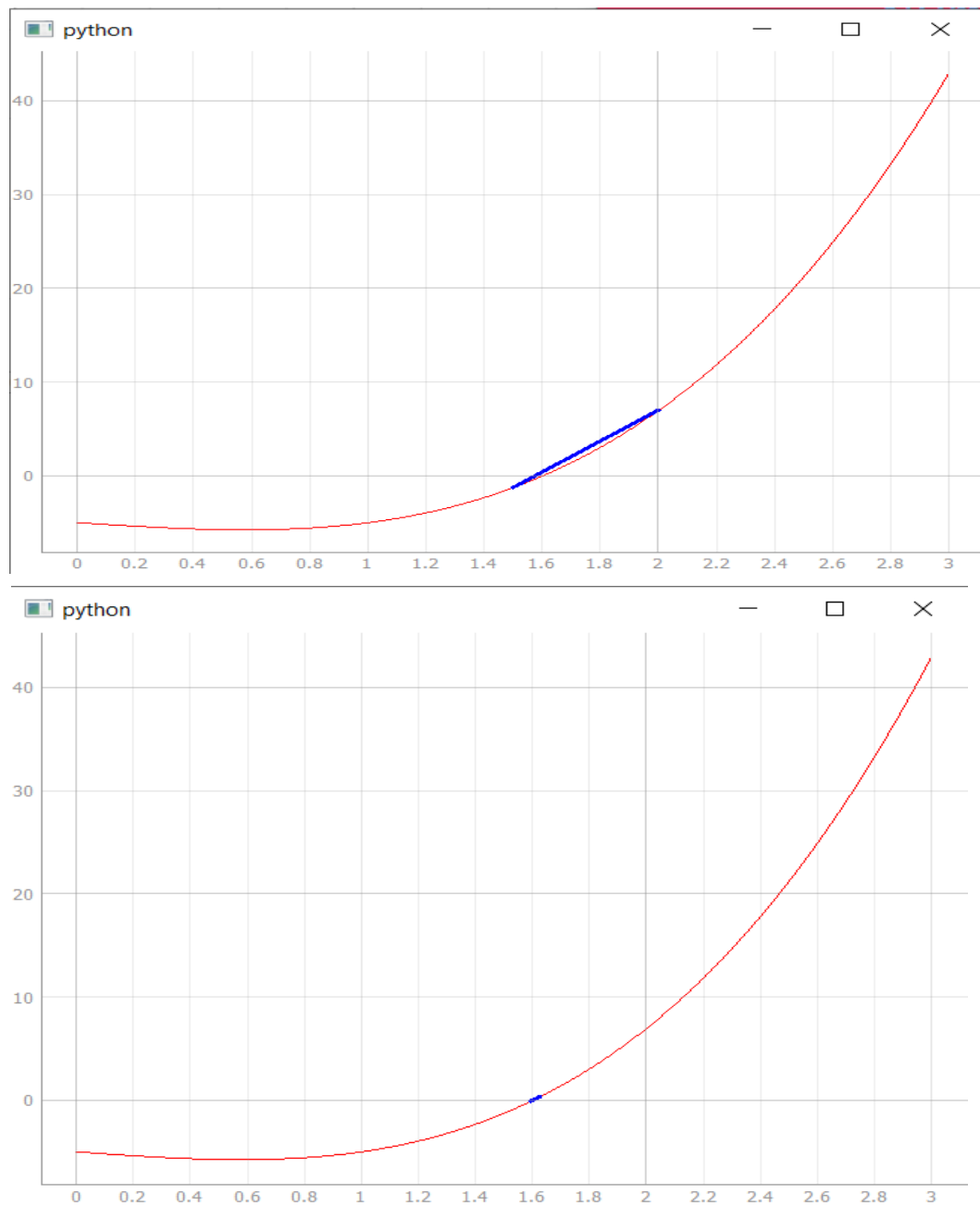
Iteration(5): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
                1.562500   -0.495605   1.625000   0.332031   1.562500   -0.495605

Iteration(6): X(lower) | f(X-lower) | X(upper) | f(X-upper) | X(mid) | f(X-mid)
                1.593750   -0.091125   1.625000   0.332031   1.593750   -0.091125
```

Graphs of this function:

Dynamic graph of Iterations :

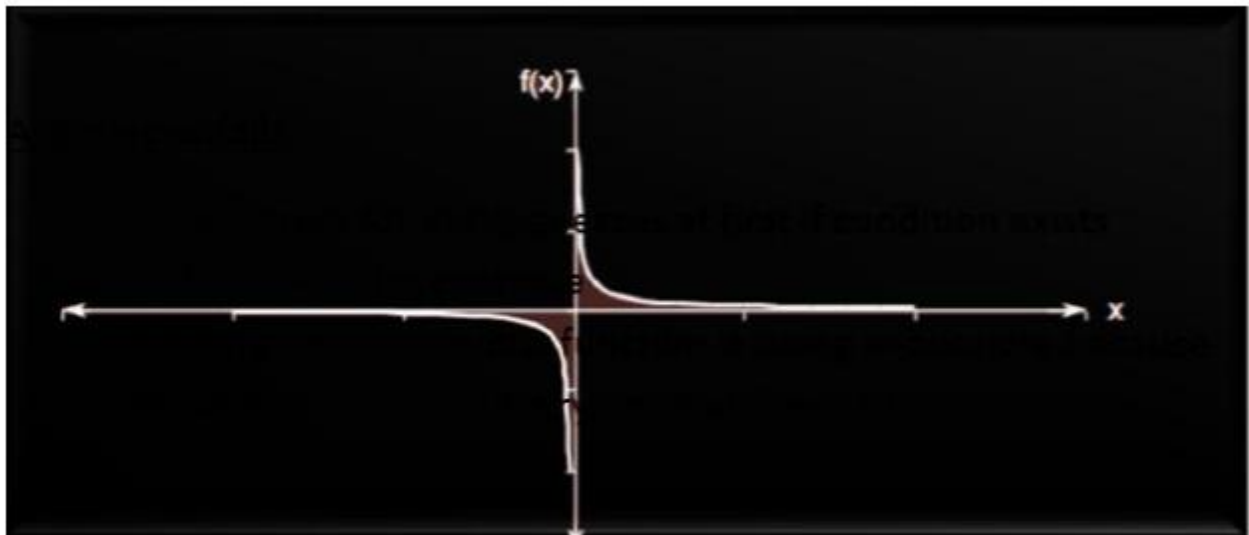




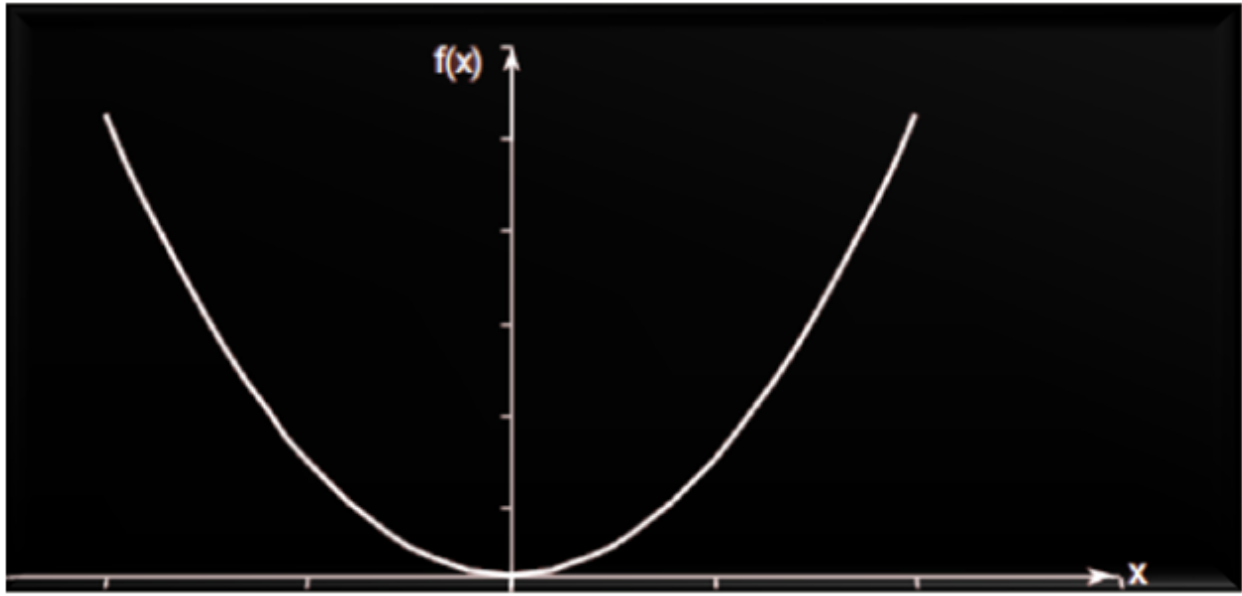
Reaching exact root as number of iterations increases

D)Pitfalls:

1. Slow method .
2. Need to find suitable initial guesses (x_l, x_u). "sometimes it is impossible to find the initial guesses "
3. No account is taken of the fact that $f(x_1)$ is closer to zero ,it is likely that the root is closer to x_1 .
4. Doesn't work with non-continuous functions even the initial guesses are correct .



- Functions that changes sign but does not have roots.



If the function just touches the x-axis , we will be unable to find the initial guesses.

- **Avoiding pitfalls :**

1. we can check for initial guesses at first . If condition exists continue ,else notify the user of the error.
2. Avoiding non-continuous functions is impossible because we have to check for every point at the given interval.

- **Divergence and convergence :** -It depends on the choice of the initial guesses :

-If initial guesses are correct then, it converges with number of iterations equals :

$$k \geq \log_2 \left| \frac{L_o}{x_l * \epsilon_{es}} \right|$$

-If initial guesses are not correct then, it diverges.

- **FIXED POINT METHOD:**

A) The code of the fixed point method:

```
# Fixed Point Iteration Method
# Importing math to use sqrt function
import math
from sympy import *
#function of f(x) el3aedya
#we use exp.subs to substitute the x with the real value we will take from user
def func(expr, value, x):
    return expr.subs(x, value)

# Re-writing f(x)=0 to x = g(x)
def funconverte(expr, value, x):
    return expr.subs(x, value)

# Implementing Fixed Point Iteration Method
#x0--> initial guess
#e-->tolerance
#N-->number of iteration
#x-->el x elgedeeda
def fixedPointIteration(expr, gx, x0, e, N, x):
    print('\n\n*** FIXED POINT ITERATION ***')
    step = 1
    flag = 1
    condition = True
    while condition:
        x1 = funconverte(gx, x0, x)
        print('Iteration-%d, x1 = %0.6f and f(x1) = %0.6f' % (step, x1, func(expr, x1, x)))
        x0 = x1
        step = step + 1

        if step > N:
            flag = 0
            break

        condition = abs(func(expr, x1, x)) > e

    if flag == 1:
        # print('\nRequired root is: %0.8f' % x1)
        return "%d ): %0.6f" % (step, x1)
    else:
        print('\nNot Convergent.')

# Main code
def mainFunc(function, g_x, N, e, x0):
    x = var('x') # the possible variable names must be known beforehand...
    expr = sympify(function)
    g_x = sympify(g_x)
    return fixedPointIteration(expr, g_x, x0, e, N, x)
```

B)Sample runs & Analysis :

1) first function :

$$E^{-x-x}$$

-lower bound: 0

-upper bound:2

-number of iterations:9

-Epsilon: 10^{-2}

magic function: E^{-x}

NUMERICAL PROJECT

Numerical Project

Function: Max Iteration:
☐ Select from File Epsilon:
Method:

Bisection

Fixed point

False-position

Newton-Raphson

Secant

First Guess: g(x):
Root Result: X(10): 0.56079
Exact Root X: 0 Execution Time: 0.065428 s

Calculate

Show Iterations

Show graph

Iterations:

```
*** FIXED POINT ITERATION ***
Iteration-1, x1 = 0.049787 and f(x1) = 0.901645
Iteration-2, x1 = 0.951432 and f(x1) = -0.565244
Iteration-3, x1 = 0.386188 and f(x1) = 0.293455
Iteration-4, x1 = 0.679643 and f(x1) = -0.172845
Iteration-5, x1 = 0.506798 and f(x1) = 0.095624
Iteration-6, x1 = 0.602422 and f(x1) = -0.054937
Iteration-7, x1 = 0.547484 and f(x1) = 0.030919
Iteration-8, x1 = 0.578403 and f(x1) = -0.017610
Iteration-9, x1 = 0.560793 and f(x1) = 0.009963
```

2)second function :

Now we will choose an equation that will diverge:

NUMERICAL PROJECT

Numerical Project

Function: Max Iteration:
☐ Select from File Epsilon:
Method:

Bisection Fixed point False-position Newton-Raphson Secant

First Guess: g(x):
Root Result: X(None)
Exact Root X: 0 Execution Time: 0.016954 s

Calculate Show Iterations Show graph

Iterations:

```
*** FIXED POINT ITERATION ***
Iteration-1, x1 = -3.000000 and f(x1) = 6.125000
Iteration-2, x1 = -3.000000 and f(x1) = 6.125000
Iteration-3, x1 = -3.000000 and f(x1) = 6.125000
Iteration-4, x1 = -3.000000 and f(x1) = 6.125000
Iteration-5, x1 = -3.000000 and f(x1) = 6.125000
Iteration-6, x1 = -3.000000 and f(x1) = 6.125000
Iteration-7, x1 = -3.000000 and f(x1) = 6.125000
Iteration-8, x1 = -3.000000 and f(x1) = 6.125000
Iteration-9, x1 = -3.000000 and f(x1) = 6.125000
Iteration-10, x1 = -3.000000 and f(x1) = 6.125000
Iteration-11, x1 = -3.000000 and f(x1) = 6.125000
Iteration-12, x1 = -3.000000 and f(x1) = 6.125000
Iteration-13, x1 = -3.000000 and f(x1) = 6.125000
Iteration-14, x1 = -3.000000 and f(x1) = 6.125000
Iteration-15, x1 = -3.000000 and f(x1) = 6.125000
Iteration-16, x1 = -3.000000 and f(x1) = 6.125000
Iteration-17, x1 = -3.000000 and f(x1) = 6.125000
Iteration-18, x1 = -3.000000 and f(x1) = 6.125000
Iteration-19, x1 = -3.000000 and f(x1) = 6.125000
Iteration-20, x1 = -3.000000 and f(x1) = 6.125000
```

3)Third Function:

NUMERICAL PROJECT

Numerical Project

Function: Max Iteration
☐ Select from File Epsilon:
Method:

Bisection Fixed point False-position Newton-Raphson Secant

First Guess: g(x):
Root Result: X(None)
Exact Root X: 0 Execution Time: 0.015006 s

Calculate Show Iterations Show graph

Iterations:

```

*** FIXED POINT ITERATION ***
Iteration-1, x1 = 7.000000 and f(x1) = 40.000000
Iteration-2, x1 = 47.000000 and f(x1) = 2160.000000
Iteration-3, x1 = 2207.000000 and f(x1) = 4068640.000000
Iteration-4, x1 = 4870847.000000 and f(x1) = 23725145626560.000000
Iteration-5, x1 = 23725150497407.000000 and f(x1) = 562882766124587916017532928.000000
Iteration-6, x1 = 562882766124611624237906848.000000 and f(x1) = 316837008400094212218388701986061763188239223924719616.000000
Iteration-7, x1 = 316837008400094212218388701986061763188239223924719616.000000 and f(x1) = 10038568989192137831263254868877825182418807118937521146752061522947470902804586034205
Iteration-8, x1 = 10038568989192137831263254868877825182418807118937521146752061522947470902804586034205 and f(x1) = 10077286735077085396351131720386
Iteration-9, x1 = 1007728673507708539635113172038675285881841645052077687125157578078235611948605336870275456861393858116301056958181492324208571887581952984868093050549672746797
Iteration-10, x1 = inf and f(x1) = inf
Iteration-11, x1 = inf and f(x1) = inf
Iteration-12, x1 = inf and f(x1) = inf
Iteration-13, x1 = inf and f(x1) = inf
Iteration-14, x1 = inf and f(x1) = inf
Iteration-15, x1 = inf and f(x1) = inf
Iteration-16, x1 = inf and f(x1) = inf
Iteration-17, x1 = inf and f(x1) = inf
Iteration-18, x1 = inf and f(x1) = inf
Iteration-19, x1 = inf and f(x1) = inf
Iteration-20, x1 = inf and f(x1) = inf

```

C) Observation

As the number of iterations increase we reaches value near the exact solution .

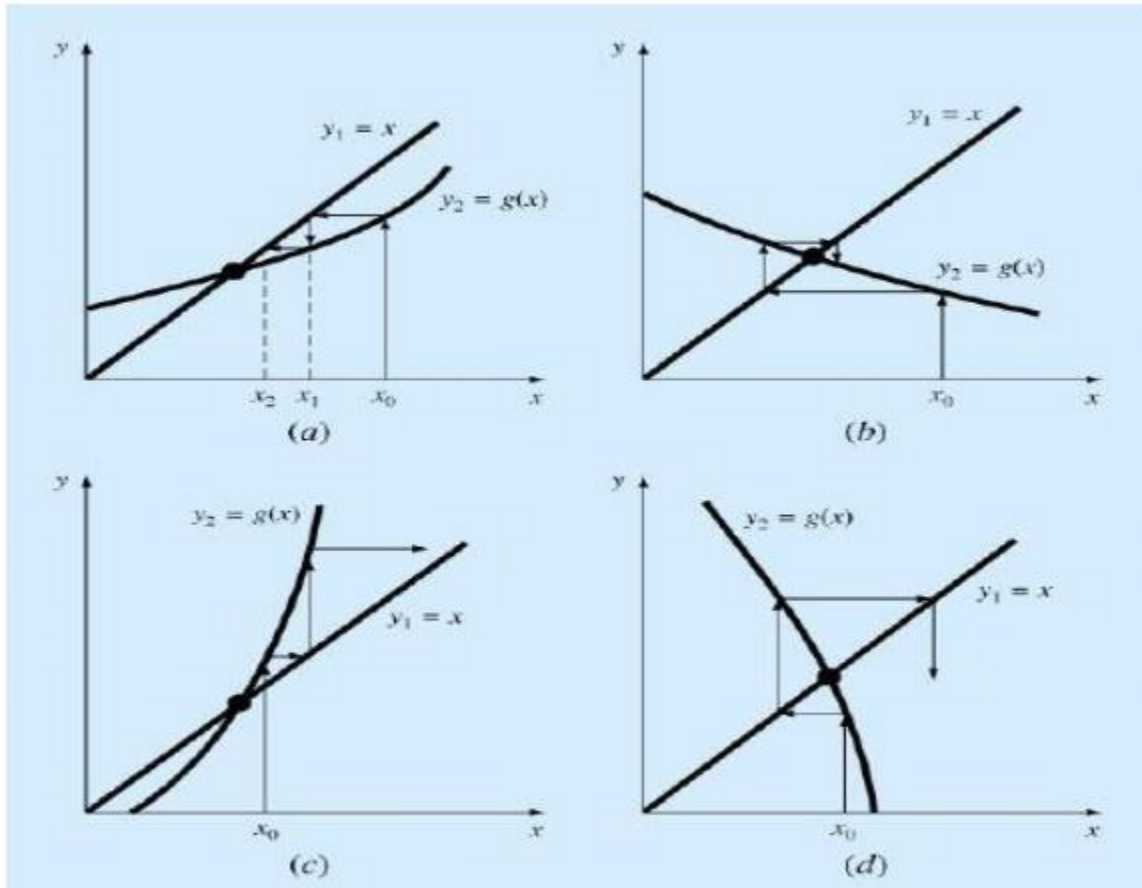
D) Pit falls :

- Finding the magical formula that will converge is the only pitfall .

- **Avoiding Pit falls :** -By finding the first derivative then substitute by the initial guess ,If the result is greater than 1 then it will converge .

• **Divergence and convergence :**

- a. $g'(x) < 1$, $g'(x)$ is positive . -----> converge , monotonic.
- b. $g'(x) < 1$, $g'(x)$ is negative -----> converge , oscillate.
- c. $g'(x) > 1$, $g'(x)$ is positive . -----> diverge , monotonic.
- d. $g'(x) > 1$, $g'(x)$ is negative . -----> diverge , monotonic



❖ FALSE POSITION METHOD:

A) The code of the false position method:

```
from sympy import var
from sympy import sympify
from sympy import *
import time

def func(expr, value, x):
    return expr.subs(x, value)

# An example function whose solution
# is determined using Bisection Method.
# The function is  $x^3 - x^2 + 2$ 

# Prints root of func(x) in interval [a, b]
def regulaFalsi(a, b, expr, x, MAX_ITER, EPSILON):
    if func(expr, a, x) * func(expr, b, x) >= 0:
        print("You have not assumed right a and b")
        return -1
    c = a
    # Initialize result
    i = 1
    old_c = int(0)
    start_time = time.time()
    step=0
    while i < MAX_ITER:
        # Find the point that touches x a/xis
        c = (a * func(expr, b, x) - b * func(expr, a, x)) / (func(expr, b, x) - func(expr,
a, x))
        print('Iteration-%d, Xr = %0.6f and f(Xr) = %0.6f' % (step, c, func(expr, c,x)))
        step=step+1
        if abs(abs(c - old_c) / c) < EPSILON:
            break
        # Check if the above found point is root
        if func(expr, c, x) == 0:
            break
        # Decide the side to repeat the steps
        elif func(expr, c, x) * func(expr, a, x) < 0:
            b = c
        else:
            a = c
        old_c = c
        i = i + 1
    end_time = time.time()
    t3 = end_time - start_time
    print("execution time for RegularFalse=", "%.6f" " sec" % t3)

    return "%d ): %0.6f" % (i, c)

def mainFunc(function, maxIteration, epsilon, a, b):
    x = var('x') # the possible variable names must be known beforehand...
    expr = sympify(function)
    return regulaFalsi(a, b, expr, x, maxIteration, epsilon)
```

B)Sample runs & Analysis :

1) first function :

$$E^{-x-x}$$

-lower bound: 0

-upper bound:2

-number of iterations:9

-Epsilon: 10^{-2}

The screenshot shows a software window titled "NUMERICAL PROJECT". The main area has a pink background with the title "Numerical Project" in blue. Below the title, there are input fields for "Function:" (containing E^{-x-x}), "Max Iteration:" (20), and "Epsilon:" (0.010000). A checkbox labeled "Select from File" is checked. Below these is a "Method:" label. A row of tabs at the bottom includes "Bisection", "Fixed point", "False-position", "Newton-Raphson", and "Secant". The "Fixed point" tab is selected. Below the tabs, there are fields for "Lower Bound:" and "Upper Bound:". The "Root Result:" is displayed as $x(4) : 0.567320$. Below that, "Exact Root X:" is 0 and "Execution Time:" is 0.001995 s. At the bottom, there are three buttons: "Calculate", "Show Iterations", and "Show graph".

ROOT RESULT: is at $x_4 = 0.56732$

Iterations:

```
Iteration-0, Xr = 0.698162 and f(Xr) = -0.200663
Iteration-1, Xr = 0.581480 and f(Xr) = -0.022410
Iteration-2, Xr = 0.568735 and f(Xr) = -0.002494
Iteration-3, Xr = 0.567320 and f(Xr) = -0.000277
execution time for RegularFalse= 0.000997 sec
```

2)second function:

NUMERICAL PROJECT

Numerical Project

Function:

☐ Select from File

Max Iteration:

Epsilon:

Method:

BisectionFixed pointFalse-positionNewton-RaphsonSecant

Lower Bound: Upper Bound:

Root Result: X(0): 0

Exact Root X: 0Execution Time: 0 ms

Calculate

Show Iterations

Show graph

Iterations :

```
The value of the root is : 1.6006
execution time for NewtonRaphson= 0.004989 sec
Iteration-0, Xr = -1.776316 and f(Xr) = -12.656979
Iteration-1, Xr = -1.612878 and f(Xr) = -10.165646
Iteration-2, Xr = -1.483581 and f(Xr) = -8.563599
Iteration-3, Xr = -1.376041 and f(Xr) = -7.458956
Iteration-4, Xr = -1.283409 and f(Xr) = -6.661088
Iteration-5, Xr = -1.201504 and f(Xr) = -6.066001
Iteration-6, Xr = -1.127588 and f(Xr) = -5.612176
Iteration-7, Xr = -1.059772 and f(Xr) = -5.260953
Iteration-8, Xr = -0.996698 and f(Xr) = -4.986859
Iteration-9, Xr = -0.937354 and f(Xr) = -4.772473
Iteration-10, Xr = -0.880965 and f(Xr) = -4.605503
Iteration-11, Xr = -0.826922 and f(Xr) = -4.477054
Iteration-12, Xr = -0.774736 and f(Xr) = -4.380546
Iteration-13, Xr = -0.724008 and f(Xr) = -4.311016
Iteration-14, Xr = -0.674406 and f(Xr) = -4.264660
Iteration-15, Xr = -0.625650 and f(Xr) = -4.238506
Iteration-16, Xr = -0.577498 and f(Xr) = -4.230200
Iteration-17, Xr = -0.529743 and f(Xr) = -4.237835
Iteration-18, Xr = -0.482204 and f(Xr) = -4.259837
execution time for RegularFalse= 0.093667 sec
```

3-Third equation:

3)Third function:

NUMERICAL PROJECT

Numerical Project

Function:

☐ Select from File

Max Iteration:

Epsilon:

Method:

Bisection

Fixed point

False-position

Newton-Raphson

Secant

Lower Bound:

Upper Bound:

Root Result: x(20): -0.3703

Exact Root X: 0

Execution Time: 0.029263 s

Calculate

Show Iterations

Show graph

Iterations:

```
Iteration-0, Xr = -0.950000 and f(Xr) = -0.907375  
Iteration-1, Xr = -0.904975 and f(Xr) = -0.836181  
Iteration-2, Xr = -0.863771 and f(Xr) = -0.780689  
Iteration-3, Xr = -0.825553 and f(Xr) = -0.737093  
Iteration-4, Xr = -0.789692 and f(Xr) = -0.702770  
Iteration-5, Xr = -0.755701 and f(Xr) = -0.675868  
Iteration-6, Xr = -0.723195 and f(Xr) = -0.655044  
Iteration-7, Xr = -0.691864 and f(Xr) = -0.639315
```

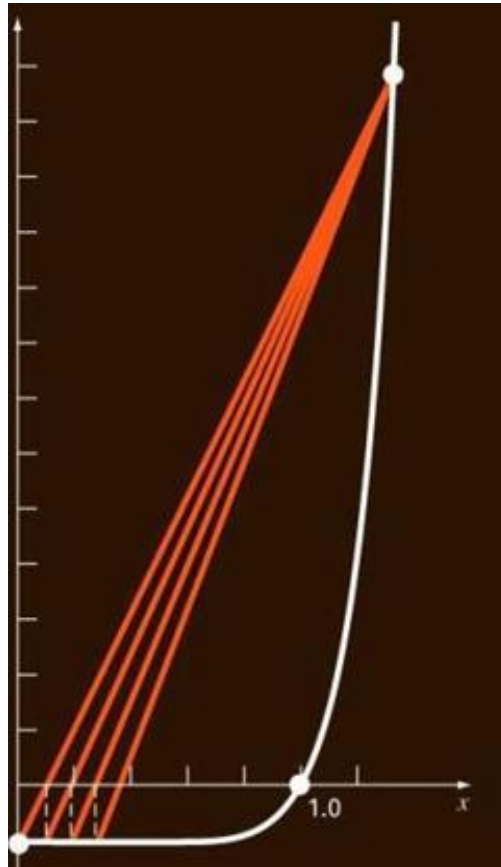
```
Iteration-8, Xr = -0.661449 and f(Xr) = -0.627945  
Iteration-9, Xr = -0.631731 and f(Xr) = -0.620383  
Iteration-10, Xr = -0.602523 and f(Xr) = -0.616213  
Iteration-11, Xr = -0.573661 and f(Xr) = -0.615123  
Iteration-12, Xr = -0.544999 and f(Xr) = -0.616879  
Iteration-13, Xr = -0.516402 and f(Xr) = -0.621307  
Iteration-14, Xr = -0.487750 and f(Xr) = -0.628286  
Iteration-15, Xr = -0.458929 and f(Xr) = -0.637729  
Iteration-16, Xr = -0.429830 and f(Xr) = -0.649583  
Iteration-17, Xr = -0.400351 and f(Xr) = -0.663817  
Iteration-18, Xr = -0.370394 and f(Xr) = -0.680421  
execution time for RegularFalse= 0.027268 sec
```

C)Observation

As the number of iterations increase we reaches value near the exact solution .

D) Pit falls :

- Works well but not always .
- Cannot detect continuous functions .
- Initial guesses are required .
- one of bounds might get stuck .



● Avoiding Pit falls :

One way to mitigate the “one-sided” nature of the false position is to have the algorithm detect when one of the bounds is stuck.

If this occurs,
then the original formula of bisection can be used.



NEWTON RAPHSON METHOD:

A) the code for newton Raphson method

```
from sympy import var
from sympy import sympify
from sympy import *
import time

def func(expr, value, x):
    return expr.subs(x, value)

# An example function whose solution
# is determined using Bisection Method.
# The function is  $x^3 - x^2 + 2$ 

# Prints root of func(x) in interval [a, b]
def regulaFalsi(a, b, expr, x, MAX_ITER, EPSILON):
    if func(expr, a, x) * func(expr, b, x) >= 0:
        print("You have not assumed right a and b")
        return -1
    c = a
    # Initialize result
    i = 1
    old_c = int(0)
    start_time = time.time()
    step=0
    while i < MAX_ITER:
        # Find the point that touches x a/xis
        c = (a * func(expr, b, x) - b * func(expr, a, x)) / (func(expr, b, x) - func(expr,
a, x))
        print('Iteration-%d, Xr = %0.6f and f(Xr) = %0.6f' % (step, c, func(expr, c,x)))
        step=step+1
        if abs(abs(c - old_c) / c) < EPSILON:
            break
        # Check if the above found point is root
        if func(expr, c, x) == 0:
            break
        # Decide the side to repeat the steps
        elif func(expr, c, x) * func(expr, a, x) < 0:
            b = c
        else:
            a = c
        old_c = c
        i = i + 1
    end_time = time.time()
    t3 = end_time - start_time
    print("execution time for RegularFalse=", "%.6f" " sec" % t3)

    return "%d ): %.6f" % (i, c)

def mainFunc(function, maxIteration, epsilon, a, b):
    x = var('x') # the possible variable names must be known beforehand...
    expr = sympify(function)
    return regulaFalsi(a, b, expr, x, maxIteration, epsilon)
```


B)Sample runs & Analysis :

1) first function :

- E^{-x-x}

-lower bound: 0

-upper bound:2

-number of iterations:9

-Epsilon: 10^{-2}

The screenshot shows a software window titled 'NUMERICAL PROJECT'. The main area has a pink background with the title 'Numerical Project' in blue. Below the title, there are input fields for 'Function:' (containing E^{-x-x}), 'Max Iteration' (20), and 'Epsilon:' (0.010000). A checkbox labeled 'Select from File' is checked. Below these is a 'Method:' label. At the bottom, there is a tabbed interface with five tabs: 'Bisection', 'Fixed point', 'False-position', 'Newton-Raphson' (which is selected), and 'Secant'. The 'Newton-Raphson' tab is active, showing a 'First Guess:' of 3, a 'Root Result: x(4): 0.567143', and 'Exact Root X: 0'. The 'Execution Time: 0.029241 s' is also displayed. At the bottom of the active tab are three buttons: 'Calculate', 'Show Iterations', and 'Show graph'.

NUMERICAL PROJECT

Numerical Project

Function: E^{-x-x} Max Iteration: 20
☒ Select from File Epsilon: 0.010000

Method:

Bisection Fixed point False-position **Newton-Raphson** Secant

First Guess: 3

Root Result: x(4): 0.567143

Exact Root X: 0 Execution Time: 0.029241 s

Calculate Show Iterations Show graph

Iterations:

```
Iteration-1, xi = 0.000000 and f(xi) = 1.000000  
Iteration-2, xi = 0.500000 and f(xi) = 0.106531  
Iteration-3, xi = 0.566311 and f(xi) = 0.001305  
The value of the root is : 0.5671  
execution time for NewtonRaphson= 0.003960 sec
```

2)second function:

NUMERICAL PROJECT

Numerical Project

Function:

☐ Select from File

Max Iteration

Epsilon:

Method:

Bisection

Fixed point

False-position

Newton-Raphson

Secant

First Guess:

Root Result: $x(4)$: 1.324718

Exact Root X: 0

Execution Time: 0.026659 s

Calculate

Show Iterations

Show graph

Iterations:

```
Iteration-1, xi = 1.500000 and f(xi) = 0.875000
Iteration-2, xi = 1.347826 and f(xi) = 0.100682
Iteration-3, xi = 1.325200 and f(xi) = 0.002058
The value of the root is : 1.3247
execution time for NewtonRaphson= 0.003988 sec
```

3)Third function:

NUMERICAL PROJECT

Numerical Project

Function:

☐ Select from File

Max Iteration:

Epsilon:

Method:

Bisection

Fixed point

False-position

Newton-Raphson

Secant

First Guess:

Root Result: X(3): 1.600645

Excact Root X: 0 Execution Time: 0.105027 s

Calculate

Show Iterations

Show graph

Iterations:

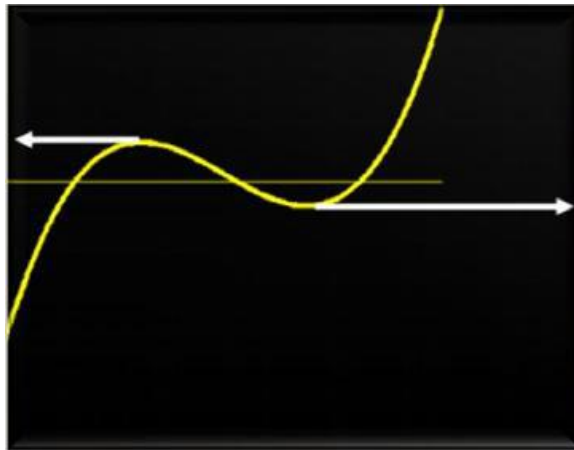
```
Iteration-1, xi = 1.500000 and f(xi) = -1.250000  
Iteration-2, xi = 1.608696 and f(xi) = 0.108901  
The value of the root is : 1.6006  
execution time for NewtonRaphson= 0.004989 sec
```

C)Observation

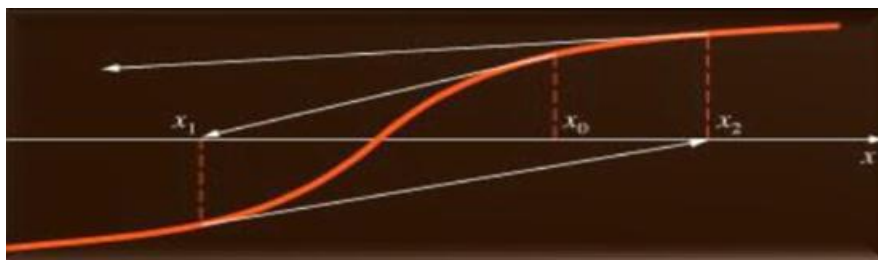
As the number of iterations increase we reaches value near the exact solution .

D)Pit Falls :

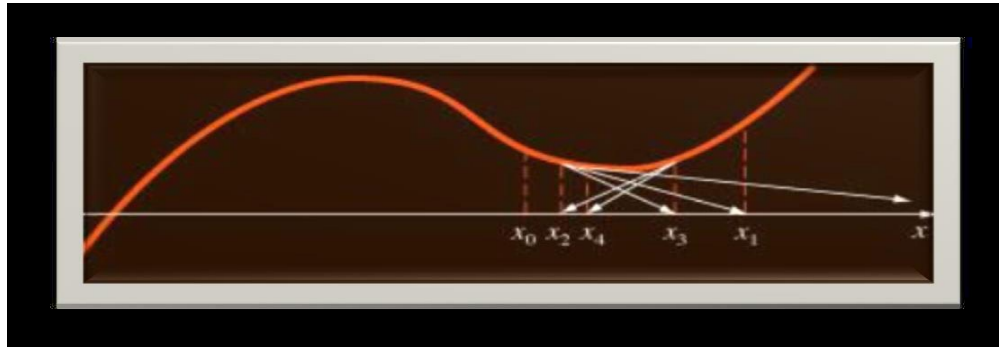
- Division by zero



- An inflection point ($f''(x)=0$) at the vicinity of a root.



-A local maximum or minimum causes oscillations.



❖ SECANT METHOD:

```
from sympy import *
import time

# Defining Function
def f(y, Function):
    x = var('x')
    return Function.subs(x, y)

# Implementing Secant Method

def secant(x0, x1, e, N, Function):
    print('\n\n*** SECANT METHOD IMPLEMENTATION **')
    step = 1
    condition = True
    start_time = time.time()
    while condition:
        if f(x0, Function) == f(x1, Function):
            print('Divide by zero error!')
            break

        x2 = x0 - (x1 - x0) * f(x0, Function) / (f(x1, Function) - f(x0, Function))
        fx = f(x2, Function)
        print('Iteration-%d, x2 = %.6f and f(x2) = %.6f' % (step, x2, f(x2, Function)))
        x0 = x1
        x1 = x2
        step = step + 1

        if step > N:
            print('Not Convergent!')
            break

        condition = abs((x1 - x0) / x1) >= e
    end_time = time.time()
    t4 = end_time - start_time
    print("execution time for Secant=", "%.6f" % t4)
    return "%d ): %.6f" % (step, x2)

def mainFunc(expr, N, e, x0, x1):
    Function = sympify(expr)
    x = var('x')
    sol = solve(Function, x)
    if sol == LambertW(1):
        sol = 0.5671432904097

    # Starting Secant Method
    return secant(x0, x1, e, N, Function)
```

1)first function:

NUMERICAL PROJECT

Numerical Project

Function: Max Iteration:
☒ Select from File Epsilon:
Method:

Bisection Fixed point False-position Newton-Raphson **Secant**

First Guess: Second Guess:
Root Result: $x(5)$: 0.567146
Exact Root X: 0 Execution Time: 0.807896 s

Iterations:

```
Iteration-2, x2 = 0.541172 and f(x2) = 0.040893  
Iteration-3, x2 = 0.567749 and f(x2) = -0.000949  
Iteration-4, x2 = 0.567146 and f(x2) = -0.000004  
execution time for Secant= 0.001991 sec
```

2)second equation:

NUMERICAL PROJECT

Numerical Project

Function: Max Iteration:
☐ Select from File Epsilon:
Method:

Bisection

Fixed point

False-position

Newton-Raphson

Secant

First Guess: Second Guess:
Root Result: $x(5)$: 1.414259
Exact Root X: 0 Execution Time: 0.026922 s

Calculate

Show Iterations

Show graph

Iterations:

```
Iteration-2, x2 = 1.375000 and f(x2) = -0.109375  
Iteration-3, x2 = 1.410959 and f(x2) = -0.009195  
Iteration-4, x2 = 1.414259 and f(x2) = 0.000130  
execution time for Secant= 0.002991 sec
```

3)third equation:

The screenshot shows a software window titled "NUMERICAL PROJECT". The main title "Numerical Project" is displayed in a large, stylized font. Below it, the "Function:" field contains the equation $x^3 - x^2 - 10x + 7$. To the right, "Max Iteration" is set to 20 and "Epsilon" is set to 0.010000. A checkbox labeled "Select from File" is present. The "Method:" section has five tabs: "Bisection", "Fixed point", "False-position", "Newton-Raphson", and "Secant", with "Secant" currently selected. Below the tabs, the "First Guess:" is 3.4 and the "Second Guess:" is 3.3. The "Root Result:" is displayed as $X(3) : 3.357503$. Below this, it shows "Exact Root X: 0" and "Execution Time: 0.033726 s". At the bottom, there are three buttons: "Calculate", "Show Iterations", and "Show graph".

Iterations:

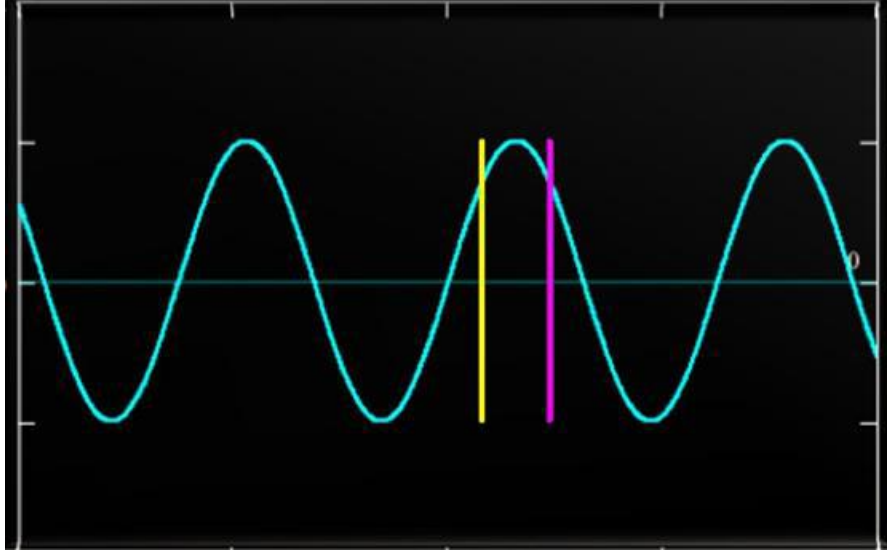
```
*** SECANT METHOD IMPLEMENTATION *  
Iteration-1, x2 = 3.356158 and f(x2) = -0.022297  
Iteration-2, x2 = 3.357503 and f(x2) = 0.000697  
execution time for Secant= 0.003989 sec
```

C)Observation

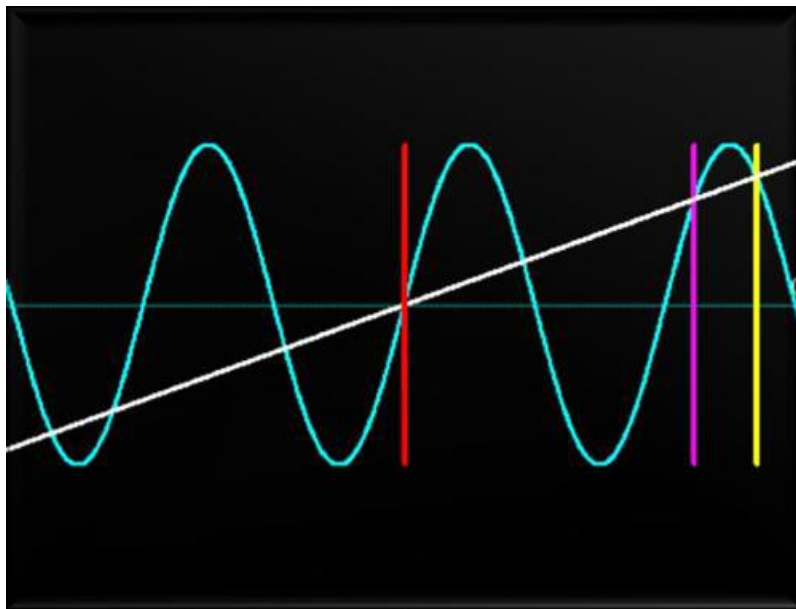
As the number of iterations increase we reaches value near the exact solution .

Pit falls :

- Division by zero .



- Root jumping .



E)Problematic functions:

Problems we have faced :

1-while passing the values (substitution) of

`x` → the value of either the upper or lower bound.

to the main function ,but we have solve it by using

func.sub(x,value).

2-To make the user have the option to pass the function and its inputs directly from the GUI or to read from file.

We have fixed this by making a check box linked with a condition (if else),and this has worked properly.

F)Data structure:

No data structure used.

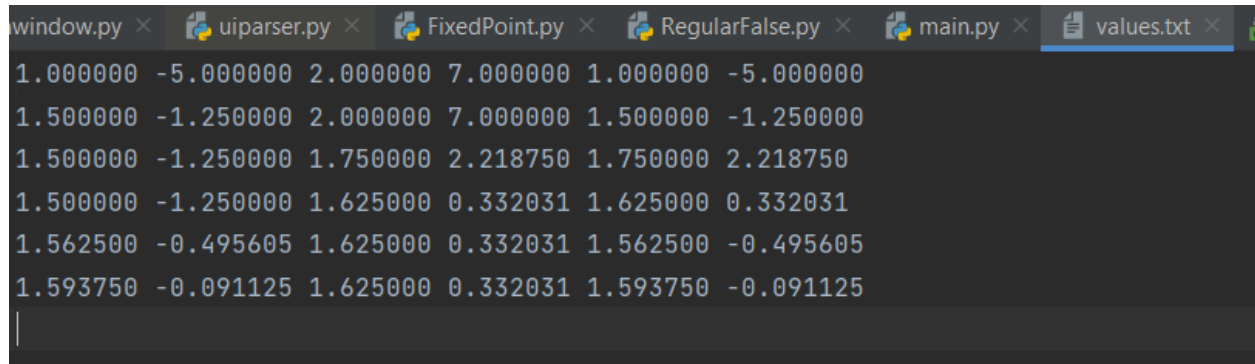
G)User manual :

The use can enter the function in the GUI or he can select the function from file then he can choose the desired method entering the initial guess then press calculate to get the results ,the user can also press on graph to see the graph of the function.

NOTE: calculations results are in the output file

NOTE:in each method we print our output in an output file.

Example:



The screenshot shows a code editor with several tabs: window.py, uiparser.py, FixedPoint.py, RegularFalse.py, main.py, and values.txt. The values.txt tab is active, displaying a terminal window with the following output:

```
1.000000 -5.000000 2.000000 7.000000 1.000000 -5.000000
1.500000 -1.250000 2.000000 7.000000 1.500000 -1.250000
1.500000 -1.250000 1.750000 2.218750 1.750000 2.218750
1.500000 -1.250000 1.625000 0.332031 1.625000 0.332031
1.562500 -0.495605 1.625000 0.332031 1.562500 -0.495605
1.593750 -0.091125 1.625000 0.332031 1.593750 -0.091125
```