

Problem 2

Describe why the base Ackermann function is so heavy to compute

Answer:

Because we use recursion to be able to recall the function and Ackermann function have 3 cases, first is the base case and don't case to call function but it really hard to reach it because to solve it we use second and third case and the problem in second and third cases that second recall function once ($n=0 \rightarrow \text{ack}(m-1, 1)$) and third case recall the function twice

($n \& m > 0 \rightarrow \text{ack}(m-1, \text{ack}(m, n-1))$) so its many calling for the function to reach to base case that make small numbers as $\text{ask}(1, 3)$ for example case lot of calling

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$$\begin{aligned} \text{ask}(1, 3) \\ \because n > 0 \quad m > 0 & \quad \therefore \text{use eq 3} \rightarrow \text{ask}(m-1, \text{ask}(m, n-1)) \\ \text{ask}(1, 3) = \text{ask}(0, \text{ask}(1, 2)) \\ \text{ask}(1, 2) & \quad \because n > 0 \quad m > 0 \quad \therefore \text{use eq 3} \\ \text{ask}(1, 2) = \text{ask}(0, \text{ask}(1, 1)) \\ \text{ask}(1, 1) & \quad \because n > 0 \quad m > 0 \quad \therefore \text{use eq 3} \\ \text{ask}(1, 1) = \text{ask}(0, \text{ask}(1, 0)) \\ \text{ask}(0, 1) & \quad m = 0 \quad \therefore \text{eq 1} \quad n+1 \\ \text{ask}(0, 1) = 2 \\ \text{ask}(1, 1) = \text{ask}(0, 2) = 3 \\ \text{ask}(1, 2) = \text{ask}(0, 3) = 4 \\ \text{ask}(1, 3) = \text{ask}(0, 4) = \boxed{5} \end{aligned}$$

and in numbers like $\text{ask}(4, 1), \text{ask}(4, 2) \dots$ etc

Will case many and many of calling that make normal computer not able to return a correct output for example $\text{ask}(4, 1)$ may take 3 minute or more to execute so it takes thousands of minutes to execute $\text{ask}(4, 2)$ and in some cases it will may take days or months to execute other examples and that's because its complexity of using recursion in it is exponential (2^n) so in many cases computer cannot be able to make all of this calling that case to take a lot of time to excute or memory not able to handle all this calling or excute incorrect result

We can solve the recursion problem of Ackermann function by dynamic programming by making matrix (double array) equal to $m+1 \& n+1$ to store values on it but this solution may not be accurate in $m > 4$