Q1:

Exponential model

Sometimes in least square method, we will use exponential curve to achieve *minimum error* with the given data.

Let the required curve has the form:

$$y = ae^{bx}$$

To use the least square method of a straight line, first we must make a transformation to the required equation, this transformation called ***linearization*** by taking the inverse of the required equation of the curve as the following:

$$lny = lna + bx$$

This equation can be written as:

$$\boxed{Y = A + Bx} \qquad (2.4)$$

With the changes:

$$\boxed{Y = lny, \;\; A = lna, \;\; B = b, \;\; x = x.}$$

Then we can apply least square equations to equation (2.4) as:

$$\boxed{\begin{aligned} \sum Y_i &= A(N) + B\sum x_i, \\ \sum x_i Y_i &= A\sum x_i + B\sum x_i^2. \end{aligned}}$$

After solving the previous equations and getting the constants (A, B) we make a reverse operation to get (a, b) again, to express the equation $y = ae^{bx}$.

A polynomial of order (2) model

Now, consider any an experiment produces a set of N data points $(x_1, y_1), \dots, (x_N, y_N)$, or table as:

x	x_1	x_2	$\cdots$	x_{N-1}	x_N
y	y_1	y_2	$\cdots$	y_{N-1}	y_N

And we want to find the best curve (second order polynomial)

$$y(x) = a_0 + a_1 x + a_2 x^2,$$

that meets these data by using Least square method. By the same way mention in previous section:

Let S_r is called the sum of the square of the errors defined by the following

$$S_r = \sum_{i=1}^{N} \varepsilon_i^2 = \sum_{i=1}^{N} [y_i - (a_0 + a_1 x_i + a_2 x_i^2)]^2.$$

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To find a_0, a_1 and a_2, we minimize S_r with respect to a_0, a_1 and a_2:

1) $\dfrac{\partial Sr}{\partial a_0} = 0 \Rightarrow \sum y_i = a_0(N) + a_1 \sum x_i + a_2 \sum x_i^2,$

2) $\dfrac{\partial Sr}{\partial a_1} = 0 \Rightarrow \sum x_i y_i = a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3,$

3) $\dfrac{\partial Sr}{\partial a_1} = 0 \Rightarrow \sum x_i^2 y_i = a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4,$

The resulting normal equations are:

$$\boxed{\begin{aligned} \sum y_i &= a_0(N) + a_1 \sum x_i + a_2 \sum x_i^2, \\ \sum x_i y_i &= a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3, \\ \sum x_i^2 y_i &= a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4, \end{aligned}} \qquad (2.3)$$

By the same way of this method, we can find the used equations for any polynomial of any order greater than one.

+ they don't give the same result because the relative error

Q2:

1. Logistic Regression
2. Decision Tree
3. SVM
4. Naive Bayes
5. kNN
6. K-Means
7. Random Forest
8. Dimensionality Reduction Algorithms
9. Gradient Boosting algorithms
    1. GBM
    2. XGBoost
    3. LightGBM
    4. CatBoost