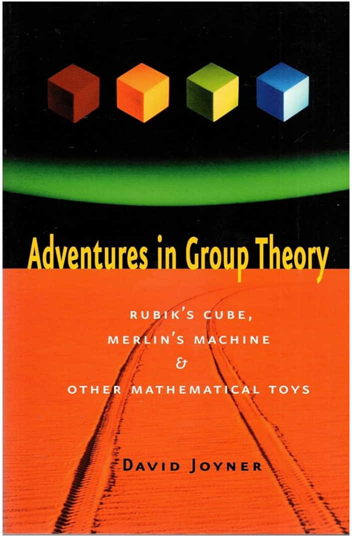
Lights out!

Menno Bartels
Formal System Analysis, TU/e

23-04-2025



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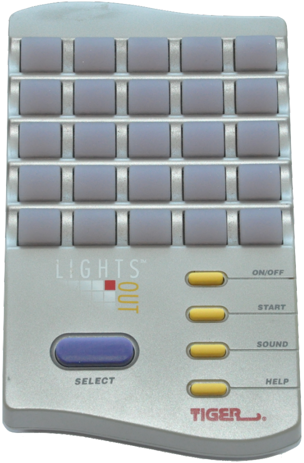

- Tiger Electronics 1995
- Popular puzzle in video games
- One player game with (presumably) symmetries

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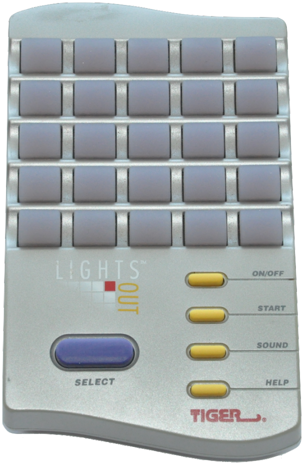
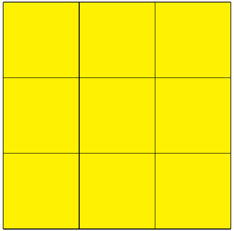


- Tiger Electronics 1995
- Popular puzzle in video games
- One player game with (presumably) symmetries
- Not in the `examples/` folder of mCRL2

Lights Out



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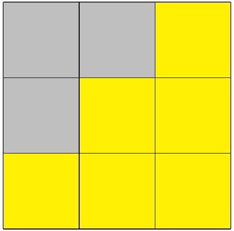

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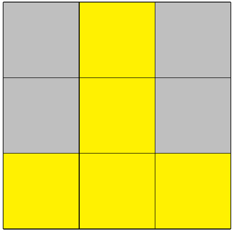

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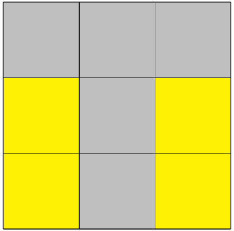

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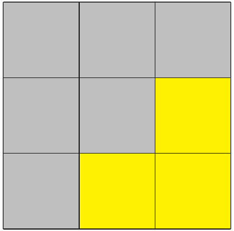

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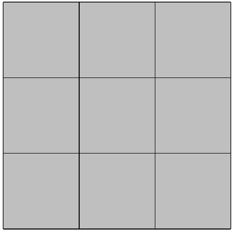

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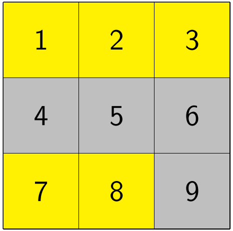

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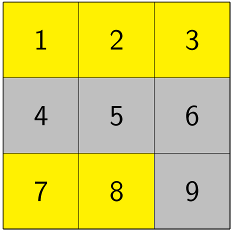
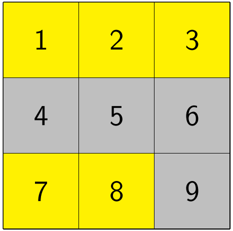


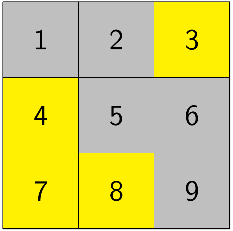
$$\left(1,\quad 1,\quad 1,\quad 0,\quad 0,\quad 0,\quad 1,\quad 1,\quad 0\right)$$

Lights Out



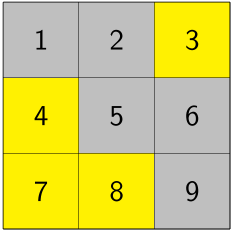
$$
\begin{array}{rl}
 & (1,\ 1,\ 1,\ 0,\ 0,\ 0,\ 1,\ 1,\ 0) \\
\oplus & (1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0)
\end{array}
$$

Lights Out



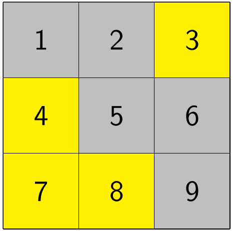
$$\begin{array}{cc} & (1,\ 1,\ 1,\ 0,\ 0,\ 0,\ 1,\ 1,\ 0) \\ \oplus & (1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0) \\ = & (0,\ 0,\ 1,\ 1,\ 0,\ 0,\ 1,\ 1,\ 0) \end{array}$$

Lights Out



$$(0,\ 0,\ 1,\ 1,\ 0,\ 0,\ 1,\ 1,\ 0)$$

Lights Out



$$
\begin{array}{rl}
 & (0,\ 0,\ 1,\ 1,\ 0,\ 0,\ 1,\ 1,\ 0) \\
\oplus & (1,\ 1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0)
\end{array}
$$

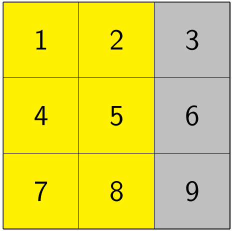
Lights Out

1	2	3
4	5	6
7	8	9

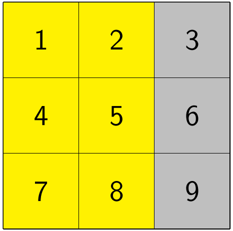
$$
\begin{array}{rl}
 & (0,\ 0,\ 1,\ 1,\ 0,\ 0,\ 1,\ 1,\ 0) \\
\oplus & (1,\ 1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0) \\
= & (1,\ 1,\ 0,\ 1,\ 1,\ 0,\ 1,\ 1,\ 0)
\end{array}
$$

Lights Out

$$\begin{pmatrix}1, & 1, & 0, & 1, & 1, & 0, & 1, & 1, & 0\end{pmatrix}$$

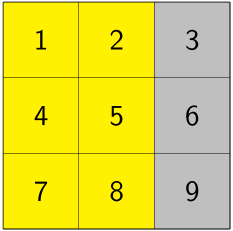


Lights Out



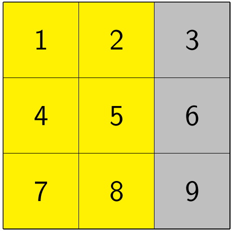
$$\begin{array}{rr} & (1,\ 1,\ 0,\ 1,\ 1,\ 0,\ 1,\ 1,\ 0) \\ \oplus & x_1\,(1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0) \end{array}$$

Lights Out



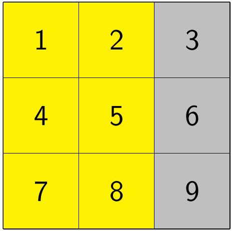
$$
\begin{array}{rrl}
 & & (1,\ 1,\ 0,\ 1,\ 1,\ 0,\ 1,\ 1,\ 0) \\
\oplus & x_1 & (1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0) \\
\oplus & x_2 & (1,\ 1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0)
\end{array}
$$

Lights Out



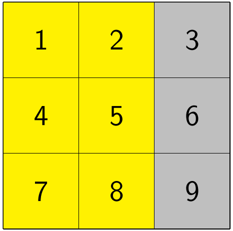
$$
\begin{array}{ccc}
 & & (1,\ 1,\ 0,\ 1,\ 1,\ 0,\ 1,\ 1,\ 0) \\
\oplus & x_1 & (1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0) \\
\oplus & x_2 & (1,\ 1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0) \\
\oplus & x_3 & (0,\ 1,\ 1,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0) \\
\oplus & x_4 & (1,\ 0,\ 0,\ 1,\ 1,\ 0,\ 1,\ 0,\ 0) \\
\oplus & x_5 & (0,\ 1,\ 0,\ 1,\ 1,\ 1,\ 0,\ 1,\ 0) \\
\oplus & x_6 & (0,\ 0,\ 1,\ 0,\ 1,\ 1,\ 0,\ 0,\ 1) \\
\oplus & x_7 & (0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 1,\ 1,\ 0) \\
\oplus & x_8 & (0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 1,\ 1,\ 1) \\
\oplus & x_9 & (0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 1,\ 1)
\end{array}
$$

Lights Out



$$
\begin{array}{ccl}
 & & (1,\ 1,\ 0,\ 1,\ 1,\ 0,\ 1,\ 1,\ 0) \\
\oplus & x_1 & (1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0) \\
\oplus & x_2 & (1,\ 1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0) \\
\oplus & x_3 & (0,\ 1,\ 1,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0) \\
\oplus & x_4 & (1,\ 0,\ 0,\ 1,\ 1,\ 0,\ 1,\ 0,\ 0) \\
\oplus & x_5 & (0,\ 1,\ 0,\ 1,\ 1,\ 1,\ 0,\ 1,\ 0) \\
\oplus & x_6 & (0,\ 0,\ 1,\ 0,\ 1,\ 1,\ 0,\ 0,\ 1) \\
\oplus & x_7 & (0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 1,\ 1,\ 0) \\
\oplus & x_8 & (0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 1,\ 1,\ 1) \\
\oplus & x_9 & (0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 1,\ 1) \\
= & & (0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0)
\end{array}
$$

Lights Out



$$
\begin{array}{ccl}
 & & (1,\ 1,\ 0,\ 1,\ 1,\ 0,\ 1,\ 1,\ 0) \\
\oplus & x_1 & (1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0) \\
\oplus & x_2 & (1,\ 1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0) \\
\oplus & x_3 & (0,\ 1,\ 1,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0) \\
\oplus & x_4 & (1,\ 0,\ 0,\ 1,\ 1,\ 0,\ 1,\ 0,\ 0) \\
\oplus & x_5 & (0,\ 1,\ 0,\ 1,\ 1,\ 1,\ 0,\ 1,\ 0) \\
\oplus & x_6 & (0,\ 0,\ 1,\ 0,\ 1,\ 1,\ 0,\ 0,\ 1) \\
\oplus & x_7 & (0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 1,\ 1,\ 0) \\
\oplus & x_8 & (0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 1,\ 1,\ 1) \\
\oplus & x_9 & (0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 1,\ 1) \\
= & & (0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0)
\end{array}
$$

An assignment that makes this equality true is $x = (1,\ 1,\ 1,\ 1,\ 0,\ 0,\ 1,\ 1,\ 1)$

Lights Out

$$x = \big(1, \quad 1, \quad 1, \quad 1, \quad 0, \quad 0, \quad 1, \quad 1, \quad 1\big)$$

1	2	3
4	5	6
7	8	9

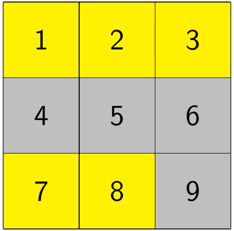
Lights Out

$$x = \big(1,\quad 1,\quad 1,\quad 1,\quad 0,\quad 0,\quad 1,\quad 1,\quad 1\big)$$

1	2	3
4	5	6
7	8	9

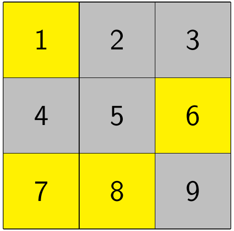
Lights Out

$$x = \big(1,\quad 1,\quad 1,\quad 1,\quad 0,\quad 0,\quad 1,\quad 1,\quad 1\big)$$



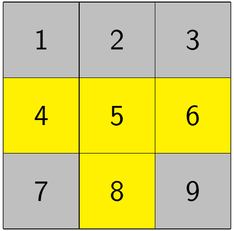
Lights Out

$$x = (1, \quad 1, \quad 1, \quad 1, \quad 0, \quad 0, \quad 1, \quad 1, \quad 1)$$



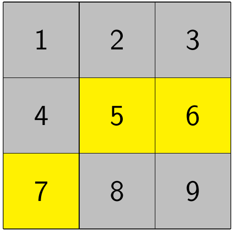
Lights Out

$$x = \begin{pmatrix} 1, & 1, & 1, & 1, & 0, & 0, & 1, & 1, & 1 \end{pmatrix}$$



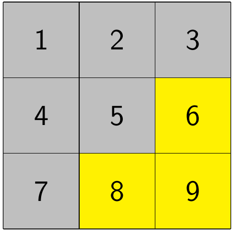
Lights Out

$$x = \big(1, \quad 1, \quad 1, \quad 1, \quad 0, \quad 0, \quad 1, \quad 1, \quad 1\big)$$



Lights Out

$$x = \begin{pmatrix} 1, & 1, & 1, & 1, & 0, & 0, & 1, & 1, & 1 \end{pmatrix}$$



Lights Out

$$x = \big(1,\quad 1,\quad 1,\quad 1,\quad 0,\quad 0,\quad 1,\quad 1,\quad 1\big)$$

1	2	3
4	5	6
7	8	9

Lights Out

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Lights Out

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Lights Out

$$A_3 \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Lights Out

$$A_3 \cdot \bar{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Lights Out

$$A_3 \cdot \bar{x} = \bar{s}$$

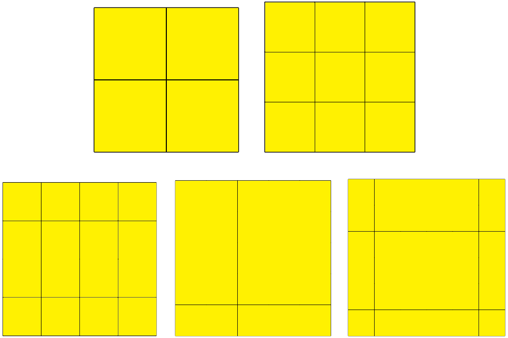
Lights Out

$$A_3 \cdot \bar{x} = \bar{s}$$

- Linear algebra tells us we can find $\bar{x}$ if $\mathrm{Det}(A_3) \neq 0 \bmod 2$.
- As $\mathrm{Det}(A_3) = 1 \bmod 2$, we know that any arrangement of 3x3 Lights Out is solvable.

Lights Out

- $\text{Det}(A_2) = 1 \mod 2$
- $\text{Det}(A_3) = 1 \mod 2$
- $\text{Det}(A_4) = 0 \mod 2$
- $\text{Det}(A_5) = 0 \mod 2$
- $\text{Det}(A_6) = 1 \mod 2$



Lights Out

- $\mathrm{Det}(A_2) = 1 \bmod 2$
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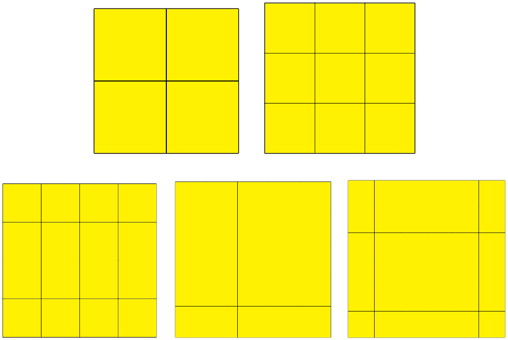
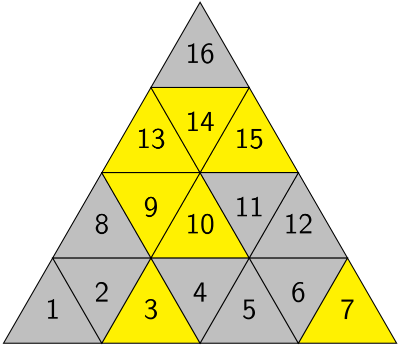
- $\mathrm{Det}(A_5) = 0 \bmod 2$
- $\mathrm{Det}(A_6) = 1 \bmod 2$

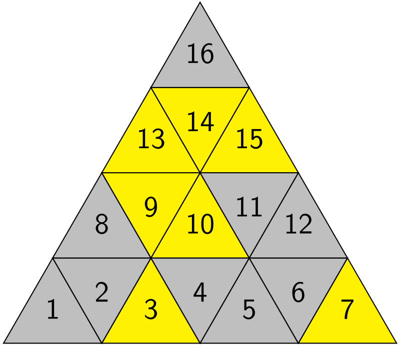


Possible alternatives: more colors, change switching rules

Lights Out

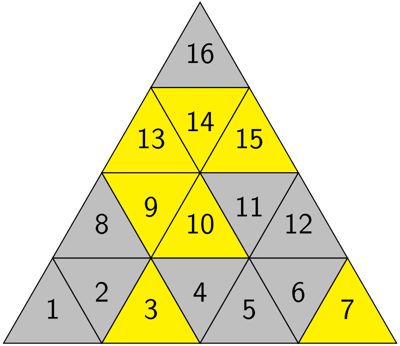


Lights Out



$$
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1
\end{bmatrix}
$$

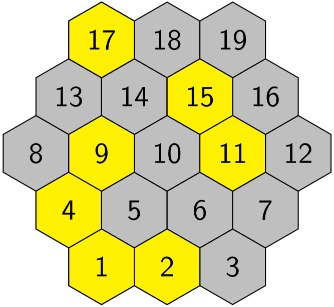
Lights Out



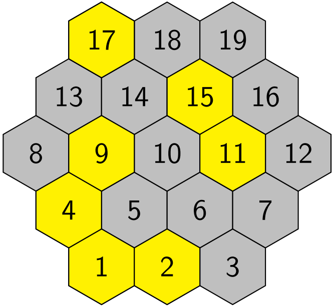
$$\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1
\end{bmatrix}$$

$$\mathrm{Det}(T) = 1 \bmod 2,$$

Lights Out

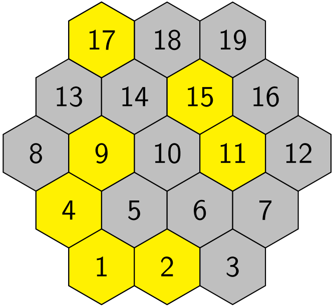


Lights Out



$$
\begin{bmatrix}
1&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&1&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&1&1&0&0&1&1&0&0&0&0&0&0&0&0&0&0\\
1&1&0&1&1&1&0&0&1&1&0&0&0&0&0&0&0&0&0\\
0&1&1&0&1&1&1&0&0&1&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&1&1&0&0&0&1&1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&1&1&0&0&0&1&0&0&0&0&0&0\\
0&0&0&1&1&0&0&1&1&1&0&0&1&1&0&0&0&0&0\\
0&0&0&0&1&1&0&0&1&1&1&0&0&1&1&0&0&0&0\\
0&0&0&0&0&1&1&0&0&1&1&1&0&0&1&1&0&0&0\\
0&0&0&0&0&0&1&0&0&0&1&1&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1&1&0&0&0&1&1&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&1&0&0&1&1&1&0&1&1&0\\
0&0&0&0&0&0&0&0&0&1&1&0&0&1&1&1&0&1&1\\
0&0&0&0&0&0&0&0&0&0&1&1&0&0&1&1&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&1&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&1&1
\end{bmatrix}
$$

Lights Out



$$\begin{bmatrix}
1&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&1&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&1&1&0&0&1&1&0&0&0&0&0&0&0&0&0&0\\
1&1&0&1&1&1&0&0&1&1&0&0&0&0&0&0&0&0&0\\
0&1&1&0&1&1&1&0&0&1&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&1&1&0&0&0&1&1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&1&1&0&0&0&1&0&0&0&0&0&0\\
0&0&0&1&1&0&0&1&1&1&0&0&1&1&0&0&0&0&0\\
0&0&0&0&1&1&0&0&1&1&1&0&0&1&1&0&0&0&0\\
0&0&0&0&0&1&1&0&0&1&1&1&0&0&1&1&0&0&0\\
0&0&0&0&0&0&1&0&0&0&1&1&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1&1&0&0&0&1&1&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&1&0&0&1&1&1&0&1&1&0\\
0&0&0&0&0&0&0&0&0&1&1&0&0&1&1&1&0&1&1\\
0&0&0&0&0&0&0&0&0&0&1&1&0&0&1&1&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&1&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&1&1
\end{bmatrix}$$

$$\mathrm{Det}(H) = 0 \bmod 2,$$

Lights Out - the mCRL2 model

Lights Out - the mCRL2 model

Brace for some mCRL2 code.

Lights Out - the mCRL2 model

```
sort   Button = struct on | off;
       Row = List(Button);
       Game = List(Row);
map    Press:         Game#Nat#Nat -> Game;
       PressMiddle:   Game#Nat -> Game;
       PressBottom:   Game#Nat -> Game;
       rowflip:       Row#Nat -> Row;
       flip:          Row#Nat -> Row;
eqn    Press([],i,j) = [];
       Press(r|>g,i,0) = rowflip(r,i) |> PressBottom(g,i);
       Press(r|>g,i,1) = flip(r,i) |> PressMiddle(g,i);
       j>1 -> Press(r|>g,i,j) = r |> Press(g,i,max(0,j-1));
```

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Lights Out - the mCRL2 model

```
sort  Button = struct on | off;
      Row = List(Button);
      Game = List(Row);
map   Press:         Game#Nat#Nat -> Game;
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      PressBottom:   Game#Nat -> Game;
      rowflip:       Row#Nat -> Row;
      flip:          Row#Nat -> Row;
eqn   Press([],i,j) = [];
      Press(r|>g,i,0) = rowflip(r,i) |> PressBottom(g,i);
      Press(r|>g,i,1) = flip(r,i) |> PressMiddle(g,i);
      j>1 -> Press(r|>g,i,j) = r |> Press(g,i,max(0,j-1));
      PressBottom([], i) = [] ;
      PressBottom(r|>g, i) = flip(r,i) |> g ;
      PressMiddle([], i) = [] ;
      PressMiddle(r|>g, i) = rowflip(r,i) |> PressBottom(g,i);
```

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Lights Out - the mCRL2 model

```
act   win;
      press:Nat#Nat;
proc  LightsOut(game:Game)=
        (all_off(game) -> win.delta) +
        sum i,j:Nat.(i<N && j<M)->
          press(i,j).
          LightsOut(Press(game,i,j));

init  LightsOut(on_game);
```

```
mu X . <win> true || <true> X
```

Lights Out - the mCRL2 model

size	explicit		symbolic	
2x2	0.16s	$<$1000	0.06s	18
3x3	0.22s	$<$1000	1.24s	468
4x4	1.97s	$\pm$4000	41.4s	3526
4x5	12min	$\pm$1048000	$>$108min?	431910?
5x5	-	-	-	-

Thanks for your attention!

Thanks for your attention!

Jan Friso: enjoy the gift