Symmetries in Predicate Formulas

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Formal System Analysis, TU/e

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Introduction

Introduction

$$
\begin{tikzcd}
 & (Red, Red) \arrow[dl, bend right] \arrow[dr, bend left] & \\
(Green, Red) \arrow[ur, bend right] \arrow[dr, bend right] & & (Red, Green) \arrow[ul, bend left] \arrow[dl, bend left] \\
 & (Green, Green) \arrow[ul, bend right] \arrow[ur, bend left] &
\end{tikzcd}
$$

Introduction

$$(Color_1, Color_2) \mapsto (Color_2, Color_1)$$

$$\begin{tikzcd}
\{(Red, Red)\} \arrow[d, bend right] \\
\{(Green, Red), (Red, Green)\} \arrow[u, bend right] \arrow[d, bend right] \\
(Green, Green) \arrow[u, bend right]
\end{tikzcd}$$

Introduction

$$
\begin{tikzcd}
& (R, R, R) \arrow[dl, shift right] \arrow[dl, leftarrow, shift left] \arrow[d, shift right] \arrow[d, leftarrow, shift left] \arrow[dr, shift right] \arrow[dr, leftarrow, shift left] & \\
(G, R, R) \arrow[d, shift right] \arrow[d, leftarrow, shift left] \arrow[dr, shift right] \arrow[dr, leftarrow, shift left] & (R, G, R) \arrow[dl, shift right] \arrow[dl, leftarrow, shift left] \arrow[dr, shift right] \arrow[dr, leftarrow, shift left] & (R, R, G) \arrow[dl, shift right] \arrow[dl, leftarrow, shift left] \arrow[d, shift right] \arrow[d, leftarrow, shift left] \\
(G, G, R) \arrow[dr, shift right] \arrow[dr, leftarrow, shift left] & (G, R, G) \arrow[d, shift right] \arrow[d, leftarrow, shift left] & (R, G, G) \arrow[dl, shift right] \arrow[dl, leftarrow, shift left] \\
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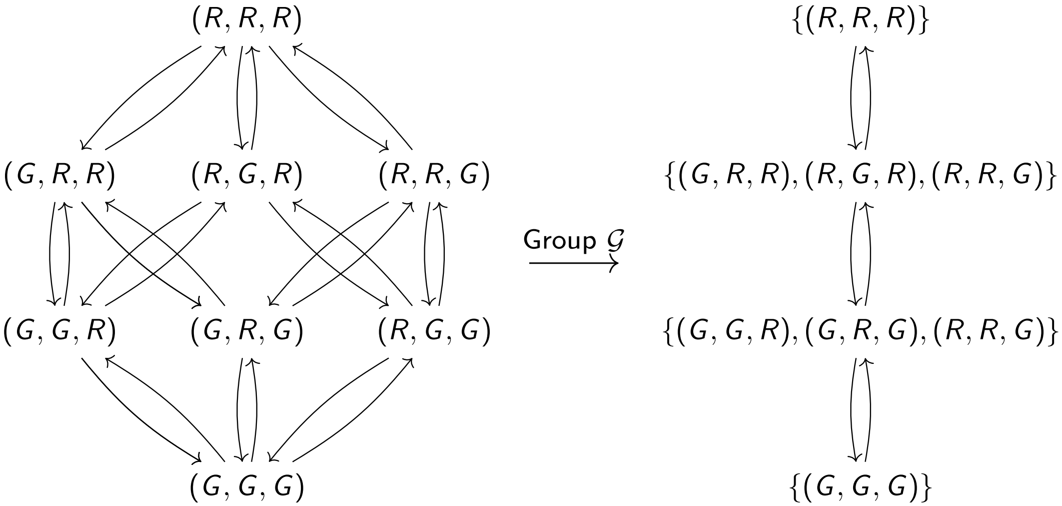
\end{tikzcd}
$$

Introduction



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- What is a (proper) definition of symmetry for predicate formulas?
- How can we extend this to the setting of PBESs?
- How do we benefit from symmetries?

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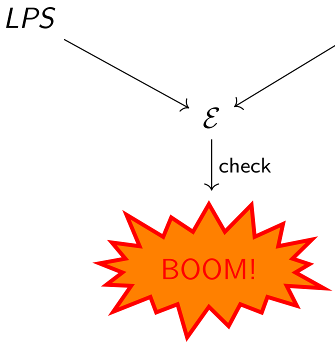

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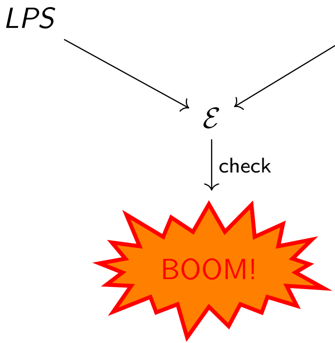

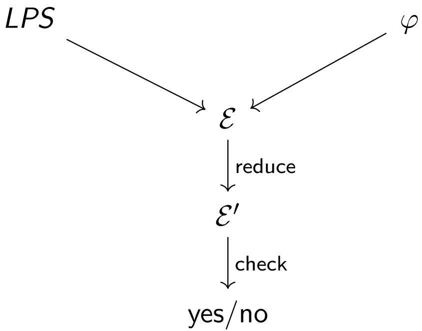

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Preliminaries

Group

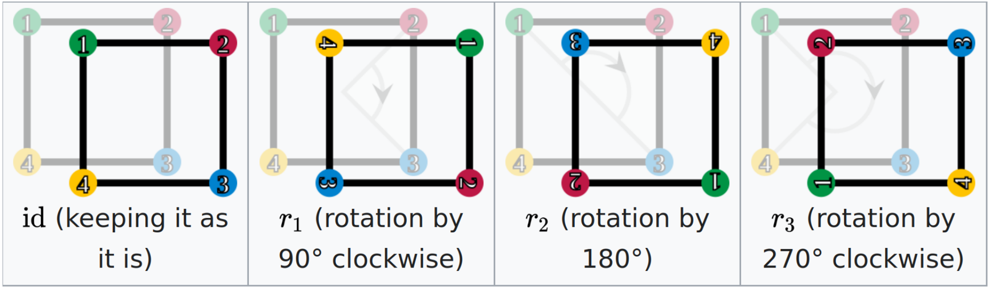
A *group* is a pair $(\mathcal{G}, \circ)$ consisting of a non-empty set $\mathcal{G}$ and a binary operator $\circ \colon \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ such that for any $\alpha, \beta, \gamma \in G$ we have

- $\alpha \circ (\beta \circ \gamma) = (\alpha \circ \beta) \circ \gamma$ (Associativity)
- $\alpha \circ \mathrm{id} = \alpha = \mathrm{id} \circ \alpha$ (Identity element)
- $\alpha \circ \alpha^{-1} = \mathrm{id} = \alpha^{-1} \circ \alpha$ (Inverse element)

The pair $(\mathbb{Z}, +)$ forms a group as for any $a, b, c \in \mathbb{Z}$ we have $a + (b + c) = (a + b) + c$, the identity element is 0 and for any $a \in \mathbb{Z}$ we have inverse $-a$.

Preliminaries

The pair $(D_4, \circ)$ where $D_4 = \{id, r_1, r_2, r_3\}$ and for any $a, b \in D_4$, $a \circ b$ is defined by composition, forms a group.



Preliminaries

Permutation groups

Let X be a set. A *permutation* is a bijection $\pi \colon X \to X$. Let $\mathrm{per}(X)$ be the set of all permutations on X, then the pair $(\mathrm{per}(X), \circ)$ where $\circ$ is defined as function composition, is a group.

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Consider the set $[3] = \{1, 2, 3\}$, then $\mathrm{per}([3]) = \{\mathrm{id}, (12), (13), (23), (123), (132)\}$

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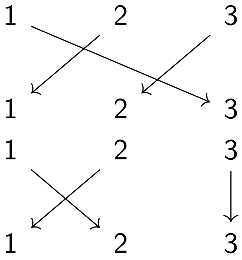
$\pi_1 = (132)$

$\pi_2 = (12)(3)$



$\pi_1 \circ \pi_2 =$

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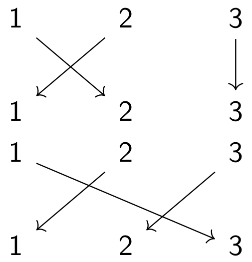

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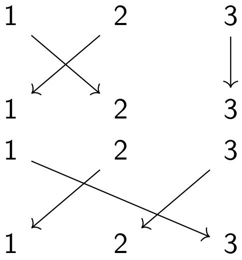
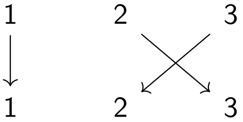


$\pi_1 \circ \pi_2 = (1)(23)$



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Permutations

Predicate Formulas

A predicate formula is generated by the following grammar

$$\varphi \quad := \quad b \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid X(f_1, \ldots, f_n) \mid \forall e : D.\varphi \mid \exists e : D.\varphi$$

The set of all predicate formulas is denotes as Φ. We assume a set of parameters $P = \{d_1, \ldots, d_n\}$ that are never bound.

$$\varphi = (\neg(d_1 = R) \vee X(G, d_2))) \wedge (\neg(d_1 = G) \vee X(R, d_2)))$$

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$$\pi = (12)$$

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Permutations on Predicate Formulas

Let $\varphi \in \Phi$ be a predicate formula and let $\pi \in \mathrm{per}([n])$ be a permutation. We define $\pi(\varphi)$ by induction on the structure of φ.

$$
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\pi(b) & = & b[d_i := d_{\pi(i)}]_{i\in[n]} \\
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$$\pi = (12) \qquad \begin{array}{r} \varphi = ((d_1 = R) \Rightarrow X(G, d_2)) \land ((d_1 = G) \Rightarrow X(R, d_2)) \\ \pi(\varphi) = ((d_2 = R) \Rightarrow X(d_1, G)) \land ((d_2 = G) \Rightarrow X(d_1, R)) \end{array}$$

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Permutations

Lemma

Let $\pi \in \mathrm{per}([n])$ be a permutation for $\varphi \in \Phi$. Let η and δ be some predicate and data environment. Then we have that $[\![\pi(\varphi)]\!]\eta\delta = [\![\varphi]\!]\pi(\eta)\pi(\delta)$.

Permutations also "behave nicely" regarding semantics.
Filling in a value for a permutated predicate formula is equal to filling in the permutated value to the original predicate formula.

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Corollary:
$[\![\varphi]\!]\eta\delta = [\![\pi(\varphi)]\!]\pi^{-1}(\eta)\pi^{-1}(\delta)$

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Symmetries

Symmetry for Predicate Formulas

Let $\pi \in \mathrm{per}([n])$ be a permutation and $\varphi \in \Phi$ be a predicate formula. We say that π is a symmetry for φ if and only if

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$$\pi(\varphi) \not\equiv \varphi$$

So π not a symmetry for φ.

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$$\pi(\psi) \equiv \psi$$

So π is a symmetry for ψ.

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Preserving Solutions

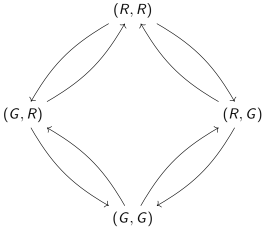
PBES

A Parametrized Boolean Equation System (PBES) is a sequence of equations generated by the following grammar.

$$\mathcal{E} \quad := \quad \emptyset \mid (\sigma X(d_1 : D, \ldots, d_n : D) = \varphi_X)\mathcal{E}$$

Here $\sigma \in \{\nu, \mu\}$ and $\varphi_X \in \Phi$.

$$\begin{aligned}\nu X(d_1 : D, d_2 : D) = {} & (d_1 = G) \Rightarrow X(R, d_2) \wedge \\ & (d_1 = R) \Rightarrow X(G, d_2) \wedge \\ & (d_2 = G) \Rightarrow X(d_1, R) \wedge \\ & (d_2 = R) \Rightarrow X(d_1, G)\end{aligned}$$



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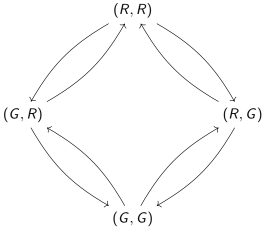

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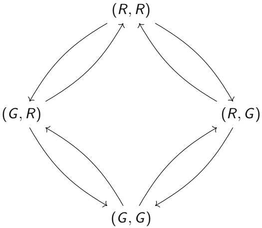

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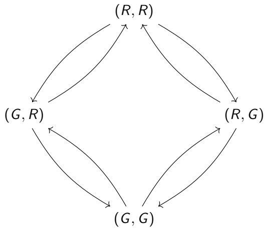

If we start from (R, G), can we reach (G, G)?

Preserving Solutions

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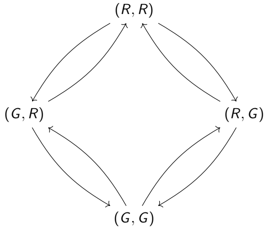

If we start from (R, G), can we reach (G, G)? Yes, so

$$(R, G) \notin [\![\mathcal{E}]\!]\eta\delta(X)$$

Preserving Solutions

Symmetry for PBES

Let $\mathcal{E}$ be a PBES let $\pi \in \mathsf{per}([n])$. Then π is a symmetry for $\mathcal{E}$ if it is a symmetry for every right hand side in $\mathcal{E}$.

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\end{aligned}
\quad \xrightarrow{\pi=(12)} \quad
\begin{aligned}
\pi(\varphi_X) = {} & (d_2 = G) \Rightarrow X(d_1, R) \land \\
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Preserving Solutions

$$\begin{tikzcd}
 & (R,R) \arrow[dl, bend right] \arrow[dr, bend left] & & & & \{(R,R)\} \arrow[d, bend right] \\
(G,R) \arrow[ur, bend right] \arrow[dr, bend right] & & (R,G) \arrow[ul, bend left] \arrow[dl, bend left] & \arrow[r, "\pi=(12)"] & {} & \{(G,R),(R,G)\} \arrow[u, bend right] \arrow[d, bend right] \\
 & (G,G) \arrow[ul, bend right] \arrow[ur, bend left] & & & & (G,G) \arrow[u, bend right]
\end{tikzcd}$$

$$(R,G) \notin [\![\mathcal{E}]\!]\eta\delta(X) \quad \Longleftrightarrow \quad (G,R) \notin [\![\mathcal{E}]\!]\eta\delta(X)$$

Preserving Solutions

Main takeaway: If we know the solution for some value we also know the solution to all its symmetric counterparts.

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Theorem

Let $\mathcal{E}$ be a PBES and $\pi \in \mathrm{per}([n])$ be a symmetry for $\mathcal{E}$. Then for all predicate variables X and values $\bar{v} = (v_1, \ldots, v_n) \in \mathbb{D}^n$ it holds that

$$\bar{v} \in [\![\mathcal{E}]\!]\eta\delta(X) \iff \pi(\bar{v}) \in [\![\mathcal{E}]\!]\eta\delta(X)$$

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Future work

- Up until now we have assumed that we have a symmetry
- The next challenge is in finding symmetries, without looking at the solution beforehand.
- We aim to achieve this with the use of control flow parameters

$$
\begin{tikzcd}
d_1 = R \arrow[d, bend right, "1"'] & d_2 = R \arrow[d, bend right, "2"'] \\
d_1 = G \arrow[u, bend right, "3"'] & d_2 = G \arrow[u, bend right, "4"']
\end{tikzcd}
$$

- Implement the use of symmetries

Thanks!

Multiple Equations

$$
\begin{aligned}
\nu X(d_1 : D, d_2 : D) &= (d_1 = G) \Rightarrow Y(R, d_2) \wedge \\
&\quad\, (d_2 = G) \Rightarrow Y(d_1, R) \\
\nu Y(d_1 : D, d_2 : D) &= (d_1 = R) \Rightarrow X(G, d_2) \wedge \\
&\quad\, (d_2 = R) \Rightarrow X(d_1, G)
\end{aligned}
$$