VIDEO 1 — WEEK 1

considero $a(t)y''+b(t)y'+c(t)y=f(t)$ , $t\in I$ cerco la soluzione → f derivabile 2 volte che risolve l'equazione

es. $y=e^{2t}$ $y''-y'-2y=0 \to 4e^{2t}-2e^{2t}-2e^{2t}=0$ ✓

se $y=t^2$ la candidata ha solo 2 soluzioni

→ $y''=0$ : $y(t)=c_1t+c_2$ funz. lineare quindi sol. ∞

es. nella caduta libera: $y''=g$ integro due volte e ottengo l'eq. del moto

$$y'=gt+c_1 \to y(t)=\frac{gt^2}{2}+c_1t+c_2$$

integ. generale → totalità delle soluzioni in dipendenza da 2 parametri

→ $\begin{cases} \text{eq. diff} \\ y(t_0)=y_0 \\ y'(t_0)=y'_0 \end{cases}$ PROBLEMA DI CAUCHY ha 1 sola soluzione ⟹ 1. det integrale generale 2. impongo le cond. iniziali 3. sostituisco nell'integrale generale

es. $\begin{cases} y''-3y'+2y=t \\ y(0)=1 \\ y'(0)=-1 \end{cases}$ integ. generale: $y(t)=c_1e^t+c_2e^{2t}+\frac{t}{2}+\frac{3}{4}$ dato $c_{1,2}\in\mathbb{R}$

2. $\begin{cases} y(0)=c_1+c_2+\frac{3}{4}=1 \\ y'(0)=c_1+2c_2+\frac{1}{2}=-1 \end{cases} \to \begin{cases} c_1=2 \\ c_2=-\frac{7}{4} \end{cases}$ le uniche che soddisfano

3. $y(t)=2e^t-\frac{7}{4}e^{2t}+\frac{t}{2}+\frac{3}{4}$ è la soluzione al pb.

VIDEO 2

→ $a(t)y''+b(t)y'+c(t)y=f(t)$ , funz. continue cerco strutt. integrale generale

↳ $a\frac{d^2}{dt^2}+b\frac{d}{dt}+c=f(t) \to Ly=f$ caratterizzate da principio di sovrapposizione

→ $c_1y_1 + c_2y_2$ $\boxed{L}$ $c_1Ly_1 + c_2Ly_2$ lineare ⟹ $y_1\to (Ly_1=f_1)\cdot c_1$, $y_2\to(Ly_2=f_2)\cdot c_2$ ⟹ $L(y)=c_1f_1+c_2f_2$

TH. 1 se ho $f_1,f_2$ la sol sarà $f(t)=c_1f_1+c_2f_2$ cl delle due f applicate

→ eq. omogenea: $\boxed{Ly=0}$

l'insieme S delle soluzioni di un'equazione lineare omogenea forma uno spazio vettoriale

→ se $a(t)\neq 0$ la dim del s.v. è 2

TH 2. di struttura: l'int. generale $a(t)y''+b(t)y'+c(t)y=0$ $a,b,c$ continue in I e $a(t)\neq0$

↳ è dato da tutte le cl $y(t)=c_1y_1(t)+c_2y_2(t)$ $c_{1,2}\in\mathbb{R}$; $y_1,y_2$ sol. l.i.

es. $y''+4y=0$ $y(t)=c_1\cos(2t)+c_2\operatorname{sen}(2t)$

verifico $y_1(t)=\cos(2t)$ e $y_2(t)=\operatorname{sen}(2t)$

$y'_1(t)=-2\operatorname{sen}(2t)$ $y'_2(t)=2\cos(2t)$

$y''_1(t)=-4\cos(2t)$ $y''_2(t)=-4\operatorname{sen}(2t) \to -4\operatorname{sen}2t+4\operatorname{sen}(2t)=0$ $\forall t\in\mathbb{R}$ ✓

$-4\cos(2t)+4\cos(2t)=0$ $\forall t\in\mathbb{R}$ ✓

$y_1,y_2$ l.i.? $\frac{y_2(t)}{y_1(t)}=\frac{\operatorname{sen}2t}{\cos2t}=\operatorname{tg}2t$ → non è costante ✓

→ eq. completa: $a(t)y''+b(t)y'+c(t)y=f(t)$ $\begin{cases} y_0 \text{ sol. omog.} \\ y_p \text{ sol comp.} \end{cases}$ $\overset{\text{th.2}}{\Longrightarrow}$ $y(t)=y_0(t)+y_p(t)$

→ TH 3. di struttura per eq. complete:

l'integ generale: $a(t)y''+b(t)y'+c(t)y=f(t)$ [hp sopra] è dato da tutte le funz.

$y(t)=c_1y_1(t)+c_2y_2(t)+y_p(t)$ dove $y_1,y_2$ sono sol. l.i. dell'omogenea

## VIDEO 3

coeff. costanti: $ay''+by'+cy=0$

**es.** $y''+4y'+3y=0$ $\quad y(t)=e^{\lambda t}$ risolve?

↳ $\lambda^2 e^{\lambda t}+4\lambda e^{\lambda t}+3e^{\lambda t}=0 \to \underbrace{e^{\lambda t}}_{\neq 0}(\lambda^2+4\lambda+3)=0 \to$ trovo $\lambda_1,\lambda_2$ soluzioni

↳ da sopra: $e^{\lambda t}(\underbrace{a\lambda^2+b\lambda+c}_{\text{POLINOMIO CARATTERISTICO}})=0$ $\quad$ e $y(t)=e^{\lambda t}$

POLINOMIO CARATTERISTICO $\to P(\lambda)=0$

$\to \Delta>0$: trovo $y_1(t), y_2(t)$ l.i. $\left(y_1(t)/y_2(t)\neq k\right)$

**es.** $2y''-y'-y=0$ $\quad P(\lambda)=2\lambda^2-\lambda-1=0 \to \Delta=b^2-4ac=1+8=9>0$

$\lambda_1=1$ $\quad \lambda_2=-\frac{1}{2}$ reali e distinte

$y_1(t)=e^t$

$y_2(t)=e^{-\frac{t}{2}}$

$\frac{y_1(t)}{y_2(t)}=e^{t+\frac{t}{2}}=e^{\frac{3t}{2}}\neq k$

## VIDEO 4

$\to ay''+by'+cy=0$ e $\Delta<0$

**es.** $y''+2y'+10y=0$ $\quad y(t)=e^{\lambda t}$

↳ $\lambda^2+2\lambda+10=0 \to \Delta=-36$

$\lambda_1=-1+3i \to y_1(t)=e^{(-1+3i)t}$

$\lambda_2=-1-3i \to y_2(t)=e^{(-1-3i)t}$

da Eulero: $e^{\alpha+i\beta}=e^{\alpha}(\cos\beta+i\,\mathrm{sen}\,\beta)$

$y_1(t)=e^{-t}(\cos 3t+i\,\mathrm{sen}\,3t)$

$y_2(t)=e^{-t}(\cos 3t-i\,\mathrm{sen}\,3t)$

$\to \frac{y_1(t)}{y_2(t)}=\cot(3t)\neq k \to y(t)$ cl $y_1$ e $y_2$

$\to$ sol complesse coniugate: $\alpha=\frac{-b}{2a}$ $\quad \beta=\frac{\sqrt{4ac-b^2}}{2a}$ $\Rightarrow$ $y_1(t)=e^{\alpha t}\cos(\beta t)$

$y_2(t)=e^{\alpha t}\,\mathrm{sen}\,(\beta t)$

$\to y(t)=e^{\alpha t}(C_1\cos(\beta t)+C_2\,\mathrm{sen}\,\beta t)$

**es.** $y''+y'+y=0 \to \lambda^2+\lambda+1=0 \to \Delta=1-4=-3<0$

$\lambda_1=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ $\quad \lambda_2=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

$u_1(t)=e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t$

$u_2(t)=e^{-\frac{1}{2}t}\,\mathrm{sen}\,\frac{\sqrt{3}}{2}t$

$y(t)=e^{-\frac{t}{2}}\left(C_1\cos\frac{\sqrt{3}}{2}t+C_2\,\mathrm{sen}\,\frac{\sqrt{3}}{2}t\right)$ $\quad C_{1,2}\in\mathbb{R}$

## VIDEO 5

$\to ay''+by'+cy=0$ e $\Delta=0$

**es.** $y''-6y'+9y=0 \to \lambda^2-6\lambda+9=0 \to \Delta=0$, $\lambda_1=\lambda_2=3 \to y_1(t)=Ce^{3t}$ $\quad C\in\mathbb{R}$

$\to$ come trovo $y_2(t)$? $\quad y_2(t)=C(t)e^{3t}$ $\quad$ l.i. da $y_1$

$y_2'(t)=e^{3t}(C'(t)-3C(t))$

$y_2''(t)=e^{3t}(C''(t)+6C'(t)+9C(t))$

$\to \cancel{e^{3t}}[C''(t)+6C'(t)+9C(t)]-6e^{3t}[C'(t)+3C(t)]+9C(t)e^{3t}=0 \to C''(t)=0 \quad \forall t\in\mathbb{R}$

$C'(t)=C_1$

$C(t)=C_1t+C_2$

$\to y_2(t)=te^{3t} \to [y_1(t)/y_2(t)]=\frac{1}{t}\neq k$

$\to y(t)=C_1y_1(t)+C_2y_2(t)=e^{3t}(C_1t+C_2)$

$\to$ 2 radici reali coincidenti: $\lambda_1=\lambda_2=-\frac{b}{2a}$

**es.** $y''+4y'+4y=0$ $\quad P(\lambda)=\lambda^2+4\lambda+4=0 \to \Delta=0 \to (\lambda+2)^2=0 \to \lambda_1=\lambda_2=-2$

$\begin{cases} y_1=e^{-2t} \\ y_2=te^{-2t}\end{cases} \to y(t)=e^{-2t}(C_1+C_2t)$ in $\mathbb{R}$

$\to ay''+by'+cy=0$ con $y_1(t)=e^{\lambda_1 t}$ e $y_2(t)=e^{\lambda_2 t} \to C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}$

$C_1=-\frac{1}{\lambda_2-\lambda_1}$

$C_2=\frac{1}{}$

scelti

$\rightarrow \phi(t) = -\frac{1}{\lambda_2-\lambda_1} e^{\lambda_1 t} + \frac{1}{\lambda_1-\lambda_2} e^{\lambda_2 t}$ con $\lambda_2 \to \lambda_1+\varepsilon$ : $\phi(t) = \frac{e^{\lambda_2 t} - e^{\lambda_1 t}}{\varepsilon} = \frac{e^{\varepsilon t}-1}{\varepsilon t} e^{\lambda_1 t}\, t \searrow e^{\lambda_1 t} t$ ; $\frac{e^{\varepsilon t}-1}{\varepsilon t} \to 1$

## VIDEO 6

→ problema di Cauchy eq. omogenea

**es.** $\begin{cases} y''+2y'+5y=0 \\ y(0)=1 \\ y'(0)=-1 \end{cases} \longrightarrow \lambda^2+2\lambda+5=0 \to \lambda_1=-1+2i, \quad \lambda_2=-1-2i$ $\quad u_1(t)=e^{-t}\cos(2t)$, $\quad u_2(t)=e^{-t}\operatorname{sen}(2t)$

$\rightarrow y(t) = e^{-t}(C_1\cos 2t + C_2 \operatorname{sen} 2t)$

$y(0) = e^0(C_1\cos 0 + C_2 \operatorname{sen} 0) = 1 \quad C_1 = 1$

$y'(0) = -1(C_1+0) + 1(2 \cdot C_2) = -1 \to C_2 = 0$

$\hookrightarrow y(t) = e^{-t}\cos 2t$

## VIDEO 7

$f: \mathbb{R} \to \mathbb{C} \longrightarrow f(t) = f_1(t) + i f_2(t) \qquad f_1, f_2 : \mathbb{R} \to \mathbb{R}$

$\rightarrow \lambda = \alpha + i\beta$ con $\alpha, \beta \in \mathbb{R} \Rightarrow f(t) = e^{\lambda t} = e^{(\alpha+i\beta)t} = \underbrace{e^{\alpha t}\cos\beta t}_{f_1} + i\underbrace{e^{\alpha t}\operatorname{sen}\beta t}_{f_2}$

def: $f'(t) = f_1'(t) + i f_2'(t)$ se $f_1, f_2$ derivabili

→ dim: $f'(t) = f_1' + i f_2' \longrightarrow f'(t) = e^{\alpha t}(\alpha\cos\beta t - \beta\operatorname{sen}\beta t) + i e^{\alpha t}(\alpha\operatorname{sen}\beta t + \beta\cos\beta t)$

$\lambda e^{\lambda t} = (\alpha+i\beta)e^{(\alpha+i\beta)t} = (\alpha+i\beta)e^{\alpha t}(\cos\beta t + i\operatorname{sen}\beta t) = e^{\alpha t}(\alpha\cos\beta t - \beta\operatorname{sen}\beta t) + i e^{\alpha t}(\alpha\operatorname{sen}\beta t + \beta\cos\beta t)$

$\hookrightarrow (e^{\lambda t})'' = \lambda^2 e^{\lambda t} \longrightarrow$ risultati : $e^{(\alpha+i\beta)t}, e^{(\alpha-i\beta)t}$

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WEEK 2

## VIDEO 1

$ay''+by'+cy = f(t)$, $a \neq 0 \longrightarrow y = C_1 y_1(t) + C_2 y_2(t) + y_p(t)$ eq. completa

**METODO DI SOMIGLIANZA:** ricordo che posso scrivere : $y \to Ly = ay''+by'+cy$

→ cerco $y$ t.c. $Ly = f(t)$

**es.** $y(t) = Ce^{\alpha t} \longrightarrow Ly = K e^{\alpha t}$ ($K$ ↳ costante)

$y(t) = t^2+t \longrightarrow Ly = 2a + t(2b+c) + ct^2$, $c \neq 0$

→ si basa sul fatto che se term. noto è esp, polin, trig cerco la $y$ simile

## VIDEO 2

come cerco $y_p$ se $f(t) = Ae^{\alpha t}$ (term. noto)

**es.** $y''+2y'-3y = e^{2t} \qquad y_p(t) = Ce^{2t} \to \bar{y}_p' = 2Ce^{2t}, \ \bar{y}_p'' = 4Ce^{2t}$

sost ↳ $4Ce^{2t} + 4Ce^{2t} - 3Ce^{2t} = e^{2t} \longrightarrow e^{2t} 5c = e^{2t} \to c = \frac{1}{5} \longrightarrow \bar{y}_p(t) = \frac{1}{5}e^{2t}$

→ per trovare $\bar{y}_p$ cerco simile a $f(t)$ MET. DI SOMIGLIANZA

**es.** $y''+2y'-3y = e^{-3t} \to 0 \cdot C = 1$ non ha soluzioni

→ cerco $y_p(t) = Cte^{-3t} \underset{\text{sost.}}{\longrightarrow} C = -\frac{1}{4} \to y_p(t) = -\frac{1}{4}te^{-3t}$

→ met. di somiglianza non funziona sse la forzante è sol. dell' omogenea associata

- $\alpha$ non è radice di $P(\lambda) = 0 \to \bar{y}_p(t) = Ce^{\alpha t}$
- $\alpha$ è radice di $P(\lambda) = 0 \to \bar{y}_p(t) = Cte^{\alpha t}$

## VIDEO 3

cercare $\bar{y}_p(t)$ se $f(t)$ è un polinomio: $\underset{gr.\,m}{y} \longrightarrow \underset{gr.\,m}{Ly}$

**es.** $y''+2y'-3y=t^2-2$ $\qquad \bar{y}_p(t)=At^2+Bt+C$, $\bar{y}_p'=2At+B$, $\bar{y}_p''=2A$

sost $\hookrightarrow 2A+2(2At+B)-3(At^2+Bt+C)=t^2-2 \rightarrow -3At^2+t(4A-3B)+2A+2B-3C=t^2-2$ $\rightarrow A=-\frac{1}{3}$, $B=-\frac{4}{9}$, $C=\frac{4}{27}$

$\bar{y}_p(t)=-\frac{1}{3}t^2-\frac{4}{9}t+\frac{4}{27}$

$\rightarrow$ in generale: $y_p(t)=\beta_m t^m+\ldots+\beta t+\beta_0$ $\qquad m$: grado $f$

$\left\{\begin{array}{l}\text{sis. lineare di} \\ m+1 \text{ eq e} \\ m+1 \text{ inc. in } \beta_i\end{array}\right.$

$\rightarrow \exists$ 1 caso in cui non funziona!

**es.** $y''+2y'=3t \rightarrow y_p(t)=At+B$, $y_p'(t)=A$, $y_p''(t)=0$

$\hookrightarrow 2A=3t \rightarrow 3t-2A=0 \rightarrow \nexists$ $A$ non dip. da $t$

se uso $\bar{y}_p(t)=At^2+Bt+C \Rightarrow A=\frac{3}{4}$, $B=-\frac{3}{4}$ $\quad \forall C \rightarrow \bar{y}_p(t)=\frac{3}{4}t^2-\frac{3}{4}t$

$\rightarrow a,c\neq 0$ ho $\bar{y}_p(t)=p_m(t)$

$a,b\neq 0$ ho $\bar{y}_p(t)=t\,p_m(t)$

## VIDEO 4

$\rightarrow$ cerco $\bar{y}_p(t)$ se $f(t)$ trig.

**es.** $y''+2y'-3y=\cos 2t-3\,\text{sen}\,2t$ $\qquad y_p(t)=C_1\cos 2t+C_2\,\text{sen}\,2t$, $y_p'(t)=-2C_1\,\text{sen}\,2t+2C_2\cos 2t$

$y_p''(t)=-4C_1\cos 2t-4C_2\,\text{sen}\,2t$

sost $\downarrow$ ..... $\rightarrow (-7C_1+4C_2)\cos 2t+(-7C_2-4C_1)\,\text{sen}\,2t=\bar{y}_p(t)$ $\qquad C_1=\frac{1}{13}$ $\quad C_2=\frac{5}{13}$

$\rightarrow ay''+by'+cy=A\cos(\nu t)+B\,\text{sen}(\nu t)$ e $\bar{y}_p(t)=C_1\cos(\nu t)+C_2\,\text{sen}(\nu t)$

**es.** $y''+9y=2\,\text{sen}\,3t$ $\qquad y_p(t)=C_1\cos(3t)+C_2\,\text{sen}(3t)$

$L(2\,\text{sen}\,3t)=0 \rightarrow$ non ho soluzioni $\rightarrow \bar{y}_p=\bar{y}_0$ **risonanza**

$\hookrightarrow y_p(t)=t(C_1\cos(3t)+C_2\,\text{sen}(3t)) \rightarrow C_1=-\frac{1}{3}$, $C_2=0$

$\rightarrow$ il metodo di somiglianza non funziona se $f(t)$ risolve eq. omogenea

$ay''+cy=0$, $a\cdot c>0$, $b=0$ $\qquad$ pongo $\omega^2=\frac{c}{a}$ $\rightarrow y''+\nu y=\alpha\cos(\nu t)+\beta\,\text{sen}(\nu t)$, $\nu=\omega$

## VIDEO 5

$\rightarrow$ ricerca $y_p(t)$ se $f(t)$ è particolare

- se $f(t)=f_1(t)+f_2(t) \longrightarrow y_p(t)=y_{1,p}(t)+y_{2,p}(t)$
- $y(t)=C_1y_1(t)+C_2(t)y_2(t)$ $\qquad y_1, y_2$ l.i. soluzioni dell'eq. omogenea

cerco poi $Ly=f$ facendo variare $C_1, C_2 \rightarrow$ cerco $C_1(t)$ e $C_2(t)$

**es.** $y''+y=\frac{1}{\text{sen}t}$ $\qquad I=(0,\pi)$

$y''+y=0 \rightarrow \lambda^2+1=0$ $\quad \lambda=\pm i$ $\qquad$ considero $y_1(t)=\cos t$ e $y_2(t)=\text{sen}(t)$

$y_p(t)=C_1(t)\cos(t)+C_2(t)\,\text{sen}(t)$ $\qquad y_p'=\underbrace{C_1'(t)\cos t+C_2'(t)\,\text{sen}t}_{=0 \text{ per semplicità} \;\; \text{I cond.}}-C_1(t)\,\text{sen}t+C_2(t)\cos t$

$\hookrightarrow y_p''(t)=-C_1'(t)\,\text{sen}t-C_1(t)\,\text{sen}t+C_2'(t)\cos t-C_2(t)\,\text{sen}(t)$

$\rightarrow$ sost in eq: $C_2'(t)\cos t-C_1'\,\text{sen}t=\frac{1}{\text{sen}t} \rightarrow$ II cond.

$\Rightarrow \left\{\begin{array}{l}\text{I cond.} \\ \text{II cond.}\end{array}\right.$ $\qquad$ trovo $C_1'(t)$, $C_2'(t)$

→ integro per trovare $c_1(t)$ e $c_2(t)$

**METODO VAR COSTANTI:** cerco sol. omogenea $y_1$ e $y_2$, determino $c_1(t)$ e $c_2(t)$, ris. sistema $c_1', c_2'$
integro per trovare $c_1(t), c_2(t)$

→ $W(t) = a(y_2' y_1 - y_1' y_2)$ sempre diversa da 0, la uso per det. integrale

$$y_p(t) = -y_1(t)\int_{t_0}^{t}\frac{f(s)y_2(s)}{W(s)}ds + y_2(t)\int_{t_0}^{t}\frac{f(s)y_1(s)}{W(s)}ds \quad \text{per } t_0 \in I$$

$$= \boxed{\int_{t_0}^{t}\frac{y_1(s)y_2(t)-y_1(t)y_2(s)}{W(s)}f(s)ds = \int_{t_0}^{t}G(t,s)f(s)ds}$$ funz. di Green

→ **es.** trasformata di Laplace

## VIDEO 6

→ **forzante esp-trig:** $f(t) = e^{\alpha t}(A\cos(\nu t) + B\,\text{sen}(\nu t))$ $\quad A, B, \alpha, \nu$ assegnati

**es.** $y'' - 2y' - 3y = 4e^{t}\cos(2t)$ idealizzo → $y(t) \to \boxed{\frac{d^2}{dt^2} - 2\frac{d}{dt} - 3} \to y''(t) - 2y'(t) - 3y(t)$

se $y(t) = c_1 e^{-t}\cos 2t$ ($o = c_2 e^{-t}\text{sen} 2t$) $\longmapsto e^{-t}(c_1\cos 2t + c_2\,\text{sen} 2t) = 4e^{-t}\cos(2t)$ .... $c_1 = -\frac{1}{5}$, $c_2 = -\frac{2}{5}$

→ generalizzo: $\bar{y}_p(t) = e^{\alpha t}(c_1\cos(\nu t) + c_2\,\text{sen}(\nu t)) \to \bar{y}_p, \bar{y}_p', \bar{y}_p''$ → impongo condizioni → $c_1, c_2$

↳ se $P(\lambda) = 0$ ha sol. complex $(\alpha \pm i\nu)$ → $y_p(t) = te^{\alpha t}(c_1\cos(\nu t) + c_2\,\text{sen}(\nu t))$
(2° membro eq. completa è soluzione dell'omog)

## VIDEO 7

→ **esp. complesso metodo di som:** $ay'' + by' + cy = Ae^{\alpha t}\cos\beta t$ (o $\text{sen}\beta t$)

**• oss:** $e^{\alpha t}\cos\beta t = Re(e^{(\alpha+i\beta)t})$
$e^{\alpha t}\text{sen}\beta t = Im(e^{(\alpha+i\beta)t})$
$\Rightarrow f \to L \to Lf$ se $f = f_1 + if_2 \to a(f_1'' + if_2'') + \ldots = Lf_1 + iLf_2$

**es.** $y'' - 2y' - 3y = \underbrace{4e^{-t}\cos(2t)}_{Re(4e^{(-1+2i)t})}$ considero $w'' - 2w' - 3w = 4e^{(-1+2i)t}$ cerco $w(t) = ce^{(-1+2i)t}$

↳ $w'(t) = c(-1+2i)e^{(-1+2i)t}$, $w''(t) = c(-1+2i)^2 e^{(-1+2i)t}$ $\xrightarrow[\text{sost. in eq.}]{}$ $c(-1+2i)^2e^{(\alpha-1)t} - 2c(-1+2i)e^{(\alpha-1)t} - 3ce^{(\alpha-1)t} = 4e^{(-1\alpha)t}$

↳ $c(-1+2i)^2 - 2c(-1+2i) - 3c = 4 \to c = \frac{-1+2i}{5}$ $\quad\Rightarrow w(t) = \frac{-1+2i}{5}e^{-t}(\cos 2t + i\,\text{sen} 2t)$

↳ $y(t) = Re(w(t)) = \frac{e^{-t}}{5}(-\cos 2t - 2\,\text{sen} 2t)$

→ se fosse stato sen avrei tenuto immag.

→ in genere: $ay'' + by' + cy = Ae^{\alpha t}\text{sen}(\omega t) \to aw'' + bw' + cw = Ae^{(\alpha+i\beta)t}$ e $w(t) = ce^{(\alpha-1)t}$

→ se $\lambda_{1,2}$ tc $P(\lambda) = 0 \Rightarrow w(t) = cte^{(\alpha-1)t}$

## VIDEO 8

→ **forzante somma:** $f = f_1 + f_2$

**es.** $y'' + y = \underbrace{7e^{t}}_{f_1} - \underbrace{5\,\text{sen} 2t}_{f_2}$ → per $f_1$: $\bar{y}_p(t) = Ae^{t}$ sost: $y'' = Ae^{t}$, $y' = Ae^{t} \to Ae^{t} + Ae^{t} = 7e^{t} \to A = \frac{7}{2}$
$\bar{y}_{1,p}(t) = \frac{7}{2}e^{t}$

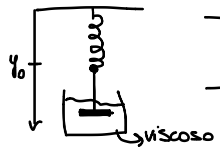
→ $f_2$: $\bar{y}_p(t) = A\cos\beta t + B\,\text{sen}\beta t$, $\bar{y}'(t) = -A\beta\,\text{sen}\beta t + B\beta\cos\beta t$ e $y'' = -A\beta^2\cos\beta t - B\beta^2\,\text{sen}\beta t$

trovo $B = \frac{5}{4}$ e $A = 0$ ↘ $\bar{y}_p(t) = -\frac{5}{4}\,\text{sen} 2t$

unisco risultati → $\bar{y}_p(t) = \frac{7}{2}e^{t} - \frac{5}{4}\,\text{sen} 2t$

WEEK 3

## VIDEO 1

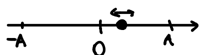



$\to y'' = -\frac{k}{m}y - \frac{h}{m}y' \to y'' = -\omega^2 y - 2\delta y' \to y'' + 2\delta y' + \omega^2 y = 0$ **EQ. DELLE OSCILLAZIONI SMORZATE**

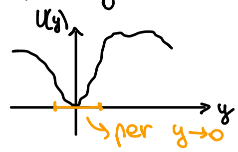
$\to$ se $\delta = 0$: $y'' + \omega^2 y = 0$ **EQ. DELLE OSCILLAZIONI LIBERE**

$\to$ considero pto mat. che si muove lungo un asse avanti e indietro:

$my'' = -U'(y)$ (en. potenziale, sistema conservativo)




$\to y = 0$ ho pto di minimo $\to U(y) \sim \frac{1}{2}U''(0)y^2$



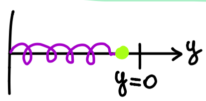

$\to U'(y) \sim U''(y)\, y$

$\to$ pongo $k = U''(0) \to y'' = -\frac{k}{m}y \to \omega^2 = k/m$ **oscillatore armonico**

per $y \to 0$ approssimo a parabola

## VIDEO 2

$\to$ **oscillazioni libere non smorzate**:




$my'' + ky = 0 \to y'' + \omega^2 y = 0 \to P(\lambda) = 0 \to \lambda^2 + \omega^2 = 0$ ↗ $\lambda_1 = i\omega$ ↘ $\lambda_2 = -i\omega$

$\hookrightarrow y(t) = C_1\cos(\omega t) + C_2\,\text{sen}(\omega t)$ $\qquad A = \sqrt{C_1^2 + C_2^2}$

$y(t)\cdot\frac{A}{A} = [C_1\cos(\omega t) + C_2\,\text{sen}(\omega t)]\frac{\sqrt{C_1^2+C_2^2}}{\sqrt{C_1^2+C_2^2}} \to y(t) = A\left[\frac{C_1}{\sqrt{C_1^2+C_2^2}}\cos(\omega t) + \frac{C_2}{\sqrt{C_1^2+C_2^2}}\,\text{sen}(\omega t)\right]$

$\frac{C_1^2}{C_1^2+C_2^2} + \frac{C_2^2}{C_1^2+C_2^2} = 1 \sim \cos(\varphi)^2 + \text{sen}^2(\varphi)$

$\hookrightarrow y(t) = A(\cos(\varphi)\cos(\omega t) + \text{sen}(\varphi)\,\text{sen}(\omega t))$

$\hookrightarrow y(t) = A\,\text{sen}(\omega t + \varphi)$ fase

ampiezza

$T = \frac{2\pi}{\omega} \qquad f = \frac{\omega}{2\pi}$

## VIDEO 3

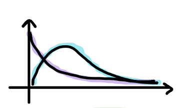
$\to$ **oscillazioni smorzate**: $y'' + 2\delta y' + \omega^2 y = 0$, $\delta > 0 \to P(\lambda) = 0 \to \lambda^2 + 2\delta\lambda + \omega^2 = 0$

$\Delta = \delta^2 - \omega^2$
- $\delta > \omega$ sovracarico
- $\delta = \omega$ critico
- $\delta < \omega$ subcritico

$\to$ **CASO $\delta > \omega$**: **smorz. sovracritico** [grande in prop. a k molla]

$\lambda_1 = -\delta + \sqrt{\delta^2 - \omega^2} \qquad \lambda_2 = -\delta - \sqrt{\delta^2 - \omega^2} \longrightarrow y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$

$\lambda_1, \lambda_2 < 0 \to$ per $t \to +\infty$ $f(t) \to 0 \Rightarrow$ NO OSCILLAZIONI

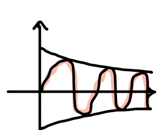


$\leftarrow$ esempi

$\to$ **CASO $\delta < \omega$**: $\lambda_1 = -\delta + i\sqrt{\omega^2 - \delta^2}$, $\lambda_2 = -\delta - i\sqrt{\omega^2 - \delta^2}$ $\to y(t) = e^{-\delta t}(C_1\cos(\sqrt{\omega^2-\delta^2}\,t) + C_2\,\text{sen}(\sqrt{\omega^2-\delta^2}\,t))$

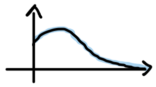
$\hookrightarrow y(t) = Ae^{-\delta t}\,\text{sen}(\sqrt{\omega^2-\delta^2}\,t + \varphi)$

$\to \lim_{t\to\infty} e^{-\delta t} = 0 \qquad f = \frac{\sqrt{\omega^2-\delta^2}}{2\pi} < \frac{\omega}{2\pi} \Rightarrow$ OSCILLAZIONI INFINITE E INF. PICCOLE



$\leftarrow$ esempio

→ CASO $\delta=\omega$: $\lambda_1=\lambda_2=-\delta \rightarrow y(t)=e^{-\delta t}(C_1+tC_2)$ eq. rapidamente



$C_1=1,\ C_2=3$
a $\infty$ non oscilla

## VIDEO 4

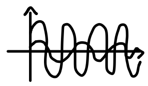
→ **oscillazioni forzate e risonanza**: $y''+\omega^2 y=\underbrace{\alpha\cos(\nu t)}_{\text{FORZANTE}}$ senza attrito

$y_o(t)=C_1\,\text{sen}(\omega t)+C_2\cos(\omega t)$

↳ $y_p$? met di somiglianza ↗ $\omega\neq\nu$ ↘ $\omega=\nu$

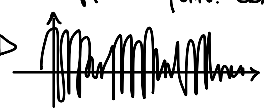
↳ $\omega\neq\nu$: cerco $y_p(t)=b\cos(\nu t)+c\,\text{sen}(\nu t)$ → $b=\frac{\alpha}{\omega^2-\nu^2}$ calcolando le derivate seconde
↘ $c=0$

→ $y(t)=y_o(t)+y_p(t)$
↓
non dip. dalle cond. iniz.



→ sovrapp. 2 funz. con periodi diversi

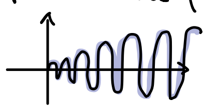
⇒



↳ $\omega=\nu$: $y''+\omega^2 y=\alpha\cos(\omega t)$ non posso usare met di somiglianza

$y_p(t)=bt\cos(\omega t)+ct\,\text{sen}(\omega t)$ → $y_p(t)=ct\,\text{sen}(\omega t)$ derivo e sost. → $y_p(t)=\frac{\alpha}{2\omega}t\,\text{sen}(\omega t)$
↓
0 perché darebbe f dispari

→ $y(t)=y_o(t)+y_p(t)$



→ risonanza

## VIDEO 5

→ **oscillazioni smorzate e forzate**:

$y''+2\delta y'+\omega^2 y=\alpha\cos(\nu t)$ $\delta>0$ e $y_p(t)=b\cos(\nu t)+c\,\text{sen}(\nu t)$, met. di somiglianza
con attrito → non risolve mai $y_o(t)$

→ $\delta=\omega$ risolve l'eq. dell'omogenea

→ Calcolo $y'', y'$ di $y_p(t)$ → impongo l'identità e trovo $b$ e $c$

→ trovo $b^2+c^2=1$ quindi $b^2=\cos\theta$ e $c^2=\text{sen}\,\theta$ → $y_p(t)=\frac{\alpha}{\sqrt{(\omega^2-\nu^2)^2+4\delta^2\nu^2}}\cos(\nu t+\theta)$ INTEG. GEN.

$y(t)=y_o(t)+y_p(t)$

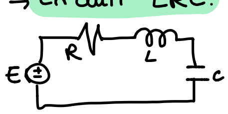
↳ $K(\nu)=\frac{1}{\sqrt{(\omega^2-\nu^2)^2+4\delta^2\nu^2}}$ con $\omega,\delta$ costanti

$K_{max}\rightarrow D(\nu)_{min}$ pongo $x=\nu^2$: $g(x)=(\omega^2+x)^2+4\delta^2\omega^2=\omega^4+x^4+2\omega^2x+4\delta^2\omega^2$ per $x>0$

$x_o=\omega^2-2\delta^2 \rightarrow \nu_{max}=\sqrt{\omega^2-2\delta^2} \rightarrow A_{max}=\frac{\alpha}{2\delta\sqrt{\omega^2-\delta^2}}$
↳ PULS. DI RISONANZA

## VIDEO 5

→ **circuiti LRC**: $V(t)=V_o\,\text{sen}(\phi t) \xrightarrow[\text{eq. diff}]{} Li'(t)+Ri(t)+\frac{q(t)}{C}=V_o\,\text{sen}(\phi t)=V(t)$





→ so che $q'(t)=i(t)$ → $Li''(t)+Ri'(t)+\frac{i(t)}{C}=V_o\phi\cos(\phi t)$ eq. diff. II ord. con resistenza ($\delta\neq0$)

↳ $i''(t)+\underbrace{\frac{R}{L}}_{2\delta}i'(t)+\underbrace{\frac{1}{LC}}_{\omega^2}i(t)=\underbrace{V_o\phi}_{\frac{V_o\phi}{L}}\cos(\phi t)\cdot\frac{1}{L}$