

→ prove semplici:

$$\cdot Y_{12} = \frac{\bar{V}_1}{\bar{V}_2} \Big|_{\bar{V}_2=0, \bar{I}_2=0} \rightarrow \text{trovo } \bar{I}_1 = p(\bar{V}_1) : \bar{V}_1 - \bar{I}_1 \cdot 1 = 0 \rightarrow \bar{I}_1 = \frac{\bar{V}_1}{1}$$

$$\rightarrow Y_{12} = \frac{\bar{V}_1}{\bar{V}_2} = 1$$

Mom USO prove semplici:

$$\rightarrow \bar{V}_1 - R_1 i_1 - \bar{X} = 0 \rightarrow \bar{X} = \bar{V}_1 - R_1 \bar{i}_1$$

$$\rightarrow V_c = \bar{X} + g\bar{X} = 10\bar{X} \rightarrow \bar{I}_2 - \frac{\bar{X}}{R_2} - 10\bar{X} j\omega C = 0 \rightarrow \bar{I}_2 = \bar{V}_2 - \bar{I}_1 - 10(\bar{V}_1 - \bar{I}_1) j\omega \cdot 0.1$$

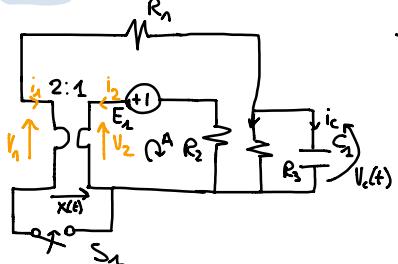
$$\rightarrow \bar{I}_2 = (1-j\omega)\bar{V}_2 - (1-j\omega)\bar{I}_1 \rightarrow \bar{I}_2 = \frac{1-j\omega}{2-j\omega} \bar{V}_2$$

$\downarrow Y_{22}$

$$\rightarrow \bar{I}_2 = 4\bar{I}_1 = 4 \cdot \frac{1-j\omega}{2-j\omega} \bar{V}_1$$

$$\begin{bmatrix} \frac{1-j\omega}{2-j\omega} & 0 \\ 4 \frac{1-j\omega}{2-j\omega} & 0 \end{bmatrix}$$

N° 4



$t_0 = 0$  l'interruttore  $S_1$  (aperto e opera a regime) viene chiuso.

→  $X(t)$  per  $t < t_0$ .

$$\rightarrow \begin{cases} V_1 = 2V_2 \\ i_1 = -\frac{1}{2}i_2 \end{cases} \rightarrow i_1 = 0 \rightarrow i_2 = 0$$

$$X(t) = V_2 - E_1 + V_{R_2} + V_c(t) - V_{R_1} = V_2 - E_1 + 0 + 0 - 0$$

→ So che  $V_2 = E_1$

$$\rightarrow X(t) = -V_1 = -2E_1 = -12V$$

→  $V_c(t)$  per  $-1 < t < \infty$

tra  $-1 < t < 0$ :  $S_1$  aperto e  $V_c = 0$

$$t > 0 : S_1$$
 chiuso → trovo  $i_c = p(\bar{V})$ :  $i_c = -i_1 - \frac{V_c}{R_3}$

$$\rightarrow \text{cerco } i_1: \text{KVL A)} V_2 - E_1 + R_2 i_2 = 0 \rightarrow V_2 = E_1 - R_2 i_2 = E_1 - R_2 (-2i_1)$$

$$\rightarrow 2V_2 + R_2 i_2 - R_3(i_c - i_1) = 0 \rightarrow 2E_1 + 4R_3 i_1 + R_3 i_c + R_3 i_2 = 0$$

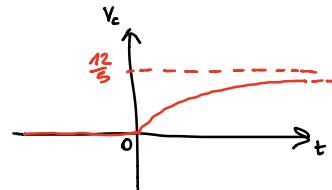
$$\rightarrow i_1 = \frac{-2E_1 - R_3 i_c}{5R_3}$$

$$\rightarrow i_c = -\frac{V_c}{R_3} + \frac{2E_1}{5R_3} + \frac{i_1}{5} \rightarrow (1 - \frac{1}{5})i_c = -\frac{V_c}{R_3} + \frac{2E_1}{5R_3} \rightarrow i_c = -\frac{5V_c}{4R_3} + \frac{E_1}{2R_3} \rightarrow \dot{V}_c = -\frac{5}{4 \cdot 10^{-3}} + \frac{6}{2 \cdot 10^{-3}}$$

$$\rightarrow \dot{V}_c = -125V_c + 300$$

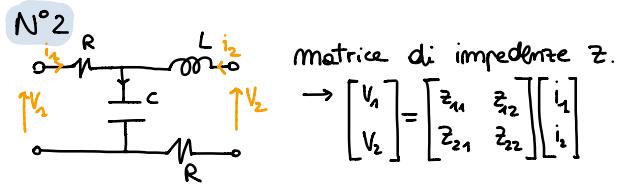
$$\text{trovo } V_c(\infty): 0 = -125V_c(\infty) + 300 \rightarrow V_c(\infty) = \frac{300}{125} = \frac{12}{5}$$

$$\text{trovo K: } 0 = K + \frac{12}{5} \rightarrow K = -\frac{12}{5} \rightarrow V_c(t) = -\frac{12}{5} + \frac{12}{5} e^{-125t}$$



## APPELLO SETT-2015

N°1



→ prove semplici:

$$z_{11} = \frac{V_1}{i_1} \Big|_{i_2=0, V_2=0} \rightarrow z_{11} = R + \frac{1}{j\omega C}$$

$$z_{12} = \frac{V_1}{i_2} \Big|_{i_1=0, V_2=0} \rightarrow V_1 = V_c \rightarrow \frac{1}{j\omega C} i_2 + j\omega L i_2 + R i_2 = 0$$

$$V_1 = V_c = (j\omega L + R) i_2 \rightarrow z_{12} = -(j\omega L + R)$$

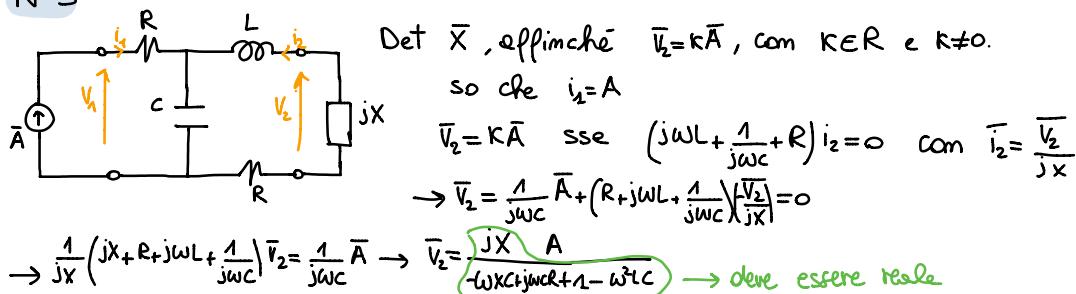
$$z_{21} = \frac{V_2}{i_1} \Big|_{V_1=0, i_2=0} \rightarrow V_2 = V_c \rightarrow V_c + R i_1 = 0$$

$$V_2 = -R i_1 \rightarrow z_{21} = -R$$

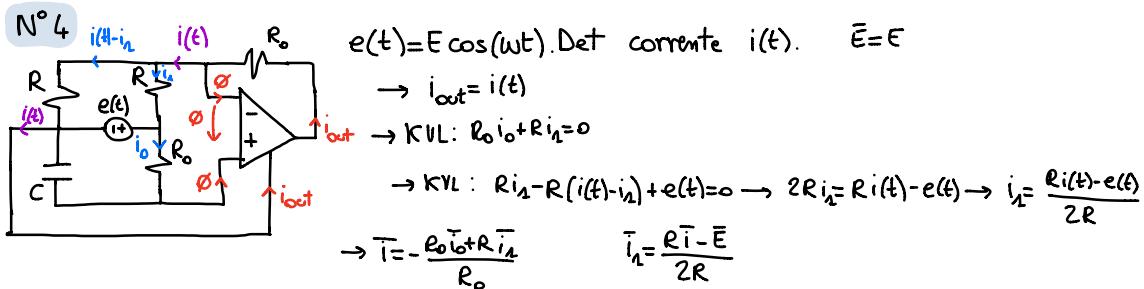
$$z_{22} = \frac{V_2}{i_2} \Big|_{V_1=0, i_1=0} \rightarrow z_{22} = j\omega L + \frac{1}{j\omega C} + R$$

$$Z = \begin{bmatrix} R + \frac{1}{j\omega C} & -(j\omega L + R) \\ -R & j\omega L + \frac{1}{j\omega C} + R \end{bmatrix}$$

N°3



N°4

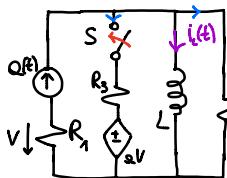


$$\rightarrow \text{KVL: } \bar{E} - R_o \bar{i}_o - \frac{1}{j\omega C} \bar{i}_o = 0 \rightarrow \bar{i}_o = \frac{\bar{E} j\omega C}{j\omega R_o C + 1}$$

$$\rightarrow R_o \frac{\bar{E} j\omega C}{j\omega R_o C + 1} + R \frac{\bar{R} - \bar{E}}{2R} = 0 \rightarrow \frac{R \bar{i}}{2} = \frac{\bar{E} - R_o \bar{E} j\omega C}{j\omega R_o C + 1} = \frac{\bar{E}}{R} \left( 1 - \frac{R_o j\omega C}{j\omega R_o C + 1} \right) = \frac{\bar{E}}{R} \left( \frac{1}{j\omega R_o C + 1} \right)$$

N° 5

$t=0^-$  a regime,  $q(t)=A$ .  $t=0$ , S si chiude. Det eq. di stato,  $i_L(t)$  per  $t \geq 0$



$\rightarrow t \leq 0^-$ : L e c.c.

$$i_L - A + i_3 + i_2 = 0 \rightarrow i_L(0^-) = A$$

$$V_i = 0 \rightarrow i_2 = 0 \text{ mentre } i_3 = -\frac{qV}{R_3} = 0$$

$\rightarrow t \geq 0^+$ : cerco  $V_L = f(i_L)$

$$\rightarrow V_L = R_3 i_3 - qV \quad \rightarrow V = q(t) R_2$$

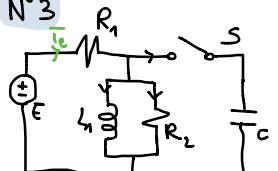
$$\text{cerco } i_3 = q(t) - i_L - \frac{V_L}{R_2}$$

$$\rightarrow V_L = R_3 \left( q(t) - i_L - \frac{V_L}{R_2} \right) \rightarrow V_L = R_3 q(t) - R_3 i_L - \frac{R_3 V_L}{R_2} - q(t) R_2$$

$$\rightarrow \left( 1 + \frac{R_3}{R_2} \right) V_L = -R_3 i_L + (R_3 - qR_2) q(t) \rightarrow V_L = -\frac{R_3 R_2}{R_2 + R_3} i_L + \frac{(R_3 - qR_2)}{R_2 + R_3} q(t)$$

$$\rightarrow i_L = -\frac{R_3 R_2}{L(R_2 + R_3)} i_L + \frac{R_2(R_3 - qR_2)}{L(R_2 + R_3)} A \text{ eq. di stato}$$

N° 3



$P_a^e = 2W$ ,  $P_e^e = 10 + 10j$ ,  $R_2 = 1\Omega$ . Det  $|I_L|$  com S aperto

$$\rightarrow 10 = \sum P = 2 + P_a^e \rightarrow P_a^e = 8W = \frac{1}{2} R_2 |I_L|^2 \rightarrow |I_L| = \sqrt{\frac{16}{R_2}} = 4$$

$$j10 = Q_a = \frac{1}{2} j\omega L |I_L|^2$$

$$\rightarrow j\omega L |I_L|^2 = j20 \rightarrow |I_L| = \sqrt{\frac{20}{\omega L}} = \sqrt{\frac{20}{4}} = j5 \rightarrow |I_L| = |I_2 + I_c| = 4 + j5$$

$$\rightarrow |I_L| = \sqrt{16 + 25} = \sqrt{41}$$

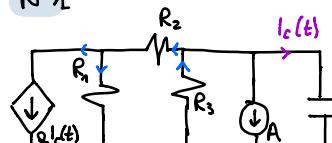
$\rightarrow S$  chiuso. Det c applichc com  $\omega = 1$ ,  $P_e^e = 10$ .

$$Q = 0 \rightarrow P_s^e = -j10 = \frac{1}{2} \frac{1}{j\omega C} |I_L|^2 \rightarrow 10 = \frac{1}{2} j\omega C |I_L|^2 \rightarrow 10 = \frac{1}{2} 1 C 16 \rightarrow C = \frac{20}{16} = \frac{5}{4}$$

$$V_c = V_{R_2} = 1 \cdot 4 = 4$$

ITE LUGLIO 2015

N° 1



Det eq. di stato.

$$\rightarrow i_c = C \frac{dV_c}{dt}$$

$$i_c = i_3 - i_2 - A$$

$$i_3 = -\frac{V_c}{R_3} \quad i_2 = i_n - \alpha i_c t = -\frac{i_2 R_2}{R_1} + \frac{V_c}{R_1} - \alpha i_c t$$

$$\rightarrow i_2 = \left( \frac{V_c}{R_1} - \alpha i_c t \right) \frac{R_1}{R_1 + R_2} = \frac{V_c}{R_1 + R_2} - \frac{\alpha i_c t R_1}{R_1 + R_2}$$

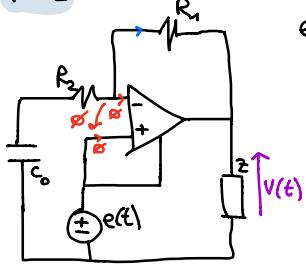
$$\rightarrow i_c = -\frac{V_c}{R_2} - \frac{V_c}{R_1 + R_2} + \frac{\alpha i_c t R_1}{R_1 + R_2} \rightarrow i_c = -\frac{R_1 R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} V_c = -\frac{R_1 + R_2 + R_3}{R_1 + R_2 + R_3} V_c$$

$$\rightarrow V_c + \frac{R_1 + R_2 + R_3}{R_3 C (R_2 + (1+\alpha)R_1)} V_c = 0 \rightarrow \text{eq. di stato}$$

N° 2

$$\begin{aligned} &\rightarrow \text{es. stabile sse: } R_3(R_2 + (1+\alpha)R_1) > 0 \rightarrow R_2 + (1+\alpha)R_1 > 0 \rightarrow (1+\alpha) > -\frac{R_2}{R_1} \rightarrow \alpha > -\frac{R_2}{R_1} - 1 \\ &\rightarrow V_c(1) = V_0, V_c(t) \text{ per } t > 0 \\ &\rightarrow V_0 = K \rightarrow V_c(t) = V_0 e^{-\frac{R_2 + R_3}{R_3 C (R_2 + (1+\alpha)R_1)} t} \quad \text{+ A...} \\ &\rightarrow t \rightarrow \infty: P_e^c = \frac{1}{2} \bar{V}_c \bar{T}_e = 0 \text{ poiché } V_c \in \text{im scerice} \end{aligned}$$

N° 3



$$e(t) = E \sin(\omega t), \text{ si det } V(t).$$

$$\bar{E} = E e^{j\frac{\pi}{2}} = -j\bar{E}$$

$$\rightarrow \bar{V} + R_1 \bar{i}_1 - \bar{E} = 0 \rightarrow \bar{V} = \bar{E} - R_1 \bar{i}_1 \quad \text{KVL}$$

$$\rightarrow \bar{i}_1 R_2 + \bar{E} + \frac{1}{j\omega C_0} \bar{i}_1 = 0 \rightarrow \bar{i}_1 = \frac{-\bar{E} j\omega C_0}{j\omega C_0 R_2 + 1} \quad \text{KVL}$$

$$\rightarrow \bar{V} = \bar{E} + \frac{\bar{E} j\omega C_0 R_1}{j\omega C_0 R_2 + 1}$$

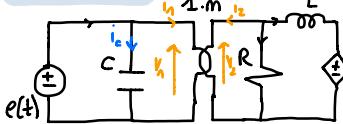
$$\rightarrow V(t) = \operatorname{Re} \left\{ \bar{V} e^{j\omega t} \right\} = \operatorname{Re} \left\{ \frac{j\omega C_0 R_2 + j\omega C_0 R_1}{j\omega C_0 R_2 + 1} \bar{E} e^{j\omega t} \right\} = \operatorname{Re} \left\{ \frac{\sim (1 - j\omega C_0 R_1)}{\omega^2 C_0^2 R_2^2 + 1} \bar{E} e^{j\omega t} \right\} =$$

$\pm$

N° 4

$$Z \in \mathbb{C} \quad P_e^E = P^E + jQ^E; \quad Z = -jQ^E$$

ES. 10.16



$$\text{Det. } C \in \mathbb{C} \quad P_e^E = P \rightarrow Q_e^E = 0.$$

$$P_e^E = \frac{1}{2} \bar{E} \bar{i}_e^*$$

$$\text{cerco } \bar{T}_e: \bar{T}_e = \bar{i}_h + \bar{i}_c$$

$$\bar{i}_h = -\frac{1}{m} \bar{i}_2$$

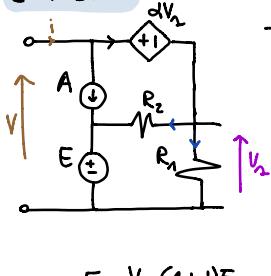
$$\text{so che: } \bar{i}_2 = -\bar{i}_h - \bar{i}_L = -\frac{m\bar{E}}{R} - \frac{\bar{E}(m - rj\omega C)}{j\omega L}$$

$$\frac{m\bar{E}}{R}$$

$$\rightarrow \bar{i}_e = \frac{\bar{E}}{m} \left( \frac{m - rj\omega C}{j\omega L} \right) + \bar{E} j\omega C = \frac{\bar{E}}{m} \left( \frac{m - jmwC}{j\omega L} - \frac{r\omega^2 LC + j\omega C}{\omega^2 L^2} \right)$$

$$\Im \{ P_e^E \} = 0 \rightarrow -\frac{m}{WL} + \omega C = 0 \rightarrow C = \frac{m}{\omega^2 L}$$

ES. 3.17



$$\rightarrow i_{mr} = C_{mr} V + A_{mr}$$

$$\rightarrow i = (i_1 + i_2) - A$$

$$\text{KVL: } E + R_2 i_2 - R_1 i_1 = 0 \rightarrow i_1 = \frac{E + R_2 i_2}{R_1}$$

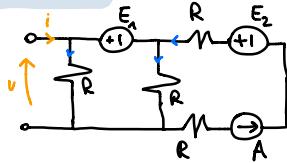
$$\text{KVL: } R_2 i_2 + dV_1 - V + E = 0 \rightarrow R_2 i_2 + d(E + R_2 i_2) - V + E = 0$$

$$\rightarrow R_2 (1 + d) i_2 = V - (1 + d) E \rightarrow i_2 = \frac{V - (1 + d) E}{R_2 (1 + d)}$$

$$\rightarrow i_2 = \frac{E}{R_1} + \frac{V - (1+\alpha)V}{R_1(1+\alpha)} \rightarrow i = \left( \frac{E}{R_1} + \frac{V - (1+\alpha)V}{(1+\alpha)} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \right) - A$$

$$\rightarrow i = \frac{R_1+R_2}{R_1 R_2 (1+\alpha)} V + \frac{E}{R_2} - A \quad \exists \neq -1$$

ES. 2.15



$$i_{mr} = G_{mr} V + A_{mr}$$

$$\rightarrow \text{spegno generatori: } G_{mr} = \frac{i_{mr}}{V} \rightarrow i = i_{R_1} + i_{R_2}$$

$$i_{R_1} = \frac{V}{R} = i_{R_2} \rightarrow i = \frac{2V}{R} \rightarrow G_{mr} = \frac{2}{R}$$

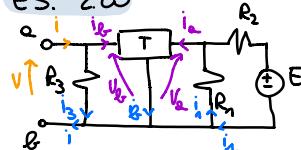
$$\rightarrow \text{spegno } V: i_{mr} = A_{mr} \rightarrow i = i_{R_1} + i_{R_2} - A$$

$$i_{R_1} = 0 \text{ perché } // \text{ a.c.} \rightarrow i = A_{mr} = -\frac{E_1}{R} - A = -\frac{E_1 + AR}{R}$$

$$i_{R_2} = \frac{-E_1}{R}$$

$$\rightarrow i_{mr} = \frac{2}{R} V - \frac{E_1 + AR}{R}$$

ES. 2.20



$$V_{Th} = R_{Th} i + E_{Th}$$

$$\begin{cases} i_a = 0 \\ i_b = gV_a \end{cases}$$

$$\rightarrow \text{spegno generatori: } R_{Th} = \frac{V_{Th}}{i} \rightarrow V_{Th} = R_3 i_3 = R_3 i$$

$$\rightarrow E = 0 \rightarrow V_{eq} = 0 \rightarrow V_a = 0 \rightarrow i_b = 0 \rightarrow i_3 = i$$

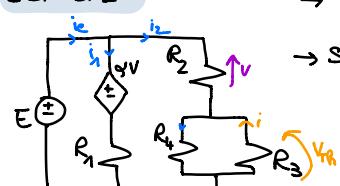
$$\rightarrow R_{Th} = R_3$$

$$\rightarrow \text{spegno } i: V_{Th} = E_{Th} = R_3 i_b$$

$$i_b = gV_a \rightarrow V_a = -R_1 i_a = -\frac{R_1 E}{R_1 + R_2} \xrightarrow{\text{partitore}} \rightarrow i_b = -\frac{gR_1 E}{R_1 + R_2} \rightarrow E_{Th} = -\frac{gR_1 R_3 E}{R_1 + R_2}$$

$$\rightarrow V_{Th} = R_3 i - \frac{gR_1 R_2 E}{R_1 + R_2} \rightarrow \text{se } R_3 \text{ è c.a. } V_{Th} = E_{Th} \text{ generatore indip. di tensione}$$

ES. 3.15



$$\rightarrow R_3 | P_a^3 \text{ max.} \rightarrow \text{sse } R_3 = R_{Th}$$

$$\rightarrow \text{spegno generatori: } R_{Th} = \frac{V_{Th}}{i}$$

$$\rightarrow V_{Th} = R_2 i_2$$

$$E = 0 \rightarrow \partial V + R_1 i_1 = 0 \rightarrow \partial R_2 i_2 = -R_1 i_1$$

$$\rightarrow i_4 R_4 + R_2 i_2 = 0 \rightarrow R_4 (i_2 - i) + R_2 i_2 = 0 \rightarrow -i R_4 + (R_4 + R_2) i_2 = 0 \rightarrow i_2 = \frac{R_4}{R_4 + R_2} i$$

$$V_{Th} = R_2 i_2 = \frac{R_2 R_4}{R_2 + R_4} i \rightarrow R_{Th} = \frac{R_2 R_4}{R_2 + R_4}$$

$$\rightarrow P_a^3 \text{ max.} \rightarrow = \frac{1}{2} \frac{E^2}{4R_2^2 (R_2 + R_4)^2} = \frac{1}{8} \frac{(R_2 + R_4) R_4^2 E^2}{R_2^2 (R_2 + R_4)^2} = \frac{R_4^2 E^2}{8 R_2 (R_2 + R_4)}$$

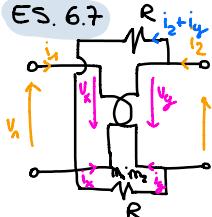
$$\hookrightarrow R_{Th} = V_{Th} |_{i=0} \rightarrow R_2 i_2 + R_4 i_2 = E \rightarrow i_2 = \frac{E}{R_2 + R_4}$$

$$V_{Th} = \frac{R_4 E}{R_2 + R_4}$$

ES. 1

$$\bar{E}_{\text{ca}} = \frac{2Q}{4\pi\epsilon_0 d^2} u_x \quad \bar{E}_a = \frac{Q}{4\pi\epsilon_0} (u_y)$$

$$\bar{E} = \frac{2Q}{4\pi\epsilon_0} u_x - \frac{Q}{4\pi\epsilon_0} u_y$$



$$M_1 = L \quad M_2 = 2 \quad R = 1 \Omega$$

$$\begin{cases} V_1 = R_{21} i_1 + R_{12} i_2 \\ V_2 = R_{22} i_2 + R_{12} i_1 \end{cases}$$

$$\cdot R_{21} = \frac{V_1}{i_2} \Big|_{V_2=i_1=0} \rightarrow i_x = i_1 \rightarrow \text{KVL} \quad V_2 = V_x \rightarrow V_x - R i_y - V_y - R i_y = 0 \rightarrow V_x - 2R \left( \frac{M_1}{M_2} i_1 \right) - V_y = 0 \rightarrow V_x - 4i_2 - V_x \frac{1}{2} = 0 \rightarrow \frac{1}{2} V_x = 4i_2 \rightarrow V_x = 8i_2$$

$$\rightarrow V_x = 8i_2 \rightarrow R_{21} = 8$$

$$\cdot R_{12} = \frac{V_1}{i_2} \Big|_{V_2=i_1=0} \rightarrow V_1 = V_x \rightarrow V_x - R(i_2 + i_y) - V_y - R(i_2 + i_y) = 0 \rightarrow \frac{V_x}{2} = 2R(i_2 + i_y) = 4i_2 \rightarrow R_{12} = 4$$

$$\cdot R_{22} = \frac{V_2}{i_2} \Big|_{V_1=i_1=0} \rightarrow V_2 = V_y \rightarrow V_y + 2R(i_2 + i_y) - 2V_y = 0 \rightarrow V_y = -2Ri_y = 2R \cdot 2i_2 = 4i_2 \rightarrow R_{22} = 4$$

$$\cdot R_{11} = \frac{V_1}{i_2} \Big|_{V_2=i_1=0} \rightarrow V_1 = -V_y \rightarrow -V_y + 2Ri_2 = 0 \rightarrow V_y = 2i_2 = V_2 \rightarrow R_{11} = 2$$

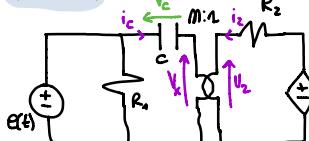
$$\rightarrow \text{Det } V_1 = i_2 + i_c \quad i_c = 0 \wedge V_2 = 2V$$

$$V_1 = 4i_2 \rightarrow V_1 = 4V$$

$$V_2 = 2 = 2i_2 \rightarrow i_2 = 1A$$

$$P_a^{\Delta B} = 2 \cdot 1 (1)^2 = 2W$$

ES. 8.7



$e(t) = E \sin(\omega t)$ ,  $V_c \left( \frac{T}{\omega} \right) = 0$ . Det equazione di stato.

$\rightarrow i_c$  in funz. di  $V_c$ :

$$\rightarrow V_x + V_c - e(t) = 0 \rightarrow \text{cerco } i_c: \quad V_x + R_2 i_2 - r i_c = 0 \rightarrow \frac{V_x}{m} - R_2 m i_c - r i_c = 0$$

$$V_x = m V_2 \quad i_2 = -m i_c$$

$$\rightarrow -\frac{V_c}{m} + \frac{e(t)}{m} = (R_2 m + r) i_c \rightarrow i_c = -\frac{V_c}{m(R_2 m + r)} + \frac{e(t)}{m(R_2 m + r)} \rightarrow \text{eq. di stato} \rightarrow i_c' + \frac{V_c}{m c (R_2 m + r)} - \frac{e(t)}{m c (R_2 m + r)} = 0 \quad r > R_2 m$$

$\rightarrow V_c(t)$  per  $t \geq \frac{T}{\omega}$

$$\cdot \text{cerco } V_c(t): \rightarrow \frac{dV_c}{dt} = -\frac{V_c}{m c (R_2 m + r)} - \frac{jE}{m c (R_2 m + r)} \rightarrow \left( j\omega + \frac{1}{m c (R_2 m + r)} \right) V_c = -\frac{jE}{m c (R_2 m + r)}$$

$$\rightarrow V_c = -\frac{jE}{j\omega m c (R_2 m + r)} = \frac{E \omega m c (R_2 m + r) + jE}{1 - \omega m^2 c^2 (R_2 m + r)^2}$$

$$V_c(t) = \operatorname{Re} \left\{ V_c e^{j\omega t} \right\} = \operatorname{Re} \left\{ \gamma (E \omega m c (R_2 m + r) + jE) (\cos(\omega t) + j \sin(\omega t)) \right\} = \gamma [E \omega m c (R_2 m + r) \cos(\omega t) - E \sin(\omega t)]$$

$$\cdot \text{cerco } K: \quad 0 = K + V_{c,0} \quad \rightarrow V_c(t) = K e^{-\gamma t} + H$$