$\rightarrow W = \Delta U + Q \longrightarrow -W_{IRR} > -W_{REV} \longrightarrow W_{IRR} < W_{REV}$

N°5

$m_G = 10\ \text{kg} \longrightarrow T_1 = 12°, T_2 = 27°C$, $c = 2{,}38 \cdot 10^3 \frac{J}{kg\,K}$. ΔS_{TOT}?

$\rightarrow$ uso $\Delta S = \int_i^f \frac{\delta Q}{T}\Big|_{REV}$: $\Delta S_{TOT} = \Delta S_{GLIC} + \Delta S_{T_2}$

$\rightarrow \Delta S_{GLIC} = \int_i^f \frac{\delta Q}{T}\Big|_{REV} \rightarrow$ la glicerina sarà messa a contatto con T_2 molto lentamente per avere trasf. rev

$\hookrightarrow \delta Q = mc\,dT \rightarrow \Delta S = \int_i^f \frac{mc\,dT}{T}\Big|_{REV} = mc \ln\left(\frac{T_f}{T_i}\right) = mc \ln\left(\frac{T_2}{T_1}\right) > 0$ perchè $T_2 > T_1$

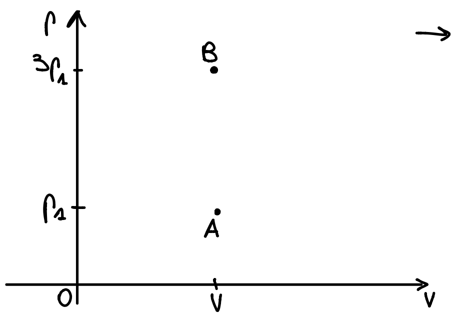
è irreversibile

$\rightarrow \Delta S_{T_2} = \int_i^f \frac{\delta Q}{T}\Big|_{REV} = \frac{Q_{CED}}{T_2} = + \frac{mc(T_1 - T_2)}{T_2} < 0$ $\longrightarrow S_{TOT} = \Delta S_G + \Delta S_{T_2}$

$\hookrightarrow Q = -mc(T_1 - T_2)$

N°6

$p_1 = 10^6\ \text{Pa}$, $V_1 = 5\ \ell$, $p_2 = 3p_1$. $m = 2$ moli $\longrightarrow$ non so se è irr o rev



$\longrightarrow W$, Q perchè non sono funzioni di stato e non posso calcolarli

$\rightarrow$ indip. da trasf. ma da pto; e pto$_f$

$\rightarrow \Delta U = m c_v \Delta T = m \cancel{c_v}^{\frac{3}{2}} \frac{\Delta p V}{\cancel{mR}} = \frac{3}{2} \Delta p V = 3 p_1 V$

$\rightarrow \Delta S_G = \int_1^2 \frac{\delta Q}{T}\Big|_{REV} = \int_1^2 \frac{dU + \delta W^{\to 0}}{T}\Big|_{rev} = \int_1^2 \frac{m c_v dT}{T} = m c_v \ln\left(\frac{T_f}{T_i}\right)\Big|_1^2$

$1°$ PR. TD.

posso usare isocora rev.

$\hookrightarrow \Delta S_G = \frac{3}{2} m R \ln\left(\frac{p_f}{p_i}\right) > 0$

$\hookrightarrow$ da eq. di stato

GAS $\neq$ UNIV. (perchè $Q \neq 0$)

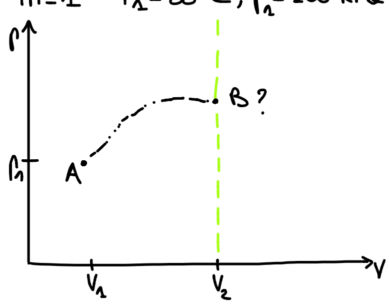
$\hookrightarrow$ se $Q = 0$ il sist è isolato e il gas è l'univ.

$\hookrightarrow$ non so se è rev o meno

ESERCITAZIONE 14

N°1

$m = 1$ $T_1 = 20°C$, $p_1 = 100\ \text{kPa}$, $V_1 = 0{,}1\ m^3$, $V_2 = 0{,}2\ m^3$. TRASF. IRR . Det $p_2, T_2, \Delta S_{GAS}$



$\longrightarrow T_2 : pV = mRT \rightarrow T_2 = \frac{p_2 V_2}{mR}$

$\rightarrow \Delta U = m c_v \Delta T = m c_v (T_2 - T_1) = m c_v \left(\frac{p_2 V_2}{mR} - T_1\right) = Q - W = 0$

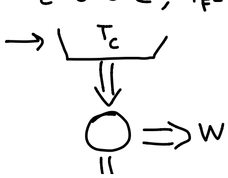
non scamb Q e W con ambiente

$\hookrightarrow p_2 = \frac{2T_1}{V_2 \cdot 3} \rightarrow T_2 = \frac{2T_1 \cancel{V_2}}{3\cancel{V_2} mR} = \frac{2T_1}{3mR}$

$\rightarrow \Delta S_G = mR \ln\left(\frac{V_2}{V_1}\right) - m c_v \ln\left(\frac{T_2}{T_1}\right)$

N°2

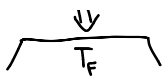
$T_c = 900°C$, $T_F = 400°C$, $\langle P_m \rangle = 20$ MW, $\eta = 50\%$. Det $\Delta S_{UNI}/h$, Q



$\rightarrow \langle P_m \rangle = \frac{W}{t} \rightarrow W = 20 \cdot 10^6 \cdot 1\,h = 20 \cdot 10^6$ j/h $(= 7{,}2 \cdot 10^{10}$ j$)$

$\rightarrow 0{,}5\,\eta_c \rightarrow \eta = 0{,}5 \cdot \left(1 - \frac{T_F}{T_c}\right) = 0{,}5\left(\frac{5}{9}\right) = \frac{5}{18} = \frac{W}{Q_{ASS}} \longrightarrow Q_c = \frac{18}{\cancel{5}} \cdot \cancel{20}^{4} \cdot 10^6 \cdot 3600\,\text{s}/h$

$\rightarrow Q = 2.59 \cdot 10^{11}$ j



$\rightarrow \eta = 1 - \frac{Q_F}{Q_c} \rightarrow Q_F = Q_c(1-\eta) = 1{,}89 \cdot 10^{11}\ J$

$\rightarrow \Delta S_{UNI} = \underbrace{-\frac{Q_c}{T_c}}_{\text{ceduto da } T_c} + \underbrace{\frac{Q_F}{T_F}}_{\text{assorbito da } T_F}$

N°3

$m = 2g$, $T_0 = 60K$, $T_A = 300K$. Det ΔS_{O_2}

$\Delta S_G = m c_p \ln\left(\frac{T_f}{T_i}\right) - mR\ln\left(\frac{p_f}{p_i}\right) \rightarrow$ non ho abb. dati vado per passaggi

T_0, T_A ; p_i

$\rightarrow$ 1 – passa da 60K a 90K

$\Delta S_1 = \int \frac{\delta Q}{T} = mc \ln\left(\frac{T_f}{T_i}\right) = 2\cdot10^{-3}\cdot 1{,}66 \ln\left(\frac{3}{2}\right) = 1{,}35\ \frac{j}{K}$

2 – passaggio di stato $\rightarrow$ irrev. $\Delta S_2 = \frac{m\lambda}{T} = \frac{2\cdot10^{-3}\cdot 2{,}13\cdot 10^{5}}{90\cancel{K}} = 4{,}73\ J/K$

3 – passa da 90K a 300K

$n = \frac{m}{P_{MOL}}$

$\Delta S_3 = n c_p \ln\left(\frac{300\cancel{K}}{90\cancel{K}}\right) = 2{,}19\ J/K$

$\rightarrow \Delta S_{TOT} = 2{,}19 + 4{,}73 + 1{,}35 = 8{,}27\ J/K$

N°4

$\# = 4$ moli $n = 1$, $C_\gamma = c_v + \alpha T$

a) dim$[\alpha] \rightarrow C_\gamma = \frac{1}{n}\frac{\delta Q}{dT} = \left[\frac{J}{mol\cdot K}\right] \rightarrow \alpha = \frac{C_\gamma - c_v}{T} \Rightarrow [\alpha] = \left[\frac{J}{mol\cdot K^2}\right]$; $[c_v] = \left[\frac{J}{K\cdot mol}\right]$

b) $T_1, T_2 \rightarrow \delta W = \delta Q - dU = n C_\gamma dT - n c_v dT = n dT (c_\gamma - c_v) = n\alpha T dT \xrightarrow{\text{integro}} W = n\alpha \frac{T^2}{2}\Big|_{T_1}^{T_2} = n\alpha \frac{(T_2 - T_1)^2}{2}$

c) eq. trasformazione:

$\begin{cases} \delta W = n\alpha T dT \\ \delta W = p dV \end{cases} \rightarrow p dV = n\alpha T dT \rightarrow \frac{\cancel{n}R\cancel{T}}{V} dV = \cancel{n}\alpha \cancel{T} dT \rightarrow \frac{R dV}{V} = \alpha dT \xrightarrow{\text{integro}} R\ln\left(\frac{V}{V_0}\right) = \alpha(T - T_0)$

$\rightarrow \ln\left(\frac{V}{V_0}\right) = \frac{\alpha}{R}(T - T_0) \rightarrow \frac{V}{V_0} = e^{\frac{\alpha T}{R}} e^{-\frac{\alpha T_0}{R}} \rightarrow V e^{\frac{\alpha T}{R}}$

N°5

1 mol, $n = 1$, $p_1 = 1{,}2$ atm, $T_1 = 300K$

a – $p_2 = 1$ atm $\rightarrow p_1 T_1^{\frac{\gamma}{1-\gamma}} = p_2 T_2^{\frac{\gamma}{1-\gamma}} \rightarrow T_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} T_1$

$\rightarrow W = -\Delta U = n c_v \Delta T = n c_v T_1\left(\left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} - 1\right)$

b – diventa rev se tolgo pistone gradualmente (dV in dT)

c – $W_{IRR} = n c_v \Delta T\ (= -\Delta U)$

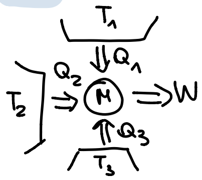
$V_1 = \frac{nRT_1}{p_1}$ $V_2 = \frac{nRT_2}{p_2}$ • cerco T_2: $nc_v(T_2 - T_1) = p_2(V_2 - V_1) \rightarrow nc_v(T_2 - T_1) = p_2\left(\frac{nRT_2}{p_2} + p_2\frac{nRT_1}{p_1}\right)$

$\hookrightarrow \frac{3}{2}T_2 - \frac{3}{2}T_1 = -T_2 + \frac{p_2 T_1}{p_1} \rightarrow \frac{5}{2}T_2 = T_1\left(\frac{3}{2} + \frac{p_2}{p_1}\right) \rightarrow T_2 = \frac{2T_1}{5}\left(\frac{3}{2} + \frac{p_2}{p_1}\right)$

$\hookrightarrow W = -\Delta U = -n c_v T_1\left(\frac{3p_1 + 2p_2}{2} - 1\right)$

N°6

Moti, T_m, Q_3, W



$\rightarrow \Delta U = Q - W = 0 \rightarrow W = Q_1 + Q_2 + Q_3$

$\Delta S_{UNIV} = \Delta S_M + \Delta S_1 + \Delta S_2 + \Delta S_3 = 0$ (perché rev)

$\Delta S_M \rightarrow 0$ n° intero di cicli

$\rightarrow Q_1 = T_1\left(-\frac{Q_2}{T_2} - \frac{Q_3}{T_3}\right) \xrightarrow{\text{sost. in 2}} W = -\frac{Q_2 T_1}{T_2} - \frac{T_1 Q_3}{T_3} + Q_2 + Q_3 \rightarrow Q_2 = \left[W + Q_3\left(\frac{T_1}{T_3} - 1\right)\right]\left(1 - \frac{T_1}{T_2}\right)$

$\rightarrow Q_2 = T_1\left(-\frac{Q_2}{T_2} - \frac{Q_3}{T_3}\right)$

APPELLO 2 2019

N°1

$\longrightarrow$ Hp: $\rightarrow$ omog. $\rightarrow$ in eq. $\rightarrow$ incomp.

Def: $\bar{f} = \frac{d\bar{F}_v}{dm}$ CAMPO DI FORZE DI VOL ($\frac{N}{Kg}$)

$\rightarrow$ si ha: $\bar{\nabla} p = \rho \bar{f}$ dove $\bar{\nabla} p = \frac{\partial p}{\partial x}\bar{u}_x + \frac{\partial p}{\partial y}\bar{u}_y + \frac{\partial p}{\partial z}\bar{u}_z = \rho f_x \bar{u}_x + \rho \ldots$

↳ la pressione varia a seconda della dir, se vi agisce una $\bar{F}_v$

↳ se $d\bar{F}_v = 0 \rightarrow p$ cost

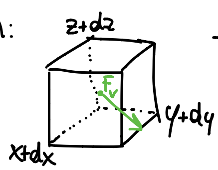
↳ se F_v conservative $\xrightarrow[\text{da rel forza-energ.}]{} \bar{F}_v = -\bar{\nabla} U \rightarrow \frac{\bar{F}_v}{m} = -\frac{\bar{\nabla} U}{m} \xrightarrow[\text{per singolo}]{} \bar{f} = -\bar{\nabla} u_m$

$dU = \bar{\nabla} U \cdot d\bar{r} \wedge dU = \bar{F} \cdot d\bar{r}$

↳ $\bar{\nabla} U d\bar{r} = \bar{F} d\bar{r} \rightarrow \bar{F} = \bar{\nabla} U$

↳ $\bar{\nabla} p = \rho \bar{f}$ per dimostrare considero 1 dir (es. $\bar{u}_x$), per le altre la dim è analoga

DIM:



$\rightarrow$ siccome in eq: $\Sigma \bar{F}_i = 0 \xrightarrow[\text{considero } \bar{u}_x]{} \Sigma \bar{F}_{ix} = 0$

$\rightarrow \bar{F}_{v_x} + \Sigma \bar{F}_{S_x} = 0 \xrightarrow[\text{da def.}]{} d\bar{F}_x + \Sigma d\bar{F}_{Sx} = \bar{f}_x dm + p(x) dS - p(x+dx) dS = 0$

$\rightarrow \bar{f} \rho dV - p(dx) dS = \bar{f} \rho dV - dp\, dS = 0$

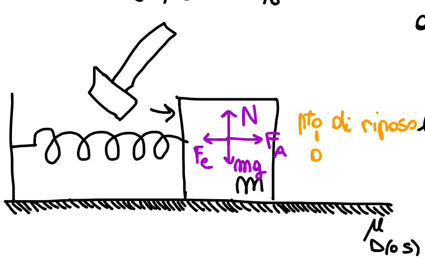
$dV = dr\, dS$

$\rightarrow \bar{f} \rho dr = dp \rightarrow \bar{f} \rho = \frac{dp}{dx}$

$dr = dx$

N°2

$m = 1$ Kg $\mu_s = 0{,}5$ $\mu_D = 0{,}4$ $K = 50$ N/m $\Delta x = 9$ cm.



a) F_s prima del mov

$F_s = \Delta x K = 9 \cdot 10^{-2} \cdot 50 = 4{,}5$ N $< \mu_s mg (= 0{,}5 \cdot 1 \cdot 9{,}8 = 4{,}9)$ ragionevole

b) ΔS_{max}?

$\rightarrow$ uso $\Delta E = -\int_0^{\ell} \bar{F}_A \cdot d\bar{r} \rightarrow E_i = \frac{1}{2} K \Delta x^2$

$E_f = \frac{1}{2} K (\ell + 0{,}09)^2$

$\rightarrow \frac{1}{2} K (\ell + 0{,}09)^2 - \frac{1}{2} K (0{,}09)^2 = -\mu_D mg (\ell + 0{,}09)^2$

$\rightarrow \frac{1}{2} K \ell^2 + \cancel{\frac{1}{2} K (0{,}09)^2} + \frac{1}{2} K (0{,}18 \ell)^2 = \mu_D mg (\ell^2 + (0{,}09)^2 + 0{,}18 \ell) \rightarrow 25 \ell^2 + 0{,}0162 K \ell^2 = 3{,}92 \ell^2 + 0{,}0317 + 0{,}705 \ell$

$\cancel{-\frac{1}{2} K (0{,}09)^2}$

$\rightarrow 21{,}48 \ell^2 - 0{,}705 \ell - 0{,}0317 = 0 \rightarrow \ell = \frac{0{,}705 \pm \sqrt{0{,}497 + 2{,}723}}{42{,}96}$ ↗ $0{,}058 \cong 6$ cm ↘ N.A.

c) $I = \Delta p = p_f - p_i = p_f = m v_0$ ($\hookrightarrow v_i = 0$)

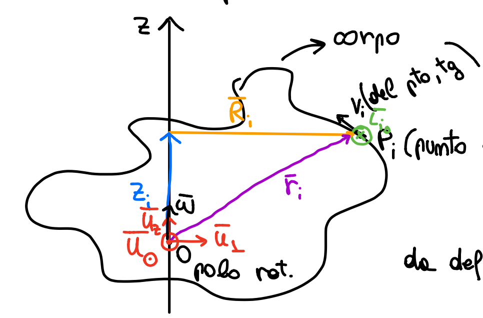
$\to$ cerco v_0 (d'inizio tc si fermi in o)

$$U_{EQ} = 0 \to -\frac{1}{2}k\Delta x^2 - \frac{1}{2}m v_0^2 = -\mu_d m g \Delta x \to \frac{1}{2}m v_0^2 = -\frac{1}{2}k\Delta x^2 - \mu_d m g \Delta x^2 \to v_0 = \sqrt{\frac{k\Delta x^2 - 2\mu_d m g \Delta x^2 \cdot \cancel{2}}{\cancel{2}m}} = \Delta x\sqrt{\frac{k - 2\mu_d m g}{m}}$$

$$\hookrightarrow v_0 = 0{,}09\sqrt{\frac{50 - 2\cdot 0{,}4\cdot 9{,}8}{1}} = 0{,}58\ m/s \to I = m v_0 = 0{,}58\ kg\frac{m}{s}$$

N°3

$\hookrightarrow$ considero corpo rigido generico (forma indef.) che ruota attorno ad asse fisso, con pos casuale rispetto a c.m.



Def: $\bar R_i = R_i \bar u_\perp$

$\bar z_i = z \bar u_z$

$\bar L_{i_0} = -L_{i_0}\bar u_0$

$\bar r_i = r_i \bar u_\perp + r_i \bar u_z = z_i \bar u_z + R_i \bar u_\perp$

si ha: $\mathbb{L}_o = \mathbb{L}_{o_{\perp z}} + \mathbb{L}_{o_{//z}}$

da def mom ang: $\bar L_{o_i} = \bar r_i \times m \bar v_i$ per singolo pto

$\mathbb{L}_o = \sum_i \bar r_i \times m \bar v_i$ per insieme di pti

$\longrightarrow$ dato che: $\bar r_i = z_i \bar u_z + R_i \bar u_\perp$ (da def) $\longrightarrow \mathbb{L}_o = \sum (z_i \bar u_z + R_i \bar u_\perp) \times m[\omega \bar u_z \times (z_i \bar u_z + R_i \bar u_\perp)]$, con $\underbrace{\bar u_z \times z_i \bar u_z}_{0}$, $\underbrace{\bar u_z \times R_i \bar u_\perp}_{-\bar u_0}$

$\bar v_i = \bar\omega \times \bar r_i$ (da desc. vel ang.)

$$\to \mathbb{L}_o = \sum_i \underbrace{(z_i \bar u_z + R_i \bar u_\perp) \times m\omega R_i(-\bar u_0)}_{z_i\bar u_z \times (-\bar u_0) = -\bar u_\perp;\ \bar u_\perp \times (-\bar u_0) = \bar u_z} = -\sum_i z_i m \omega R_i u_\perp + \sum m\omega R_i^2 \bar u_z = \underbrace{m\sum R_i^2 \bar\omega}_{\text{lungo }//} - \underbrace{m\omega \sum \bar R_i}_{\text{lung }\perp} = \mathbb{L}_{o_{//z}} + \mathbb{L}_{o_{\perp z}}$$

N°4

$m = 1$ $\qquad$ $600°C = 873{,}15$

a) $W = Area = \frac{(p_B + p_A)(V_B - V_A)}{2} = \frac{(p_B + p_A) m R T_A}{2 p_A} = \frac{1\cdot 1\cdot 10^{\cancel{3}}\cdot 8{,}314\cdot 873{,}14}{2\cdot 10^{4}} = 3992{,}6 J$ $\qquad$ $V_A = \frac{mRT_A}{p_A}$

$\hookrightarrow \Delta U = Q - W \to Q = \Delta U + W$

$\Delta U = m c_v \Delta T = m c_v\left(-T_A + \frac{p_B 2 V_A}{mR}\right)$

b) $\Delta S_{GAS} = m c_v \ln\left(\frac{T_B}{T_A}\right) + mR\ln\left(\frac{2V_A}{V_A}\right) = \frac{5}{2}\ln\left(\frac{T_B}{T_A}\right) + R\ln(2)$

c) $\Delta S_{UNI} = 0 \to$ non è sist tot isolato

N°1 ITE 2 2019

INIZIO: $p_0 V_0^\gamma = p_f V_f^\gamma$ $\qquad \to p_f = p_0\left(\frac{V_0}{V_f}\right)^\gamma$

$p_0 T_0^{\frac{\gamma}{1-\gamma}} = p_f T_f^{\frac{\gamma}{1-\gamma}}$

$T_0 V_0^{\gamma-1} = T_f V_f^{\gamma-1}$ $\qquad \to V_f = V_0^{\gamma-1}\frac{T_0}{T_f}$

$\to p_f = p_0\left(\frac{T_0}{T_f}\right)^{\frac{\gamma}{1-\gamma}}$

so che: $\Delta U = -W \to m c_v \Delta T = -W$

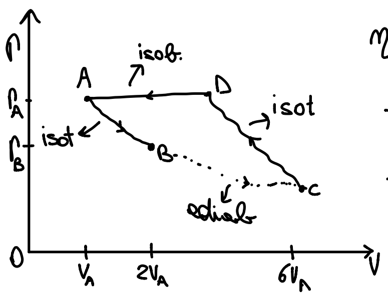
$\to \left(\frac{T_0}{T_f}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{V_0}{V_f}\right)^\gamma \to \left(\frac{T_0}{T_f}\right)^{\frac{1}{1-\gamma}} = \frac{V_0}{V_f}$

$\hookrightarrow \frac{V_f}{T_f^{\sim}} = \frac{V_0}{T_0^{\sim}}$

$\rightarrow W = \int F_v\, ds = \int p_0 V \cdot \frac{dV}{S} =$
$\hookrightarrow p_0 V$

P. 11.4

$g = 3$



$T_B = 2T_C$ $\quad \eta$?

$\eta = 1 - \frac{|Q_c|}{Q_F}$

$\rightarrow$ tra A-B: $Q = nRT_B \ln\left(\frac{2V_A}{V_A}\right) \geq 0$

$\rightarrow$ tra B-C : $Q = 0$

$\rightarrow$ tra C-D: $Q = nR\frac{T_B}{2}\ln\left(\frac{V_D}{6V_A}\right) \leq 0$

$\rightarrow$ tra A-D: $Q = \Delta U + W = nc_V\left(T_B - \frac{T_B}{2}\right) + \underbrace{p_A(V_D - V_A)}_{nR\frac{T_B}{2} - nRT_B}$

$= n\frac{3}{4}RT_B - nR\frac{T_B}{2} = \frac{n}{4}RT_B \geq 0$

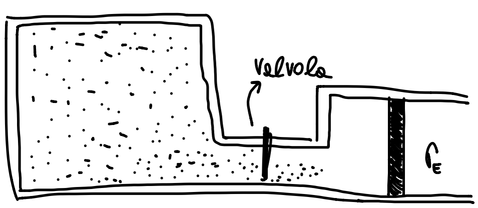
cerco $V_D = \frac{nRT_B}{2p_A} = \frac{V_A}{2}$ $\quad (\frac{nRT_B}{p_A} \rightarrow V_A)$

$Q_C = nRT_B \ln(2) + \frac{n}{4}RT_B$ $\qquad Q_F = nR\frac{T_B}{2}\ln\left(\frac{1}{12}\right)$

$\eta = 1 - \frac{\left|\cancel{nR}\frac{T_B}{2}\ln\left(\frac{1}{12}\right)\right|}{\cancel{nRT_B}\left(\ln(2) + \frac{1}{4}\right)} = 1 + \frac{2\ln(12)}{4\ln(2) + 1}$

P. 10.23

$p_0 = 2$ atm, $p_E = 1$ atm, $T_1 = 300$ K, $g = 3$



$\rightarrow$ trasf. quasi-statica ma non rev.

$\rightarrow T_f = ?$

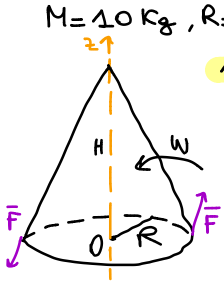
• eq. : $p_f = p_E$

$\rightarrow V_0 = \frac{nRT_1}{p_0}$

$\Delta U = \overset{\rightarrow 0}{Q} - W$
$\hookrightarrow nc_V\Delta T = p_E(V_f - V_0)$ $\quad (V_f \rightarrow \frac{nRT_f}{p_E})$

ES. 1

$M = 10$ kg, $R = 0{,}5$ m, $t = 0$ s, $|\bar{F}| = 10$ N, $V(0) = 0$ m/s :



1 $\rightarrow$ $I_z = \int r^2 dm = \int r^2 \rho\, \pi h (= \frac{3MR^2}{10}$

2 $\rightarrow$ $\omega(5) = ?$

$\rightarrow \Sigma \mathcal{T} = 0 \rightarrow 2FR = I_z\alpha \rightarrow \alpha = \frac{2F\cancel{R}\cdot 10}{3M R^{\cancel{2}}} = \frac{20}{3\cdot 0{,}5} = 13{,}3$ rad/s²

$\omega(5) = \alpha t = 13{,}3 \cdot 5 = 66{,}6$ rad/s

3 $\rightarrow$ W da t=0s a t=5s

$\hookrightarrow \theta(t) = \frac{1}{2}\alpha t^2 = \frac{1}{2}\cdot 13{,}3 \cdot 25 = 166{,}67$ rad

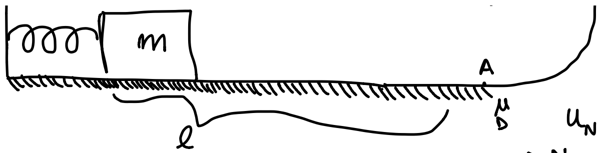
$W = \int \mathcal{T}_0 \cdot d\theta = 1666{,}67$ J

4 $\rightarrow$ $\Delta E = W$

ES. 2

$m = 0{,}5$ kg, $K = 150$ N/m, $\ell = 0{,}2$ m, $\mu_D = 0{,}5$,
$R = 0{,}4$ m . ΔX t_c in B cada

$\frac{1}{2}K\Delta X^2 + \frac{1}{2}mv^2 = \int F \cdot d\bar{s}$ ... tra ... V



$\frac{1}{2} k \Delta x^2 + \frac{1}{2} m v_A^2 \ldots \to v_A$ ... v_A

$\to$ inizia MCUD e in B deve avere $N=0$

u_N: $N - m\omega^2 R - mg\cos\theta = 0$

$\to N = m\frac{v_B^2}{R} + mg\cos\theta = 0$ in B $\underset{\cos\theta=-1}{\to}$ $v_B = \sqrt{\frac{\cancel{m}gR}{\cancel{m}}} = \sqrt{gR}$

$\Delta E_{AB} = 0 \to \frac{1}{2}m\left(\frac{kx^2}{m} - 2\mu g\ell\right)^2 = \frac{1}{2}mgR + mg2R \to$

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$Q_1 = Q_2 \to M_1 c(T_{eq} - T_1) = M_2 c(T_{eq} - T_2) \to M_1 \alpha T_1^2 (T_{eq} - T_1) = M_2 \alpha T_2^2 (T_{eq} - T_2) \to c = \alpha T^2$ non è costante

$\to \Delta S = \Delta S_1 + \Delta S_2 = \int \frac{M_1 c\, dT}{T} + \int \frac{M_2 c\, dT}{T}$

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$1 \to \theta_{eq}$: $\Sigma \vec{\tau}_O = 0 \to k_1\overbrace{\left(\frac{\ell}{2}\operatorname{sen}\theta\right)}^{\Delta x_1}\cancel{\cos\theta}\cdot\frac{\ell}{2} + k_2\ell(\operatorname{sen}\theta)\,\ell\cancel{\cos\theta} = mg\frac{\ell}{2}\cancel{\cos\theta}$

$\to \frac{k_1\ell}{4}\operatorname{sen}\theta + k_2\ell\operatorname{sen}\theta = \frac{mg}{2} \to \operatorname{sen}\theta = \frac{mg}{\cancel{2}} \cdot \frac{\overset{2}{\cancel{4}}}{k_1\ell + 4k_2\ell} = \frac{2mg}{\ell(k_1+4k_2)}$

$\to \theta = \arcsin\left(\frac{2mg}{\ell(k_1+4k_2)}\right) = 3{,}12°$

$2 \to T = \frac{2\pi}{\omega}$

$\hookrightarrow -k_1\frac{\ell^2}{4}\operatorname{sen}\theta\cos\theta - k_2\ell^2\operatorname{sen}\theta\cos\theta + mg\frac{\ell}{2}\cos\theta = \frac{1}{12}M\ell^2\frac{d^2\theta}{dt^2}$

$\to \frac{m\cancel{\ell^2}}{\underset{6}{\cancel{12}}}\frac{d^2\theta}{dt^2} + \left(\frac{k_1}{4} + k_2\right)\frac{\cancel{\ell^2}\operatorname{sen}(2\theta)}{\cancel{2}} = mg\frac{\cancel{\ell}}{\cancel{2}}\cos\theta \to \frac{d^2\theta}{dt^2} + \frac{k_1+4k_2}{\underset{2}{\cancel{4}}}\cdot\frac{\overset{3}{\cancel{6}}\cancel{\ell}}{m\cancel{\ell}}\operatorname{sen}(2\theta) = \frac{\cancel{m}g6\cos\theta}{\cancel{m}\ell}$

piccole oscillazioni: $\frac{d^2\theta}{dt^2} + \underbrace{\frac{3(k_1+4k_2)}{2m}}_{\omega^2}\theta = \frac{6g}{\ell} \to T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{3(k_1+4k_2)}{2m}}}$

$\operatorname{sen} \to \theta$ $\quad \cos \to 1$

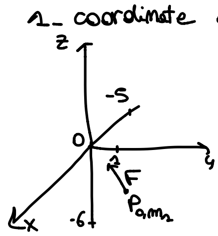
$\theta(t) = \theta(t)_{omog} + \theta(t)_{particolare} = \theta_0\operatorname{sen}(\omega t + \varphi) + \frac{6g}{\ell}$

$\begin{cases} \theta(0) = 0 \\ \dot\theta(0) = 0 \end{cases}$

ES. 2

$m_1 = 0{,}5$ kg, $t = 0$ s, $v(0) = 0$, $r_1 = -5\hat{x} + \hat{y} - 6\hat{z}$, $F = t\hat{x} + 2t\hat{z}$. $t = 3$ s $\to m_2 = 2m_1$, $v_2 = \frac{3}{4}\hat{x} - 3\hat{z}$

1 _ coordinate dove avviene l'urto



$a \to v_1 \to r_1$ (pto a $t=3$) $\to \Delta p = 0$

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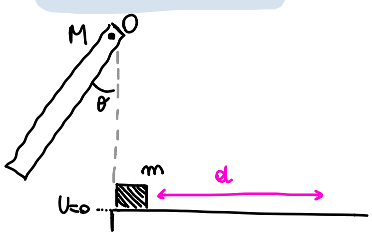

$\ell = 0{,}5$ m, $M = 0{,}4$ kg, $\theta = 45°$, $\mu_D = 0{,}4$

1 → ω_i $\quad$ {i: prima dell'urto; f: dopo urto}

$\to \Delta E = 0 \to \frac{1}{2}\overset{\frac{1}{12}M\ell^2}{I_z}\omega^2 = mg(\ell - \ell\cos\theta) \to \omega_i = \sqrt{\frac{24\cancel{m}g\cancel{\ell}(1-\cos\theta)}{\cancel{m}\ell^{\cancel{2}}}} \to \omega_i = \sqrt{\frac{24g(1-\cos\theta)}{\ell}}$

2 → URTO ELASTICO ($\Delta K = 0$, $\Delta p = 0$)

$$\rightarrow \begin{cases} K_i = K_f \rightarrow \frac{1}{24} M \ell^2 \omega_i^2 = \underbrace{0}_{\text{M ferma}} + \frac{1}{2} m V_f^2 \rightarrow \frac{1}{\cancel{24}_{12}} \cancel{M} \cancel{\ell^2} \cancel{\omega^2} = \frac{1}{\cancel{2}} \cancel{m} \frac{M^{\cancel{2}} \cancel{\omega_i^2} \ell^{\cancel{4}\,2}}{m^{\cancel{2}}} \\ M v_i = m V_f \rightarrow V_f = \frac{M v_i}{m} = \frac{M \omega_i \ell^2}{m} \end{cases}$$

$$\hookrightarrow m = 12 M \ell^2$$

3 → $V_f = \frac{M \omega_i \ell^2}{m} = \frac{\cancel{M} \omega_i \cancel{\ell^2}}{12 \cancel{M} \cancel{\ell^2}} = \sqrt{\frac{\cancel{24}^{2} g(1-\cos\theta)}{\cancel{12}_{6} \cdot \cancel{12}\, \ell}} = \sqrt{\frac{g(1-\cos\theta)}{6\ell}}$

4 → $\Delta E = -\int_0^d F_A \cdot dr \rightarrow \cancel{-} \frac{1}{\cancel{2}} \cancel{12 M \ell^2} \cdot \frac{\cancel{g}(1-\cos\theta)}{\cancel{6}_{3}\ell} = \cancel{-} \mu_D \cancel{12 M \ell^2} \cancel{g} (d-0) \rightarrow d = \frac{1-\cos\theta}{3\ell\mu_D}$

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$M, V_0, t=0 \rightarrow V_s = \lambda$ Kg/s

1 → $t>0 \rightarrow$ mg non costante → V non costante $m(t) = M + \lambda t$

2 → lungo x : $F = \frac{dP}{dt} = 0 = \frac{d(m(t)V)}{dt} = M\frac{dV}{dt} + \lambda V + \lambda \frac{dV}{dt} t = 0 \rightarrow \frac{dV}{dt} = -\frac{\lambda V(t)}{M+\lambda t} \rightarrow a(t) = -\frac{\lambda V(t)}{M+\lambda t}$

$\hookrightarrow V(t) =$