RIPASSONE

SISTEMA DINAMICO:

T.C.

$$\begin{cases} \dot{x} = A(t)x(t) + B(t)u(t) \\ y(t) = C(t)x(t) + Du(t) \end{cases}$$

T.D.

$$\begin{cases} x(k+1) = f(x(k), u(k), k) \\ y(k) = g(x(k), u(k), k) \end{cases}$$

VAR. CONTROLLATA: y

VAR. DI CONTROLLO: u

- SEGNALE DI RIF: w

- SISO: 1 ing, 1 uscita
- MIMO: moltep. ing e uscite
- STRETT. PROPRIO: $y = g(x(t), t)$, no $u(t)$
- STAZIONARIO: $y = g(x(t), u(t))$, no t
- NON-DINAMICO: $y = (u(t), t)$, no $x(t)$

es. $p_1(t), p_2(t)$ posizione $\quad K_1, K_2$ cost. elastiche

$F_1(t), F_2(t)$ su carrelli

1_ sistema MIMO, strett. proprio, stazionario, dinamico

su m_1: $F_1 + K_2(p_2(t) - p_1(t)) - K_1 p_1(t) = m_1 \cdot \frac{d^2 p_1(t)}{dt^2} \rightarrow \ddot{p}_1(t) = \frac{K_2}{m_1} p_2(t) - \frac{K_2+K_1}{m_1} p_1(t) + \frac{F_1}{m_1}$

su m_2: $F_2 - K_2(p_2 - p_1) = m_2 \frac{d^2 p_2}{dt^2}$

$\rightarrow \ddot{p}_2(t) = \frac{K_2}{m_2} p_1(t) - \frac{K_2}{m_2} p_2(t) + \frac{F_2}{m_2}$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_2+K_1}{m_1} & 0 & \frac{K_2}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{m_2} & 0 & -\frac{K_2}{m_2} & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$D = 0$

$K_{1,2} = 1$; $m_1 = 1$, $m_2 = 2$

$x_2 = \dot{p}_1$, $x_1 = p_1$, $x_3 = p_2$, $x_4 = \dot{p}_2$, $u_1 = F_1$, $u_2 = F_2$

$$\begin{cases} \dot{x}_1 = x_2(t) \\ \dot{x}_2(t) = -2x_1(t) + x_3(t) + u_1 \\ \dot{x}_3 = x_4(t) \\ \dot{x}_4(t) = \frac{1}{2}x_1 - \frac{1}{2}x_3 + \frac{u_2}{2} \\ y_1(t) = x_1(t) \\ y_2(t) = x_3(t) \end{cases}$$

→ **SISTEMA LINEARE:**

T.C.

$$\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)u(t) \\ y(t) = C(t)x(t) + D(t)u(t) \end{cases}$$

T.D.

$$x(k+1) = A(k)x(k) + B(k)u(k)$$
$$y(k) = C(k)x(k) + D(k)u(k)$$

→ **EQUILIBRIO:**

T.C.: $u(t) = \bar{u}$, $\dot{x}(t) = 0$, $\bar{y} = g(\bar{x}, \bar{u})$

T.D.: $u(k) = \bar{u}$, $\bar{x}(k+1) = \bar{x}(k)$, $y(k) = \bar{y}$

$\bar{x} = f(\bar{x}, \bar{u})$

$\bar{y} = g(\bar{x}, \bar{u})$ trovo pti di equilibrio

→ **FORMULA LAGRANGE:**

T.C.:

$$x(t) = x_{t_0} e^{a(t-t_0)} + \int_{t_0}^{t} e^{a(t-\tau)} b u(\tau) \, d\tau$$

$y(t) = c\,x(t) + d\,u(t)$

da la soluzione $a, b, c, d \in \mathbb{R}$

SE A, B, C, D matrici: $x(t)$ = uguale ma con mat.

$$y(t) = Ce^{A(t-t_0)} x_{t_0} + C\int_{t_0}^{t} e^{A(t-\tau)} Bu(\tau)\, d\tau + Du(t)$$

$\hookrightarrow e^{At} = I + At + A^2\frac{t^2}{2!} + A^3\frac{t^3}{3!} + \ldots$

T.D.:

$$x(k) = A^{k-k_0} x(k_0) + \sum_{i=k_0}^{k-1} A^{k-i-1} Bu(i)$$

$$y(k) = CA^{k-k_0} x_{k_0} + C\sum_{i=k_0}^{k-1} A^{k-i-1} Bu(i) + Du(k)$$

• guad. statico è: $\frac{\bar{y}}{\bar{u}} = C(I-A)^{-1}B + D$

PER ENTRAMBI

$= \mathcal{L}^{-1}[(sI-A)^{-1}]$

→ LINEARIZZAZIONE:

$$\delta\dot{x}(t) = f(\bar{x}+\delta x(t), \bar{u}+\delta u(t))$$

$$\bar{y}+\delta y(t) = g(\bar{x}+\delta x(t), \bar{u}+\delta u(t))$$

sost. $x(t) = \bar{x}+\delta x(t)$ → eq.

$$\delta x(k+1) = \frac{\partial f}{\partial x}\Big|_{\substack{x=\bar{x}\\u=\bar{u}}} \delta x(k) + \frac{\partial f}{\partial u}\Big|_{\sim} \delta u(k)$$

$$\delta y(k) = \frac{\partial g}{\partial x}\Big|_{\sim} \delta x(k) + \frac{\partial g}{\partial u}\Big|_{\sim} \delta u(k)$$

$\bar{x}, \bar{u}$ pto di equilibrio

→ SEGNALI: periodico

$$f(t) = \sum_{m=-\infty}^{+\infty} F_m e^{jm\omega_0 t} \quad \hookrightarrow \text{coeff.} \qquad f(t)=f(t+T) \quad \omega_0 = \frac{2\pi}{T}$$

$$F_m = \frac{1}{T}\int_T f(t) e^{-jm\omega_0 t} dt$$

$$f(k) = \sum_N F_m e^{jm\theta_0 t} \qquad f(k)=f(k+N) \quad \theta_0 = \frac{2\pi}{N}$$

N ↪ tipo il periodo

$$F_m = \frac{1}{N}\sum_N f(k) e^{-jm\theta_0 k}$$

→ TRASF. DI FOUR:

$$F(j\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(j\omega) e^{j\omega t} dt$$

$$F(e^{j\theta}) = \mathcal{F}^*[f(k)] = \sum_{k=-\infty}^{+\infty} f(k) e^{-j\theta k}$$

$$f(k) = \frac{1}{2\pi}\int_{2\pi} F(e^{j\theta}) e^{j\theta k} d\theta$$

→ TRASF. LAPLACE:

$$F(s) = \int_{-\infty}^{+\infty} f(t) e^{-st} dt$$

$$f(t) = \frac{1}{2\pi}\int_{\sigma-i\infty}^{\sigma+i\infty} F(s) e^{st} ds \quad \text{inversa}$$

→ TRASF. ZETA:

$$F(z) = \sum_{-\infty}^{+\infty} f(k) z^{-k}$$

$$f(k) = \frac{1}{2\pi j}\oint F(z) z^{k-1} dz$$

→ RITARDO:

$$\mathcal{L}[f(t)] = F(s) \rightarrow \mathcal{L}[f(t-\tau)] = e^{-\tau s} F(s)$$

$$\mathcal{L}[e^{\alpha t} f(t)] = F(s-\alpha)$$

$$\mathcal{Z}[f(k)] = F(z) \rightarrow \mathcal{Z}[f(k-1)] = z^{-1}F(z)$$

→ ANTICIPO: $\mathcal{L}[f(k)] = F(z) \longrightarrow \mathcal{L}[f(k+1)] = zF(z)$

→ DERIVAZ:

$$\mathcal{L}[\dot{f}(t)] = sF(s) - f(0) \quad \left(\text{se + volte } -\sum_{i=1}^{m} s^{m-i} \frac{d^{i-1}f(t)}{dt^{i-1}}\Big|_{t=0}\right)$$

$$\mathcal{L}[k f(k)] = -z\frac{dF(z)}{dz}$$

→ INTEG: $\mathcal{L}[\int f(\tau)dt] = \frac{1}{s}F(s)$

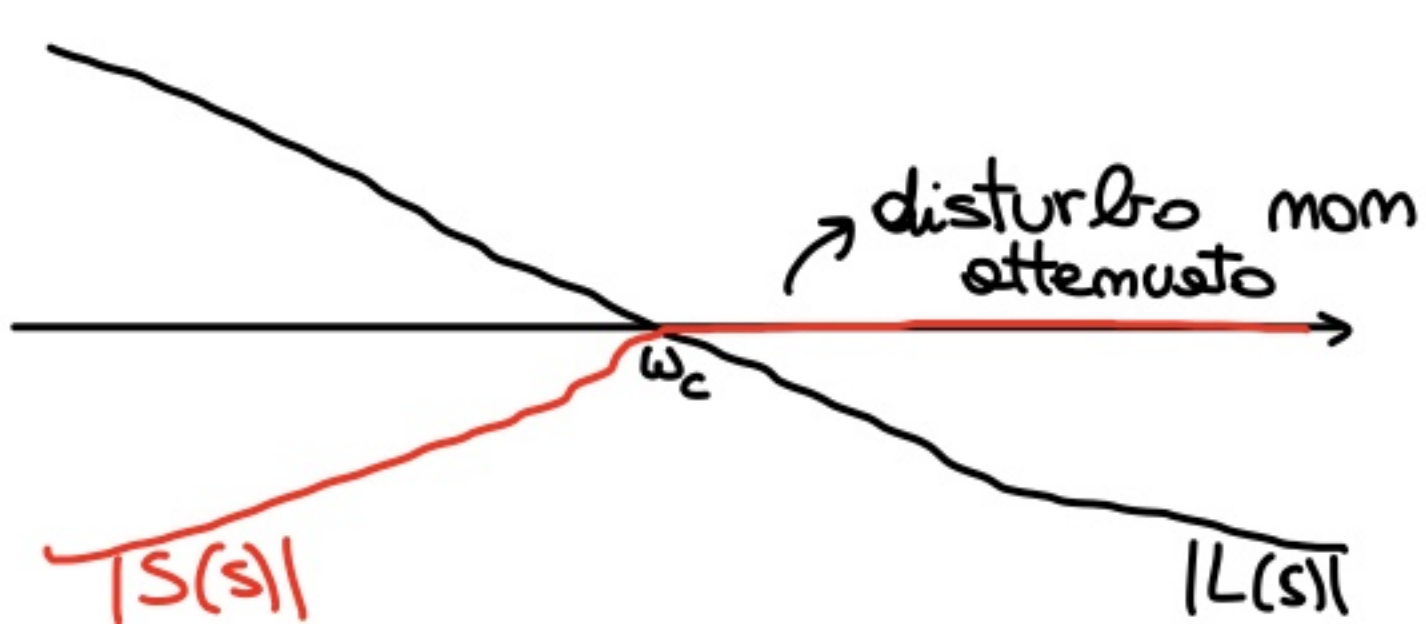
→ FdT: $G(s) = C(sI-A)^{-1}B+D$ $\qquad$ $G(z) = C(zI-A)^{-1}B+D$

→ F GENERALE: $\mathcal{L}[t^m e^{\alpha t} sca(t)] = \dfrac{m!}{(s-\alpha)^{m+1}}$

→ FUNZIONI DI SENSIBILITÀ:

• **sensitività:** $\dfrac{Y(s)}{D(s)} = S(s) = \dfrac{1}{1+L(s)}$ $\qquad$ idealmente $S(s)=0$

$$\lim_{s\to 0} S(s) = \begin{cases} \frac{1}{1+\mu} & , g=0 \\ 0 & , g>0 \\ 1 & , g<0 \end{cases} \qquad |S(j\omega)| = \frac{1}{1+|L(j\omega)|} = \begin{cases} \frac{1}{|L(j\omega)|} & \omega \ll \omega_c \\ 1 & \omega \gg \omega_c \end{cases}$$



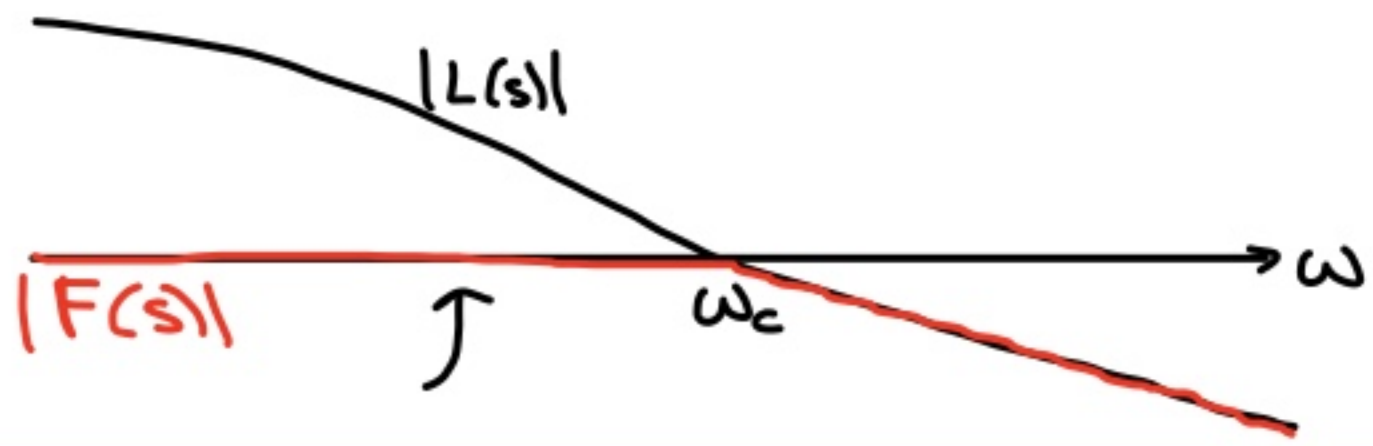
[FILTRO PASSA-ALTO]

• **sensit. complement.**: $\dfrac{Y(s)}{N(s)} = F(s) = \dfrac{-L(s)}{1+L(s)}$ idealmente $F(s)=1$

disturbo su retroaz. $\rightarrow N(s)$

$$\lim_{s\to 0} F(s) = \begin{cases} \dfrac{\mu}{1+\mu} & , g=0 \\ 0 & , g<0 \\ 1 & , g>0 \end{cases}$$

$$|F(j\omega)| = \frac{|L(j\omega)|}{|1+L(j\omega)|} = \begin{cases} 1 & \omega \ll \omega_c \\ |L(j\omega)| & \omega \gg \omega_c \end{cases}$$



$\rightarrow$ se $\varphi_m < 75°$: $T_a = \dfrac{5}{\xi \omega_c}$ $\xi = \dfrac{\varphi_m}{100}$

$$F(s) \approx \begin{cases} \dfrac{1}{1+s/\omega_c} & \varphi_m > 50 \\ \dfrac{\omega_c^2}{s^2+2\xi\omega_c s+\omega_c^2} & \varphi_m < 50° \end{cases}$$

• **sensit. del controllo**: $\dfrac{U(s)}{Y^o(s)} = Q(s) = \dfrac{R(s)}{1+R(s)G(s)} = R(s)S(s) = F(s)G^{-1}(s)$

$$|Q(j\omega)| = \frac{|R(j\omega)|}{|1+L(j\omega)|} \begin{cases} \dfrac{1}{|G(j\omega)|} & \omega \ll \omega_c \\ |R(j\omega)| & \omega \gg \omega_c \end{cases}$$

$\rightarrow$ CONTROLLORI P, PI, PID, PD:

• **PID**: $R(s) = K_P + \dfrac{K_I}{s} + K_D s = K_P\left(1 + \dfrac{1}{sT_I} + sT_D\right)$ $\rightarrow$ aggiungo polo in alta freq. per compensare 2 zeri

$\rightarrow$ semplicità di realizzazione, efficacia, affidabilità ed economicità, semplicità taratura parametri o taratura automatica.

• **PI**: $R(s) = \dfrac{K_P s + K_I}{s} = K_P \dfrac{1+T_I s}{sT_I}$ cancello polo lento