Digital technology

Numbering systems

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Numbering systems

Conversion from decimal system to some other system

- The conversion method depends on the number – is it an integer or fraction
- If the number consists of both parts, it has to be converted in two parts
- Integers
  - The converted number is divided by the base number of the wanted system
    - Remainder is least significant number in the new numbering system
    - Quotient is divided again by the base number of the wanted system
    - The remainder is the next least significant number in the new numbering system
  - Etc. Repeated … until the quotient of the division is zero -> job done.
    - The latest quotient is most significant number in the new numbering system

Example of conversion of an integer to binary system

- Converting number 47_{10} into binary system i.e., 47_{10} -->X_2,

- 47/2=23 and remainder **1** LSB
- 23/2=11 and remainder 1
- 11/2=5 and remainder 1
- 5/2=2 and remainder 1
- 2/2=1 and remainder 0
- 1/2=**0** and remainder **1** MSB

- LSB=Least Significant Bit
- MSB=Most Significant Bit
- The conversion results in 47_{10}=**1**0111**1**$_2$

Example of conversion of an integer to hexadecimal system

- Converting 47_{10} to hexadecimal system i.e., 47_{10} -->X_{16},

- 47/16=2 and remainder **15 or F** LSB
- 2/16=**0** and remainder **2** MSB

- LSB=Least Significant Bit
- MSB=Most Significant Bit

- The conversion results in 47_{10}=**2F**$_{16}$

Conversion of a fraction from decimal system to some other system

- The number to be converted is multiplied by the base number of the wanted numbering system
- The integer of the product is most significant number in the new numbering system
- The fraction of the product is multiplied again by the base number of the new wanted numbering system
- The integer of the product is the next most significant number in the new numbering system etc. Repeated until the product of multiplication is 1, and the conversion has thus been completed.
- The last integer is least significant number in the new numbering system
  - Sometimes the conversion results in an infinite succession of numbers -> the conversion is cut in a suitable cut-off point
  - Suitable cut-off point is e.g., three numbers longer succession of numbers than the original number to be converted

Conversion of a fraction from decimal system to some other system, example

- E.g.: Convering $0{,}485_{10}$ --> X_2,

- 0,485*2=**0**,97 integer is **0** MSB
- 0,97*2=**1**,94 integer is **1**
- 0,94*2=1,88 integer is 1
- 0,88*2=1,76 integer is 1
- 0,76*2=1,52 integer is 1
- 0,52*2=1,04 integer is 1 (Note, result very near the correct one)
- 0,04*2=0,08 integer is 0
- 0,08*2=0,16 integer is 0
- Etc.

- The conversion results in $0{,}485_{10}$=0,01111100,,,$_2$ //0.485 englannissa desimaalipiste

- Checking the result by converting it back to decimal system ,
- $0{,}011111_2 = 0*2^{-1}+1*2^{-2}+1*2^{-3}+1*2^{-4}+1*2^{-5}+1*2^{-6} = 0+0{,}25+0{,}125+0{,}0625+0{,}03125 = 0{,}484375_{10}$

Conversion of a fraction from decimal system to some other system, example

- Example: Convering $0{,}485_{10}$ --> X_{16}, **//desimaalipiste englannissa! Huomio tämä**

- 0,485*16=**7**,76 integer is **7** MSB
- 0,76*16=**12**,16 integer is **12** i.e., in hexademical system **C**
- 0,16*16=2,56 integer is 2
- 0,56*16=8,96 integer is 8
- 0,96*16=15,36 integer is 15 i.e. F
- 0,36*16=5,76 integer is 5 (note, the result is already very close to correct one)
- Etc.

- Conversion results in $0{,}485_{10}=0{,}7C28F5\ldots_{16}$

- Checking the result by converting back to decimal system ,
- $0{,}7C28F5_{16} = 7*16^{-1}+C*16^{-2}+2*16^{-3}+8*16^{-4}+F*16^{-5}+5*16^{-6} = 0{,}484999954700469970 7_{10}$

Exercises

2. Convert

a) 835_{10} --> X_2

b) 835_{10} --> X_8

c) 835_{10} --> X_{16}

d) $0{,}835_{10}$ --> X_2

e) $8{,}35_{10}$ --> X_8

f) $83{,}5_{10}$ --> X_{16}