Mathematics in Economics

How is mathematics present in the Nash Equilibrium?

Subject: Mathematics

Word Count: 2562 words

Contents

Introduction:

Mathematics are necessary for economics for two big reasons: clarity of argument and quantitative prediction.

The reason why economists adopted mathematics as the language to create their models is that it is super super hard to be imprecise with mathematics. Sure, it can be hard for a layman or even fellow economists to undersrand, but when you use mathematics as a language to model economic phenomena, it forces you to be very explicit about your (often unrealistic) assumptions and how you prove your results. It's really easy to claim in blocks of text that certain things cause certain economic phenomena and obfuscate it with the English language to hide any logical fallacies/inconsistencies. However, it is really difficult to do this with mathematics, so when writing a proof of something, it's really easy to pick up where a person's model is actually false. had always been aware of the mathematical nature of Economics as a subject. A subject like Economics, which serves as a description of the entire economy, has to be quantitative for it to be useful. Sure, we can always make qualitative statements about the behaviors of consumers and producers but, at the end of the day, what we really want is a quantitative model of the economy. That is where mathematics comes into the picture. There is one fundamental concept in Economics that is the basis for everything else. Every other quantity you define is going to be affected by this concept in several ways. The key concept that I am referring to is Optimization. Suppose that you take some form of action. It doesn't matter what that action is. The point is that when you do that, there will both be a Benefit and Cost to that action. Now, the individual nature of these two quantities is not important. We know what they are intuitively.

Background:

Economics is the study of the use of economic resources in the marketplace. Free market economies often rely on this information to gauge current economic conditions. Economic analysis is a primary tool used to evaluate a nation's economy. Economic analysis is commonly defined as a systematic approach to determining the optimum use of scarce or limited economic resources. The analysis often includes several assumptions or constraints found in the economic marketplace.

Calculus is the most common type of math found in economics. Calculus includes the use of various formulas to measure limits, functions and derivatives. Many economists use differential calculus when measuring economic information. Differential calculus is the specific measuring of a derivative that relates to a specific function. In basic terms, a function usually represents a straight line known as a tangent. This represents a functions normal operation. The derivative is any change in the tangent that represents a deviation (up or down) in the original line.

Business owners can use economic information to help forecast expected sales for their business operations. The use of economic analysis in a business is an important management tool when making business plans and decisions. Business owners do not usually require the heavy use of technical math concepts when breaking down economic information. Owners can use the information provided by economists and make basic decisions regarding business operations from these economic models.

One of the important theories from Economics that includes Mathematics is the Game theory that is the Nash Equilibrium or also known as the prisoner's dilemma. This game theory has become an important tool in microeconomics and is based heavily on the work of American mathematician and economist John F. Nash. It is a mathematical technique analysing the behaviour of decision-makers who are dependent on each other, and who display strategic behaviour. The general idea for the game is that it shows how two rational decision-makers, who use strategic behaviour to maximise profits by trying to guess the rival's behaviour, may end up being collectively worse off. The final position that results from the game is called a Nash equilibrium.

Body:

Taking two oligopolistic firms in the same industry: ‘A’ and ‘B’. Each firm must decide on a pricing strategy, i.e. what price to charge consumers for its space travel services, and can choose either a high-price or a low-price strategy. Each firm is interested in making its own profit as large as possible, but its profit will depend on the particular combination of pricing strategies that the two forms choose.

The below figure shows four possible combinations of pricing strategies and their corresponding profit outcomes (called ‘payoffs’) for the two firms. This figure represents a payoff matrix. For example, of both A and B choose the high-price strategy, in box 4, each will have the profit of 40 lakhs Rupees (taking the currency as Rupees). Box 3 shows the profit outcomes of different price strategies; A with a low-price strategy makes 70 lakhs Rupees, and B with a high-price strategy makes 10 lakhs. The reason why the low-price firm makes much higher profits is that by charging a low price it captures a large portion of sales from its rival.

Suppose the two firms begin in box 1, where they are competing with each other on the basis of price (price competition) and therefore have a low price, leading to a low profit of 20 lakhs Rupees each. Realising that they will both be better off if they enter into a collusive agreement and charge a high price, they collaborate and agree to adopt a high-price strategy, thus entering box 4 where each one earn profits of 40 lakhs Rupee.

Now each firm faces a dilemma. From firm A’s point of view (though B is thinking along the same lines). A realises that by sticking to the agreement it will continue to earn 40 lakhs Rupees. On the other hand, A also realises that by cheating, in other words secretly breaking the agreement, and charging a lower price, it can earn the much higher profit of 70 lakhs Rupees, while B earns only 10 million Rupees. In addition, A realises that B might break the agreement, in which case A will find itself making only 10 lakhs Rupees (worse than even when It was in competition with B, making 20 lakhs Rupees).

As A tries to ‘outguess’ B, it is likely to cut its price to beat B to the higher profits, but since B is thinking along the exactly same lines, they are both likely to adopt the low-price strategy, in which case they will end up in box 1 where they both have low prices and low profits. This is the Nash equilibrium, in which both firms become worse off.

The Nash equilibrium shows that there is sometimes a conflict between the pursuit of individual self-interest and the collective firm interest. This conflict is the prisoner’s dilemma. Although the firms could be better off by cooperating, each firm, trying to make itself better off, ends up making both itself and its rivals worse off.

The game also shows real world aspects of oligopolistic firms.

The Nash equilibrium is useful not just when it is itself an accurate predictor of how people will behave in a game but also when it is not, because then it identifies situations in which there is a tension between individual incentives and other motivations. A class of problems that have received a good deal of study from this point of view is the family of “social dilemmas,” in which there is a socially desirable action that is not a Nash equilibrium. Indeed, one of the first responses to Nash's definition of equilibrium gave rise to one of the best known models in the social sciences, the Prisoners' Dilemma. This model began life as a simple experiment conducted in January 1950 at the Rand Corporation by mathematicians Melvin Dresher and Merrill Flood, to demonstrate that the Nash equilibrium would not necessarily be a good predictor of behaviour. Each of the two players in that game had to choose one of two decisions, which, for expositional purposes, we will call “cooperate” or “defect.” The game specifies the payoffs for each player for each of the four possible outcomes: (cooperate, cooperate), (cooperate, defect), (defect, cooperate), and (defect,

defect). The payoffs used were such that each player's best counter to either of the other's choices was to defect, but both players would earn more if they both cooperated than if they both chose their equilibrium decision and defected.

A Nash Equilibrium is a set of strategies that players act out, with the property that no player benefits from changing their strategy. Intuitively, this means that if any given player were told the strategies of all their opponents, they still would choose to retain their original strategy. For example, in the game of trying to guess 2/3 of the average guesses, the unique Nash equilibrium is (counterintuitively) for all players to choose 0.

The simplest example of Nash equilibrium is the coordination game, in which both players benefit from coordinating but may also hold individual preferences. For instance, suppose two friends wish to plan an evening around either partying or watching a movie. Both friends prefer to engage in the same activity, but one prefers partying to movies by a factor of 2, and the other prefers movies to partying by the same ratio. This can be modelled by the following payoff matrix:

	Party	Movie
Party	2,1	0,0
Movie	0,0	1,2

where the payoff vector is listed under the appropriate strategy profile (the first player's strategies are listed on the left). In this case, both {Party, Party} *and* {Movie, Movie} are Nash equilibria, as neither side would choose to deviate when informed of the other's choice.

The most famous example of Nash equilibrium, however, is the *Prisoner's dilemma* problem, in which each of two prisoners have the choice of "cooperating" with the other prisoner by keeping quiet, or "defecting" by confessing. If both prisoners cooperate, they will face little jail time, but if exactly one of them defects, the defector will immediately go free and the co-operator will face lots of jail time. The catch is that if both prisoners choose to defect, they will both face a moderate amount of jail time. This can be modelled by the payoff matrix where lower jail sentences have higher payoffs. In this scenario, there is exactly one Nash equilibrium: both players choose to defect -- in any other case, a cooperating prisoner would choose instead to defect. This is despite the fact that both prisoners would improve their situation by both cooperating, meaning that the Nash equilibrium is globally inferior to the "both cooperate" strategy.

We begin with an example of a game. We have two players, Alice (abbreviated as A and referred to by the pronoun “she”) and Bob (B, “he”) each of which has the choice between two actions. For the choice ai of A and bj of B, the (i, j) entry in the table lists the pay-off of A before, and the one of B after the comma.

	b_1	b_2
a_1	3, 2	1, 3
a_2	2, 1	0, 0

So, how should the two players reason and act in order to maximize their payoff, assuming that both know the structure of the game and the pay-off matrix? We assume at this point that the two players play simultaneously. The first observation is that for player A, given an action of B, the first row is always better than the second. One says that action a1 dominates a_2. So, let us reason that therefore she should disregard action a_2 and play a_1 in any case. When B realizes this, he should play b_2. Thus, A will get the pay-off 1, whereas B gets 3. This represents a so-called Nash equilibrium, meaning that neither player can unilaterally change her/his action without reducing her/his pay-off. If A changed from a_1 to a_2, but B keeps b_2, her pay-off would be reduced from 1 to 0. If B switched from b_2 to b_1, while A continues to play a_1, his pay-off would be reduced from 3 to 2. Obviously, this equilibrium leaves B better off than A. If the game were played sequentially instead of simultaneously, with A playing first, she should choose a_2 in place of a_1, even though that action is dominated in the simultaneous game, as this would force B to play b_1, giving A the pay-off 2 which is higher than 1 as achieved in the Nash equilibrium.

This observation reveals a problem with the dominance argument that A should disregard a_2 in the simultaneous game. After all, if she can make it plausible to B that she will play a_2, this would force him to play b_1. The dominance argument compares the outcome of each action against the same action of the opponent. The opponent, however, will react differently to the different actions of A. He will play b_2 against a_1, but b_1 against a_2, and the latter is better for A. When, however, B can move first, he should play b_2, forcing A to play a_1, which is the above Nash equilibrium which is optimal for B.

We now consider the situation where A can move first, but B cannot observe A's move, B might reason that A had played a_2 in order to force him to play b_1, and he should correspondingly do so. Now, however, A could think that because of this reasoning, B will play b_1 anyway, and he could therefore decide to play a_2 to maximize her pay-off. When then, in turn, B anticipates that reasoning, he could then play b_2. However, whenever A believes B to play b_2, she should play a_1. And hence the consistent beliefs repeat themselves. Whenever B believes A to play a_1, he should play b_2, and whenever A expects that B will play b_2, she should play a_1. And since any chain of consistent higher order beliefs will eventually arrive at that point, it seems that (a_1, b_2) will be the only equilibrium. We need a formal definition of an equilibrium here, however, in order to substantiate that claim. Incidentally, in this example, it helps B to be ignorant about A's move, as otherwise A could have forced him to play b_1 which leads to a worse pay-off for him. Thus, in such games, it can be disadvantageous to acquire more information about the opponent. – One should be careful with the interpretation of this finding, however. What harms B is not that he has the information, but the fact that A knows that he has that information. That is, A also possesses some additional information. If A did not know that B knows her move, then B would not be at a disadvantage. – Conversely, A should try to transmit the information about her move to B, but when B cannot verify the correctness of that information, we are again in a situation to which the preceding analysis applies. We can gain further insight by modifying the pay-off matrix. For instance, when we consider

	b_1	b_2
a_1	1, 2	1, 3
a_2	2, 1	0, 0

that is, we lower the pay-off for A when playing a_1 against b_1, then it is perfectly reasonable for A to play a_2 to which B should react with b_1. Thus, when we remove the temptation for A to play a_1 instead of a_2 against b_1, we improve her position.

Bibliography:

1.
https://brilliant.org/wiki/nash-equilibrium/

2.
https://www.linkedin.com/learning/managerial-economics/game-theory

3.
https://www.haverford.edu/mathematical-economics

4.
https://www.lse.ac.uk/study-at-lse/Undergraduate/Degree-programmes-2022/BSc-Mathematics-with-Economics

5.
https://corporatefinanceinstitute.com/resources/knowledge/economics/nash-equilibrium-game-theory/

6.
Theory of Games and Economic Behavior by John von Neumann and Oskar Morgenstern

7.
Existence and Stability of Nash Equilibrium by Guilherme Carmona

8.
Game Theory: A Very Short Introduction by Ken Binmore

9.
Game Theory: An Introduction by Steven Tadelis