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Signal Processing Lab

Subject- Digital Signal Processing

EXPERIMENT NO. 3

Aim: Study of Discrete Fourier Transform (DFT) and its inverse.

Software Used: V-Labs Portal (IIT Bombay)

Theory: The discrete-time Fourier transform (DTFT) of a sequence is a continuous function of ω, and repeats with period 2π. In practice we usually want to obtain the Fourier components using digital computation, and can only evaluate them for a discrete set of frequencies. The discrete Fourier transform (DFT) provides a means for achieving this.

The DFT is itself a sequence, and it corresponds roughly to samples, equally spaced in frequency, of the Fourier transform of the signal. The discrete Fourier transform of a length N signal x[n], n = 0, 1... N - 1 is given by

$$x[k] = \sum_{n=0}^{n-1} x[n] e^{-j\left(2\frac{\pi}{N}\right)kn}$$

This is the analysis equation. The corresponding inverse equation is

$$x[n] = \frac{1}{N} \sum_{n=0}^{n-1} x[k] e^{j\left(2\frac{\pi}{N}\right)kn}$$

With this notation the DFT analysis-inverse pair becomes

$$W_N = e^{-j\left(2\frac{\pi}{N}\right)}$$

With this notation the DFT analysis-inverse pair becomes

$$x[k] = \sum_{n=0}^{n-1} x[n] w_n^{kn}$$

$$x[n] = \frac{1}{N} \sum_{n=0}^{n-1} x[k] w_n^{-(kn)}$$

An important property of the DFT is that it is cyclic, with period N, both in the discrete-time and discrete-frequency domains. For example, for any integer r

$$X[K+rN] = \sum_{n=0}^{n-1} x[n] w_N^{K+rN}.\, n = \sum_{n=0}^{n-1} W_N^{Kn} (W_N^N)^{rn}$$
$$= \sum_{n=0}^{n-1} x[n] W_N^{Kn} = X[K],$$

since

$$WN^N = e^{-j\left(2\frac{\pi}{N}\right)N} = e^{-j2\pi} = 1$$

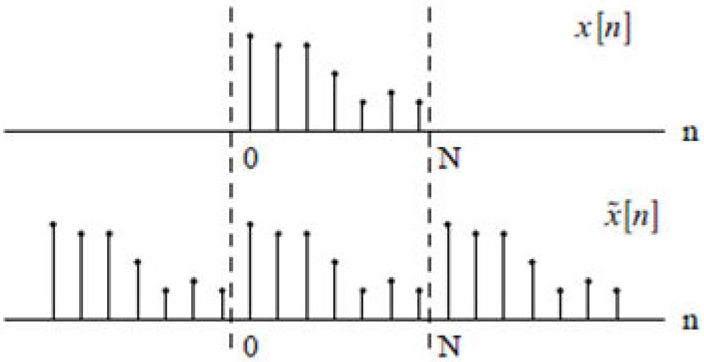
Similarly, it is easy to show that x[n + rN] = x[n], implying periodicity of the inverse equation. This is important - even though the DFT only depends on samples in the interval 0 to N - 1, it is implicitly assumed that the signals repeat with period N in both the time and frequency domains.

To this end, it is sometimes useful to define the periodic extension of the signal X[n] to be

$$x[n] = x[n \bmod N] = x[((n))N]$$

Here n mod N and((n))N are taken to mean n modulo N, which has the value of the remainder after n is divided by N. Alternatively, if n is written in the form n = kN + l for 0 ≤ l ≤ N, then

n mod N = ((n))N = l



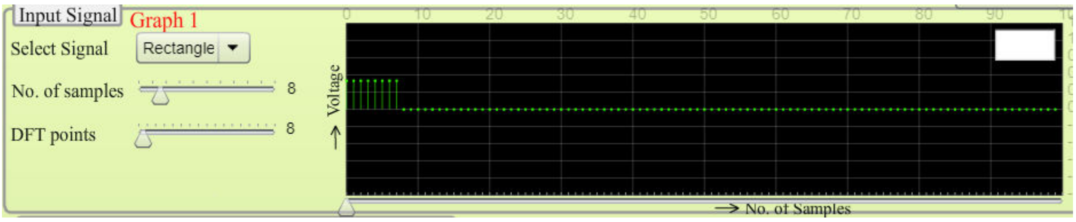
Similarly, the periodic extension of X[k] is defined to be

$$X[k] = X[k \bmod N] = X[((K))n]$$

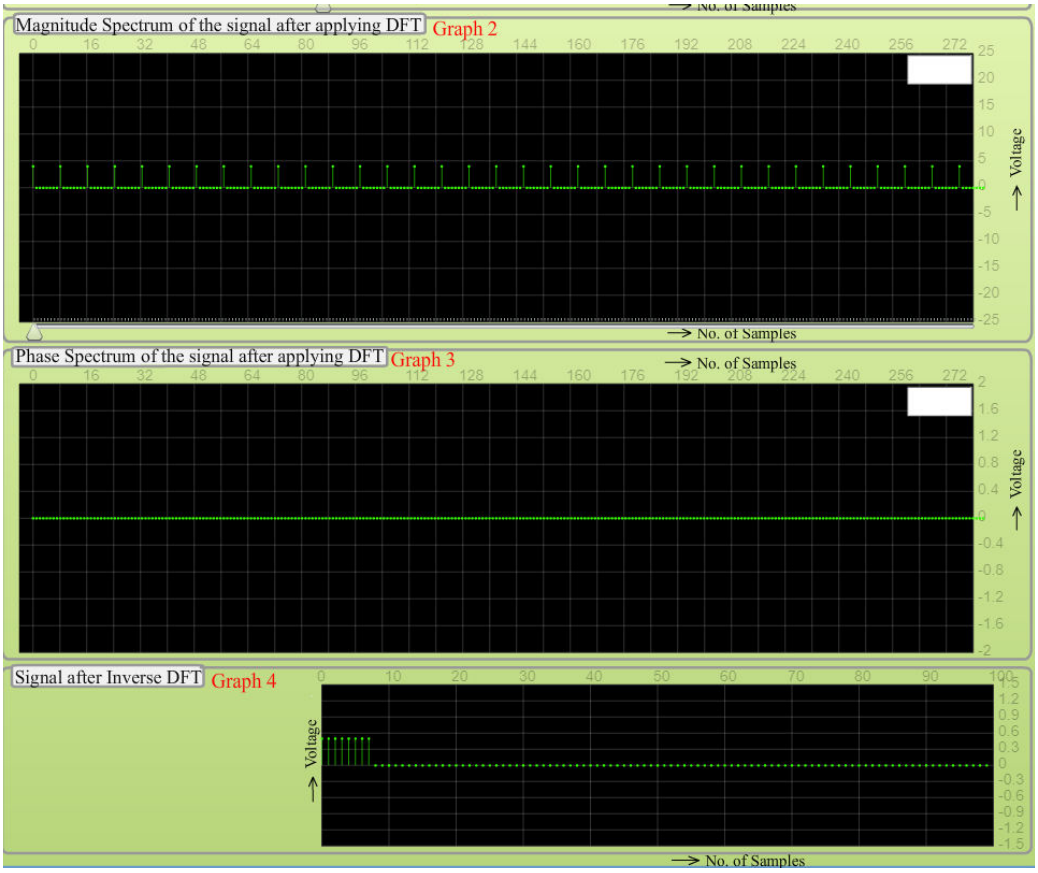
It is sometimes better to reason in terms of these periodic extensions when dealing with the DFT. Specifically, if X[k] is the DFT of x[n], then the inverse DFT of X[k] is [n]. The signals x[n] and [n] are identical over the interval 0 to N-1, but may differ outside of this range. Similar statements can be made regarding the transform X[k].

Output:

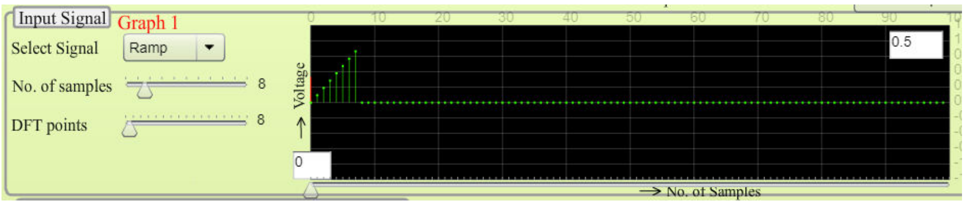
a) Signal Type - Rectangular
Number of Samples = 8
Number of DFT Points = 8



Number of Samples	Input Signal	Magnitude Spectrum of the signal after applying DFT	Phase Spectrum of the signal after applying DFT
0	0.5	4	0
1	0.5	0	0
2	0.5	0	0
3	0.5	0	0
4	0.5	0	0
5	0.5	0	0
6	0.5	0	0
7	0.5	0	0
8	0	4	0
9	0	0	0
10	0	0	0
11	0	0	0
12	0	0	0
13	0	0	0
14	0	0	0
15	0	0	0

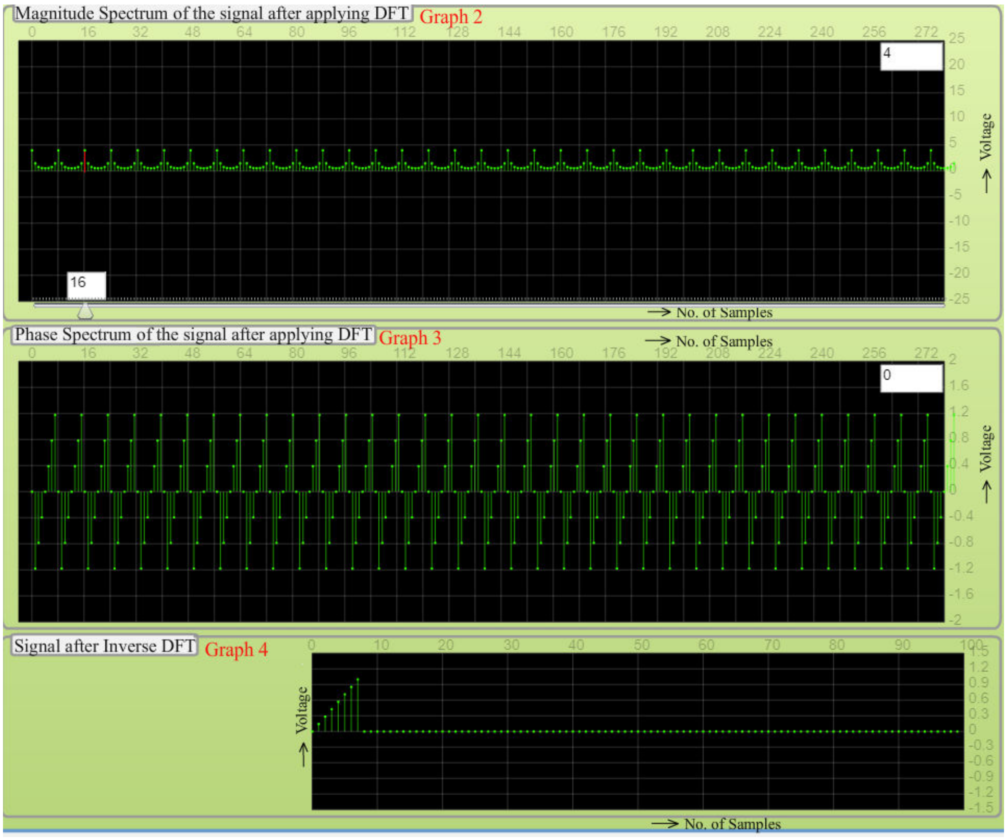


b) Signal Type - Ramp
Number of Samples = 8
Number of DFT Points = 8

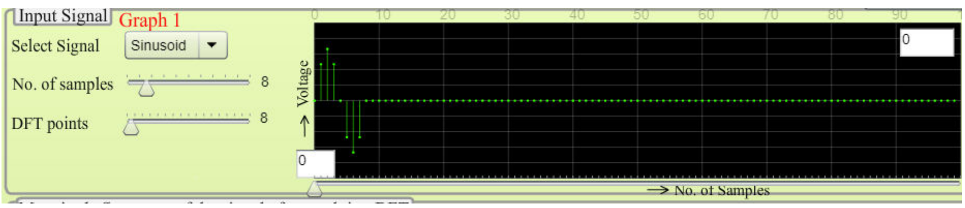


Number of Samples	Input Signal	Magnitude Spectrum of the signal after applying DFT	Phase Spectrum of the signal after applying DFT
0	0	4	0
1	0.142	1.493	-1.1781
2	0.285	0.808	-0.7854
3	0.428	0.618	-0.3927
4	0.571	0.571	0
5	0.714	0.618	0.3927
6	0.857	0.808	0.7854
7	1	1.493	1.1781

8	0	4	0
9	0	1.493	-1.1781
10	0	0.808	-0.7854
11	0	0.618	-0.3927
12	0	0.571	0
13	0	0.618	0.3927
14	0	0.808	0.7854
15	0	1.493	1.1781

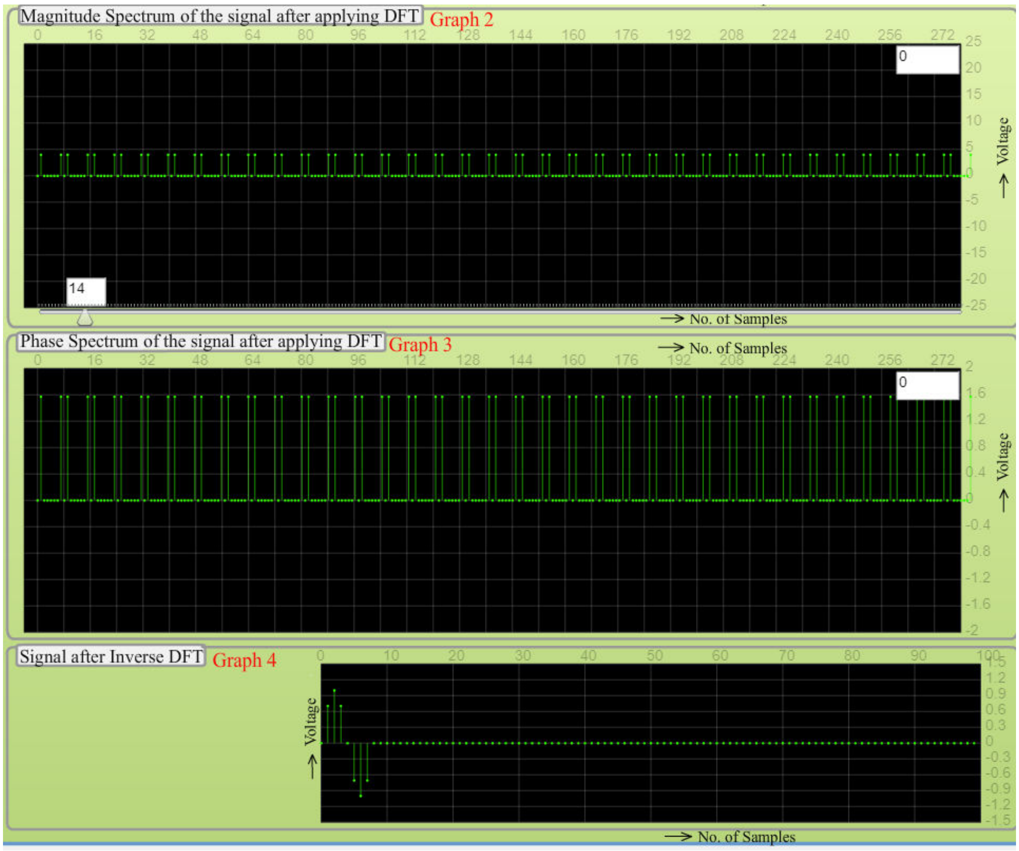


c) Signal Type - Sinusoid
Number of Samples = 8
Number of DFT Points = 8



Number of	Input Signal	Magnitude	Phase Spectrum

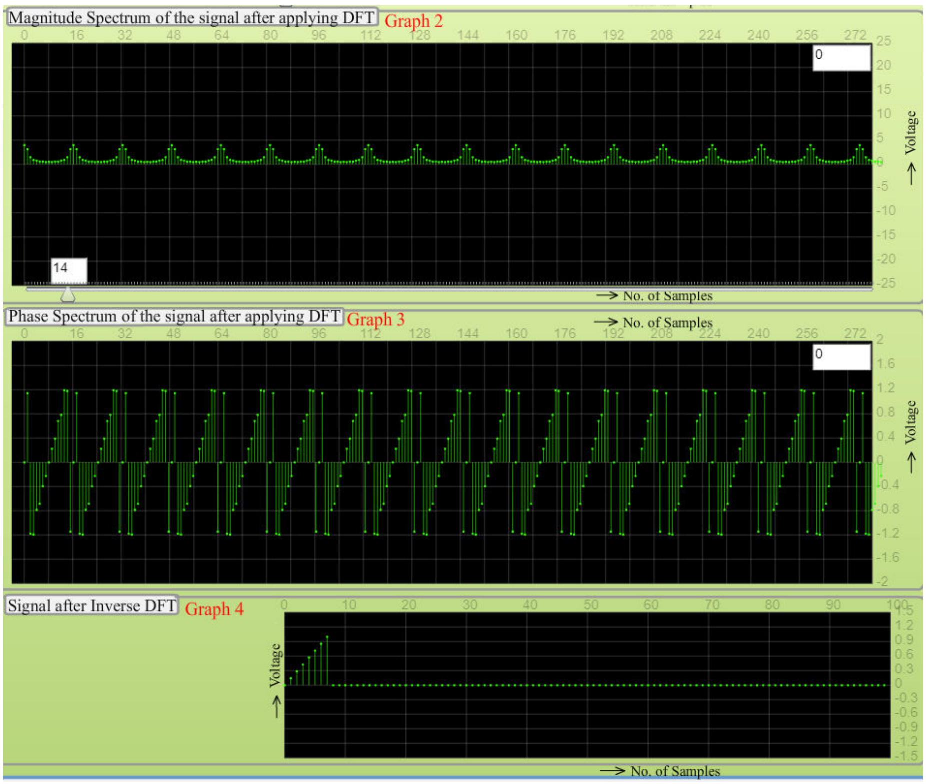
Samples		Spectrum of the signal after applying DFT	of the signal after applying DFT
0	0	0	0
1	0.707	4	1.5708
2	1	0	0
3	0.707	0	0
4	0	0	0
5	-0.707	0	0
6	-1	0	0
7	-0.707	4	1.5708
8	0	0	0
9	0	4	1.5708
10	0	0	0
11	0	0	0
12	0	0	0
13	0	0	0
14	0	0	0
15	0	4	1.5708



d) Signal Type - Ramp

Number of Samples = 8
Number of DFT Points = 16

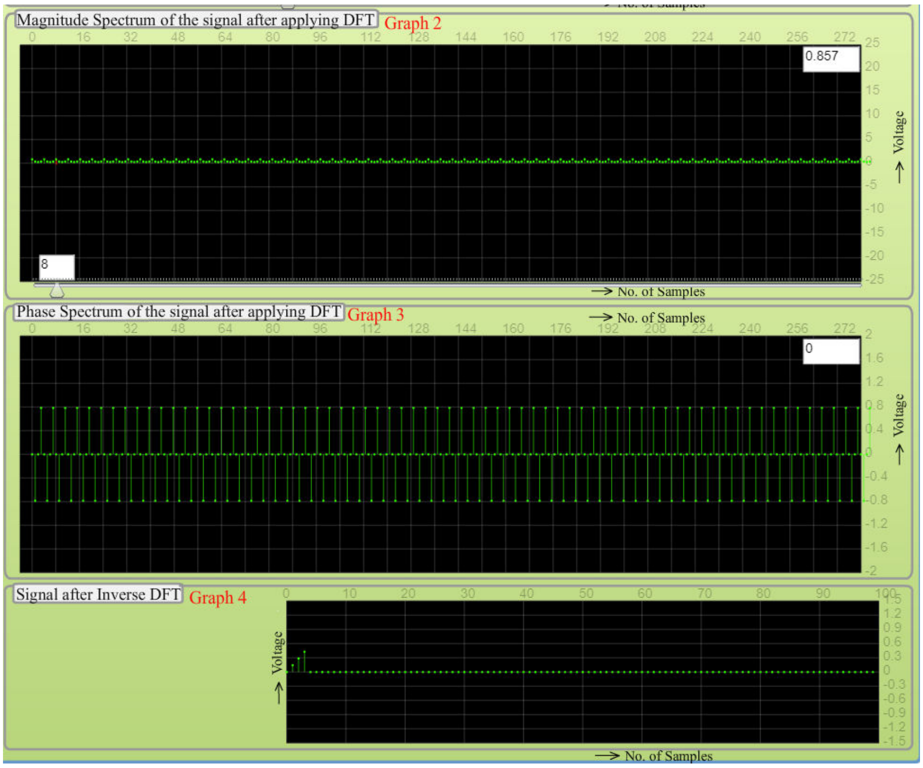
Number of Samples	Input Signal	Magnitude Spectrum of the signal after applying DFT	Phase Spectrum of the signal after applying DFT
0	0	4	0
1	0.142	3.155	1.1443
2	0.285	1.493	-1.1781
3	0.428	0.92	-1.1924
4	0.571	0.808	-0.7854
5	0.714	0.604	-0.6843
6	0.857	0.618	-0.3927
7	1	0.51	-0.225
8	0	0.571	0
9	0	0.51	0.225
10	0	0.618	0.3927
11	0	0.604	0.6843
12	0	0.808	0.7854
13	0	0.92	1.1924
14	0	1.493	1.1781
15	0	3.155	1.1443



e) Signal Type - Ramp

Number of Samples = 8
Number of DFT Points = 4

Number of Samples	Input Signal	Magnitude Spectrum of the signal after applying DFT	Phase Spectrum of the signal after applying DFT
0	0	0.857	0
1	0.142	0.404	-0.7854
2	0.285	0.285	0
3	0.428	0.404	0.7854
4	0.571	0.857	0
5	0.714	0.404	-0.7854
6	0.857	0.285	0
7	1	0.404	0.7854
8	0	0.857	0
9	0	0.404	-0.7854
10	0	0.285	0
11	0	0.404	0.7854
12	0	0.857	0
13	0	0.404	-0.7854
14	0	0.285	0
15	0	0.404	0.7854



Conclusion: This experiment helped us understand what is DFT and inverse DFT by visualizing the amplitude and phase spectrum of the signal in frequency domain.