

| $\#\text{III}(E/\mathbb{Q})$ | Group Invariants | Frequency |
|------------------------------|--------------------|-----------|
| 1 | [1] | 1366261 |
| 4 | [2, 2] | 209051 |
| 9 | [3, 3] | 68948 |
| 16 | [2, 2, 2, 2] | 1110 |
| 16 | [4, 4] | 23418 |
| 25 | [5, 5] | 10422 |
| 36 | [2, 2, 3, 3] | 3287 |
| 49 | [7, 7] | 2519 |
| 64 | [2, 2, 2, 2, 2, 2] | 1 |
| 64 | [2, 2, 4, 4] | 311 |
| 64 | [8, 8] | 1298 |
| 81 | [3, 3, 3, 3] | 25 |
| 81 | [9, 9] | 656 |
| 100 | [2, 2, 5, 5] | 350 |
| 121 | [11, 11] | 307 |
| 144 | [2, 2, 2, 2, 3, 3] | 53 |
| 144 | [3, 3, 4, 4] | 149 |
| 169 | [13, 13] | 147 |
| 196 | [2, 2, 7, 7] | 59 |
| 225 | [3, 3, 5, 5] | 49 |
| 256 | [2, 2, 8, 8] | 30 |
| 256 | [4, 4, 4, 4] | 2 |
| 256 | [16, 16] | 37 |
| 289 | [17, 17] | 26 |
| 324 | [2, 2, 3, 3, 3, 3] | 1 |

Table 1: Frequency of Tate-Shafarevich group orders and types of rank 0 elliptic curve

| $\#\text{III}(E/\mathbb{Q})$ | Group Invariants | Frequency |
|------------------------------|--------------------|-----------|
| 324 | [2, 2, 9, 9] | 6 |
| 361 | [19, 19] | 22 |
| 400 | [2, 2, 2, 2, 5, 5] | 8 |
| 400 | [4, 4, 5, 5] | 11 |
| 441 | [3, 3, 7, 7] | 4 |
| 484 | [2, 2, 11, 11] | 6 |
| 529 | [23, 23] | 11 |
| 576 | [2, 2, 3, 3, 4, 4] | 1 |
| 576 | [3, 3, 8, 8] | 4 |
| 625 | [25, 25] | 7 |
| 625 | Unknown | 3 |
| 676 | [2, 2, 13, 13] | 3 |
| 729 | [3, 3, 9, 9] | 1 |
| 729 | [27, 27] | 1 |
| 784 | [2, 2, 2, 2, 7, 7] | 1 |
| 784 | [4, 4, 7, 7] | 5 |
| 841 | [29, 29] | 2 |
| 961 | [31, 31] | 4 |
| 1024 | [4, 4, 8, 8] | 2 |
| 1089 | [3, 3, 11, 11] | 1 |
| 1225 | [5, 5, 7, 7] | 1 |
| 1681 | [41, 41] | 1 |
| 2209 | [47, 47] | 2 |
| 2500 | [2, 2, 25, 25] | 1 |
| 5625 | [3, 3, 5, 5, 5, 5] | 1 |

Table 2: Frequency of Tate-Shafarevich group orders and types of rank 0 elliptic curve

| $\#\text{III}(E/\mathbb{Q})$ | Grp Inv | Frequency | $\#\text{III}(E/\mathbb{Q})$ | Grp Inv | Frequency |
|------------------------------|--------------------|-----------|------------------------------|--------------------|-----------|
| 1 | [0] | 1366261 | 169 | [13, 13] | 147 |
| 4 | [2, 2] | 209051 | 196 | [2, 2, 7, 7] | 59 |
| 9 | [3, 3] | 68948 | 225 | [3, 3, 5, 5] | 49 |
| 16 | [2, 2, 2, 2] | 1110 | 256 | [2, 2, 8, 8] | 30 |
| 16 | [4, 4] | 23418 | 256 | [4, 4, 4, 4] | 2 |
| 25 | [5, 5] | 10422 | 256 | [16, 16] | 37 |
| 36 | [2, 2, 3, 3] | 3287 | 289 | [17, 17] | 26 |
| 49 | [7, 7] | 2519 | 324 | [2, 2, 3, 3, 3] | 1 |
| 64 | [2, 2, 2, 2, 2] | 1 | 324 | [2, 2, 9, 9] | 6 |
| 64 | [2, 2, 4, 4] | 311 | 361 | [19, 19] | 22 |
| 64 | [8, 8] | 1298 | 400 | [2, 2, 2, 2, 5, 5] | 8 |
| 81 | [3, 3, 3, 3] | 25 | 400 | [4, 4, 5, 5] | 11 |
| 81 | [9, 9] | 656 | 441 | [3, 3, 7, 7] | 4 |
| 100 | [2, 2, 5, 5] | 350 | 484 | [2, 2, 11, 11] | 6 |
| 121 | [11, 11] | 307 | 529 | [23, 23] | 11 |
| 144 | [2, 2, 2, 2, 3, 3] | 53 | 576 | [2, 2, 3, 3, 4, 4] | 1 |
| 144 | [3, 3, 4, 4] | 149 | 576 | [3, 3, 8, 8] | 4 |
| | | | | Total | 1688626 |

Table 3: Frequency of Tate-Shafarevich group orders and types of rank 0 elliptic curve