

$\#\text{III}(E/\mathbb{Q})$	Group Invariants	Frequency
1	[1]	1366261
4	[2, 2]	209051
9	[3, 3]	68948
16	[2, 2, 2, 2]	1110
16	[4, 4]	23418
25	[5, 5]	10422
36	[2, 2, 3, 3]	3287
49	[7, 7]	2519
64	[2, 2, 2, 2, 2, 2]	1
64	[2, 2, 4, 4]	311
64	[8, 8]	1298
81	[3, 3, 3, 3]	25
81	[9, 9]	656
100	[2, 2, 5, 5]	350
121	[11, 11]	307
144	[2, 2, 2, 2, 3, 3]	53
144	[3, 3, 4, 4]	149
169	[13, 13]	147
196	[2, 2, 7, 7]	59
225	[3, 3, 5, 5]	49
256	[2, 2, 8, 8]	30
256	[4, 4, 4, 4]	2
256	[16, 16]	37
289	[17, 17]	26
324	[2, 2, 3, 3, 3, 3]	1

Table 1: Frequency of Tate-Shafarevich group orders and types of rank 0 elliptic curve

$\#\text{III}(E/\mathbb{Q})$	Group Invariants	Frequency
324	[2, 2, 9, 9]	6
361	[19, 19]	22
400	[2, 2, 2, 2, 5, 5]	8
400	[4, 4, 5, 5]	11
441	[3, 3, 7, 7]	4
484	[2, 2, 11, 11]	6
529	[23, 23]	11
576	[2, 2, 3, 3, 4, 4]	1
576	[3, 3, 8, 8]	4
625	[25, 25]	7
625	Unknown	3
676	[2, 2, 13, 13]	3
729	[3, 3, 9, 9]	1
729	[27, 27]	1
784	[2, 2, 2, 2, 7, 7]	1
784	[4, 4, 7, 7]	5
841	[29, 29]	2
961	[31, 31]	4
1024	[4, 4, 8, 8]	2
1089	[3, 3, 11, 11]	1
1225	[5, 5, 7, 7]	1
1681	[41, 41]	1
2209	[47, 47]	2
2500	[2, 2, 25, 25]	1
5625	[3, 3, 5, 5, 5, 5]	1

Table 2: Frequency of Tate-Shafarevich group orders and types of rank 0 elliptic curve

$\#\text{III}(E/\mathbb{Q})$	Grp Inv	Frequency	$\#\text{III}(E/\mathbb{Q})$	Grp Inv	Frequency
1	[0]	1366261	169	[13, 13]	147
4	[2, 2]	209051	196	[2, 2, 7, 7]	59
9	[3, 3]	68948	225	[3, 3, 5, 5]	49
16	[2, 2, 2, 2]	1110	256	[2, 2, 8, 8]	30
16	[4, 4]	23418	256	[4, 4, 4, 4]	2
25	[5, 5]	10422	256	[16, 16]	37
36	[2, 2, 3, 3]	3287	289	[17, 17]	26
49	[7, 7]	2519	324	[2, 2, 3, 3, 3]	1
64	[2, 2, 2, 2, 2]	1	324	[2, 2, 9, 9]	6
64	[2, 2, 4, 4]	311	361	[19, 19]	22
64	[8, 8]	1298	400	[2, 2, 2, 2, 5, 5]	8
81	[3, 3, 3, 3]	25	400	[4, 4, 5, 5]	11
81	[9, 9]	656	441	[3, 3, 7, 7]	4
100	[2, 2, 5, 5]	350	484	[2, 2, 11, 11]	6
121	[11, 11]	307	529	[23, 23]	11
144	[2, 2, 2, 2, 3, 3]	53	576	[2, 2, 3, 3, 4, 4]	1
144	[3, 3, 4, 4]	149	576	[3, 3, 8, 8]	4
				Total	1688626

Table 3: Frequency of Tate-Shafarevich group orders and types of rank 0 elliptic curve