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Multivariate Analysis

Economic theories and models used to calculate equilibrium FX rates

There is not a single common definition of equilibrium exchange (FX) rate. The concepts and models can differ by timeframe (long-run, media and short, by the metric employed to assess different approaches and ultimately by the goal pursued with the use of the rate [1].

In the extreme short term horizon, the concept of market equilibrium exchange rate refers to the rates continuously defined/formed on FX markets. I this sense it is argued that FX rate is at its equilibrium value all the time, at least in the absence of market interventions (e.g. from the central banks).

In medium to long term the notion of equilibrium exchange rate is tied to the models used to determine it.

Balance of Payments Model

$$ca_t + ka_t = 0 \quad \textit{describes statistical equilibrium}$$

$$ca_t = nx_t + i_t\, n\, f\, a_t$$

$$nx_t = \alpha_1(s_t + p^*_t - p_t) - \alpha_2\, y_t + \alpha_3 y^*_t\ , \ \ldots \quad \alpha_1, \alpha_2, \alpha_3 > 0$$

$$ka_t = \mu\, (i_t - i^*_t - \Delta s^e_{t+k}), \quad \mu < \infty$$

where

ca_t is current accounts of the balance of payments
ka_t is capital accounts of the balance of payments
nx_t denotes net exports (export minus imports),
$i_t n\, f\, a_t$ is net interest payments on net foreign assets; changes in FX exchange rate may have influence through this term
s_t is the log of the spot exchange rate (domestic currency price of a unit of foreign currency)
p_t is the log of domestic price level
y_t is the log of domestic income
$*$ (asterics) market a foreign magnitude
α_i alphas are elasticities
i_t interest rate yield
Δ fist difference operator

this term $\alpha_1(s_t + p^*_t - p_t)$ is a measure of competitiveness and assumed to be positively associated with net exports

Above produces the following representation of balance of payments equation for exchange rate:

$$s_t = p_t - p^*_t + (\alpha_2 / \alpha_1)\, y_t - (\alpha_3 / \alpha_1)\, y^*_t - \alpha_{-11}(i_t\, n\, f\, a_t) - \mu/\alpha_1(i_t - i_{*t} - \Delta s_{et+k}) \qquad \textbf{(eq.1)}$$

By definition the real exchange rate $q_t \equiv s_t - p_t + p^*_t$, hence

$q_t = \beta'_1 Z_{1t} + \beta'_2 Z_{2t} + \tau' T_t + \epsilon_t$, where β'_1, β'_2 and τ' are vectors of coefficients, T is a set of transitory, short-run variables and ϵ_t is a random error term;

The equilibrium exchange rate is given by the above equation with transitory and random terms equal to zero:

$q'_t = \beta'_1 Z_{1t} + \beta'_2 Z_{2t}$, here

Z_{1t} is a set of fundamentals which are expected to have persistent effects on the long-run real exchange rate (e.g. relative output terms and net foreign assets);

Z_{2t} is a set of fundamentals which have persistent effects in the medium-term (interest rate yields)

Purchasing Power Parity - PPP

The oldest theory of exchange rate determination. Based on idea that exchange rate should evolve to neutralise competitiveness changes caused by movements in price indexes across nations, so that real exchange rates are mean reverting processes. [2]

It was not intended as a short-run rate determination method but rather a long term concept [5].

In term of equation 1 the strict (or "absolute") version of PPP theory assumes $\alpha_1 \to \infty$, $\mu = 0$, rendering $s_t = p_t - p^*_t$.

Thus the model relies on the forces of arbitrage to level off the prices on goods for similar goods where price discrepancy exists; it assumes no impediment to international trade, and ignores the capital flows and their effect on exchange rate, as well as net foreign assets positions. The absolute model is too restrictive to be useful;

The relative version of PPP theory does not assume that prices in two economies are exactly the same, but rather that the relative purchasing power of two different monetary units remains broadly unchanged over the long run, which means that the sample mean of the real exchange rate is a good proxy for PPP-implied equilibrium exchange rate.

Behavioral Equilibrium Exchange Rate (BEER)

The PPP-implied mean reversion of real exchange rates to their longer-term equilibrium is in practice very slow (if it happens at all) [2]. Among theories explaining this phenomenon is so-called Behavioural Equilibrium Exchange Rate Model, that attempts to explain the dynamics of the rates in terms of other economic fundamentals within a cointegrating framework.

For example, the rise in per-capita GDP leads to increase in demand for non-traded goods (services), ie. to increase in their relative price.

The Balassa-Samuelson effect suggests that an increase in wages in the tradable goods sector of an emerging economy will also lead to higher wages in the non-tradable (service) sector of the economy.

The rise in net foreign assets (n f a) improves the interest income on the current account and therefore requires counterbalancing by of the trade balance. Which in turn requires domestic currency appreciation (i.e. the rise of exchange rates).

The terms of trade (*tot*) is another explanatory variable commonly used in BEER. *tot* is the ratio between a country's export prices and its import prices.

$rer_{itBEER} = \mu + \alpha_1 gdp_{it} + \alpha_2 nfa_{it} + \alpha_3 tot_{it}$

Other Models for Equilibrium Exchange Rate

There are other approaches for exchange change valuation, such as

Macroeconomic Balance Approach
The Cundamental Equilibrium Exchange Rate (FEER)
The Desired Equilibrium Exchange Rate (DEER)
The Permanent Equilibrium Exchange Rate (PEER)
The Natural Rate of Exchange (NATREX)

and many others. They differ in relevant time horizon, statistical and theoretical assumptions, variables used.

A good summary and a table of comparison can be found at [Driver and Westaway], page 26.

Connection between linear regression and Vector Error Correction (VEC)

The concept of stationarity is of the utmost significance in most statistical analysis as in linear regression. The assumption is that the error term is a white noise process and thus, stationary.

The vector error correction (VEC) model is just a special case of the VAR for variables that are stationary in their differences (i.e., I(1)). The VEC can also take into account any cointegrating relationships among the variables. The VEC has cointegration relations built into the specification so that it restricts the long-run behavior of the endogenous variables to converge to their cointegrating relationships while allowing for short-run adjustment dynamics. The cointegration term is known as the error correction term since the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments.

To take the simplest possible example, consider two time-series variables, y_t and x_t. Generalizing the discussion about dynamic relationships to these two interrelated variables yields a system of equations:

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} x_{t-1} + \varepsilon_t^y$$

$$x_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} x_{t-1} + \varepsilon_t^x$$

The equations describe a system in which each variable is a function of its own lag, and the lag of the other variable in the system. In this case, the system contains two variables y and x. Together the equations constitute a system known as a vector autoregression (VAR). In this example, since the maximum lag is of order one, we have a VAR(1).

If y and x are stationary, the system can be estimated using least squares applied to each equation. If y and x are not stationary in their levels, but stationary in differences (i.e., I(1)), then take the differences and estimate:

$$\Delta y_t = \beta_{10} + \beta_{11}\Delta y_{t-1} + \beta_{12}\Delta x_{t-1} + \varepsilon_t^{\Delta y}$$

$$\Delta x_t = \beta_{20} + \beta_{21}\Delta y_{t-1} + \beta_{22}\Delta x_{t-1} + \varepsilon_t^{\Delta x}$$

using least squares. If y and x are I(1) and cointegrated, then the system of equations is modified to allow for the cointegrating relationship between the I(1) variables. Introducing the cointegrating relationship leads to a model known as the vector error correction (VEC) model.

Estimation of Equilibrium Exchange Rate using VEC Model

D.Stephen [6] considers a hybrid PPP-UIP (Uncovered Interest Rate Parity) model for equilibrium exchange rate, where price levels of PPP and Interest Rates from UIP models are used as co-integrated variables with Exchange Rate

The main equation is

$s_t + p_t - p^*_t + (1 / \alpha)(i_t - i^*_t) + k = q_t$,

where $k = c + u / \alpha$,

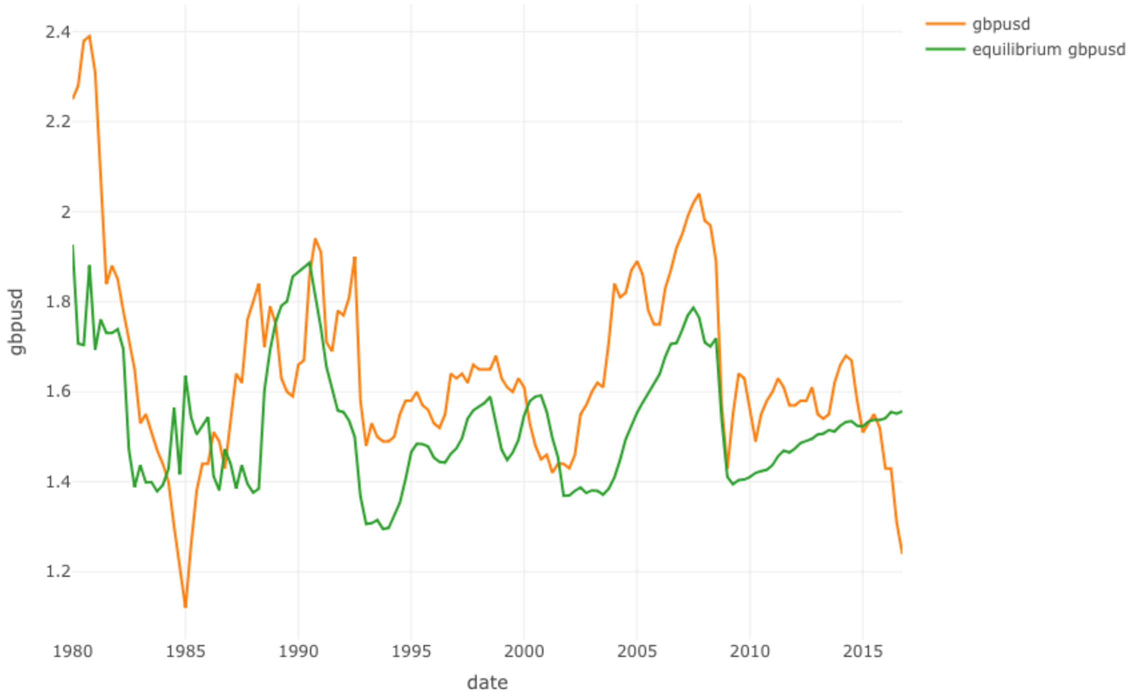
u is risk premium associated with holding domestic currency
q_t is stationary and represents the deviation from the equilibrium PPP-UIP condition
p_t, p^*_t are price levels (domestic and foreign), as per respective CPI data
i_t , I^*_t are interest rates levels (domestic and foreign, respectively)

Above equation provides an opportunity to estimate the long run value around which q_t varies. The estimated difference between q_t and this long run value serves as an indication of how far the five variables are from equilibrium with one another. From this the "exchange rate misalignment" - the adjustment to exchange rate required to restore q_t to its long run average at any point in the sample period can be inferred. After this the equilibrium nominal exchange rate can be calculated as the exchange rate that would have restored the equilibrium between PPP and UIP.

We follow the path of assessment for non-stationarity, fitting VAR model to select appropriate number of lags, then testing for cointegration and fitting appropriate VECM model with relevant number of cointegration relationships. The eigen vectors within the module will be the estimates for the coefficients such as that when they used for the components of the main equation above they make the right part q(t) stationary, i.e. I(0)

Pls refer to the code in multivariate.R for the final steps of fitting the model and constructing the equilibrium rate.

This is the chart of equilibrium GBPUSD rate against actual historical values of GBPUSD. The chart shows that equilibrium rate estimate is broadly consistent with main economic cycles observed in historical GBPUSD rate.



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