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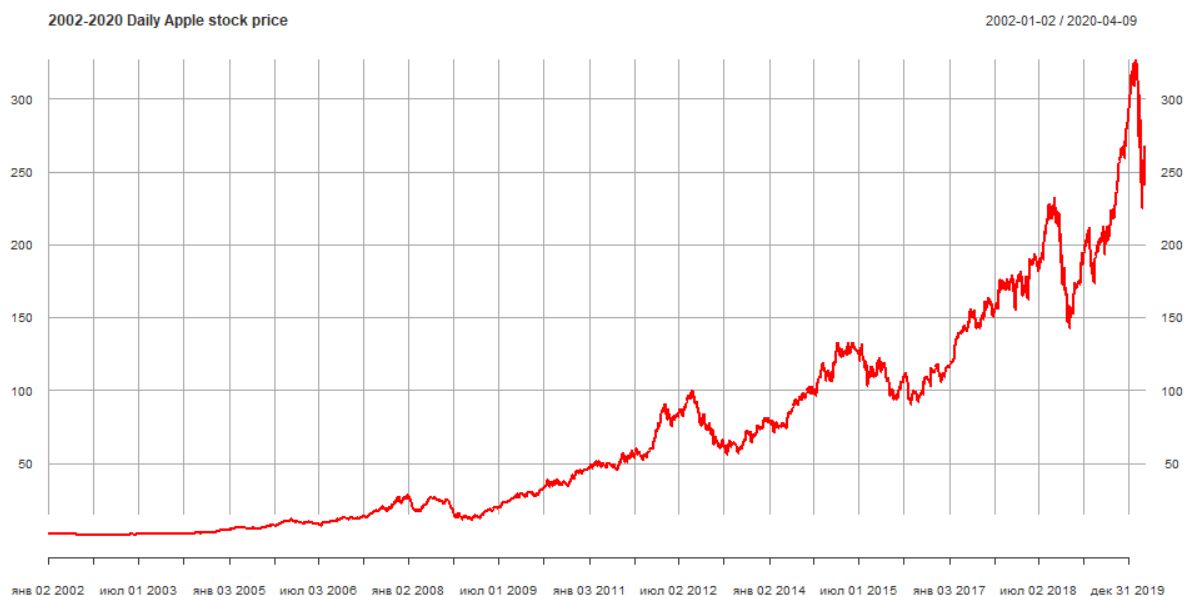
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Volatility Analysis

Forecast Apple daily stock return using a GARCH model

The GARCH process is often preferred by financial modeling professionals because it provides a more real-world context than other forms when trying to predict the prices and rates of financial instruments. In this work we will analyze data using one of the GARCH model. As the final output we will forecast the daily stock return of Apple using a GARCH model. Apple stocks are taken from Yahoo Finance from 1st January 2002 up to now.

Firstly, let's have a look on the raw data and its basic statistics:



AAPL.Close

nobs 4.600000e+03

NAs 0.000000e+00

Minimum 9.371430e-01

Maximum 3.272000e+02

1. Quartile 1.037178e+01

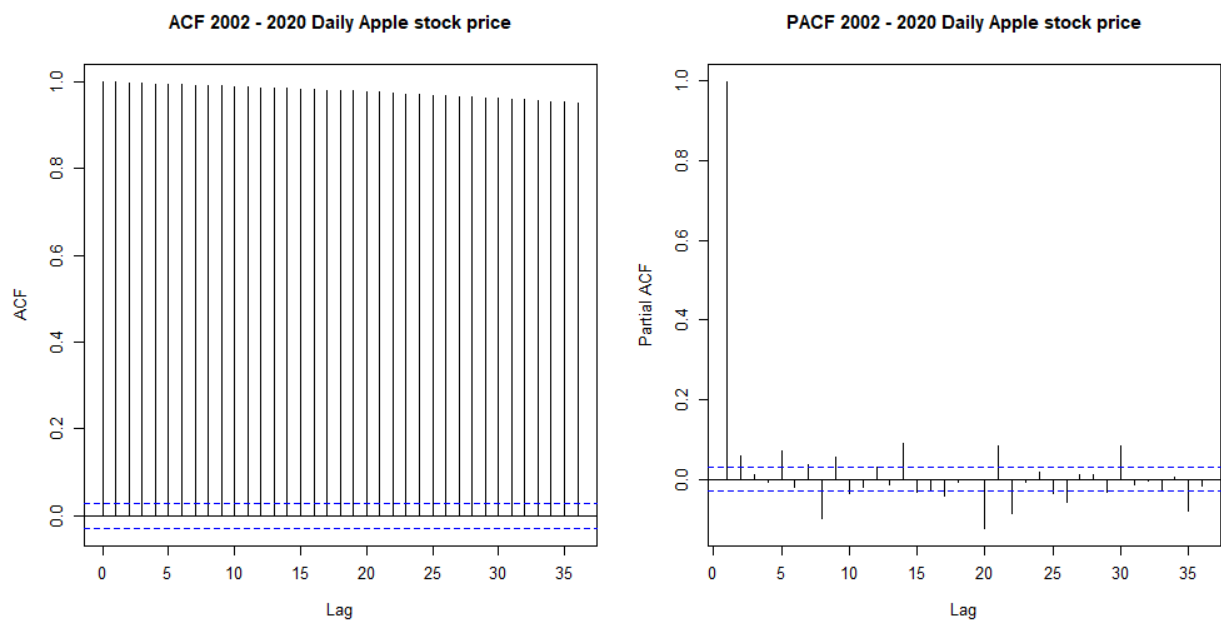
3. Quartile 1.098350e+02

Mean 6.917472e+01

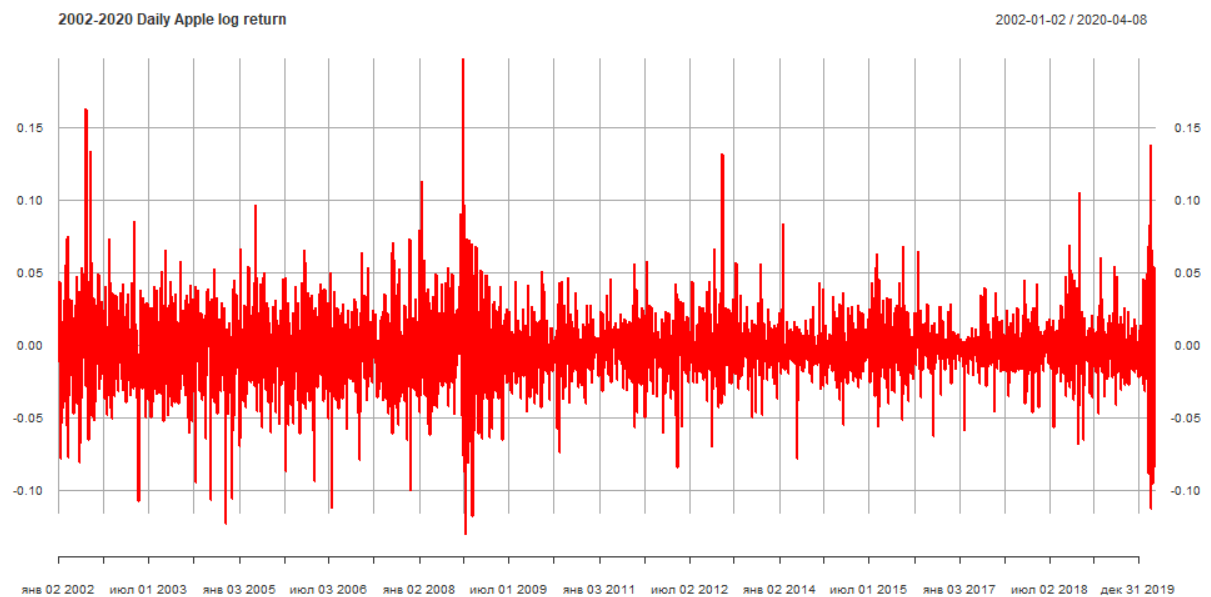
Median 4.783143e+01

Sum	3.182037e+05
SE Mean	1.037954e+00
LCL Mean	6.713983e+01
UCL Mean	7.120960e+01
Variance	4.955801e+03
Stdev	7.039745e+01
Skewness	1.133867e+00
Kurtosis	7.208980e-01

Mean value is not zero and the variance is very high. This indicates that the time series is non-stationary with varying mean and variance. Looking at ACF/PACF plots we can confirm that plot decays to 0 slowly, which means that the shock affects the process permanently.



Thus, to make process stationary, we will observe the log return of the stock price. Below you will find graph and basic statistics of log returns:

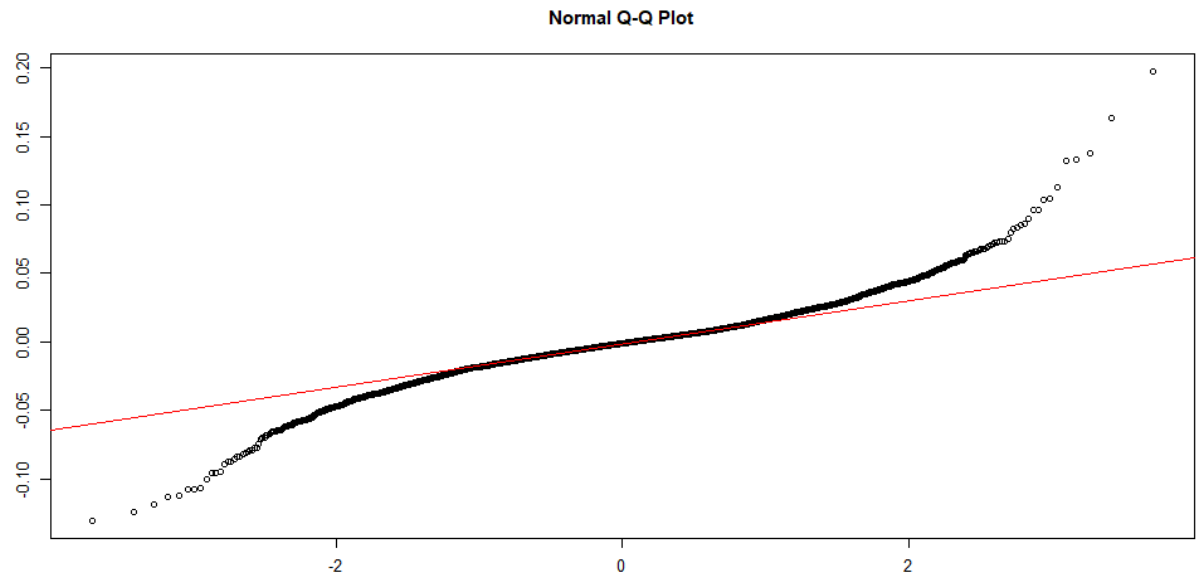


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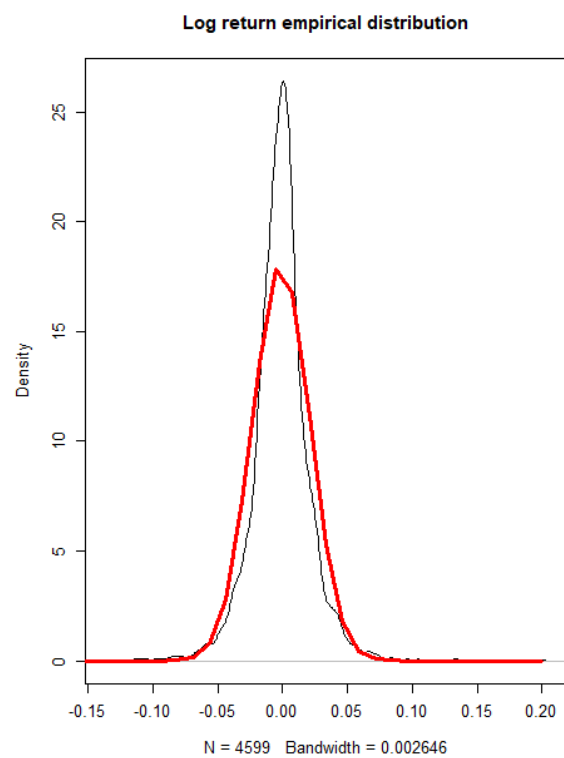
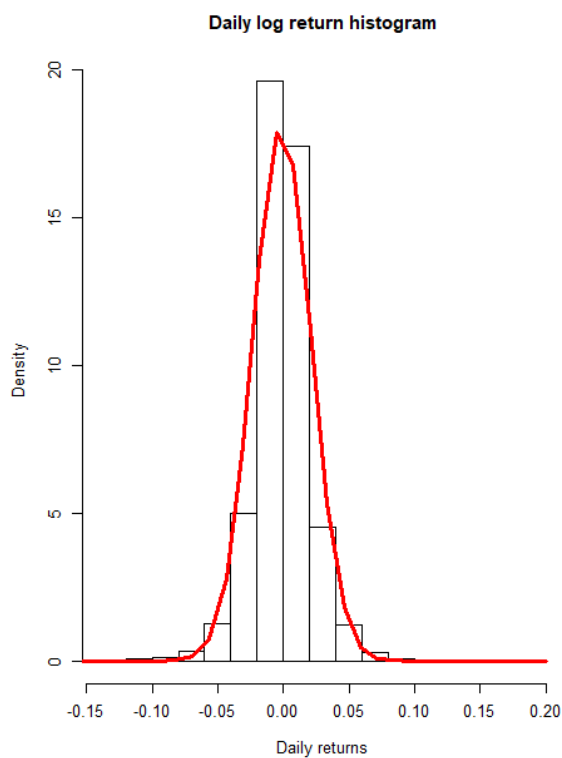
AAPL.Close
nobs      4599.000000
NAs       0.000000
Minimum   -0.130194
Maximum    0.197470
1. Quartile -0.012083
3. Quartile  0.009200
Mean      -0.001105
Median     -0.000943
Sum        -5.081553
SE Mean    0.000324
LCL Mean   -0.001740
UCL Mean   -0.000470
Variance  0.000482
Stdev      0.021955
Skewness   0.213676
Kurtosis   5.849132

```

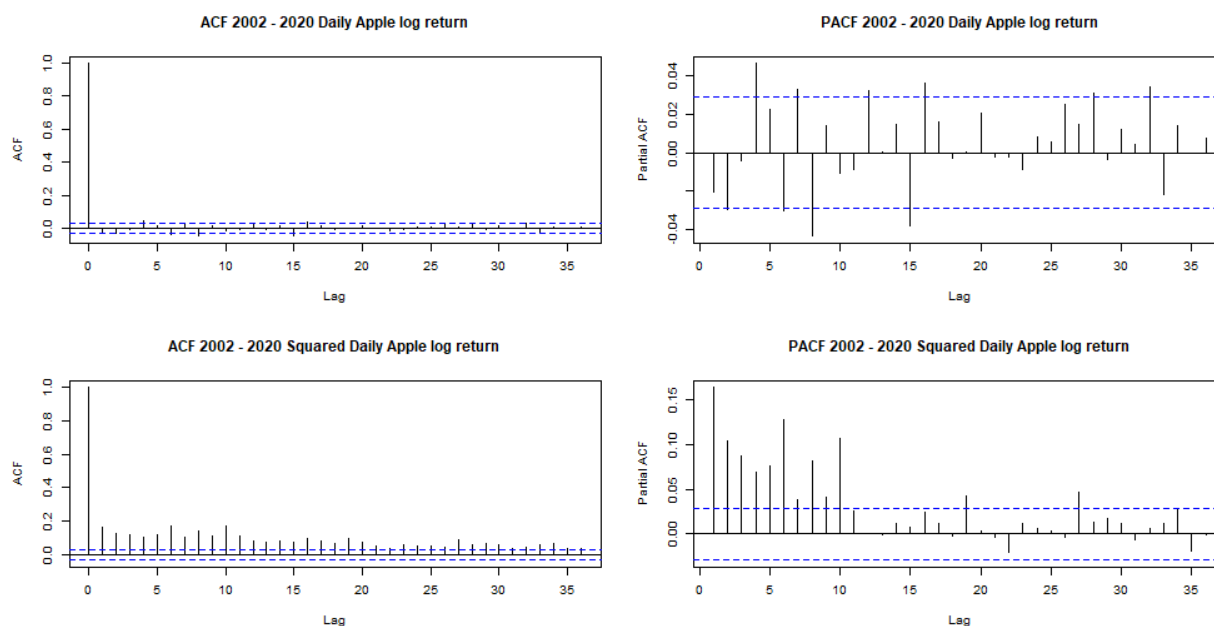
Mean is now 0 and the distribution of log returns has excess kurtosis and fat tails. It can also be observed on the QQ-plot:



This also could be observed on below histogram plots. Red line on these plots represents normal distribution of the same mean and standard deviation.



Looking on the ACF and PACF plots of the daily return and squared daily return we can see that log returns are serially uncorrelated. But the squared log returns show significant autocorrelations, which implies that log returns are not correlated but independent:



Now let's model out data by simple GARCH model. Here are results:

```
*-----*
*           GARCH Model Fit           *
*-----*
```

Conditional Variance Dynamics

GARCH Model : sGARCH(1,1)

Mean Model : ARFIMA(0,0,0)

Distribution : norm

Optimal Parameters

	Estimate	Std. Error	t value	Pr(> t)
mu	-0.001807	0.000181	-9.99063	0.000000
omega	0.000007	0.000011	0.68824	0.491300
alpha1	0.072379	0.016145	4.48302	0.000007

beta1 0.914349 0.015781 57.94080 0.000000

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(> t)
mu	-0.001807	0.003888	-0.464697	0.642149
omega	0.000007	0.000217	0.033971	0.972900
alpha1	0.072379	0.346486	0.208896	0.834530
beta1	0.914349	0.348947	2.620306	0.008785

LogLikelihood : 11512.4

Information Criteria

Akaike	-5.0047
Bayes	-4.9991
Shibata	-5.0047
Hannan-Quinn	-5.0028

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.9468	0.3305
Lag[2*(p+q)+(p+q)-1][2]	1.5877	0.3414
Lag[4*(p+q)+(p+q)-1][5]	5.4711	0.1195
d.o.f=0		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.1390	0.7092
Lag[2*(p+q)+(p+q)-1][5]	0.8748	0.8873
Lag[4*(p+q)+(p+q)-1][9]	1.9220	0.9141

d.o.f=2

Weighted ARCH LM Tests

```
-----  
                Statistic Shape Scale P-Value  
ARCH Lag[3]      0.2963 0.500 2.000 0.5862  
ARCH Lag[5]      1.3095 1.440 1.667 0.6438  
ARCH Lag[7]      1.5796 2.315 1.543 0.8053
```

Nyblom stability test

```
-----  
Joint Statistic: 2.9878  
Individual Statistics:  
mu      0.2582  
omega   1.0765  
alpha1  0.9598  
beta1   1.4137
```

Asymptotic Critical Values (10% 5% 1%)

```
Joint Statistic:      1.07 1.24 1.6  
Individual Statistic: 0.35 0.47 0.75
```

Sign Bias Test

```
-----  
                t-value    prob sig  
Sign Bias      0.9266 0.35416  
Negative Sign Bias 0.3579 0.72043  
Positive Sign Bias 1.3555 0.17532  
Joint Effect    7.9692 0.04665 **
```

Adjusted Pearson Goodness-of-Fit Test:

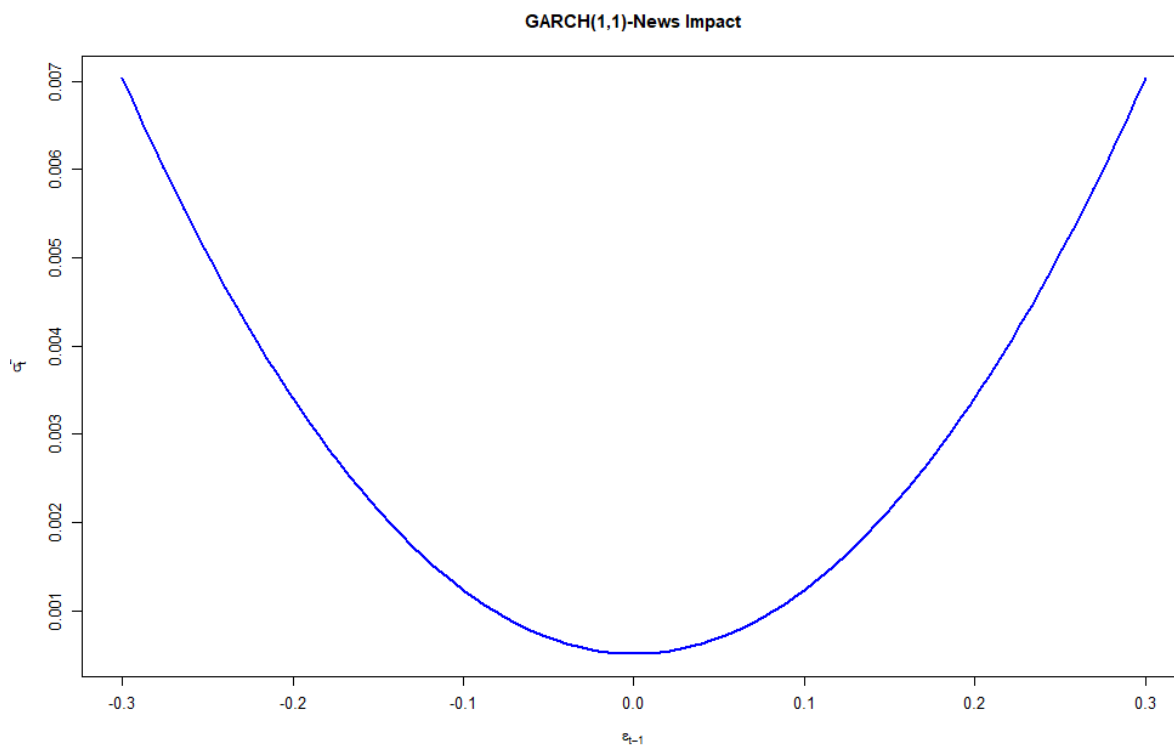
```
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group statistic p-value(g-1)
```

1	20	173.6	5.716e-27
2	30	194.6	1.930e-26
3	40	200.4	1.394e-23
4	50	222.7	5.469e-24

Elapsed time : 0.2390001

	mu	omega	alpha1	beta1	gamma1
	-0.001470696	-0.208622281	0.068963455	0.972304324	0.160610843

The GARCH model able to capture fat tails and volatility clustering. However, to explain asymmetries caused by the leverage effect, we need to consider more advanced model.



Above plot shows no asymmetries in response to positive / negative shocks. Thus, we decided to take a model which will consider asymmetric effects also. One of the GARCH models which able to capture this effect is EGARCH (exponential) model. Processing same analysis we get following results:

```

*-----*
*          GARCH Model Fit          *
*-----*

```


Conditional Variance Dynamics

GARCH Model : eGARCH(1,1)

Mean Model : ARFIMA(0,0,0)

Distribution : norm

Optimal Parameters

 Estimate Std. Error t value Pr(>|t|)
mu -0.001471 0.000252 -5.8291 0
omega -0.208622 0.016717 -12.4794 0
alpha1 0.068963 0.008043 8.5742 0
beta1 0.972304 0.002077 468.0894 0
gamma1 0.160611 0.012167 13.2003 0

Robust Standard Errors:

 Estimate Std. Error t value Pr(>|t|)
mu -0.001471 0.000289 -5.0908 0
omega -0.208622 0.025117 -8.3060 0
alpha1 0.068963 0.013369 5.1586 0
beta1 0.972304 0.002994 324.7807 0
gamma1 0.160611 0.023450 6.8492 0

LogLikelihood : 11569.24

Information Criteria

Akaike -5.0290

Bayes -5.0220

Shibata -5.0290

Hannan-Quinn -5.0266

Weighted Ljung-Box Test on Standardized Residuals

```

-----
                                statistic p-value
Lag[1]                          2.148 0.14280
Lag[2*(p+q)+(p+q)-1][2]        2.658 0.17372
Lag[4*(p+q)+(p+q)-1][5]        6.466 0.06975
d.o.f=0
H0 : No serial correlation

```

Weighted Ljung-Box Test on Standardized Squared Residuals

```

-----
                                statistic p-value
Lag[1]                          0.2986 0.5848
Lag[2*(p+q)+(p+q)-1][5]        1.7579 0.6766
Lag[4*(p+q)+(p+q)-1][9]        2.2251 0.8765
d.o.f=2

```

Weighted ARCH LM Tests

```

-----
                Statistic Shape Scale P-Value
ARCH Lag[3]    0.6960 0.500 2.000 0.4041
ARCH Lag[5]    0.6991 1.440 1.667 0.8237
ARCH Lag[7]    0.7599 2.315 1.543 0.9492

```

Nyblom stability test

```

-----
Joint Statistic: 3.0359
Individual Statistics:
mu      0.6954
omega   2.0174
alpha1  0.7536
beta1   1.9225
gamma1  0.4611

```

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.28 1.47 1.88

Individual Statistic: 0.35 0.47 0.75

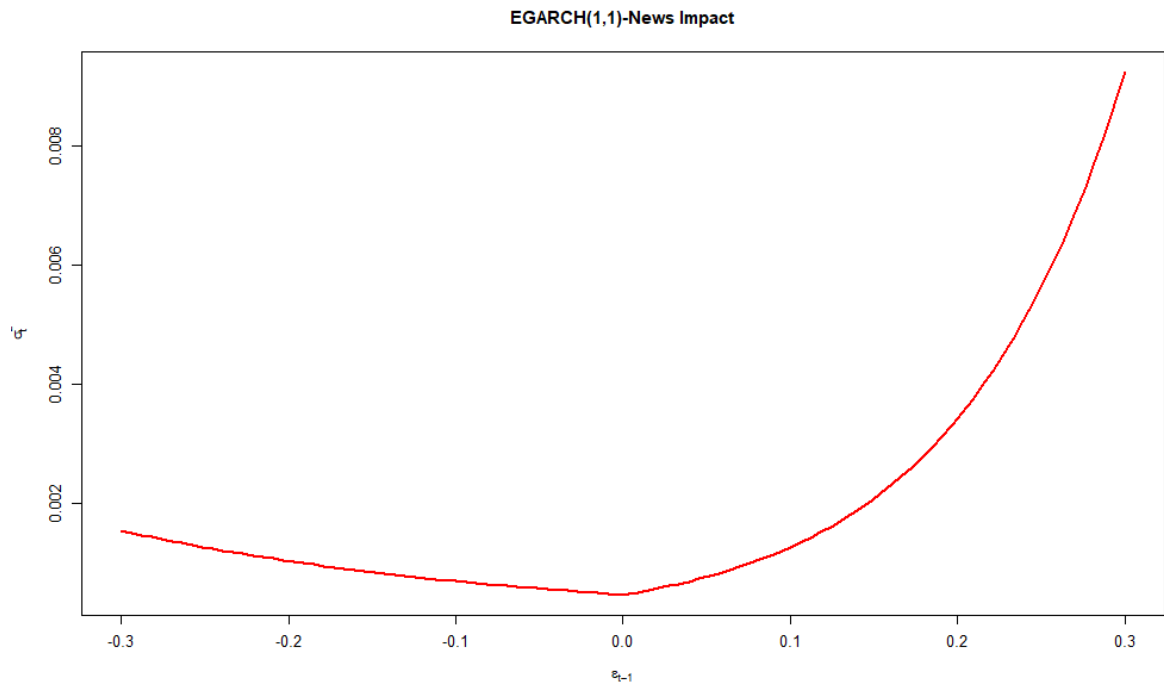
Sign Bias Test

	t-value	prob	sig
Sign Bias	1.4437	0.1489	
Negative Sign Bias	0.7073	0.4794	
Positive Sign Bias	0.2176	0.8277	
Joint Effect	2.4182	0.4903	

Adjusted Pearson Goodness-of-Fit Test:

	group	statistic	p-value (g-1)
1	20	153.4	4.900e-23
2	30	168.8	1.138e-21
3	40	187.3	2.873e-21
4	50	199.3	5.043e-20

Elapsed time : 0.3719978



Now we can see strong asymmetry in response of conditional volatility to positive or negative shocks. Based on the above analysis we consider using EGARCH model for forecasting as well. Details below:

```
*-----*
*      GARCH Model Forecast      *
*-----*
```

Model: eGARCH

Horizon: 10

Roll Steps: 10

Out of Sample: 10

0-roll forecast [T0=2020-03-11]:

	Series	Sigma
T+1	-0.001466	0.05194
T+2	-0.001466	0.05082
T+3	-0.001466	0.04975
T+4	-0.001466	0.04873
T+5	-0.001466	0.04776
T+6	-0.001466	0.04683
T+7	-0.001466	0.04594
T+8	-0.001466	0.04510
T+9	-0.001466	0.04429
T+10	-0.001466	0.04352

