GTAP Technical Paper

The <u>En</u>vironmental <u>I</u>mpact and <u>S</u>ustainability <u>A</u>pplied <u>G</u>eneral <u>E</u>quilibrium (ENVISAGE) Model, Version 11.0

by Dominique van der Mensbrugghe

Center for Global Trade Analysis
Purdue University

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Dominique van der Mensbrugghe[1]

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Abstract: This document's main purpose is to provide a full description of the ENVISAGE Model. ENVISAGE is a global recursive dynamic computable general equilibrium model developed originally at the World Bank. ENVISAGE has been developed to assess the interactions between economies and the global environment as affected by human-based emissions of greenhouse gases. At its core, ENVISAGE is a relatively standard recursive dynamic multi-sector multi-region CGE model. It is complemented with an emissions and climate module that links directly economic activities to changes in global mean temperature. And it incorporates a feedback loop that links changes in temperature to impacts on economic variables such as agricultural yields or damages created by sea level rise [tbd]. One of the overall objectives of the development of ENVISAGE has been to provide a greater focus on the economics of climate change for a more detailed set of developing countries as well as greater attention to the potential economic damages. The model remains a work in progress as there are several key features of the economics of climate change that are planned to be incorporated.

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[1] The Center for Global Trade Analysis, Purdue University, 403 Mitch Daniels Boulevard, West Lafayette, IN 47906. Corresponding e-mail: vandermd@purdue.edu.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

The purpose of this document is to provide a complete specification of the equations of the ENVIRONMENTAL IMPACT AND SUSTAINABILITY APPLIED GENERAL EQUILIBRIUM (ENVISAGE) MODEL. The ENVISAGE Model is designed to analyze a variety of issues related to the economics of climate change:

- Baseline emissions of CO_2 and other greenhouse gases
- Impacts of climate change on the economy
- Adaptation by economic agents to climate change
- Greenhouse gas mitigation policies—taxes, caps and trade
- The role of land use in future emissions and mitigation
- The distributional consequences of climate change impacts, adaptation and mitigation—at both the national and household level.
- Assessing projections of nutritional indicators and policies that impact nutrition pathways.

ENVISAGE is intended to be flexible in terms of its dimensions. The core database—that includes energy volumes and CO_2 emissions—is the GTAP database, currently version 11B with a 2017 base year (see Aguiar et al. (2023)). The latter divides the world into 160 regions, of which 141 are individual countries and the other are region-based aggregations.[1] The database divides global production into 65 sectors—with extensive details for agriculture and food and energy (coal mining, crude oil production, natural gas production, refined oil, electricity, and distributed natural gas). Appendix E provides more detail. Due to numerical and algorithmic constraints, a typical model is limited to some 20-30 sectors and 20-30 regions.

This document describes the current version of ENVISAGE, which is still in a developmental stage. This current version includes the following:

- Capital vintage production technology that permits analysis of the flexibility of economies.
- Two-level decomposition of domestic absorption into demand for domestic production and bilateral imports. This ranges from a simple version with a national agent optimizing each stage, to a full MRIO version with bilateral sourcing at the agent level. [Added 31-Oct-2020] The three specifications can co-exist, i.e. the model allows for any one of the three specifications to operate simultaneously and the choice can be both region- and sector-specific.
- A detailed specification of energy demand in each economy.
- [Added 31-OCT-2020]. The energy bundles—both in production and final demand—have been adjusted to allow for the traditional specification using a standard nested CES structure as well as the so-called additive version of the CES. The latter, which represents a different formulation of the logit specification, implies both volume and value additivity. It is more fully described in Appendix A and in van der Mensbrugghe and Peters (2020). The Armington trade structure and the CET allocation of domestic supply have also been adapted to allow for the additive version of the CES and CET, respectively.
- A resource depletion module for coal, oil and natural gas.
- The ability to introduce future alternative energy (or backstop) technologies [tbd]

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[1]   The countries defined in GTAP cover well over 90 percent of global GDP and population.

- CO_2 emissions that are fuel and demand specific.

- Incorporation of the main Kyoto greenhouse gases (methane, nitrous oxide and the fluoridated gases) as well as a set of additional emissions (such as particulates, black carbon, etc.).

- A flexible system for incorporating any combination of carbon taxes, emission caps and tradable permits.

- [Added 31-Oct-2020] A simplified climate module that links greenhouse gas emissions to atmospheric concentrations combined with a carbon cycle that leads to radiative forcing and temperature changes.

- A water module that incorporates water use for irrigation by crop, and aggregate water demand in other parts of the economy.

- [Added 21-Apr-2018] Government to government transfers have been incorporated. As more small developing countries are being included in the GTAP database and models, a more detailed accounting of the balance of payments is important to reflect macro balances.

- [Added 01-Dec-2018] The trade module now includes the *ad valorem* equivalent (AVE) of non-tariff measures (NTMs). Unlike tariff revenues, the revenues generated by the NTMs can be allocated to private households and government—either domestic or foreign.

- [Added 01-Dec-2018] The make module has a new technology parameter that can be used to capture changes in post-harvest (or production) losses.

- [Added 01-Dec-2018] Household demand includes a new waste module, with the possibility of taxing household waste.

- [Added 20-Jan-2019] A research and development (R&D) module has been added. R&D expenditures add to the knowledge stock, however with a distributed lag structure. The knowledge stock influences an endogenous component of labor productivity growth.

- [Added 20-Jan-2019] A global social welfare function has been introduced. This is still provisional.

- [Added 01-Apr-2020] A resource depletion module has been added for coal, oil and natural gas resources.

- [Added 01-Apr-2020] The carbon mitigation regimes have been modified to deal with multiple regimes within a same country/region—for example Europe's Emission Trading Scheme (ETS). The model also includes the possibility of inter-regional transfers to equate burden sharing among coalition countries (measured by equivalent variation).

- [Added 1-Sep-2021] Cap and trade regimes are now defined over GHG emission bundles—that may include one or more GHGs.

- [01-Nov-2021] Process emissions have been added to the database and the model. Process emissions are linked to the top-level GHG bundle in production. Work is ongoing to calibrate the GHG bundles to external MAC curve estimates.

- [Added 01-Apr-2020] Fiscal closure has been modified to allow for different re-cycling schemes for carbon revenues.

- [Added 01-Apr-2020] 'Phantom' taxes have been added to shape changes in preferences in the power bundle and household demand.

- [Added 01-Apr-2020] A savings equation has been added that links the domestic private saving rate to changes in the rate-of-return to capital.

- [Added 01-Apr-2020] Labor market closure now has three specifications: (1) Fixed employment and equilibrium wages; (2) An endogenous minimum (or) reservation wage that moves with the rate of unemployment and allows for endogenous regime shifts between full and un-employment; (3) [New] Short-term deviations in employment from long-run trends similar to the wage formulation in Monash-style models (Dixon and Rimmer (2002)).

- [Added 01-Apr-2020] Two new productivity parameters have been introduced. One (TFP) applies to all inputs—materials and value added. The second ($FTFP$) applies only to value added.

- [01-Nov-2021] The natural resource bundle in production has two possible locations—bundled with capital at the bottom of the production nest or as a share of output (net of GHG) at the top of the nest.

- [20-Apr-2022] The model allows for multiple land types and can be coupled to GTAP's land-use database that has agricultural and forestry land-use classified by 18 agro-ecological zones (AEZ). See Baldos and Corong (2020).

- [08-Aug-2023] The model was coupled to the GTAP nutrition database see Chepeliev (2022).

The current work program includes the following tasks:

■ The climate change damage module

The Envisage Model is a descendant of a family of models that originated at the OECD in the late 1980s and early 1990s, WALRAS, RUNS and GREEN.[2] Envisage was initially developed at the World Bank in 2007 and was a re-coded version of the World Bank's Linkage model (van der Mensbrugghe (2011)), which had a trade focus. It is designed specifically to analyze climate change issues and thus incorporates a more developed energy sector, a climate module (that makes integrated assessment an option), and climate change impact feedbacks. Envisage is coded using the GAMS/MCP package.

The current (draft) version is a complete re-coding of the original Envisage Model largely undertaken at the behest of the OECD's Environment Directorate, whose ENV-Linkages model also descended from the above-mentioned OECD models. [The current version has yet to incorporate all of the features of the original Envisage Model, but work is ongoing to complete the task.] This version, developed and residing at the Center for Global Trade Analysis (GTAP), thus shares a core code with the OECD ENV-Linkages model, though further development of the individual models will diverge some over time due to differing priorities and preferences. The version developed for the OECD, ENV-Linkages, is a relatively faithful translation of their original specification (Chateau et al. (2014)), but with a clean interface, streamlined code and improvements in its numerical properties. The water module was developed specifically for the OECD. Note that the ENV-Linkages team at the OECD was a core contributor to the quantification of the Shared Socio-Economic Pathways that will frame the analysis of the economics of climate change for at least the next decade (Dellink et al. (2017)).

The following items have been added to this version of the model compared to the previous:

■ A new capital flow module has been implemented, but is still undergoing testing and should be used with appropriate caution. It is inspired by the domestic capital allocation specification of the USAGE Model (Dixon and Rimmer (2005)).

■ Preference shifters have been added to the power bundles.

■ The model interface has been revamped to couple with the new GTAP database format. The latter is described in more detail in Corong et al. (2017). The standard GTAP model has undergone its first major facelift in over a decade and this has entailed a number of changes to the database—though mostly in terms of naming conventions as opposed to changes in functionality. The two most important changes include the introduction of a 'make' matrix, i.e. the explicit differentiation of activities from commodities; and the conversion of all taxes to be measured relative to base prices—in other words, all taxes are positive and subsidies negative. This affects output and export taxes.

■ To hone more closely to the new GTAP standard model, two sets of taxes have been modified. The first relates to output taxes. These are now identified by two indices—source activity and supplied commodity. The current database, in the absence of additional tax information, assumes these are uniform across source activities, though need not be after aggregation. The second refers to taxes on factor income. These are now identified by source activity as well as by receiving factor. Thus taxes on capital (or any other factor) can differ depending on the activity where the factor is used.

■ [31-Dec-2019] The entire model has been re-coded such that all (or almost all) variables have been normalized in the base year, i.e. are set to the value 1. A key purpose for implementing the model in normalized levels is that it should help in dealing with scaling problems. It also simplifies many expressions, notably the CES/CET expressions as the share parameters can mostly be dropped. The CES/CET dual price expressions are expressed in terms of the base year value shares.[3] The description of the model herein is based on a version of the model calibrated in levels, thus the normalization is implicit. In the code, all formulas adhere to the normalized levels of the variables that are scaled when needed, such as for accounting relations.

■ [31-Mar-2025] The front-end of the model, i.e., the preparation of the database, has been extensively modified and the former workflow, which was a complex Windows-based command file[4] has been converted to a workflow written as a GAMS file. The latter increases transparency and also makes the workflow independent of the operating system.

The next section provides a brief summary of the main model features. This is followed by a much longer section with a full-blown description of the Envisage Model.

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[2] OECD (1989/1990), Burniaux (1987), Burniaux and van der Mensbrugghe (1994), Burniaux et al. (1992), and van der Mensbrugghe (1994). Note that WALRAS and RUNS were initially developed at Stanford University and the Université Libre de Bruxelles, respectively.

[3] This comes close to the GEMPACK implementation of CGE models, albeit the interpretations of the share parameters differ as in the case of the model in levels, the share parameters are fixed at base year levels, whereas in the GEMPACK models, the share parameters reflect the contemporaneous value shares.

[4] Also available as a MacOS shell script.

Chapter 2

The Envisage Model in a nutshell

The Envisage Model at its core is a recursive dynamic and global computable general equilibrium model (CGE). It follows the circular flow of an economy paradigm. Firms purchase input factors (for example labor and capital) to produce goods and services. Households receive the factor income and in turn demand the goods and services produced by firms. And equality of supply and demand determine equilibrium prices for factors, goods and services. The model is solved as a sequence of comparative static equilibria where the factors of production are exogenous for each time period and linked between time periods with accumulation expressions.

Production is implemented as a series of nested constant-elasticity-of-substitution (CES) functions the aim of which is to capture the substitutability across all inputs. Three production archetypes are implemented. The first is for crops that reflects intensification of inputs versus land extensification. The second is for livestock that reflects range-fed versus ranch-fed production. The final, also referred to as the default, revolves largely around capital/labor substitutability. Some production activities highlight specific inputs (for example agricultural chemicals in crops and feed in livestock) and all activities include energy and its components as part of the cost minimization paradigm. Production is also identified by vintage—divided into *Old* and *New*—with typically lower substitution possibilities associated with *Old* capital.

Each production activity is allowed to produce more than one commodity—for example the ethanol sector can produce ethanol and distiller's dried grains with solubles (DDGS). And commodities can be formed by the output of one or more activities (for example electricity). Envisage therefore uses a different classification of activities and commodities.[1] One of the features of the model is that it integrates the GTAP power data base that disaggregates GTAP's electricity sector ('ely') into 11 different power sources plus electricity transmission and distribution. Though the database has both the supply and demand side for all 11 power sources, the aggregation facility permits the aggregation of electricity demand into a single commodity and the 'make' matrix specification combines the output from the different power activities into a single electricity commodity.

Income accrues from payments to factors of production and is allocated to households (after taxes). The government sector accrues all net tax payments and purchases goods and services. The model incorporates multiple utility functions for determining household demand. There is a set of three household demand functions linked to the ubiquitous linear expenditure system (LES): the standard LES, the extended LES (ELES) that incorporates household saving into the utility function, and 'an implicitly directly additive demand system' (AIDADS), that allows for non-linear Engel curves in the LES framework.[2] The fourth option uses the constant differences in elasticity (CDE) utility function that is used in the core GTAP model (Hertel (1997) and Corong et al. (2017)). The ELES incorporates the decision to save in a top level utility function. The other demand systems assume savings is a proportion of disposable income in the default closure. The consumer utility function determines consumer demand bundles that are subsequently converted to produced goods using a consumer demand 'make' or transition matrix. Investment is savings driven and equal to domestic saving adjusted by net capital flows.

Trade is modeled using the so-called Armington specification that posits that demand for goods are differentiated by region of origin. The model allows for domestic/import sourcing at the aggregate level (after aggregating domestic absorption across all agents), or at the agent-level. In the standard specification, a second Armington nest allocates aggregate import demand across all exporting regions using a representative agent specification. Note that a newer, though minimally tested version, allows for sourcing imports by agent—also known as the MRIO specification. Exports are modeled in an analogous fashion using a nested constant-elasticity-of-transformation (CET) specification.

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[1] Production activities are indexed with a and commodities are indexed with i.
[2] Users can also specify implementing a Cobb-Douglas (CD) utility function, which can be considered part of the LES framework.

The domestic supply of each commodity is supplied to the domestic market and an aggregate export bundle using a top-level CET function. The latter is allocated across regions of destination using a second-level CET function.[3] Each bilateral trade node is associated with four prices: 1) the producer price; 2) the export border price, also referred to as the free-on-board (FOB) price; 3) the import border price, also referred to as the cost, insurance and freight (CIF) price; and 4) the end-user price that includes all applicable trade taxes (but before domestic sales or VAT taxes). The wedge between the producer price and the FOB price is represented by the export tax (or subsidy if negative) and the wedge between the CIF and end-user prices represents the import tariff (and perhaps other import related distortions). The wedge between the CIF and FOB prices represents the international trade and transport margins. These margins represent the use of real resources that are supplied by each region. The global international trade and transport sector purchases these services from each region so as to minimize the aggregate cost.

The model has two fundamental markets for goods and services. Domestically produced goods sold on the domestic market, and domestically produced goods sold by region of destination. All other goods and services are composite bundles of these goods. Two market equilibrium conditions are needed to clear these two markets.[4]

The model incorporates five types of production factors: 1) labor (of which there can be up to 5 types); 2) capital; 3) land (of which there can be multiple types); 4) a sector specific natural resource (such as fossil fuel energy reserves); and 5) water. The labor market is allowed to be segmented (though not required). The model allows for regime switching between full and partial wage flexibility. Capital is allocated across sectors so as to equalize rates of returns. If all sectors are expanding, *Old* capital is assumed to receive the economy-wide rate of return. In contracting sectors, *Old* capital is sold on secondary markets using an upward sloping supply curve. This implies that capital is only partially mobile across sectors. Aggregate land (by type) and water supply are specified using supply curves. Though there are several options, the preferred supply curve is a logistic function that has an upper bound. Water demand also includes exogenous components for environmental uses and groundwater recharge. Land and water are allocated across activities using a nested CET specification.[5] Natural resources are supplied to each sector using an iso-elastic supply function with the possibility differentiated elasticities depending on market conditions.

Envisage incorporates the main greenhouse gases—carbon, methane, nitrous oxides and fluorinated gases, as well as an additional set of 10 emissions, such as particulate matter and black carbon. Emissions are generated by consumption of commodities (such as fuels), factor use (for example land in rice production and herds in livestock production) and there are also processed base emissions such as methane from landfills.[6] A number of carbon control regimes are available in the model. Carbon taxes can be imposed exogenously—potentially differentiated across regions. The incidence of the carbon tax allows for partial or full exemption by commodity and end-user. For example households can be exempted from the carbon tax on natural gas consumption. The model allows for emission caps in a flexible manner—where regions can be segmented into coalitions on a multi-regional or global basis. The model allows for countries/regions to be in multiple trading systems simultaneously—such as Europe's Emission Trading System (ETS). In addition to the standard cap system, a cap and trade system can be defined where each region within a coalition is assigned an initial emission quota.

Dynamics involves three elements. Labor supply (by skill level) grows at an exogenously determined rate. The aggregate capital supply evolves according to the standard stock/flow motion equation, i.e. the capital stock at the beginning of each period is equal to the previous period's capital stock, less depreciation, plus the previous period's level of investment. The third element is technological change. The standard version of the model assumes labor augmenting technical change—calibrated to given assumptions about GDP growth and inter-sectoral productivity differences. A portion of labor productivity growth can be linked to R&D expenditures. In policy simulations, technology is typically assumed to be fixed at the calibrated levels, though can be influenced by changes in R&D expenditures.

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[3]   The model allows for perfect transformation, which is the standard specification in the GTAP model.

[4]   If there are N commodities and R regions, there will be $R \times N$ market clearing conditions for domestic goods and $R \times N \times R$ market clearing conditions for bilateral trade.

[5]   Land is only implemented for agricultural activities. Water demand by activity is only present in irrigated crop sectors. Other water demand is based on aggregate demand functions with market clearing, but is not part of the cost structure.

[6]   The current version of the model does not include carbon emissions from deforestation—an important source of global carbon emissions.

Chapter 3

Model specification

The next section describes the key model dimensions. Subsequent sections describe the various blocks or modules of the model using the traditional circular flow scheme of economics, i.e. starting with production and factor incomes, income distribution, demand, trade, and macro closure. At the end, there is a discussion on the model dynamics.

A note on normalization. As noted above, the new version of the model is fully normalized. Normalization is in part intended to improve the numeric scale of the model. It also simplifies many formulas, such as the CES or iso-elastic supply and demand functions. In general, these formulas no longer require calibration. In the case of the CES price expression, the calibrated share parameters are replaced with the base year cost or budget shares.

In its most generic formulation, the CES expressions can be reduced to:

$$X_i = \alpha_i \left(\lambda_i\right)^{\sigma-1} V \left(\frac{C}{P_i}\right)^{\sigma}$$

and

$$C = \left[\sum_i \alpha_i \left(\frac{P_i}{\lambda_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)}$$

where X_i are the components of the CES composite, V, with prices P_i, and with an aggregate cost of C. The α_i are the (dual) cost share parameters that can be calibrated to an arbitrary initialization of all the variables and λ_i are shift parameters (e.g. technology or preferences). The normalized expressions (see Appendix A) are:

$$X_i^n = \left(\lambda_i\right)^{\sigma-1} V^n \left(\frac{C^n}{P_i^n}\right)^{\sigma}$$

and

$$C^n = \left[\sum_i s_i \left(\frac{P_i^n}{\lambda_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)}$$

where s_i is the base year cost share and the superscripted variables are the normalized values. Consistent accounting is expressed by:

$$\left(C_0 V_0\right) C^n V^n = \sum_i \left(P_{i,0} X_{i,0}\right) P_i^n X_i^n \iff C^n V^n = \sum_i s_i P_i^n X_i^n$$

where the '0' subscript represents the base year level.

The model as described below is based on level expressions, i.e. not the normalized version of the model. Note as well, that in principle the CES dual share parameters can be dropped from the CES expressions (and are replaced by the base year cost shares in the case of the price expression). However, in some cases the dual share parameters may be retained as they could be modified during the course of a simulation, for example recalibration of *'Old'* technology in production, or preference shifters. However, these parameters, if retained, are not calibrated using base year data and instead are initialized to 1.

3.1 Model dimensions

The model specification in GAMS is mostly as general as possible, i.e. virtually all variables and equations are specified using set-based indices that are defined at run-time when the data is read in. The model can thus be configured for any number of regions (up to 141 with the latest GTAP release) and any number of sectors (up to 65 with the standard GTAP database and 84 with the power/water database).

Table 3.1 lists the main indices used by the model. On the demand side the key dimension is the number of Armington agents (indexed by aa). The Armington agents include all production activities (indexed by a), and final demand agents (indexed by fd). The latter are split into households (h), government (gov) and investment (gov). Activities, i.e. production sectors, are separated from commodities. The former are indexed by a and the latter by i (with a possible alias of j). Thus intermediate demand, $XA_{i,a}$ represents the demand for commodity i by activity a.[1] The ENVISAGE model allows for a non-diagonal make matrix that is user-determined at the time of aggregating the GTAP database. This has proven useful, for example, for the power sector. The new power database has 11 power generating technologies with a complementary transmission and distribution activity. These can be aggregated to a single electricity commodity using the make matrix (with a nested CES specification).

The model incorporates three different production structures—crops, livestock and other. These will be indexed by acr, alv and ax, respectively—all subsets of a. Activities are also sub-divided into two 'geographic' zones—rural and urban. Typically, agricultural activities are assigned to the rural zone and all other activities to the urban zone. The model allows for segmented labor markets by geographic zone with a Harris-Todaro-type migration function (Harris and Todaro (1970)). Most of the other indices are relatively self-explanatory and will be further elaborated on in the model description.

Three key indices are not part of the table—an index for power (pb), land (lb) and water bundles ($wbnd$). These indices are user-defined and are used to aggregate power supply in the power module, allocate land across sectors in the land supply module and allocate water across uses in the water supply module. They will be further described in their respective sections.

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[1] In the 'classic' GTAP database there is a one-to-one mapping between activities and commodities. The new 'standard' GTAP database differentiates activities and commodities.

Table 3.1: **Sets used in model definition**

Set	Description
aa	Armington agents
a	Activities (a subset of aa)
acr	Crop activities (a subset of a)
alv	Livestock activities (a subset of a)
ax	All other activities (a subset of a)
$elya$	Power activities (a subset of a)
z	Set of zones—just rural (`rur`) and urban (`urb`)
i	Produced (or supplied) goods
$inum$	Set of manufacturing sectors (subset of i, used in definition of numéraire)
fp	Factors of production
l	Labor categories (subset of fp)
ul	Unskilled labor categories (subset of l)
sl	Skilled labor categories (subset of l)
cap	Capital account (subset of fp)
lnd	Land account(s) (subset of fp)
nrs	Natural resource account (subset of fp)
wat	Water account (subset of fp)
k	Consumed commodities
$nrg(k)$	The energy bundle in consumed commodities
fd	Final demand (subset of aa)
fdc	Final demand excluding households (subset of fd)
h	Households (subset of fd)
gov	Government account (subset of fd)
inv	Investment account (subset of fd)
gy	Government revenue accounts
$itax$	Indirect taxes (subset of gy)
$ptax$	Production tax account (subset of gy)
$mtax$	Import tariff account (subset of gy)
$etax$	Export tax account (subset of gy)
$vtax$	Tax on factors of production account (subset of gy)
$ptax$	Production tax account (subset of gy)
$ctax$	Carbon tax (subset of gy)
$dtax$	Direct taxes (subset of gy)
r	Regions
s, d	Aliases with r (for *source* and *destination* regions)
$rnum$	Set of regions used in definition of numéraire (subset of r)
$rres$	Residual region (subset of r, must be of single dimension)
em	Emission types

3.2 Production block

Production is implemented using a nested CES structure—rather standard in many applied general equilibrium models.[2] Three production structure prototypes are introduced—crops, livestock and all other (the default production structure). Figures 1, 2 and 3 depict the production structures for each of the three prototypes. These figures are supplemented by the nesting for the so-called *KEF* and energy bundles in respectively figures 4 and 5. This section will provide a full description of all the nests starting from the top nest. At the end of each of the terminal nodes will be the derived demand for the basic components (i.e. inputs) of production—intermediate goods and factors of production.

Each nest is re-produced for each of the possible vintages. In the comparative static version of the model there is usually only a single vintage. The default in the dynamic version of the model is to have two vintages—so-called *Old* vintage (or installed capital), and *New* that represents the most recent supply of capital. The former is meant to be only partially mobile across sectors, whereas the latter is fully mobile—detailed further below.

Production for each vintage is thus associated with a unit (or marginal) cost of production, represented by *uc*. The post tax unit cost of production, *PXv*, is equal to the tax-adjusted pre-tax unit cost of production, equation (P-2), where τ^{uc} is the tax on the cost of production. The aggregate unit cost of production across vintages, *PX*, is given in equation (P-1) and is the weighted sum of the vintage-specific costs of production, with weights given by the production volume shares where *XPv* and *XP* represent respectively output by vintage and aggregate output.[3]

$$PX_{r,a}XP_{r,a} = \sum_v PXv_{r,a,v}XPv_{r,a,v} \tag{P-1}$$

$$PXv_{r,a,v} = UC_{r,a,v}\left(1 + \tau^{uc}_{r,a,v}\right) \tag{P-2}$$

The remainder of this section describes the various CES nests that represent the production structure by vintage. The purpose of these CES nests is to replicate the substitution and complementarity relations across all of the inputs. The top level CES nest represents the combination of output, *XPX*, with a bundle of non-CO_2 greenhouse gas (GHG) emissions, *XGHG*. The latter is a special bundle that is used to simulate the marginal cost of abating non-CO_2 greenhouse gases. As the price of this bundle increases (e.g. from a tax on emissions), producers substitute away from this relatively more costly input.[4]

Equations (P-3) and (P-4) represent the derived demands for the output and GHG bundles respectively. These are the standard CES demand functions where *PXP* and *PXGHG* represent the price of the component bundles and *UC* is the price of the composite (or aggregate bundle). In this case, *UC*, is the unit cost of production, including the price associated with the GHG emissions. The parameters α^{xp} and α^{ghg} represent the standard CES share parameters, A^{xpv} is a tech-neutral shift in the production nest and σ^{xp} is the elasticity of substitution between production and the emissions of GHGs. The production nest also allows for input specific technological change as represented by the parameters λ^{xp} and λ^{ghg}. These are typically exogenous. Equation (P-5) defines the component price of the CES bundle, *UC*. The expression uses the CES dual price formula and could be replaced by the zero-profit condition.[5]

$$XPX_{r,a,v} = \alpha^{xp}_{r,a,v}\left(A^{xpv}_{r,a,v}\lambda^{xp}_{r,a,v}\right)^{\sigma^{xp}_{r,a,v}-1}\left(\frac{UC_{r,a,v}}{PXP_{r,a,v}}\right)^{\sigma^{xp}_{r,a,v}}XPv_{r,a,v} \tag{P-3}$$

$$XGHG_{r,a,v} = \alpha^{ghg}_{r,a,v}\left(A^{xpv}_{r,a,v}\lambda^{ghg}_{r,a,v}\right)^{\sigma^{xp}_{r,a,v}-1}\left(\frac{UC_{r,a,v}}{PXGHG_{r,a,v}}\right)^{\sigma^{xp}_{r,a,v}}XPv_{r,a,v} \tag{P-4}$$

$$UC_{r,a,v} = \frac{1}{A^{xpv}_{r,a,v}}\left[\alpha^{xp}_{r,a,v}\left(\frac{PXP_{r,a,v}}{\lambda^{xp}_{r,a,v}}\right)^{1-\sigma^{xp}_{r,a,v}} + \alpha^{ghg}_{r,a,v}\left(\frac{PXGHG_{r,a,v}}{\lambda^{ghg}_{r,a,v}}\right)^{1-\sigma^{xp}_{r,a,v}}\right]^{\frac{1}{1-\sigma^{xp}_{r,a,v}}} \tag{P-5}$$

The second level nest decomposes aggregate production net of the GHG bundle into two bundles, *ND¹* and *VA*. The first bundle includes all intermediate goods with the exception of energy goods and other intermediate goods that

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[2] The main alternative is to implement some form of a single-level flexible functional form such as the translog (see for example Jorgenson et al. (2013)).

[3] An earlier version of the model had an output price that was equal to the unit cost of production plus a (fixed) markup and adjusted by a production tax. The output tax has been moved to the supply side, see below, and is now both activity- and commodity specific. The markup has been eliminated for the moment.

[4] The reference GTAP Data Base does not have any revenues related to process emissions—and thus calibration of the share parameters is a challenge in the absence of prices. One way of resolving this is to modify the database, for example with the use of Altertax, to introduce a small price on GHG emissions associated with the GHG bundle. This is explained in section F.1.11 of the User Guide.

[5] The zero profit condition (without indices) is $UC.XPv = PXP.XPX + PXGHG.XGHG$.

are treated specially in a given activity. In the default configuration of the model, activity-specific intermediate goods include fertilizers for crop activities and feed for livestock activities.[6] The *VA* contains all factors of production, the energy goods and activity-specific goods where applicable. Equation (P-6) determines the demand for the top level intermediate demand bundle, ND^1. Equation (P-7) determines the demand for the *VA* bundle. The bundle prices are respectively PND^1 and PVA and the substitution elasticity is given by σ^p. It is worth noting that the equation for ND^1 is summed over all vintages. This is because the further decomposition of the ND^1 bundle is assumed to be independent of the vintage, whereas the decomposition of the *VA* is vintage specific as the substitution elasticities further down the nest are allowed to vary by vintage. Equation (P-8) determines the price of XPX, PXP.

[17-MAR-2020] The total factor productivity *TFP* parameter was added.

$$ND^1_{r,a} = \sum_v \alpha^{nd1}_{r,a,v} \left(TFP_{r,a,v}\right)^{\sigma^p_{r,a,v}-1} \left(\frac{PXP_{r,a,v}}{PND^1_{r,a}}\right)^{\sigma^p_{r,a,v}} XPX_{r,a,v} \tag{P-6}$$

$$VA_{r,a,v} = \alpha^{va}_{r,a,v} \left(TFP_{r,a,v}\right)^{\sigma^{xp}_{r,a,v}-1} \left(\frac{PXP_{r,a,v}}{PVA_{r,a,v}}\right)^{\sigma^p_{r,a,v}} XPX_{r,a,v} \tag{P-7}$$

$$PXP_{r,a,v} = \frac{1}{TFP_{r,a,v}} \left[\alpha^{nd1}_{r,a,v} \left(PND^1_{r,a}\right)^{1-\sigma^p_{r,a,v}} + \alpha^{va}_{r,a,v} \left(PVA_{r,a,v}\right)^{1-\sigma^p_{r,a,v}}\right]^{\frac{1}{1-\sigma^p_{r,a,v}}} \tag{P-8}$$

[15-APR-2021] The GHG bundle is formed of all GHG process emissions, i.e. emissions not linked to the combustion of fossil fuels. They are measured in common units of CO_2e. A CES specification is used to combine the distinct GHG emissions into a single GHG bundle. This is captured in equation (P-9) where *prEmi* represents the process emission linked to GHG *ghg*, that is linked to the price *PprEmi*. The elasticity of substitution is given by σ^{prEmi}. Equation (P-10) determines the price of the aggregate GHG bundle. The price of process emissions is given by $PprEmi = PCarb + prEmiTax$, where *PCarb* is an exogenous carbon price, typically set to a small value in the reference year, and *prEmiTax* is a price markup used for mitigating greenhouse gas emissions. It could be exogenous, or endogenously determined subject to one or more emission constraints.[7]

$$prEmi_{r,ghg,a} = \sum_v \alpha^{prEmi}_{r,ghg,a,v} \left(\lambda^{prEmi}_{r,ghg,a,v}\right)^{\sigma^{prEmi}_{r,a,v}-1} \left(\frac{PXGHG_{r,a,v}}{PprEmi_{r,ghg,a}}\right)^{\sigma^{prEmi}_{r,a,v}} XGHG_{r,a,v} \tag{P-9}$$

$$PXGHG_{r,a,v} = \left[\sum_{ghg} \alpha^{prEmi}_{r,ghg,a,v} \left(PprEmi_{r,ghg,a}\right)^{1-\sigma^{prEmi}_{r,a,v}}\right]^{\frac{1}{1-\sigma^{prEmi}_{r,a,v}}} \tag{P-10}$$

The middle CES nests are activity specific and reflect the three production structure prototypes described above— crops, livestock and all other activities. The crop production structure is designed to capture production characterized by the choice between intensification versus extensification, i.e. where land is abundant and cheap, production will tend to expand using more land, and vice versa if land is scarce and expensive. Livestock production is characterized by feed versus land substitution. In the default production structure, the main characteristic of production is the standard capital and labor substitution.

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[6]   The standard GTAP database does not include fertilizers. These are associated with the 'chm' good, i.e. or the chemical sector. The standard feed goods are wheat, coarse grains, oil seeds, and other processed foods respectively 'wht', 'gro', 'osd' and 'ofd' in the GTAP database.

[7]   *PCarb* is multiplied by the model numéraire to ensure price homogeneity.

Crop output by vintage (XPv)

σ^{xp}

Output by vintage excl. GHG (XPX)

σ^{p}

Process emissions GHG bundle ($XGHG$)

σ^{prEmi}

$ProcEmi_{ghg}$

Intermediate demand bundle excl. $FERT$ and NRG ($ND1$)

σ^{n1}

Armington demand (XA_i)

Value added bundle (VA)

σ^{v}

Type 1 labor bundle ($LAB1$)

σ^{l1}

Type 1 labor demand (XF_l)

Value added bundle excl. Type 1 labor ($VA1$)

σ^{v1}

Value added bundle excl. $LAB1$ and $FERT$ ($VA2$)

σ^{v2}

Value added bundle incl. capital, Type 2 labor and NRG (KEF)

See Figure 4

Land demand bundle ($Land^d$)

Land demand by type (XF_{lnd})

$FERT$ bundle ($ND2$)

σ^{n2}

Armington demand (XA_i)

Figure 1: **Crop production nest**

These different structures will be captured by a set of intermediate bundles, represented by VA^1 and VA^2 that represent different composition of factors and activity-specific intermediate goods where appropriate. The top of the nest is represented by the bundle VA^1. The bottom of this set of nests are the bundles LAB^1, XF_{lnd}, KEF and ND^2, where appropriate. The bundle LAB^1 represents the demand for the Type 1 labor bundle.[8] The variable XF_{lnd} represents the activity's demand for the land factor(s), and will be derived from the demand for the land bundle, $Land^d$. The bundle KEF represents the nested combination of capital, Type 2 labor, energy and the natural resource factor, where appropriate. The bundle ND^2 represents the activity-specific demand for intermediates—fertilizers in the case of crops and feed in the case of livestock activities. The decomposition of these three bundles is the same for all activities and described below. Table 3.2 summarily describes the composition of these middle nests for the three production prototypes.

Figures 1, 2 and 3 provide an illustrative description of the production nests.

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[8]   The user determines what labor types are considered Type 1 labor. The user can decide to have all labor types in this bundle, in which case the Type 2 labor bundle, LAB^2, will be empty.

Table 3.2: **Intermediate CES nests in production**

Activity	Bundle composition
Crops	
VA	$CES\left(LAB^1, VA^1\right)$
VA^1	$CES\left(ND^2, VA^2\right)$
VA^2	$CES\left(Land^d, KEF\right)$
Livestock	
VA	$CES\left(VA^1, VA^2\right)$
VA^1	$CES\left(LAB^1, KEF\right)$
VA^2	$CES\left(Land^d, ND^2\right)$
Default	
VA	$CES\left(LAB^1, VA^1\right)$
VA^1	$CES\left(Land^d, KEF\right)$

The equations for the intermediate nests will be described according to the demand for the individual component bundles, with the price equations described subsequently. The first set of equations determines the bundles VA^1 and VA^2. The former is a share of VA for all activities. The latter is a share of VA^1 in the case of crops and of VA in the case of livestock. The default production structure does not require the VA^2 bundle.

[17-MAR-2020] Added a total factor productivity factor, $FTFP$ to the VA bundle. Note that this will affect demand for any intermediate good in either the energy or $ND2$ bundle.
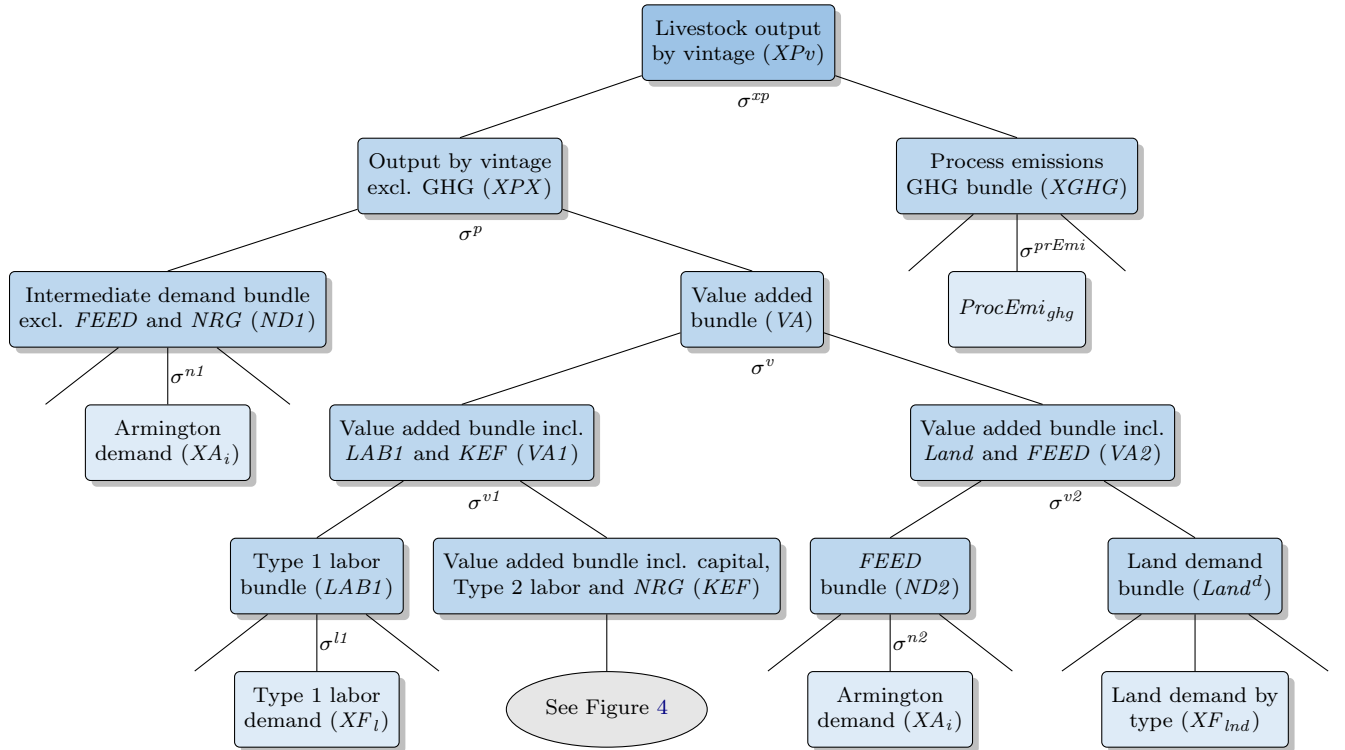
Figure 2: **Livestock production nest**

$$VA^1_{r,a,v} = \alpha^{va1}_{r,a,v} \left(FTFP_{r,a,v}\right)^{\sigma^v_{r,a,v}-1} \left(\frac{PVA_{r,a,v}}{PVA^1_{r,a,v}}\right)^{\sigma^v_{r,a,v}} VA_{r,a,v} \tag{P-11}$$

$$VA^2_{r,a,v} = \begin{cases} \alpha^{va2}_{r,a,v} \left(\dfrac{PVA^1_{r,a,v}}{PVA^2_{r,a,v}}\right)^{\sigma^{v1}_{r,a,v}} VA^1_{r,a,v} & \text{if } a \in \{Crops\} \\[12pt] \alpha^{va2}_{r,a,v} \left(FTFP_{r,a,v}\right)^{\sigma^v_{r,a,v}-1} \left(\dfrac{PVA_{r,a,v}}{PVA^2_{r,a,v}}\right)^{\sigma^v_{r,a,v}} VA_{r,a,v} & \text{if } a \in \{Livestock\} \end{cases} \tag{P-12}$$

The next set of equations determine the bundles LAB^1, KEF and ND^2. The subsequent decomposition of these three bundles will be identical for all activities. The LAB^1 bundle is a share of VA^1 for *Livestock* and VA for all other activities. The KEF bundle is a share of VA^2 for *Crops* and VA^1 for all other activities. The ND^2 bundle is a share of VA^1 for *Crops* and VA^2 for *Livestock*. The ND^2 bundle is not used in the *Default* production structure.

$$LAB^1_{r,a} = \begin{cases} \displaystyle\sum_v \alpha^{l1}_{r,a,v} \left(FTFP_{r,a,v}\right)^{\sigma^v_{r,a,v}-1} \left(\dfrac{PVA_{r,a,v}}{PLAB^1_{r,a}}\right)^{\sigma^v_{r,a,v}} VA_{r,a,v} & \text{if } a \in \{Default\} \\[14pt] \displaystyle\sum_v \alpha^{l1}_{r,a,v} \left(\dfrac{PVA^1_{r,a,v}}{PLAB^1_{r,a}}\right)^{\sigma^{v1}_{r,a,v}} VA^1_{r,a,v} & \text{if } a \in \{Livestock\} \end{cases} \tag{P-13}$$

$$KEF_{r,a,v} = \begin{cases} \alpha^{kef}_{r,a,v} \left(\dfrac{PVA^2_{r,a,v}}{PKEF_{r,a,v}}\right)^{\sigma^{v2}_{r,a,v}} VA^2_{r,a,v} & \text{if } a \in \{Crops\} \\[14pt] \alpha^{kef}_{r,a,v} \left(\dfrac{PVA^1_{r,a,v}}{PKEF_{r,a,v}}\right)^{\sigma^{v1}_{r,a,v}} VA^1_{r,a,v} & \text{if } a \in \{Default\} \end{cases} \tag{P-14}$$

$$ND^2_{r,a} = \begin{cases} \displaystyle\sum_v \alpha^{nd2}_{r,a,v} \left(\dfrac{PVA^1_{r,a,v}}{PND^2_{r,a}}\right)^{\sigma^{v1}_{r,a,v}} VA^1_{r,a,v} & \text{if } a \in \{Crops\} \\[14pt] \displaystyle\sum_v \alpha^{nd2}_{r,a,v} \left(\dfrac{PVA^2_{r,a,v}}{PND^2_{r,a}}\right)^{\sigma^{v2}_{r,a,v}} VA^2_{r,a,v} & \text{if } a \in \{Livestock\} \end{cases} \tag{P-15}$$

[Modified 20-APR-2022] The final demand equation in this set of intermediate nests determines the demand for the land bundle, $Land^d$.[9] It is a share of VA^2 for *Crops* and *Livestock* and a share of VA^1 for the *Default* activities.

$$Land^d_{r,a} = \begin{cases} \displaystyle\sum_v \alpha^{land}_{r,a,v} \left(\dfrac{PVA^2_{r,a,v}}{PLand^d_{r,a}}\right)^{\sigma^{v2}_{r,a,v}} VA^2_{r,a,v} & \text{if } a \in \{Crops\} \\[14pt] \displaystyle\sum_v \alpha^{land}_{r,a,v} \left(\dfrac{PVA^2_{r,a,v}}{PLand^d_{r,a}}\right)^{\sigma^{v2}_{r,a,v}} VA^2_{r,a,v} & \text{if } a \in \{Livestock\} \\[14pt] \displaystyle\sum_v \alpha^{land}_{r,a,v} \left(\dfrac{PVA^1_{r,a,v}}{PLand^d_{r,a}}\right)^{\sigma^{v1}_{r,a,v}} VA^1_{r,a,v} & \text{if } a \in \{Default\} \end{cases} \tag{P-16}$$

The aggregate land bundle for activity a is disaggregated into demand for land by type, $XF_{r,lnd,a}$. The relevant price of land is $PF^p_{r,lnd,a}$ that represents the user (or agent) price of land. It is equal to the market price of land adjusted for an activity-specific tax or subsidy. The specification allows for either the standard CES, or the additive version (ACES). Equation (P-18) determines the composite price of the land bundle. The average price of the land bundle is determined by the explicit specification of the zero-profit condition, equation (P-19).

$$XF_{r,lnd,a} = \begin{cases} \alpha^{lnd}_{r,lnd,a} \left(\dfrac{\lambda^f_{r,lnd,a} PLand^c_{r,a}}{PF^p_{r,lnd,a}}\right)^{\sigma^{lnd}_{r,a}} \dfrac{Land^d_{r,a}}{\lambda^f_{r,lnd,a}} & \text{if } ifLandCET \\[14pt] \alpha^{lnd}_{r,lnd,a} \left(\dfrac{PLand^c_{r,a}}{\lambda^f_{r,lnd,a} PF^p_{r,lnd,a}}\right)^{\sigma^{lnd}_{r,a}} Land^d_{r,a} & \text{if } \neg ifLandCET \end{cases} \tag{P-17}$$

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[9] The code was modified to allow for multiple land types, for example agro-ecological zones (AEZ). An additional CES nest is specified. The former nest assumed a single land-type (per activity). The new nest, has a top-level land bundle, $Land^d$, which is further disaggregated across land-types (see for example Hertel et al. (2009)).

$$PLand_{r,a}^c = \begin{cases} \left[\displaystyle\sum_{lnd} \alpha_{r,lnd,a}^{lnd} \left(\dfrac{PF_{r,lnd,a}^p}{\lambda_{r,lnd,a}^f}\right)^{1-\sigma_{r,a}^{lnd}}\right]^{1/(1-\sigma_{r,a}^{lnd})} & \text{if } ifLandCET \\[3em] \left[\displaystyle\sum_{lnd} \alpha_{r,lnd,a}^{lnd} \left(PF_{r,lnd,a}^p \lambda_{r,lnd,a}^f\right)^{-\sigma_{r,a}^{lnd}}\right]^{-1/\sigma_{r,a}^{lnd}} & \text{if } \neg ifLandCET \end{cases} \tag{P-18}$$

$$PLand_{r,a}^d = \left[\sum_{lnd} PF_{r,lnd,a}^p PF_{r,lnd,a}\right] / Land_{r,a}^d \tag{P-19}$$

The description of the intermediate nests is finished with the determination of the prices of the intermediate bundles VA, VA^1 and VA^2. Equation (P-20) determines the price of the VA bundle, PVA. Equation (P-21) determines the price of the VA^1 bundle, PVA^1. Equation (P-21) determines the price of the VA^2 bundle, PVA^2.

$$PVA_{r,a,v} = \begin{cases} \dfrac{1}{FTFP_{r,a,v}}\left[\alpha_{r,a,v}^{l1}\left(PLAB_{r,a}^1\right)^{1-\sigma_{r,a,v}^v} \right. \\[1em] \left. + \alpha_{r,a,v}^{va1}\left(PVA_{r,a,v}^1\right)^{1-\sigma_{r,a,v}^v}\right]^{\frac{1}{1-\sigma_{r,a,v}^v}} & \text{if } a \in \{Default\} \\[2em] \dfrac{1}{FTFP_{r,a,v}}\left[\alpha_{r,a,v}^{va1}\left(PVA_{r,a,v}^1\right)^{1-\sigma_{r,a,v}^v}\right. \\[1em] \left. + \alpha_{r,a,v}^{va2}\left(PVA_{r,a,v}^2\right)^{1-\sigma_{r,a,v}^v}\right]^{\frac{1}{1-\sigma_{r,a,v}^v}} & \text{if } a \in \{Livestock\} \end{cases} \tag{P-20}$$

$$PVA_{r,a,v}^1 = \begin{cases} \left[\alpha_{r,a,v}^{nd2}\left(PND_{r,a}^2\right)^{1-\sigma_{r,a,v}^{v1}}\right. \\[1em] \left. + \alpha_{r,a,v}^{va2}\left(PVA_{r,a,v}^2\right)^{1-\sigma_{r,a,v}^{v1}}\right]^{\frac{1}{1-\sigma_{r,a,v}^{v1}}} & \text{if } a \in \{Crops\} \\[2em] \left[\alpha_{r,a,v}^{l1}\left(PLAB_{r,a}^1\right)^{1-\sigma_{r,a,v}^{v1}}\right. \\[1em] \left. + \alpha_{r,a,v}^{kef}\left(PKEF_{r,a,v}\right)^{1-\sigma_{r,a,v}^{v1}}\right]^{\frac{1}{1-\sigma_{r,a,v}^{v1}}} & \text{if } a \in \{Livestock\} \\[2em] \left[\alpha_{r,a,v}^{land}\left(PLand_{r,a}^d\right)^{1-\sigma_{r,a,v}^{v1}}\right. \\[1em] \left. + \alpha_{r,a,v}^{kef}\left(PKEF_{r,a,v}\right)^{1-\sigma_{r,a,v}^{v1}}\right]^{\frac{1}{1-\sigma_{r,a,v}^{v1}}} & \text{if } a \in \{Default\} \end{cases} \tag{P-21}$$

$$PVA_{r,a,v}^2 = \begin{cases} \left[\alpha_{r,a,v}^{land}\left(PLand_{r,a}^d\right)^{1-\sigma_{r,a,v}^{v2}}\right. \\[1em] \left. + \alpha_{r,a,v}^{kef}\left(PKEF_{r,a,v}\right)^{1-\sigma_{r,a,v}^{v2}}\right]^{\frac{1}{1-\sigma_{r,a,v}^{v2}}} & \text{if } a \in \{Crops\} \\[2em] \left[\alpha_{r,a,v}^{land}\left(PLand_{r,a}^d\right)^{1-\sigma_{r,a,v}^{v2}}\right. \\[1em] \left. + \alpha_{r,a,v}^{nd2}\left(PND_{r,a}^2\right)^{1-\sigma_{r,a,v}^{v2}}\right]^{\frac{1}{1-\sigma_{r,a,v}^{v2}}} & \text{if } a \in \{Livestock\} \end{cases} \tag{P-22}$$
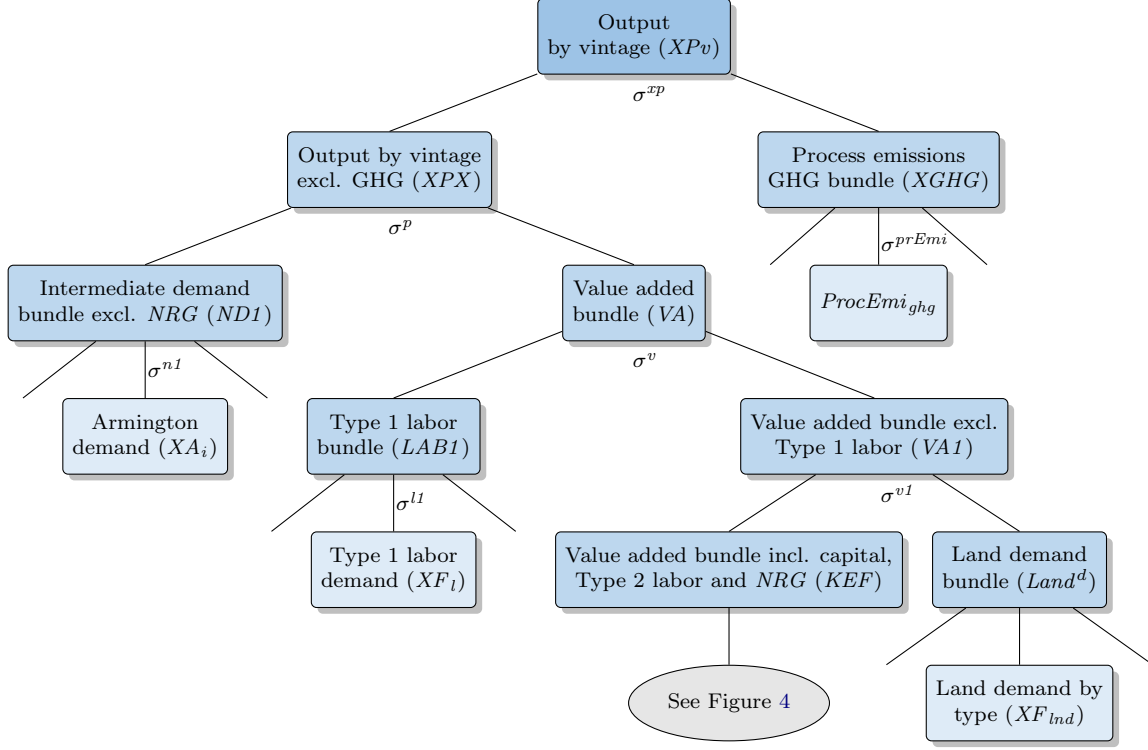
Figure 3: **Default production nest**

The subsequent production nests are identical for all three production prototypes. The *KEF* bundle is composed of the *KF* bundle, a composition of capital, Type 2 labor, water and natural resources, and the energy bundle, *XNRG*. Equations (P-23) and (P-24) represent the derived demands for the *KF* and *XNRG* bundles respectively, with the bundle prices represented by *PKF* and *PNRG*. The main substitution elasticity is σ^{kef}. The parameter $\lambda^{p,nrg}$ represents energy efficiency, normalized to 1 in the reference year. It is applied uniformly across all energy carriers but is activity and vintage specific. Equation (P-25) represents the CES dual price expression for the price of the *KEF* bundle, *PKEF*.
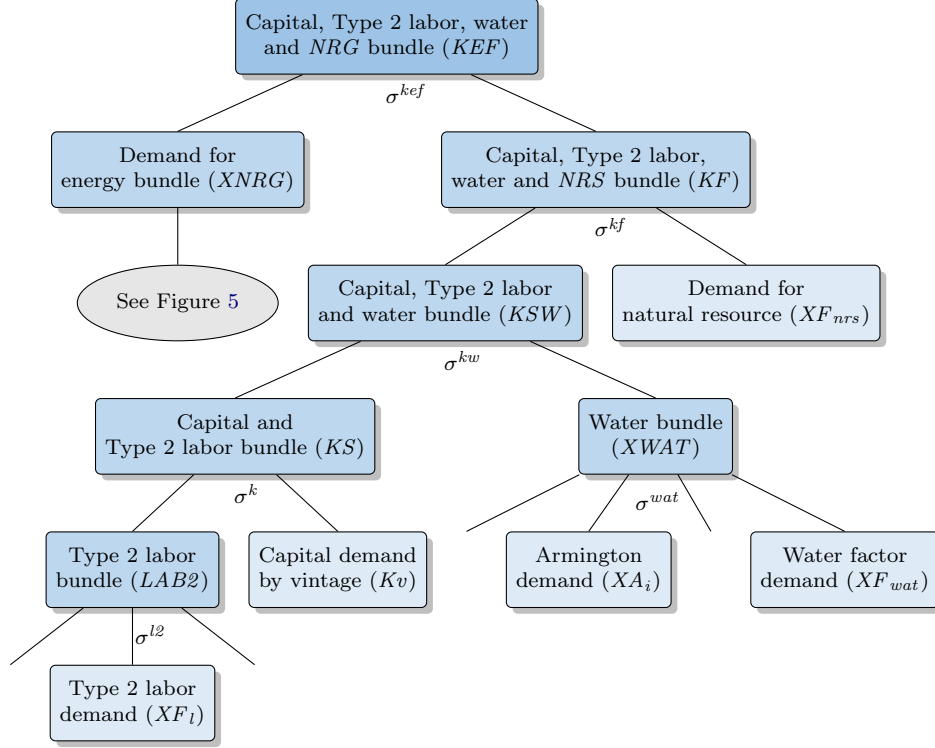
Figure 4: **KEF bundle nest**

$$KF_{r,a,v} = \alpha_{r,a,v}^{kf} \left(\frac{PKEF_{r,a,v}}{PKF_{r,a,v}} \right)^{\sigma_{r,a,v}^{kef}} KEF_{r,a,v} \tag{P-23}$$

$$XNRG_{r,a,v} = \alpha_{r,a,v}^{e} \left(\lambda_{r,a,v}^{p,nrg} \right)^{\sigma^{kef}-1} \left(\frac{PKEF_{r,a,v}}{PNRG_{r,a,v}} \right)^{\sigma_{r,a,v}^{kef}} KEF_{r,a,v} \tag{P-24}$$

$$PKEF_{r,a,v} = \left[\alpha_{r,a,v}^{kf} \left(PKF_{r,a,v} \right)^{1-\sigma_{r,a,v}^{kef}} + \alpha_{r,a,v}^{e} \left(\frac{PNRG_{r,a,v}}{\lambda_{r,a,v}^{p,nrg}} \right)^{1-\sigma_{r,a,v}^{kef}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{kef}}} \tag{P-25}$$

The KF bundle is composed of the KSW bundle, a composition of capital, Type 2 labor and water and the sector-specific natural resource, XF_{nrs}. Equations (P-26) and (P-27) represent the derived demands for the KSW bundle and the sector-specific natural resource, XF_{nrs} (when present), respectively, with the respective prices represented by $PKSW$ and PF_{nrs}^{p}. The main substitution elasticity is σ^{kf}. The natural resource factor has an efficiency factor, λ_{nrs}^{f}, that is typically exogenous. Equation (P-28) represents the CES dual price expression for the price of the KF bundle, PKF.

$$KSW_{r,a,v} = \alpha_{r,a,v}^{ksw} \left(\frac{PKF_{r,a,v}}{PKSW_{r,a,v}} \right)^{\sigma_{r,a,v}^{kf}} KF_{r,a,v} \tag{P-26}$$

$$XF_{r,nrs,a} = \sum_{v} \alpha_{r,a,v}^{nrs} \left(\frac{\lambda_{r,nrs,a}^{f} PKF_{r,a,v}}{PF_{r,nrs,a}^{p}} \right)^{\sigma_{r,a,v}^{kf}} \frac{KF_{r,a,v}}{\lambda_{r,nrs,a}^{f}} \tag{P-27}$$

$$PKF_{r,a,v} = \left[\alpha_{r,a,v}^{ksw} \left(PKSW_{r,a,v} \right)^{1-\sigma_{r,a,v}^{kf}} + \alpha_{r,a,v}^{nrs} \left(\frac{PF_{r,nrs,a}^{p}}{\lambda_{r,nrs,a}^{f}} \right)^{1-\sigma_{r,a,v}^{kf}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{kf}}} \tag{P-28}$$

The KSW bundle is composed of the KS bundle, a composition of capital and Type 2 labor and the water bundle, $XWAT$. Equations (P-29) and (P-30) represent the derived demands for the KS bundle and the water bundle, respectively, with the respective prices represented by PKS and $PWAT$. The main substitution elasticity is σ^{kw}. Equation (P-31) represents the CES dual price expression for the price of the KSW bundle, $PKSW$.

16

$$KS_{r,a,v} = \alpha_{r,a,v}^{ks} \left(\frac{PKSW_{r,a,v}}{PKS_{r,a,v}} \right)^{\sigma_{r,a,v}^{kw}} KSW_{r,a,v} \tag{P-29}$$

$$XWAT_{r,a} = \sum_v \alpha_{r,a,v}^{wat} \left(\frac{PKSW_{r,a,v}}{PWAT_{r,a}} \right)^{\sigma_{r,a,v}^{kw}} KSW_{r,a,v} \tag{P-30}$$

$$PKSW_{r,a,v} = \left[\alpha_{r,a,v}^{ks} \left(PKS_{r,a,v} \right)^{1-\sigma_{r,a,v}^{kw}} + \alpha_{r,a,v}^{wat} \left(PWAT_{r,a} \right)^{1-\sigma_{r,a,v}^{kw}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{kw}}} \tag{P-31}$$

The *KS* bundle is composed of capital (by vintage), K^v, and the Type 2 labor bundle LAB^2. Equations (P-32) and (P-33) represent the derived demands for capital and the Type 2 labor bundle, respectively, with the respective prices represented by PK^p and $PLAB^2$. The main substitution elasticity is σ^k. The capital factor has an efficiency factor, λ_{cap}^f, that is typically exogenous. Equation (P-34) represents the CES dual price expression for the price of the *KS* bundle, *PKS*.

$$K_{r,a,v}^v = \alpha_{r,a,v}^k \left(\frac{\lambda_{r,cap,a}^f PKS_{r,a,v}}{PK_{r,a,v}^p} \right)^{\sigma_{r,a,v}^k} \frac{KS_{r,a,v}}{\lambda_{r,cap,a}^f} \tag{P-32}$$

$$LAB_{r,a}^2 = \sum_v \alpha_{r,a,v}^{l2} \left(\frac{PKS_{r,a,v}}{PLAB_{r,a}^2} \right)^{\sigma_{r,a,v}^k} KS_{r,a,v} \tag{P-33}$$

$$PKS_{r,a,v} = \left[\alpha_{r,a,v}^k \left(\frac{PK_{r,a,v}^p}{\lambda_{r,cap,a}^f} \right)^{1-\sigma_{r,a,v}^k} + \alpha_{r,a,v}^{l2} \left(PLAB_{r,a}^2 \right)^{1-\sigma_{r,a,v}^k} \right]^{\frac{1}{1-\sigma_{r,a,v}^k}} \tag{P-34}$$

The next set of CES nests decomposes the two labor bundles, Type 1 and Type 2 labor, respectively given by LAB^1 and LAB^2.[10] The model specification allows for considerable user flexibility. There are intermediate labor bundles between *LAB1* and *LAB2* and the final labor demand, indexed by *wb*. These bundles are user defined. In the standard configuration where all labor types are bundled together, there is only need for a single intermediate labor demand bundle that will be mapped to *LAB1*, and all labor categories will be mapped to it. There are alternative configurations. One could define two intermediate labor demand bundles, for example unskilled ('nsk') and skilled ('skl') and map GTAP's three unskilled labor types to the 'nsk' bundle and GTAP's two skilled labor types to the 'skl' bundle. The user can then choose whether to map both bundles to LAB^1, or to map the 'skl' bundle to LAB^2. Beyond this added flexibility, this configuration can be used to assess the impacts of additional labor characteristics, such as gender or region of origin.

Equation (P-35) provides the demand equations for the intermediate labor demand bundles. The equations depend on the user-based mapping defined by *wb*. The intermediate bundles are either mapped to LAB^1 or LAB^2, and we allow for differentiated elasticities given be σ^{l1} and σ^{l2}, respectively. Equations (P-36) and (P-37) provide respectively the prices of the LAB^1 and LAB^2 bundles.

$$LABB_{r,wb,a} = \begin{cases} \alpha_{r,wb,a}^{lbb} \left(\frac{PLAB_{r,a}^1}{PLABB_{r,wb,a}} \right)^{\sigma_{r,a}^{l1}} LAB_{r,a}^1 & \text{if } wb \in \{LAB1\} \\ \alpha_{r,wb,a}^{lbb} \left(\frac{PLAB_{r,a}^2}{PLABB_{r,wb,a}} \right)^{\sigma_{r,a}^{l2}} LAB_{r,a}^2 & \text{if } wb \in \{LAB2\} \end{cases} \tag{P-35}$$

$$PLAB_{r,a}^1 = \left[\sum_{wb \in \{LAB1\}} \alpha_{r,wb,a}^{lbb} \left(PLABB_{r,wb,a} \right)^{1-\sigma_{r,a}^{l1}} \right]^{\frac{1}{1-\sigma_{r,a}^{l1}}} \tag{P-36}$$

$$PLAB_{r,a}^2 = \left[\sum_{wb \in \{LAB2\}} \alpha_{r,wb,a}^{lbb} \left(PLABB_{r,wb,a} \right)^{1-\sigma_{r,a}^{l2}} \right]^{\frac{1}{1-\sigma_{r,a}^{l2}}} \tag{P-37}$$

The next stage determines the demand for labor type *l*, the fundamental labor type—normally up to 5 in the standard GTAP database. These labor types are mapped to one, and only one intermediate demand bundle. Equation (P-38) provides the decomposition of the bundles where the key substitution elasticity is given by σ^l and is specific to each intermediate bundle. The producer cost of labor is given by PF_l^p, and λ_l^f represents labor efficiency. Equation (P-39) determines the price of the of the intermediate demand labor bundles, *PLABB*.

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[10]   [NEW 01-May-2019] The labor demand decomposition has been reformulated.

$$XF_{r,l,a} = \alpha^f_{r,l,a} \left(\frac{\lambda^f_{r,l,a} PLABB_{r,wb,a}}{PF^p_{r,l,a}} \right)^{\sigma^l_{r,wb,a}} \frac{LABB_{r,wb,a}}{\lambda^f_{r,l,a}} \qquad \text{if } l \in \{wb\} \tag{P-38}$$

$$PLABB_{r,wb,a} = \left[\sum_{l \in \{wb\}} \alpha^f_{r,l,a} \left(\frac{PF^p_{r,l,a}}{\lambda^f_{r,l,a}} \right)^{1-\sigma^l_{r,wb,a}} \right]^{\frac{1}{1-\sigma^l_{r,wb,a}}} \tag{P-39}$$

Similar to the labor bundles, the next set of CES nests decomposes the three intermediate demand bundles, ND^1, ND^2 and $XWAT$. Recall that ND^2 contains activity-specific inputs such as fertilizers in crops and feed in livestock. The ND^1 bundle contains all of the other intermediate goods—with the exception of water and energy goods. The $XWAT$ bundle contains all designated water commodities from intermediate demand as well as the water factor in some sectors—such as irrigated agriculture. Equation (P-40) provides the decomposition of the bundles where the key substitution elasticities are σ^{nd1}, σ^{nd2} and σ^{wat}, the producer cost of intermediate goods is given by PA^a, and λ^{io} represents an efficiency factor for the use of intermediate goods. Equation (P-41) determines the price of the ND^1 bundle, PND^1 bundle, and equation (P-42) determines the price of the ND^2 bundle, PND^2. The user determines the composition of the intermediate demand bundles by mapping the specific intermediate commodities to one of the two bundles.[11]

$$XA_{r,i,a} = \begin{cases} \alpha^{io}_{r,i,a} \left(\frac{\lambda^{io}_{r,i,a} PND^1_{r,a}}{PA^a_{r,i,a}} \right)^{\sigma^{nd1}_{r,a}} \frac{ND^1_{r,a}}{\lambda^{io}_{r,i,a}} & \text{if } i \in \{ND1\} \\[4mm] \alpha^{io}_{r,i,a} \left(\frac{\lambda^{io}_{r,i,a} PND^2_{r,a}}{PA^a_{r,i,a}} \right)^{\sigma^{nd2}_{r,a}} \frac{ND^2_{r,a}}{\lambda^{io}_{r,i,a}} & \text{if } i \in \{ND2\} \\[4mm] \alpha^{io}_{r,i,a} \left(\frac{\lambda^{io}_{r,i,a} PWAT_{r,a}}{PA^a_{r,i,a}} \right)^{\sigma^{wat}_{r,a}} \frac{XWAT_{r,a}}{\lambda^{io}_{r,i,a}} & \text{if } i \in \{iw\} \end{cases} \tag{P-40}$$

$$PND^1_{r,a} = \left[\sum_{i \in \{ND1\}} \alpha^{io}_{r,i,a} \left(\frac{PA^a_{r,i,a}}{\lambda^{io}_{r,i,a}} \right)^{1-\sigma^{nd1}_{r,a}} \right]^{\frac{1}{1-\sigma^{nd1}_{r,a}}} \tag{P-41}$$

$$PND^2_{r,a} = \left[\sum_{i \in \{ND2\}} \alpha^{io}_{r,i,a} \left(\frac{PA^a_{r,i,a}}{\lambda^{io}_{r,i,a}} \right)^{1-\sigma^{nd2}_{r,a}} \right]^{\frac{1}{1-\sigma^{nd2}_{r,a}}} \tag{P-42}$$

Equation (P-43) determines the demand for the water factor.[12] Equation (P-44) determines the price of the $XWAT$ bundle, where the subset iw spans the set of water commodities.[13]

$$XF_{r,wat,a} = \alpha^f_{r,wat,a} \left(\frac{\lambda^f_{r,wat,a} PWAT_{r,a}}{PF^p_{r,wat,a}} \right)^{\sigma^{wat}_{r,a}} \frac{XWAT_{r,a}}{\lambda^f_{r,wat,a}} \tag{P-43}$$

$$PWAT_{r,a} = \left[\sum_{i \in \{iw\}} \alpha^{io}_{r,i,a} \left(\frac{PA^a_{r,i,a}}{\lambda^{io}_{r,i,a}} \right)^{1-\sigma^{wat}_{r,a}} + \alpha^f_{r,wat,a} \left(\frac{PF^p_{r,wat,a}}{\lambda^f_{r,wat,a}} \right)^{1-\sigma^{wat}_{r,a}} \right]^{\frac{1}{1-\sigma^{wat}_{r,a}}} \tag{P-44}$$

The final set of nests in production concern the energy bundle, $XNRG$. It will be decomposed into demand for the energy commodities, figure 5. The energy bundle is first decomposed into electric and non-electric bundles. The latter is then decomposed into a coal bundle and a non-coal bundle (or the oil & gas bundle). The oil & gas bundle is then split into a gas bundle and an oil bundle. The four remaining bundles—electric, coal, oil and gas—represent a combination of existing or future energy sources. In the case of the original GTAP database, the electric bundle would hold the 'ely' commodity, the coal bundle would hold the 'coa' commodity, the oil bundle would hold the 'oil' and 'p_c' commodities and the gas bundle would hold the 'gas' and 'gdt' commodities. Non-GTAP commodities would be mapped to one of the existing bundles. For example, 'p_c' could be split into gasoline and diesel, and/or could include ethanol or bio-diesel.

[NEW 31-OCT-2020]. The energy bundle has been modified to allow for the so-called additive version of the CES (ACES). The ACES preserves both volume and value additivity and replicates the logit specification of energy

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[11]   In the GAMS code the two mappings are driven by the sets `mapi1` and `mapi2`.

[12]   At the moment, only irrigated crops have any water demand.

[13]   In a standard aggregation of the GTAP database, the subset iw would be composed of the GTAP commodity labeled `wtr`.

demand, which is used in a number of models including GCAM.[14] While the standard CES requires two equations—demand and the zero profit condition—the ACES requires three equations—demand, the composite price index and the zero profit condition.[15] In the expressions below, the description will include all three equations for both the standard and additive CES. In the model code, the zero profit condition is dropped for the standard CES.[16]
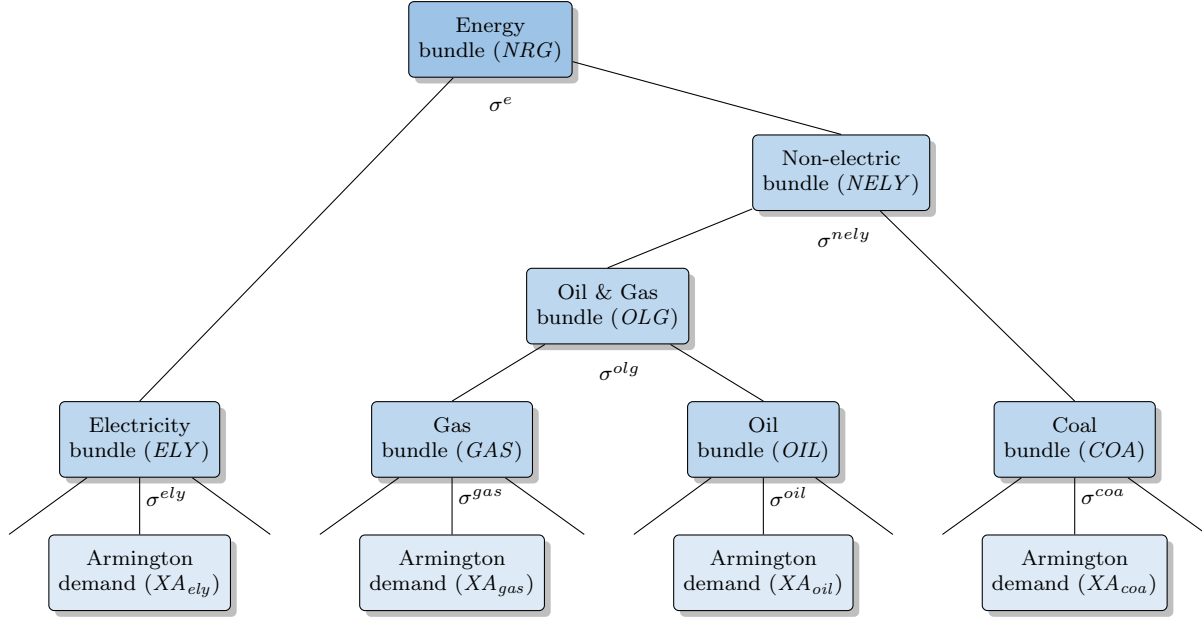
Figure 5: **Energy bundle nest**

Equation (P-45) determines the demand for the electric bundle, XA^{ely}.[17] Equation (P-46) determines the demand for the non-electric bundle, $XNELY$. In both equations, the key substitution elasticity is given by σ^e. As stated above, the demand equations rely on the composite price index, which has a different specification for the standard and additive versions of the CES. Equation (P-47) describes the aggregate composite price index of energy, $PNRG^c$, as a function of the IFNRGACES flag. The latter is true for the additive CES and false otherwise. Equation (P-48) describes the zero profit condition and determines the average price of the energy bundle. In the case of the standard CES, the zero-profit condition is substituted out of the model code as it is identical to the composite price index.

$$XA_{r,a,v}^{ely} = \alpha_{r,a,v}^{ely} \left(\frac{PNRG_{r,a,v}^c}{PA_{r,a,v}^{ely}} \right)^{\sigma_{r,a,v}^e} XNRG_{r,a,v} \tag{P-45}$$

$$XNELY_{r,a,v} = \alpha_{r,a,v}^{nely} \left(\frac{PNRG_{r,a,v}^c}{PNELY_{r,a,v}} \right)^{\sigma_{r,a,v}^e} XNRG_{r,a,v} \tag{P-46}$$

$$PNRG_{r,a,v}^c = \begin{cases} \left[\alpha_{r,a,v}^{ely} \left(PA_{r,a,v}^{ely} \right)^{1-\sigma_{r,a,v}^e} + \alpha_{r,a,v}^{nely} \left(PNELY_{r,a,v} \right)^{1-\sigma_{r,a,v}^e} \right]^{\frac{1}{1-\sigma_{r,a,v}^e}} & \text{if } \neg\text{IFNRGACES} \\[2ex] \left[\alpha_{r,a,v}^{ely} \left(PA_{r,a,v}^{ely} \right)^{-\sigma_{r,a,v}^e} + \alpha_{r,a,v}^{nely} \left(PNELY_{r,a,v} \right)^{-\sigma_{r,a,v}^e} \right]^{-\frac{1}{\sigma_{r,a,v}^e}} & \text{if } \text{IFNRGACES} \end{cases} \tag{P-47}$$

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[14] GCAM derived from an earlier model developed by Edmonds and Reilly (Edmonds and Reilly (1985)).

[15] It is easy to show that for the standard CES, the composite price expression and the zero profit equation are identical. This is not the case for the ACES.

[16] While the zero profit condition is needed for the ACES/ACET it will normally be implemented using the relation that links the average bundle price with the composite price index. See equations (A-20) and (A-17) in Annex A. The standard zero-profit condition can lead to numeric difficulties in some situations.

[17] In the GAMS code, the demand for the four energy bundles ('ely', 'gas', 'oil' and 'coa') are represented by the variable xaNRG that has an additional dimension representing the four main energy carriers. This simplifies the decomposition of these bundles that only requires one set of equations rather than four. In principle, it also makes it somewhat easier to increase the number of energy bundles.

$$PNRG_{r,a,v} = \left[PA_{r,a,v}^{ely} XA_{r,a,v}^{ely} + PNELY_{r,a,v} XNELY_{r,a,v} \right] \Big/ XNRG_{r,a,v} \tag{P-48}$$

Equation (P-49) determines the demand for the coal bundle, XA^{coa}. Equation (P-50) determines the demand for the oil & gas bundle, $XOLG$. In both equations, the key substitution elasticity is given by σ^{nely}. Equation (P-51) defines the composite price index of the non-electric bundle, $PNELY^c$ and equation (P-52) describes the aggregate price of the non-electric bundle, $PNELY$.

$$XA_{r,a,v}^{coa} = \alpha_{r,a,v}^{coa} \left(\frac{PNELY_{r,a,v}^{c}}{PA_{r,a,v}^{coa}} \right)^{\sigma_{r,a,v}^{nely}} XNELY_{r,a,v} \tag{P-49}$$

$$XOLG_{r,a,v} = \alpha_{r,a,v}^{olg} \left(\frac{PNELY_{r,a,v}^{c}}{POLG_{r,a,v}} \right)^{\sigma_{r,a,v}^{nely}} XNELY_{r,a,v} \tag{P-50}$$

$$PNELY_{r,a,v}^{c} = \begin{cases} \left[\alpha_{r,a,v}^{coa} \left(PA_{r,a,v}^{coa} \right)^{1-\sigma_{r,a,v}^{nely}} + \alpha_{r,a,v}^{olg} \left(POLG_{r,a,v} \right)^{1-\sigma_{r,a,v}^{nely}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{nely}}} & \text{if } \neg\texttt{IFNRGACES} \\[2ex] \left[\alpha_{r,a,v}^{coa} \left(PA_{r,a,v}^{coa} \right)^{-\sigma_{r,a,v}^{nely}} + \alpha_{r,a,v}^{olg} \left(POLG_{r,a,v} \right)^{-\sigma_{r,a,v}^{nely}} \right]^{-\frac{1}{\sigma_{r,a,v}^{nely}}} & \text{if } \texttt{IFNRGACES} \end{cases} \tag{P-51}$$

$$PNELY_{r,a,v} = \left[PA_{r,a,v}^{coa} XA_{r,a,v}^{coa} + POLG_{r,a,v} XOLG_{r,a,v} \right] \Big/ XNELY_{r,a,v} \tag{P-52}$$

The remaining two energy bundles are oil and gas and emanate from the $XOLG$ bundle. Equation (P-53) determines the demand for the oil bundle, XA^{oil}. Equation (P-54) determines the demand for the gas bundle, XA^{gas}. In both equations, the key substitution elasticity is given by σ^{OLG}. Equation (P-55) describes the composite price index of the oil & gas bundle, $POLG^c$. Equation (P-56) describes the aggregate price of the oil & gas bundle, $POLG$.

$$XA_{r,a,v}^{oil} = \alpha_{r,a,v}^{oil} \left(\frac{POLG_{r,a,v}^{c}}{PA_{r,a,v}^{oil}} \right)^{\sigma_{r,a,v}^{olg}} XOLG_{r,a,v} \tag{P-53}$$

$$XA_{r,a,v}^{gas} = \alpha_{r,a,v}^{gas} \left(\frac{POLG_{r,a,v}^{c}}{PA_{r,a,v}^{gas}} \right)^{\sigma_{r,a,v}^{olg}} XOLG_{r,a,v} \tag{P-54}$$

$$POLG_{r,a,v}^{c} = \begin{cases} \left[\alpha_{r,a,v}^{oil} \left(PA_{r,a,v}^{oil} \right)^{1-\sigma_{r,a,v}^{olg}} + \alpha_{r,a,v}^{gas} \left(PA_{r,a,v}^{gas} \right)^{1-\sigma_{r,a,v}^{olg}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{olg}}} & \text{if } \neg\texttt{IFNRGACES} \\[2ex] \left[\alpha_{r,a,v}^{oil} \left(PA_{r,a,v}^{oil} \right)^{-\sigma_{r,a,v}^{olg}} + \alpha_{r,a,v}^{gas} \left(PA_{r,a,v}^{gas} \right)^{-\sigma_{r,a,v}^{olg}} \right]^{-\frac{1}{\sigma_{r,a,v}^{olg}}} & \text{if } \texttt{IFNRGACES} \end{cases} \tag{P-55}$$

$$POLG_{r,a,v} = \left[PA_{r,a,v}^{oil} XA_{r,a,v}^{oil} + PA_{r,a,v}^{gas} XA_{r,a,v}^{gas} \right] \Big/ XOLG_{r,a,v} \tag{P-56}$$

The final nest in the energy bundle is to decompose the four aggregate energy bundles into their constituent parts that represent the Armington demand for the energy commodities. Equation (P-57) reflects the Armington demand for energy commodity e, XA, where the cost to producers is given by PA^a. The key substitution elasticity for each energy bundle is given by σ^{NRG}. Equation (P-59) represents the composite price index of the aggregate energy bundles, $PA^{c,NRG}$. Equation (P-59) represents the price of the aggregate energy bundles, PA^{NRG}. Note that the parameter λ has a different role in the two versions. Under the standard CES, λ is used to change the efficiency of energy use (by carrier, activity and vintage). In the case of the additive CES, λ is a preference shifter. For the moment, the core shift in energy efficience is embedded in the demand for the energy bundle, equation (P-24), which is activity and vintage specific, but not carrier specific.

$$XA_{r,e,a} = \begin{cases} \displaystyle\sum_v \alpha_{r,e,a,v}^{eio} \left(\frac{\lambda_{r,e,a,v}^{e} PA_{r,a,v}^{c,NRG}}{PA_{r,e,a}^{a}} \right)^{\sigma_{r,a,v}^{NRG}} \frac{XA_{r,a,v}^{NRG}}{\lambda_{r,e,a,v}^{e}} & \text{if } e \in \{NRG\} \,\&\text{if } \neg\texttt{IFNRGACES} \\[3ex] \displaystyle\sum_v \alpha_{r,e,a,v}^{eio} \left(\frac{PA_{r,a,v}^{c,NRG}}{\lambda_{r,e,a,v}^{e} PA_{r,e,a}^{a}} \right)^{\sigma_{r,a,v}^{NRG}} XA_{r,a,v}^{NRG} & \text{if } e \in \{NRG\} \,\&\text{if } \texttt{IFNRGACES} \end{cases} \tag{P-57}$$

$$PA_{r,a,v}^{c,NRG} = \begin{cases} \left[\displaystyle\sum_{e \in \{NRG\}} \alpha_{r,e,a,v}^{eio} \left(\frac{PA_{r,e,a}^{a}}{\lambda_{r,e,a,v}^{e}} \right)^{1-\sigma_{r,a,v}^{NRG}} \right]^{\frac{1}{1-\sigma_{r,a,v}^{NRG}}} & \text{if } \neg\texttt{IFNRGACES} \\[4ex] \left[\displaystyle\sum_{e \in \{NRG\}} \alpha_{r,e,a,v}^{eio} \left(\lambda_{r,e,a,v}^{e} PA_{r,e,a}^{a} \right)^{-\sigma_{r,a,v}^{NRG}} \right]^{-\frac{1}{\sigma_{r,a,v}^{NRG}}} & \text{if } \texttt{IFNRGACES} \end{cases} \tag{P-58}$$

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$$PA_{r,a,v}^{NRG} = \sum_{e \in \{NRG\}} PA_{r,e,a}^{a} XA_{r,e,a} \bigg/ XA_{r,a,v}^{NRG} \tag{P-59}$$

3.2.1 Alternative nesting for the natural resource factor

[01-Aug-2020] The model allows for the user to change the nesting for the natural resource factor. Figure 6, shows a simplified version of the standard nesting. In the standard nesting the natural resource factor enters as a factor bundled with capital. In the alternative nesting, the demand for the natural resource is at the top of the nest (but after the output/GHG nest), where the variable XPN represents the output net of GHG and XPX is equal to XPN in the standard case, but is output net of the natural resource in the case of the alternate nesting—see Figure 7.
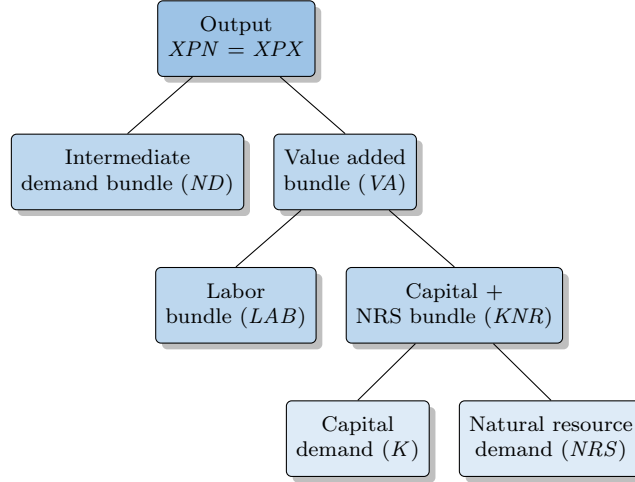
Figure 6: **Simplified standard nest**
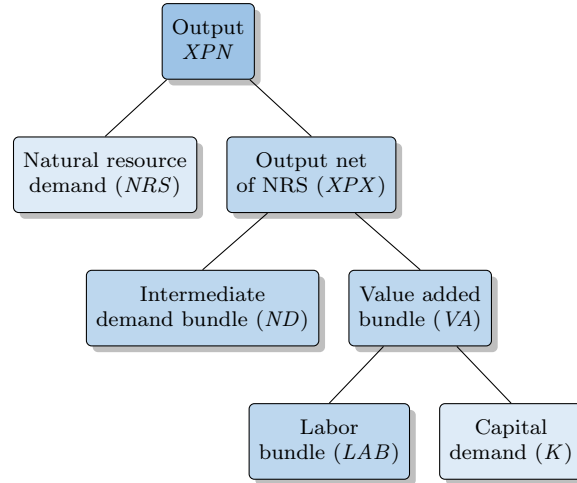
Figure 7: **Simplified alternative nest**

The nesting is a user-specified option governed by the parameter `ifNRSTop`. If its value is set to 1, the alternative nesting is used, else the model uses the standard nesting. The modified nesting introduces a new substitution elasticity, σ^{nrs}. Default values are given, but it is up to the user to verify and modify the value if needed.[18]

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[18]   The standard nesting uses the CES elasticity σ^{kf} to deduce the demand for the natural resource factor.

3.3   Commodity supply

The model allows for each activity a to produce one or more commodities (indexed by i). For example, the ethanol industry could produce both ethanol as well as distiller's dried grain with solubles (or DDGS). Similarly, a single commodity can be produced by one or more activities. For example, the electricity commodity can be produced by multiple generation activities—thermal, nuclear, hydro, renewables, etc. Joint production is captured by a CET transformation function, with the possibility of perfect transformation. The aggregation of output from multiple activities is captured with a CES preference function, again with the possibility of commodity homogeneity, in which case the law-of-one-price holds.[19]

As an option to the standard model, power generation can be treated in a special fashion compared to other goods and services. The aggregation of production across activities is normally determined using a single CES nest. For the power activities, aggregation to the single electricity commodity uses a nested CES structure that is explained in the subsequent section.

The latest version of ENVISAGE hones to the new GTAP standard model, see Corong (2017), that applies the output tax to the 'make' commodities, i.e. the output tax is now both activity- and commodity-specific. This introduces a wedge between the basic price of commodity X, P, and the tax-inclusive price, PP.[20]

3.3.1   Non-electric goods

This section describes the make matrix for all commodities with the exception of electricity that is described subsequently. The equations thus hold for all commodities indexed by i except for the electricity commodity. The allocation of output, $XP_{r,a}$, from activity a, to supply one or more commodity i, is described in equation (S-1) that is the standard CET supply allocation expression. The variable X represents the supply of commodity i by activity a. The transformation elasticity is given by ω^s. The model allows for perfect transformation, in which case the law-of-one-price holds. Equation (S-2) is in essence an equilibrium condition that determines the aggregate output of activity a, XP. In the case of perfect transformation, it simply aggregates the individual supplies. The CET expression incorporates an efficiency parameter, λ^s that allows for changes between the quantity produced and the quantity to reach the market.[21] This can be used, for example, to reduce post-harvest losses.

$$\begin{cases} X_{r,a,i} = \gamma^p_{r,a,i} \left(\dfrac{1}{\lambda^s_{r,a,i}}\right)^{1+\omega^s_{r,a}} \left(\dfrac{P_{r,a,i}}{PX_{r,a}}\right)^{\omega^s_{r,a}} XP_{r,a} & \text{if } \omega^s_{r,a} \neq \infty \\ P_{r,a,i} = \lambda^s_{r,a,i} PX_{r,a} & \text{if } \omega^s_{r,a} = \infty \end{cases} \tag{S-1}$$

$$PX_{r,a} XP_{r,a} = \sum_i P_{r,a,i} X_{r,a,i} \tag{S-2}$$

The output tax, τ^p, is applied to price P and generates the supply price of commodity i produced by activity a, equation (S-3). The supply of commodity i is the (CES) aggregation of output of one or more activities a. A CES preference function is used for the aggregation. Equation (S-4) determines the 'demand' for output a to compose commodity i, X.[22] The substitution elasticity is given by σ^s. The model allows for perfect substitution, in which case the law-of-one-price holds. Equation (S-5) determines the (market) price of commodity i, PS.

$$PP_{r,a,i} = \left(1 + \tau^p_{r,a,i}\right) P_{r,a,i} \tag{S-3}$$

$$\begin{cases} X_{r,a,i} = \alpha^s_{r,a,i} \left(\dfrac{PS_{r,i}}{PP_{r,a,i}}\right)^{\sigma^s_{r,i}} XS_{r,i} & \text{if } \sigma^s_{r,i} \neq \infty \\ PP_{r,a,i} = PS_{r,i} & \text{if } \sigma^s_{r,i} = \infty \end{cases} \tag{S-4}$$

$$PS_{r,i} XS_{r,i} = \sum_a PP_{r,a,i} X_{r,a,i} \tag{S-5}$$

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[19]   In the standard GTAP database, there is a one-to-one correspondence between activities and commodities, i.e. the make matrix is diagonal. The aggregation facility for the ENVISAGE model allows the user to determine the mapping from activities to commodities. For example, it is possible to have a single agricultural production function that produces a number of agricultural commodities. Similarly, it is possible to have a number of agricultural production activities that produce a single agricultural commodity.

[20]   Note that the variable PP is substituted out of all equations in the model specification.

[21]   [NEW: 4-OCT-2018]

[22]   Equation (S-1) determines supply and equation (S-4) determines demand. The supply/demand equilibrium condition is substituted out.

3.3.2   Domestic supply of electricity

The bundling of electricity uses a nested CES structure instead of a single nest, see figure 8. The top nest combines aggregate power supply with distribution and transmission services to form aggregate domestic electric supply. The power nest combines a number of different (and user-determined) power bundles. Subsequently, each of these power bundles are formed by the different power activities that are segmented into the different power bundles (under user-mapping). For example, the power bundles may be composed of coal-, oil- and gas-generation, nuclear, and all other. Using the GTAP power database, base load coal could be mapped to the coal power bundle, base and peak load oil could be mapped to the oil power bundle, base and peak load gas could be mapped to the gas power bundle, base load nuclear would be mapped to the nuclear power bundle, and all other power activities (wind, solar, hydro and other) could be mapped to the other power bundle. The strategy for future technologies would be to bundle them in power bundles. For example coal capture and storage could be in the coal power bundle, and advanced nuclear could be incorporated in the nuclear power bundle.
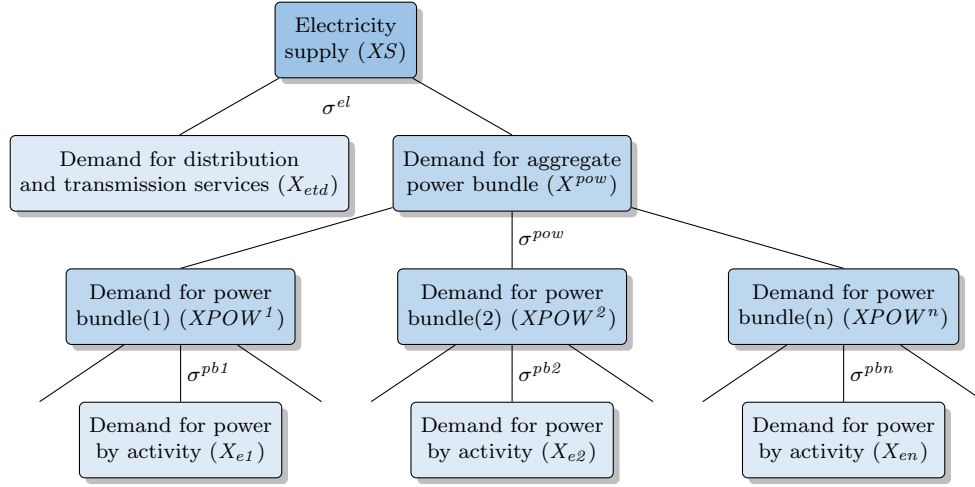
Figure 8: **CES nest for power bundle**

Equation (S-6) determines the demand for electricity services—indexed by activities *etd*—used to produce one or more electric commodities—indexed by *ely*.[23] It is linked to the total supply of power, XS^{pow}, in a CES bundle. The normal specification assumes a Leontief technology, i.e. a substitution elasticity of zero. Equation (S-7) determines the demand for the power bundle, it is a bundle of all electricity generation, and excludes the transmission and distribution services. Equation (S-8) determines the supply price of aggregate electricity.

$$X_{r,etd,ely} = \alpha^s_{r,etd,ely} \left(\frac{PS_{r,ely}}{PP_{r,etd,ely}} \right)^{\sigma^{el}_{r,ely}} XS_{r,ely} \tag{S-6}$$

$$XPOW_{r,ely} = \alpha^{pow}_{r,ely} \left(\frac{PS_{r,ely}}{PPOW_{r,ely}} \right)^{\sigma^{el}_{r,ely}} XS_{r,ely} \tag{S-7}$$

$$PS_{r,ely} = \left[\alpha^s_{r,etd,ely} \left(PP_{r,etd,ely} \right)^{1-\sigma^{el}_{r,ely}} + \alpha^{pow}_{r,ely} \left(PPOW_{r,ely} \right)^{1-\sigma^{el}_{r,ely}} \right]^{1/(1-\sigma^{el}_{r,ely})} \tag{S-8}$$

The following stage decomposes aggregate demand for power into a user-determined number of power bundles—indexed by *pb*. Equation (S-9) determines the demand for the power bundles. Notice that the aggregate price used in the expression is *PPOWN* and not *PPOW*. The latter is the aggregate, or average price of the power bundle. The former is a price index that is defined in equation (S-10). In the standard CES, the two price concepts are identical. The power decomposition uses the so-called adjusted CES, which preserves the additivity of the CES components (see Appendix A). The demand expressions in both versions of the CES are similar. However, the expresssion for the aggregate price index differs and the price index is not equal to the average price (as calculated using the zero profit condition). Thus, equation (S-11) is added that evaluates the average price, *PPOW*.

[17-MAR-2020] Control of targeted power bundles has been added using so-called *phantom* taxes. See Section 3.11 for a description of this model feature.

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[23]   Typically there is a single transmission and distribution activity and a single electricity commodity.

$$XPB_{r,pb,ely} = \alpha^{pb}_{r,pb,ely} \left(\lambda^{pow}_{r,pb,ely}\right)^{-\sigma^{pow}_{r,ely}} \left(\frac{PPOWN_{r,ely}}{PPB_{r,pb,ely}}\right)^{\sigma^{pow}_{r,ely}} XPOW_{r,ely} \tag{S-9}$$

$$PPOWN_{r,ely} = \left[\sum_{pb} \alpha^{pb}_{r,pb,ely} \left(\lambda^{pow}_{r,pb,ely} PPB_{r,pb,ely}\right)^{-\sigma^{pow}_{r,ely}}\right]^{-1/\sigma^{pow}_{r,ely}} \tag{S-10}$$

$$PPOW_{r,ely} XPOW_{r,ely} = \sum_{pb} PPB_{r,pb,ely} XPB_{r,pb,ely} \tag{S-11}$$

The subsequent nest decomposes the various power bundles into component power activities. Each power activity is mapped to one of the aggregate power bundles. Equation (S-12) determines the demand for power generated by power activity *elya* that is mapped to power bundle *pb*. Equations (S-13) and (S-14) determine respectively the price index for the power bundle *pb*, as derived from the adjusted CES price index expression, and the average price of the power bundle *pb* using the zero profit condition.

$$X_{r,elya,ely} = \alpha^{s}_{r,elya,ely} \left(\frac{PPBN_{r,pb,ely}}{\lambda^{pb}_{r,elya,ely} PP_{r,elya,ely}}\right)^{\sigma^{pb}_{r,pb,ely}} XPB_{r,pb,ely} \quad \text{if } elya \in pb \tag{S-12}$$

$$PPBN_{r,pb,ely} = \left[\sum_{elya \in pb} \alpha^{s}_{r,elya,ely} \left(\lambda^{pb}_{r,elya,ely} PP_{r,elya,ely}\right)^{-\sigma^{pb}_{r,pb,ely}}\right]^{-1/\sigma^{pb}_{r,pb,ely}} \tag{S-13}$$

$$PPB_{r,pb,ely} XPB_{r,pb,ely} = \sum_{elya \in pb} PP_{r,elya,ely} X_{r,elya,ely} \tag{S-14}$$

3.4   Income block

There are three domestic final demand agents—households (h), an aggregate government sector (gov) and an aggregate investment sector (inv). All factor income, net of taxes, accrues to the private household, government revenues are generated by the various indirect taxes in the economy as well as direct taxes, and investment income is equated to the sum of domestic and foreign savings. A portion of capital income flows to a 'global' holder of equity that then portions out profits from the global fund. Remittances are also incorporated and are fully bilateral.

3.4.1   Region-wide accounts

The depreciation allowance is calculated as the replacement cost of the estimated depreciation, equation Y-1. The parameter δ^f is allowed to differ from the physical rate of depreciation, δ, though in most cases it will be identical. The variable PFD_{inv} is the unit cost of investment and K^s is the non-normalized level of the aggregate capital stock. The normalized level of the capital stock, further described below, is scaled to the initial aggregate remuneration of capital, i.e. it's price in the base year is 1. The non-normalized level is needed for calculating the depreciation allowance and in the dynamic equation for updating the aggregate capital stock.

$$DeprY_r = \delta^f_r PFD_{r,inv} K^s_r \tag{Y-1}$$

The income variable $ntmY$ represents the total income generated by the *ad valorem* equivalent (AVE) of non-tariff measures (NTMs). The latter is measured by the trade instrument τ^{ntm}, which is an additive component to the standard bilateral tariff. Equation Y-2 defines the total NTM income—that can be distributed to both domestic and foreign agents.[24] The standard GTAP database does not incorporate NTMs. These have been introduced using a version of Altertax (Malcolm (1998)) using AVE estimates from Kee et al. (2009).

$$ntmY_r = \begin{cases} \sum_s \sum_i \tau^{ntm}_{s,i,r} PWM_{s,i,r} \lambda^w_{s,i,r} \lambda^x_{s,i,r} XW_{s,i,r} & \text{if default specification} \\ \sum_s \sum_i \sum_{aa} \tau^{ntma}_{s,i,r,aa} PWM_{s,i,r} XW^a_{s,i,r,aa} & \text{if MRIO specification} \end{cases} \tag{Y-2}$$

The model incorporates some level of international income flow from profits. A portion of each region's profits (net of taxes) flows to a global equity fund that disburses its aggregate income across regions. Equation Y-3 represents the flow of a region's profits net of taxes, $YQTF$, to the global equity fund. Total income for the global equity fund, $TrustY$, is provided by equation Y-4. Foreign profit inflows, $YQHT$, are represented by equation Y-5. Equation Y-6

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[24]   [NEW: 4-OCT-2018] The MRIO version of the model has not yet been modified to handle NTMs.

determines remittances from country r to country s for labor of skill l.[25] Remittances are calculated relative to after tax labor income.

$$YQTF_r = \chi_r^f \sum_a \left(1 - \kappa_{r,cap,a}^f\right) PF_{r,cap,a} XF_{r,cap,a} \tag{Y-3}$$

$$TrustY = \sum_r YQTF_r \tag{Y-4}$$

$$YQHT_r = \chi_r^h TrustY \tag{Y-5}$$

$$Remit_{s,l,r} = \chi_{s,l,r}^r \sum_a \left(1 - \kappa_{r,l,a}^f\right) PF_{r,l,a} XF_{r,l,a} \tag{Y-6}$$

The model incorporates government to government transfers.[26] These have been labeled as Official Development Assistance (ODA), but in fact cover a fairly broad range of inter-government transfers. Equation (Y-7) determines outward ODA. The basic assumption is that outward ODA grows at the same rate as GDP, where the parameter χ^{OO} represents the initial outward ODA to GDP ratio. The equation allows for the ratio to increase with the growth in per capita income, where η^{ODA} represents the elasticity of ODA relative to GDP with respect to per capita income growth.[27] Equation (Y-8) represents aggregate ODA, i.e. the sum of ODA across all regions. Global ODA is allocated in fixed proportions across all regions, i.e. Equation (Y-9) determines inward ODA.

$$ODAOut_r = \chi_r^{OO} GDPMP_r \left(\frac{RGDPpc_r}{RGDPpc_{r,0}}\right)^{\eta^{ODA}} \tag{Y-7}$$

$$ODAGbl = \sum_r ODAOut_r \tag{Y-8}$$

$$ODAIn_r = \chi_r^{OI} ODAGbl \tag{Y-9}$$

3.4.2 Household income and savings

Equation (Y-10) describes household income, YH. It is the sum across all activities of factor income, at market prices and net of taxes and depreciation. Household income also includes net foreign capital income, net remittances and some portion of NTM-related income from all modeled regions.[28] Factor returns at the price producers pay have a superscript p, which is absent from the factor returns at market prices. Equation (Y-11) describes disposable income, YD, where κ^h is the marginal (and average) rate of tax on household income. Macro closure is discussed below.

$$
\begin{aligned}
YH_r &= \sum_f \sum_a \left(1 - \kappa_{r,f,a}^f\right) PF_{r,f,a} XF_{r,f,a} + YQHT_r - YQTF_r \\
&+ \sum_d \sum_l Remit_{r,l,d} - \sum_s \sum_l Remit_{s,l,r} - DeprY_r + \sum_s \chi_{s,r}^{hNTM} ntmY_s
\end{aligned} \tag{Y-10}
$$

$$YD_r = \left(1 - \kappa_r^h\right) YH_r \tag{Y-11}$$

Household savings is treated differently across the demand systems. In the case of the ELES, savings is incorporated in the utility function. In the case of the other demand systems, a separate assumption is made about the savings propensity and savings is subtracted from disposable income with the residual allocated to consumer goods.

Equation (Y-12) determines savings for all utility function with the exception of the ELES. Savings is simply a proportion of household disposable income.

[17-MAR-2020] Equation (Y-13) determines the savings rate, aps. It is equal to a shifter, χ^s, which is set to 1 initially and can also reflect changes in the rate of return. By default, the savings rate elasticity is equal to zero. Depending on closure, the variable χ^s can be endogenous to target some other objective, for example, the level of investment relative to GDP. In other closures, it could be exogenous, in which case investment is savings driven. The equation is not active in the case of the ELES, since savings is determined simultaneously with consumption. In the

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[25]   In the GMig model, remittances are linked to the labor income of migrants by region of origin, i.e. s. Herein, the remittances are linked to the entire wage bill (by skill), not the wage bill of migrants from region s.

[26]   This feature was added 21-Apr-2018.

[27]   The default value is 0.

[28]   [NEW: 4-OCT-2018] The relevant revenue shares related to NTM income must add to 1.

case of the ELES, equation (Y-14) determines household savings as the difference between disposable income and total expenditures on goods and services.[29] In the case of the other demand systems, the equation is dropped.

$$S_{r,h}^h = aps_r YD_r \tag{Y-12}$$

$$aps_r = \chi_r^s \left(\frac{TR_{r,t}/TR_{r,t-1}}{PGDP_r} \right)^{\eta_r^s} \tag{Y-13}$$

$$S_{r,h}^h = YD_r - \sum_k PC_{r,k,h} XC_{r,k,h} \tag{Y-14}$$

[17-Mar-2020] Finally, equation (Y-15) determines household expenditures on goods and services, i.e. disposable income net of savings.

$$YC_{r,h} = YD_r - S_{r,h}^h \tag{Y-15}$$

3.4.3 Government revenues

The next set of equations describe government revenues, contained in the variable $YGOV$, which has an additional index for the different revenue streams (gy). Equation (Y-16) describes revenues from production and cost taxes. The production tax is applied on commodity i produced by activity a. The revenue index is given by ptx. Equation (Y-17) describes revenues generated on the factors of production—labor, capital, land and natural resources.[30] The relevant factor prices are the market prices, i.e. prices received by the factors themselves. The revenue index is given by vtx.

$$YGOV_{r,ptx} = \sum_a \left[\sum_i \tau_{r,a,i}^p P_{r,a,i} X_{r,a,i} + \sum_v \tau_{r,a,v}^{uc} UC_{r,a,v} XPv_{r,a,v} \right] \tag{Y-16}$$

$$YGOV_{r,vtx} = \sum_a \sum_f \tau_{r,f,a}^v PF_{r,f,a} XF_{r,f,a} \tag{Y-17}$$

Equation (Y-18) describes revenues generated by consumption of goods, essentially a sales tax. The sum is over all domestic agents (indexed by aa). The relevant price is the market price of good i. The equation incorporates the two different Armington options. In the first case, the sourcing of goods is made at the national (aggregate) level and thus all users face a common Armington price, PAT, that is then adjusted by the end-user sales tax.[31] The second option assumes a top level Armington sourcing by agent in which case the domestic sales tax is differentiated by source. Sales taxes on import consumption is based on agents' prices of imports, which is the tariff-inclusive price of imports given by the variable PM^a. If bilateral imports are determined at the national level, the standard model assumption, agents' prices are uniform and represented by the variable PMT_i, which represents the average tariff-inclusive price of imports. In the MRIO version of the model, agents' import prices are agent-specific and represent potentially differences in both the preference structure and tariffs. The revenue index is given by itx.

$$YGOV_{r,itx} = \begin{cases} \displaystyle\sum_{aa} \sum_i \tau_{r,i,aa}^a \gamma_{r,i,aa}^{eda} PAT_{r,i} XA_{r,i,aa} & \text{if } ArmFlag = 0 \\ \displaystyle\sum_{aa} \sum_i \tau_{r,i,aa}^{ad} \gamma_{r,i,aa}^{edd} PDT_{r,i} XD_{r,i,aa} \\ \qquad + \tau_{r,i,aa}^{am} \gamma_{r,i,aa}^{edm} PM_{r,i,aa}^a XM_{r,i,aa} & \text{if } ArmFlag \neq 0 \end{cases} \tag{Y-18}$$

Equation (Y-19) describes revenues generated by import tariffs. This equation has two versions depending on whether or not the MRIO version is implemented. In the standard specification, tariffs are uniform across all agents and the tax collection is done at the border relative to the aggregate level of bilateral tariffs and summed across all source countries (s), where the first regional index is always the exporting region and the second regional index is

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[29]   To target investment, in the case of the ELES it is possible to target a given level of investment (as a share of GDP for example) and to endogenize the β^c parameter that in effect will find a level of savings consistent with targeted investment.

[30]   [17-Mar-2020] In the model code, τ^v, can be an endogenous variable used to implement alternative fiscal closures. For example, an additive (or multiplicative) shifter can be endogenous in order to recycle carbon tax revenues. The shifter is both activity and factor specific.

[31]   Goods at this level are assumed to be additive though allowances are made for different prices captured by the γ^e coefficients.

always the destination (or importing) region. The tariffs are applied to the border (or CIF) price of imports, PWM.[32] In the MRIO version, tariffs are agent specific, τ^{ma}, and applied on the variable XW^a, which represents the volume of imports by agent aa sourced in region s. It is assumed that all agents pay the same CIF price for their imports. The revenue index is given by mtx.

$$YGOV_{r,mtx} = \begin{cases} \sum_s \sum_i \tau^m_{s,i,r} PWM_{s,i,r} \lambda^w_{s,i,r} \lambda^x_{s,i,r} XW^d_{s,i,r} & \text{if default specification} \\ \sum_s \sum_i \sum_{aa} \tau^{ma}_{s,i,r,aa} PWM_{s,i,r} XW^a_{s,i,r,aa} & \text{if MRIO specification} \end{cases}$$ (Y-19)

Equation (Y-20) describes revenues generated by export taxes and subsidies. These are summed over all destination countries (d). They are applied to the producer price of exports, PE. The revenue index is given by etx.

$$YGOV_{r,etx} = \sum_d \sum_i \tau^e_{r,i,d} PE_{r,i,d} XW^s_{r,i,d}$$ (Y-20)

Equation (Y-21) describes revenues generated by a tax on waste generated by household consumption.[33] The tax instrument is τ^w and XA^w represents the level of household waste. The revenue index is given by wtx.

$$YGOV_{r,wtx} = \sum_h \sum_i \tau^w_{r,i,h} PA_{r,i,h} XA^w_{r,i,h}$$ (Y-21)

Equation (Y-22) describes revenues generated by carbon taxes. This equation holds for either Armington specification. Details on the carbon tax are described further below. The revenue index is given by ctx. Section 3.11.7 describes alternatives to the standard fiscal closure that allows for targeted recycling of carbon tax revenues.

$$YGOV_{r,ctx} = \begin{cases} \sum_{em} \sum_i \sum_{aa} \chi^{Emi}_{em} \rho^{Emi}_{r,em,i,aa} \phi^{Emi}_{r,em,i,aa} \tau^{Emi}_{r,em,aa} XA_{r,i,aa} & \text{if } ArmFlag = 0 \\ \sum_{em} \sum_i \sum_{aa} \chi^{Emi}_{em} \rho^{Emi,d}_{r,em,i,aa} \phi^{Emi}_{r,em,i,aa} \tau^{Emi}_{r,em,aa} XD_{r,i,aa} & \\ \quad + \chi^{Emi}_{em} \rho^{Emi,m}_{r,em,i,aa} \phi^{Emi}_{r,em,i,aa} \tau^{Emi}_{r,em,aa} XM_{r,i,aa} & \text{if } ArmFlag \neq 0 \end{cases}$$ (Y-22)

Finally, direct taxes are described in equation (Y-23), where the revenue index is given by dtx. Direct taxes are imposed on specific factor incomes and there is a net direct tax (that can be negative) on total household income after factor taxes that balances the government account (for a given deficit level).

$$YGOV_{r,dtx} = \sum_f \sum_a \kappa^f_{r,f,a} PF_{r,f,a} XF_{r,f,a} + \kappa^h_r YH_r$$ (Y-23)

3.4.4 Investment/savings balance

Equation (Y-24) describes the financing of gross investment. The variable YFD represents final demand expenditures, in value terms, for the final demand agents, indexed by fd that takes on values of h, gov and inv respectively for households, government and investment.[34] Gross investment is equated to the sum of all savings, domestic from households (S^h) and government (S^g), and foreign (S^f), where the latter is evaluated using a global price index, PW^{sav}, and the depreciation allowance. Macro closure defines what variable this equation determines. In the default closure, investment is savings driven and therefore this equation determines the nominal level of investment. If investment is fixed, then this equation could determine either household or public savings. Macro closure is further discussed below. This equation is defined for all regions, but one should be dropped due to Walras' Law. In fact, the equation includes an additive shifter labeled *Walras*, that is added to the equation for a residual region. It should evaluate to zero.[35]

$$YFD_{r,inv} = S^h_r + S^g_r + PW^{sav} S^f_r + DeprY_r$$ (Y-24)

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[32] The model allows for iceberg trade costs using the parameters λ^x and $\lambda^w_{s,i,r}$. The former is applied to exports and the latter to imports. These parameters generate a wedge between the volume exported and the volume imported. The import volume is given by XW^d and the export volume by XW^s. At equilibrium, the following expression holds: $XW^d = \lambda^w \lambda^x XW^s$ and this expression is used to substitute out XW^d. Thus in the model implementation, the import tax revenue expression also contains the iceberg parameters as the model only carries the variable XW without a superscript and it represents pre-border export supply.

[33] [NEW 4-OCT-2018] The model also includes the possibility of an excise tax on waste as well as the *ad valorem* tax.

[34] With the introduction of the R & D module, it will also include r_d for R & D expenditures.

[35] In the model implementation, the residual region is a subset of r and is designated as `rres`.

3.5 Final demand

3.5.1 Household demand

The commodity space for household demand has a different classification than the product commodity space (indexed by i). Household demand for commodities is indexed by k. A transition matrix approach is used to convert the household commodities into the produced commodities.[36] In value terms, the transition matrix is an $i \times k$ matrix. The sum across the columns equals the value of consumption for commodity i and must line up with the values from the consumer demand vector of the SAM. The sum down the columns equals the value of consumption of commodity k. Table 3.3 provides and example from the GREEN database.[37]

Table 3.3: **Example of a consumer transition matrix**

	Food&Bev	Energy	Trp&Comm	Other	Total
Agric	15.9	0.1	0.0	5.2	21.3
Coal	0.0	0.3	0.0	0.0	0.3
Oil	0.0	0.0	0.0	0.0	0.0
Gas	0.0	12.8	0.0	0.0	12.8
Energy int manu	0.0	0.0	0.1	50.8	50.9
Refined oil	0.0	17.5	50.0	0.0	67.5
Electricity	0.0	73.1	0.0	22.9	96.0
Other	347.9	0.1	366.2	1,644.0	2,358.3
Total	363.9	103.8	416.3	1,723.0	2,607.0

There are several advantages to the transition matrix approach.

1. Potentially, it provides a more accurate picture of consumer demand. For example, it is possible to connect different energy bundles with different sources of demand. The demand for transport is likely to be dominated by liquid fuels, whereas the demand for energy in households is likely to be a mix of electricity, gas and coal.

2. It is likely to provide a better link with household surveys.

3. It may be easier to generate more plausible demand behavior. For example, in the standard LES demand system, a rise in the price of energy (due to a carbon tax for example) will normally lead to a rise in demand for automobiles. If fuel and cars are bundled in a transport bundle, the carbon tax will lead to a drop in the demand for the transport bundle and thus also a drop in the demand for cars.

With the original GTAP database, the transition matrix approach is largely limited to mapping specific supplied commodities to consumer commodities. At a minimum there would be a one-to-one mapping of all non-energy commodities, i.e. a diagonal transition matrix for the non-energy commodities, and all energy commodities mapped to a single energy consumer good.[38] However, with appropriate mapping it is possible to limit the number of consumer goods. For example, the user can map the GTAP commodities `tex`, `wap` and `lea` to a single 'clothing' consumer good and yet retain the production of all three commodities individually. In a similar vein, certain agriculture and food products can be aggregated together for the purposes of the top level consumer utility function, yet continue to be produced individually. The consumer demand nesting is then used to decompose the aggregate demand, for example for food or clothing, into their components.

The consumer demand system is thus implemented as a nested structure that starts with disposable income, see figure 9. A top nest allocates disposable income between savings and consumer commodities (indexed by k). Each commodity k is decomposed into demand for the various product commodities (indexed by i) using a series of nested CES functions. The top CES nest decomposes demand for commodity k into a non-energy aggregate bundle

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[36] The consumer demand transition matrix approach was utilized in the OECD GREEN model, see Burniaux et al. (1992) and van der Mensbrugghe (1994). See also Cardenete et al. (2012), section 5.3, for a more recent description of the approach in GE modeling.

[37] The data is from the GREEN model database and represents the transition matrix for the USA in $1985 million at the Armington level of aggregation.

[38] The user provides the relevant mappings between the GTAP goods (after aggregation) and the user-defined consumer goods. This mapping is part of the bridge file used by the aggregation facility.

and an energy bundle. The former is decomposed into demand for non-energy Armington goods with a single nest (and added across all consumer goods k). The latter is decomposed using the same energy nesting as in production with energy demand similarly aggregated across all consumer goods k to generate total energy demand by carrier by households.
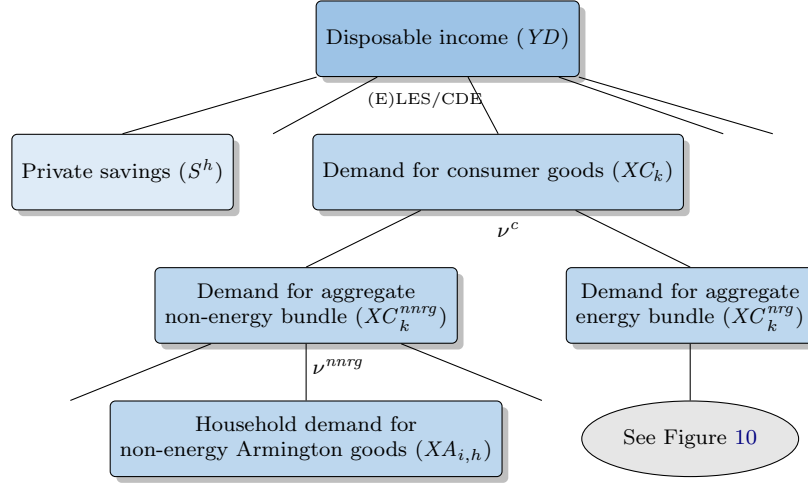
Figure 9: **Consumer demand nest**

The latest version of the ENVISAGE Model has four possible top-level consumer demand utility functions. Each can determine household demand for goods and services (at the level of consumer goods, i.e. indexed by k). Three are in a class of models linked to the oft-used linear expenditure system (LES)—one of which, the ELES, integrates the savings decision in the utility function. The fourth utility function is the *constant differences in elasticity* (CDE) utility function—widely popularized in the GTAP model and its variants.[39]

The Linear Expenditure System and its offshoots

This section describes use of the linear expenditure system (LES) and two of its offshoots. The first offshoot is the extended linear expenditure system (ELES) developed by Lluch (1973) and Howe (1975). The ELES integrates the savings decision into top level utility, i.e. the savings decision is handled with demand for the other goods and services. The second offshoot is the so-called "An Implicitly Direct Additive Demand System", or AIDADS, that leaves the possibility of non-linear Engel curves, unlike the LES—see Rimmer and Powell (1992a) and Rimmer and Powell (1996). All three demand systems share the same underlying specification with some deviations.

Under the LES-type demand system, household demand is the sum of two components—the so-called subsistence minimum, or floor consumption, and a marginal share of supernumerary income. The latter is equal to income less expenditures on the subsistence minima. Equation (D-1) defines supernumerary income on a per capita basis, Y^{sup}.[40] The first term is per capita income. In the case of the ELES, it is disposable income. In the case of the LES and AIDADS, it is disposable income less savings. The second term is the total amount spent on the subsistence minima where γ represents the per capita subsistence minimum and expenditures are evaluated at consumer prices, all taxes included, PC. Equation (D-2) determines consumer demand for good k, where μ^c represents the marginal budget share. Utility and parameters of the LES are defined in per capita terms. Hence, per capita demand is multiplied by population to get aggregate household demand. Equation (D-3) defines the utility level for the LES-type demand specification. In the case of the ELES, the variable P^{sav} represents the perceived 'price' of savings. Given the overall myopic expectations, it is equated to the consumer price index as it represents future claims on consumption. The parameter μ^s represents the marginal propensity to save out of disposable income.

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[39] See Hertel (1997) and Corong et al. (2017).

[40] The consumer demand system is written to allow for multiple households. However, at the moment, disposable income and population are only provided at an aggregate level. To convert the model to multiple households would require the development of a more complete income distribution specification.

$$Y_{r,h}^{sup} = \begin{cases} \dfrac{YD_r}{Pop_r} - \sum_k PC_{r,k,h}\gamma_{r,k,h} & \text{if ELES} \\[3mm] \dfrac{YC_r}{Pop_r} - \sum_k PC_{r,k,h}\gamma_{r,k,h} & \text{if LES or AIDADS} \end{cases} \tag{D-1}$$

$$XC_{r,k,h} = Pop_r \left(\gamma_{r,k,h} + \frac{\beta_{r,h}^c \mu_{r,k,h}^c}{PC_{r,k,h}} Y_{r,h}^{sup} \right) \tag{D-2}$$

$$u_{r,h} = \begin{cases} \dfrac{Y_{r,h}^{sup}}{\left(\dfrac{P_{r,h}^{sav}}{\mu_{r,h}^s}\right)^{\mu_{r,h}^s} \prod_k \left(\dfrac{PC_{r,k,h}}{\beta_{r,h}^c \mu_{r,k}^c}\right)^{\beta_{r,h}^c \mu_{r,k}^c}} & \text{if ELES} \\[5mm] -1 - \ln\left(A_{r,h}^{ad}\right) + \sum_k \mu_{r,k,h}^c \ln\left[\dfrac{XC_{r,k,h}}{Pop_r} - \gamma_{r,k,h}\right] & \text{if LES or AIDADS} \end{cases} \tag{D-3}$$

The key difference between AIDADS and the standard LES is that the marginal budget share parameter, μ^c, is a function of utility in the case of AIDADS, but is constant in the case of the LES. This allows for non-linear Engel curves as income expands. Equation (D-4) defines the marginal budget share specification. With α^{ad} equal to β^{ad}, the function reverts back to the LES with constant marginal budget shares.

$$\mu_{r,k,h}^c = \frac{\alpha_{r,k,h}^{ad} + \beta_{r,k,h}^{ad} e^{u_{r,h}}}{1 + e^{u_{r,h}}} \tag{D-4}$$

The Constant Differences in Elasticity demand system

The CDE demand system is composed of four equations. The first equation, (D-5), evaluates an auxiliary variable, ZC, that is used in subsequent formulas and allows for some simplification in implementation of the CDE. The e^h parameters are referred to as the expansion parameters and are closely linked to the income elasticies (see Appendix B). The b^h parameters are referred to as the substitution parameters and are linked to the price elasticities. The variable u is the utility level and is required for the CDE demand system. The α^h parameters are calibrated shifters. Equation (D-6) determines the budget shares for the CDE. Equation (D-7) implicitly determines utility that is required to calculate the budget shares. And equation (D-8) is used to back out consumption levels using the budget shares. Note that equation (D-8) is also used in the LES-type demand systems to calculate the budget shares—though these are not strictly needed.[41]

$$ZC_{r,k,h} = \alpha_{r,k,h}^h b_{r,k,h}^h u_{r,h}^{e_{r,k,h}^h b_{r,k,h}^h} \left(\frac{PC_{r,k,h}}{YC_r/Pop_r} \right)^{b_{r,k,h}^h} \tag{D-5}$$

$$s_{r,k,h}^h = \frac{ZC_{r,k,h}}{\sum_{k'} ZC_{r,k',h}} \tag{D-6}$$

$$\sum_k \frac{ZC_{r,k,h}}{b_{r,k,h}^h} \equiv 1 \tag{D-7}$$

$$s_{r,k,h}^h = \frac{PC_{r,k,h} XC_{r,k,h}}{YC_r} \tag{D-8}$$

Conversion of consumer goods to producer goods

The subsequent nests decompose consumer demand XC into Armington demand and the latter are summed across all consumer categories k. The first nest allocates XC between a non-energy bundle and an energy bundle. Equations (D-9) and (D-10) describe respectively the demand for the aggregate non-energy and energy bundles, XC^{nnrg} and XC^{nng}, with a substitution elasticity given by ν^c. Energy demand can be impacted by changes in energy efficiency captured by the parameter $\lambda^{c,nrg}$. Equation (D-11) represents the price of the consumer good defined by index k, PC.

$$XC_{r,k,h}^{nnrg} = \alpha_{r,k,h}^{cnnrg} \left(\frac{PC_{r,k,h}}{PC_{r,k,h}^{nnrg}} \right)^{\nu_{r,k,h}^c} XC_{r,k,h} \tag{D-9}$$

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[41] Note that the budget shares reflect the shares net of savings. In the case of the ELES, the shares needed for calibration are relative to disposable income, i.e. inclusive of savings.

$$XC_{r,k,h}^{nrg} = \alpha_{r,k,h}^{cnrg} \left(\lambda_{r,k,h}^{c,nrg} \right)^{\nu_{r,k,h}^c - 1} \left(\frac{PC_{r,k,h}}{PC_{r,k,h}^{nrg}} \right)^{\nu_{r,k,h}^c} XC_{r,k,h} \tag{D-10}$$

$$PC_{r,k,h} = \left[\alpha_{r,k,h}^{cnnrg} \left(PC_{r,k,h}^{nnrg} \right)^{1-\nu_{r,k,h}^c} + \alpha_{r,k,h}^{cnrg} \left(\frac{PC_{r,k,h}^{nrg}}{\lambda_{r,k,h}^{c,nrg}} \right)^{1-\nu_{r,k,h}^c} \right]^{1/(1-\nu_{r,k,h}^c)} \tag{D-11}$$

Aggregate non-energy demand is decomposed next into Armington demand and summed across all consumer goods. Equation (D-12) represents demand for non-energy Armington goods by private households, where the across-goods substitution is given by ν^{nnrg}.[42] The price of the non-energy demand bundle is given by equation (D-13).[43]

$$XA_{r,in,h} = \sum_{k} \left[\alpha_{r,in,k,h}^c \left(\frac{PC_{r,k,h}^{nnrg}}{PA_{r,in,h}^h} \right)^{\nu_{r,k,h}^{nnrg}} XC_{r,k,h}^{nnrg} \right] \tag{D-12}$$

$$PC_{r,k,h}^{nnrg} = \left[\sum_{in} \alpha_{r,in,k,h}^c \left(PA_{r,in,h}^h \right)^{1-\nu_{r,k,h}^{nnrg}} \right]^{1/(1-\nu_{r,k,h}^{nnrg})} \tag{D-13}$$

The decomposition of the energy bundle is similar to the energy nest in production, see figure 10. The energy bundle is first decomposed into electric and non-electric bundles. The latter is then decomposed into a coal bundle and a non-coal bundle (or the oil & gas bundle). The oil & gas bundle is then split into a gas bundle and an oil bundle. The four remaining bundles—electric, coal, oil and gas—represent a combination of existing or future energy sources. In the case of the original GTAP database, the electric bundle would hold the 'ely' commodity, the coal bundle would hold the 'coa' commodity, the oil bundle would hold the 'oil' and 'p_c' commodities and the gas bundle would hold the 'gas' and 'gdt' commodities. Non-GTAP commodities would be mapped to one of the existing bundles. For example, 'p_c' could be split into gasoline and diesel, and/or could include ethanol or bio-diesel. Similar to the energy nesting in production, the decomposition of energy demand has two possible specifications: the normal CES and the additive version of the CES. In the case of the latter, the model specification requires both the composite price index as well as the average price.
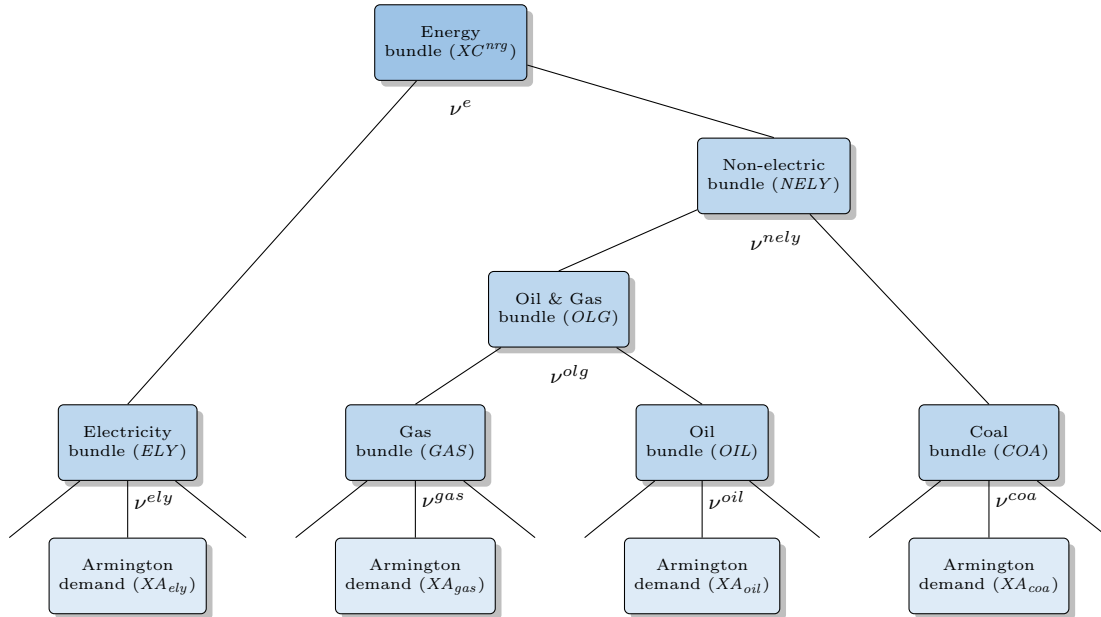
Figure 10: **Energy bundle nest in household demand**

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[42] The index in covers the non-energy commodities and is a subset of i.

[43] [NEW: 4-OCT-2018] Consumer prices are now represented by the variable PA^h instead of PA^a. The difference between the two prices reflects the introduction of a waste tax.

Equation (D-14) determines the demand for the electric bundle, XC^{ely}.[44] Equation (D-15) determines the demand for the non-electric bundle, XC^{nely}. In both equations, the key substitution elasticity is given by ν^e. Equation (D-16) describes the composite price index of energy, $PC^{c,nrg}$. Equation (D-17) then describes the aggregate price of energy, PC^{nrg}.

$$XC^{ely}_{r,k,h} = \alpha^{cely}_{r,k,h} \left(\frac{PC^{c,nrg}_{r,k,h}}{PC^{ely}_{r,k,h}} \right)^{\nu^e_{r,k,h}} XC^{nrg}_{r,k,h} \tag{D-14}$$

$$XC^{nely}_{r,k,h} = \alpha^{cnely}_{r,k,h} \left(\frac{PC^{c,nrg}_{r,k,h}}{PC^{nely}_{r,k,h}} \right)^{\nu^e_{r,k,h}} XC^{nrg}_{r,k,h} \tag{D-15}$$

$$PC^{c,nrg}_{r,k,h} = \begin{cases} \left[\alpha^{cely}_{r,k,h} \left(PC^{ely}_{r,k,h} \right)^{1-\nu^e_{r,k,h}} + \alpha^{cnely}_{r,k,h} \left(PC^{nely}_{r,k,h} \right)^{1-\nu^e_{r,k,h}} \right]^{\frac{1}{1-\nu^e_{r,k,h}}} & \text{if } \neg\texttt{IFNRGACES} \\ \left[\alpha^{cely}_{r,k,h} \left(PC^{ely}_{r,k,h} \right)^{-\nu^e_{r,k,h}} + \alpha^{cnely}_{r,k,h} \left(PC^{nely}_{r,k,h} \right)^{-\nu^e_{r,k,h}} \right]^{-\frac{1}{\nu^e_{r,k,h}}} & \text{if } \texttt{IFNRGACES} \end{cases} \tag{D-16}$$

$$PC^{nrg}_{r,k,h} = \left[PC^{ely}_{r,k,h} XC^{ely}_{r,k,h} + PC^{nely}_{r,k,h} XC^{nely}_{r,k,h} \right] \Big/ XC^{nrg}_{r,k,h} \tag{D-17}$$

Equation (D-18) determines the demand for the coal bundle, XC^{coa}. Equation (D-19) determines the demand for the oil & gas bundle, XC^{olg}. In both equations, the key substitution elasticity is given by ν^{nely}. Equation (D-20) describes the composite price index of the non-electric bundle, $PC^{c,nely}$. Equation (D-21) then describes the aggregate price of the non-electric bundle, PC^{nely}.

$$XC^{coa}_{r,k,h} = \alpha^{ccoa}_{r,k,h} \left(\frac{PC^{c,nely}_{r,k,h}}{PC^{coa}_{r,k,h}} \right)^{\nu^{nely}_{r,k,h}} XC^{nely}_{r,k,h} \tag{D-18}$$

$$XC^{olg}_{r,k,h} = \alpha^{colg}_{r,k,h} \left(\frac{PC^{c,nely}_{r,k,h}}{PC^{olg}_{r,k,h}} \right)^{\nu^{nely}_{r,k,h}} XC^{nely}_{r,k,h} \tag{D-19}$$

$$PC^{c,nely}_{r,k,h} = \begin{cases} \left[\alpha^{ccoa}_{r,k,h} \left(PC^{coa}_{r,k,h} \right)^{1-\nu^{nely}_{r,k,h}} + \alpha^{colg}_{r,k,h} \left(PC^{olg}_{r,k,h} \right)^{1-\nu^{nely}_{r,k,h}} \right]^{\frac{1}{1-\nu^{nely}_{r,k,h}}} & \text{if } \neg\texttt{IFNRGACES} \\ \left[\alpha^{ccoa}_{r,k,h} \left(PC^{coa}_{r,k,h} \right)^{-\nu^{nely}_{r,k,h}} + \alpha^{colg}_{r,k,h} \left(PC^{olg}_{r,k,h} \right)^{-\nu^{nely}_{r,k,h}} \right]^{-\frac{1}{\nu^{nely}_{r,k,h}}} & \text{if } \texttt{IFNRGACES} \end{cases} \tag{D-20}$$

$$PC^{nely}_{r,k,h} = \left[PC^{coa}_{r,k,h} XC^{coa}_{r,k,h} + PC^{olg}_{r,k,h} XC^{olg}_{r,k,h} \right] \Big/ XC^{nely}_{r,k,h} \tag{D-21}$$

The remaining two energy bundles are oil and gas and emanate from the XC^{olg} bundle. Equation (D-22) determines the demand for the oil bundle, XC^{oil}. Equation (D-23) determines the demand for the gas bundle, XC^{gas}. In both equations, the key substitution elasticity is given by ν^{olg}. Equation (D-24) then describes the aggregate price of the oil & gas bundle, $PC^{c,olg}$. Equation (D-25) then describes the aggregate price of the oil & gas bundle, PC^{olg}.

$$XC^{oil}_{r,k,h} = \alpha^{coil}_{r,k,h} \left(\frac{PC^{c,olg}_{r,k,h}}{PC^{oil}_{r,k,h}} \right)^{\nu^{olg}_{r,k,h}} XC^{olg}_{r,k,h} \tag{D-22}$$

$$XC^{gas}_{r,k,h} = \alpha^{cgas}_{r,k,h} \left(\frac{PC^{c,olg}_{r,k,h}}{PC^{gas}_{r,k,h}} \right)^{\nu^{olg}_{r,k,h}} XC^{olg}_{r,k,h} \tag{D-23}$$

$$PC^{c,olg}_{r,k,h} = \begin{cases} \left[\alpha^{coil}_{r,k,h} \left(PC^{oil}_{r,k,h} \right)^{1-\nu^{olg}_{r,k,h}} + \alpha^{cgas}_{r,k,h} \left(PC^{gas}_{r,k,h} \right)^{1-\nu^{olg}_{r,k,h}} \right]^{\frac{1}{1-\nu^{olg}_{r,k,h}}} & \text{if } \neg\texttt{IFNRGACES} \\ \left[\alpha^{coil}_{r,k,h} \left(PC^{oil}_{r,k,h} \right)^{-\nu^{olg}_{r,k,h}} + \alpha^{cgas}_{r,k,h} \left(PC^{gas}_{r,k,h} \right)^{-\nu^{olg}_{r,k,h}} \right]^{-\frac{1}{\nu^{olg}_{r,k,h}}} & \text{if } \texttt{IFNRGACES} \end{cases} \tag{D-24}$$

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[44]   In the GAMS code, the demand for the four energy bundles ('ely', 'gas', 'oil' and 'coa') are represented by the variable `xacNRG` that has an additional dimension representing the four main energy carriers. This simplifies the decomposition of these bundles that only requires one set of equations rather than four. In principle, it also makes it somewhat easier to increase the number of energy bundles.

$$PC_{r,k,h}^{olg} = \left[PC_{r,k,h}^{oil} XC_{r,k,h}^{oil} + PC_{r,k,h}^{gas} XC_{r,k,h}^{gas} \right] \Big/ XC_{r,k,h}^{olg} \tag{D-25}$$

The final nest in the energy bundle is to decompose the four aggregate energy bundles into their constituent parts that represent the Armington demand for the energy commodities for each commodity k and are then summed across all consumer commodities. Equation (D-26) reflects the Armington demand for energy commodity e, XA_h, where the cost to consumers is given by PA^h.[45] The key substitution elasticity for each energy bundle is given by ν^{NRG}. The parameter λ^{ce} plays a different role in the two variants. In the case of the standard CES, it is an energy efficiency parameter that is specif to the energy carrier and consumer bundle (k). In the case of the additive CES, it is a preference shifter. Equation (D-27) represents the composite price index of the aggregate energy bundles, $PC^{c,NRG}$. Equation (D-28) represents the price of the aggregate energy bundles, PC^{NRG}.

$$XA_{r,e,h} = \begin{cases} \displaystyle\sum_k \alpha_{r,e,k,h}^c \left(\frac{\lambda_{r,e,k,h}^{ce} PC_{r,k,h}^{c,NRG}}{PA_{r,e,h}^h} \right)^{\nu_{r,k,h}^{NRG}} \frac{XC_{r,k,h}^{NRG}}{\lambda_{r,e,k,h}^{ce}} & \text{if } e \in \{NRG\} \,\&\, \text{if } \neg\texttt{IFNRGACES} \\[2em] \displaystyle\sum_k \alpha_{r,e,k,h}^c \left(\frac{PC_{r,k,h}^{c,NRG}}{\lambda_{r,e,k,h}^{ce} PA_{r,e,h}^h} \right)^{\nu_{r,k,h}^{NRG}} XC_{r,k,h}^{NRG} & \text{if } e \in \{NRG\} \,\&\, \text{if } \texttt{IFNRGACES} \end{cases} \tag{D-26}$$

$$PC_{r,k,h}^{c,NRG} = \begin{cases} \displaystyle\left[\sum_{e\in\{NRG\}} \alpha_{r,e,k,h}^c \left(\frac{PA_{r,e,h}^h}{\lambda_{r,e,k,h}^{ce}} \right)^{1-\nu_{r,k,h}^{NRG}} \right]^{\frac{1}{1-\nu_{r,k,h}^{NRG}}} & \text{if } \neg\texttt{IFNRGACES} \\[2em] \displaystyle\left[\sum_{e\in\{NRG\}} \alpha_{r,e,k,h}^c \left(\lambda_{r,e,k,h}^{ce} PA_{r,e,h}^h \right)^{-\nu_{r,k,h}^{NRG}} \right]^{-\frac{1}{-\nu_{r,k,h}^{NRG}}} & \text{if } \texttt{IFNRGACES} \end{cases} \tag{D-27}$$

$$PC_{r,k,h}^{NRG} = \left[\sum_{e\in\{NRG\}} PA_{r,e,h}^h XA_{r,e,h} \right] \Big/ XC_{r,k,h}^{NRG} \tag{D-28}$$

Consumer waste module

[4-OCT-2018: This section is provisional and is being tested.] Total household consumption of commodity i, $XA_{r,i,h}$, is split into actual consumption, $XA_{r,i,h}^c$, and waste, $XA_{r,i,h}^w$, using the additive form of the CES function (ACES). Equations (D-29) and (D-30) determine the respective demand for actual consumption and waste, where the substitution elasticity is given by σ^{ac}. The demand functions allow for preference shifters given by the λ parameters. Equation (D-31) determines the composite price, which represents the price of utility.

$$XA_{r,i,h}^c = \alpha_{r,i,h}^{ac} XA_{r,i,h} \left(\frac{PACC_{r,i,h}}{\lambda_{r,i,h}^{ac} PA_{r,i,h}^c} \right)^{\sigma_{r,i,h}^{ac}} \tag{D-29}$$

$$XA_{r,i,h}^w = \alpha_{r,i,h}^{aw} XA_{r,i,h} \left(\frac{PACC_{r,i,h}}{\lambda_{r,i,h}^{aw} PA_{r,i,h}^w} \right)^{\sigma_{r,i,h}^{ac}} \tag{D-30}$$

$$PACC_{r,i,h} = \left[\alpha_{r,i,h}^{ac} \left(\lambda_{r,i,h}^{ac} PA_{r,i,h}^c \right)^{-\sigma_{r,i,h}^{ac}} + \alpha_{r,i,h}^{aw} \left(\lambda_{r,i,h}^{aw} PA_{r,i,h}^w \right)^{-\sigma_{r,i,h}^{ac}} \right]^{-1/\sigma_{r,i,h}^{ac}} \tag{D-31}$$

It is assumed that both actual consumption and waste are both priced at the market price, $PA_{r,i,h}$. However, the price of waste is allowed to deviate using a 'waste' tax.[46] The government revenue equation has been modified to incorporate the waste tax. The final equation defines the final consumer price of total consumption, $PA_{r,i,h}^h$. In the absence of a waste tax, it is equal to the market price of consumption.

$$PA_{r,i,h}^c = PA_{r,i,h} \tag{D-32}$$

$$PA_{r,i,h}^w = PA_{r,i,h} \left(1 + \tau_{r,i,h}^w \right) \tag{D-33}$$

$$PA_{r,i,h}^h XA_{r,i,h} = PA_{r,i,h}^c XA_{r,i,h}^c + PA_{r,i,h}^w XA_{r,i,h}^w \tag{D-34}$$

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[45]   [NEW: 4-OCT-2018] Consumer prices are given by PA^h instead of PA^a to reflect the introduction of the tax on waste.

[46]   The model also allows for an excise tax on waste instead of the *ad valorem* tax.

[17-MAR-2020] *Phantom* taxes have been added to the consumer demand module at the level of the i commodities, not the k commodities. It currently only works for the normal demand system, without the waste module. See section 3.11 for its implementation.

3.5.2 Other final demand

In the standard model there are two other domestic agents—government and investment. The model allows for a number of other domestic final demand agents, though this typically requires supplemental information as the standard database only has public and investment expenditures. One option, for example, is to include expenditures on research and development. The same expenditure function is assumed, albeit with a different cost structure and substitution elasticity. The demand for goods and services for each is governed by a CES expenditure function. Equation (D-35) represents the demand, therefore, for the remaining final demand agents, indexed by *fdc*. The substitution elasticity across goods is given by σ^{fd} and will typically be either 0 or 1, i.e. either constant volume shares or constant value shares. Equation (D-36) determines the other final demand aggregate price deflator, PFD_{fdc}, using the ubiquitous CES dual price expression.

$$XA_{r,i,fdc} = \alpha^{fd}_{r,i,fdc} \left(\frac{PFD_{r,fdc}}{PA^a_{r,i,fdc}} \right)^{\sigma^{fd}_{r,fdc}} XFD_{r,fdc} \tag{D-35}$$

$$PFD_{r,fdc} = \left[\sum_i \alpha^{fd}_{r,i,fdc} \left(PA^a_{r,i,fdc} \right)^{1-\sigma^{fd}_{r,fdc}} \right]^{1/(1-\sigma^{fd}_{r,fdc})} \tag{D-36}$$

The expenditure price deflator for government and investment expenditures is well-defined as the CES dual price. There is no similar expression of household consumption. A CPI index is created using the Fisher price index. More formally, for a price index over set i, the Fisher price index is equal to the following:

$$F^p = \sqrt{\frac{\sum_i P_{i,t}Q_{i,t-1}}{\sum_i P_{i,t-1}Q_{i,t-1}} \cdot \frac{\sum_i P_{i,t}Q_{i,t}}{\sum_i P_{i,t-1}Q_{i,t}}} = \sqrt{L^p \cdot P^p}$$

The Fisher price index represents the geometric mean of the Laspeyres (L^p) and Paasche price indices (P^p), with the former using lagged volume weights and the latter using current volume weights.

$$L^p = \frac{\sum_i P_{i,t}Q_{i,t-1}}{\sum_i P_{i,t-1}Q_{i,t-1}} \qquad P^p = \frac{\sum_i P_{i,t}Q_{i,t}}{\sum_i P_{i,t-1}Q_{i,t}}$$

These indices represent percent change relative to the previous period and thus the price indices are chain weighted. The price index itself is thus defined as:

$$PI_t = PI_{t-1}F^p_t$$

where typically the price index will be set to 1 or 100 in some base period.

The Fisher volume index has a similar definition using different price weights:

$$F^q = \sqrt{\frac{\sum_i P_{i,t-1}Q_{i,t}}{\sum_i P_{i,t-1}Q_{i,t-1}} \cdot \frac{\sum_i P_{i,t}Q_{i,t}}{\sum_i P_{i,t}Q_{i,t-1}}} = \sqrt{L^q \cdot P^q}$$

The Fisher volume index represents the geometric mean of the Laspeyres (L^q) and Paasche volume indices (P^q), with the former using lagged price weights and the latter using current price weights.

$$L^q = \frac{\sum_i P_{i,t-1}Q_{i,t}}{\sum_i P_{i,t-1}Q_{i,t-1}} \qquad P^q = \frac{\sum_i P_{i,t}Q_{i,t}}{\sum_i P_{i,t}Q_{i,t-1}}$$

It is easy to show that the aggregate nominal value is equal to the product of the price and volume indices:

$$P_t \cdot Q_t = \sum_i P_{i,t} \cdot Q_{i,t} = F^p_t \cdot F^q_t \cdot P_{t-1} \cdot Q_{t-1}$$

Equation (D-37) describes the CPI using the Fisher price index where we define the indicator $QFD_{fd,tp,tp}$ to be:[47]

$$QFD_{r,h,tp,tq} = \sum_i PA^h_{r,i,h,tp} XA_{r,i,h,tq}$$

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[47] [NEW: 4-OCT-2014] Use of the waste-inclusive consumer price, PA^h instead of PA^a.

In other words the indicator is the value of aggregate absorption (for agent *fd*) using prices of period *tp* and quantities period *tq*. Equation (D-38) defines *YFD* for households, i.e. total nominal expenditure.

$$PFD_{r,h,t} = PFD_{r,h,t-1}\sqrt{\frac{QFD_{r,h,t,t-1}}{QFD_{r,h,t-1,t-1}} \cdot \frac{QFD_{r,h,t,t}}{QFD_{r,h,t-1,t}}} \tag{D-37}$$

$$YFD_{r,h} = PFD_{r,h}XFD_{r,h} \tag{D-38}$$

Equation (D-39) is an identity relating nominal expenditures to real expenditures. What equation (D-39) identifies will depend on macro closure. In the standard model, it provides a definition of the volume of consumer expenditures (where the price index is defined above), the nominal level of government expenditures where by default the volume of government expenditures is exogenous, and the volume of investment expenditures, where the nominal level is determined by the investment-savings balance equation, equation (Y-24) above.

$$YFD_{r,fd} = PFD_{r,fd}XFD_{r,fd} \tag{D-39}$$

3.6 International trade

3.6.1 Import demand

Similar to most CGE models, import demand uses the Armington specification that assumes product differentiation between domestic and imported goods. A nested CES structure first differentiates between domestic goods and aggregate imports. A second nest differentiates imports across source regions. The GTAP database and the standard GTAP model assume that the top level Armington nest is agent-specific, i.e. the allocation of Armington demand between domestic goods and aggregate imports. The ENVISAGE model allows for 'national' sourcing, i.e. that the top level Armington disaggregation is done after aggregating Armington demand across all agents. While the former is perhaps a more plausible assumption under certain circumstances, this formulation requires significantly more computation. The top level sourcing is governed by *ArmFlag*. If it is set to zero, national sourcing is assumed. Any other value implies agent-specific sourcing. The second level decision nest decomposes import demand by region of origin. The default specification assumes this is done by a representative national agent—in common with the GTAP model and consistent with the standard GTAP database. ENVISAGE incorporates agent-specific sourcing of imports, consistent with the multi-regional input-output (MRIO) model. If used with the standard GTAP database, the initial agent-specific share parameters are uniform across agents, as are the agent-specific import tax rates. The Armington elasticities could, in principle, be differentiated across agents, and simulations could assess the impacts of changes to agent-specific tariffs.

[31-OCT-2020] The additive CES specification has been incorporated as an option in the Armington system—this allows for regional and global consistency of the energy balances as it guarantees additivity across the entire value chain. N.B. The additive CES has not (yet) been incorporated in the MRIO of the Armington trade system.

National sourcing of aggregate imports

This first section describes the top level Armington equations in the case of national sourcing. Equation (T-1) adds up Armington demand (or domestic absorption for the Armington good) across all Armington agents to measure aggregate Armington demand, *XAT*. The latter is then decomposed into demand for domestically produced goods, XDT^d, equation (T-2), and aggregate import demand, *XMT*, equation (T-3). The parameter σ^{mt} represents the key substitution elasticity and is often referred to as the Armington elasticity. Each region in the model provides international trade and transport services *XTT*, further described below. It is assumed that these services are only provided using domestically produced goods. Hence, equation (T-2) also includes the region's provision of these services, in addition to the demand for domestic goods coming from the other agents in the economy. To allow for the additive version of the CES, the Armington system includes the two different versions of the composite price index, PAT^c. Equation (T-5) determines the Armington price (of the aggregate Armington bundle), *PAT*, a reflection of the zero-profit condition.

$$XAT_{r,i} = \sum_{aa} \gamma^{eda}_{r,i,aa}XA_{r,i,aa} \tag{T-1}$$

$$XDT^d_{r,i} = \alpha^{dt}_{r,i}\left(\frac{PAT^c_{r,i}}{PDT_{r,i}}\right)^{\sigma^{mt}_{r,i}}XAT_{r,i} + XTT_{r,i} \tag{T-2}$$

$$XMT_{r,i} = \alpha_{r,i}^{mt} \left(\frac{PAT_{r,i}^c}{PMT_{r,i}} \right)^{\sigma_{r,i}^{mt}} XAT_{r,i} \tag{T-3}$$

$$PAT_{r,i}^c = \begin{cases} \left[\alpha_{r,i}^{dt} \left(PDT_{r,i} \right)^{1-\sigma_{r,i}^{mt}} + \alpha_{r,i}^{mt} \left(PMT_{r,i} \right)^{1-\sigma_{r,i}^{mt}} \right]^{1/(1-\sigma_{r,i}^{mt})} & \text{if } \neg\texttt{IFNRGACES} \\ \left[\alpha_{r,i}^{dt} \left(PDT_{r,i} \right)^{-\sigma_{r,i}^{mt}} + \alpha_{r,i}^{mt} \left(PMT_{r,i} \right)^{-\sigma_{r,i}^{mt}} \right]^{-1/\sigma_{r,i}^{mt}} & \text{if } \texttt{IFNRGACES} \end{cases} \tag{T-4}$$

$$PAT_{r,i} = \left[PDT_{r,i} \left(XDT_{r,i}^d - XTT_{r,i} \right) + PMT_{r,i} XMT_{r,i} \right] \Big/ XAT_{r,i} \tag{T-5}$$

Agents pay *PA* for Armington goods, as defined by equation (T-6), where τ^a is the agent specific tax on Armington consumption. The economy-wide Armington price, *PAT*, is allowed to vary across end-users using a price wedge represented by γ^{eda}. The emissions tax is treated as a Pigouvian tax that is added to the after-tax Armington price. The parameter ρ^{Emi} represents the rate of emissions per unit of consumption and τ^{Emi} is the emissions tax per unit of emissions. The parameter χ^{Emi} is a global shifter on the emission rates. And the parameter ϕ^{Emi} allows for partial or full exemption of the tax incidence. By default, ϕ^{Emi} is set to 1. Any valid value is between 0 (full exemption) to 1 (zero exemption). For example specific activities and/or households can be exempted from the emissions tax and for specific fuels as well—for example natural gas.

$$PA_{r,i,aa} = \left(1 + \tau_{r,i,aa}^a \right) \gamma_{r,i,aa}^{eda} PAT_{r,i} + \sum_{em} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} \tag{T-6}$$

This section determines *XAT*, *XDT*, *XMT*, *PAT^c*, *PAT*, and *PA*. The variables *XAT*, *PAT^c*, and *PAT* are specific to the national-sourcing assumption and are not part of the model specification in the case of agent-sourcing. The other variables are included in the agent-sourcing specification, but have an alternative specification.

Agent sourcing of aggregate imports

In the agent-sourcing of imports, each agent has a unique preference for domestic goods and (aggregate) imports. This specification is more computationally intensive, but is likely to be a more plausible specification. Equations (T-7) and (T-8) determine respectively the agents' demand for domestic goods and the aggregate import bundle, respectively *XD* and *XM*. All relevant variables and parameters are indexed by *aa*, which represents the set of all Armington agents. Equation (T-9) the agent-specific composite Armington price index. Equation (T-10) defines the agent-specific Armington price.

$$XD_{r,i,aa} = \alpha_{r,i,aa}^d \left(\frac{PA_{r,i,aa}^c}{PD_{r,i,aa}} \right)^{\sigma_{r,i,aa}^m} XA_{r,i,aa} \tag{T-7}$$

$$XM_{r,i,aa} = \alpha_{r,i,aa}^m \left(\frac{PA_{r,i,aa}^c}{PM_{r,i,aa}} \right)^{\sigma_{r,i,aa}^m} XA_{r,i,aa} \tag{T-8}$$

$$PA_{r,i,aa}^c = \begin{cases} \left[\alpha_{r,i,aa}^d \left(PD_{r,i,aa} \right)^{1-\sigma_{r,i,aa}^m} + \alpha_{r,i,aa}^m \left(PM_{r,i,aa} \right)^{1-\sigma_{r,i,aa}^m} \right]^{1/(1-\sigma_{r,i,aa}^m)} & \text{if } \neg\texttt{IFNRGACES} \\ \left[\alpha_{r,i,aa}^d \left(PD_{r,i,aa} \right)^{-\sigma_{r,i,aa}^m} + \alpha_{r,i,aa}^m \left(PM_{r,i,aa} \right)^{-\sigma_{r,i,aa}^m} \right]^{-1/\sigma_{r,i,aa}^m} & \text{if } \texttt{IFNRGACES} \end{cases} \tag{T-9}$$

$$PA_{r,i,aa} = \left[PD_{r,i,aa} XD_{r,i,aa} + PM_{r,i,aa} XM_{r,i,aa} \right] \Big/ XA_{r,i,aa} \tag{T-10}$$

The next two equations define the end-user, or agent-specific, sales price, equations (T-11) and (T-12). Agents are assumed to face a common price of domestic goods, *PDT*. Under the MRIO specification, agents could face distinct import prices—the preference structure of bilateral imports can differ across agents and import tariffs could be agent-specific. The agents' tariff-inclusive price of imports is given by the variable PM^a. In the standard specification, this price is uniform across agents and is given by the variable *PMT*. Prices can be adjusted by exogenous wedges (to allow for differences in quality for example) and sales taxes differ between domestic and imported goods. The carbon tax is a Pigouvian tax added to the tax-inclusive sales price, where the emission coefficient is source-specific.

$$PD_{r,i,aa} = \left(1 + \tau_{r,i,aa}^{ad} \right) \gamma_{r,i,aa}^{edd} PDT_{r,i} + \sum_{em} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,d} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} \tag{T-11}$$

$$PM_{r,i,aa} = \left(1 + \tau_{r,i,aa}^{am} \right) \gamma_{r,i,aa}^{edm} PM_{r,i,aa}^a + \sum_{em} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,m} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} \tag{T-12}$$

The final two equations determine aggregate demand for domestic goods, XDT, and the import bundle, XMT. These correspond to equation (T-1) above and represent a simple aggregation of the agent-specific demand (adjusted by the agent-specific coefficients to allow for exogenous price relatives). Note that domestic supply of international trade and transport margins is added to the (Armington) agents' demand for domestic goods. The variable XMT is not calculated for the MRIO version of the model.[48]

$$XDT^d_{r,i} = \sum_{aa} \gamma^{edd}_{r,i,aa} XD_{r,i,aa} + XTT_{r,i} \tag{T-13}$$

$$XMT_{r,i} = \sum_{aa} \gamma^{edm}_{r,i,aa} XM_{r,i,aa} \tag{T-14}$$

This section determines XD, XM, PA, PD, PM, XDT^d and XMT. The variables PA, XDT^d and XMT are common to both specifications albeit with different expressions. The variables XD, XM, PD and PM are only included with the agent-sourcing specification. The variable XMT is not part of the MRIO model specification.

Second level Armington nest—standard specification

The second CES nest decomposes the aggregate demand for imports, XMT, into demand for imports by source region, XW^d. Equation (T-15) describes demand for imports by region r for imports from region s for good i.[49] The variable PDM represents the end-user price of imports, i.e. it includes bilateral tariffs. The key substitution elasticity is given by σ^w. The composite import price index, PMT^c, is defined in equation (T-16). The aggregate import price, PMT, is defined in equation (T-17).

$$XW^d_{s,i,r} = \alpha^w_{s,i,r} \left(\frac{PMT^c_{r,i}}{PDM_{s,i,r}} \right)^{\sigma^w_{r,i}} XMT_{r,i} \tag{T-15}$$

$$PMT^c_{r,i} = \begin{cases} \left[\sum_s \alpha^w_{s,i,r} \left(PDM_{s,i,r} \right)^{1-\sigma^w_{r,i}} \right]^{1/(1-\sigma^w_{r,i})} & \text{if } \neg\texttt{IFNRGACES} \\ \left[\sum_s \alpha^w_{s,i,r} \left(PDM_{s,i,r} \right)^{-\sigma^w_{r,i}} \right]^{-1/\sigma^w_{r,i})} & \text{if } \texttt{IFNRGACES} \end{cases} \tag{T-16}$$

$$PMT_{r,i} = \left[\sum_s \left(PDM_{s,i,r} XW^d_{s,i,r} \right) \right] \Big/ XMT_{r,i} \tag{T-17}$$

Second level Armington nest—MRIO specification

Under the MRIO specification, the allocation of agent-specific import demand, XM, across source regions is also agent-specific—unlike the standard specification where agents' import demand is summed across all agents, and then allocated across trading partners. Equation (T-18) represents the agent specific allocation of XM across trading partners, XW^a. The relevant prices include PDM^a, which is the tariff-inclusive price for agent aa for commodity i sourced in region s, and PM^a, which represents the average tariff inclusive price for agent aa for commodity i, averaged over all source countries. We allow for the substitution elasticity to be agent-specific, though there is currently little empirical evidence to support this. Equation (T-19) defines the agent-specific average price of imports using the zero-profit condition. Equation (T-20) defines the tariff-inclusive price of imports for agent aa sourced in region s. Total import demand, that must be reconciled with export supply, is provided in equation (T-21).

$$XW^a_{s,i,r,aa} = \alpha^{wa}_{s,i,r,aa} \left(\frac{PM^a_{r,i,aa}}{PDM^a_{s,i,r,aa}} \right)^{\sigma^{wa}_{r,i,aa}} XM_{r,i,aa} \tag{T-18}$$

$$PM^a_{r,i,aa} XM^a_{r,i,aa} = \sum_s PDM^a_{s,i,r,aa} XW^a_{s,i,r,aa} \tag{T-19}$$

$$PDM^a_{s,i,r,aa} = \left(1 + \tau^{ma}_{s,i,r,aa} \right) PWM_{s,i,r} \tag{T-20}$$

$$XW^d_{s,i,r} = \sum_{aa} XW^a_{s,i,r,aa} \tag{T-21}$$

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[48]   The MRIO model only operates if the top-level Armington specification is agent-specific.
[49]   As mentioned above, the XW^d is substituted out and thus this expression carries the iceberg parameter in the model implementation.

Summary

The standard specification requires variables PMT and PDM that represent respectively the average tariff-inclusive price of imports—uniform across all agents—, and the bilateral tariff-inclusive price of imports. In addition, the variable XMT represents aggregate import demand, summed across all agents. These three variables are not used in the MRIO version of the model. The MRIO version of the model uses instead PM^a, PDM^a and XW^a. Both models require the variable XW^d. Note that the MRIO model significantly expands the size of the model—even allowing for some substitution. The 10x10 model, with no substitution has around 45,000 endogenous variables. This can be pared down to some 26,000 variables with judicious substitution. The model with the standard trade specification (and substitution) solves for some 13,000 variables. The agent- and source-specific import variable requires some 12,000 variables alone for the 10x10 model.

3.6.2 Export supply

Domestic supply of good i is given by XS. It will be allocated between domestic and export markets analogously to the Armington assumption on the demand side. At the top level, the national supplier decides the optimal allocation of total supply between the domestic market and aggregate exports. At the second level, aggregate exports are allocated across all regions of destination. The allocation functions are based on the CET transformation specification. The implementation does allow for perfect transformation, in which case the law-of-one-price holds. [31-OCT-2020] The specification also allows for the additive form of the CET (ACET), which can be used in particular for energy commodities to preserve the energy balances across regions and globally.

Equation (T-22) determines the optimal supply of domestic production to be allocated to the domestic market, XDT^s. The transformation elasticity is given by ω^x. In the case of perfect transformation, the market price of goods sold domestically is equal to the average supply price adjusted by an exogenous price wedge, γ^{esd}. Equation (T-23) determines the optimal supply of aggregate exports, XET. Equation (T-24) determines the composite price index for domestic supply, PS^c. Equation (T-25) represents the zero-profit condition and determines PS, i.e. the aggregate price of domestic supply.

$$\begin{cases} XDT^s_{r,i} = \gamma^d_{r,i} \left(\gamma^{esd}_{r,i}\right)^{-\omega^x_{r,i}-1} \left(\dfrac{PDT_{r,i}}{PS^c_{r,i}}\right)^{\omega^x_{r,i}} XS_{r,i} & \text{if } \omega^x_{r,i} \neq \infty \\ PDT_{r,i} = \gamma^{esd}_{r,i} PS_{r,i} & \text{if } \omega^x_{r,i} = \infty \end{cases} \tag{T-22}$$

$$\begin{cases} XET_{r,i} = \gamma^e_{r,i} \left(\gamma^{ese}_{r,i}\right)^{-\omega^x_{r,i}-1} \left(\dfrac{PET_{r,i}}{PS^c_{r,i}}\right)^{\omega^x_{r,i}} XS_{r,i} & \text{if } \omega^x_{r,i} \neq \infty \\ PET_{r,i} = \gamma^{ese}_{r,i} PS_{r,i} & \text{if } \omega^x_{r,i} = \infty \end{cases} \tag{T-23}$$

$$PS^c_{r,i} = \begin{cases} \left[\gamma^d_{r,i}\left(PDT_{r,i}\right)^{1+\omega^x_{r,i}} + \gamma^e_{r,i}\left(PET_{r,i}\right)^{1+\omega^x_{r,i}}\right]^{1/(1+\omega^x_{r,i})} & \text{if } \omega^x_{r,i} \neq \infty \ \& \text{ if } \neg\texttt{IFNRGACES} \\ \left[\gamma^d_{r,i}\left(PDT_{r,i}\right)^{\omega^x_{r,i}} + \gamma^e_{r,i}\left(PET_{r,i}\right)^{\omega^x_{r,i}}\right]^{1/\omega^x_{r,i}} & \text{if } \omega^x_{r,i} \neq \infty \ \& \text{ if } \texttt{IFNRGACES} \end{cases} \tag{T-24}$$

$$PS_{r,i} = \left[PDT_{r,i}XDT^s_{r,i} + PET_{r,i}XET_{r,i}\right] \Big/ XS_{r,i} \tag{T-25}$$

The second level nest is represented by equation (T-26), which determines the quantity of exports allocated to each of the destination markets (d), XW^s. The key transformation elasticity is given by ω^w. Equation (T-27) determines the composite price index for aggregate exports, PET^c. Equation (T-28) determines the aggregate export price, PET. Similar to the top-level CET nest, the second level nest allows or an exogenous wedge in the bilateral prices relative to the average, or aggregate, price.

$$\begin{cases} XW^s_{r,i,d} = \gamma^w_{r,i,d} \left(\gamma^{ew}_{r,i,d}\right)^{-\omega^w_{r,i}-1} \left(\dfrac{PE_{r,i,d}}{PET^c_{r,i}}\right)^{\omega^w_{r,i}} XET_{r,i} & \text{if } \omega^w_{r,i} \neq \infty \\ PE_{r,i,d} = \gamma^{ew}_{r,i,d} PET_{r,i} & \text{if } \omega^w_{r,i} = \infty \end{cases} \tag{T-26}$$

$$PET^c_{r,i} = \begin{cases} \left[\displaystyle\sum_d \gamma^w_{r,i,d}\left(PE_{r,i,d}\right)^{1+\omega^w_{r,i}}\right]^{1/(1+\omega^w_{r,i})} & \text{if } \omega^w_{r,i} \neq \infty \ \& \text{ if } \neg\texttt{IFNRGACES} \\ \left[\displaystyle\sum_d \gamma^w_{r,i,d}\left(PE_{r,i,d}\right)^{\omega^w_{r,i}}\right]^{1/\omega^w_{r,i}} & \text{if } \omega^w_{r,i} \neq \infty \ \& \text{ if } \texttt{IFNRGACES} \end{cases} \tag{T-27}$$

$$PET_{r,i} = \sum_d PE_{r,i,d}XW^s_{r,i,d} \Big/ XET_{r,i} \tag{T-28}$$

3.6.3 Bilateral trade prices

Each bilateral trade node is associated with four prices. PE represents the price domestic producers in region r receive for goods exported to destination region d. An export tax or subsidy, τ^e, is introduced between the producer price and the border (of FOB) price, PWE, equation (T-29). A transportation margin represents the wedge between the border (or FOB) price of the exporting region and the border (or CIF) price of the importing region, PWM. The variable ζ^{mg} represents the per unit transportation margin that is valued at the average price of $PWMG$, equation (T-30). The CIF import price is adjusted by the iceberg parameter, λ^w. The final price of imports for the end-user is the border (or CIF) price plus trade protection measures of which there are two: bilateral import tariffs, $\tau^m_{r,i,d}$, and the *ad valorem* equivalent of non tariff measures, τ^{ntm}.[50] This price is represented by PDM, equation (T-31).[51]

$$PWE_{r,i,d} = \left(1 + \tau^e_{r,i,d}\right) PE_{r,i,d} \tag{T-29}$$

$$PWM_{r,i,d} = \left[PWE_{r,i,d} + PWMG_{r,i,d}\zeta^{mg}_{r,i,d}\right] \Big/ \left[\lambda^x_{r,i,d}\lambda^w_{r,i,d}\right] \tag{T-30}$$

$$PDM_{r,i,d} = \left(1 + \tau^m_{r,i,d} + \tau^{ntm}_{r,i,d}\right) PWM_{r,i,d} \tag{T-31}$$

3.6.4 Trade margins

International trade is associated with transport margins that capture the wedge between the price at the source port and the price at the destination port, i.e. the CIF/FOB price wedge. Thus trade engenders a demand for services. The supply of these services is assumed to emanate from a global supplier that chooses the lowest cost supply subject to a CES preference function.

Equation (T-32) provides the demand for international trade and transport services per bilateral node, $XWMG$, a simple linear technology. The bilateral demand for trade and transport services is allocated across margin commodities (m) using a similar linear technology, equation (T-33). The specification allows for technological change as rendered by the variable λ^{mg}. The average price of transportation per bilateral node, $PWMG$, is given by equation (T-34).

$$XWMG_{r,i,d} = \zeta^{mg}_{r,i,d} XW^s_{r,i,d} \tag{T-32}$$

$$XMG^m_{m,r,i,d} = \alpha^{mg}_{m,r,i,d} \frac{XWMG_{r,i,d}}{\lambda^{mg}_{m,r,i,d}} \tag{T-33}$$

$$PWMG_{r,i,d} = \sum_m \alpha^{mg}_{m,r,i,d} \frac{PTMG_m}{\lambda^{mg}_{m,r,i,d}} \tag{T-34}$$

Total global demand (and therefore supply) for trade and transport margins, $XTMG$, for each commodity m, is the sum across all potential bilateral nodes equation (T-35). The global supplier allocates this demand across potential suppliers using a CES preference function. Equation (T-36) determines region r's supply of trade and transport services, XTT for commodity m. The global average price for trade and transport service m, $PTMG$, is given by equation (T-37).

$$XTMG_m = \sum_r \sum_i \sum_d XMG^m_{m,r,i,d} \tag{T-35}$$

$$XTT_{r,m} = \alpha^{tt}_{r,m} \left(\frac{PTMG_m}{PD_{r,m}}\right)^{\sigma^{mg}_m} XTMG_m \tag{T-36}$$

$$PTMG_m XTMG_m = \sum_m PD_{r,m} XTT_{r,m} \tag{T-37}$$

3.7 Goods market equilibrium

There are two sets of equilibrium conditions. The domestic market for goods produced domestically and bilateral trade. Equation (E-1) determines PD, the market or equilibrium price equating supply of domestically produced goods to the demand for domestic goods. Equation (E-2) determines PE, the producer price of exported goods by

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50   [NEW: 4-OCT-2018] NTMs are new to the model. See introduction above in the income section (3.4).
51   Note that in the model implementation, these equations are typically substituted out of the model in order to reduce its overall size.

region of origin r destined for region d. The equilibrium condition allows for iceberg trade costs that are represented by the parameters λ^x and λ^w. The former relates to changes related to the exporter and the latter relates to changes related to the importer. The parameters are initialized at 1. An improvement in trade, i.e. a reduction in iceberg costs, is represented by an increase from 1. Thus a 10 percent improvement is represented by setting λ^w to 1.1. In model implementation, these equilibrium conditions are substituted out of the model specification.

$$XDT_{r,i}^d = XDT_{r,i}^s \tag{E-1}$$

$$XW_{r,i,d}^d = \lambda_{r,i,d}^w \lambda^x XW_{r,i,d}^s \tag{E-2}$$

3.8 Factor markets

3.8.1 Labor market

Equilibrium on the labor markets allows for labor market segmentation—rural, equated with agricultural sectors, and urban—or full mobility across all sectors. In the case of labor market segmentation, the model also implements a migration function that allows for some adjustment in the respective labor supplies depending on expected relative wage rates. The two labor market segments are indexed by z, or zone. When market segmentation is assumed, the index z has two elements, `rur` and `urb`. For nationally integrated labor markets, the index z has a single element, `nsg` (i.e. not-segmented). Note that the model allows for mixed specifications. For example, market segmentation can be assumed for unskilled workers but skilled workers could be fully mobile.

Within each zone (in either the segmented or non-segmented specifications), the model allows for three wage formation mechanisms. (1) The default is that wages clear by equating labor demand with (fixed) labor supply. (2) The second specification implements a reservation (or minimum) wage that puts a lower bound on market wages. With sufficiently high demand the equilibrium wage will likely exceed the reservation wage and the wage will be determined by equality of demand and supply (allowing for 'natural' unemployment). If labor demand is low, the market wage would be determined by the reservation wage and labor supply will exceed labor demand generating unemployment. (3) The third specification is inspired from the Monash-class of models (Dixon and Rimmer (2002)). It allows for short-run deviations from full-employment (induced by a shock), but long-run equilibrium between labor supply an demand.

Equation (F-1) defines labor demand within the relevant labor market zone.[52]

$$LD_{r,l,z}^z = \sum_{a \in z} XF_{r,l,a} \tag{F-1}$$

Labor market closure with reservation wage specification

This labor market closure allows for endogenous regime shifting between full employment with the equilibrium wage determined by the equality of supply and demand and a regime of less than full employment. A reservation (or minimum) wage puts a floor on wages. If labor demand is low relative to supply, the market wage will be determined by the reservation wage. In the case of high labor demand, wages will be determined by equilibrium conditions. The two regimes are specified using a mixed complementarity expression that takes the following form:

$$\left(W^e - W^{res}\right)\left(UE - UE^{min}\right) = 0$$

with

$$W^e \geq W^{res} \quad \text{and} \quad UE \geq UE^{min}$$

where W^e is the market wage, W^{res} is the reservation wage, UE is the rate of unemployment and UE^{min} is a floor for the unemployment rate. For the complementarity condition to hold either the unemployment rate is at the minimum and the market wage is greater than the reservation wage, or, the unemployment level is greater than the minimum level and the market wage is equal to the reservation wage.

Figure 11 provides an illustration of the two different market closures. The vertical line labeled L^s represents total labor supply, however effective labor supply is represented by the other vertical line, $(1 - UE^{min})L^s$, and represents labor supply adjusted by the minimal (or natural) rate of unemployment (that is exogenous). The labor

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[52]   All equations relating to labor market zones are conditioned on a set flag represented by `lsFlag`. For segmented labor markets, the flag will be active for `rur` and `urb`. For non-segmented labor markets, the flag will only be active for `nsg`.
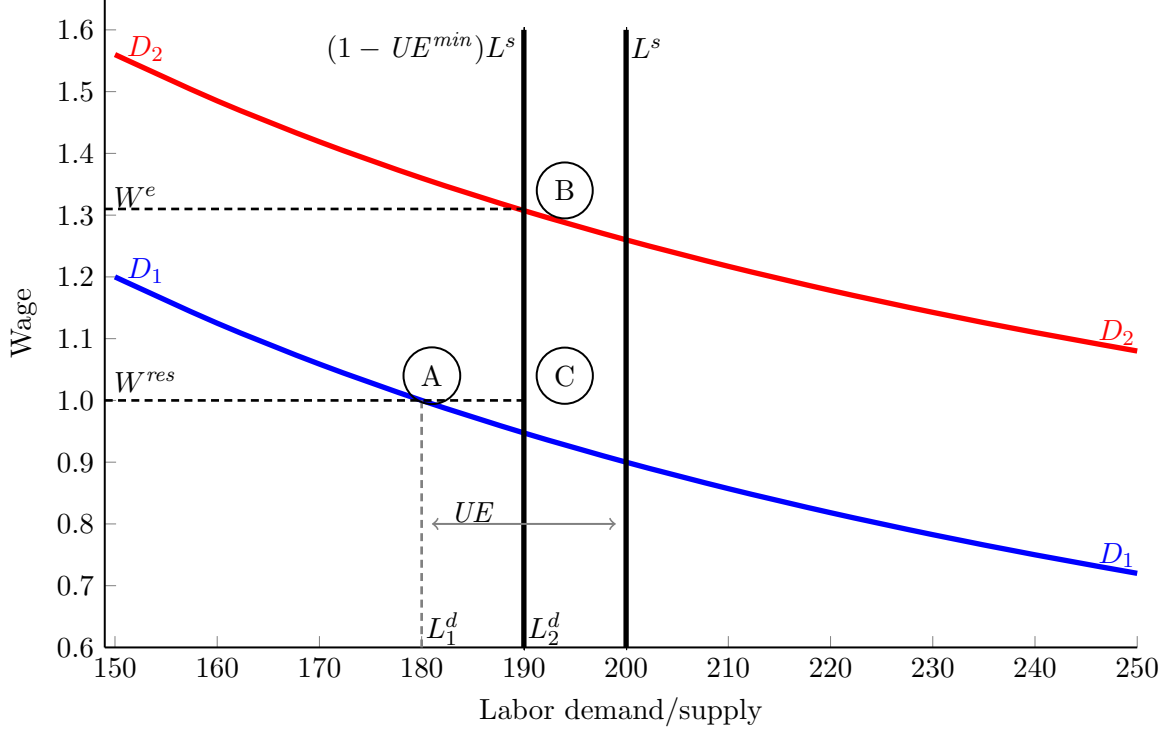
Figure 11: **Labor market closure**

demand curves D_1D_1 and D_2D_2 represent two different situations with the former having relatively low labor demand and the latter relatively high labor demand. The D_1D_1 curve crosses the effective labor supply curve below the reservation wage and hence the market wage will be given by the reservation wage as indicated at point A. The rate of unemployment is measured as the distance between total labor supply and effective labor demand (L_1^d), of which some part reflects the minimum rate of unemployment. The D_2D_2 curve, on the other hand, crosses the effective labor supply curve at a wage greater than the reservation wage, in which case the market wage is given by supply/demand equality at point B and with a market wage of W^e. Effective labor demand is L_2^d and equal to effective labor supply. As the demand curve D_1D_1 shifts up and to the right, the market wage will be determined by the reservation wage until point C when the regime shift towards full employment occurs.

Equation (F-2) defines the reservation wage, W^{res}. It is a positive function of income growth per capita and the CPI. It is a negative function of the unemployment rate, i.e. if the rate of unemployment increases, this would tend to have a downward pressure on the reservation wage. The (positive) elasticities ω^{rwg} and ω^{rwp} reflect the sensitivity of the reservation wage with respect to income growth and the CPI, respectively. The (positive) elasticity ω^{rwue} reflects the sensitivity of the reservation wage with respect to the rate of unemployment.[53] The market wage, W^e is determined by equation (F-3) reflecting the complementarity condition as the expression uses the '=g=' relational operator of GAMS. This equation will be paired with the complementarity condition on the unemployment level. Equation (F-5) defines the unemployment level.

$$W_{r,l,z,t}^{res} = \chi_{r,l,z}^{rw} \left(1 + g_{r,t}^y\right)^{\omega_{r,l,z}^{rwg}} \left(1 - UE_{r,l,z,t}^z\right)^{\omega_{r,l,z}^{rwue}} \left(\frac{PFD_{r,h,t}}{PFD_{r,h,t-1}}\right)^{\omega_{r,l,z}^{rwp}} \tag{F-2}$$

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[53]   To deal with the possibility of zero unemployment, i.e. full employment, the reservation wage equation is formulated in terms of changes to 1 minus the unemployment rate, i.e. the rate of employment. The elasticity in this case should be positive, i.e. as we approach full employment, the reservation wage is likely to increase. The respective elasticities are linked by the formula: $\eta = -\omega \left(1 - UE\right)/UE$, where ω is the elasticity with respect to the unemployment rate and η is the elasticity with respect to the employment rate. Note that this relation is not constant and depends on the rate of unemployment. For example, if the elasticity between the reservation wage and the unemployment rate is -0.5 and the initial unemployment rate is 10 percent, the elasticity with respect to the employment rate is 4.5 ($= -(-0.5) \cdot 0.9/0.1$).

$$W^e_{r,l,z,t} \geq W^{res}_{r,l,z,t} \perp UE^z_{r,l,z,t} \geq UEMin^z_{r,l,z,t} \tag{F-3}$$

Monash-style labor market closure

Equation (F-4) defines the motion equation for the real wage, RW, where the superscript b refers to the baseline levels.[54] The baseline scenario is used to calibrate real wages and full employment is assumed. In shock scenarios, both the real wage and labor demand may deviate from baseline values in the short-run. The parameter χ^l determines the speed of adjustment back to baseline employment levels. In the standard specification, η^l is set to zero implying that there is no permanent deviation from the baseline employment level.[55] The residual term can be used to shift the real wage formulation—Dixon and Rimmer (2002) use this term to smooth out cycles in the real wage as it converges towards its long-term path.

$$\left[\frac{RW_{r,l,z,t}}{RW^b_{r,l,z,t}} - 1 \right] = \left[\frac{RW_{r,l,z,t-1}}{RW^b_{r,l,z,t-1}} - 1 \right] + \chi^l_{r,l,z,t} \left[\frac{LD^z_{r,l,z,t}}{LD^{z,b}_{r,l,z}} - \left(\frac{RW_{r,l,z,t-1}}{RW^b_{r,l,z,t-1}} \right)^{\eta^l_{r,l,z}} \right] + \varepsilon^w_{r,l,z,t} \tag{F-4}$$

Equation (F-5) holds for all model specifications. In the case of full-employment, UE^z is exogenous and this equation in effect is the equilibrium equation that determines the equilibrium wage rate. This also holds for the Monash closure for the baseline. It could also hold for the baseline for the reservation wage closure, where trend unemployment is fixed. In this case, the shifter on the reservation wage equation, χ^{rw}, is endogenous. In policy simulations, the shifter is fixed and unemployment is endogenous. For the Monash closure, equation (F-4) is normally only active for policy simulations—though it could also be active for other closures with the residual term, ε^w, then endogenous.

$$\left(1 - UE^z_{r,l,z,t} \right) LS^z_{r,l,z,t} = LD^z_{r,l,z,t} \tag{F-5}$$

The model allows for inter-sectoral wage differentials. These are represented by the π^w parameters that are calibrated using base year wage differentials and held fixed afterwards. This implies that all wages (for a particular skill and within a zone) move proportionately but allowing for wage differentials. Note that this assumption has potential implications for model results as any inter-sectoral shifts in labor may imply shifts in aggregate labor productivity (irrespective of technological change). Equation (F-6) sets the sectoral wage equal to the equilibrium wage (by zone) adjusted by the sector-specific wage differential.

$$PF_{r,l,a} = \pi^w_{r,l,a} W^e_{r,l,z} \quad \text{for } a \in z \tag{F-6}$$

If the rural to urban migration module is active, it depends on the so-called urban premium, i.e. the extent to which the urban wage exceeds the rural wage. Equation (F-7) defines the average wage in each of the zones, W^a (by skill). Equation (F-8) defines the urban premium, π^{Urb}. It is the ratio of the expected urban wage to the expected rural wage, where the actual average wage is adjusted by the probability of employment. Migration, which will be a function of the urban premium, is described below as part of the dynamics of the model. In the comparative static version of the model, it is assumed that labor supply is fixed (by zone).

$$W^a_{r,l,z} = \frac{\sum_{a \in z} PF_{r,l,a} XF_{r,l,a}}{\sum_{a \in z} XF_{r,l,a}} \tag{F-7}$$

$$\pi^{Urb}_{r,l} = \frac{\left(1 - UE^z_{r,l,Urb} \right) W^a_{r,l,Urb}}{\left(1 - UE^z_{r,l,Rur} \right) W^a_{r,l,Rur}} \tag{F-8}$$

Another potentially useful concept is the notion of the skill premium, i.e. the extent to which skilled wages exceed unskilled wages.[56] Equation (F-9) defines the average economy-wide wage for each skill type, W^t.[57] Equation (F-10) defines the so-called skill premium, π^s. It is the percentage by which the reference wage exceeds other wages, where

---

[54]   N.B. There is a subtle difference with the original Monash formulation. In ENVISAGE, the equation is specified for each labor type, l (and potentially for each zone). In Monash, the equation holds for aggregate labor demand. Wages by skill and sector are given as a proportion of the prevailing wage.

[55]   Dixon and Rimmer (2002) argue that, at least in the case of Australia, this parameter ranges from 0 to 0.15 and typically set the parameter to 0.

[56]   In a dynamic calibration scenario, assumptions regarding the evolution of the skill premium can be used to calibrate the relative growth of skilled vs. unskilled labor or to calibrate productivity differentials across skills.

[57]   In the case of non-segmented markets, $W^t_{r,l}$ should be equal to $W^a_{r,l,nsg}$.

the reference wage is equal to a weighted average of wages over a subset of labor skills (where the subset is indexed by lr)—the subset is typically the set of skilled labor types.

$$W_{r,l}^t = \frac{\sum_a PF_{r,l,a} XF_{r,l,a}}{\sum_a XF_{r,l,a}} \tag{F-9}$$

$$\pi_{r,l,t}^s = \frac{\sum_{lr} W_{r,lr,t}^t LAB_{r,lr,t}^s / \sum_{lr} LAB_{r,lr,t}^s}{W_{r,l,t}^t} - 1 \tag{F-10}$$

The final two sets of equations define the total labor supply by skill, equation (F-11), and the total labor supply, equation (F-12), respectively, L^s and TL^s. In comparative static simulations, LS^z is fixed by zone. In dynamic simulations, LS^z grows at some 'natural' rate of growth, but may also be adjusted by rural to urban migration. This is further described below.

$$L_{r,l}^s = \sum_z LS_{r,l,z}^z \tag{F-11}$$

$$TL_r^s = \sum_l L_{r,l}^s \tag{F-12}$$

3.8.2 Capital market

The model can operate in recursive dynamic mode or in comparative static mode. Capital market closure differs in each. In the comparative static mode, there is a single vintage and aggregate capital is allocated across activities using a CET transformation function. The latter allows two extreme assumptions—fixed capital per activity, i.e. no mobility, and perfect mobility and a uniform rates of return across activities. In dynamic mode, with vintage capital, *New* capital is fully mobile across sectors. *Old* capital is released on the secondary capital market if installed capital exceeds demand, for example in declining sectors.

Comparative static closure

A CET transformation function is used to allocate aggregate capital in the comparative static version of the model. Equation (F-13) describes the supply of capital to activity a, K^s, where ω^k determines the degree of mobility across activities.[58] With perfect mobility, the rate of return across activities is uniform. Equation (F-14) determines the aggregate rate of return to capital, TR. Equation (F-15) is the equilibrium condition for capital—supply of capital to each activity is equal to demand. In the model implementation the equilibrium condition is substituted out.

$$\begin{cases} K_{r,a,v}^s = \gamma_{r,a,v}^k \left(\dfrac{PK_{r,a,v}}{TR_r} \right)^{\omega_r^k} TK_r^s & \text{if } \omega_r^k \neq \infty \\ PK_{r,a,v} = TR_r & \text{if } \omega_r^k = \infty \end{cases} \tag{F-13}$$

$$TR_r TK_r^s = \sum_a \sum_v PK_{r,a,v} K_{r,a,v}^s \tag{F-14}$$

$$K_{r,a,v}^s = K_{r,a,v}^v \tag{F-15}$$

Vintage capital closure

There are two types of capital in the vintage specification, *Old*, or installed capital, and *New* capital. Initial installed capital is set at the beginning of the period and is equal to the depreciated level of the previous period's aggregation of both *Old* and *New* capital. The demand for *New* capital will depend on market conditions, substitution and other factors. A sector is in expansion if demand for *New* capital exceeds the initial installed capital. A sector is in contraction if demand for capital in the sector is less than its initial installed capital. This can come about, for example, if public policies affect demand for a good—such as a carbon tax leading to reduced demand for coal or oil. If a sector is in contraction, it will release its *Old* capital using an upward sloping supply schedule, potentially with an infinite elasticity.

Figure 12 represents the two possible regimes each under two possible capital demand schedules. The left panel shows the case for a flat supply curve, i.e. an infinite supply curve for *Old* capital. In that case, even if capital demand is contracting, the rate of return to *Old* capital is equated to the economy-wide rental rate, i.e. the ratio of returns, *Old* over *New* is 1. The right panel shows the case where the supply of *Old* capital follows an upward

---

[58]   The vintage index is for convenience only. In the comparative static version there is only a single vintage.

sloping schedule. If capital demand is in decline, the rate of return to *Old* capital will be less than the economy-wide return. In the case of an expanding demand, the rate of return to *Old* capital is equal to the economy-wide return and the supply of *Old* capital is capped at the initially installed level, i.e. K_0.

(a) $\eta^I = \infty$ (b) $\eta^I \neq \infty$
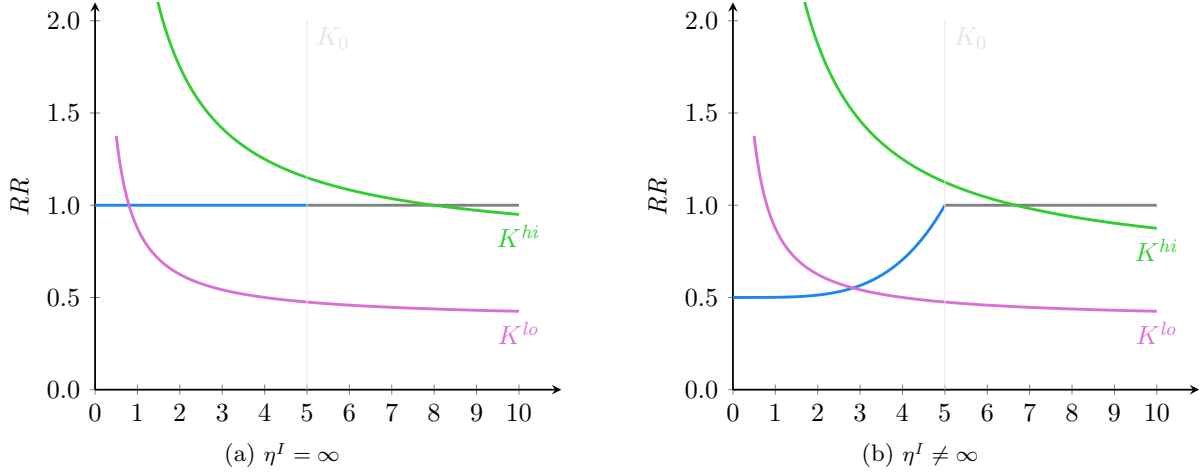
Figure 12: **Capital supply regimes**

Equation (F-16) represents the mixed complementarity condition for the supply of old capital. If demand is contracting, the upward sloping portion of the supply schedule is determined by the iso-elastic relation with the supply elasticity given by η^I. At the inflection point, the supply of *Old* capital cannot exceed the amount of the initially installed *Old* given by K^0. The complementarity condition then translates into: (a) all of the installed capital is needed, $K^{lo} = K^0$ and the ratio of returns is equal to 1; or (b) the supply of old capital is less than the installed capital and the ratio of returns is less than 1. The second complementarity condition, equation (F-17), states that either the ratio of returns is 1 and thus supply of new capital is positive, or the ratio is less than 1 and then the supply of new capital is 0. Equation (F-18) states that total supply should meet total demand. The model allows for the possibility of a horizontal supply schedule, in which case the ratio of returns is equal to 1 irrespective of market conditions and supply will be set to demand.

$$K_{r,a}^{lo} \geq K_{r,a}^0 RR_{r,a}^{\eta_{r,a}^I} \perp RR_{r,a} \leq 1 \tag{F-16}$$

$$RR_{r,a} \leq 1 \perp K_{r,a}^{hi} \geq 0 \tag{F-17}$$

$$K_{r,a}^{lo} + K_{r,a}^{hi} = \sum_v K_{r,a,v}^v \tag{F-18}$$

Equation (F-19) determines the economy-wide rate of return, *TR*. Equation (F-20) determines the sector- and vintage-specific rate of return. If the activity uses *New* capital, *RR* is equal to 1 and both *Old* and *New* capital obtain the rate of return *TR*. If the sector is in decline, it receives a rate of return lower than the economy-wide rate of return.

$$TK_r^s = \sum_a \sum_v K_{r,a,v}^v \tag{F-19}$$

$$PK_{r,a,v} = RR_{r,a} TR_r \tag{F-20}$$

Determination of output by vintage

Aggregate output, *XP*, is determined by equilibrium conditions. Equation (F-21) represents the contemporaneous capital-output ratio for the *Old* vintage, *kxRat*. Equation (F-22) determines output with *Old* capital. *Old* output, will be equal to the capital output ratio applied to the supply of old capital, K^{lo}. Equation (F-23) ensures that the sum of output across vintages equals aggregate output. In the vintage capital version of the model, it essentially determines production with *New* capital. In the comparative static version, it trivially equates total production to production produced by the only vintage that is labeled *Old*.

44

$$kxRat_{r,a,Old} = \frac{K^v_{r,a,Old}}{XPv_{r,a,Old}} \tag{F-21}$$

$$kxRat_{r,a,Old} XPv_{r,a,Old} = K^{lo}_{r,a} \tag{F-22}$$

$$XP_{r,a} = \sum_v XPv_{r,a,v} \tag{F-23}$$

Equations (F-24) and (F-25) are added for accounting purposes as many of the various income identities are simplified with these substitutions. Equation (F-25) holds in all circumstances. If an activity is expanding the return on installed capital is equal to the return on *New* capital. If an activity is contracting, the only relevant return is on *Old* capital.

$$XF_{r,cap,a} = \sum_v K^v_{r,a,v} \tag{F-24}$$

$$PF_{r,cap,a} = PK_{r,a,Old} \tag{F-25}$$

3.8.3   Land market

The supply side of the land market has two components. The first component provides the aggregate supply of land, by land-type.[59] The second step allocates aggregate land by type across different activities allowing for a nested CET structure, the possibility of perfect mobility, and the use of the adjusted CET that preserves land additivity.

The aggregate land supply curve is allowed to have four shapes—constant elasticity, a logistic curve with an upward asymptote, a generalized hyperbola also with an upward asymptote, and perfectly horizontal. The aggregate land supply curve is given by equation (F-26). The initial elasticity of aggregate land is given by η^t.

$$
\begin{cases}
TLand_{r,lnd} = \chi^t_{r,lnd} \left(\dfrac{PTLand_{r,lnd}}{PGDP_r} \right)^{\eta^t_{r,lnd}} & \text{iso-elastic} \\[3ex]
TLand_{r,lnd} = \dfrac{TLand^{Max}_{r,lnd}}{1 + \chi^t_{r,lnd} e^{-\gamma^{tl}_{r,lnd}(PTLand_{r,lnd}/PGDP_r)}} & \text{logistic} \\[3ex]
TLand_{r,lnd} = TLand^{Max}_{r,lnd} - \chi^t_{r,lnd} \left(\dfrac{PTLand_{r,lnd}}{PGDP_r} \right)^{-\gamma^{tl}_{r,lnd}} & \text{hyperbola} \\[3ex]
PTLand_{r,lnd} = PGDP_{r,lnd} & \text{if } \eta^t_{r,lnd} = \infty
\end{cases} \tag{F-26}
$$

The iso-elastic curve by definition has a constant elasticity given by η^t. The logistic function, in a simplified form is given by:

$$S = \frac{\overline{S}}{1 + \chi e^{-\gamma P}}$$

where S is supply, the land maximum is given by $\overline{S}$ and P is the real price of land. The supply elasticity is given by the following formula:

$$\eta = \gamma P \left(\frac{\overline{S} - S}{\overline{S}} \right)$$

The γ coefficient can be calibrated using the expression above given initial values for η, S, $\overline{S}$ and P. The calibration formula is given by:

$$\gamma = \frac{\eta}{P} \left(\frac{\overline{S}}{\overline{S} - S} \right)$$

Calibration of the χ parameter is given by the following expression and requires first calibrating the γ parameter:

$$\chi = e^{\gamma P} \left(\frac{\overline{S} - S}{\overline{S}} \right)$$

---

[59]   [20-Apr-2022] The code has been modified to allow for multiple land types, for example AEZs. The specification for land supply is identical to the previous version except that all relevant equations and variables are now indexed by land type, i.e., *lnd*. With the default GTAP database, there is a single land-type. With GTAP's AEZ database, there can be up to 18 land-types for each land-using activity in each region.

The generalized hyperbola, using the same notation is given by:

$$S = \overline{S} - \chi P^{-\gamma}$$

The supply elasticity is given by the following formula:

$$\eta = \gamma \left(\frac{\overline{S} - S}{S} \right)$$

The γ coefficient can be calibrated using the expression above given initial values for η, S, $\overline{S}$ and P. The calibration formula is given by:

$$\gamma = \eta \left(\frac{S}{\overline{S} - S} \right)$$

Calibration of the χ parameter is given by the following expression and requires first calibrating the γ parameter:

$$\chi = P^{\gamma} \left(\overline{S} - S \right)$$

The user chooses the functional form by setting the global parameter named `TASS` towards the top of a simulation file. Valid options are `KELAS` for the iso-elastic form, `LOGIST` for the logistic specification, `HYPERB` for the generalized hyperbola and `INFTY` for the horizontal supply function.[60]

Total land is allocated across different uses according to the relative rates of return across uses. A nested CET structure is implemented that allows for a degree of user control.[61] Figure 13 provides a graphic depiction of the land allocation specification. The number of land bundles (XLB) is under user control, but typically there should be at least one. The first land bundle is in the top CET nest with an intermediate land bundle (XLB). All other land bundles are nested within the intermediate land bundle. Some possible configurations from the literature include:

- To mimic MAGNET's specification, `v_f`, `ocr`, `oap` are mapped to land bundle 1, `sug`, `ctl` and `rmk` are mapped to land bundle 2 and `ric`, `wht`, `gro` and `osd` are mapped to land bundle 2. The MAGNET model has two intermediate bundles and only three elasticities. In this specification, there are four intermediate bundles. Thus the top elasticity and the elasticity for land bundle 1 (horticulture, etc.), in MAGNET, are identical (and set to σ^1 in the MAGNET documentation, see edepot.wur.nl/310764, page 74). The elasticity for the intermediate land bundle and the second land bundle (pasture, etc.) would also be identical (and set to σ^2). And the third land bundle (cereals) would have the elasticity given by σ^3.

- The original version of the GTAP-AEZ model, (Hertel et al., 2009) has forestry in land bundle 1. The intermediate bundle is called agriculture—under which there are two additional bundles: crops and grazing. All crops are then assigned to the *Crops* bundle and all livestock activities are assigned to the *Grazing* bundle.[62] The AEZ-based land-use database removes land-use from the `oap` sector.

- A different version of the GTAP-AEZ model has altered the original nesting. It has crops in bundle 1, and a grazing plus forestry bundle as the intermediate bundle (Stevenson et al., 2013).

It is possible to mimic the GTAP standard nesting, a single level nest, by defining only one land bundle and use the GTAP transformation elasticity for that bundle. The top level elasticity is essentially irrelevant since in this case the intermediate land bundle would not be used.

The top level nest decomposes total land supply into land bundle 1 and the intermediate land bundle. Equations (F-27) and (F-28) provide the first order conditions and hold irrespective of the CET specification used—either the standard, with `ifLandCET` set to 1, or the adjusted, volume preserving, CET specification. Equation (F-29) defines the aggregate price index of land, $PTLandN$. The index expression depends on the choice of CET specification. For the standard CET, the price index will be identical to the average price (defined next). For the adjusted CET, the price index will not generally be equal to the average price. Note that the price index expressions are not incorporated in the model under the assumption of perfect transformation, i.e. when the transformation elasticity is infinite. The average price of aggregate land, $PTLand$ is given in equation (F-30) and holds for both versions of the CET.

---

[60] These options are defined in the `model.gms` file with a set named `landAssumptions`. The subset named `tass` contains the user-chosen specification.

[61] The implementation was first developed to mimic the CET structure of the land allocation specification in the MAGNET model.

[62] That version of the GTAP-AEZ model had a fixed aggregation for grazing that included cattle (`ctl`), raw milk (`rmk`) and wool (`wol`).
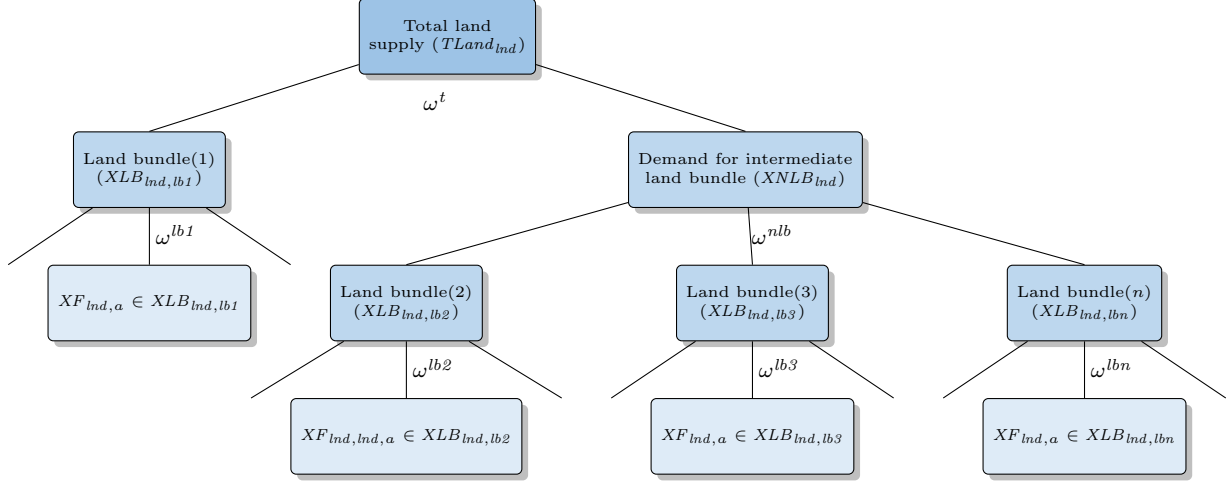
Figure 13: **CET nest for land allocation across activities by land-type**

$$
\begin{cases}
XLB_{r,lnd,lb1} = \gamma_{r,lnd,lb1}^{lb} \left(\dfrac{PLB_{r,lnd,lb1}}{PTLandN_{r,lnd}} \right)^{\omega_{r,lnd}^{t}} TLand_{r,lnd} & \text{if } \omega_{r,lnd}^{t} \neq \infty \\
PLB_{r,lnd,lb1} = PTLand_{r,lnd} & \text{if } \omega_{r,lnd}^{t} = \infty
\end{cases}
\tag{F-27}
$$

$$
\begin{cases}
XNLB_{r,lnd} = \gamma_{r,lnd}^{nlb} \left(\dfrac{PNLB_{r,lnd}}{PTLandN_{r,lnd}} \right)^{\omega_{r,lnd}^{t}} TLand_{r,lnd} & \text{if } \omega_{r,lnd}^{t} \neq \infty \\
PNLB_{r,lnd} = PTLand_{r,lnd} & \text{if } \omega_{r,lnd}^{t} = \infty
\end{cases}
\tag{F-28}
$$

$$
\begin{cases}
PTLandN_{r,lnd} = \left[\gamma_{r,lnd,lb1}^{lb} PLB_{r,lnd,lb1}^{1+\omega_{r,lnd}^{t}} + \gamma_{r,lnd}^{nlb} PNLB_{r,lnd}^{1+\omega_{r,lnd}^{t}} \right]^{1/(1+\omega_{r,lnd}^{t})} & \text{if } ifLandCET \\
PTLandN_{r,lnd} = \left[\gamma_{r,lnd,lb1}^{lb} PLB_{r,lnd,lb1}^{\omega_{r,lnd}^{t}} + \gamma_{r,lnd}^{nlb} PNLB_{r,lnd}^{\omega_{r,lnd}^{t}} \right]^{1/\omega_{r,lnd}^{t}} & \text{if } \neg ifLandCET
\end{cases}
\tag{F-29}
$$

$$
PTLand_{r,lnd} TLand_{r,lnd} = PLB_{r,lnd,lb1} XLB_{r,lnd,lb1} + PNLB_{r,lnd} XNLB_{r,lnd}
\tag{F-30}
$$

The second level nest decomposes the intermediate land bundle (if it is defined) into a number of user determined land bundles, excluding the first land bundle. Equation (F-31) defines the demand for the land bundles as a share of the intermediate land bundle.[63] The price index of the intermediate land bundle is defined in equation (F-32), allowing for both CET specifications and is only included if the transformation elasticity at this nest is finite. The average price of the land bundle is defined in equation (F-33).

$$
\begin{cases}
XLB_{r,lnd,lb} = \gamma_{r,lnd,lb}^{lb} \left(\dfrac{PLB_{r,lnd,lb}}{PNLBN_{r,lnd}} \right)^{\omega_{r,lnd}^{nlb}} XNLB_{r,lnd} & \text{if } \omega_{r,lnd}^{nlb} \neq \infty \\
PLB_{r,lnd,lb} = PNLB_{r,lnd} & \text{if } \omega_{r,lnd}^{nlb} = \infty
\end{cases}
\tag{F-31}
$$

$$
\begin{cases}
PNLBN_{r,lnd} = \left[\displaystyle\sum_{\{lb:lb \neq lb1\}} \gamma_{r,lnd,lb}^{lb} PLB_{r,lnd,lb}^{1+\omega_{r,lnd}^{nlb}} \right]^{1/(1+\omega_{r,lnd}^{nlb})} & \text{if } ifLandCET \\
PNLBN_{r,lnd} = \left[\displaystyle\sum_{\{lb:lb \neq lb1\}} \gamma_{r,lnd,lb}^{lb} PLB_{r,lnd,lb}^{\omega_{r,lnd}^{nlb}} \right]^{1/\omega_{r,lnd}^{nlb}} & \text{if } \neg ifLandCET
\end{cases}
\tag{F-32}
$$

$$
PNLB_{r,lnd} XNLB_{r,lnd} = \sum_{\{lb:lb \neq lb1\}} PLB_{r,lnd,lb} XLB_{r,lnd,lb}
\tag{F-33}
$$

The bottom land nest(s) decompose each of the land bundles into the primary land allocation to each activity a—each activity a (that uses land) must be mapped to one, and only one, land bundle. Equation (F-34) defines the

---

[63]   The index lb excludes the first land bundle.

supply of land to activity a that has been mapped to land bundle lb. Equation (F-35) defines the price index for the bundles that depends on the CET specification. And equation (F-36) defines the average price of the land bundles.

$$
\begin{cases}
Land^s_{r,lnd,a} = \gamma^t_{r,lnd,a} \left(\dfrac{PF_{r,lnd,a}}{PLBN_{r,lnd,lb}} \right)^{\omega^{lb}_{r,lnd,lb}} XLB_{r,lnd,lb} & \text{if } \omega^{lb}_{r,lnd,lb} \neq \infty \\
PF_{r,lnd,a} = PLB_{r,lnd,lb} & \text{if } \omega^{lb}_{r,lnd,lb} = \infty
\end{cases} \quad a \in \{lb\} \tag{F-34}
$$

$$
\begin{cases}
PLBN_{r,lnd,lb} = \left[\displaystyle\sum_{a \in \{lb\}} \gamma^t_{r,lnd,a} PF^{1+\omega^{lb}_{r,lnd,lb}}_{r,lnd,a} \right]^{1/(1+\omega^{lb}_{r,lnd,lb})} & \text{if } ifLandCET \\[3ex]
PLBN_{r,lnd,lb} = \left[\displaystyle\sum_{a \in \{lb\}} \gamma^t_{r,lnd,a} PF^{\omega^{lb}_{r,lnd,lb}}_{r,lnd,a} \right]^{1/\omega^{lb}_{r,lnd,lb}} & \text{if } \neg ifLandCET
\end{cases} \tag{F-35}
$$

$$
PLB_{r,lnd,lb} XLB_{r,lnd,lb} = \sum_{a \in \{lb\}} PF_{r,lnd,a} Land^s_{r,lnd,a} \tag{F-36}
$$

The final equation in the land market module, equation (F-37) equates land supply to activity a to its respective demand thus determining the equilibrium and sector-specific price for land, PF_{lnd}. The equation is substituted out of the model specification.

$$
Land^s_{r,lnd,a} = XF_{r,lnd,a} \tag{F-37}
$$

3.8.4 Market for natural resources

Natural resources are assumed to be activity-specific, for example crude oil reserves. The supply of natural resources, $XNRS^s$, is governed by a kinked iso-elastic supply curve. The model allows the supply response to differ according to whether market conditions are leading to increased supply, the 'hi' elasticity, versus a contracting market, the 'lo' elasticity. Ideally, we could implement this with a regime shift using continuous functions, but discontinuous derivatives. Instead, we add an equation to the model that determines the supply elasticity using the sigmoid function, also known as a logistic function.[64] The sigmoid function is defined by:

$$
f(x) = \frac{1}{1 + e^{-x}}
$$

Equation (F-38) defines the applied elasticity as a function of market conditions. The user enters the 'lo' and 'hi' elasticities in the parameters η^{nrsx}. One would typically assume that the 'lo' elasticity is higher than the 'hi' elasticity, i.e. it is harder to expand output than to contract output. If the two are identical, the equation is trivial. The argument in the sigmoid function reflects market growth and will be positive for expansion and negative for contraction. The κ parameter captures the 'speed' of adjustment between the 'lo' and 'hi' elasticities. The greater the value, the more rapid the transition. We have used a value of 30 with few numerical problems.[65] Figure 14 shows the kinked elasticity and the resulting supply curve s a function of price. It is centered at a price of 1. The 'lo' elasticity is assumed to be 2 and the 'hi' elasticity is 0.5. The 'kink' parameter is been set at 10 in this example.

Equation (F-39) reflects the supply function for the natural resource. For finite elasticities, it is an iso-elastic function, however, where the supply elasticity is determined by market conditions as described above. It is a function of the growth of the real price of the natural resource and the supply curve shifts according to lagged supply.[66] The (finite) supply curve has two shifters that can be used to target either the regional output price of the commodity being produced or the global price. To target the regional price, exogenize the output price, PX, and endogenize the regional shifter, χ^{nrs}. To target the global price, exogenize the average producer price, PW,[67] and endogenize the global shifter, $\chi^{w,nrs}$. The supply curve can also be horizontal, in which case the supply price is fixed relative o an anchor price, herein the GDP deflator. Equation (F-40) determines the equilibrium price. The latter equation is substituted out of the model.

$$
\eta^{nrs}_{r,a,t} = \eta^{nrsx}_{r,a,lo} + \frac{\eta^{nrsx}_{r,a,hi} - \eta^{nrsx}_{r,a,lo}}{1 + exp\left(-\kappa \left[\dfrac{XF_{r,nrs,a,t}}{XF_{r,nrs,a,t-1}} - 1 \right] \right)} \tag{F-38}
$$

---

[64]   This is used for example in the CAPRI model of global agriculture with a focus on the EU, Britz et al. (2007).

[65]   The sigmoid function is an intrinsic function in GAMS.

[66]   This is somewhat different from the previous specification where the supply shifter was constant unless endogenous for calibration purposes.

[67]   Defined in equation (M-32).
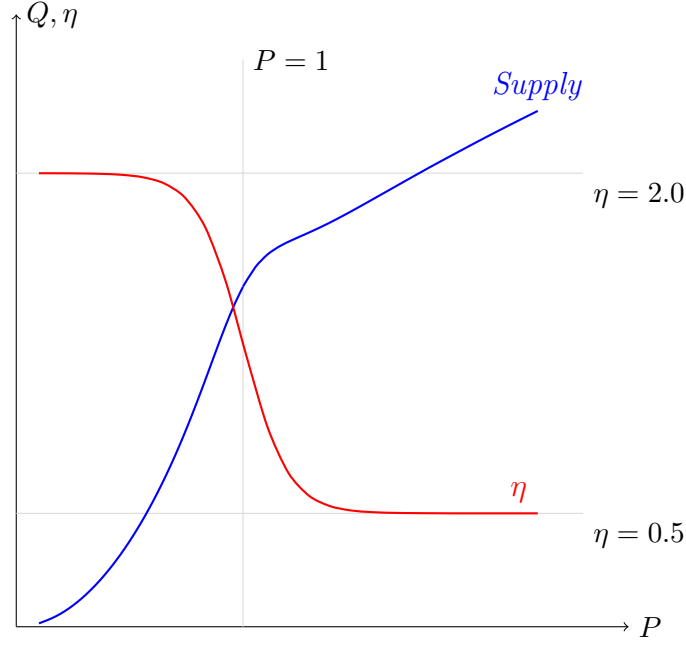
Figure 14: **Example of a kinked supply curve using the sigmoid function**

$$
\begin{cases}
XNRS^s_{r,a,t} = \chi^{w,nrs}_a \chi^{nrs}_{r,a} XNRS^s_{r,a,t-1} \left(\dfrac{\chi^{nrsp}_{r,a} PF_{r,nrs,a,t}/PGDPMP_{r,t}}{PF_{r,nrs,a,t-1}/PGDPMP_{r,t-1}} \right)^{\eta^{nrs}_{r,a}} & \text{if } \eta^{nrs}_{r,a} \neq \infty \\
\chi^{nrsp}_{r,a} PF_{r,nrs,a} = PGDPMP_r & \text{if } \eta^{nrs}_{r,a} = \infty
\end{cases}
\tag{F-39}
$$

$$
XNRF^s_{r,a} = XF_{r,nrs,a}
\tag{F-40}
$$

3.8.5 Water market

The market for water is somewhat similar to the land market. An aggregate supply curve is provided—and aggregate supply is then allocated to different uses using a nested CET structure. However, unlike land, water is in direct use in only the irrigated crop sectors. Demand for water in other sectors is specified using aggregate demand functions.

The aggregate water supply curve is allowed to have four shapes—constant elasticity, a logistic curve with an upward asymptote, a generalized hyperbola also with an upward asymptote, and perfectly horizontal. The aggregate water supply curve is given by equation (F-41). The initial elasticity of aggregate water is given by η^w.

$$
\begin{cases}
TH2O_r = \chi^{h2o}_r \left(\dfrac{PTH2O_r}{PGDP_r} \right)^{\eta^w_r} & \text{iso-elastic} \\[2mm]
TH2O_r = \dfrac{TH2O^{Max}_r}{1 + \chi^{h2o}_r e^{-\gamma^{tw}_r (PTH2O_r/PGDP_r)}} & \text{logistic} \\[2mm]
TH2O_r = TH2O^{Max}_r - \chi^{h2o}_r \left(\dfrac{PTH2O_r}{PGDP_r} \right)^{-\gamma^{tw}_r} & \text{hyperbola} \\[2mm]
PTH2O_r = PGDP_r & \text{if } \eta^w_r = \infty
\end{cases}
\tag{F-41}
$$

The iso-elastic curve by definition has a constant elasticity given by η^w. The calibration and other formulas for the logistic and hyperbola functions are the same as the land market expressions and are not repeated here.

Water use at the sub-aggregate level is represented by a number of 'bundles', *H2OBnd*. Table 3.4 lists the bundles and their relationships. The marketed water bundles are allocated in two CET nests. The top nest allocates water between agriculture and non-agriculture. These bundles are indexed by *wb1*. The second level nests include crops and livestock linked to the agriculture bundle, and industrial and municipal water use linked to the non-agriculture bundle. The second level bundles are indexed by *wb2*. The second level bundles are also segmented into two groups— *wba* and *wbi*. The former allocates water across different activities—for example irrigated crops—and the latter is

49

linked to an aggregate water demand—not explicitly linked to specific activities. The set *wbx* refers to exogenous water demand.

Table 3.4: **Aggregate water bundles**

Name	Description	Set	Dependency	Type
ENV	Environmental services		*TH2O*	*wbx*
GRD	Groundwater recharge		*TH2O*	*wbx*
AGR	All agriculture	*wb1*	$TH2O^m$	
CRP	Crops	*wb2*	AGR	*wba*
LVS	Livestock	*wb2*	AGR	*wbi*
NAG	Not agriculture	*wb1*	$TH2O^m$	
IND	Industrial water use	*wb2*	NAG	*wbi*
MUN	Municipal water use	*wb2*	NAG	*wbi*

At the top level water supply is allocated between marketed water and non-marketed water. The latter is exogenous and split into two variables—water for environmental services and water required for ground water recharge. In equilibrium, one would assume the latter is close to zero, i.e. ground water withdrawal closely matches ground water extraction. In the current database, both of these exogenous water uses are initially set at zero. A policy shock may include making these positive in a future time period—perhaps permanently for environmental services and temporarily for ground water recharge. Marketed water is then the difference between total water supply and exogenous demand. Equation (F-42) is the market supply equilibrium equation equating total water supply to marketed water plus the sum of all exogenous water demand. The variable $H2O^m$ represents marketed water.

$$TH2O_r = TH2O_r^m + \sum_{wb \in wbx} H2OBnd_{r,wb} \tag{F-42}$$

Marketed water is allocated across the various water bundles using a nested (adjusted) CET specification.[68] Figure 15 illustrates the nested structure.
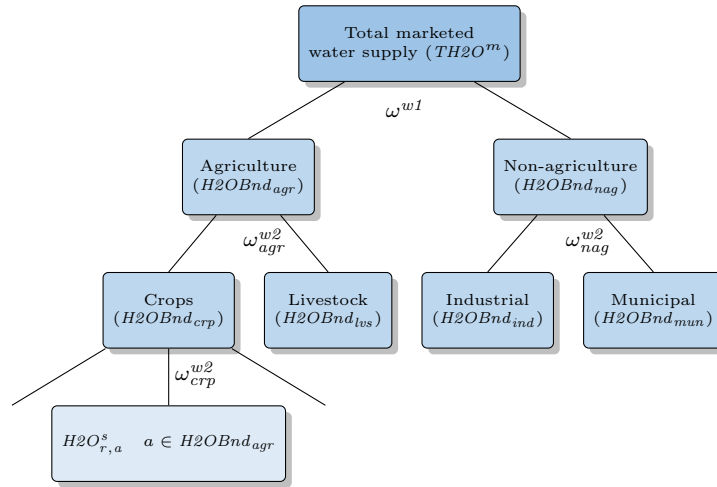
Figure 15: **CET nest for water allocation**

The first level nest allocates aggregate marketed water to the agricultural and non-agricultual bundles using an adjusted CET specification. The supply expressions are given in equation (F-43) that allows for perfect transformation. With reference to Table the index *wb1* covers the AGR and NAG water bundles.[69] If the transformation elasticity

---

[68] The adjusted CET specification preserves quantity additivity—see Appendex A.

[69] The GAMS code is written in a relatively flexible way so that the user can largely change the set definitions for all water bundles and the relevant subsets and mappings.

is finite, equation (F-44) provides the expression for the CET aggregate price index for $TH2O^m$. The aggregate water supply price is given by equation (F-45).

$$\begin{cases} H2OBnd_{r,wb1} = \gamma^{w1}_{r,wb1} \left(\dfrac{PH2OBnd_{r,wb1}}{PTH2O^n_r} \right)^{\omega^{w1}_r} TH2O^m_r & \text{if } \omega^{w1}_r \neq \infty \\ PH2OBnd_{r,wb1} = PTH2O_r & \text{if } \omega^{w1}_r = \infty \end{cases}$$
(F-43)

$$PTH2O^n_r = \left[\sum_{wb1} \gamma^{w1}_{r,wb1} PH2OBnd^{\omega^{w1}_r}_{r,wb1} \right]^{1/\omega^{w1}_r} \text{if } \omega^{w1}_r \neq \infty$$
(F-44)

$$PTH2O_r \, TH2O^m_r = \sum_{wb1} PH2OBnd_{r,wb1} H2OBnd_{r,wb1}$$
(F-45)

The next nest decomposes the AGR and LVS bundles into crops (CRP) and livestock (LVS) on the one hand and industrial (IND) and municipal (MUN) water demand on the other hand. Equation (F-46) determines the supply for the second level bundles—indexed by $wb2$. Each of the $wb2$ bundles is mapped to one, and only one, top level bundle indexed by $wb1$. Equation (F-47) defines the relevant aggregate price index that is used to determine the allocation. It is only used in the case of a finite transformation elasticity. Equation (F-48) defines the price of the aggregate $wb1$ bundles, .e. the price of the AGR and NAG bundles.

$$\begin{cases} H2OBnd_{r,wb2} = \gamma^{w2}_{r,wb2} \left(\dfrac{PH2OBnd_{r,wb2}}{PH2OBnd^n_{r,wb1}} \right)^{\omega^{w2}_{r,wb1}} H2OBnd_{r,wb1} & \text{if } \omega^{w2}_{r,wb1} \neq \infty \\ PH2OBnd_{r,wb2} = PH2OBnd_{r,wb1} & \text{if } \omega^{w2}_{r,wb1} = \infty \end{cases} wb2 \in wb1$$
(F-46)

$$PH2OBnd^n_{r,wb1} = \left[\sum_{wb2 \in wb1} \gamma^{w2}_{r,wb2} PH2OBnd^{\omega^{w2}_{r,wb1}}_{r,wb2} \right]^{1/\omega^{w2}_{r,wb1}} \text{if } \omega^{w2}_{r,wb1} \neq \infty$$
(F-47)

$$PH2OBnd_{r,wb1} H2OBnd_{r,wb1} = \sum_{wb2 \in wb1} PH2OBnd_{r,wb2} H2OBnd_{r,wb2}$$
(F-48)

The second level bundles, indexed by $wb2$, are segmented into two subsets. The first subset, indexed by wba, has an additional CET nest that maps the aggregate supply to individual production activies (a). With the current version of the database, the AGR bundle is the only bundle that maps to water demand by production activities. The water bundles in the second subset, indexed by wbi, are set equal to an aggregate demand for water over a relevant subset of activities mapped to that bundle.[70]

Equation (F-49) determines the supply of water to each activity a that is mapped to the aggregate water supply indexed by wba (currently limited to irrigated crops mapped to the AGR bundle).[71] Equation (F-50) defines the relevant aggregate price index that is used to determine the allocation. It is only used in the case of a finite transformation elasticity. Equation (F-51) defines the price of the aggregate wba bundle(s).

$$\begin{cases} H2O^s_{r,a} = \gamma^{w3}_{r,a} \left(\dfrac{PF_{r,wat,a}}{PH2OBnd^n_{r,wba}} \right)^{\omega^{w2}_{r,wba}} H2OBnd_{r,wba} & \text{if } \omega^{w2}_{r,wba} \neq \infty \\ PF_{r,wat,a} = PH2OBnd_{r,wba} & \text{if } \omega^{w2}_{r,wba} = \infty \end{cases} a \in wba$$
(F-49)

$$PH2OBnd^n_{r,wba} = \left[\sum_{a \in wba} \gamma^{w3}_{r,a} \left(PF_{r,wat,a} \right)^{\omega^{w2}_{r,wba}} \right]^{1/\omega^{w2}_{r,wba}} \text{if } \omega^{w2}_{r,wba} \neq \infty$$
(F-50)

$$PH2OBnd_{r,wba} H2OBnd_{r,wba} = \sum_{a \in wba} PF_{r,wat,a} H2O^s_{r,a}$$
(F-51)

Equation (F-52) determines the equilibrium market price of water, PF_{wat}, delivered to activity a. Water demand is provided by equation (P-43). The equilibrium condition is substituted out of the model.

$$H2O^s_{r,a} = XF_{r,wat,a}$$
(F-52)

For the aggregate water bundles indexed by wbi, demand is constructed as a function of the aggregate production volume of the constituent activities and includes a price sensitive component. Equation (F-53) defines demand for aggregate water, $H2OBnd^d$. Demand is a function of a volume index over the activities mapped to the water

---

[70] This is an ad hoc treatment of water demand that generates an equilibrium price. However, the demand for water in the relevant production activities do not form part of the cost structure of these activities—unlike the case for irrigated crops.

[71] The same elasticity parameter, ω^{w2} is used for all of the aggregate water bundles, irrespective of the nest level.

bundles—subject to an elasticity given by η^{h2ob}. In addition, demand responds to the real price of the relevant bundle. Equation (F-54) determines the equilibrium price for the aggregate bundle. The equilibrium expression is substituted out of the model.

$$H2OBnd^d_{r,wbi} = \frac{\alpha^{h2ob}_{r,wbi}}{\lambda^{h2ob}_{r,wbi}} \left(\frac{PH2OBnd^d_{r,wbi}}{PGDPMP_r} \right)^{-\epsilon^{h2ob}_{r,wbi}} \left[\frac{\sum\limits_{a \in wbi} PX_{r,a,0} XP_{r,a}}{\sum\limits_{a \in wbi} PX_{r,a,0} XP_{r,a,0}} \right]^{\eta^{h2ob}_{r,wbi}}$$ (F-53)

$$H2OBnd_{r,wbi} = H2OBnd^d_{r,wbi}$$ (F-54)

3.8.6 Factor taxes and prices

Producers pay the market price for factors adjusted by factor taxes. Equation (F-55) determines the producers purchase price of factors for all factors of production. Equation (F-56) is the relevant equation for the price of capital across vintages.

$$PF^p_{r,f,a} = \left(1 + \tau^v_{r,f,a}\right) PF_{r,f,a}$$ (F-55)

$$PK^p_{r,a,v} = \left(1 + \tau^v_{r,cap,a}\right) PK_{r,a,v}$$ (F-56)

3.9 National accounts and model closure

3.9.1 National accounts

Definition of the national accounts would normally be part of the post-processing portion of a simulation, i.e. it is not necessarily part of the core definition of a CGE model. However, when running the model in dynamics, it is typically useful to have some of the key national accounting identities available for targeting purposes, for example to target the growth of per capita GDP, or a macro ratio such as the share of government or investment expenditures relative to GDP.

Equation (M-1) defines nominal GDP at market price, *GDPMP*. The expression uses the indicator *QGDP* that represents the value of GDP using prices and quantities evaluated at different periods—similar to the absorption indicator above:

$$QGDP_{r,tp,tq} = \sum_{fd} \sum_i PA^a_{r,i,fd,tp} XA_{r,i,fd,tq} + \sum_m PD_{r,m,tp} XTT_{r,m,tq}$$
$$+ \sum_i \left\{ \sum_d PWE_{r,i,d,tp} XW^s_{r,i,d,tq} - \sum_s PWM_{s,i,r,tp} XW^d_{s,i,r,tq} \right\}$$

i.e. it is the sum of domestic absorption, including the exports of international trade and transport services and the net trade balance for all other goods and services evaluated at their respective border prices.

$$GDPMP_{r,t} = QGDP_{r,t,t}$$ (M-1)

Equation (M-2) defines the GDP at market price deflator, *PGDPMP*, which is calculated as the Fisher price index of GDP.[72] Equation (M-3) then defines the volume of GDP at market price.

$$PGDPMP_{r,t} = PGDPMP_{r,t-1} \sqrt{\frac{QGDP_{r,t,t-1}}{QGDP_{r,t-1,t-1}} \cdot \frac{QGDP_{r,t,t}}{QGDP_{r,t-1,t}}}$$ (M-2)

$$RGDPMP_r = GDPMP_r / RGDPMP_r$$ (M-3)

Equation (M-4) defines real per capita GDP, *RGDPpc*. Equation (M-5) defines the growth in real per capita GDP, g^y. In comparative static exercises, it measures the difference in real per capita GDP between the baseline and the shock simulation.[73] In dynamic simulations the equation allows for multi-year time steps as measured by n. In most dynamic baseline or reference simulations, the growth rate of real per capita GDP will be exogenous

---

[72] When prices and quantities are evaluated at the same year, nominal GDP can replace the relevant *QGDP* indicator.

[73] The value of n is always 1 in comparative static simulations.

and an economy-wide variable, for example a uniform labor productivity shifter, is endogenous and will serve as the instrument to target GDP growth. In policy or alternative simulations, the economy-wide factor would normally be exogenous and the growth rate of GDP would be endogenous. Equation (M-6) defines the average capital labor ratio in efficiency units. Under a balanced growth scenario, this ratio is intended to be uniform across time. It could be used to target the growth of capital productivity independently of the growth of labor productivity.

$$RGDPpc_r = \frac{RGDPMP_r}{Pop_r} \tag{M-4}$$

$$RGDPpc_{r,t} = \left(1 + g^y_{r,t}\right)^n RGDPpc_{r,t-n} \tag{M-5}$$

$$KLRat_r = \frac{\sum_a \sum_v PK_{r,a,v,0} \lambda^f_{r,cap,a} Kv_{r,a,v}}{\sum_a \sum_l PF_{r,l,a,v,0} \lambda^f_{r,l,a} XF_{r,l,a}} \tag{M-6}$$

3.9.2 Closure

Government accounts

Equation (M-7) defines nominal government savings, S^g, as the difference between government revenues and government's current expenditures on goods and services. In the standard model, there is only one stream of government expenditures. Alternative versions allow for additional streams, such as expenditures on research and development, public investment, etc. The subset Gov includes all final demand expenditures linked to government accounts.[74] When an emissions quota regime is put into place, revenues from the quotas[75] accrue to the government and eventually to households using the default closure with a lump sum tax on households. Note that the counterpart account of the quota income appears in the balance of payments expression, i.e. as a flow from the rest of the world. The equation also includes net government to government transfers and the government's income from NTM-related measures—both domestic and foreign.[76] Equation (M-7) defines real government savings, RS^g. In the standard closure, the public deficit is held fixed in real terms (in order to preserve price homogeneity). The direct tax rate, κ^h, adjusts to the target for the public deficit. Because there is a single household, this is equivalent to a change in lump sum taxes.

$$
\begin{aligned}
S^g_r &= \sum_{gy} YGOV_{r,gy} + \sum_{em} QuotaY^{Emi}_{r,em} + \sum_s \chi^{gNTM}_{s,r} ntmY_s \\
&+ \quad ODAIn_r - ODAOut_r - \sum_{fd \in Gov} YFD_{r,fd}
\end{aligned} \tag{M-7}
$$

$$RS^g_r = \frac{S^g_r}{PGDPMP_r} \tag{M-8}$$

Macro closure

Equations (M-9) and (M-10) define respectively the real and nominal share of aggregate domestic final demand expenditures with respect to GDP, respectively real and nominal. These equations are often used to calibrate the model in the dynamic baseline. For example, real government and research and development expenditures may be fixed relative to real GDP in the baseline. In shock simulations, they may be fixed at baseline levels, and then these equations determine how the shares change with shocks. For investment, the baseline may target the ratio of investment relative to GDP, with an endogenous savings rate. The savings rate would then be exogenous in shock simulations.

$$rFDShr_{r,fd} = \frac{XFD_{r,fd}}{RGDPMP_r} \tag{M-9}$$

$$nFDShr_{r,fd} = \frac{YFD_{r,fd}}{GDPMP_r} \tag{M-10}$$

---

[74]   [NEW: 20-JAN-2019] In the current model code, the subset Gov does not exist. Instead the sum is over all fd that equal either gov or r_d.

[75]   Revenues are positive if a region is a net seller and negative if a region is a net buyer.

[76]   [NEW: 4-OCT-2018]

Capital account

The model has four closures for the capital account. The standard closure fixes foreign savings in real terms. Equation (M-11) determines S^f. For all but one region, the residual region, foreign saving is equal to a fixed level. The second part of the equation, guarantees that the sum across regions is identically zero, and thus sets the level for the residual region.[77]

$$\begin{cases} S_r^f = \overline{S}_r^f & \text{if } r \neq rres \\ \sum_r S_r^f \equiv 0 & \text{if } r = rres \end{cases} \tag{M-11}$$

In addition, equation (M-12) is added to the model specification in the case of the fixed capital account closure and simply evaluates R^g as a weighted average of regional expected returns, with the weights given in equation (M-13).

$$R^g = \sum_r \varphi_r^r R_r^e \tag{M-12}$$

$$\varphi_r^r = \frac{PFD_{r,inv}\left(XFD_{r,inv} - \delta_r\, TK_r^s\right)}{\sum_s PFD_{s,inv}\left(XFD_{s,inv} - \delta_s\, TK_s^s\right)} \tag{M-13}$$

The second alternative fixes the capital account relative to GDP, equation (M-14). Note that in this case the capital flows are deflated by the world price of savings since the variable S^f is a volume, not a value. The residual region becomes a borrower or lender of last resort as there is nothing that guarantees that if the ratio is fixed for all regions that their sum will add to zero. As in the case of the first closure, equation (M-12) is part of the model specification for this capital account closure.

$$\begin{cases} S_r^f = \chi_r^{sf}\, GDPMP_r / PW^{sav} & \text{if } r \neq rres \\ \sum_r S_r^f \equiv 0 & \text{if } r = rres \end{cases} \tag{M-14}$$

The third alternative, inspired from the GTAP model (Hertel (1997)), allows foreign savings to adjust to relative differentials in expected rates of return—with a specific definition of what is meant by the expected rate of return.

We start with the definition of the expected end-of-period capital stock, equation (M-15). This is simply the standard motion equation for the capital stock based on this period's beginning-of-period capital stock and investment level.

$$TK_{r,t}^e = (1 - \delta_{r,t})\, TK_{r,t} + XFD_{r,inv,t} \tag{M-15}$$

Equation (M-16) defines the aggregate after-tax rate of return. The factor χ^k converts the normalized rate of return to the non-normalized rate of return.[78] Equation (M-17) defines the net current rate of return, the rental rate adjusted for the price of replacing capital, less the depreciation rate. Therefore, a tariff reduction, that would lower the price of investment goods, would reduce the replacement price of capital and increase the rate of return (all else equal). The expected rate of return is given by equation (M-18). Regions with relatively high investment rates, i.e, that would lead to relatively high growth in the capital stock and thus depress the capital rate of return, would tend to see a reduction in the expected rate of return. The degree of adjustment is controlled by the parameter ϵ^{ror}.

$$R_{r,t} = \frac{\sum_a \sum_v \left(1 - \kappa_{r,cap,a}^f\right) PK_{r,a,v} K_{r,a,v}^v}{PFD_{r,inv,t}\, TK_{r,t}^s} \tag{M-16}$$

$$R_{r,t}^c = \frac{R_{r,t}}{PFD_{r,inv,t}} - \delta_{r,t} \tag{M-17}$$

$$R_{r,t}^e = R_{r,t}^c \left[\frac{TK_{r,t}^e}{TK_{r,t}}\right]^{-\epsilon^{ror}} \tag{M-18}$$

The level of foreign saving for each region will then be determined by equation (M-19), which equates the expected regional rates of return to the global average, R^g, adjusted for any risk premium that is embedded in the base year

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[77] It is best if the user makes sure that the sum of exogenous foreign savings equals 0 to avoid unintended consequences, but at least with this formulation, a small deviation will not create any problems.

[78] In the base year, TR is set to 1, and the non-normalized return will be a percent rate between 0 and perhaps up to 30%.
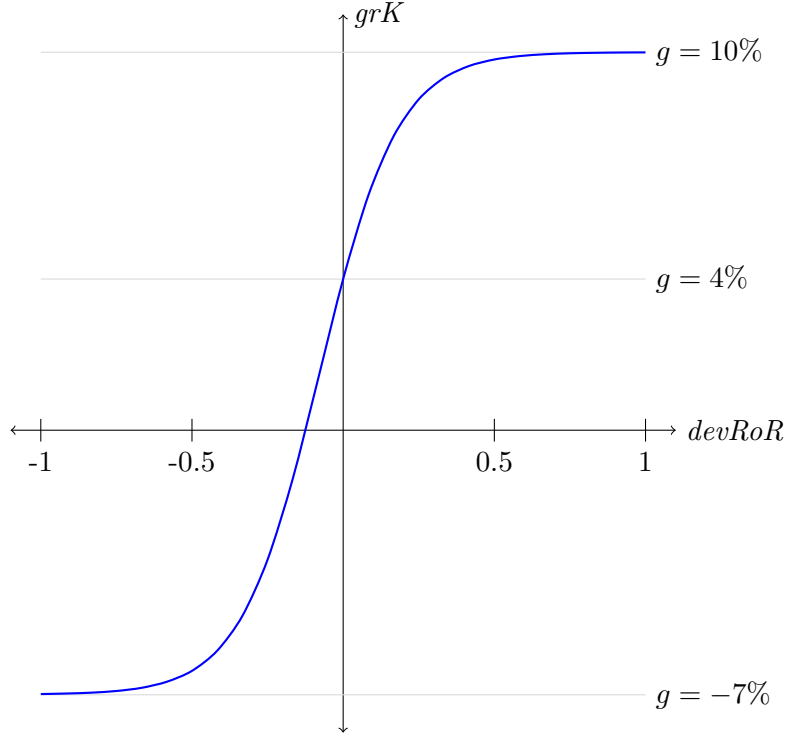
Figure 16: **Capital supply as a function of deviations in the rate of return**

data. Equation (M-20) guarantees that global savings equals global investment and in essence determines R^g. High values for ϵ^{ror} lead to relatively low adjustment in foreign savings. The default value for the GTAP model is 10.[79]

$$\pi^p_{r,t} R^c_{r,t} = R^g_t \tag{M-19}$$

$$\sum_r S^f_r \equiv 0 \tag{M-20}$$

The fourth specification is inspired by the capital allocation mechanism in the Monash and USAGE models (Dixon and Rimmer (2002) and Dixon and Rimmer (2005)).[80] The MONASH uses a capital supply schedule that links the growth in capital (for each region), grK, to deviations in the region's rate of return, ΔRoR, where the deviation is relative to an exogenous normal rate of return, R^n. A positive deviation leads to growth in capital above trend, $grTrend$. Growth is bounded by $grKMin$ at the lower end and $grKMax$ at the upper end. A logistic function describes the capital growth schedule where the curvature parameter, χ^k, is calibrated at $\Delta RoR = 0$ to an econometric estimate. Figure 16 shows the shape of the logistic curve with the growth of capital on the vertical axis and the deviation in the rate of return on the horizontal axis. The curve crosses at the normal or trend growth rate (4% in the graph) when the deviation in the rate of return is equal to its long-term average (i.e. 0%). The maximum growth in the example is 10% and the minimum is -7%.

We start by describing the expected rate of return. The expected rate of return is the sum of two components. The first is the expected nominal gross return in year $t + 1$, R,[81] relative to the unit cost of investing in year t, PFD_{inv}. The second term represents the capital gains on the unit of investment, i.e. the difference in the unit cost of investing between t and $t+1$ (adjusted by depreciation). Both terms are discounted by the real rate of interest, r.

$$E\left(R^e_{r,t}\right) = \frac{1}{1+r}\left[E\left(\frac{R_{r,t+1}}{PFD_{r,inv,t}}\right) + (1 - \delta_{r,t}) E\left(\frac{PFD_{r,inv,t+1}}{PFD_{r,inv,t}}\right)\right] - 1$$

---

[79] Users are urged to read the discussion in Hertel (1997), page 57.
[80] See also the discussion in the freely available Monash notes—Chapter 5, pages 317–326. The notes can be downloaded from https://www.copsmodels.com/ftp/monbook1/m1-chap5.pdf (accessed 28-May-2017).
[81] See equation (M-16).

With myopic expectations, the expected future values of R and PFD_{inv} are the current values and the expression above simplifies to equation (M-21), that defines the expected rate of return, R^e, at time t for region r. Equation (M-22) defines the deviation of the expected rate of return from its 'normal' or trend path, ΔRoR, which is used to determine the desired growth rate of the stock of capital in region r, where the expected rate of return is given by R^n and is an exogenous input. If the deviation is 0, the desired growth rate of the stock of capital is the trend growth rate (see equation (M-23) below. The equation allows for region-specific, R^d and a global shifter, R^g that allow for deviations from the normal trend.

$$R^e_{r,t} = \frac{1}{1+r}\left[\frac{R_{r,t}}{PFD_{r,inv,t}} + (1-\delta_{r,t})\right] - 1 \tag{M-21}$$

$$\Delta RoR_{r,t} = R^e_{r,t} - R^n_{r,t} - R^d_{r,t} - R^g_t \tag{M-22}$$

Equation (M-23) defines the growth in the capital stock, grK, using the doubly bounded logistic curve as a function of the deviation in the real rate of return from its long-run average.[82] Equation (M-24) defines the demand for new capital, i.e. investment, XFD_{inv}, for each region r. With the standard motion equation that sets end-of-period capital stock equal to the depreciated level of the beginning-of-period capital stock plus investment and the notional growth in the capital stock as given by grK, 'desired' investment is equal to the beginning-of-period capital stock multiplied by the sum of the growth rate and depreciation. It is bounded below by zero since $grKMin$ is set at minus the depreciation rate. Given that we also have equation (Y-24), this equation can be thought of as determining foreign savings. Equation (M-25) ensures the global savings/investment balance and in essence determines the R^g_t.[83]

$$grK_{r,t} = \frac{grKMax_{r,t}e^{\chi^k_{r,t}\Delta RoR_{r,t}} + grKMin_{r,t}\left[\dfrac{grKMax_{r,t} - grKTrend_{r,t}}{grKTrend_{r,t} - grKMin_{r,t}}\right]}{e^{\chi^k_{r,t}\Delta RoR_{r,t}} + \left[\dfrac{grKMax_{r,t} - grKTrend_{r,t}}{grKTrend_{r,t} - grKMin_{r,t}}\right]} \tag{M-23}$$

$$K_{r,t+1} = (1-\delta_{r,t})K_{r,t} + XFD_{r,inv,t} = \left(1 + grK_{r,t}\right)K_{r,t}$$
$$\Updownarrow \tag{M-24}$$
$$XFD_{r,inv,t} = K_{r,t}\left(grK_{r,t} + \delta_{r,t}\right)$$

$$\sum_r S^f_r \equiv 0 \tag{M-25}$$

The user provides values for $grKMin$, $grKMax$, $grKTrend$, χ^k and R^n. The variables grK and R^e are initialized to base year data. Equation (M-23) can be inverted to yield the initial level for ΔRoR. And finally, equation (M-22) is used to initialize R^d.

The capital closure specification is governed by setting the `savfFlag`. The valid options are:

1. `capFix`. With this setting the capital account is fixed.

2. `capRFix`. With this setting the capital account is fixed relative to GDP for all but a residual region, which becomes the lender or borrower of last resort.

3. `capFlexGTAP`. With this setting the capital account is flexible and uses the GTAP specification.

4. `capFlexUSAGE`. With this setting the capital account is flexible and uses an allocation mechanism similar in spirit to the domestic allocation mechanism used in the USAGE model.

Price anchors

Equation (M-26) defines the so-called manufactures unit value index, $PMUV$ and it is based on the Fisher index of export prices of selected goods (often manufactures) over selected exporters (often high-income regions). The $QMUV$ indicator is based on the following expression:

---

[82] Further discussion is available in Dixon and Rimmer (2002), pp. 190-193. The version described herein inverts the functional form, but is otherwise identical. The inverted function yields the following, which can be used to initialize ΔRoR:

$$\Delta RoR_{r,t} = \frac{1}{\chi^k_{r,t}}\ln\left[\left(\frac{grKMax_{r,t} - grKTrend_{r,t}}{grKTrend_{r,t} - grKMin_{r,t}}\right)\left(\frac{grK_{r,t} - grKMin_{r,t}}{grKMax_{r,t} - grK_{r,t}}\right)\right]$$

[83] Though R^g is used for all closures it has a different significance across closures. In the USAGE closure, one would anticipate that in the long-run it is 0.

$$QMUV_{tp,tq} = \sum_{r \in RMUV} \sum_{i \in IMUV} \sum_d PWE_{r,i,d,tp} XW^s_{r,i,d,tq}$$

and represents the an export indicator using time dependent prices and quantities.

$$PMUV_t = PMUV_{t-1} \sqrt{\frac{QMUV_{t,t-1}}{QMUV_{t-1,t-1}} \cdot \frac{QMUV_{t,t}}{QMUV_{t-1,t}}} \tag{M-26}$$

The model has a flexible formula for calculating various versions of a consumer price index (CPI). The user can choose the subset of consumer items to be included in any consumption bundle—for example food, energy, other. The CPIs are indexed by 'CPINDX', which define specific consumer bundles. Like most other indices in the model, the CPI indices are derived from a Fisher ideal price expression that is based on the following formula:

$$QCPI_{tp,tq,ndx} = \sum_{k \in \{ndx\}} PC_{r,k,h,tp} XC_{r,k,h,tq}$$

Equation (M-27) defines the various versions of the CPI.

$$CPI_{r,h,ndx,t} = CPI_{r,h,ndx,t-1} \sqrt{\frac{QCPI_{t,t-1,ndx}}{QCPI_{t-1,t-1,ndx}} \cdot \frac{QCPI_{t,t,ndx}}{QCPI_{t-1,t,ndx}}} \tag{M-27}$$

In a similar vein, the model has a factor price index—one per region—that can be used to target the real exchange rate. The regional factor price index is based on the following formula:

$$QFACT_{tp,tq} = \sum_f \sum_a PF_{r,f,a,tp} XF_{r,f,a,tq}$$

Equation (M-28) defines the regional factor price index. It is the Fisher price index of regional factor prices aggregated over all factors and activities.[84]

$$PFACT_{r,t} = PFACT_{r,t-1} \sqrt{\frac{QFACT_{t,t-1}}{QFACT_{t-1,t-1}} \cdot \frac{QFACT_{t,t}}{QFACT_{t-1,t}}} \tag{M-28}$$

Foreign savings flows, S^f, are determined in real terms and are evaluated at the price PW^{sav}, that is set equal to $PMUV$, equation (M-30). The model requires at least one price to be fixed and treated as the numéraire. Equation (M-31) defines the numéraire, $PNUM$. Herein it is set equal to a world GDP price deflator, but it could easily be set to $PMUV$. The left-out equation, as discussed above, is one of the investment/savings balance equations. A variable called *Walras* has been added to the investment/savings balance equation for a single region. It should evaluate to zero.

$$PWGDP_t = \frac{\sum_r GDPMP_t}{\sum_r RGDPMP_t} \tag{M-29}$$

$$PW^{sav} = PMUV \tag{M-30}$$

$$PNUM = PWGDP \tag{M-31}$$

In summary, the standard macro closure has household savings determined by the ELES demand system, government savings are fixed (in real terms) with the direct household tax schedule endogenous, and foreign savings are fixed (with the real exchange rate adjusting to *ex ante* changes in the trade balance. Investment is savings determined. The model numéraire is the export price index of high-income manufactured exports.

---

[84]   We have done some limited testing with fixing the real exchange rate—using the factor price index as a measure of the real exchange rate. If the real exchange rate is fixed, then the capital account must adjust. In this case, the set `fixER` must be set to `'YES'` for the region with the fixed exchange rate (under the `'capFix'` capital account closure). In this case the region will be excluded from the capital account closure equation and the capital account adjustments will be assumed by the residual region. This closure should be used with the appropriate caution.

3.9.3 Targeting prices

It is often the case, particularly in developing a baseline, that it is desirable (or required) to target price trends for key commodities, for example crude oil. The first step is to define what is meant by a 'world' price. The current version of the model defines the 'world' price to be the average producer price—defined using a Fisher price index. The key reason is that typically the commodities to be targeted have a sector-specific resource base that can be adjusted to target an exogenous price.[85] Equation (M-32) defines the world price for activity a. It is based on the following indicator:

$$QX_{a,tp,tq} = \sum_r PX_{r,a,tp} XP_{r,a,tq}$$

$$PW_{a,t} = PX_{a,t-1} \sqrt{\frac{QX_{a,t,t-1}}{QX_{a,t-1,t-1}} \cdot \frac{QX_{a,t,t}}{QX_{a,t-1,t}}} \tag{M-32}$$

To target a price, the variable PW needs to exogenized and a counterpart needs to be endogenized. The most logical choice is to endogenize the shifter on the natural resource supply curve, $\chi_a^{w,nrs}$.

3.9.4 Regional and global welfare

[NEW 20-Jan-2019] This section will be revised in the future.

Equivalent variation

Equivalent variation is defined to be the level of income needed to attain utility level u at base year prices. The formulation depends on the specific utility function—and in many cases will need to be derived using a solver given implicit functional forms. For the CDE utility function, equivalent variation, EV, is defined implicitly by the following expression:

$$\sum_k \alpha_{r,k,h}^h b_{r,k,h}^h u_{r,h}^{e_{r,k,h}^h b_{r,k,h}^h} \left(\frac{PC_{r,k,h,0}}{EV_{r,h}/Pop_r} \right)^{b_{r,k,h}^h} = 1 \tag{M-33}$$

For the AIDADS class of utility functions, which includes the LES and Cobb-Douglas, the expression for equivalent variation is provided by the following:

$$\ln \left(\frac{EV_{r,h}}{Pop_r} - \sum_k PC_{r,k,h,0} \gamma_{r,k,h} \right) = 1 + \ln \left(A_{r,h}^{ad} \right) + u_{r,h} + \sum_k \mu_{r,k,h}^c \ln \left(\frac{PC_{r,k,h,0}}{\mu_{r,k,h}^c} \right) \tag{M-34}$$

Evaluation of EV is incorporated in the model code and is a model output.

Utility can also be derived from other final demand expenditures. These all rely on some form of a CES expenditure function, for which the expenditure function is relatively straightforward. Equation (M-35) defines the equivalent variation for each of the other final demand expenditure components—the real value of current expenditures, times the initial aggregate price. Equation (M-36) defines the expenditure function linked to domestic savings. In the absence of capital flows, it will be equal to the expenditure function of investment. The GTAP model (McDougall (2003) and Corong et al. (2017)) defines total equivalent variation as the sum of equivalent variation of households, public expenditures and savings. In the GREEN Model (Burniaux et al. (1992)), total equivalent variation is equated to private and public expenditures in each year. These are discounted to the base year to evaluate the net discounted change in equivalent variation and used as one metric to summarize the net impacts of a policy change. In GREEN, savings (or investment) are not included in the definition of welfare, as the welfare from savings are derived in future periods (as increments to private or public consumption). This does not take into consideration the utility derived from savings in periods beyond the model's time horizon.

$$EVF_{r,fdc} = PFD_{r,fdc,0} \frac{YFD_{r,fdc,t}}{PFD_{r,fdc,t}} \tag{M-35}$$

$$EVS_r = PSAVE_{r,0} \frac{\sum_h S_{r,h,t}^h + S_{r,t}^g}{PSAVE_{r,t}} \tag{M-36}$$

The price of savings formula is derived from an adaptation of the price of savings from the GTAP model (van der Mensbrugghe (2018)). In the absence of capital flows, the price of savings is equal to the price of investment. Equation (M-37) adjusts the definition to take into account foreign savings.

---

[85] With a diagonal make matrix, targeting the producer price and the commodity price would be identical.

$$PSAVE_{r,t} = \chi_t^s \frac{PSAVE_{r,t-1}}{PFD_{r,inv,t-1}} PFD_{r,inv,t} \tag{M-37}$$

The adjustment factor χ^s is updated each period and is given by the following expression:

$$\chi_t^s = \frac{\sum_r \varphi_{r,t}^i \frac{PFD_{r,inv,t}}{PFD_{r,inv,t-1}}}{\sum_r \varphi_{r,t}^s \frac{PSAVE_{r,t}}{PSAVE_{r,t-1}}}$$

where φ^i represents the regional share in global net investment:

$$\varphi_{r,t}^i = \frac{PFD_{r,inv,t-1} \left(XFD_{r,inv,t-1} - \delta_{r,t-1} KStock_{r,t-1} \right)}{\sum_s PFD_{s,inv,t-1} \left(XFD_{s,inv,t-1} - \delta_{s,t-1} KStock_{s,t-1} \right)}$$

and φ^s represents the regional share in global savings:

$$\varphi_{r,t}^s = \frac{\sum_h S_{r,h,t-1}^h - S_{r,t}^g}{\sum_s \left(\sum_h S_{s,h,t-1}^h - S_{s,t}^g \right)}$$

Global welfare function

The model includes an objective function, which for most simulations is redundant as the model is intended to be square with a unique solution. In principle, it would be possible to run the model with more variables than equations and to use an optimizing package to maximize an objective, with all model equations acting as constraints. In the current version, the social welfare function is a CES aggregate of per capita real income, as measured by equivalent variation, that is population weighted and with welfare weights—currently initialized at the value of 1. For a value of $\epsilon^w = 0$ the social welfare function is simply the average global level of real income. The objective function in any given time period is set to the level of the social welfare function. The model also calculates a two additional global social welfare functions: (1) one that takes into account the utility derived from public expenditures, equation (M-39); and (2) one consistent with the GTAP definition of welfare that includes private and public expenditures and domestic savings, equation (M-40).

$$SW = \sum_r \frac{\varphi_r Pop_r}{1 - \epsilon^w} \left(\frac{\sum_h EV_{r,h}}{Pop_r} \right)^{1-\epsilon^w} \bigg/ \sum_r Pop_r \tag{M-38}$$

$$SWT = \sum_r \frac{\varphi_r Pop_r}{1 - \epsilon^w} \left(\frac{\sum_h EV_{r,h} + EVF_{r,gov}}{Pop_r} \right)^{1-\epsilon^w} \bigg/ \sum_r Pop_r \tag{M-39}$$

$$SWT2 = \sum_r \frac{\varphi_r Pop_r}{1 - \epsilon^w} \left(\frac{\sum_h EV_{r,h} + EVF_{r,gov} + EVS_r}{Pop_r} \right)^{1-\epsilon^w} \bigg/ \sum_r Pop_r \tag{M-40}$$

3.10   Emissions

3.10.1   Emissions

The model incorporates a number of greenhouse gas (GHG) emissions and non-greenhouse gas emissions, indexed by em.

The standard database includes the following greenhouse gas emissions: carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and an aggregate emissions bundle of fluorinated gases (F-gases). The input data for carbon is in millions of metric tons of CO_2 and the data for the other GHGs are both in physical units (metric tons) as well as in CO2-equivalent.[86] The model allows for conversion between CO_2e and Ce, i.e. carbon equivalent, depending on the needs of the user.[87]

---

[86]   The conversion from tons to CO2-equivalent uses the standard global warming potential (gwp) coefficients defined by the Intergovernmental Panel on Climate Change (IPCC).

[87]   The `ifCEQ` flag determines the units. If set to 1, the model will use Ce, else CO_2e. The main impact will be on the level of any endogenously determined carbon tax. The factor $(44/12)$ is used to convert from Ce to CO_2e in volume terms. The inverse ratio is used to convert prices and taxes.

The non-greenhouse gases are described in Table 3.5.[88] These emissions have no corresponding 'carbon equivalent' component, though they can impact on climate and are an input to many climate models, for example MAGICC.[89]. They can also have an impact on morbidity and mortality—see for example Chepeliev and van der Mensbrugghe (2017) and OECD (2016).

<div align="center">

Table 3.5: **Non-greenhouse gas emissions**

</div>

Gas	*Description*
BC	Black carbon
CO	Carbon monoxide
NH_3	Ammonia
NMVOC	Non-methane volatile organic compounds
NO_x	Nitrogen oxides
OC	Organic carbon
PM10	Particulate matter 10
PM2.5	Particulate matter 2.5
SO_2	Sulfur dioxide

Emissions are generated by three sources: 1) direct consumption of a commodity; 2) factor-based emissions (e.g. capital, i.e. herds, in the livestock sectors); and 3) output or processed based emissions (e.g. methane from landfills). The bulk of carbon emissions are generated by the combustion of fossil fuels—coal, oil (crude and refined) and natural gas, but the greenhouse gas bundle $XGHG$ does include process-based carbon emissions, see equation (P-4. Other emissions can be a combination of all three sources of emissions. Equation (E-1) reflects the consumption based emissions over all Armington agents (aa) and for all commodities (i). Emissions are based on the consumption level multiplied by the rate of emission per unit of consumption (ρ^{Emi}). The specification allows for both national- and agent-specific sourcing of imports. In the case of the latter, the relevant volume of consumption is XA. In the case of the latter, the relevant consumption volumes are XD and XM and the emission rates are allowed to vary across bundles.[90] Equation (E-2) reflects the factor based emissions for labor, capital, land and natural resources. Finally, equation (E-3) represents the processed based emissions.[91]

$$Emi_{r,em,i,aa} = \begin{cases} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi} XA_{r,i,aa} & \text{if } ArmFlag = 0 \\ \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,d} XD_{r,i,aa} + \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,m} XM_{r,i,aa} & \text{if } ArmFlag \neq 0 \end{cases} \tag{E-1}$$

$$Emi_{r,em,f,a} = \chi_{em}^{Emi} \rho_{r,em,f,a}^{Emi} XF_{r,f,a} \tag{E-2}$$

$$Emi_{r,em,tot,a} = \chi_{em}^{Emi} \rho_{r,em,tot,a}^{Emi} XP_{r,a} \tag{E-3}$$

[11-AUG-2025] Equation (E-4) is the aggregate level of emissions for region r. It is the sum across all potential sources of emissions represented by the indices is and aa. The equation allows for a region specific level of emissions from exogenous sources—that could be used for example to introduce forest-based emissions. [15-Apr-2021] The formula also includes non-combustion based process emissions. Equation (E-5) defines the global level of emissions, the sum of total regional emisssions, and also allows for an exogenous level of emissions that is not region-specific.

$$EmiTot_{r,em} = \sum_{is} \sum_{aa} Emi_{r,em,is,aa} + \sum_{a} prEmi_{r,em,a} + EmiOth_{r,em} \tag{E-4}$$

---

[88] With the release of V10 of the GTAP Data Base, the non-GHG emission database has been recompiled, Chepeliev (2020). There are some differences with the previous version, Chepeliev (2018). The key source database, EDGAR, has combined the previous 2 NMVOC emissions into a single pollutant.

[89] Meinshausen et al. (2011)

[90] The CO_2 emissions database distinguishes between domestic and import emissions, however the non-CO_2 database does not and the emission rates are assumed to be identical.

[91] There is a master set, is, that represents all of the labels of the SAM—including one for the total that is represented by the subset tot.

$$EmiGbl_{em} = \sum_r EmiTot_{r,em} + EmiOthGbl_{em} \tag{E-5}$$

Equation (E-6) represents the total emissions of type emq generated by production of activity a. The first term is the sum of emissions across all 'input' types, which includes commodities (i), production factors (f) and output (tot).The second term represents the activity-based process emissions. Equation (E-7) defines the emissions intensity per unit of output, i.e., total activity-based emissions divided by output. The unit are in tons per dollar of activity (in base year prices). Equation (E-8) defines the total level of emissions for the ETS agents for region r. The first term represents emissions related to activities, and the second term represents emissions related to final demand agents that are mapped to the ETS regime.

$$EmiAct_{r,emq,a} = \sum_{is} \sum_{em \in emq} Emi_{r,em,is,a} + \sum_{em \in emq} prEmi_{r,em,a} \tag{E-6}$$

$$EmiInt_{r,emq,a} = \frac{EmiAct_{r,emq,a}}{EmiAct_{r,a}} \tag{E-7}$$

$$EmiETS_{r,emq,aets} = \sum_{a \in \{aets\}} EmiAct_{r,emq,a} + \sum_{em \in emq} \sum_{fd \in \{aets\}} \sum_{is} Emi_{r,em,is,fd} \tag{E-8}$$

3.10.2 Emission price regimes

[17-MAR-2020] The emission price regimes have been completely revised in the current version of the model. The revisions were necessitated to incorporate coalitions with multiple regimes, such as the European Emission Trading System (ETS). Emission coalitions can now work with multiple regions covering different activities. Examples are provided in the User Guide. One significant change is that the emission tax variable is now agent-specific. This was possible in the old specification as well by using the participation flag, ϕ^{Emi}.

Exogenous emission taxes

The simplest option is to set an emissions tax exogenously and all the required equations have already been described. If the model is being solved in terms of CO_2, a tax of \$x/TCO_2 is implemented as:[92]

```
emiTax.fx(r,"co2",aa,t) = cscale*x ;
```

Emission caps

An emissions cap requires two additional equations—one that calculates the carbon tax for all regions within a coalition of regions (that may be as small as one region), specified over a subset of agents denoted by $aets$ and over a superset of emissions, denoted by emq.

The set emq is composed of one or more emissions and the mapping `mapEM` maps from the model's individual emissions to emq.[93] For example, taxes on carbon emissions alone could be setup by creating a set item `CO2` in emq and the relevant mapping is `mapEQ("CO2","CO2") = yes`. To target all GHG, the setup could be to create a set item `CO2Eq` and the relevant mapping is:

```
mapEQ("CO2Eq", "CO2") = yes ;
mapEQ("CO2Eq", "CH4") = yes ;
mapEQ("CO2Eq", "N2O") = yes ;
mapEQ("CO2Eq", "FGAS") = yes ;
```

The tax specific to coalition rq for agents mapped to $aets$ for the emission combination emq will be generated by the emissions constraint relationship given in equation (E-9). The coalition rq is a subset of ra that represents a user-defined set of aggregate regions that can be used by the model (and the output routines). It normally includes all of

---

[92] The parameter ϕ^{Emi} can be used to control the incidence of the tax, or the user can limit its application by subsetting aa.

[93] Aggregation of emissions in this context assumes that they are somehow defined as 'equivalent'. All GHGs in the model are specified in terms of global warming potential (GWP). The GWP factor converts the relevant emissions in physical units, for example millions of tons, into their GWP equivalent, normally denoted as CO2eq, as GWP is measured by the effect of the emissions on temperature relative to carbon dioxide emissions.

the separate regions of the model—so that each modeled region can be its own coalition—plus a number of additional aggregate regions to which the individual regions are mapped. Coalitions are then setup by defining the subset ra to which the regional mappings will already have been specified. On the left-hand side is the (exogenous) cap, that is defined for each active pair rq and $aets$. A world cap, for example, would map each region to a world coalition (for example 'WLD') and include all agents (for example 'ALL'). On the right-hand side is the level of emissions for each active rq and $aets$ pair summed over all possible sources of emissions—also including process emissions (15-Apr-2021). The emissions cap equation implicitly determines the coalition-wide emissions tax, $\tau^{EmiQ}_{rq,em,aets}$.

Equation (E-10) maps the coalition-wide tax to the emissions tax of each individual region that is part of the rq coalition and for each agent aa mapped to the aggregation of agents denoted by $aets$. In the absence of quotas, the emission tax revenues collected in each region fully accrue to the regional government.[94]

$$Emi^{Cap}_{rq,emq,aets} \geq \sum_{r \in \{rq\}} \left[\sum_{em \in \{emq\}} \left(\sum_{aa \in \{aets\}} \sum_{is} Emi_{r,em,is,aa} + \sum_{a \in \{aets\}} prEmi_{r,em,a} \right) \right] \tag{E-9}$$

$$\tau^{Emi}_{r,em,aa} = \tau^{EmiQ}_{rq,emq,aets} \qquad \text{if } r \in \{rq\}, \ em \in \{emq\} \text{ and } aa \in \{aets\} \tag{E-10}$$

The following code snippet shows one way how to implement the European ETS regime. The set of agents is divided into two: (a) those that participate in the ETS—mostly large energy intensive sectors such as power and metals—though in the example, transport services are included; and (b) all other sectors and final demand agents. There is trading among the ETS sectors, so a European-wide coalition is defined ('EUR'). The trading regime is setup and there will be a coalition-wide tax that will be uniform across ETS sectors and across countries/regions within the coalition. Outside of the ETS sectors, each country in Europe has an emissions cap on non-ETS activities. The example assumes no trading and thus the carbon tax for the non-ETS sectors will diverge across countries within Europe.

Listing 3.1: **Example of ETS regime**

```
1   *  Define modeled regions
2   set r / "DEU" "Germany",   "FRA" "FRANCE",      "ITA" "Italy",
3         "XEU" "Other EU", "XOE" "Other OECD", "ROW" "Rest of the World" / ;
4   *  Define potential coalitions
5   set ra / "EUR", "NEU", "WLD", "DEU", "FRA", "ITA", "XEU" / ;
6   *  Map the coalitions
7   set mapr(ra,r) / "EUR".("DEU", "FRA", "ITA", "XEU"),
8                    "NEU".("XOE", "ROW"),
9                    "FRA"."FRA", "DEU"."DEU",
10                   "ITA"."ITA", "XEU"."XEU" / ;
11  mapr("WLD",r) = yes ;
12  *  Define model agents
13  set aa / "AGR" "Agriculture",            "NMN" "Energy intensive manufacturing",
14        "XMN" "Other manufacturing",      "TRP" "Transport services",
15        "XSV" "Other goods and services", "HHD" "Households",
16        "GOV" "Government",               "INV" "Investment" / ;
17  *  Define aets regimes
18  set aets / "ETS" "ETS agents", "NONETS" "Non-ETS agents", "ALL" "All agents" / ;
19  * Map agents to aets regimes
20  set mapets(ra, aets, aa) / "EUR"."ETS".("NMN", "TRP") / ;
21  mapets("EUR","NONETS",aa)$(not mapets("EUR","ETS",aa)) = yes ;
22  mapets("EUR","ALL",aa) = yes ;
23  *  Define emission aggregation
24  set emq / "CO2" / ;
25  set mapEM("CO2","CO2") = yes ;

27  *  Setup the "ETS" as a EU-wide trading regime
28  rq("EUR") = yes ;

30  *  Set the CAP flags for the coalition and the regions in the coalition
31  ifEmiCap("EUR","CO2","ETS") = yes ;
32  ifEmiRCap(r,"CO2",aa)$(mapr("EUR",r) and mapets("EUR","ETS",aa)) = yes ;

34  *  Set the exogenous cap (calculated elsewhere)
35  emiCap.fx("EUR","CO2","ETS",tsim) = EmiCurETS(tsim) ;
36  *  Endogenize the taxes: coalition and the regions in the coalition
```

---

[94]   01-SEP-2025. The *aets* map is now coalition specific.

```
37  emiCTAX.lo("EUR","CO2","ETS",tsim) = 0 ;
38  emiCTAX.up("EUR","CO2","ETS",tsim) = +inf ;
39  emiTax.lo(r,"CO2",aa,tsim)$(mapr("EUR",r) and mapets("EUR","ETS",aa)) = -inf ;
40  emiTax.up(r,"CO2",aa,tsim)$(mapr("EUR",r) and mapets("EUR","ETS",aa)) = +inf ;

42  *  Setup the cap constraints for each region/country for the non-ETS sectors
43  *  Assume no trading
44  *  Activate 'rq' for each country/region in "EUR"
45  loop((r,ra)$(mapr("EUR",r) and sameas(r,ra)),
46      rq(ra) = yes ;
47  ) ;
48  *  Initialize the cap flags
49  loop((r,ra)$(sameas(r,ra) and mapr("EUR",r)),
50      ifEmiCap(ra,"CO2","NONETS")$mapr("EUR",r)  = yes ;
51      ifEmiRCap(r,"CO2",aa)$(mapr("EUR",r) and mapets("EUR","NonETS",aa)) = yes ;
52  ) ;
53  *  Set the exogenous cap (calculated elsewhere)
54  loop((r,ra)$(sameas(r,ra) and mapr("EUR",r)),
55      emiCap.fx(ra,"CO2","NONETS",tsim)$mapr("EUR",r) = EmiCurNETS(r,tsim) ;
56  ) ;
57  *  Endogenize the relevant taxes
58  loop(mapr("EUR",r),
59      emiCTax.lo(rq,"CO2","NONETS",tsim)$sameas(rq,r)  =    0 ;
60      emiCTax.up(rq,"CO2","NONETS",tsim)$sameas(rq,r)  =  inf ;
61      emiTax.lo(r,"CO2",aa,tsim)$(mapets("NONETS",aa)) = -inf ;
62      emiTax.up(r,"CO2",aa,tsim)$(mapets("NONETS",aa)) = +inf ;
63  ) ;
```

The implementation of caps is somewhat more complex in order to allow for comparability across policy simulations. For example, one may wish to compare the cost of the ETS regime with the optimal regime, i.e. where there is a single carbon tax that is uniform across all agents and all countries within the coalition. In the case of the latter, one could establish a target—say a 40 percent decline in coalition emissions by the year 2030 relative to 2005 levels. The model could be used to assess the required uniform carbon tax to reach that target in optimal fashion. A subsequent policy introduces the ETS with its agent segmentation and trading only across the ETS sectors. If the ETS cap is binding, one can only guess what the non-ETS caps would need to be to achieve the same target as in the optimal case. The model allows for an adjustment factor on the non-ETS caps so that the total decline in emissions across policy regimes are identical.

For all trade regimes with a hard cap, equation (E-9) continues to hold. For trade regimes with a 'soft' cap, i.e. where we allow for adjustments to achieve some emission target, we add the uniform adjustment target. Equation (E-9) is then written as:

$$\chi_{rqt,emq,aetstgt}^{Cap} Emi_{rq,emq,aets}^{Cap} \geq \sum_{r \in \{rq\}} \left[\sum_{em \in \{emq\}} \left(\sum_{aa \in \{aets\}} \sum_{is} Emi_{r,em,is,aa} + \sum_{a \in \{aets\}} prEmi_{r,em,a} \right) \right] \quad \text{(E-11)}$$

where the index rqt targets the regions that are part of the overall target region and $aetstgt$ targets the appropriate sector. The adjustment factor will be determined by equation (E-12) that defines the overall cap for the coalition region.

$$Emi_{rqt,emq,aetstgt}^{Cap} = \sum_{r \in \{rq\}} \left[\sum_{em \in \{emq\}} \left(\sum_{aa \in \{aetstgt\}} \sum_{is} Emi_{r,em,is,aa} + \sum_{a \in \{aetstgt\}} prEmi_{r,em,a} \right) \right] \quad \text{(E-12)}$$

The following code snippet shows how to extend the example above to include an overall cap on EU emissions. In this case, the cap on the non-ETS emissions is a 'soft' cap and the adjustment factor is endogenous and will adjust uniformly the emissions cap on non-ETS agents to achieve a given overall target on EU emissions.

Listing 3.2: **Example of ETS regime with a hard overall cap**

```
1  *  Target total EU emissions
2  *  Set the overall cap (detemined elsewhere)
3  emiCap.fx("EUR","CO2","ALL",tsim) = emiRed0(tsim) ;
4  *  Set the flag for activating the target equation
```

```
5   ifRQCap("EUR","CO2","ALL") = yes ;
6   *  Endogenize the adjustment factor
7   chiCap.lo("EUR","CO2","ALL",tsim) = -inf ;
8   chiCap.up("EUR","CO2","ALL",tsim) = +inf ;
9   *  Initialize the country/region specific flags
10  loop((r,ra)$(mapr("EUR",r) and sameas(r,ra)),
11     ifXCAP(ra,"CO2","NONETS") = no ;
12     maprq("EUR",ra) = yes ;
13  ) ;
```

Emissions trading is setup by specifying coalitions and allowing for a uniform carbon price across the regions in the coalition (over a set of agents, potentially all). In the default specification all carbon tax revenues accrue to the source region of emissions. One could instead allocate initial emission quotas across regions in a coalition.[95] The sum of the quotas within a coalition must add up to the emissions cap for the coalition. Equation (E-13) determines the value of emissions trade for each region within a coalition. It is equal to the level of the coalition-wide emission tax times the difference between the quota allocation and actual emissions. If the difference is positive, the value of trade will be positive. It is assumed that the value of the trading rights goes directly to the government (with the counterbalancing account in the omitted balance of payments constraint), see equation (M-7).

$$EmiQY_{r,emq,aets} = \tau^{Emi}_{r,em,aets}\left(EmiQ_{r,emq,aets} - EmiTot_{r,emq,aets}\right) \tag{E-13}$$

The code snippet below shows how to set up a cap and trade regime at the global level. It is assumed that developing countries are allocated quotas equivalent to their emission level from the previous period. The residual quotas, for developed countries, are allocated based on their share of emissions of the previous period. Note that in lines 26-31 where the quotas are set for the regions in the high-income coalition the quotas for the developing regions is subtracted from the overall cap using the lagged value of emissions and not the variable `emiQuota`. This is because the order of the loop over `ra` is undetermined and it is possible the quotas for the developed regions will be set before the quotas for the developing regions.

Listing 3.3: **Example of a cap and trade regime**

```
1   loop((ra,em,aets)$(sameas(ra,"WLD") and sameas(em,"CO2") and sameas(aets,"ALL")),
2   *  Set up the global trading regime covering all regions and all agents
3      rq(ra)                = yes ;
4      ifEmiCap(ra,em,aets) = 1 ;
5      ifEmiRCap(r,em,aa)$(mapr(ra,r) and mapets(aets,aa)) = yes ;
6   *  Cap is a reduction of 20% wrt to previous period's emissions
7      emiCap.fx(ra,em,aets,tsim) = 0.8*sum((r,aa,is)$(mapr(ra,r) and mapets(aets,aa)),
8         emi.l(r,em,is,aa,tsim-1)*emi0(r,em,is,aa)) ;

10     emiCTax.lo(ra, em, aets, tsim) = -inf ;
11     emiCTax.up(ra, em, aets, tsim) = +inf ;
12     emiTax.lo(r, em, aa, tsim)     = -inf ;
13     emiTax.up(r, em, aa, tsim)     = +inf ;

15  *  Setup the quota
16     ifEmiRQuota(r,em,aets)$mapr(ra,r) = yes ;
17     loop(ra,
18        if(sameas(ra, "lmy"),
19  *        Quotas for developing countries is that same as prior emissions
20           emiQuota.fx(r,em,aets,tsim)$mapr(ra,r)
21              = sum((aa,is)$mapets(aets,aa), emi.l(r,em,is,aa,tsim-1)*emi0(r,em,is,aa) ;
22        elseif(sameas(ra,"hic")),
23  *        Quotas for developed countries is residual
24  *           apportioned according to prior emission shares
25           emiQuota.fx(r,em,aets,tsim)$mapr(ra,r)
26              = (emiCap.l("WLD", em, aets, tsim)
27              -  sum(rp$mapr("lmy",rp),sum((aa,is)$mapets(aets,aa),
28                    emi.l(rp,em,is,aa,tsim-1)*emi0(rp,em,is,aa)))
29              * sum((aa,is)$mapets(aets,aa), emi.l(r,em,is,aa,tsim-1)*emi0(r,em,is,aa)
30              / sum(rp$mapr(ra,rp), sum((aa,is)$mapets(aets,aa),
31                 emi.l(rp,em,is,aa,tsim-1)*emi0(rp,em,is,aa)) ;
32        ) ;
```

---

[95] This implies that in the default specification the implicit quotas are exactly the actual level of emissions.

```
33        ) ;
34        emiQuotaY.lo(r, em, aets, tsim) = -inf ;
35        emiQuotaY.up(r, em, aets, tsim) = +inf ;
36    ) ;
```

3.11 Special modules

This section describes some additional modules that have been implemented for specific scenarios, though not needed for standard simulations.

3.11.1 Targeted production subsidies

The idea behind this simulation is to promote certain activities by introducing a subsidy to production. The target is a level of total subsidies relative to GDP, for example 1 percent, and the model determines a subsidy level consistent with this target. For this, we first introduce a production subsidy, τ^s, which is uniform over a set of targeted activities, at, which is a subset of a. This requires changing all equations that depend on the production tax, to take into account the subsidy. Equation (S-3) includes the new subsidy, which is assumed to be positive. The coefficient φ^s takes the value 1 for the subset at, i.e. all targeted sectors, and the value 0 for all other sectors.[96] The second equation that needs modification is the government revenue equation, (Y-16).

$$PP_{r,a,i} = \left(1 + \tau^p_{r,a,i} - \varphi^s_a \tau^s_r\right) P_{r,a,i} \tag{S-3}$$

$$YGOV_{r,ptx} = \sum_a \left[\sum_i \left(\tau^p_{r,a,i} - \varphi^s_a \tau^s_r\right) P_{r,a,i} X_{r,a,i} + \sum_v \tau^{uc}_{r,a,v} UC_{r,a,v} XPv_{r,a,v}\right] \tag{Y-16}$$

Beyond the modifications to the equations that include the production tax, we introduce two new equations. The first, equation (Z-1), calculates the total value of the cost of the subsidy in the targeted sectors, $PSubsY$. Equation (Z-2) defines the share of the total cost of the subsidy relative to (nominal) GDP. In the policy scenario, the share can be fixed, for example 1 percent. In this case, the subsidy rate, τ^s_r, is endogenous and is evaluated to line up with the subsidy cost as a share of GDP. If this is part of the baseline, the calibrated subsidy rate becomes exogenous in subsequent scenarios.

$$PSubsY_r = \sum_{at} \sum_i \tau^s_r P_{r,i,at} X_{r,i,at} \tag{Z-1}$$

$$PSubsYShrGDP_r = \frac{PSubsY_r}{GDPMP_r} \tag{Z-2}$$

3.11.2 Sector-specific emission rights

Certain carbon price policy scenarios allow for full or partial compensation of the sector-specific costs of carbon pricing, i.e., sectors may be given an allocation of carbon permits, which they are free to use and/or sell on carbon markets. The allocation permits can either be exogenous, e.g. some percentage of past or baseline emissions, or endogenous, e.g. a share of actual emissions. Equation (Z-3) determines the revenues generated by by the permit allocation. It is equal to the carbon price level times the amount of the allocation. If the allocation is exogenous, the relevant variable is $emiRebateX$ and normally χ^{rbt} would be set to 1. If the allocation is based on actual emissions, the relevant variable is Emi, where χ^{rbt} is the (exogenous) share of actual emissions. The allocation rebate is assumed to shift the aggregate output price in each activity as provided in a revised equation (P-1). All activities, therefore, perceive the marginal cost of carbon pricing, but can be partially or fully compensated for these costs with a lump sum payment. The allocation rights reduce the emission revenues accruing to the government and thus a change to equation (Y-22).

$$EmiRebate_{r,a} = \begin{cases} \sum_{em} \sum_i \tau^{Emi}_{r,em,a} \chi^{rbt}_{r,em,i,a} emiRebateX_{r,em,i,a} & \text{if } emiRebateFlag_{r,a} = \texttt{Exogenous} \\ \sum_{em} \sum_i \tau^{Emi}_{r,em,a} \chi^{rbt}_{r,em,i,a} Emi_{r,em,i,a} & \text{if } emiRebateFlag_{r,a} = \texttt{Endogenous} \end{cases} \tag{Z-3}$$

---

[96]   In the model code, the subsidy rate is conditioned by the subset `asubs` that represents the subset of targeted activities.

$$PX_{r,a}XP_{r,a} = \sum_v PXv_{r,a,v}XPv_{r,a,v} - EmiRebate_{r,a} \tag{P-1}$$

$$YGOV_{r,ctx} = \begin{cases} \sum_{em}\sum_i\sum_{aa} \chi_{em}^{Emi}\rho_{r,em,i,aa}^{Emi}\phi_{r,em,i,aa}^{Emi}\tau_{r,em,aa}^{Emi}XA_{r,i,aa} \\ \quad - \sum_a EmiRebate_{r,a} & \text{if } ArmFlag = 0 \\[2ex] \sum_{em}\sum_i\sum_{aa} \chi_{em}^{Emi}\rho_{r,em,i,aa}^{Emi,d}\phi_{r,em,i,aa}^{Emi}\tau_{r,em,aa}^{Emi}XD_{r,i,aa} \\ \quad + \chi_{em}^{Emi}\rho_{r,em,i,aa}^{Emi,m}\phi_{r,em,i,aa}^{Emi}\tau_{r,em,aa}^{Emi}XM_{r,i,aa} \\ \quad - \sum_a EmiRebate_{r,a} & \text{if } ArmFlag \neq 0 \end{cases} \tag{Y-22}$$

Listing 3.4 shows how to implement the emission rights for both an endogenous and and exogenous rebate. In the case of the former the rights are set to 80% of actual emissions. In the case of the latter, emission rights are set to 80% of baseline emissions (which could potentially exceed actual emissions). The allocation rights are limited to all regions mapped to `EUR` and sectors eligible for the rebate as determined by the `etsRebate` mapping, and starting in year 2021. The rebate flag (`emiRebateFlag`) is set to `YES` in both cases. In the case of an endogenous emission right level, the exogenous rebate flag (`EmiRebateExog`) is set to `NO`. The rebate variable, *emiRebate*, is made endogenous in both cases.

Listing 3.4: **Implementing emission rights**

```
1   loop(tsim$(years(tsim) ge 2021),

3   *   Implement the rebates

5       if(1,
6           loop((r,a)$(mapr("EUR",r) and etsRebate(a)),
7               emiRebateFlag(r,a,tsim) = yes ;
8               emiRebateExog(r,a,t)    = no ;
9               chiRebate(r,"CO2",i,a,tsim)  = 0.8 ;
10              emiRebate.lo(r,a,tsim) = -inf ;
11              emiRebate.up(r,a,tsim) = +inf ;
12          ) ;
13      else
14          loop((r,a)$(mapr("EUR",r) and etsRebate(a)),
15              emiRebateFlag(r,a,tsim) = yes ;
16              emiRebateExog(r,a,t)    = yes ;
17              chiRebate(r,"CO2",i,a,tsim)  = 1 ;
18              emiRebateX(r,"CO2",i,a,tsim) = 0.8*emibau0(r,i,a)*emibau(r,i,a,tsim) ;
19              emiRebate.lo(r,a,tsim) = -inf ;
20              emiRebate.up(r,a,tsim) = +inf ;
21          ) ;
22      ) ;
23  ) ;
```

3.11.3 Well-head based carbon taxes

An alternative to end-user carbon taxes, i.e. imposing the tax at the point of combustion, is to impose the tax at the point of extraction—the coal mine, the oil or gas well. We implement this by evaluating the standard end-user based carbon tax, but then rebate the collected revenues to the producing countries based on their production share of global extraction.

Equation (Z-4) defines the share of output for activity a in region r as a share of global production.[97] This equation can be used in the standard model. Equation (Z-5) is similar to the carbon tax revenue equation, (Y-22), though the revenues from consumption are mapped to the fossil fuel source. Coal and gas are mapped respectively to the coal and gas extractive industries. In the case of oil, all carbon revenues linked to oil and refined oil combustion are mapped back to the oil extraction industry.[98] Equation (Z-6) determines the rebated revenues to the source producers. It is equal to the global amount of collected carbon tax revenues, which is allocated to the producers

---

[97] Output is evaluated at base year prices.
[98] N.B. Most of oil will be combusted in the form of refined oil.

using their share of global extraction, less the revenues collected from domestic carbon tax revenues. Summing the recycled revenues over all regions should sum to zero. Equations (Z-5) and (Z-6) are only active if the flag `ifCO2Tax` is activated. The carbon revenue rebate is only active if the `ifRecycle` flag is active. The recycled revenues accrue to government accounts, but with the default closure these revenues are rebated back to the representative household. Equation (M-7) reprises the government closure equation and includes the recycled revenues, subject to the `ifRecycle` flag being activated.

$$\varphi_{r,a}^p = \frac{PX_{r,a,0} XP_{r,a}}{\sum_s PX_{s,a,0} XP_{s,a}} \tag{Z-4}$$

$$EmiTaxRevFF_{r,affl} = \begin{cases} \displaystyle\sum_{em=\text{CO}_2} \sum_{i \in affl} \sum_{aa} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} XA_{r,i,aa} \\ \qquad\qquad \text{if } ArmFlag = 0 \\[2ex] \displaystyle\sum_{em=\text{CO}_2} \sum_{i \in affl} \sum_{aa} \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,d} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} XD_{r,i,aa} \\ \qquad + \chi_{em}^{Emi} \rho_{r,em,i,aa}^{Emi,m} \phi_{r,em,i,aa}^{Emi} \tau_{r,em,aa}^{Emi} XM_{r,i,aa} \\ \qquad\qquad \text{if } ArmFlag \neq 0 \end{cases} \tag{Z-5}$$

$$cTaxRecycle_r = \sum_{affl} \varphi_{r,affl}^p \sum_s EmiTaxRevFF_{r,affl} - YGOV_{r,ctx} \tag{Z-6}$$

$$\begin{aligned} S_r^g &= \sum_{gy} YGOV_{r,gy} + \sum_{em} QuotaY_{r,em}^{Emi} + cTaxRecycle_r + \sum_s \chi_{s,r}^{gNTM} ntmY_s \\ &+ \quad ODAIn_r - ODAOut_r - \sum_{fd \in Gov} YFD_{r,fd} \end{aligned} \tag{M-7}$$

3.11.4 Controlling preferences in the power bundle

[17-Mar-2020] Two modifications have been added to the power bundle. The first is an exogenous preference shifter, λ^{pow}. This is typically adjusted in the baseline or reference scenario to shift preferences to one or more power bundles, for example the renewable bundle.

The second provides a more direct control over preferences using so-called *phantom taxes*.[99] The implementation is fairly straightforward. The price of each power bundle is adjusted by a price wedge, i.e. the phantom tax, τ^{fpb}. The power bundles are split into two sets, those whose level is under control, i.e. fixed, and all others. The first set is defined over *IPHPB*. By default, the set is empty and the phantom taxes are all set to zero. When under direct control, demand for the relevant bundles in the set *IPHPB* is fixed, i.e. exogenous, and equation (Z-7) is used to determine the phantom tax for each fixed bundle. Equation (Z-10) determines total revenues generated by the phantom taxes. When phantom taxes are implemented, total revenues are exogenized and set to 0, i.e. the net fiscal implication of the phantom taxes is neutralized. In essence, this equation determines the variable χ^{fpb}, which is the uniform phantom tax shifter on all of the non-controlled power bundles. Equation (Z-11) sets the actual phantom tax to the uniform shifter for all non-controlled power bundles.

To implement phantom taxes, the user must specify the set *IPHPB* (set `iphpb` in the GAMS code) and initialize the `phTaxpbFlag` flag to 'yes' (for each region being targeted). The relevant phantom taxes need to be endogenized as well as the variable χ^{fpb} (`chiphpb` in the GAMS code). Demand for the relevant power bundles needs to be exogenized. Finally, the phantom tax revenues need to be exogenized and set to 0. An example is provided in the user guide.

$$XPB_{r,pb,ely} = \alpha_{r,pb,ely}^{pb} \left(\frac{PPOWN_{r,ely}}{\left(1 + \tau_{r,pb,ely}^{fpb}\right) \lambda_{r,pb,ely}^{pow} PPB_{r,pb,ely}} \right)^{\sigma_{r,ely}^{pow}} XPOW_{r,ely} \tag{Z-7}$$

$$PPOWN_{r,ely} = \left[\sum_{pb} \alpha_{r,pb,ely}^{pb} \left(\left(1 + \tau_{r,pb,ely}^{fpb}\right) \lambda_{r,pb,ely}^{pow} PPB_{r,pb,ely} \right)^{-\sigma_{r,ely}^{pow}} \right]^{-1/\sigma_{r,ely}^{pow}} \tag{Z-8}$$

$$PPOW_{r,ely} XPOW_{r,ely} = \sum_{pb} \left(1 + \tau_{r,pb,ely}^{fpb}\right) PPB_{r,pb,ely} XPB_{r,pb,ely} \tag{Z-9}$$

---

[99] Dixon et al. (2019) use phantom taxes to adjust capital allocation in a modified and dynamic version of the GTAP model.

$$PHTaxY_r = \sum_{pb} \sum_{ely} \tau_{r,pb,ely}^{fpb} PPB_{r,pb,ely} XPB_{r,pb,ely} \tag{Z-10}$$

$$\tau_{r,pb,ely}^{fpb} = \chi_r^{fpb} \qquad pb \notin IPHPB \tag{Z-11}$$

3.11.5   Controlling consumer demand preferences

[17-Mar-2020] Similar to the power bundle, phantom taxes have been added to control consumer preferences. The control is done at the i commodity level, not the k commodity level. It has been setup to work with the standard consumer demand module, not the demand module that includes waste. Equation (Z-12) determines the final consumer price of goods and incorporates the so-called phantom tax, τ^{fhh}.[100] Equation (Z-13) determines the revenues generated by the phantom taxes. The final equation, equation (Z-14) determines the uniform shifter, χ^{fhh}, for the phantom tax on the non-targeted commodities. Note that it is formulated somewhat differently from the power bundle phantom taxes. There are two sets of phantom taxes, τ^{fhh} and τ_0^{fhh}. Equation (Z-14) sets the phantom tax to τ_0^{fhh} for the targeted commodities, the latter determined by fixing demand for the relevant commodity. For the non-targeted commodities, τ_0^{fhh} is fixed (by default to -0.01) and the uniform shifter, χ^{fhh}, is a multiplicative factor with respect to τ_0^{fhh}. It is determined by exogenizing total revenues from the phantom taxes, typically set to 0.

Implementation requires setting a regional flag, `phhTaxFlag` to 'yes'. A flag for the targeted commodities, `iphh`, also needs to be set to 'yes'. The phantom tax, `phhTax`, needs to be endogenized for *all* commodities.[101] The uniform shifter, `chiPhh`, needs to be endogenized. The target(s) for demand, `xa(r,i,h)`, needs to be exogenized and the corresponding other phantom tax, `phhTax0`, needs to be endogenous (which is the case by default). The same phantom tax needs to be exogenous for the non-targeted commodities and initialized, for example -0.01. Revenues from the phantom taxes need to be exogenized (which determines the uniform shifter) and normally set to 0. An example is provided in the user guide.

$$PA_{r,i,h}^h = PA_{r,i,h} \left(1 + \tau_{r,i}^{fhh}\right) \tag{Z-12}$$

$$PHHTaxY_r = \sum_i \sum_h \tau_{r,i}^{fhh} PA_{r,i,h} XA_{r,i,h} \tag{Z-13}$$

$$\tau_{r,i}^{fhh} = \begin{cases} \tau_{r,i,0}^{fhh} & \text{if } i \in IPHHH_{r,i} \\ \chi^{fhh} \tau_{r,i,0}^{fhh} & \text{if } i \notin IPHHH_{r,i} \end{cases} \tag{Z-14}$$

3.11.6   Nutrition

The GTAP nutrition database has a number of indicators derived from the FAO's Food Balance Sheet database (Chepeliev (2022)). Table 3.6 summarizes the core indicators, which will be indexed by *sua*, or supply utilization account. Another key index (u) is the use category. This is composed of all of the production activities, final demand use and a *loss* category. The database also tracks the source of supply, which is the union of the set of all regions (s) and the domestic economy using the label 'LCL'. The union will be designated *srcr*. The nutrition accounts are given by the model parameter $NUTR$, which has five dimensions: (1) region (r); (2) commodity (i); (3) use category (u); (4) supply utilization account (sua); and (5) source region ($srcr$).[102]

---

[100]   The module would have to be re-configured somewhat to allow for household specific shifters. The current code assumes that there is a single representative household per region.

[101]   Conditional on initial consumer demand being greater than zero.

[102]   The ordering of the indices is somewhat different in the input database.

Table 3.6: **Supply utilization accounts in the nutrition database**

Element	Description
FEED_KT	Feed demand in kilotons
SEED_KT	Seed demand in kilotons
FOOD_KT	Food demand in kilotons
OTHU_KT	Other uses in kilotons
LOSS_KT	Losses in kilotons
FOOD_TC	Food demand in total calories
FATS_KT	Fats in kilotons
PROT_KT	Proteins in kilotons
CARB_KT	Carbohydrates in kilotons

The nutrition accounts are currently updated after a solve statement as there is currently no interaction between nutrition and the model equations.[103] Equations (Z-15) and (Z-16) describe the update formulas for respectively domestically sourced products and imported products. Demand for all *sua* accounts that are domestically sourced are assumed to change at the same rate as domestic demand for the relevant commodity. Import demand, by source, similarly changes at the rate of total import demand with an adjustment by source determined by the change in the bilateral trade flow. These two equations hold for all uses—production activities and final demand.

$$NUTR_{r,i,u,sua,lcl,t} = NUTR_{r,i,u,sua,lcl,t-1} \frac{XD_{r,i,aa,t}}{XD_{r,i,aa,t-1}} \qquad \text{for } u = aa \tag{Z-15}$$

$$
\begin{aligned}
NUTR_{d,i,u,sua,s,t} &= NUTR_{d,i,u,sua,s,t-1} \\
&\times \left[\frac{XM_{d,i,aa,t}}{XM_{d,i,aa,t-1}} + \left(\frac{XW_{s,i,d,t}}{XW_{s,i,d,t-1}} - \sum_r s^m_{r,i,d} \frac{XW_{r,i,d,t}}{XW_{r,i,d,t-1}} \right) \right] \qquad \text{for } u = aa
\end{aligned}
\tag{Z-16}
$$

The updating of the loss accounts use aggregate indicators. Equations (Z-17) and (Z-18) describe the updating of the *loss* accounts. In the case of domestically sourced losses, they change at the same rate as total domestic demand for domestic production. In the case of imported losses by source, the rate of change is identified with the change in the bilateral flow.

$$NUTR_{r,i,lss,loss\_kt,lcl,t} = NUTR_{r,i,lss,loss\_kt,lcl,t-1} \frac{XDT_{r,i,t}}{XD_{r,i,t-1}} \tag{Z-17}$$

$$NUTR_{d,i,lss,loss\_kt,s,t} = NUTR_{d,i,lss,loss\_kt,s,t-1} \frac{XW_{s,i,d,t}}{XW_{s,i,d,t-1}} \tag{Z-18}$$

The aggregation facility will aggregate the nutrition accounts along the standard aggregation dimensions: (1) commodity (2) region of destination ; and (3) source region of imports. The nutrition indicators are also output in per capita and daily terms (with a transformation of the labels). In the case of food-based calories, the indicator is also multiplied by 1000.

[N.B. We need to resolve the issue of discrepancies between the FAO data and the SAM. For example, the SAM may indicate no consumption, but the FAO has positive consumption. For the moment, we assume that the rate of change is zero, i.e., we hold fixed the FAO levels.]

3.11.7   Factor taxes and subsidies and fiscal closure

In the default model, factor taxes and subsidies are exogenous and represented by the tax τ^v. The default closure fixes the net government fiscal position, i.e. government savings, and the adjusting fiscal instrument is κ^h, i.e. net fiscal transfers between households and the government or lump sum taxation. In some policy scenarios it would be useful to assess the use of alternative fiscal instruments, for example in the case of carbon re-cycling. For this purpose, the factor tax has been converted to a GAMS macro that takes the following form:

---

[103]   If one wanted to constrain caloric intake, for example, an equation would be need to be added to the model that measures calories endogenously.

$$\tau_{r,f,a}^{vm} = \left[\chi_r^f \varphi_{r,f,a}^m + (1 - \varphi_{r,f,a}^m) \right] \tau_{r,f,a}^v + \alpha_r^f \varphi_{r,f,a}^a$$

where the default specification sets χ_r^f and α^f to zero, i.e. the factor tax is simply equal to the exogenous tax, τ^v, which is initially calibrated to the GTAP database. The user can choose to *endogenize* factor taxes using a multiplicative scalar, χ^f, i.e. all taxes adjust in the same proportion, or an additive scalar, α^f, i.e. all taxes adjust in the same percentage point amount. Closure can be defined over specific factors (for example labor only), and/or specific sectors. For example, to change from the default closure to an additive factor tax on labor the user needs to exogenize κ^h, endogenize α^f and set $\varphi_{r,l,a}^a$ to 1. In the code, this appears as:

Listing 3.5: **Labor tax closure**

```
2   *  Apply labor tax closure to all countries in "EUR"
3   *  and across all activities

5      loop(mapr("EUR",r),
6          kappah.fx(r,tsim) = kappah.l(r,tsim-1) ;
7          alphaFtax.lo(r,tsim) = -inf ;
8          alphaFtax.up(r,tsim) = +inf ;
9          ifAfTaxFlag(r,l,a) = 1 ;
10     ) ;
```

To apply the closure to all factors, use the index f. To apply the closure to capital, use the index *cap*.

The model includes an equation that measures the difference between carbon tax revenues and the subsidies paid to factors from these revenues. In the standard closure the gap is maximal, as the carbon tax revenues are re-cycled directly to households. The equation below measures the gap. The second term on the right-hand side is the level of the revenues from the subsidies measured as the difference between the endogenous (net) tax on factors and the original tax. The sum is over all activities a and factors f that are the target of the subsidies in region r as defined by the set FS.[104] The first term is some share of the carbon tax revenues, by default, α^y is one. The two revenue streams are summed, rather than subtracted, as the second term would normally be a negative number. In the standard closure, $CTAXGap$ is endogenous and is equal to the carbon tax revenues, as the factor subsidies are zero. In the re-cycling closures, $CTAXGap$ is exogenous and typically set to zero. This equation is then used to determine the endogenous factor tax shifter, α^f, in the case of an additive shifter. It is possible to target a share of total tax revenues by setting the parameter α^y to a value different from 1.

$$CTAXGap_r = \alpha_r^y YGov_{r,ctx} + \sum_a \sum_{f \in FS_r} \left(\tau_{r,f,a}^{vm} - \tau_{r,f,a}^v \right) PF_{r,f,a} XF_{r,f,a}$$

Listing 3.6 shows how to implement carbon revenue recycling. It is similar to the listing above, but the fiscal closure is different. In the listing above, direct taxes are exogenized (for example to the baseline level) and total changes in fiscal revenues are used to subsidize labor. If *ex post* revenues increase beyond those generated by the carbon taxes, these are used to add additional subsidies to labor use. In the listing below, the direct tax rate is still endogenous and is the main fiscal target, and carbon tax revenues are matched one-for-one with the subsidies to labor, assuming full recycling.

Listing 3.6: **Carbon tax recycling to lower tax on labor**

```
2   *  Apply labor tax closure to all countries in "EUR"
3   *  and across all activities

5      loop(mapr("EUR",r),
6          ctaxgap.fx(r,tsim)   = 0 ;
7          alphaFtax.lo(r,tsim) = -inf ;
8          alphaFtax.up(r,tsim) = +inf ;
9          ifAfTaxFlag(r,l,a)   = 1 ;
10     ) ;
```

---

[104] In the model code, this is identified with the parameter `ifAFTaxFlag`, see above.

Targeting labor subsidies

One potential policy simulation is to assess different closures that have (roughly) the same fiscal impact. For example, one may want to compare a policy scenario that targets subsidies for renewable technologies with using those subsidies to lower the cost of labor. The model includes one new equation that targets a labor subsidy amount using the same mechanism described above.[105]

$$LSubsY_r = \sum_a \sum_l \left[\tau_{r,l,a}^{vm} - \tau_{r,l,a}^v \right] PF_{r,l,a} XF_{r,l,a}$$

A first policy scenario is run that calculates some level of subsidy, for example to renewable technologies. That amount is then saved and is used as input to the alternative scenario where the tax on labor is endogenous to target the exogenous level of the aggregate subsidies using the same mechanism as described above though the default fiscal closure still holds, i.e. the additive labor tax factor, α^f, is used to target the (exogenous) variable $LSubsY_r$.[106]

3.11.8 Specific tariffs

Tariffs in the standard model are assumed to be *ad valorem* and calibrated to the GTAP database. The code allows for specific tariffs, which are calculated on a volume basis, for example per ton of apples. Specific tariffs have been used in practice to tax the carbon content of imports—the latter of which is calculated outside of the model and typically has the units of tons of CO_2 per dollar (or millions of dollars) of imports.

Two equations in the standard model are modified to deal with specific tariffs—the tariff inclusive (bilateral) price of imports and revenues generated by import taxes.[107]

$$PDM_{r,i,d} = \left(1 + \tau_{r,i,d}^m + \tau_{r,i,d}^{ntm}\right) PWM_{r,i,d} + \tau_{s,i,r}^{st} \tag{Z-19}$$

$$YGOV_{r,mtx} = \sum_s \sum_i \left(\tau_{s,i,r}^m PWM_{s,i,r} + \tau_{s,i,r}^{st}\right) \lambda_{s,i,r}^w \lambda_{s,i,r}^x XW_{s,i,r}^d \tag{Z-20}$$

At the moment, the specific tariff is implemented as a GAMS macro and is always assumed to be exogenous. In the case of taxing of the carbon content of imports, the macro has been set equal to:

$$\rho_{s,i}^x \tau_{s,i,d}^{cx}$$

where ρ^x is the quantity of carbon embedded in imports from source region s (measured in tons of CO_2 per dollar of imports) and τ^{cx} is dollars per ton of CO_2 and exogenous.[108] Note that the tax would typically only be applied to export from regions outside a carbon constrained coalition. Also note that even if the specific tariff is uniform per unit of carbon per source region, it will not necessarily have the same level per average unit across countries within a coalition for the same commodity. One country may largely be importing from a less carbon intensive country compared to another. For other uses of the specific tariff, users may need to redefine the macro, or make a clever use of the underlying instruments.

3.11.9 Revenue neutral inter-regional transfers

This section describes policy scenarios that involve a coalition of regions where the net gains/losses are unevenly distributed. Inter-regional transfers, that sum to zero, are introduced such that the net/gains losses (in percent terms) are equalized across the regions in the coalition. Equation (Z-21) determines the ratio of equivalent variation, EV, for each country r in a coalition C relative to baseline EV and compares this with the coalition-wide average. In the default model, β^{EV} is endogenous and typically would differ from 1, i.e. the net gains/losses across regions in the coalition are not necessarily uniform. If β^{EV} is exogenously set to 1, this equation will determine for each region with the exception of a residual region the level of transfers that equalizes the net gains/losses (in percent terms). Equation (Z-22) imposes the neutrality of gross transfers, and *de facto* determines net transfers for the residual region. The listing below shows one implementation for a European coalition. The variable β^{EV} is fixed at 1, the

---

[105] Note that the difference between τ^{vm} and τ^v can be used more broadly post-simulation to calculate the net cost of a policy in terms of the impacts on factor taxes.

[106] The labor subsidy cost equation is only active when the flag `lTaxFlag` is set to `yes`.

[107] For the moment specific tariffs apply only for the default trade specification and not for the MRIO version of the model.

[108] In some applications, for example border tax adjustments, the tax on embedded emissions should be equal to the domestic carbon tax, which could be endogenous. In this case an additional equation would be necessary. It turns out that a small set of model iterations can achieve the desired result without an explicit equation.

`evRatioFlag` is set to `yes` for all regions in the coalition except for the residual region, Germany in this case. The transfers are endogenized for all regions in the coalition and equation (Z-22) is activated.

$$\frac{\sum_h EV_{r,h}}{\sum_h EV_{r,h}^b} = \beta_r^{EV} \frac{\sum_{s\in\{C\}}\sum_h EV_{s,h}}{\sum_{s\in\{C\}}\sum_h EV_{s,h}^b} \qquad \forall r \in \{C\} \tag{Z-21}$$

$$\sum_{r\in\{C\}} ITRNSF_r \equiv 0 \tag{Z-22}$$

Listing 3.7: **Neutral inter-regional transfers**

```
2   *   Setup inter-regional transfers to equalize the
3   *   'pain' from the European carbon tax. Assume that
4   *   'the left-out region is Germany

6       evRatioFlag(r,"EUR")$(mapr("EUR",r) and not sameas(r,"DEU")) = yes ;
7       betaEV.fx(r,tsim) = 1 ;
8       itransfers.lo(r,tsim)$mapr("EUR",r) = -inf ;
9       itransfers.up(r,tsim)$mapr("EUR",r) = +inf ;
10      iTransferFlag("EUR") = yes ;
```

3.11.10   Border tax adjustment

The following describes how border tax adjustments are implemented in ENVISAGE.

Data and methodology

To provide an accounting of CO_2 emissions embodied in bilateral trade, we follow an approach outlined in Peters (2008) and applied to the GTAP-based ENVISAGE Model.[109] Country-specific CO_2 emissions per unit of output by sector are used to estimate emissions associated with bilateral trade flows. This method assumes that in a given sector and country, the same production technology is used to produce domestic and exported commodities. This allows to decompose emissions from domestic output into components associated with the domestic demand and exports. CO_2 emissions associated with fossil-fuel combustion and embodied in trade flows from region s to region d (Emi^m) for each commodity i are estimated as:

$$Emi_{i,sd}^m = \rho_{i,s}^I E_{i,sd}$$

where ρ^I is a vector of region-specific CO_2 (direct and indirect) emissions per unit of output across sectors and E represents the level of bilateral trade between s and d for commodity i. The total emission rate ρ^I is calculated from $\left(\rho_s^D\right)'[I - A_s]^{-1}$, where ρ^D is the rate of direct emissions to produce commodity i in region s, I is the identity matrix and A is the technological matrix, which represents the industry requirements of domestically produced products in region s. Value flows are estimated using producer prices. The parameter ρ^I is an estimate of the emission rate encompassing the entire domestic value chain. For instance, CO_2 emitted by a coal power plant to produce electricity, which was further used to produce iron and steel, would be embodied into the exports of iron and steel.

These parameters are related to model variables in the following way. The rate of direct emissions (per unit of output) is based on:

$$\rho_{r,j}^D = 1000 \times \frac{\sum_i Emi_{r,em,i,j}}{XS_j}$$

Note that these formulas are based on a diagonal make matrix where activity a maps directly to output j. For the power version of the model, all electric activities are collapsed into a single activity for the purposes of these calculations (i.e. emissions are added across all power activities). Also note that XS includes the value of production

---

[109]   This section benefited from significant inputs from Maksym Chepeliev, including the development of the original code.

taxes. The scale factor 1000 converts the rate of direct emissions from metric ton per unit of output, to kg per unit of output.[110]

The matrix of (domestic input use) technical coefficients (A) is calculated as:

$$A_{r,i,j} = \frac{PDT_{r,i,0} XD_{r,i,j}}{XS_j}$$

where again the coded formulas take into account non-diagonality and scaling issues.

There are three so-called scope levels for emissions. Scope 1 refers to the direct emissions for each region and sector j, i.e. $\rho^1 = \rho^D$, measured in kg per unit of output.[111] Scope 3 emissions refer to both the direct and indirect emissions, i.e. $\rho^3 = \rho_s^I = \left(\rho_s^D\right)' [I - A]^{-1}$. Scope 2 emissions are an intermediate level of emissions that include the direct emissions and indirect emissions linked to the use of electricity. Hence we have:

$$\rho_{r,i}^2 = \rho_{r,i}^1 + \rho_r^{ely} \frac{XA_{r,ely,i}}{XS_{r,i}}$$

where the second term measures the use of electricity per unit of output multiplied by the average level of emissions per unit of domestic electricity produced:

$$\rho_r^{ely} = 1000 \times \frac{\sum_j \sum_{a \in elya} EMI_{r,CO2,j,a}}{XS_{r,ely}}$$

The program also calculates a parameter that represents the level of emissions embedded in bilateral trade, named 'EMITRD', with levels in million metric tons. It is not currently used in the model.

$$EMITRD_{s,i,d,es} = \frac{\rho_{s,i}^{es}}{1000} \left[XW_{s,i,d} + XTT_{s,i} \frac{XW_{s,i,d}}{\sum_{r'} XW_{s,i,r'}} \right]$$

Calculating the carbon embedded in trade

[01-SEP-2025] The calculations for the embodied emissions are calculated after each time period in a post-simulation routine contained in the file `EmiIntOutput.gms`, which is included in `runsim.gms`.[112] The matrix inverse is calculated using a solver (the default is CNS), using a single equation (`A_Inveq`), which is defined towards the bottom of `model.gms`. When implementing a border tax adjustment, users are free to use a linear combination of the baseline indicator and the lagged value from the current simulation. This allows for capturing structural changes in the exporting country if the latter is implementing mitigation policies.

Border tax adjustment based on exporter's carbon content

The border tax adjustment is implemented as an excise tax on imports (τ^{mx}). Thus equations (T-31) and equations (T-20) become respectively:

$$PDM_{s,i,d} = \left(1 + \tau_{s,i,d}^m + \tau_{s,i,d}^{ntm}\right) PWM_{s,i,d} + \tau_{s,i,d}^{mx} \tag{Z-23}$$

$$PDM_{s,i,d,aa}^a = \left(1 + \tau_{s,i,d,aa}^{ma}\right) PWM_{s,i,d} + \tau_{s,i,d}^{mx} \tag{Z-24}$$

where the former is used for the standard Armington model and the latter for the MRIO version of bilateral trade. The BTA tax, i.e. the excise tax is composed of two components—the level of CO_2 emissions[113] per dollar imported ρ^I and the imposed carbon tax, τ^{EmiX}.[114] The first term represents a linear combination of the embodied emissions from the baseline and the (lagged) embodied emissions from the current simulation. If the parameter φ^{BTAlag} is set to 1, only the current (lagged) indicator will be used, if it is set to 0, only the baseline indicator will be used. The flag

---

[110]   The calculations in the code adjust for any differences in input scale so that emissions revert back to million tons and output to million dollars.

[111]   The embodied emission factors (ρ^{es}) are represented by the parameter `EMIXSC` defined over regions (r), commodity (i), scope (es) and time (t).

[112]   Earlier versions calculated the embodied emissions using a separate program and only intended to calculate the indicators for the baseline scenario.

[113]   Converted back to tons.

[114]   The current implementation only holds for CO_2 taxes.

is designed to be coalition specific. The choice of scope (es) and which imports to tax (i) are also coalition specific and user-designated governed by the set mapping `mapScope`.

$$\tau_{s,i,d,t}^{mx} = \left[(1 - \varphi_R^{BTAlag})\rho_{s,i,t}^{es,BaU} + \varphi_R^{BTAlag}\rho_{s,i,t-1}^{es}\right]\tau_{rq,s,i,d,t}^{EmiX} \qquad \text{for } d \in \{R\}, (i, es) \in \{\texttt{mapScope}(R, i, es)\} \qquad \text{(Z-25)}$$

Finally, we need an equation that links τ^{EmiX} to the actual carbon tax, potentially adjusted for the level of effort by the exporter—thus we have:

$$\tau_{R,s,i,d,t}^{EmiX} = \chi_{R,s,i,d}^{BTA,imp} \frac{\displaystyle\sum_{a \in \{i\}} \tau_{d,CO2,a}^{Emi}}{\displaystyle\sum_{a \in \{i\}} 1} - \chi_{R,s,i,d}^{BTA,exp} \frac{\displaystyle\sum_{a \in \{i\}} \tau_{s,CO2,a}^{Emi}}{\displaystyle\sum_{a \in \{i\}} 1} \qquad \text{(Z-26)}$$

which holds for all exporters to the coalition region R.[115] The user is allowed to determine the *gauge* ($\chi^{BTA,imp}$) for the power of the carbon price of the importer (d in coalition R) as well as the gauge for the power of the carbon price implemented in the exporting region (s). The formula takes into account the mapping of the tax from activities (a) to commodities (i).

An example is provided in the following listing.

Listing 3.8: **Border tax adjustment based on the carbon content of the exporter**

```
2   *  Introduce CBAM

4   if(tsim.val ge 2021,
5   *  BTA on non-WEU imports into WEU, excluding refined oil
6      ifBTA("WEU",s,i,d)$(mapra("WEU",d) and not sameas(s,"WEU") and not sameas(i,"P_C-c")) = yes ;

8   *  Tax only ETS sectors
9      BTAGauge("WEU",s,i,d,"imp",tsim) = 0 ;
10     BTAGauge("WEU",s,i,d,"imp",tsim)$sum(a$(mapets("WEU","ETS",a) and mapmak(a,i)),1) = 1 ;
11  *  Do not take into account emission price of exporters
12     BTAGauge("WEU",s,i,d,"exp",tsim) = 0 ;
13     emiTaxX.lo("WEU",s,i,d,tsim)$ifBTA("WEU",s,i,d) = 0 ;
14     emiTaxX.up("WEU",s,i,d,tsim)$ifBTA("WEU",s,i,d) = +INF ;
15  ) ;
```

---

[115]   This equation is governed by the flag `ifBTA(ra,s,i,d)`. Thus it is set to 1 for all regions s exporting to region d that are part of coalition ra.

Chapter 4

Model dynamics

The model as defined above represents the fully static version.[1] The factor stocks and technology are largely exogenous. This section describes the additional equations needed for the standard dynamics.

4.1 Labor supply

Labor is assumed to grow at some 'natural' rate for each skill level and by zone (in the case of segmented labor markets). At the moment, the natural rate of growth is equated to some measure of the growth in the relevant population and is assumed to be uniform for all skills and zones. The relevant population growth is defined by the following expression:

$$TLAB_{r,t} = \frac{\sum_p \phi^l_{r,p,t} Pop^t_{r,p,t}}{\sum_p \phi^l_{r,p,t-n} Pop^t_{r,p,t-n}} TLAB_{r,t-n}$$

where ϕ^l is the relevant labor force participation rate. Population growth by cohort is assumed to be exogenous and is typically taken from a population scenario such as the UN or from those developed by the IAMC community known as the shared socio-economic pathways, or SSPs.

In the case of labor market segmentation with rural to urban migration, the level of migration is defined by equation (G-1). This simple migration equation is similar in spirit to the so-called Harris-Todaro model of migration.[2] Migration is a function of the expected urban wage relative to the expected rural wage, where ω^m represents the migration elasticity.[3] Labor supply in each zone is equal to some exogenous component based on the 'natural' growth rate adjusted by migration, equation (G-2). The growth rate g^{lz} is typically exogenous and for the moment uniform across skills and zones. The parameter δ^m is equal to -1 in the case of the rural zone and is equal to 1 for the urban zone. The parameter μ^m is a multiplier for migration that accounts for migration occurring between periods in a multi-year time step. The level of the multiplier is given in equation (G-3). It is equal to 1 for single-year time steps. It is approximately equal to n for time steps of n years. Equation (G-4) calculates implicitly the growth rate of labor by skill at the economy-wide level.

$$MIGR_{r,l} = \chi^m_{r,l} \left(\pi^{Urb}_{r,l} \right)^{\omega^m_{r,l}} \tag{G-1}$$

$$L^s_{r,l,z,t} = \left(1 + g^{lz}_{r,l,z,t} \right)^n L^s_{r,l,z,t-n} + \delta^m \mu^m_{r,l,z,t} MIGR_{r,l,t} \tag{G-2}$$

---

[1]   In the model implementation, all of the equations listed till now are part of the `core` model—save for those that are substituted out.

[2]   See Harris and Todaro (1970).

[3]   There are more complex specifications of the migration function. See for example Burniaux (1987) and Burniaux and van der Mensbrugghe (1991).

$$\mu_{r,l,z,t}^m = \frac{\left(1 + g_{r,l,z,t}^{lz}\right)^n \left(\frac{\pi_{r,l,t-n}^{Urb}}{\pi_{r,l,t}^{Urb}}\right)^{\omega_{r,l}^m} - 1}{\left(1 + g_{r,l,z,t}^{lz}\right)\left(\frac{\pi_{r,l,t-n}^{Urb}}{\pi_{r,l,t}^{Urb}}\right)^{\omega_{r,l}^m/n} - 1} \tag{G-3}$$

$$LAB_{r,l,t}^s = \left(1 + g_{r,l,t}^l\right)^n LAB_{r,l,t-n}^s \tag{G-4}$$

4.2 Capital supply

The total capital stock, K^s evolves according to the standard capital accumulation formula, equation (G-5). Since both terms on the right-hand side are lagged, with a single year time step the supply of capital is in effect exogenous. Annex C.1 explains how the equation is implemented for multi-year time steps. In this case, the equation depends on the contemporaneous level of investment and the capital stock is therefore no longer pre-determined.

$$K_{r,t}^s = (1 - \delta_t) K_{r,t-1}^s + XFD_{r,Inv,t-1} \tag{G-5}$$

Equation (G-6) defines the normalized capital stock. It is a fixed scalar of the non-normalized level where the scaling factor is initialized using base year data, i.e. $\chi_r^k = TK_{r,0}^s / K_{r,0}^s$. The normalized capital stock is equal to aggregate capital remuneration in the base year, i.e. it is the level of the capital stock with the price of aggregate capital set to 1.

$$TK_{r,t}^s = \chi_r^k K_{r,t}^s \tag{G-6}$$

4.3 Climate module

[31-Oct-2020: This module is being tested and undergoing further development. For the moment, this module is ignoring all emissions other than carbon.] The climate module is composed of three components:

1. Emissions are released in the atmosphere. These emissions lead to changes in their concentration in the atmosphere (and potentially other earth systems such as soils, oceans, etc.). Carbon emissions are treated differently from the other emissions as the carbon cycle involves a complex set of relations across different earth systems.

2. Atmospheric concentration of gases leads to variegated impacts on radiative forcing.

3. In turn radiative forcing affects temperature and this is captured by an energy balance model (EBM) that reflects the energy fluxes between different earth systems.

4.3.1 The carbon cycle

The carbon cycle described herein is derived from Millar et al. (2017) and **?** as implemented by Hänsel et al. (2020) and is known as the *Finite Amplitude Impulse Response* (FAIR) model.

The carbon cycle depends on total CO_2 emissions: fossil-fuel combustion-based, other industrial CO_2 emissions, $EmiX^C$, and changes in land-use emissions, $EmiLand^C$. For the moment, the latter two sources of CO_2 emissions are exogenous. Total global CO_2 emissions are provided in equation (G-7).

The carbon cycle also depends on cumulative carbon emissions—expressed in gtC. Equation (G-8) represents the standard cumulative expression, where industrial emissions are converted from gtCO2 to gtC. Equation (G-9) takes into account the exogenous cumulative emissions from land-use changes. The starting point for 2015 is 400 gtC of cumulative industrial emissions and 197 gtC of cumulative land-use emissions.[4]

$$Emi_t^C = EmiGbl_{co2,t} + EmiX_t^C + EmiLand_t^C \tag{G-7}$$

$$CumEmiInd_t = CumEmiInd_{t-1} + (12/44)\left(EmiGbl_{co2,t-1} + EmiX_{t-1}^C\right) \tag{G-8}$$

$$CumEmi_t^T = CumEmiInd_t + CumEmiLand_t \tag{G-9}$$

---

[4]   Hänsel et al. (2020) use a value of 197 gtC, which is nearly double the level in the 2016 version of the DICE model, see Nordhaus (2016).

The FAIR model uses anthropogenic fossil and land use CO_2 emissions as input and partitions them into four boxes R_b—with partition fraction ϕ and τ representing the differing timescales of carbon uptake by geological processes (`GEOPR`), the deep ocean (`DPOCN`), the biosphere (`BIOSF`) and the ocean mixed layer (`MXOCN`). The one-step motion equation is given by equation (G-10). The carbon reservoir receives a fraction of the CO_2 emissions impulse and it is adjusted by some level of 'decay'.[5] The α parameter is endogenous and is calibrated dynamically so that the overall behavior of the carbon cycle mimics other earth systems models (ESMs). This is detailed further below.

$$\Delta R_{b,t} = \phi_b(12/44)E_{t-1} - \frac{R_{b,t-1}}{\alpha\tau_b} \iff R_{b,t} = R_{b,t-1}\left(1 - \frac{1}{\alpha\tau_b}\right) + \phi_b(12/44)Emi_{t-1}^C \tag{G-10}$$

The calibration of α is based on Millar et al. (2017): "To identify a suitable state dependence, we focus on parameterising variations in the 100-year integrated impulse response function, *iIRF100*. A focus on the integrated impulse response (average airborne fraction over a period of time, multiplied by the length of time period), as opposed to the airborne fraction at a particular point in time, is more closely related to the impact of CO_2 emissions on the global energy budget, and also to other metrics such as global warming potential. With other coefficients fixed, iIRF100 is a monotonic (but nonlinear) function of α." Equation (G-11) expresses this relation.

In addition from the same authors, "We assume *iIRF100* is a function of the accumulated perturbation carbon stock in the land and ocean (equivalent to the amount of emitted carbon that no longer resides in the atmosphere), ..., and of the global mean temperature anomaly from preindustrial conditions, T^{atm}." Equation (G-12) expresses this relation, where MAT is the carbon stock residing in the atmosphere. In the model code, the two expressions for *iIRF100* are substituted out and this relation determines the α parameter dynamically. Finally, equation (G-13) defines the atmospheric concentration (in gTC) for each time period.

$$iIRF100 = \sum_b \alpha_t\phi_b\tau_b\left[1 - \exp\left(-\frac{100}{\alpha_t\tau_b}\right)\right] \tag{G-11}$$

$$iIRF100 = r_0 + r_C\left(CumEmi_t^T - (MAT_t - 588)\right) + r_T T_t^{atm} \tag{G-12}$$

$$MAT_t = \sum_b R_{b,t} + 588 \tag{G-13}$$

4.3.2 Radiative forcing

Radiative forcing is the sum of the impacts of individual gases on radiative forcing. In the case of carbon, it takes a logarithmic form where the key parameter is κ, which represents the equilibrium level of forcing for a doubling of carbon concentration relative to its pre-industrial level. The default parameterization sets $\kappa = 3.6813$. At the moment, all other forcings are collapsed into a single exogenous time-varying level.[6]

$$FORC_{CO2,t} = \kappa\frac{\ln\left[MAT_t/MAT^e\right]}{\ln(2)} \tag{G-14}$$

$$RF_t = \sum_{frc} FORC_{em,t} + FORC_t^X \tag{G-15}$$

4.3.3 The energy balance model (EBM)

The energy balance model (EBM) is derived from **?**. This EBM has a 3-box model: land and atmosphere (`ATMOS`), shallow (`UPOCN`) and deep oceans (`DPOCN`). A common representation of this physical process is outlined by Geoffroy et al. (2013). The **?** paper extends the 2-box model to three boxes to better represent the transient response.

The motion equation for the change in temperature across the three boxes is represented by the following matrix differential equation:

$$\dot{\mathbf{T}} = A\mathbf{T} + \mathbf{M}RF$$

where $\dot{\mathbf{T}}' = \begin{bmatrix} \dot{T}_{atmos} & \dot{T}_{upocn} & \dot{T}_{upocn} \end{bmatrix}$, $\mathbf{T}' = \begin{bmatrix} T_{atmos} & T_{upocn} & T_{upocn} \end{bmatrix}$, RF is radiative forcing, $\mathbf{M}' = \begin{bmatrix} 1/C_1 & 0 & 0 \end{bmatrix}$ and:

---

[5]   Appendix C describes the multi-step version of the carbon cycle.

[6]   Future model versions will add endogenous forcings for some of the other gases that are generated by Envisage.

$$A = \begin{pmatrix} -\left(\lambda + \kappa_2\right)/C_1 & \kappa_2/C_1 & 0 \\ \kappa_2/C_2 & -\left(\kappa_2 + \varepsilon\kappa_3\right)/C_2 & \varepsilon\kappa_3/C_2 \\ 0 & \kappa_3/C_3 & -\kappa_3/C_3 \end{pmatrix}$$

The 'C' parameters represent the heat capacity of each of the boxes. The 'κ' parameters represent the heat exchange coefficients between the boxes. The 'λ' parameter is called the climate feedback parameter. And the 'ϵ' parameter is an efficacy factor that affects transient warming. In the form given in equation (G-16), the change in atmospheric temperature can be solved as a difference equation, which also includes the temperature of the two ocean layers.[7]

$$\mathbf{Temp}_t = (I + A)\mathbf{Temp}_{t-1} + \mathbf{M}RF_t \tag{G-16}$$

There are different strategies for converting these equations for use in models with varying time steps. In the new GAMS code, the model implements equation (G-16) using an annual time step, i.e. the time dimension for this equation differs from the time step of most of the other equations. The only modification is for the forcing impulsion. In the current code, the average forcing is used for any individual year between two model periods. For example, if the year is 2023, the relevant forcing level is the average of 2020 and 2025 if using 5-year time steps from 2015 forwards. Forcing is thus uniform between two model periods. Given the relatively slow evolution of forcing, the choice is unlikely to have a major impact especially with relatively small step sizes such as 5-years.[8]

Parameterization is based on the latest parameter values from ?. Hänsel et al. (2020) uses an earlier vesion of FAIR (v1.5), which is based on a two box model. In that case, κ_3 is set to zero and there is no need for ϵ. There is a different interpretation of the labels. The label ATMOS represents land, atmosphere and the shallow oceans and the label UPOCN represents the deep ocean.

4.4 Climate-induced economic impacts

[30-Apr-2023: NEW] The climate impacts module incorporates a number of channels for the impacts—mostly negative but potentially positive—of rising temperatures on economic potential. The current channels include:

- Total factor productivity in each activity
- Labor productivity by skill and activity
- Health-related impacts to labor productivity
- Household energy demand
- Land supply
- Tourism revenues
- Capital stock

Each is specified using a generic quadratic function:

$$I_{r,agt} = \left[1 + \chi_{r,agt}^d \left(\beta_{r,agt}^0 + \beta_{r,agt}^1 \left(T_{r,t} - T_{r,0}\right) + \beta_{r,agt}^2 \left(T_{r,t} - T_{r,0}\right)^2\right)\right]$$

where *agt* refers to a specific agent with potentially an additional qualifier, for example labor in a particular activity. The shifter χ^d is an adjustment factor that relies on expenditures on adaptation that can dampen the effects of temperature change and is described further below. The equation allows for the temperature change, T, to be region specific, though it will often be equated to the change in global mean surface temperature. The intercept term is normally calibrated to 0 so that the indicator takes the value of 1 in the reference year. The indicator, I, is then applied as an adjustment factor in the main model, for example labor productivity. The specific channels and their indicators are provided in equations (G-17)–(G-23). These equations cover respectively, impacts to total factor productivity (`tfp`) by activity, labor productivity by activity and labor-type (`lab`), health-related labor productivity

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[7]   In the FAIR model, the EBM model is converted to an impulse-response model that only solves for the change in atmospheric temperature. In essence, this involves diagonalizing the A matrix by calculating the eigenvalues and eigenvectors. Both the EBM and IR versions yield very similar results. Results available from the author.

[8]   The relevant forcing for any year tt is governed by a mapping between tt and the model period denoted by t. In the code, this is governed by the mapping named `mapt`. It is initialized in the climate data file, for example `ClimDataV10` for Version 10 of the GTAP Data Base. The user can change the flag `ifFORCAVG` to the value 0 to use only the contemporaneous forcing, rather than the average over two simulation periods.

also by activity and labor-type (`hht`), household energy demand (`nrg`), land supply by type of land (`lnd`), (export) tourism revenues (`tou`), and the aggregate capital stock (`cap`).

$$IDAM_{r,a}^{tfp} = \left[1 + \chi_{r,a}^{d,tfp}\left(\beta_{r,a}^{d,tfp,0} + \beta_{r,a}^{d,tfp,1}\left(T_{r,t} - T_{r,0}\right) + \beta_{r,a}^{d,tfp,2}\left(T_{r,t} - T_{r,0}\right)^2\right)\right] \tag{G-17}$$

$$IDAM_{r,l,a}^{lab} = \left[1 + \chi_{r,l,a}^{d,lab}\left(\beta_{r,l,a}^{d,lab,0} + \beta_{r,l,a}^{d,lab,1}\left(T_{r,t} - T_{r,0}\right) + \beta_{r,l,a}^{d,lab,2}\left(T_{r,t} - T_{r,0}\right)^2\right)\right] \tag{G-18}$$

$$IDAM_{r,l,a}^{hht} = \left[1 + \chi_{r,l,a}^{d,hht}\left(\beta_{r,l,a}^{d,hht,0} + \beta_{r,l,a}^{d,hht,1}\left(T_{r,t} - T_{r,0}\right) + \beta_{r,l,a}^{d,hht,2}\left(T_{r,t} - T_{r,0}\right)^2\right)\right] \tag{G-19}$$

$$IDAM_{r,k,h}^{nrg} = \left[1 + \chi_{r,k,h}^{d,nrg}\left(\beta_{r,k,h}^{d,nrg,0} + \beta_{r,k,h}^{d,nrg,1}\left(T_{r,t} - T_{r,0}\right) + \beta_{r,k,h}^{d,nrg,2}\left(T_{r,t} - T_{r,0}\right)^2\right)\right] \tag{G-20}$$

$$IDAM_{r,lnd}^{lnd} = \left[1 + \chi_{r,lnd}^{d,lnd}\left(\beta_{r,lnd}^{d,lnd,0} + \beta_{r,lnd}^{d,lnd,1}\left(T_{r,t} - T_{r,0}\right) + \beta_{r,lnd}^{d,lnd,2}\left(T_{r,t} - T_{r,0}\right)^2\right)\right] \tag{G-21}$$

$$IDAM_{r,i}^{tou} = \left[1 + \chi_{ri}^{d,tou}\left(\beta_{r,i}^{d,tou,0} + \beta_{r,i}^{d,tou,1}\left(T_{r,t} - T_{r,0}\right) + \beta_{r,i}^{d,tou,2}\left(T_{r,t} - T_{r,0}\right)^2\right)\right] \tag{G-22}$$

$$IDAM_r^{cap} = \left[1 + \chi_r^{d,cap}\left(\beta_r^{d,cap,0} + \beta_r^{d,cap,1}\left(T_{r,t} - T_{r,0}\right) + \beta_r^{d,cap,2}\left(T_{r,t} - T_{r,0}\right)^2\right)\right] \tag{G-23}$$

4.5   Adaptation expenditures

The shifters in the impact functions can be adjusted through expenditures on adaptation measures. Each channel is associated with its own adaptation expenditure with its own cost structure. However, the impact adjustment factor is impacted by aggregate adaptation expenditure, i.e., the expenditures are not 'agent' specific, accept to the extent the shifter on the adjustment equation is 'agent' specific. The generic adaptation shifter function has the following form:

$$\chi_{r,a}^{d,tgt} = \chi_{r,agt}^{a,tgt}\left(XFD_{r,tgt}\right)^{-\rho_r^{a,tgt}}$$

where $\chi^{a,tgt}$ is the 'agent'-specific shifter for the targeted impact channel ('tgt') and XFD is the aggregate amount of adaptation expenditures for the targeted channel. The cost structure for the adaptation expenditures must be specified—similarly to the cost structure of government and investment expenditures, i.e., with top-level budget shares and second-level sourcing shares and respective substitution elasticities. The standard model takes care of implementing the demand for goods and services once the cost structure is known and the level of aggregate expenditure is specified.[9] The full set of adaptation adjustment factors is given by equations equations (G-24)–(G-30).

$$\chi_{r,a}^{d,tfp} = \chi_{r,a}^{a,tfp}\left(XFD_{r,tfp}\right)^{-\rho_r^{a,tfp}} \tag{G-24}$$

$$\chi_{r,l,a}^{d,lab} = \chi_{r,l,a}^{a,lab}\left(XFD_{r,lab}\right)^{-\rho_r^{a,lab}} \tag{G-25}$$

$$\chi_{r,l,a}^{d,hht} = \chi_{r,l,a}^{a,hht}\left(XFD_{r,hht}\right)^{-\rho_r^{a,hht}} \tag{G-26}$$

$$\chi_{r,k,h}^{d,nrg} = \chi_{r,k,h}^{a,nrg}\left(XFD_{r,nrg}\right)^{-\rho_r^{a,nrg}} \tag{G-27}$$

$$\chi_{r,lnd}^{d,lnd} = \chi_{r,lnd}^{a,lnd}\left(XFD_{r,lnd}\right)^{-\rho_r^{a,lnd}} \tag{G-28}$$

$$\chi_{r,i}^{d,tou} = \chi_{r,i}^{a,tou}\left(XFD_{r,tou}\right)^{-\rho_r^{a,tou}} \tag{G-29}$$

$$\chi_r^{d,cap} = \chi_r^{a,cap}\left(XFD_{r,cap}\right)^{-\rho_r^{a,cap}} \tag{G-30}$$

The source of funding for the adaptation expenditures is from one of three sources: households, government and investment. The user specifies the sourcing shares for each of the adaptation expenditure categories. Equation (G-31)

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[9]   In the current version of the damages/adaptation module there are seven additional final demand agents designated by `adptfp`, `adplab`, `adphht`, `adpnrg`, `adplnd`, `adptou` and `adpcap`, representing respectively adaptation expenditures on adjusting total factor productivity, labor productivity, health-related labor productivity, energy demand, land supply, tourism revenues and capital supply.

describes the allocation mechanism. $YFD_{r,adp}$ represents the nominal value of the aggregate expenditure spent on adaptation measure adp.[10] The coefficient $\chi_{r,fd,adp}^{adp}$ represents the allocation shares. For each expenditure category adp, the shares must sum to 1 over the fd agents.[11]

$$YFD_{r,fd}^{adp} = \sum_{adp} \chi_{r,fd,adp}^{adp} YFD_{r,adp} \tag{G-31}$$

The expenditures are subtracted from the relevant accounting identities for the agents. In the case of households the expenditures will be subtracted from disposable income, equation (Y-15). In the case of the government, the expenditures will be subtracted from the government fiscal balance expresion, equation (M-7). In the case of investment, the expenditures will be subtracted from the investment/savings balance, equation (Y-24).

4.6   Resource depletion

[17-MAR-2020—still needs editing.] This section describes the potential supply of resources for the coal, oil and gas extraction sectors. Oil and gas are treated alike with two sets of reserves, proven and unproven (or yet-to-discover). In the case of coal, the model assumes that all reserves are proven. The model is based on two key parameters: the rate of extraction from proven reserves, ρ^x, and the rate of discovery of new reserves, ξ, also called the conversion rate. Potential supply in any given time period is a fraction of proven reserves:

$$XF_t^{Pot} = \rho_t^x Res_t$$

Reserves at the beginning of a period are equal to the previous period's reserves, less actual extraction, plus new discoveries:

$$Res_t = Res_{t-1} - XF_{t-1} + \xi_{t-1} YTD_{t-1}$$

where YTD represents the level of the yet-to-discover reserves and ξ is the conversion rate. If ρ^x and ξ are invariant over time, one can trace out the time profiles for proven and yet-to-discover reserves under the assumption that producers are on their reserve profile. Figure 17 provides two potential profiles. In the left panel, production (measured as a given share of proven reserves, R) is in decline as production out of proven reserves exceeds new discoveries. The right panel shows a strong increase in production, followed by a peak and decline.

(a) ρ=**6.25%**, ξ=**6%**, R_0=**1446**, YTD_0=**2087**   (b) ρ=**6%**, ξ=**8.61%**, R_0=**1172**, YTD_0=**4184**
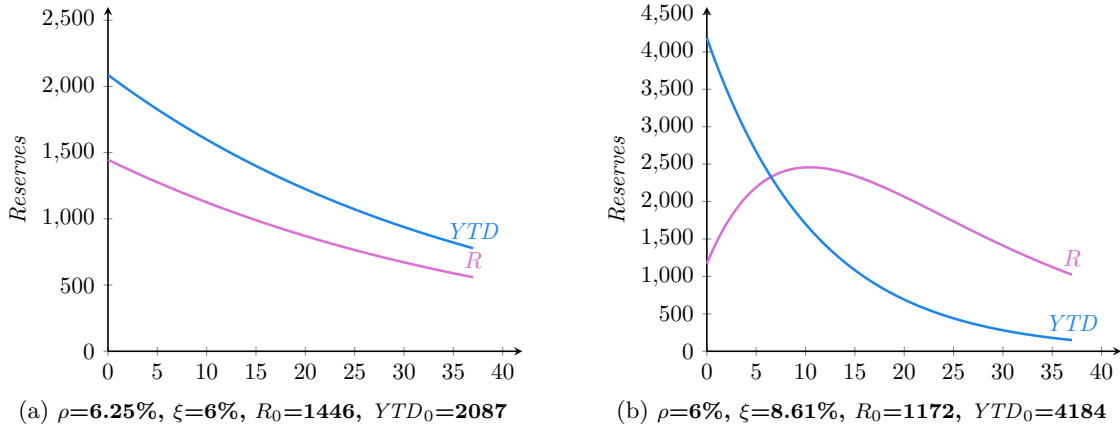
Figure 17: **Resource profiles**

Following the assumptions from the GREEN model[12] Envisage assumes that the rate of extraction for oil and gas is exogenous, i.e. invariant to market conditions, but that the discovery rate is affected by market conditions. The discovery rate will be a function of the deviation of the market price from a given pre-determined price trend, where the market price is the output price for individual supplies. The elasticity is allowed to differ depending on

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10   With *XFD* exogenous, the revenue will be measured by equation (D-39).
11   We may want to generalize the allocation mechanism to allow activities to spend resources on adaptation.
12   Burniaux et al. (1992) and van der Mensbrugghe (1994).

market conditions. This kink in the discovery rate elasticity is implemented using the sigmoid function. In addition, we allow suppliers to be off their potential supply curve, i.e. they can supply less than their maximum given the exogenous extraction rate.

The key price relation that determines the elasticity of discovery is given by equation (G-32). The exogenous trend, *PTrend*, is indexed at 1 in the reference year. If output prices track the trend exactly, the *PRatio* variable will always have the value of 1. If production prices exceed the exogenous trend, the price ratio will be above 1, i.e. market conditions are buoyant, and vice versa if the ratio falls below 1. Equation (G-33) defines the elasticity according to the price regime. As the variable *PRatio* rises, the denominator tends towards 1 and at the limit, the active elasticity is the high elasticity. The limit of the denominator as prices decline is $1 + e^\kappa$, which for suitably large κ implies that the ratio is zero and the active elasticity is the low elasticity.

The conversion rate, ξ, responds to the price signal, *PRatio*, where the elasticity can differ depending on market conditions. The curve is anchored to the reference conversion rate, ξ^{Ref}. The shifter, $\chi^d_{r,a,t}$, can be used to calibrate the conversion rate, to an exogenous price scenario for example. Note that the conversion rate is bounded on both ends with user supplied lower and upper bounds. Figure 18 provides an illustration of the elasticity and conversion rate curves. Note that the elasticity curve is almost virtually kinked at the price reference point, i.e. where *PRatio* is equal to 1. On either side, the elasticity quickly moves to either the 'low' elasticity (0.3 in the figure), or the 'high' elasticity (1.2 in the figure). The conversion rate curve has three inflection points. With very low prices the conversion rate is bound below at 4% and at very high prices it is bound above at 10%. The third inflection point is where *PRatio* equals 1. The figure clearly shows the difference in the elasticity around this inflection point, with a steeper curve when prices are increasing.

$$PRatio_{r,a,t} = \left[\frac{PX_{r,a,t}/PGDP_{r,t}}{PX_{r,a,t-1}/PGDP_{r,t-1}} \right] \bigg/ PTrend_{r,a,t} \tag{G-32}$$

$$\omega^r_{r,a,t} = \omega^{rLo}_{r,a,t} + \left(\omega^{rHi}_{r,a,t} - \omega^{rLo}_{r,a,t} \right) / \left[1 + \exp\left(-\kappa(PRatio_{r,a,t} - 1) \right) \right] \tag{G-33}$$

$$\xi_{r,a,t} = \chi^d_{r,a,t} \xi^{Ref}_{r,a} \left(PRatio_{r,a,t} \right)^{\omega^r_t} \tag{G-34}$$
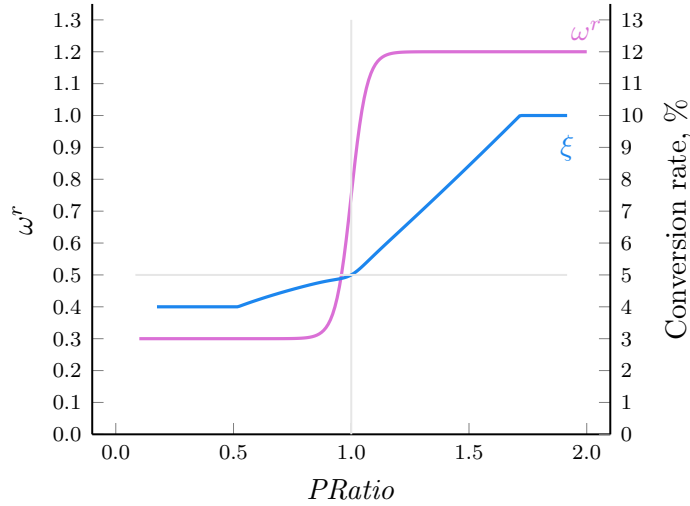
Figure 18: **Conversion rate curves**

The extraction rate of coal is assumed to be price sensitive and uses the same elasticity, ω^r, to drive the extraction rate. In the case of oil and gas, the extraction rate is allowed to vary with a constant elasticity functional form, though generally the elasticity will be zero. Equation (G-35) defines the extraction rate, ρ^x, for coal, oil and gas reserves. For coal, the elasticity is potentially kinked and thus there will be an inflection point for the extraction curve. For oil and gas, the extraction elasticity is given by ω^e.

$$\rho^x_{r,a,t} = \begin{cases} \chi^x_{r,a,t} \rho^x_{r,a,t-1} \left(PRatio_{r,a,t} \right)^{\omega^r_{r,a,t}} & \text{if } a \in \{'Coal'\} \\ \chi^x_{r,a,t} \rho^x_{r,a,t-1} \left(PRatio_{r,a,t} \right)^{\omega^e_{r,a,t}}_{r,a,t} & \text{if } a \notin \{'Coal'\} \end{cases} \tag{G-35}$$

Equation (G-36) determines the actual reserve level. It is equal to the previous period's reserve less less actual extraction, *Extr*, plus the addition of converted unproven reserves. Equation (G-37) is the level of reserves that

correspond to being on the reserve profile, i.e. where the actual extraction is equal to the extraction rate times the level of (lagged) reserves. If market conditions warrant it, producers may wish to be off their reserve profile and keep the resource underground. In this case, actual production may be less than potential production. Equation (G-38) determines the gap between actual reserves and reserves along the resource profile, which is always a positive number. Potential supply, equation (G-39) is equal to the normal level of extraction from the reserve profile, plus what might be available from the reserve gap, for regions that are not forced to be on their depletion profile.

$$Res_{r,a,t} = Res_{r,a,t-1} - Extr_{r,a,t-1} + \xi_{r,a,t-1} YTD_{r,a,t-1} \tag{G-36}$$

$$Res^p_{r,a,t} = \left(1 - \rho^x_{r,a,t-1}\right) Res^p_{r,a,t-1} + \xi_{r,a,t-1} YTD_{r,a,t-1} \tag{G-37}$$

$$ResGap_{r,a,t} \geq Res_{r,a,t} - Res^p_{r,a,t} \tag{G-38}$$

$$XF^{Pot}_{r,a,t} = \rho^x_t Res^p_{r,a,t} + ResGap_{r,a,t} \tag{G-39}$$

Equation (G-40) is the motion equation for yet-to-discover reserves—simply previous unproven reserves less those converted to proven.

$$YTD_{r,a,t} = (1 - \xi_{r,a,t-1}) YTD_{r,a,t-1} \tag{G-40}$$

4.6.1 Linking depletion variables to natural resource supply

The standard supply equation for natural resources is modified for coal, oil and gas reserves when the depletion module is on. Instead of determining XF^s as in the standard specification, it determines, XF^{Not}, i.e. the notional supply of the natural resource. So we have two variables, the notional supply that comes from the standard natural resource supply curve, and the potential supply, XF^{Pot}, which comes from the depletion module. Equation (G-41) determines the gap between notional and potential supply. The gap variable is bounded below by zero, in other words if potential supply is greater than notional supply, the gap variable is set to 0. If notional supply exceeds potential supply, producers would like to supply more, but are constrained to the potential supply and in this case, actual supply is equal to potential supply. On the other hand, if notional supply is below potential supply, producers would prefer to sell less than their potential and will supply the notional amount. The gap between the actual and potential supply is then kept in the ground. These conditions are captured in equation (G-42). If the gap is zero, actual supply is equal to notional supply and below potential. If the gap is positive, actual supply is equal to potential supply. The level of extraction will track the actual supply of natural resources.

$$XF^{Gap}_{r,a,t} \geq XF^{Not}_{r,a,t} - XF^{Pot}_{r,a,t} \tag{G-41}$$

$$XF^s_{r,a,t} = XF^{Not}_{r,a,t} - XF^{Gap}_{r,a,t} \tag{G-42}$$

$$Extr_{r,a,t} = Extr_{r,a,t-1} \left(\frac{XF_{r,nrs,a,t}}{XF_{nrs,a,t-1}} \right) \tag{G-43}$$

4.7 Knowledge module

[NEW 20-JAN-2019] The model includes a knowledge module that operates somewhat similarly to investment and the capital stock. Each year there is aggregate expenditure on research and development (R&D), that is captured by the variable XFD_{r_d}. This expenditure flow increases knowledge, though knowledge also depreciates each year. The main difference with the usual capital/investment dynamic, is that the impact of R&D expenditures happens with a distributed lag.[13] The latter is governed by a *Gamma* distribution function, the parameters and length of which are region specific.[14]

Equation (G-44) describes the motion equation for knowledge. The variable KN represents the stock of knowledge. It is equal to the previous period's knowledge stock—adjusted by depreciation, δ^k—, plus the distributed lag of current and previous expenditures on R&D, where γ^k represents the lag coefficients.[15] Annex C.4 describes how the model

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[13]   This formulation is inspired by Smeets Křístková et al. (2016).
[14]   Additional details and example distribution functions are provided in Annex C.4.
[15]   If $\gamma^k_0 = 0$ and $\gamma^k_1 = 1$, the equation would have the same assumption as the standard capital accumulation equation.

implements the knowledge module. Implementation requires taking into account two features. (1) It is implemented using the variable time step that is a central part of the model. Because of the distributed lags, this also requires that the R&D expenditures be evaluated each *year*, independently of the time step. (2) The first years of the simulation require a backward projection of the R&D expenditures, i.e. prior to the initial year. If N is 50 for example, it requires projecting R&D expenditures back some 50 years.

$$KN_{r,t} = \left(1 - \delta_r^k\right) KN_{r,t-1} + \sum_{i=0}^{N} \gamma_{r,i}^k XFD_{r,r\_d,t-i} \tag{G-44}$$

where

$$\gamma_i^k = \chi \left(i+1\right)^{\delta/(1-\delta)} \lambda^i$$

In its default mode, the user inputs R&D expenditures as a (percent) share of reference year GDP. The default closure fixes the (real) GDP share of R&D expenditures at the reference year level. The model specification assumes that R&D expenditures are paid for from government revenues.

The knowledge module is designed to impact on labor productivity growth. Equation (G-45) describes the relation between the endogenous component of labor productivity in sector a, π^k, relative to the growth rate of the stock of knowledge. The elasticity is given by ϵ^r and is allowed to be sector specific. The shifter, γ^r, is initialized at 1.

$$\pi_{r,a,t}^k = \gamma_{r,a,t}^r \epsilon_{r,a,t}^r \left(\frac{KN_{r,t}}{KN_{r,t-1}} - 1\right) \tag{G-45}$$

4.8 Technology shifts

By default, changes in technology are largely exogenous, for example autonomous energy efficiency improvement (AEEI) and improvements in international trade and transport services. The one exception is labor productivity. Equation (G-46) describes the growth in labor productivity. In the base year, λ^f is initialized to 1, and is set to grow at a rate of γ^l each year. If α^{gl} is set to 0 and β^{gl} is set to 1, then labor productivity grows at a uniform rate across all activities. There are multiple strategies to put values on these parameters. These may include setting β^{gl} to 0 for a subset of activities and initializing α^{gl} to some exogenous level based on external studies—for example setting α^{gl} to 2 percent in the agricultural sectors. The remaining sectors could be segmented into two subsets—one with high productivity, e.g. set α^{gl} to 2 percent, and one with low productivity, e.g. set α^{gl} to 0, with β^{gl} set to 1 in both cases.

Two other factors can influence sector-specific labor productivity. The first is a skill-specific shifter, χ^{gl}, that introduces biased technical change across skill types. In typical simulations, this factor is initialized at 0 and there is no difference in labor productivity growth across skills. One alternative strategy is to have some (exogenous) assumption on the skill premium, for example no change in the skill premium. It is then possible to endogenize skill-biased technical change to see how much this parameter must change to maintain the exogenous assumption on the skill premium. The second factor is the knowledge-induced endogenous technical change, π^k. This formulation is described in the knowledge module, see for example equation (G-45).

In the baseline or reference scenario, γ^l will be calibrated to target a given level of per capita GDP growth and subject to the assumptions made on α^{gl} and β^{gl}. In subsequent scenarios, λ^f will be exogenous having been calibrated in the reference scenario, subject to any changes in the two other factors: χ^{gl} and π^k.

$$\lambda_{r,l,a,t}^f = \left(1 + \chi_{r,l,t}^{gl} + \pi_{r,a,t}^k + \alpha_{r,l,a,t}^{gl} + \beta_{r,l,a,t}^{gl} \gamma_{r,t}^l\right)^n \lambda_{r,l,a,t-n}^f \tag{G-46}$$

Because *Old* capital is malleable, the cost structure of *Old* capital is adjusted between periods in such a way that the aggregate inputs of the previous period (i.e. summed over vintages) can produce aggregate output. There is no such adjustment of the cost structure of *New* capital. In generic terms, for any nest in the production structure we have inputs $X_{i,v,t}$ at price $P_{i,v,t}$ to form the bundle $V_{v,t}$ at price $R_{v,t}$, with technological shifters given by $\lambda_{i,v,t}$. The basic re-calibration formula is:

$$\alpha_{i,Old,t+1} = \lambda_{i,Old,t}^{(1-\sigma_{Old})} \frac{\sum_v X_{i,v,t}}{\sum_v V_{v,t}} \left[\frac{\sum_v P_{i,v,t} X_{i,v,t} \Big/ \sum_v X_{i,v,t}}{\sum_v R_{v,t} V_{v,t} \Big/ \sum_v V_{v,t}}\right]^{\sigma_{Old}}$$

where σ is the relevant substitution elasticity for *Old* capital. The formula remains the same for the normalized version of the CES function.

4.9 Servitization

[Added 13-FEB-2022] Code has been introduced to allow for the 'servitization' of the economy, essentially defined as the change in the production structure that rotates towards services demand away from manufactured demand. The implementation is based on the description from Chateau et al. (2020).

Let k represent the 'services' inputs, m represent the manufactured inputs, and c represent the union of the two.[16] The basic idea is to raise the share of services inputs each year by a growth factor, for example 1%, and to adjust downwards the the share of manufactured inputs. A cap is introduced on the share, for example 60%.[17] In ENVISAGE, this is done in a cost-neutral fashion, i.e. using the 'twist' methodology—further described in the next section and in Appendix A.

The initial share of services in activity a with respect to all relevant inputs, i.e. c, is given by the following formula. Note that the denominator does not quite have the same value as the *ND1* bundle, as it excludes some inputs such as transport services.

$$s_{a,t0}^t = \frac{\sum_k PA_{k,a,t0} XA_{k,a,t0}}{\sum_c PA_{c,a,t0} XA_{c,a,t0}}$$

The following expression provides the desired growth in the share of services in production, where gs is the user-provided growth rate. The share is capped at $smax^t$.

$$s_{a,t}^t = min \left[s_{a,t-n}^t (1 + gs_a)^n, smax_{a,t}^t \right]$$

Equations (G-47) and (G-48) represent the update formulas for the input-output technology parameters that thus incorporate the servitization assumption.

$$\lambda_{k,a,t}^{io} = \lambda_{k,a,t-1}^{io} \left(\frac{s_{a,t-1}^t}{s_{a,t}^t} \right)^{1/(1-\sigma_a^{nd1})} \tag{G-47}$$

$$\lambda_{m,a,t}^{io} = \lambda_{m,a,t-1}^{io} \left(\frac{1 - s_{a,t-1}^t}{1 - s_{a,t}^t} \right)^{1/(1-\sigma_a^{nd1})} \tag{G-48}$$

It is pretty straightforward to show that the so-called 'twist' variable takes the following form in this case:

$$tw_{a,t} = \frac{s_{a,t}^t - s_{a,t-1}^t}{s_{a,t-1}^t \left(1 - s_{a,t}^t \right)}$$

4.10 Preference shifters

4.10.1 Armington twists

In the standard version of the model, the Armington preference parameters—both top and second levels—are held fixed at their base year calibrated levels. Thus changes in import shares are linked only to changes in relative prices. Historically, we have observed large changes in import shares without a concomitant change in relative prices. Dixon and Rimmer (2002) have developed a specification for the MONASH (and USAGE) model that 'twists' the relevant share parameters. The 'twist' targets a percentage change in the ratio of imports to domestic demand, which at unchanged component prices, is cost-neutral. The mechanics of the twist methodology is described in Appendix A.

For the ENVISAGE model, we have incorporated three versions of the twist—for the dual nested sourcing decision. The first two deal with the top-level Armington decision—purchases of domestic goods and the aggregate import bundle. Since we have two possible specifications—agent-based sourcing or national sourcing, we provide two twist specifications. The third is for the second level nest that sources aggregate imports by region of origin.

National sourcing

The equations for national sourcing of domestic purchases and aggregate imports are provided in equations (T-2) and (T-3), with the domestic and import share parameters given respectively by α^{dt} and α^{mt}. Equation (G-49) provides the 'twisted' domestic and imported share parameters where tw^{t1} is the level of the desired twist, i.e. the

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[16]   In Chateau et al. (2020) k represents all non-transport services, (though it is unclear how 'ELY' and 'CNS' are treated. It is also not clear what is included in the manufactured inputs, e.g. energy?).

[17]   In Chateau et al. (2020) the 1% and 60% assumptions apply to the manufacturing activities. The relevant numbers in the services sectors are 0.2% and 75%.

percent change in the ratio of imports to domestic purchases at unchanged prices. The twist has no impact on the share parameters if it is set to its default value of 0. The twist updating uses the (lagged) value share of imports in the top-level Armington bundle.[18] Note that the formulas apply the twist factor directly to the share coefficients themselves and we drop the λ factor, i.e. preference change variable, from the formulas as these can be substituted out (refer to Appendix A).

$$
\begin{aligned}
\alpha_{r,i,t}^{dt} &= \alpha_{r,i,t-1}^{dt} \frac{1}{1 + s_{r,i,t-1}^{t1} tw_{r,i,t}^{t1}} \\
\alpha_{r,i,t}^{mt} &= \alpha_{r,i,t-1}^{mt} \frac{1 + tw_{r,i,t}^{t1}}{1 + s_{r,i,t-1}^{t1} tw_{r,i,t}^{t1}}
\end{aligned}
\tag{G-49}
$$

where

$$
s_{r,i,t-1}^{t1} = \frac{PMT_{r,i,t-1} XMT_{r,i,t-1}}{PAT_{r,i,t-1} XAT_{r,i,t-1}}
$$

Agent sourcing

The expressions for agent-based sourcing of domestic goods and aggregate imports are nearly identical with the inclusion of the index for the Armington agent, aa. The share parameters are part of equations (T-7) and (T-8). Equation (G-50) provides the relevant formulas for the 'twisted' shares using the appropriate import share level. Note that the twists are allowed to be agent-specific, though in practice, there is probably little evidence to differentiate across agents.

$$
\begin{aligned}
\alpha_{r,i,aa,t}^{d} &= \alpha_{r,i,aa,t-1}^{d} \frac{1}{1 + s_{r,i,aa,t-1}^{1} tw_{r,i,aa,t}^{1}} \\
\alpha_{r,i,aa,t}^{m} &= \alpha_{r,i,aa,t-1}^{m} \frac{1 + tw_{r,i,aa,t}^{1}}{1 + s_{r,i,aa,t-1}^{1} tw_{r,i,aa,t}^{1}}
\end{aligned}
\tag{G-50}
$$

where

$$
s_{r,i,aa,t-1}^{1} = \frac{PM_{r,i,aa,t-1} XM_{r,i,aa,t-1}}{PA_{r,i,aa,t-1} XA_{r,i,aa,t-1}}
$$

Second level Armington twists

The second level of the Armington nest allocates aggregate imports across source regions (see equation (T-15)). With the sole exception of a 2-region model, the standard twist does not apply and we thus apply the twist to a target set of source countries, i.e. we segment the source regions into 2.[19] The targeted regions will see their share increase (by the same twist), and the non-targeted regions will see their share decline.

$$
\begin{aligned}
\alpha_{s,i,r,t}^{w} &= \alpha_{s,i,r,t-1}^{w} \frac{1}{1 + s_{i,r,t-1}^{2} tw_{r,i,t}^{2}} \quad \text{if } s \notin rtwtgt(s,r) \\
\alpha_{s,i,r,t}^{w} &= \alpha_{s,i,r,t-1}^{w} \frac{1 + tw_{r,i,t}^{2}}{1 + s_{i,r,t-1}^{2} tw_{r,i,t}^{2}} \quad \text{if } s \in rtwtgt(s,r)
\end{aligned}
\tag{G-51}
$$

where

$$
s_{i,r,t-1}^{2} = \frac{\sum_{s \in rtwtgt(s,r)} PDM_{s,i,r,t-1} XW_{s,i,r,t-1}}{\sum_{s} PDM_{s,i,r,t-1} XW_{s,i,r,t-1}}
$$

4.10.2 Introducing cost curves

Changes in costs in the normal functioning of the model depend on changes in input prices and the standard assumptions regarding technology change. This section explains how to introduce an acceleration in cost reduction coming from some exogenous phenomenon such as learning by doing. The basic idea is to start from an initial price, say P_0, and to reduce costs over time, albeit with a lower limit given by *PMIN*. There also exists a target price for year T, given by P_T, that must be greater than *PMIN*. Let α represent the ratio of the final price, P_T, relative to the base year price, P_0, i.e.:

---

[18] In the MONASH and USAGE models, the twist parameters are calculated contemporaneously, i.e. they are part of the model equations. In ENVISAGE, the twists are applied between periods. This should have only minimal impact on model results.

[19] The targeted regions can include only a single region.
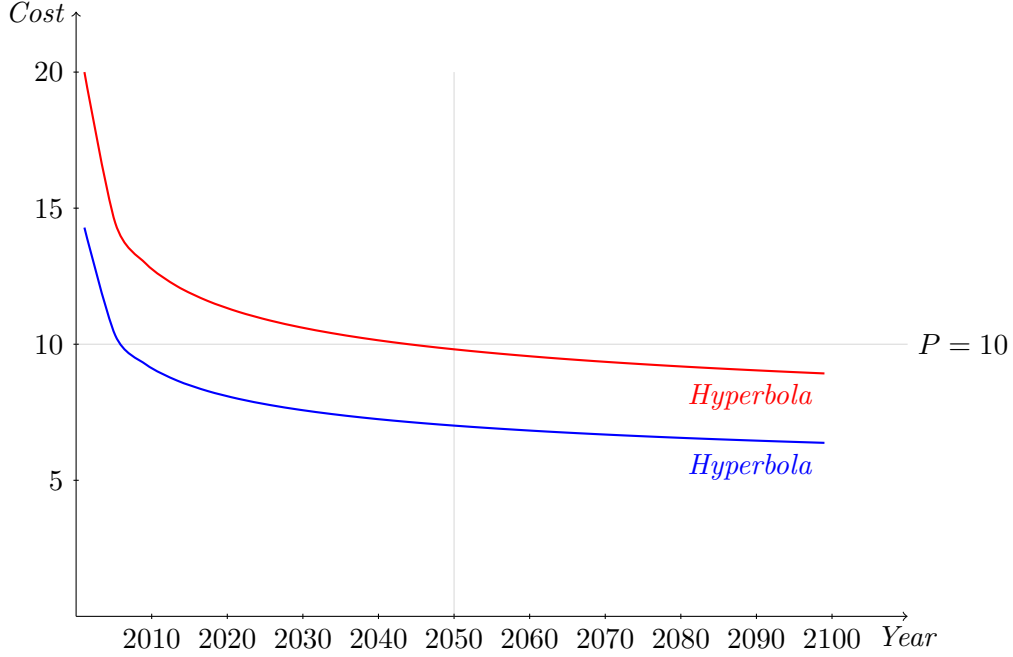
Figure 19: **Examples of cost curves**

$$P_T = \alpha P_0$$

and let γ represent the ratio of the minimum price, $PMIN$, relative to the initial price, i.e.:

$$PMIN = \gamma P_0$$

where $\alpha > \gamma$. Two functional forms are introduced to represent the cost curve—the hyperbola and the logistic functions. The hyperbola takes the following form:

$$P_t = PMIN + \chi t^{-\beta}$$

The logistic function takes the form:

$$P_t = \frac{PMIN}{1 + \chi e^{-\beta t}}$$

Calibration of the parameters for the hyperbola function involve the following expressions:

$$\beta = \frac{\ln\left(\dfrac{\alpha - \gamma}{1 - \gamma}\right)}{\ln\left(\dfrac{t_0}{T}\right)} \qquad \chi = P_0 \left(1 - \gamma\right)$$

where we define t_0 to start with 1.[20] In the case of the logistic function, the calibration formulas are:

$$\beta = -\frac{1}{T} \ln\left(\frac{\alpha - \gamma}{\alpha \left(1 - \gamma\right)}\right) \qquad \chi = \gamma - 1$$

where we define t_0 to start with 0.[21] Figure 19 depicts the shapes of the two cost curves for the same initial and end points and price limit. From the shape of the curves it is clear that costs drop sharply in the initial years using the hyperbola specification. The drop is more gradual (and constant in percentage terms) for the logistic function.

---

[20]   If the first year is 2011, t is then defined as $t - 2010$. If the final year is 2050, then T is equal to $2050 - (2011 - 1) = 40$.

[21]   If the first year is 2011, t is then defined as $t - 2011$. If the final year is 2050, then T is equal to $2050 - 2011 = 39$.

The adjustment to the cost curve is made on total factor productivity (TFP) for the relevant activity. The aggregate cost is given by the following formula:

$$P_t = \frac{1}{\lambda_t} F(P_{i,t})$$

where F is the cost function and depends on the prices of the various inputs given by P_i. In most cases, the cost function will be the dual price expression of the CES function.[22] TFP is assumed to behave according to the following expression:

$$\lambda_t = (1 + \pi_t)^n \lambda_{t-n}$$

Assuming that input prices are invariant, the growth in TFP must be equal to the following expression:

$$\pi_t = \left(\frac{P_{t-n}}{P_t} \right)^{1/n} - 1$$

We can introduce the formulas for the two cost curves to derive expressions for π. For the hyperbola, we have:

$$\pi_t = \left(\frac{PMIN + \chi(t-n)^{-\beta}}{PMIN + \chi t^{-\beta}} \right)^{1/n} - 1 \iff \left(\frac{\gamma + (1-\gamma)(t-n)^{-\beta}}{\gamma + (1-\gamma)t^{-\beta}} \right)^{1/n} - 1$$

For the logistic function, the expression for π is given by:

$$\pi_t = \left(\frac{e^{\beta t} + \chi}{e^{\beta t} + \chi e^{\beta n}} \right)^{1/n} - 1$$

Both changes in preferences and introduction of the cost curves can be implemented to change the share of any technology in a demand bundle. The latter operates directly on the preference shares of the buyer. The former operates on the perceived cost of the technology. These are formulated independently of the actual cost changes that arise from the endogenous changes in input prices.

Alternative cost curves

One alternative for the hyperbolic cost curve is to calibrate the curve through three data points: the reference year and two future years—respectively t_0, t_1 and t_2. The curve can be defined by:

$$P_t = P_{t_0} + \chi (t - t_0)^{-\beta}$$

The exponent can be calibrated using the following formula:

$$\beta = \ln \left(\frac{P_{t_1} - P_{t_0}}{P_{t_2} - P_{t_0}} \right) / \ln \left(\frac{t_2 - t_0}{t_1 - t_0} \right)$$

The slope coefficient can be calibrated using:

$$\chi = (P_{t_1} - P_{t_0}) (t_1 - t_0)^{\beta}$$

4.10.3 Electrification preferences

Each agent in the economy has demand for an aggregate energy bundle. The parameters are calibrated to the base year data—and in many instances, most agents have relatively low shares for electricity demand. It might thus prove desirable to autonomously increase electrification to a user's targeted share. The formulas below are introduced assuming constant prices and ignoring the re-calibration of 'Old' share parameters that is part of a dynamic simulation. The relevant preference parameters are equal to the ratio of the target share in year t relative to the actual base year share. The target share can be some multiple of the base year share, or any other assumption desired by the user. Note that it is up to the user to make sure that the targeted share is within a normal range.

Calibrated preference parameters between electric and non-electric bundles in production.

$$\alpha_{r,a,v,t}^{ely} = \frac{s_{r,a,t}^{ely}}{s_{r,a,t_0}^{ely}}$$

---

[22] In most cases λ is initialized at 1 and is held constant. Technology is introduced as labor-augmenting technical change and with an exogenous improvement in energy efficiency.

$$\alpha_{r,a,v}^{nely} = \frac{1 - s_{r,a,t}^{ely}}{1 - s_{r,a,t_0}^{ely}}$$

Calibrated preference parameters between electric and non-electric bundles in consumption.

$$\alpha_{r,k,h,t}^{cely} = \frac{s_{r,h,t}^{cely}}{s_{r,h,t_0}^{cely}}$$

$$\alpha_{r,k,h,t}^{cnely} = \frac{1 - s_{r,k,h,t}^{cely}}{1 - s_{r,k,h,t_0}^{cely}}$$

If the user provides a target share for some future year, say T[23], the intermediate shares can be adjusted using several different smoothing assumptions, for example geometric or arithmetic growth.

$$s_{r,a,t}^{ely} = \begin{cases} s_{r,a,t_0}^{ely} \left[\frac{s_{r,a,T}^{ely}}{s_{r,a,t_0}^{ely}}\right]^{(t-t_0)/(T-t_0)} & \text{if } t < T \text{ and geometric growth} \\ s_{r,a,t_0}^{ely} + \left[s_{r,a,T}^{ely} - s_{r,a,t_0}^{ely}\right] \frac{t - t_0}{T - t_0} & \text{if } t < T \text{ and arithetic growth} \\ s_{r,a,T}^{ely} & \text{if } t \geq T \end{cases}$$

4.11   Implementation

The `core` model includes all of the equations from Chapter 3.[24]   There are two versions of the dynamic model. The reference simulation, that targets GDP growth, is defined as the `core` model plus the GDP growth equation (M-5), migration (G-1), labor supply by skill and zone (G-2), total labor supply by skill (G-4), investment growth factor (C-1), non-normalized capital stock (C-2), normalized capital stock (G-6), and labor productivity (G-46). This model is called `coreBaU` and it essentially calibrates the economy-wide labor productivity factor. All other dynamic simulations assume that labor productivity is exogenous i.e. per capita GDP growth is endogenous. This model is called `coreDyn` and includes the `core` model plus the GDP growth equation (M-5), migration (G-1), labor supply by skill and zone (G-2), total labor supply by skill (G-4), investment growth factor (C-1), non-normalized capital stock (C-2), and normalized capital stock (G-6).

4.12   Diagnostics

There are a number of post-simulation diagnostics available with the model. Most are relatively straightforward and need no additional explanation, for example the GDP deflator at market price—these are taken directly from the model solution file and aggregated where appropriate.

4.12.1   Sectoral investment

Sectoral investment can be backed out by the capital accumulation motion equation:

$$I_{a,t} = K_{a,t+1} - (1 - \delta)K_{a,t}$$

4.13   Exogenous variables

In this section, we summarize the standard list of exogenous variables in the model. These are partitioned into sub-groups depending on their role in the model specification.

4.13.1   Factors of production

Population and labor force growth are typically exogenous. The latter may be defined over rural and urban if using the migration module. In this case, the growth of labor is exogenous, but the growth per zone may be influenced by endogenous rural to urban migration. The capital stock is exogenous in the comparative static version of the model

---

[23] T is often the final year of a simulation.

[24]   With the exception of the GDP per capita growth equation as this requires a different MCP formulation.

and is in principle pre-determined in the dynamic model with annual step sizes. In the case of multi-year step size, current investment will influence the current stock level (see equation (C-2)). Natural resources are supplied with an upward sloping supply curve with two, normally fixed, shifters. The first shifter is a regional shifter. The second is a global shifter. The latter has been used to target a trend for world prices (e.g. oil or gas). In this case the world price for the relevant sector is fixed to the desired trend level and the global resource supply shifter is endogenous. This would typically be implemented only in a baseline. The land and water asymptotes are used-entered, as is the demand for exogenous water bundles.

Table 4.1: **Exogenous factor variables**

Variable	GAMS	Description
Pop^t	pop	Population
g^{lz}	glabz	Growth of labor by zone
χ^{rw}	chirw	Reservation wage shifter
K^s	kstock	Capital stock (endogenous in dynamic version)
TK^s	tkaps	Normalized capital stock (endogenous in dynamic version)
$\chi^{w,nrs}$	wchinrs	Global natural resource supply shifter
χ^{nrs}	chinrs	Region specific natural resource supply shifter
PW	pw	World price by activity
$TLand^{Max}$	landmax	Land maximum
$TH2O^{Max}$	h2oMax	Water maximum
$H2OBnd_{wbx}$	h2obnd	Exogenous water bundle demand

4.13.2   Macro closure

There are three different closure rules—all with default assumptions. The savings-investment closure fixes the household savings rate, aps, adjusted by the shift parameter χ^s. The former is always fixed. The latter can be endogenous to achieve a given target—for example the investment to GDP ratio. The latter is endogenous under the default closure rule but may be over-ridden. Typically, in the baseline, one will target some level of the investment to GDP ratio and allow the private savings rate adjust.[25] In a similar vein, the wage premium, normally the skilled wage relative to unskilled wages, is endogenous. One could target the wage premium, i.e. make it exogenous. The endogenous variable could be the relative growth of skilled and unskilled labor in levels or the relative growth in labor productivity. The default public closure is that government expenditures are fixed relative to GDP in real terms (in the baseline) and real government savings are fixed. The direct tax rate, κ^h, on household income, is the swing variable that ensures the fiscal target. The default balance of payment closure fixes net foreign saving. The real exchange rate is endogenous. Two additional closures are available. One fixes the share of foreign savings relative to GDP—similar in many ways to the default closure. The third allows for some endogeneity in foreign savings, in which case the deviation of expected capital returns is equalized across all regions.

---

[25]   The targeted investment variable is `rinvshr` in the GAMS code.

Table 4.2: **Exogenous macro closure variables**

Variable	GAMS	Description
aps	aps	Average savings propensity (not for ELES)
χ^s	chiaps	Private savings shifter (depends on investment/savings closure)
π^s	wprem	Wage premium
RS^g	rsg	Real public savings (depends on fiscal closure)
YFD_{gov}	xfd(gov)	Government expenditures
S^f	savf	Foreign saving (depends on BoP closure)
χ^{sf}	savRat	Foreign saving as share of GDP (depends on BoP closure)
$PNUM$	pnum	Model numeraire

4.13.3   Tax rates

In the default specification all tax rates are exogenous. The actual tax rate used could depend on model specification choices, for example, a national versus agent-based representation of the second level Armington specification.

Table 4.3: **Exogenous tax rate**

Variable	GAMS	Description
τ^p	ptax	Production tax
τ^{uc}	uctax	Unit cost tax
τ^v	pftax	Tax on factor use
κ^f	kappaf	Factor income tax by source activity and factor
τ^a	patax	Tax on Armington consumption (depends on ArmFlag)
τ^{ad}	pdtax	Tax on domestic sales of domestic goods (depends on ArmFlag)
τ^{am}	pmtax	Tax on domestic sales of imported goods (depends on ArmFlag)
τ^e	etax	Bilateral export tax
$\tau^m_{s,i,r}$	mtax	Bilateral import tariff
τ^{ma}	mtaxa	Import tariff for MRIO version of model

4.13.4   Technology variables

In most cases all of the technology variables are exogenous, though not necessarily fixed at base year levels—for example, the international trade margins or energy efficiency. The model calibrates labor-biased technological change (the λ^f parameter for labor) in the baseline to target GDP growth. In this case, g^y, i.e. per capita output growth, is exogenous, and γ^l is endogenous. An equation in the model evaluates λ^f for labor based on the exogenous GDP growth target. In all subsequent simulations the labor productivity variable is fixed and GDP growth is endogenous.

Table 4.4: **Exogenous technology variables**

Variable	GAMS	Description
ζ^{mg}	`tmarg`	Bilateral trade margins
A^{xpv}	`axghg`	Uniform shifter in production bundle
λ^{xp}	`lambdaxp`	Output shifter in production bundle
λ^{ghg}	`lambdaghg`	GHG shifter in production bundle
λ^{f}	`lambdaf`	Productivity parameter for primary factors
λ^{k}	`lambdak`	Capital efficiency shifter
λ^{h2ob}	`lambdah2obnd`	Water efficiency shifter in water bundle use
λ^{io}	`lambdaio`	Efficiency shifter in intermediate demand
λ^{e}	`lambdae`	lambdae
λ^{ce}	`lambdace`	lambdace
g^{y}	`grrgdppc`	Growth rate of real per capita GDP
γ^{l}	`gl`	Economywide labor productivity parameter

4.13.5 Preference variables

Preference variables are all fixed, but may depend on which utility function is implemented. They could also be adjusted between solution periods to target some trend growth, for example the food budget, as incomes change.

Table 4.5: **Exogenous preferences variables**

Variable	GAMS	Description
e^{h}	`eh`	CDE expansion parameter
b^{h}	`bh`	CDE substitution parameter
α^{h}	`alphah`	CDE shift parameter
γ	`gamma`	AIDADS/E/LES marginal propensity to consume parameter
μ^{c}	`muc`	AIDADS/E/LES Subsistence minima

4.13.6 Emission variables

The default specification for emissions is that they are endogenous and the emissions tax is exogenous (and in most cases set to zero[26]). One could target a trend in global emissions for any particular gas or pollutant and allow a global shifter—attached to the emission equations—be endogenous (χ^{Emi}). The model also allows for non-model based emissions, either at a regional (e.g. deforestation) or global level. The emission regime variables are described in the emissions section.

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[26] The exceptions could include the price on carbon emissions in the European Union's Emission Trading Scheme, or an interpretation of the Nationally Determined Commitments (NDCs) that are part of the Paris Agreeement

Table 4.6: **Emission variables**

Variable	GAMS	Description
χ^{Emi}	chiEmi	Global shifter in emissions
$EmiOth$	emiOth	Other non-modeled emissions (by region)
$EmiOthGbl$	emiOthGbl	Other non-modeled global emissions
τ^{Emi}	emitax	Emission tax
Emi^{Cap}	emiCap	Emission cap
$EmiQ$	emiQuota	Emission quota
$EmiQY$	emiQuotaY	Emission quota revenues
τ^{EmiQ}	emiRegTax	Regional emission tax
χ^{Cap}	chiCap	Emissions cap shifter

Appendix A

The CES and CET functions

This appendix describes in full detail the two functional forms most widely used in CGE models-the constant-elasticity-of-substitution (CES) and constant-elasticity-of-transformation (CET) functions.

CES functions are widely used in demand functions where substitutability across different products and/or factors is needed and where the main objective is to minimize cost. CET functions are broadly used to determine supply functions across different markets where the main objective is to maximize revenues. The two are very similar in many ways and the algebraic derivations below will be more detailed for the CES function.

A.1 The CES function

A.1.1 Basic formulas

In production, the CES function is used to select an optimal combination of inputs (either goods and/or factors) subject to a CES production function. In consumer demand, the CES is used as a utility (or sub-utility) or preference function. In either case, the purpose is to minimize the cost of purchasing the 'inputs' subject to the production or utility function. In generic terms the system takes the following form:

$$\min_{X_i} \sum_i P_i X_i$$

subject to the constraint:

$$V = A \left[\sum_i a_i (\lambda_i X_i)^\rho \right]^{1/\rho}$$

The objective function represents aggregate expenditure. The constraint expression will be referred to as the CES primal function. The parameter A is an aggregate shifter that can be used to shift the overall production function (or utility function). Each input, X_i, is multiplied by an input-specific shifter, λ_i, that can be used to implement input-specific productivity increases (for example biased technological change), or specific changes in consumer preferences. The (primal) share coefficients, a_i, are typically calibrated to some base year data and held fixed. The CES exponent, ρ, is linked to the curvature of the CES function (and will be explained further below). For given component prices, P_i, and a given level of production or utility V, solving the optimization program above will yield optimal demand functions for the inputs, X_i.

The Lagrangian can be set up as:

$$\mathcal{L} = \sum_i P_i X_i + \Lambda \left(V - A \left[\sum_i a_i (\lambda_i X_i)^\rho \right]^{1/\rho} \right)$$

Taking the partial derivative with respect to X_i and the Lagrange multiplier Λ yields the following system of equations:

$$P_i = \Lambda a_i \lambda_i^\rho X_i^{\rho-1} A \left[\sum_i a_i (\lambda_i X_i)^\rho \right]^{(1-\rho)/\rho} = \Lambda a_i A^\rho \lambda_i^\rho X_i^{\rho-1} V^{1-\rho}$$

$$V = A \left[\sum_i a_i (\Lambda_i X_i)^\rho \right]^{1/\rho}$$

Taking the first expression, it can be multiplied by X_i, and then summed. This of course is equal to the value of the bundle, i.e. $P.V$, where P is the aggregate price:

$$P.V = \sum_i P_i X_i = \Lambda V^{1-\rho} A^\rho \sum_i a_i \lambda_i^\rho X_i^\rho = \Lambda V^{1-\rho} V^\rho = \Lambda V$$

This shows that Λ, the Lagrange multiplier is the same as the aggregate price, P. We can re-arrange expression above to get an expression for optimal input demand, where Λ is replaced by P:

$$X_i = a_i^{1/(1-\rho)} A^{\rho/(1-\rho)} \left(\frac{P}{P_i} \right)^{1/(1-\rho)} \lambda_i^{\rho/(1-\rho)} V$$

We finally end up with the following expression, where the CES primal exponent, ρ, is replaced by the so-called CES elasticity of substitution, σ:

$$X_i = \alpha_i (A\lambda_i)^{\sigma-1} \left(\frac{P}{P_i} \right)^\sigma V \tag{A-1}$$

where we made the following substitutions:

$$\sigma = \frac{1}{1-\rho} \Leftrightarrow \rho = \frac{\sigma-1}{\sigma} \Leftrightarrow \frac{\rho}{1-\rho} = \sigma - 1 \Leftrightarrow \rho.\sigma = \sigma - 1$$

and

$$\alpha_i = a_i^{1/(1-\rho)} = a_i^\sigma \Leftrightarrow a_i = \alpha_i^{1/\sigma}$$

Abstracting from the technology parameters, the demand equation implies that demand for 'input' X_i is a (volume) share of total demand V. The share, with equal prices is simply equal to α_i. With a positive elasticity of substitution, the share is sensitive to the ratio of prices relative to the aggregate price index. Since the component price is in the denominator, the demand for that component declines if its price rises relative to the average and vice versa if its price declines vis-à-vis the average price. The α parameters will be referred to as the CES dual share parameters (for reasons described below), and the a parameters are the primal CES share parameters. Notice that expression (A-1) simplifies if it is expressed in terms of efficiency inputs, X^e and efficiency prices, P^e:

$$X_i^e = \alpha_i A^{\sigma-1} \left(\frac{P}{P_i^e} \right)^\sigma V$$

where

$$X_i^e = \lambda_i X_i$$

and

$$P_i^e = \frac{P_i}{\lambda_i}$$

The aggregate price P can be determined using two expressions. The first is the zero profit condition:

$$P = \frac{\sum_i P_i X_i}{V}$$

The other is by inserting the optimal demand relation X_i (equation A-1) in the zero profit condition :

$$P.V = \sum_i P_i X_i = A^{\sigma-1} \sum_i P_i \alpha_i \left(\frac{P}{P_i} \right)^\sigma \lambda_i^{\sigma-1} V = P^\sigma A^{\sigma-1} V \sum_i \alpha_i \left(\frac{P_i}{\lambda_i} \right)^{1-\sigma}$$

The V's cancel out, and the aggregate price can then be expressed by the following formula:

$$P = \frac{1}{A} \left[\sum_i \alpha_i \left(\frac{P_i}{\lambda_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)} = \frac{1}{A} \left[\sum_i \alpha_i (P_i^e)^{1-\sigma} \right]^{1/(1-\sigma)} \tag{A-2}$$

This is sometimes referred to as the dual price expression. It has virtually the same functional form as the CES primal, which is a CES aggregation of the input volumes using the primal share parameters as weights. The CES dual price formula is a CES aggregation of the input prices using the CES dual share parameters as weights and a

different exponent. In a CGE model, the zero-profit condition or the dual price formula can be used interchangeably (with the proviso that the substitution elasticity differs from 1).[1] There is a simple formula for the budget shares given by:

$$s_i = \frac{P_i X_i}{P.V} = \alpha_i (A\lambda_i)^{\sigma-1} \left(\frac{P}{P_i}\right)^{\sigma} V\left(\frac{P_i}{P}\right) \frac{1}{V} = \alpha_i (A\lambda_i)^{\sigma-1} \left(\frac{P}{P_i}\right)^{\sigma-1} \tag{A-3}$$

Notice that this expression for the budget shares is only a function of prices. With the technology parameters set to 1, this simplifies further to:

$$s_i = \alpha_i \left(\frac{P}{P_i}\right)^{\sigma-1}$$

It turns out that the parameter σ measures the elasticity of substitution for the CES function and is constant over the entire domain. The elasticity of substitution is an indication of the curvature of an isoquant, see Varian (1992), i.e. it measures the rate of change of the ratio of inputs (in a 2-input case), relative to the change in their relative prices. For example, if the CES combines capital and labor to form output, a large substitution elasticity suggests that the factor proportions will change rapidly as one of the inputs becomes cheaper relative to the other. There are two limiting cases of interest. If the substitution elasticity is zero, then there is no substitution across inputs and the optimal choice is to use them in fixed proportion. At the other extreme, if the substitution elasticity is infinite, this is equivalent to saying the inputs are identical, and in this case, in equilibrium, the two inputs would have the same price. This could potentially be the case for electricity production. If there is a regional or national buyer of electricity, the buyer is most likely indifferent about how the electricity is produced and thus will purchase from the lowest cost producer (a perhaps somewhat simplified view of electricity markets.) This implies that the cost of the electricity inputs, from all sources (e.g. thermal, nuclear, etc.) would be (nearly) identical.

The elasticity of substitution across inputs is defined by the following formula:

$$\sigma = \frac{\partial\left(\frac{X_i}{X_j}\right)}{\partial\left(\frac{P_i}{P_j}\right)} \frac{\left(\frac{P_i}{P_j}\right)}{\left(\frac{X_i}{X_j}\right)}$$

The ratio of the optimal inputs using expression (A-1) is:

$$\frac{\alpha_i}{\alpha_j} \left(\frac{P_i}{P_j}\right)^{-\sigma} \left(\frac{\lambda_i}{\lambda_j}\right)^{\sigma-1}$$

Taking the partial derivative of the expression with respect to the ratio P_i/P_j and multiplying it by the second term of the elasticity of substitution yields the conclusion that the substitution elasticity is $-\sigma$. It is logical that it is negative. If the price of one input increases, say labor, relative to the other, say capital, producers would substitute away from labor towards capital, i.e. the ratio of labor to capital would drop as the price of labor increases relative to capital. Varian (1992) in fact defines the elasticity of substitution in terms of the absolute value of the technical rate of substitution, that measures the slope of the budget line. Numerically what it represents is the relative change in the ratios. If σ is 1, for example, and the price of labor increases by 10 percent relative to capital, the labor to capital ratio would decrease by (around) 10 percent.[2] The higher is σ, the more the proportion changes.

A.1.2 Special cases

There are three special cases that require additional derivations due to numerical restrictions on the primal and dual exponents. A substitution elasticity of 0 is clearly a special case and is referred to as a Leontief technology. From the dual price formula, it is clear that σ equal to 1 is a special case and is known as a Cobb-Douglas technology (or utility function). Finally, a value of ρ equal to 1 corresponds to infinite substitution elasticity and a linear primal aggregation function. This is also referred to as a case of perfect substitution.

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[1]   We shall see below that when the substitution elasticity is 1, both primal and dual expressions take a different functional form.

[2]   The elasticity is a marginal concept that holds only approximately for large changes.

The Leontief case

The first special case is for the so-called Leontief functional form.[3] In this case the substitution elasticity is 0 and corresponds to a value for ρ that is $-\infty$. In this case the optimization program takes the following form:[4]

$$\min_{X_i} \sum_i P_i X_i$$

subject to the constraint:

$$V = \min\left(\frac{a_i}{\lambda_i X_i}\right)$$

The visual implementation has L-shaped isoquants. The Leontief technology constraint or production/utility function is discontinuous. Fortunately, the optimal demand functions are easy to implement and are just special cases of expression (A-1):

$$X_i = \frac{\alpha_i}{\lambda_i}\frac{V}{A}$$

$$P = \frac{1}{A}\sum_i \alpha_i\left(\frac{P_i}{\lambda_i}\right)$$

Thus the Leontief specification implies that inputs are always in fixed proportion relative to output and the aggregate price is simply the linear weighted aggregation of the input prices, where the weights are given by the input-output coefficients, adjusted by changes in efficiency. The efficiency parameter has a nice intuitive interpretation in this case. Say λ increases by 10 percent, then demand for the input declines by 10 percent.

The Cobb-Douglas function

Another special case is the so-called Cobb-Douglas function, very frequently used in introductory text books in microeconomics. The Cobb-Douglas function has a substitution elasticity of 1 implying that ρ is equal to 0. Clearly, this creates a problem for specifying the CES primal function as well as the CES dual price function. As with the Leontief, the optimal demand conditions are given by expression (A-1), with σ set to 1:

$$X_i = \alpha_i\left(\frac{P}{P_i}\right)V \Leftrightarrow s_i = \frac{P_i X_i}{P.V} = \alpha_i$$

The Cobb-Douglas specification has constant budget shares irrespective of relative prices (and changes in technology). Another implication of the Cobb-Douglas specification is that the dual shares must add up to 1 as they are equivalent to the budget shares. By definition, as well, the primal and dual shares are the same. The Cobb-Douglas primal and dual price functions have the following expressions:

$$V = A\prod_i (\lambda_i X_i)^{\alpha_i}$$

$$P = \frac{1}{A}\prod_i \left(\frac{P_i}{\alpha_i \lambda_i}\right)^{\alpha_i}$$

Rather than code the Cobb-Douglas function as a special case, many modelers choose to replace the elasticity of 1 with a value close to 1 such as 1.01. This would have only marginal repercussions on the results.

Perfect substitution

The third special case is for a substitution elasticity of infinity. In this case ρ takes the value of 1 and the primal function is a straight linear aggregation of the inputs. The optimal demand conditions cannot be used in the case of an infinite substitution elasticity. In its stead, the optimal demand condition is replaced with the law-of-one-price, adjusted by efficiency differentials, and the zero profit condition is replaced with the CES primal function, i.e. the linear weighted aggregation of the inputs:

$$\frac{P_i}{\alpha_i \lambda_i} = P$$

---

[3]   Leontief, winner of the 1973 Nobel prize in Economics, is renowned for his work on input-output tables, much of which focused on fixed input technologies (!!!! reference).

[4]   !!!! need a reference

$$V = \sum_i \alpha_i \lambda_i X_i$$

The aggregation function can be replaced by the zero profit condition:[5]

$$P.V = \sum_i P_i X_i$$

A.1.3 Calibration of the CES function

Calibration typically involves inverting functional forms to evaluate the value of a parameter given initial values for variables. Prices and volumes, P_i, X_i, V and P, are normally initialized to a given database or SAM. This may or may not include actual price/volume splits. If not, prices will typically be initialized at unit value—potentially adjusted for a price wedge such as a tax or a margin. The substitution elasticities are also normally inputs—either derived from econometric estimation, other data bases or models, or from a literature review. This leaves the parameters λ_i, α_i and A to calibrate. The technology parameters are normally associated with dynamics, so there is little reason not to initialize them to unit value as they can be incorporated in the initial share parameter value without any loss in generality. Thus, the only parameters left to calibrate are the α_i from which it is possible to derive the primal share parameters, a_i, if needed. The calibration formula is derived from the inversion of equation (A-1):

$$\alpha_i = \left(\frac{X_i}{V} \right) \left(\frac{P_i}{P} \right)^\sigma (A.\lambda_i)^{1-\sigma} = \left(\frac{X_i}{V} \right) \left(\frac{P_i}{P} \right)^\sigma$$

The right-most term is the most used formula where the technology parameters are explicitly set to 1.[6]

A.1.4 Normalized CES

It is sometimes the case that the CES is badly scaled, particularly when working with actual price/volume splits. One work-around is to normalize all variables such that they are all equal to 1 in the base case—$\bar{X} = X/X_0 = 1$, where X_0 is a base level. The CES equations become:

$$X_{i,0}\bar{X}_i = \alpha_i (A\lambda_i)^{\sigma-1} \left(\frac{P_0\bar{P}}{P_{i,0}\bar{P}_i} \right)^\sigma V_0\bar{V}$$

$$P_0\bar{P} = \frac{1}{A} \left[\sum_i \alpha_i \left(\frac{P_{i,0}\bar{P}_i}{\lambda_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)}$$

The initial values can be collected to yield the following expressions:

$$\bar{X}_i = \alpha_i \left(\frac{V_0}{X_{i,0}} \left(\frac{P_0}{P_{i,0}} \right)^\sigma \right) (A\lambda_i)^{\sigma-1} \left(\frac{\bar{P}}{\bar{P}_i} \right)^\sigma \bar{V}$$

$$\bar{P} = \frac{1}{A} \left[\sum_i \alpha_i \left(\frac{P_{i,0}}{P_0} \right)^{1-\sigma} \left(\frac{\bar{P}_i}{\lambda_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)}$$

Define the following parameter:

$$\chi_i = \frac{V_0}{X_{i,0}} \left(\frac{P_0}{P_{i,0}} \right)^\sigma = \frac{1}{s_{i,0}} \left(\frac{P_{i,0}}{P_0} \right)^{1-\sigma}$$

where s_i is the initial value share defined above. The demand expression can be converted to:

$$\bar{X}_i = \alpha_i \chi_i (A\lambda_i)^{\sigma-1} \left(\frac{\bar{P}}{\bar{P}_i} \right)^\sigma \bar{V}$$

---

[5] Modelers have the choice of using the primal aggregation function or the revenue function. The latter holds in all three special cases for the substitution elasticity.

[6] In many introductions to CGE models, the calibration formulas explicitly exclude the price term. This is a dangerous practice that can lead to model bugs that can be hard to detect. It is best to explicitly initialize prices to 1 and use the correct calibration formula. In fact, one way to test model calibration and specification is to initialize prices to an arbitrary value and initialize volumes subject to these prices. Simulating a counter-factual with no shocks should replicate the initial data solution. If not, there is an error in initialization, calibration and/or specification.

Given the calibration formula, it is the case that $\alpha_i \chi_i = 1$. It may be nonetheless useful to carry both terms if technology or preference shifters are embodied in the α terms.

The price index expression can be written as:

$$\bar{P} = \frac{1}{A}\left[\sum_i s_{i,0}\alpha_i\chi_i\left(\frac{\bar{P}_i}{\lambda_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)}$$

If we do not need to carry through the original calibrated share parameters, we can write the two expressions as:

$$\bar{X}_i = (A\lambda_i)^{\sigma-1}\left(\frac{\bar{P}}{\bar{P}_i}\right)^{\sigma}\bar{V}$$

$$\bar{P} = \frac{1}{A}\left[\sum_i s_{i,0}\left(\frac{\bar{P}_i}{\lambda_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)}$$

Both expressions are now only a function of the normalized variables and the base year shares. Any subsequent use of the variables requires re-scaling, for example $X_{i,0}\bar{X}_i$. Note as well that the primal function in normalized form can be reduced to the following expression that represents the dual of the price function:

$$\bar{V} = A\left[\sum_i s_{i,0}\left(\lambda_i\bar{X}_i\right)^{\rho}\right]^{1/\rho}$$

A.1.5 Comparative statics

Elasticities

This section will derive some of the key elasticities of the CES function. The first relationship is the elasticity of the aggregate price with respect to a component price:

$$\frac{\partial P}{\partial P_i}\frac{P_i}{P} = s_i = \frac{P_i X_i}{P.V}$$

The elasticity of the aggregate price relative to an input price is equal to the budget share, irrespective of the substitution elasticity. The matrix of own- and cross-price elasticities, holding the aggregate volume constant is given by the following formula:

$$\varepsilon_{ij} = \frac{\partial X_i}{\partial P_j}\frac{P_j}{X_i} = \sigma(s_j - \delta_{ij})$$

where δ_{ij} is the so-called Kronecker's delta that takes the value 1 for i equal to j, else it takes the value 0. Since σ is positive, all components are gross substitutes in the CES.

Example: Trade elasticities

In this section we derive the elasticity of demand for a good sourced in region s, using the standard dual nested Armington formulation. Let XA be the top level Armington demand for a commodity that is derived from a generic demand function—for example an LES, with an associated price PA. The Armington demand is split into two components, XD, a domestic good, and XMT, an aggregate (or composite import demand), with prices PD and PMT respectively. The aggregate import bundle, XMT, is broken out by region of origin to determine XM_s, import from region s, with a domestic import price of PM_s (i.e. the landed or CIF price tariff inclusive).

Given XA, the following equations determine the top-level demand system, where σ^m is the top-level substitution elasticity (i.e. between the domestic goods and aggregate imports):

$$XD = \alpha^d\left(\frac{PA}{PD}\right)^{\sigma^m}XA \tag{A-4}$$

$$XMT = \alpha^m\left(\frac{PA}{PMT}\right)^{\sigma^m}XA \tag{A-5}$$

$$PA = \left[\alpha^d PD^{1-\sigma^m} + \alpha^m PMT^{1-\sigma^m}\right]^{1/(1-\sigma^m)} \tag{A-6}$$

The next set of equations determines imports by region of origin, where σ^w represents the elasticity of substitution for imports across region of origin:

$$XM_s = \alpha_s^w \left(\frac{PMT}{PM_s} \right)^{\sigma^w} XMT \tag{A-7}$$

$$PMT = \left[\sum_s \alpha_s^w PM_s^{1-\sigma^w} \right]^{1/(1-\sigma^w)} \tag{A-8}$$

The elasticity we would like to derive is the following:

$$\varepsilon_s^w = -\frac{\partial XM_s}{\partial PM_s} \frac{PM_s}{XM_s} \tag{A-9}$$

In other words, by how much would demand for imports from region s change relative to a change in its own price (including possibly a reduction (or increase) in the applied tariff).

The starting point for the derivation is equation (A-7) from which it is possible to derive the following expression:

$$\begin{aligned}
\varepsilon_s^w &= -\frac{\partial XM_s}{\partial PM_s} \frac{PM_s}{XM_s} = -\left[(1-s_s)\sigma^w - \frac{\partial XMT}{\partial PM_s} \frac{PM_s}{XMT} \right] \\
&= -\left[(1-s_s)\sigma^w + s_s \left(-\frac{\partial XMT}{\partial PMT} \frac{PMT}{XMT} \right) \right]
\end{aligned} \tag{A-10}$$

Equation (A-10) shows that the trade elasticity is equal to the weighted average of the second-level Armington elasticity and the elasticity of demand for aggregate imports relative to the aggregate price of imports, where the weights are given by the import share of region s in total imports:

$$s_s = \frac{PM_s \cdot XM_s}{PMT \cdot XMT}$$

The final term in equation (A-10), not surprisingly takes the following form as derived from equation (A-5):

$$\begin{aligned}
-\frac{\partial XMT}{\partial PMT} \frac{PMT}{XMT} &= -\left[(1-s_m)\sigma^m - \frac{\partial XA}{\partial PMT} \frac{PMT}{XA} \right] \\
&= -\left[(1-s_m)\sigma^m + s_m \left(-\frac{\partial XA}{\partial PA} \frac{PA}{XA} \right) \right]
\end{aligned} \tag{A-11}$$

Similar to equation (A-10), equation (A-11) shows that the aggregate elasticity of import demand is the weighted average of the top-level Armington demand and the overall price elasticity of the (Armington) demand with respect to the (Armington) price, where the weight is given by the (aggregate) import share in total demand:

$$s_m = \frac{PMT \cdot XMT}{PA \cdot XA}$$

These equations use the following expressions that provide the elasticity of the composite CES price with respect to its individual price components:

$$\frac{\partial PMT}{\partial PM_s} \frac{PM_s}{PMT} = s_s$$

$$\frac{\partial PA}{\partial PMT} \frac{PMT}{PA} = s_m$$

Finally, the trade elasticity is given by the following expression:

$$\varepsilon_s^w = -\left[(1-s_s)\sigma^w + s_s \left((1-s_m)\sigma^m + s_m \epsilon^d \right) \right] \tag{A-12}$$

where

$$\varepsilon^d = -\frac{\partial XA}{\partial PA} \frac{PA}{XA}$$

Formulas in percent differences

It is useful in terms of comparative static analyses to convert the basic equations into percent differences. It is easy to trace out the impacts of a change in one of the 'exogenous' variables on demand and the overall price index. This is also the form of the equations used for models implemented in GEMPACK such as MONASH-style models.

The following expressions convey expressions (A-1) and (A-2) into their percent difference form:

$$\frac{\partial X_i}{X_i} = \dot{X}_i = \dot{V} + \sigma \left(\dot{P} - \dot{P}_i \right) + (\sigma - 1) \left(\dot{A} + \dot{\lambda}_i \right)$$

$$\frac{\partial P}{P} = \dot{P} = -\dot{A} + \sum_i s_i \dot{P}_i - \sum_i s_i \dot{\lambda}_i = -\dot{A} + \sum_i s_i \left(\dot{P}_i - \dot{\lambda}_i \right)$$

Thus the percent change in the unit cost, P, for a change in the input price, P_i, all else equal, is (approximately) the value share of component i—as already noted above.

A.1.6   Growth Accounting

Use can be made of the linearization above to derive the linearized growth accounting formula:

$$\frac{\Delta V}{V} = \frac{\Delta A}{A} + \sum_i s_i \frac{\Delta x_i}{x_i} + \sum_i s_i \frac{\Delta \lambda_i}{\lambda_i}$$

A.1.7   Parameter twists

The basic analytics

This final section on the CES describes how to adjust the share parameters in a dynamic scenario under a specific assumption—this is called the twist adjustment and is a core feature of the dynamic MONASH model, see Dixon and Rimmer (2002). The basic idea is to alter the share parameter, in a two-component CES, to target a given change in the ratio of the two components, however, with neutral impacts on the aggregate cost. For example, the target may be a cost-neutral increase in the capital/labor ratio by x%, or an increase in the import to domestic ratio of y%.

The ratio of the two components is given by the following expression using equation (A-1) as the starting point:

$$R = \frac{\alpha_1 \lambda_1^{\sigma-1} P_2^{\sigma}}{\alpha_2 \lambda_2^{\sigma-1} P_1^{\sigma}}$$

The idea is to move the initial ratio, R_{t-1} to R_t by tw percent.

$$\frac{R_t}{R_{t-1}} = (1 + tw_t)$$

Using the formulas above, we have:

$$\frac{R_t}{R_{t-1}} = (1 + tw_t) = \frac{\left(\frac{\lambda_{1,t}}{\lambda_{1,t-1}} \right)^{\sigma-1}}{\left(\frac{\lambda_{2,t}}{\lambda_{2,t-1}} \right)^{\sigma-1}} = \frac{(1 + \pi_{1,t})^{\sigma-1}}{(1 + \pi_{2,t})^{\sigma-1}}$$

The π variables represent the growth (either positive or negative) that will be applied to the technology parameters under the assumption of cost-neutral technological change. We can start with the dual cost function for year t, but with year $t-1$ prices:

$$
\begin{aligned}
P_{t-1}^{1-\sigma} &= \alpha_1 \left(\frac{P_{1,t-1}}{\lambda_{1,t}} \right)^{1-\sigma} + \alpha_2 \left(\frac{P_{2,t-1}}{\lambda_{2,t}} \right)^{1-\sigma} \\
&= \alpha_1 (1 + \pi_{1,t})^{\sigma-1} \left(\frac{P_{1,t-1}}{\lambda_{1,t}} \right)^{1-\sigma} + \alpha_2 (1 + \pi_{2,t})^{\sigma-1} \left(\frac{P_{2,t-1}}{\lambda_{2,t}} \right)^{1-\sigma}
\end{aligned}
$$

Recall that the share equation is given by:

$$s_{i,t-1} = \alpha_i \lambda_{i,t-1}^{\sigma-1} \left(\frac{P_t}{P_{i,t-1}} \right)^{\sigma-1}$$

Dividing through the expression above by $P_t^{1-\sigma}$ and inserting the share expressions for year $t-1$, we end up with:

$$1 = s_{1,t-1}(1 + \pi_{1,t})^{\sigma-1} + s_{2,t-1}(1 + \pi_{2,t})^{\sigma-1}$$

Solving in terms of π_1, we have:

$$(1 + \pi_{1,t})^{\sigma-1} = \frac{1 - s_{2,t-1}(1 + \pi_{2,t})^{\sigma-1}}{s_{1,t-1}}$$

and this can be inserted into the twist target formula to get:

$$1 + tw_t = \frac{1 - s_{2,t-1}(1 + \pi_{2,t})^{\sigma-1}}{s_{1,t-1}(1 + \pi_{2,t})^{\sigma-1}} = \frac{(1 + \pi_{2,t})^{1-\sigma} - s_{2,t-1}}{s_{1,t-1}}$$

100

Finally, π_2 can be isolated to yield:

$$1 + \pi_{2,t} = [s_{1,t-1}(1 + tw_t) + s_{2,t-1}]^{1/(1-\sigma)} = [1 + s_{1,t-1}tw_t]^{1/(1-\sigma)}$$

We can re-insert this into the expression above to derive an expression for π_1:

$$1 + \pi_{1,t} = \left[\frac{1 + s_{1,t-1}tw_t}{1 + tw_t}\right]^{1/(1-\sigma)}$$

Finally, the productivity update formulas that incorporate the twist adjustment take the form:

$$\lambda_{1,t} = (1 + \pi_{1,t})\lambda_{1,t-1} = \left[\frac{1 + s_{1,t-1}tw_t}{1 + tw_t}\right]^{1/(1-\sigma)} \lambda_{1,t-1}$$

$$\lambda_{2,t} = (1 + \pi_{2,t})\lambda_{2,t-1} = [1 + s_{1,t-1}tw_t]^{1/(1-\sigma)} \lambda_{2,t-1}$$

The model code implementing the twist can be simplified. The generic CES code is given by:

$$X_i = \alpha_i \lambda_i^{\sigma-1} (PV/P_i)^\sigma V$$

and

$$PV = \left[\sum_i \alpha_i \left(\frac{P_i}{\lambda_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)}$$

The parameter twist expression can be inserted directly into the standard CES functions, which reduces the number of equations and also removes a source of non-linearity:

$$X_i = \begin{cases} \alpha_i \lambda_{i,t-1}^{\sigma-1} (PV/P_i)^\sigma V \left(\dfrac{1 + tw_t}{1 + s_{1,t-1}tw_t}\right) & \text{for } i \in \{1\} \\[2ex] \alpha_i \lambda_{i,t-1}^{\sigma-1} (PV/P_i)^\sigma V \left(\dfrac{1}{1 + s_{1,t-1}tw_t}\right) & \text{for } i \in \{2\} \end{cases}$$

$$PV = \left[\sum_{i\in\{1\}} \alpha_i \left(\frac{1 + tw_t}{1 + s_{1,t-1}tw_t}\right)\left(\frac{P_i}{\lambda_{i,t-1}}\right)^{1-\sigma} + \sum_{i\in\{2\}} \alpha_i \left(\frac{1}{1 + s_{1,t-1}tw_t}\right)\left(\frac{P_i}{\lambda_{i,t-1}}\right)^{1-\sigma}\right]^{1/(1-\sigma)}$$

It is possible to generalize these formulas by partitioning the set of CES components into two sets—a set indexed by 1 that is the target set, and a set indexed by 2 that is the complement. For example, think of a set of electricity technologies that includes conventional and advanced. It is possible then to provide the same twist to all of the new technologies relative to the conventional technologies. The only change in the formulas above is that the share variable for the single component is replaced by the sum of the shares for the bundle of components:

$$\lambda_{1,t} = (1 + \pi_{1,t})\lambda_{1,t-1} = \left[\frac{1 + tw_t \sum\limits_{i\in 1} s_{i,t-1}}{1 + tw_t}\right]^{1/(1-\sigma)} \lambda_{1,t-1}$$

$$\lambda_{2,t} = (1 + \pi_{2,t})\lambda_{2,t-1} = \left[1 + tw_t \sum_{i\in 1} s_{i,t-1}\right]^{1/(1-\sigma)} \lambda_{2,t-1}$$

A different but related problem is to target one or more components (but less than n) and have the same objective, i.e., a cost neutral change in preferences. The generic solution is:

$$X_i = \alpha_i \lambda_i V \left(\frac{P}{P_i}\right)^\sigma$$

where

$$\lambda_i = \begin{cases} \dfrac{X_{i,1}}{X_{i,0}} & \text{if } i \in \{\textit{Target}\} \\[3ex] \dfrac{1 - \sum\limits_{j\in\{\textit{Target}\}} s_j \lambda_i}{1 - \sum\limits_{j\in\{\textit{Target}\}} s_j} & \text{if } i \notin \{\textit{Target}\} \end{cases}$$

i.e., the λ parameter is commodity specific for the targeted commodities, but it is uniform across all non-targeted commodities. It would also be feasible to raise the λ parameters to the power of $1/(\sigma - 1)$, in which case it could be used in the conventional form as a technology or preference shifter. It is easy to show that:

$$\sum_i s_i \lambda_i = 1$$

i.e., the weighted sum of the λ factors is equal to 1. There are restrictions on the constraints, otherwise the non-constrained quantities could take implausible, or negative, values. The relevant price index is:

$$P = \left[\alpha_i \lambda_i P_i^{1-\sigma} \right]^{1/(1-\sigma)}$$

Converting to percent differences

The π factors reflect a percentage change in the relevant productivity factors for each of the components. Using a Taylor series approximation, the formulas above can be converted to a linear equation that is used by the Monash-style models. For the first component, we have:

$$\pi_1 = F(tw) = \left[\frac{1 + s_1 tw}{1 + tw} \right]^{1/(1-\sigma)} - 1 \approx F(0) + tw.F'(0) = -tw \frac{1 - s_1}{1 - \sigma}$$

For the second component we have:

$$\pi_2 = F(tw) = [1 + s_1 tw]^{1/(1-\sigma)} - 1 \approx F(0) + tw.F'(0) = tw \frac{s_1}{1 - \sigma}$$

Note that in the Monash models, the signs are reversed because the productivity factors divide the volume components whereas in the formulation above the productivity factors are multiplicative.

Examples of twisting the share parameters

We demonstrate these concepts with two examples. The first is a CES production function of capital and labor, where the labor share is 60% and the capital/labor substitution elasticity (i.e. σ) is set to 0.9. Prices are initialized at 1, therefore the original capital/labor ratio is 2/3. The target is to raise the capital/labor ratio 10% assuming cost neutrality. Table A.1 shows the key results. Labor efficiency would increase by 48% and capital efficiency would decline by 43%.

Table A.1: **Example of capital/labor twist**

	Labor	Capital	Capital/labor ratio
Initial	60.0	40.0	0.6667
After twist	57.7	42.3	0.7333
Percent change	-3.8	5.8	10.0
Growth factor	0.48	-0.43	

The second example comes from trade and the Armington assumption. Assume an 80/20 split between domestic goods and imports in value and volume implying a ratio of imports to demand of domestic goods of 0.25. Table A.2 shows the twist parameters needed to achieve an increase in this ratio of 10 percent with an Armington elasticity of 2. The preference parameter for imports increases by nearly 8 percent, while that for domestic goods decreases by 2 percent.

Table A.2: **Example of Armington import/domestic twist**

	Domestic	Import	Import/domestic ratio
Initial	80.0	20.0	0.250
After twist	78.4	21.6	0.275
Percent change	-2.0	7.8	10.0
Growth factor	-0.02	0.08	

A.1.8 Summary

In summary, the CES functional form is often used as a production (or sub-production) function that combines two or more inputs to form output (or an intermediate composite bundle), under the assumption of cost minimization. It is also frequently used to maximize utility (or sub-utility) over a set of two or more goods, again with the assumption of cost minimization. Table A.3 highlights the two main expressions to emerge from the optimization—the derived demand functions, X_i, and the CES dual price expression, P. The top row shows the expression with all technology parameters initialized at 1, and the bottom row the most generic version.

Table A.3: **Key equations for CES implementation**

	Demand	Aggregate price
Basic	$X_i = \alpha_i V \left(\dfrac{P}{P_i} \right)^{\sigma}$	$P = \left[\sum_i \alpha_i P_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$
with full technology	$X_i = \alpha_i (A\lambda_i)^{\sigma-1} V \left(\dfrac{P}{P_i} \right)^{\sigma}$	$P = \dfrac{1}{A} \left[\sum_i \alpha_i \left(\dfrac{P}{\lambda_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$

A.2 The CET Function

A.2.1 The basic formulation

This section describes the constant-elasticity-of-transformation (CET) function. The CET function is often used to describe a transformation frontier between two or more outputs. For example, a producer may produce two or more products and decides how much of each to produce based on market conditions, i.e. relative prices. The CET is often used to represent a producer's decision on the allocation of output between domestic and foreign markets. Another example is land supply, where land will be allocated across different uses according to the relative returns. The transformation elasticity is assumed to be uniform between any pair of outputs and therefore is analogous to the demand-based CES function described in detail above. The exposition of the CET will be much more succinct than that of the CES because most of the derivations can be derived in a similar fashion.

The CET can be setup as a revenue maximization problem, subject to a transformation frontier:

$$\max_{X_i} \sum_i P_i X_i$$

subject to

$$V = A \left[\sum_i g_i (\lambda_i X_i)^v \right]^{1/v}$$

where V is the aggregate volume (e.g. aggregate supply), X_i are the relevant components (sector-specific supply), P_i are the corresponding prices, g_i are the CET (primal) share parameters, and ν is the CET exponent. The CET exponent is related to the CET transformation elasticity, ω via the following relation:

$$\nu = \frac{\omega + 1}{\omega} \Leftrightarrow \omega = \frac{1}{\nu - 1}$$

The transformation elasticity is assumed to be positive. Solution of this maximization problem leads to the following first order conditions:

$$X_i = \gamma_i (A\lambda_i)^{-1-\omega} \left(\frac{P_i}{P} \right)^{\omega} V \tag{A-13}$$

and

$$P = \frac{1}{A} \left[\sum_i \gamma_i \left(\frac{P_i}{\lambda_i} \right)^{1+\omega} \right]^{1/(1+\omega)} \tag{A-14}$$

where the γ_i parameters are related to the primal share parameters, g_i, by the following formula:

$$\gamma_i = g_i^{-\omega} \Leftrightarrow g_i = \left(\frac{1}{\gamma_i}\right)^{1/\omega}$$

From expression A-13, and ignoring the technology parameters for the moment, the clear difference with the CES expression for optimal demand (equation A-1) is that the component price is in the numerator and the aggregate price in the denominator. This is intuitively logical. If the supply price to a market goes up relative to the average market price, one would anticipate that supply would increase to that market. The greater the transformation elasticity the greater are the market shifts.

Calibration is similar to the CES case. Prices and volumes are initialized using base year data. Equation (A-13) can then be inverted to calculate the share parameters, γ_i, with typically the technology parameters initialized to the value 1. In most implementations, there is no need to carry around the primal share parameters, nor the primal exponent.

The main interesting case for the CET is the case of perfect transformation, i.e. the transformation elasticity is infinity. In this case the CET exponent is 0 and the aggregation function is a linear weighted aggregation of the components. The standard CET equations are then replaced by the law-of-one price and the linear aggregation function (or alternatively, the zero profit condition).

$$\frac{P_i}{A\lambda_i} = P \forall i$$

$$A \sum_i \lambda_i X_i = X$$

A.2.2   Converting to percent differences

It is easier to interpret or decompose the results of a simulation by looking at the CET equations in percent differences form—that is the standard form for MONASH-style models and implementation in GEMPACK. The following equations show the equations in percent difference form:

$$\frac{\partial X_i}{X_i} = \dot{X}_i = \dot{V} + \omega\left(\dot{P}_i - \dot{P}\right) - (\omega + 1)\left(\dot{\lambda}_i + \dot{A}\right)$$

$$\frac{\partial P}{P} = \dot{P} = -\dot{A} + \sum_i s_i \dot{P}_i - \sum_i s_i \dot{\lambda}_i = -\dot{A} + \sum_i s_i \left(\dot{P}_i - \dot{\lambda}_i\right)$$

where the variable s_i is the value share of component i in total revenue:

$$s_i = \frac{P_i X_i}{P \cdot V} = \gamma_i \left(\frac{P_i}{A \cdot \lambda_i \cdot P}\right)^{\omega + 1}$$

A.2.3   Twists and the CET

It is relatively easy to show that the analogous twist formulas for the CET are:

$$1 + \pi_{1,t} = \left[\frac{1 + s_{1,t-1} tw_t}{1 + tw_t}\right]^{1/(1+\omega)} \qquad 1 + \pi_{2,t} = \left[1 + s_{1,t-1} tw_t\right]^{1/(1+\omega)}$$

where the twist favors the first factor.

A.3   Modified CES and CET that incorporate additivity

The standard CET supply allocation specification does not preserve physical additivity, i.e. the sum of the volume components is not necessarily equal to the total volume. There are a number of alternative specifications that do preserve volume homogeneity, for example the multinomial logit.[7] One alternative, described below, uses a modified form of the CET preference function. This specification has been used for labor and land supply allocations (see respectively Dixon and Rimmer (2006) and Giesecke et al. (2013)).[8]

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[7]   See Fujimori et al. (2014) and Wise and Calvin (2011).
[8]   See also van der Mensbrugghe and Peters (2020).

A.3.1 The CET implementation

The CET alternative, which we will refer to as the additive CET (ACET), involves solving the following optimization:

$$\max_{X_i} U = \left[\sum_i g_i \left(\lambda_i P_i X_i\right)^\nu\right]^{1/\nu}$$

subject to the constraint:

$$V = \sum_i X_i$$

The variable definitions are similar to above, X_i are the volume components, P_i are the relevant component prices and V is aggregate volume. The λ_i parameters are preference parameters. The CET utility function is not simply a function of the volumes, but explicitly a function of the preference-adjusted revenues of the individual components. The closed-form solution to the above system is the following set of equations:

$$X_i = \gamma_i V \left(\frac{\lambda_i P_i}{P^c}\right)^\omega \tag{A-15}$$

$$P^c = \left[\sum_i \gamma_i \left(\lambda_i P_i\right)^\omega\right]^{1/\omega} \tag{A-16}$$

Both equations are similar to their standard CET counterparts, but with some differences. First, P^c is a price index, but it is not the average price of the components, i.e. $P^c X \neq \sum_i P_i X_i$. Second, this price index is based on ω not $1 + \omega$ as in the standard CET dual price expression. The revenue correct price index is defined by the following formula:

$$P = \frac{\sum_i \gamma_i \lambda_i^\omega P_i^{\omega+1}}{\sum_i \gamma_i \lambda_i^\omega P_i^\omega} = \frac{\sum_i \gamma_i \lambda_i^\omega P_i^{\omega+1}}{(P^c)^\omega} = \sum_i \gamma_i P_i \left(\frac{\lambda_i P_i}{P^c}\right)^\omega = \sum_i \frac{X_i}{V} P_i \tag{A-17}$$

The other transformations include:

$$\gamma_i = g_i^{1+\omega}$$

$$\omega = \frac{\nu}{1-\nu} \iff \nu = \frac{\omega}{1+\omega}$$

It is worth noting that the relation between ω and ν differs from the standard CET relation as the respective formula is inverted. The implication of this is that ν is bounded below by 0 instead of ∞, but is otherwise positive over the entire (positive) range of ω. And, in both the standard and additive CET ν converges to 1 as ω converges to ∞. As regards calibration, there is an extra degree of freedom as the value for utility is not specified. It is easiest to simply set P^c to 1 as for given P_i and λ_i the calibration of the γ parameters is straightforward:

$$\gamma_i = \frac{X_i}{V} \left(\frac{\lambda_i P_i}{P^c}\right)^{-\omega}$$

If prices and technology or preference parameters are initialized at 1, the calibrated γ parameters are equal to the initial volume shares.

Converting this to a Monash-style equation in percent differences, the derived supply function is:

$$\dot{X}_i = \dot{V} + \omega \left[\dot{P}_i + \dot{\lambda}_i - \sum_{j=1}^n \frac{X_j}{V} \left(\dot{P}_j + \dot{\lambda}_j\right)\right]$$

This equation uses volume shares as weights for cross-price (and cross-preference) effects. In the standard CET formulation, value shares are used as weights.

The standard specification needs some modifications for two special cases—perfect transformation and perfect immobility. The case of perfect transformation, i.e. a transformation elasticity of ∞, leads to all prices moving in unison with the aggregate price index. Thus equation (A-15) is replaced with the following expression:

$$\lambda_i P_i = P^c$$

where λ_i is calibrated to the initial price ratios. The price index expression, equation (A-16) is replaced with the volume constraint:

$$V = \sum_i X_i$$

In the model implementation of the additive CET, this latter expression can be used in all cases and can replace equation (A-16).

The case of zero mobility is readily implemented by dropping completely equation (A-16) (or its equivalent, i.e. the volume adding up constraint). With a transformation elasticity of 0, the price composite index in equation (A-15) simply drops out and the volume components are in strict proportion to the aggregate volume.

The ACET versus the logit formulation

The logit formulation for land allocation as used in Fujimori et al. (2014) and Wise and Calvin (2011) takes the following form:

$$r_i = X_i/V = \frac{\gamma_i P_i^\omega}{\sum_j \gamma_j P_j^\omega} \qquad \omega > 0$$

which is the same formula for the volume share of the ACET. Replacing the denominator by the composite price definition above, the supply expressions are identical.

A.3.2   ACET and twists

It is relatively easy to show that the analogous twist formulas for the ACET are:

$$1 + \pi_{1,t} = \left[\frac{1 + tw_t}{1 + s_{1,t-1} tw_t}\right]^{1/\omega} \qquad 1 + \pi_{2,t} = \left[\frac{1}{1 + s_{1,t-1} tw_t}\right]^{1/\omega}$$

where the volume shares replace the value shares from the standard CET twists and the exponent differs.[9].

A.3.3   The CES implementation

The additive CET (and CRETH) functions replace their counterparts for the allocation problem that preserves volume additivity. Analogous specifications exist for the CES and CRESH functions that emulate the implementation of their standard counterparts but also allow for additivity.

The CES alternative involves solving the following optimization:

$$\min_{X_i} U = \left[\sum_i a_i \left(\lambda_i P_i X_i\right)^\rho\right]^{1/\rho}$$

subject to the constraint:

$$V = \sum_i X_i$$

As in the case of the additive CET, the additive CES utility function is a function of the preference-adjusted cost components. The closed-form solution to the above system is the following set of equations:

$$X_i = \alpha_i V \left(\frac{P^c}{\lambda_i P_i}\right)^\sigma \tag{A-18}$$

$$P^c = \left[\sum_i \alpha_i \left(\lambda_i P_i\right)^{-\sigma}\right]^{-1/\sigma} \tag{A-19}$$

Both equations are similar to their standard CES counterparts, but with some differences. First, P^c is a price index, but it is not the average price of the components, i.e. $P^c X \neq \sum_i P_i X_i$. Second, this price index is based on $-\sigma$ not $1 - \sigma$ as in the standard CES dual price expression. The revenue correct price index is defined by the following formula:

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9   These formulas are consistent with no change in the ACET composite price. However, the average price may vary. Utility will be invariant as it is the product of the composite price and the (constant) aggregate volume

$$P = \frac{\sum_i \alpha_i \lambda_i^{-\sigma} P_i^{1-\sigma}}{\sum_i \alpha_i \lambda_i^{-\sigma} P_i^{-\sigma}} = \frac{\sum_i \alpha_i \lambda_i^{-\sigma} P_i^{1-\sigma}}{(P^c)^{-\sigma}} = \sum_i \alpha_i P_i \left(\frac{\lambda_i P_i}{P^c} \right)^{-\sigma} = \sum_i \frac{X_i}{V} P_i \tag{A-20}$$

The other transformations include:

$$\alpha_i = a_i^{1-\sigma}$$

$$\sigma = \frac{\rho}{\rho - 1} \iff \rho = \frac{\sigma}{\sigma - 1}$$

It is worth noting that the relation between σ and ρ differs from the standard CES relation as the respective formula is inverted. The implication of this is that ρ is bounded below by 0 instead of $-\infty$. It decreases towards $-\infty$, as σ increases towards 1, which is a discontinuity point. It decreases from ∞ towards 1 as σ increases from 1 to ∞.

It is relatively easy to show that the following simple expression holds for the utility function:

$$U = P^c V \tag{A-21}$$

As regards calibration, there is an extra degree of freedom as the value for utility is not specified. It is easiest to simply set P^c to 1 as for given P_i and λ_i the calibration of the α parameters is straightforward:

$$\alpha_i = \frac{X_i}{V} \left(\frac{\lambda_i P_i}{P^c} \right)^\sigma$$

If prices and technology or preference parameters are initialized at 1, the calibrated α parameters are equal to the initial volume shares.

Converting this to a Monash-style equation in percent differences, the derived demand function is:

$$\dot{X}_i = \dot{V} - \sigma \left[\dot{P}_i + \dot{\lambda}_i - \sum_{j=1}^n \frac{X_j}{V} \left(\dot{P}_j + \dot{\lambda}_j \right) \right]$$

This equation uses volume shares as weights for cross-price (and cross-preference) effects. In the standard CES formulation, value shares are used as weights.

The ACES versus the logit formulation

The logit specification has been used for formulating energy demand across carriers for many years.[10] The formulation assures volume additivity. The standard logit formula takes the following form:

$$r_i = X_i/V = \frac{\alpha_i P_i^{-\sigma}}{\sum_j \alpha_j P_j^{-\sigma}} \qquad \sigma > 0$$

which is the same formula for the volume share of the ACES.[11] Replacing the denominator by the composite price definition above, the demand expressions are identical.

The normalized ACES

Similar to the standard CES formulation, one can normalize the variables so that they are all 1 at some reference point. The normalized expressions are the following:

$$X_i = V \left(\frac{P^c}{\lambda_i P_i} \right)^\sigma$$

and

$$P^c = \left[\sum_i \alpha_i \left(\lambda_i P_i \right)^{-\sigma} \right]^{-1/\sigma}$$

where the share parameters on the composite price index are the initial *volume* shares, i.e. $\alpha_i = r_{i,0} = X_{i,0}/V_0$.

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[10]   See Edmonds and Reilly (1985) and Brenkert et al. (2003).

[11]   In the typical logit formulation the exponent excludes the minus sign, but the input exponent is negative.

A.3.4 Using twists with the additive CES

The 'twist' idea described for the normal CES can be applied to the additive CES. The concept is somewhat different given the type of optimization problem posed. Rather than change the share parameters in a given direction with cost neutrality, the idea is to change the share parameters with utility neutrality. The problem posed, therefore, is to change the ratio of demand for two goods by a specified amount, while maintaining the same level of utility.

The ratio of the two components is given by the following expression using equation (A-18) as the starting point:

$$R = \frac{\alpha_1 \lambda_2 P_2{}^\sigma}{\alpha_2 \lambda_1 P_1{}^\sigma}$$

The idea is to move the initial ratio, R_{t-1} to R_t by tw percent.

$$\frac{R_t}{R_{t-1}} = (1 + tw_t)$$

while holding U constant. The two expressions above imply that the preference shifters, given by the π parameters, are linked via the following expression:

$$1 + \pi_2 = (1 + \pi_1)(1 + tw)^{1/\sigma} \tag{A-22}$$

Given equation (A-21), holding U constant is equivalent to holding the price index, P^c, constant as well (for a fixed aggregate volume). Thus we can solve the following equation for the parameter π_1:

$$
\begin{aligned}
(P_{t-1}^c)^{-\sigma} &= \alpha_1 (P_{1,t-1}\lambda_{1,t-1})^{-\sigma} + \alpha_2 (P_{2,t-1}\lambda_{2,t-1})^{-\sigma} \\
&= \alpha_1 (P_{1,t-1}\lambda_{1,t-1}(1+\pi_{1,t}))^{-\sigma} + \alpha_2 (P_{2,t-1}\lambda_{2,t-1}(1+\pi_{2,t}))^{-\sigma} \\
&= \alpha_1 (P_{1,t-1}\lambda_{1,t-1}(1+\pi_{1,t}))^{-\sigma} + \alpha_2 \left(P_{2,t-1}\lambda_{2,t-1}(1+\pi_{1,t})(1+tw)^{1/\sigma}\right)^{-\sigma} \\
&= (P_t^c)^{-\sigma}
\end{aligned}
$$

The π variables represent the growth (either positive or negative) that will be applied to the preference parameters under the assumption of utility-preserving preference shifts. This formula can be written in terms of the initial volume shares, $r_i = X_i/V$, simplified and re-arranged to yield:

$$(1 + \pi_1)^\sigma = r_1 + \frac{r_2}{1 + tw}$$

and when re-inserted in equation (A-22) we get:

$$(1 + \pi_2)^\sigma = \left(r_1 + \frac{r_2}{1 + tw}\right)(1 + tw)$$

The final formulas for the two twist parameters only depend on the initial volume shares, the substitution elasticity and the level of the 'twist':

$$\pi_1 = \left[\frac{1 + r_1\tau}{1 + \tau}\right]^{1/\sigma} - 1 \tag{A-23}$$

$$\pi_2 = [1 + r_1\tau]^{1/\sigma} - 1 \tag{A-24}$$

It is possible to generalize these formulas by partitioning the set of CES components into two sets—a set indexed by 1 that is the target set, and a set indexed by 2 that is the complement. For example, think of a set of electricity technologies that includes conventional and advanced. It is possible then to provide the same twist to all of the new technologies relative to the conventional technologies. The only change in the formulas above is that the volume share variable for the single component is replaced by the sum of the volume shares for the bundle of components:

$$\lambda_{1,t} = (1 + \pi_{1,t})\lambda_{1,t-1} = \left[\sum_{i\in 1} r_{i,t-1} + \frac{\sum_{i\in 2} r_{i,t-1}}{1 + tw_t}\right]^{1/\sigma} \lambda_{1,t-1}$$

$$\lambda_{2,t} = (1 + \pi_{2,t})\lambda_{2,t-1} = \left[\sum_{i\in 1}(1 + tw_t)\, r_{i,t-1} + \sum_{i\in 2} r_{i,t-1}\right]^{1/\sigma} \lambda_{2,t-1}$$

Converting to percent differences

The π factors reflect a percentage change in the relevant productivity factors for each of the components. Using a Taylor series approximation, the formulas above can be converted to a linear equation that is used by the Monash-style models. For the first component, we have:

$$\pi_1 = F(tw) = \left[r_1 + \frac{r_2}{1 + tw}\right]^{1/\sigma} - 1 \approx F(0) + tw.F'(0) = -tw\frac{r_2}{\sigma}$$

For the second component we have:

$$\pi_2 = F(tw) = [r_1(1 + tw) + r_2]^{1/\sigma} - 1 \approx F(0) + tw.F'(0) = tw\frac{r_1}{\sigma}$$

Appendix B

The demand systems

This appendix describes in full detail the consumer demand systems available in the Envisage Model. The first section derives the main relations for the extended linear expenditure system (ELES). The second section describes the constant differences in elasticity (CDE) demand system.

B.1 The extended linear expenditure system (ELES)

Many models assume separability in household decision making between saving and current consumption. Lluch and Howe[1] introduced a relatively straightforward extension of the LES consumer demand function to include the saving decision simultaneously with the allocation of income on goods and services, this has become known as the extended linear expenditure system or the ELES. The ELES is based on consumers maximizing their intertemporal utility between a bundle of current consumption and an expected future consumption bundle represented in the form of savings.

B.1.1 Basic formulation

The utility function of the ELES has the following form:

$$u = \prod_i (x_i - \gamma_i)^{\mu_i} \left(\frac{S}{P^s} \right)^{\mu_s} \tag{B-1}$$

with

$$\sum_i \mu_i + \mu_s = 1 \tag{B-2}$$

where u is utility, x is the vector of consumption goods, S is household saving (in value), P^s is the price of saving, and μ and γ are ELES parameters.

The consumer solves the following problem:

$$\max \prod_i (x_i - \gamma_i)^{\mu_i} \left(\frac{S}{P^s} \right)^{\mu_s}$$

subject to

$$\sum_{i=1}^{n} p_i x_i + S = Y$$

where p is the vector of consumer prices, and Y is disposable income. The demand functions are:

$$x_i = \gamma_i + \frac{\mu_i}{p_i} \left(Y - \sum_{j=1}^{n} p_j \gamma_j \right) \tag{B-3}$$

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[1]   See Lluch (1973) and Howe (1975).

$$S = \mu_s \left(Y - \sum_{j=1}^{n} p_j \gamma_j \right) = Y - \sum_{j=1}^{n} p_j x_j \tag{B-4}$$

The term in parentheses is sometimes called supernumerary income, i.e. it is the income that remains after subtracting total expenditures on the so-called subsistence (or floor) expenditures as represented by the γ parameter. The parameter μ then represents the marginal budget share out of supernumerary income.

B.1.2 ELES elasticities

From the demand equation we can derive the income and price elasticities:

$$\eta_i = \frac{\mu_i Y}{p_i x_i} = \frac{\mu_i}{s_i} \quad \eta_s = \frac{\mu_s Y}{S} = \frac{\mu_s}{s} \tag{B-5}$$

$$\varepsilon_i = \frac{\gamma_i (1 - \mu_i)}{x_i} - 1 \quad \varepsilon_s = -1 \tag{B-6}$$

$$\varepsilon_{ij} = -\frac{\mu_i p_j \gamma_j}{p_i x_i} = -\frac{\mu_i p_j \gamma_j}{s_i Y} \quad \varepsilon_{sj} = -\frac{\mu_s p_j \gamma_j}{sY} = -\frac{p_j \gamma_j}{Y^*} \tag{B-7}$$

where s is the average propensity to save. Note that the matrix of elasticities can be collapsed to a single formula using the Kronecker factor:

$$\varepsilon_{ij} = -\frac{\mu_i p_j \gamma_j}{p_i x_i} - \delta_{ij} \frac{p_i x_i - p_i \gamma_i}{p_i x_i} = -\frac{\mu_i}{s_i Y} \left[\delta_{ij} Y^* + p_j \gamma_j \right] = -\eta_i \left[\delta_{ij} \frac{Y^*}{Y} + \frac{p_j \gamma_j}{Y} \right] \tag{B-8}$$

The last expression shows that there is clear linkage between the income and price elasticities. At the limit, when income is much larger than supernumerary income, the two are virtually identical in levels (with opposite signs).

B.1.3 Welfare

With the addition of saving, the indirect utility function is given by:

$$v(p, Y) = \prod_i \left(\frac{\mu_i}{p_i} Y^* \right)^{\mu_i} \left(\frac{\mu_s}{P^s} Y^* \right)^{\mu_s} \tag{B-9}$$

or

$$v(p, Y) = \frac{Y^*}{P} \tag{B-10}$$

where

$$P = \prod_i \left(\frac{p_i}{\mu_i} \right)^{\mu_i} \left(\frac{P^s}{\mu_s} \right)^{\mu_s}$$

The expenditure function is derived by minimizing the cost of achieving a given level of utility, u. It is set-up as:

$$\min \sum_{i=1}^{n} p_i x_i + S$$

subject to

$$\prod_i (x_i - \gamma_i)^{\mu_i} \left(\frac{S}{P^s} \right)^{\mu_s} = u$$

The final expression for the expenditure function is:

$$E(p, u) = \sum_{i=1}^{n} p_i \gamma_i + uP \tag{B-11}$$

where P, the aggregate price index (including the price of savings) is defined as above.

B.1.4 Calibration

Calibration of the ELES uses the budget share information from the base SAM, including the household saving share. Typically, calibration uses income elasticities for all of the n commodities represented in the demand system and uses equation (B-5) to derive the marginal budget shares, μ_i. This procedure leads to a residual income elasticity, which in this case is the income elasticity of saving. The derived savings income elasticity may be implausible, in which case adjustments need to be made to individual income elasticities for the goods, or adjustments can be made on the group of goods, assuming some target for the savings income elasticity.

The first step is therefore to calculate the marginal budget shares using the average budget shares and the initial income elasticity estimates.

$$\mu_i = \frac{\eta_i p_i x_i}{Y} = \eta_i s_i$$

The savings marginal budget share is derived from the consistency requirement that the marginal budget shares sum to 1:

$$\mu_s = 1 - \sum_{i=1}^{n} \mu_i$$

Assuming this procedure leads to a plausible estimate for the savings income elasticity, the next step is to calibrate the subsistence minima, γ. This can be done by seeing that the demand equations, (B-3), are linear in the γ parameters. Note that in the case of the ELES the system of equation are of full rank because the μ_i parameters do not sum to 1 (over the n commodities)—they only sum to 1 including the marginal saving share.[2] This may lead to calibration problems if the propensity to save is 0, which may be the case in some SAMs with poor households. The linear system can be written as:

$$C = I\gamma + MY - M\Pi\gamma$$

where I is an $n \times n$ identity matrix, M is a diagonal matrix with μ_i/P_i on the diagonal, and Π is a matrix, where each row is identical, each row being the transpose of the price vector. The above system of linear equations can be solved via matrix inversion for the parameter γ:

$$\gamma = A^{-1} C^*$$

where

$$A = I - M\,\Pi$$
$$C^* = C - MY$$

The matrices A and C^* are defined by:

$$A = [a_{ij}] = \left[\delta_{ij} - \mu_i \frac{p_j}{p_i} \right] = \begin{cases} 1 - \mu_i & \text{if} \quad i = j \\ -\mu_i \frac{p_j}{p_i} & \text{if} \quad i \neq j \end{cases}$$

$$C^* = [c_i] = x_i - \frac{\mu_i Y}{p_i}$$

The A and C^* matrices are simplified if the price vector is initialized at 1:

$$A = [a_{ij}] = \begin{cases} 1 - \mu_i & \text{if} \quad i = j \\ -\mu_i & \text{if} \quad i \neq j \end{cases}$$

$$C^* = [c_i] = x_i - \mu_i Y$$

In GAMS one could invert the system of equations embodied in equation (B-3) directly by solving for the endogenous γ while holding all of the other variables and parameters fixed.

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[2]   Note that the calibration and the setup of the ELES assume explicitly that the minimal expenditure on savings is zero.

B.2 An Implicitly Direct Additive Demand System (AIDADS)

Many commonly used utility functions typically exhibit poor Engel behavior—particularly in a dynamic framework. The CDE utility function, popularized by the GTAP model (see Hertel (1997)), has relatively constant income elasticities. The LES utility function has even worse behavior, as in the absence of any shifts in the underlying parameters, the LES converges relatively quickly to a Cobb-Douglas utility function as rapidly rising consumption tends to dominate the floor consumption parameters, even when adjusting the latter to take into account population growth. Rimmer and Powell (see Rimmer and Powell (1992b), Rimmer and Powell (1992a) and Rimmer and Powell (1996)) examine an extension to the standard LES demand system that in effect allows the marginal propensity to consumer parameter to be driven by changes in utility. Their utility function has been called An Implicitly Direct Additive Demand System, or AIDADS. The LES function is a special case of the AIDADS system where the marginal propensity variable is constant. This extension allows for more complex demand behavior, as well as providing better validation for observed changes in consumption patterns.[3]

B.2.1 Basic formulation

AIDADS starts with the implicitly additive utility function given by:

$$\sum_i U_i\left(x_i, u\right) \equiv 1 \tag{B-12}$$

Assume the following functional form for the utility function:

$$U_i = \mu_i \ln\left(\frac{x_i - \gamma_i}{Ae^u}\right) \tag{B-13}$$

where

$$\mu_i = \frac{\alpha_i + \beta_i G\left(u\right)}{1 + G\left(u\right)} \tag{B-14}$$

with the restrictions

$$\sum_i \alpha_i = \sum_i \beta_i = 1$$
$$0 \leq \alpha_i \leq 1$$
$$0 \leq \beta_i \leq 1$$
$$\gamma_i < x_i$$

Cost minimization implies the following:

$$\min \sum_i p_i x_i$$

subject to

$$\sum_i \mu_i \ln\left(\frac{x_i - \gamma_i}{Ae^u}\right) \equiv 1 \tag{B-15}$$

The first order conditions lead to:

$$\lambda \frac{\partial U_i}{\partial x_i} = p_i = \lambda \frac{\mu_i}{x_i - \gamma_i} \Rightarrow \lambda \mu_i = p_i x_i - p_i \gamma_i \tag{B-16}$$

Taking the sum over i and using the fact that the μ_i sum to unity implies:

$$\lambda = \sum_i p_i x_i - \sum_i p_i \gamma_i = Y - \sum_i p_i \gamma_i = Y^* \tag{B-17}$$

where Y is aggregate expenditure, and Y^*, sometimes referred to as supernumerary income, is residual expenditure after subtracting total expenditure on the so-called subsistence minima, γ.

Re-inserting equation (B-17) into (B-16) yields the consumer demand equations:

$$x_i = \gamma_i + \frac{\mu_i}{p_i} Y^* = \gamma_i + \frac{\mu_i}{p_i}\left[Y - \sum_j p_j \gamma_j\right] \tag{B-18}$$

---

[3] AIDADS has also been explored in the context of the GTAP model, see for example Yu et al. (2003).

Equation (B-18) is virtually identical to the LES demand equation. Similar to the linear expenditure system (LES), demand is the sum of two components—a subsistence minimum, γ, and a share, μ, of supernumerary income. Unlike the LES, the share parameter, μ, is not constant, but depends on the level of utility itself. AIDADS collapses to the LES if each α parameter is equal to the corresponding β parameter, with the ensuing function of utility, $G(u)$, dropping from equation (B-14).

B.2.2 Elasticities

This section develops the main expressions for the income and price elasticities. These formulas will be needed to calibrate the initial parameters of the AIDADS function.

Income elasticities

To derive further properties of AIDADS requires specifying a functional form for $G(u)$. Rimmer and Powell (1996) propose the following:

$$G(u) = e^u \tag{B-19}$$

The first step is to calculate the marginal budget share, ρ , defined as:

$$\rho_i = p_i \frac{\partial x_i}{\partial Y}$$

The following expression can be derived from equation (B-18):

$$\frac{\partial x_i}{\partial Y} = \frac{Y^*}{p_i} \frac{\partial \mu_i}{\partial Y} + \frac{\mu_i}{p_i} \frac{\partial Y^*}{\partial Y} = \frac{Y^*}{p_i} \frac{\partial \mu_i}{\partial u} \frac{\partial u}{\partial Y} + \frac{\mu_i}{p_i}$$

Thus:

$$\rho_i = \mu_i + Y^* \frac{\partial \mu_i}{\partial u} \frac{\partial u}{\partial Y} \tag{B-20}$$

Expression (B-20) can be expanded in two steps—first evaluating the partial derivative of the share variable, μ, with respect to utility, and then the more difficult calculation of the partial derivative of u with respect to income. The marginal share formula is:

$$\mu_i = \frac{\alpha_i + \beta_i e^u}{1 + e^u}$$

Its derivative is:

$$\frac{\partial \mu_i}{\partial u} = \frac{(1 + e^u)(\beta_i e^u) - (\alpha_i + \beta_i e^u) e^u}{(1 + e^u)^2} = \frac{e^u (\beta_i - \alpha_i)}{(1 + e^u)^2} \tag{B-21}$$

Utility and income are combined in implicit form and thus we will invoke the implicit function theorem to calculate the partial derivative of u with respect to Y. First, insert equation (B-18) into equation (B-15):

$$\sum_i \mu_i \ln \left(\frac{x_i - \gamma_i}{Ae^u} \right) = \sum_i \mu_i \ln \left(\frac{\mu_i Y^*}{Ae^u p_i} \right) = 1$$

Expanding the latter expression yields:

$$f(u, Y) = \sum_i \mu_i \ln \left(\frac{\mu_i}{p_i} \right) + \ln (Y^*) - \ln (A) - u = 1 \tag{B-22}$$

which provides the implicit relation between Y and u. The implicit function theorem states the following:

$$\frac{\partial u}{\partial Y} = -\frac{\partial f}{\partial Y} \left[\frac{\partial f}{\partial u} \right]^{-1} \tag{B-23}$$

The partial derivative of f with respect to Y is simply:

$$\frac{\partial f}{\partial Y} = \frac{1}{Y^*} \tag{B-24}$$

The partial derivative of f with respect to u is:

$$\frac{\partial f}{\partial u} = -1 + \sum_i \left[\frac{\partial \mu_i}{\partial u} \ln\left(\frac{\mu_i}{p_i}\right) + \mu_i \frac{p_i}{\mu_i} p_i \frac{\partial \mu_i}{\partial u} \right]$$

$$= -1 + \frac{e^u}{(1+e^u)^2} \sum_i \left[\left(\ln\left(\frac{\mu_i}{p_i}\right) + 1 \right) (\beta_i - \alpha_i) \right]$$

$$= \frac{e^u}{(1+e^u)^2} \left[\sum_i (\beta_i - \alpha_i) \ln(x_i - \gamma_i) - \frac{(1+e^u)^2}{e^u} \right] \tag{B-25}$$

$$= \frac{e^u}{(1+e^u)^2} \Omega^{-1}$$

where

$$\Omega = \left[\sum_i (\beta_i - \alpha_i) \ln(x_i - \gamma_i) - \frac{(1+e^u)^2}{e^u} \right]^{-1} \tag{B-26}$$

The second line uses equation (B-21). In the third line, equation (B-18) substitutes for the expression in the logarithm, and the adding up constraint allows for the deletion of non-indexed variables. Substituting equations (B-24) and (B-25) into equation (B-23) yields:

$$\frac{\partial u}{\partial Y} = -\frac{\Omega}{Y^*} \frac{(1+e^u)^2}{e^u} \tag{B-27}$$

Substituting equations (B-21) and (B-27) into equation (B-20) yields the following expression for ρ:

$$\rho_i = \mu_i - (\beta_i - \alpha_i)\,\Omega$$

The income elasticities are derived from the following expression:

$$\eta_i = \frac{\partial x_i}{\partial Y} \frac{Y}{x_i} = \frac{\partial x_i}{\partial Y} \frac{Y}{x_i} \frac{p_i}{p_i} = \frac{\rho_i}{s_i}$$

where s_i is the average budget share:

$$s_i = \frac{p_i x_i}{Y} = \frac{p_i \gamma_i}{Y} + \mu_i \frac{Y^*}{Y} = \mu_i + \left(\frac{p_i \gamma_i - \mu_i \sum_j p_j \gamma_j}{Y} \right)$$

Thus the income elasticity, η, is equal to the ratio of the marginal budget share, ρ, and the average budget share, s. Finally, equation (B-28) describes one formulation of the income elasticity:

$$\eta_i = \frac{\mu_i - (\beta_i - \alpha_i)\,\Omega}{s_i} \tag{B-28}$$

Price elasticity

The matrix of substitution elasticities is identical to the expression for the LES and has the form:

$$\sigma_{ij} = [\mu_j - \delta_{ij}] \frac{\mu_i Y^*}{s_i s_j Y} \tag{B-29}$$

where δ is the Kronecker delta:

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

It is clear that the matrix is symmetric. The matrix of substitution elasticities is also equal to:

$$\sigma_{ij} = [\mu_j - \delta_{ij}] \frac{\mu_i Y^*}{s_i s_j Y} = \frac{(x_i - \gamma_i)}{x_i} \frac{(x_j - \gamma_j)}{x_j} \frac{Y}{Y^*} - \frac{\delta_{ij}}{s_j} \frac{(x_i - \gamma_i)}{x_i}$$

The compensated demand elasticities derive from the following:

$$\xi_{ij} = s_j \sigma_{ij} = [\mu_j - \delta_{ij}] \frac{\mu_i Y^*}{s_i Y} \tag{B-30}$$

Finally, the matrix of uncompensated demand elasticities is given by:

$$\varepsilon_{ij} = \xi_{ij} - s_j \eta_i = [\mu_j - \delta_{ij}] \frac{\mu_i Y^*}{s_i Y} - s_j \eta_i \tag{B-31}$$

115

The uncompensated demand elasticities can also be written as:

$$\varepsilon_{ij} = -\frac{\mu_i}{s_i Y} \left[p_j \gamma_j + \delta_{ij} Y^* \right] + \frac{s_j}{s_i} \left(\beta_i - \alpha_i \right) \Omega \tag{B-32}$$

The first term on the right-hand side is always negative. The second term differs from the LES expression for the uncompensated demand elasticities.[4] We can see from expression (B-32) that the AIDADS specification allows for both gross complementarity and substitution. As well, it allows for luxury goods, i.e. positive own-price demand elasticities should the second term be positive and greater than the first term.

B.2.3 Implementation

Implementation of AIDADS is somewhat more complicated than the LES since the marginal propensity to consume out of supernumerary income is endogenous, and utility is defined implicitly. The following four equations are needed for model implementation:

$$Y^* = Y - \sum_i p_i \gamma_i \tag{B-33}$$

$$x_i = \gamma_i + \frac{\mu_i}{p_i} Y^* \tag{B-34}$$

$$\mu_i = \frac{\alpha_i + \beta_i e^u}{1 + e^u} \tag{B-35}$$

$$u = \sum_i \mu_i \ln \left(x_i - \gamma_i \right) - 1 - \ln \left(A \right) \tag{B-36}$$

Equations (B-33) and (B-34) are identical to their LES and ELES counterparts.[5] Equation (B-35) determines the level of the marginal propensity to consume out of supernumerary income, μ, which is a constant in the case of the LES (ELES). It requires however the calculation of the utility level, u, which is defined in equation (B-36).

B.2.4 Calibration

[To be updated] Calibration requires more information than the LES. Where the LES has $2n$ parameters to calibrate (subject to consistency constraints), AIDADS has $3n$ parameters (less the consistency requirements)—α, β and γ. The calibration system includes equations (B-33) through (B-36) which have $2 + 2n$ endogenous variables (Y^*, γ, μ, and A). There are no equations for calibrating the α and β parameters. If we have knowledge of the income elasticities, we can add the following equations:

$$\Psi = \frac{1}{\Omega} = \left[\sum_i \left(\beta_i - \alpha_i \right) \ln \left(x_i - \gamma_i \right) - \frac{(1 + e^u)^2}{e^u} \right] \tag{B-37}$$

$$\eta_i = \frac{\rho_i}{s_i} = \frac{\mu_i - \left(\beta_i - \alpha_i \right) \Omega}{s_i} = \frac{\mu_i}{s_i} - \frac{\left(\beta_i - \alpha_i \right)}{s_i \Psi} \tag{B-38}$$

There are an additional $1 + n$ equations, solving for Ψ and α. There is need for an additional n equations. Assuming we have knowledge of at least n price elasticities, for example the own-price elasticities, we can add the following equation:

$$\varepsilon_{ii} = -\frac{\mu_i}{s_i Y} \left[p_i \gamma_i + Y^* \right] + \left(\beta_i - \alpha_i \right) \Omega \tag{B-39}$$

The α and β parameters are not independent, the following restrictions must hold:

$$\sum_i \alpha_i = 1 \tag{B-40}$$

$$\sum_i \beta_i = 1 \tag{B-41}$$

The system is under-determined, there are $5 + 4n$ equations and $3 + 4n$ variables. One solution, is to make the own-price elasticities endogenous. In this case, we are adding n variables, but then the system is over-determined. We can minimize a loss function with respect to the price elasticities:

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[4] Recall that for the LES, the α and β terms are equal and thus the second term drops.
[5] Though the definition of Y includes savings in the case of the ELES.

$$L = \sum_i \left(\varepsilon_i - \varepsilon_i^0 \right)^2$$

where ε^0 represents an initial guess of the own-price elasticities and the calibration algorithm will calculate the endogenous ε in order to minimize the loss function, subject to constraints (B-37) through (B-41) and the model equations (B-33) through (B-36). The exogenous parameters in the calibration procedure include p, x, s, Y, η, ε^0 and u.

B.3 Constant differences in elasticity (CDE) demand system

The *Constant Difference of Elasticities* (CDE) function is a generalization of the CES function, but it allows for more flexibility in terms of substitution effects across goods and for non-homotheticity.[6] The starting point is an implicitly additive indirect utility function (see Hanoch (1975)) from which we can derive demand using Roy's identity (and the implicit function theorem).

B.3.1 General form

A dual approach is used to determine the properties of the CDE function. The indirect utility function is defined implicitly via the following expression:

$$V(p, u, Y) = \sum_{i=1}^{n} \alpha_i u^{e_i b_i} \left(\frac{p_i}{y} \right)^{b_i} \equiv 1 \tag{B-42}$$

where p is the vector of commodity prices, u is (per capita) utility and y is per capita income. Using Roy's identity and the implicit function theorem[7] we can derive demand, x, where v is the indirect utility function (defined implicitly):

$$x_i = -\frac{\partial v}{\partial p_i} \Big/ \frac{\partial v}{\partial Y} = -\left(\frac{\partial V}{\partial p_i} \Big/ \frac{\partial V}{\partial u} \right) \Big/ \left(\frac{\partial V}{\partial Y} \Big/ \frac{\partial V}{\partial u} \right) = -\left(\frac{\partial V}{\partial p_i} \Big/ \frac{\partial V}{\partial Y} \right) \tag{B-43}$$

This then leads to the following demand function:

$$x_i = \frac{\alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{y} \right)^{b_i - 1}}{\sum_j \alpha_j b_j u^{e_j b_j} \left(\frac{p_j}{y} \right)^{b_j}} \tag{B-44}$$

Implementation is easier if we define the following variable:

$$ZC_i = \alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{y} \right)^{b_i} \tag{B-45}$$

Then the budget shares can be expressed as:

$$s_i = \frac{ZC_i}{\sum_j ZC_j} \tag{B-46}$$

and the demand expression is:

$$x_i = \frac{s_i}{p_i} y \tag{B-47}$$

Implementation also requires evaluating u. This can be done by implementing equation (B-42) and inserting the expression for ZC:

$$\sum_{i=1}^{n} \frac{ZC_i}{b_i} \equiv 1 \tag{B-48}$$

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[6] More detailed descriptions of the CDE can be found in Hertel et al. (1991), Surry (1993) and Hertel (1997).
[7] See Varian (1992), p. 109.

B.3.2 Elasticities

In order to calibrate the CDE system, it is necessary to derive the demand and income elasticities of the CDE. The algebra is tedious, but straightforward.

The own-price elasticity is given by the following:

$$\varepsilon_i = \frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i} = \frac{s_i\left[\sum_j s_j e_j b_j - e_i b_i\right]}{\sum_j s_j e_j} + b_i\left(1 - s_i\right) - 1 \tag{B-49}$$

In deriving the elasticity, we make use of the following formula that defines the elasticity of utility with respect to price (and again makes use of the implicit function theorem):

$$\frac{\partial u}{\partial p_i}\frac{p_i}{u} = -\frac{p_i}{u}\left(\frac{\partial V}{\partial p_i}\right) / \left(\frac{\partial V}{\partial u}\right) = -\frac{s_i}{\sum_j s_j e_j} \tag{B-50}$$

The price elasticity of utility is approximately the value share of the respective demand component as long as the weighted sum of the expansion parameters, e, is close to unity. The value (or budget) share is defined in the next equation:

$$s_i = \frac{p_i x_i}{y} \tag{B-51}$$

Letting $\sigma_i = 1 - b_i$ (or $b_i = 1 - \sigma_i$), we can also write:

$$\varepsilon_i = s_i\left[\sigma_i - \frac{e_i(1 - \sigma_i)}{\sum_j s_j e_j} - \frac{\sum_j s_j e_j \sigma_j}{\sum_j s_j e_j}\right] - \sigma_i \tag{B-52}$$

With σ uniform, we also have:

$$\varepsilon_i = -\frac{s_i e_i(1 - \sigma)}{\sum_j s_j e_j} - \sigma \tag{B-53}$$

With both e and σ uniform, the formula simplifies to:

$$\varepsilon_i = -s_i(1 - \sigma) - \sigma = \sigma(s_i - 1) - s_i \tag{B-54}$$

Equation (B-54) reflects the own-price elasticity for the standard CES utility function. Finally, with e uniform but not σ, we have:

$$\varepsilon_i = s_i\left[2\sigma_i - 1 - \sum_j s_j\sigma_j\right] - \sigma_i \tag{B-55}$$

The derivation of the cross elasticities is almost identical and will not be carried out here. Combining both the own-and cross price elasticities, the matrix of substitution elasticities takes the following form where we use the Kronecker product, δ:[8]

$$\varepsilon_{ij} = s_j\left[-b_j - \frac{e_i b_i}{\sum_k s_k e_k} + \frac{\sum_k s_k e_k b_k}{\sum_k s_k e_k}\right] + \delta_{ij}(b_i - 1) \tag{B-56}$$

Again, we replace b by $1 - \sigma$, to get:

$$\varepsilon_{ij} = s_j\left[\sigma_j - \frac{e_i(1 - \sigma_i)}{\sum_k s_k e_k} - \frac{\sum_k s_k e_k \sigma_k}{\sum_k s_k e_k}\right] - \delta_{ij}\sigma_i \tag{B-57}$$

For uniform σ, equation (B-57) takes the form:

$$\varepsilon_{ij} = -\frac{e_i s_j(1 - \sigma)}{\sum_k s_k e_k} - \delta_{ij}\sigma \tag{B-58}$$

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[8] δ takes the value of 1 along the diagonal (i.e. when $i = j$) and the value 0 off-diagonal (i.e. when $i \neq j$).

And with a uniform σ and e, i.e. the CES assumption, we have:

$$\varepsilon_{ij} = -s_j(1-\sigma) - \delta_{ij}\sigma = \sigma(s_j - \delta_{ij}) - s_j \tag{B-59}$$

Finally, for a uniform e only, the matrix of elasticities is:

$$\varepsilon_{ij} = s_j\left[\sigma_j - (1-\sigma_i) - \sum_k s_k\sigma_k\right] - \delta_{ij}\sigma_i \tag{B-60}$$

The income elasticities are derived in a similar fashion:

$$\eta_i = \frac{\partial x_i}{\partial Y}\frac{Y}{x_i} = \frac{1}{\sum_k s_k e_k}\left[e_i b_i - \sum_k s_k e_k b_k\right] - (b_i - 1) + \sum_k b_k s_k \tag{B-61}$$

For this, we need the elasticity of utility with respect to income:

$$\frac{\partial u}{\partial Y}\frac{Y}{u} = -\frac{Y}{u}\left(\frac{\partial V}{\partial Y}\right)/\left(\frac{\partial V}{\partial u}\right) = \frac{1}{\sum_k s_k e_k} \tag{B-62}$$

Note that for a uniform and unitary e, the income elasticity of utility is 1.
Replacing b with $1-\sigma$, equation (B-61) can be re-written to be:

$$\eta_i = \frac{1}{\sum_k s_k e_k}\left[e_i(1-\sigma_i) + \sum_k s_k e_k \sigma_k\right] + \sigma_i - \sum_k s_k \sigma_k \tag{B-63}$$

With a uniform σ, the income elasticity becomes:

$$\eta_i = \frac{1}{\sum_k s_k e_k}\left[e_i(1-\sigma) + \sigma\sum_k s_k e_k\right] = \frac{e_i(1-\sigma)}{\sum_k s_k e_k} + \sigma \tag{B-64}$$

With e uniform, the income elasticity is unitary, irrespective of the values of the σ parameters.
From the Slutsky equation, we can calculate the compensated demand elasticities:

$$\xi_{ij} = \varepsilon_{ij} + s_j\eta_i = -\delta_{ij}\sigma_i + s_j\left[\sigma_j + \sigma_i - \sum_k s_k\sigma_k\right] \tag{B-65}$$

The cross-Allen partial elasticities are equal to the compensated demand elasticities divided by the share:

$$\sigma_{ij}^a = \sigma_j + \sigma_i - \sum_k s_k\sigma_k - \delta_{ij}\sigma_i/s_j \tag{B-66}$$

It can be readily seen that the difference of the partial elasticities is constant, hence the name of *constant difference in elasticities*.

$$\sigma_{ij}^a - \sigma_{il}^a = \sigma_j - \sigma_l \tag{B-67}$$

With a uniform σ, we revert back to the standard CES where there is equivalence between the CES substitution elasticity and the cross-Allen partial elasticity:

$$\sigma_{ij}^a = \sigma \tag{B-68}$$

B.3.3 Calibration of the CDE

Calibration assumes that we know the budget shares, the own uncompensated demand elasticities and the income elasticities. The weighted sum of the income elasticities must equal 1, so the first step in the calibration procedure is to make sure Engel's law holds. One alternative is to fix some (or none) of the income elasticities and re-scale the others using least squares. The problem is to minimize the following objective function:

$$\sum_{i\in\Omega}\left(\eta_i - \eta_i^0\right)^2$$

subject to

$$\sum_{i \in \Omega} s_i \eta_i = 1 - \sum_{i \notin \Omega} s_i \eta_i$$

where the set Ω contains all sectors where the income elasticity is not fixed, i.e. its complement contains those sectors with fixed income elasticities. The solution is:

$$\eta_i = \eta_i^0 + s_i \frac{1 - \sum\limits_{j \notin \Omega} s_j \eta_j - \sum\limits_{j \in \Omega} s_j \eta_j^0}{\sum\limits_{j \in \Omega} s_j^2} \quad \forall i \in \Omega$$

Calibration of the σ parameters is straightforward given the own elasticities and the input value shares. The first step is to calculate the Allen partial elasticities, these are simply the income elasticity adjusted by the own elasticities divided by the budget shares:

$$\sigma_{ii}^a = \eta_i + \frac{\varepsilon_{ii}}{s_i} \tag{B-69}$$

Next, equation (B-66) is setup in matrix form:

$$\sigma_{ii}^a = A\sigma_i \tag{B-70}$$

where the matrix A has the form:

$$A = \begin{bmatrix} 2 - \dfrac{1}{s_1} - s_1 & -s_2 & \cdots & -s_n \\ -s_1 & 2 - \dfrac{1}{s_2} - s_2 & \cdots & -s_n \\ \vdots & \vdots & \ddots & \vdots \\ -s_1 & -s_2 & \cdots & 2 - \dfrac{1}{s_n} - s_n \end{bmatrix} \tag{B-71}$$

or each element of A has the following formula:

$$a_{ij} = \delta_{ij}(2 - 1/s_i) - s_j$$

We can then solve for σ (and back-out the b parameters):

$$\sigma_i = A^{-1}\sigma_{ii}^a \tag{B-72}$$

There is nothing which guarantees the consistency of the calibrated σ parameters, which are meant to be positive. The calculation of the σ parameters depends only on the budget shares and the own-price uncompensated elasticities. If the calibrated σ parameters are not all positive, one could modify the elasticities until consistency is achieved. In practice, problems have occurred when a sector's budget share dominates total expenditure.

The e parameters are derived from Equation (B-63) and normalizing them so that their share weighted sum is equal to 1. Equation (B-63) can then be converted to matrix form and inverted:

$$B = \begin{bmatrix} s_1\sigma_1 + (1 - \sigma_1) & s_2\sigma_2 & \cdots & s_n\sigma_n \\ s_1\sigma_1 & s_2\sigma_2 + (1 - \sigma_2) & \cdots & s_n\sigma_n \\ \vdots & \vdots & \ddots & \vdots \\ s_1\sigma_1 & s_2\sigma_2 & \cdots & s_n\sigma_n + (1 - \sigma_n) \end{bmatrix} \tag{B-73}$$

or

$$b_{ij} = s_j\sigma_j + \delta_{ij}(1 - \sigma_i)$$

Then the e parameters are derived from matrix inversion:

$$e_i = B^{-1}C_i = B^{-1}\left(\eta_i - \sigma_i + \sum_k s_k\sigma_k\right) \tag{B-74}$$

Calibration of the α parameters is based on equations (B-42) and (B-44). Start first with equation (B-44) and write it in terms relative to consumption of good 1, i.e.:

$$\frac{x_i}{x_1} = \frac{\alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{Y}\right)^{b_i-1}}{\alpha_1 b_1 u^{e_1 b_1} \left(\frac{p_1}{Y}\right)^{b_1-1}} \tag{B-75}$$

This equation can be used to isolate α_i:

$$\alpha_i = \frac{x_i}{x_1} \frac{\alpha_1 b_1 u^{e_1 b_1} \left(\frac{p_1}{Y}\right)^{b_1-1}}{b_i u^{e_i b_i} \left(\frac{p_i}{Y}\right)^{b_i-1}} \tag{B-76}$$

and then inserted into equation (B-44):

$$\sum_{i=1}^{n} \alpha_i u^{e_i b_i} \left(\frac{p_i}{Y}\right)^{b_i} = \alpha_1 u^{e_1 b_1} \frac{b_1}{s_1} \left(\frac{p_1}{Y}\right)^{b_1} \left[\sum_{i=1}^{n} \frac{s_i}{b_i}\right] \equiv 1 \tag{B-77}$$

The final expression in equation (B-77) can be used to solve for α_1 since the formula must equal 1 by definition:

$$\alpha_1 = u^{-e_1 b_1} \frac{s_1}{b_1} \left(\frac{Y}{p_1}\right)^{b_1} \left[\sum_{i=1}^{n} \frac{s_i}{b_i}\right]^{-1} \tag{B-78}$$

Substituting back into equation (B-77) we get:

$$\alpha_i = \frac{x_i}{b_i} u^{-e_i b_i} \left(\frac{Y}{p_i}\right)^{b_i-1} \left[\sum_{j=1}^{n} \frac{s_j}{b_j}\right]^{-1} \tag{B-79}$$

The final calibration expression is then the following:

$$\alpha_i = \frac{s_i}{b_i} \left(\frac{Y}{p_i}\right)^{b_i} \frac{u^{-e_i b_i}}{\sum_{j=1}^{n} \frac{s_j}{b_j}} \tag{B-80}$$

Utility is undefined in the base data and it is easiest to simply set it to 1.

In conclusion, for calibration we need the budget shares, initial prices, total expenditure, income elasticities and the own-price uncompensated elasticities. From this, we can derive base year consumption volumes, the Allen partial substitution elasticities through equation (B-69), σ (and therefore b) through equation (B-72) and the inversion of the A-matrix, e through equation (B-74) and inversion of the B-matrix, and finally α through equation (B-80).

It is possible that the initial shares and elasticities lead to inconsistent calibrated values for the b or e parameters. One solution, modified from Hertel (1997), is to implement some sort of maximum entropy method—explicitly imposing the constraints on the parameters. Step 1 is to calibrate the b-parameters using the following minimization problem:

$$\min L = \sum_i s_i \left(\varepsilon_{ii} - \varepsilon_{ii}^0\right)^2$$

subject to

$$\varepsilon_{ii} = (1 - b_i)(s_i - 1) - s_i \left[b_i + \eta_i - \sum_j s_j b_j\right]$$

$$0 < b_i < 1$$

The loss function is a weighted some of square errors where ε^0 represents the initial or target own-price elasticity and ε will be the estimated elasticity with the constraints holding. The first constraint is a transformation of equation (B-49) where the income elasticity is substituted into the definition of the own-price elasticity (swapping out for the yet unknown e-coefficients). One critical issue is to ascertain what income elasticities to use in the formula above. One could use the target income elasticities, or an initial transformation of the target elasticities such as described above.

The next step calibrates the e-parameters with some target income elasticities as given as well as the now calibrated b-parameters. The minimization problem is formulated as the following:

$$\min L = \sum_i s_i \left(\eta_i - \eta_i^0\right)^2$$

121

subject to

$$\eta_i = \frac{1}{\sum\limits_k s_k e_k} \left[e_i b_i - \sum_k s_k e_k b_k \right] - (b_i - 1) + \sum_k b_k s_k$$

$$\sum_i s_i \eta_i \equiv 1$$

$$(\eta_i - 1)\ (\eta_i^0 - 1) > 0$$

The final constraint insures that the estimated income elasticities preserve their relationship relative to 1, i.e. target elasticities lower than 1 remain lower than 1 in the estimation procedure.

B.3.4 CDE in first differences

It is useful to decompose changes in demand using a linearized version of the demand function, and that which is used in the standard GEMPACK version of the CDE function. The CDE implicit utility function can be used to derive a relation between changes in income, utility and prices (all in per capita terms). The first step in the differentiation of the utility function, equation (B-42), leads to the following expression:

$$
\begin{aligned}
0 \quad = \quad & \sum_i \alpha_i e_i b_i u^{e_i b_i - 1} \left(\frac{p_i}{Y} \right)^{b_i} du \\
- \quad & \sum_i \alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{Y} \right)^{b_i - 1} \frac{p_i}{Y^2} dY \\
+ \quad & \sum_i \alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{Y} \right)^{b_i - 1} \frac{1}{Y} dp_i
\end{aligned}
$$

This can be simplified by inserting the expression for the demand equation, equation (B-44), and replacing demand with the budget shares (s_i):

$$0 = \frac{du}{u} \sum_i e_i s_i - \frac{dY}{Y} \sum_i s_i + \sum_i s_i \frac{dp_i}{p_i}$$

And the final expression can be written as:

$$\dot{Y} = \sum_i e_i s_i \dot{u} + \sum_i s_i \dot{p}_i \tag{B-81}$$

where the dotted variables represent the percent change (and noting that the sum of the budget shares is equal to 1).

The differentiation of the demand function, equation (B-44) is somewhat more tedious. The first step leads to the following expression:

$$
\begin{aligned}
dx_i \quad = \quad & \alpha_i b_i e_i b_i u^{e_i b_i - 1} \left(\frac{p_i}{Y} \right)^{b_i - 1} \frac{du}{D} \\
+ \quad & \alpha_i b_i u^{e_i b_i} (b_i - 1) \left(\frac{p_i}{Y} \right)^{b_i - 2} \frac{1}{Y} \frac{dp_i}{D} \\
- \quad & \alpha_i b_i u^{e_i b_i} (b_i - 1) \left(\frac{p_i}{Y} \right)^{b_i - 2} \frac{p_i}{Y^2} \frac{dY}{D} \\
- \quad & \alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{Y} \right)^{b_i - 1} D^{-2} \sum_j \alpha_j b_j e_j b_j u^{e_j b_j - 1} \left(\frac{p_j}{Y} \right)^{b_j} du \\
- \quad & \alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{Y} \right)^{b_i - 1} D^{-2} \sum_j \alpha_j b_j b_j u^{e_j b_j} \left(\frac{p_j}{Y} \right)^{b_j - 1} \frac{1}{Y} dp_j \\
+ \quad & \alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{Y} \right)^{b_i - 1} D^{-2} \sum_j \alpha_j b_j b_j u^{e_j b_j} \left(\frac{p_j}{Y} \right)^{b_j - 1} \frac{p_j}{Y^2} dY
\end{aligned}
$$

where D is the denominator in the demand equation. This can be simplified to the following expression in terms of the percent changes:

$$
\begin{aligned}
\dot{x}_i \quad = \quad & e_i b_i \dot{u} + (b_i - 1) \dot{p}_i - (b_i - 1) \dot{Y} \\
- \quad & \sum_j e_j b_j s_j \dot{u} - \sum_j b_j s_j \dot{p}_j + \sum_j b_j s_j \dot{Y}
\end{aligned}
$$

Re-grouping terms, the expression becomes:

$$\dot{x}_i \quad = \quad (b_i - 1)\dot{p}_i - \sum_j b_j s_j \dot{p}_j$$

$$+ \quad \dot{u} \left[e_i b_i - \sum_j e_j b_j s_j \right]$$

$$+ \quad \dot{Y} \left[\sum_j b_j s_j - (b_i - 1) \right]$$

The percent change in u can be replaced with the expression above, equation (B-81), to yield the following after re-arrangement:

$$\dot{x}_i \quad = \quad (b_i - 1)\dot{p}_i - \sum_j b_j s_j \dot{p}_j - \frac{1}{\sum_k e_k s_k} \sum_j s_j \dot{p}_j \left[e_i b_i - \sum_k e_k b_k s_k \right]$$

$$+ \quad \dot{Y} \left[\sum_k b_k s_k - (b_i - 1) + \frac{1}{\sum_k e_k s_k} \left(e_i b_i - \sum_k e_k b_k s_k \right) \right]$$

The final formula inserts the formulas for the income and price elasticities from above to simplify further to the following expression:

$$\dot{x}_i = \sum_j \varepsilon_{ij} \dot{p}_j + \eta_i \dot{Y} \tag{B-82}$$

123

Appendix C

Analytics of dynamics

C.1 Capital stock

When moving to flexible time steps, cumulative investment over intermediate years needs to be accounted for. One simple way to do this is to assume a constant growth rate for investment between a past year and the current year:

$$I_t = \left(1 + g_t^I\right)^n I_{t-n}$$

After some algebra, this leads to a motion equation for the capital stock that takes the following form:[1]

$$K_t^s = (1 - \delta_t)^n K_{t-n}^s + \frac{\left(1 + g_t^I\right)^n - (1 - \delta_t)^n}{g_t^I + \delta_t} I_{t-n}$$

The resulting motion equation is no longer pre-determined as the contemporaneous level of investment is needed to measure the annual growth in investment. The equation is also quite sensitive to the relative values of g^I and δ. In the model implementation, a somewhat transformed version of the motion equation is used that includes an additional equation that captures investment growth. Equation (C-1) is the investment growth factor. The value of the expression is equal to the inverse of the annual rate of growth of investment plus the depreciation rate. For example, when both are at 5 percent, Ψ takes the value 10. Equation (C-2) is the capital stock accumulation function for multi-period time steps. Both formulas depend on the contemporaneous level of investment. Equation (G-6) defines the normalized capital stock. It is a fixed scalar of the non-normalized level where the scaling factor is initialized using base year data, i.e. $\chi_r^k = TK^s{}_{r,0}/K_{r,0}^s$

$$\Psi_{r,t} = \left[\left(\frac{XFD_{r,inv,t}}{XFD_{r,inv,t-n}}\right)^{1/n} - 1 + \delta_{r,t}\right]^{-1} \tag{C-1}$$

$$K_{r,t}^s = \left[K_{r,t-n}^s - \Psi_{r,t} XFD_{r,inv,t-n}\right](1 - \delta_{r,t})^n + \Psi_{r,t} XFD_{r,inv,t} \tag{C-2}$$

At times it is useful to measure sectoral investment needs. For a single time step, this is relatively straightforward using the following expression:

$$I_{r,a,t-1} = \frac{1}{\chi_r^k}\left[\sum_v K_{r,a,v,t}^v - (1 - \delta_{r,t})\sum_v K_{r,a,v,t-1}^v\right]$$

where it is assumed that the factor χ^k applies uniformly across all activities and not just for the aggregate capital stock. The value of investment is equal to the volume times the price of investment given by the variable *PFD*. In a multi-time step simulation, the formula can be generalized to the following:

$$I_{r,a,t-1} = \frac{1}{\chi_r^k} \frac{\delta_{r,t}}{1 - (1 - \delta_{r,t})^n}\left[\sum_v K_{r,a,v,t}^v - (1 - \delta_{r,t})^n \sum_v K_{r,a,v,t-n}^v\right]$$

under the assumption of equal investment in each year. Cumulative investment is then n times the annual level. The formula is derived from the following:

---

[1] It is easy to see that the equation collapses to the one-period motion equation when n is equal to 1.

$$\sum_v K^v_{r,a,v,t} = (1 - \delta_{r,t})^n \sum_v K^v_{r,a,v,t-n} + \chi^k_r \sum_{i=0}^{n-1} (1 - \delta_{r,t})^{n-i-1} I_{r,a,t-n+i}$$

and under the assumption that investment is equal in each year.

C.2 Climate module

Some of the key changes to the climate module are described here. First, equation (G-8) is modified to account for intra-period emissions. It simply multiplies the previous period's emissions by the step size. Other strategies are available, including using the average of emissions between two solution periods.

$$CumEmiInd_t = CumEmiInd_{t-1} + n(12/44)\left(EmiGbl_{co2,t-1} + EmiX^C_{t-1}\right) \tag{C-3}$$

Second, the one-step carbon cycle dynamics is converted to an n-step equation, (C-4). With a little bit of algebra, this can be converted to the final form, equation (C-5).

A few additional notes:

- It is easy to see that equation (C-5) collapses to equation (G-10) when n is equal to 1.

- The multi-step formulas explicitly assume that the inter-period emissions are equal to E_{t-1} for each year. One could make an alternative assumption, such as taking the mean, i.e. $0.5(Emi^C_t + Emi^C_{t-1})$. A more sophisticated formula would incorporate the inter-period growth in emissions, $(Emi^C_t/Emi^C_{t-1})^{1/n}$, which could be integrated into equation (C-5). With relatively small time steps, this is unlikely to lead to very significant changes.

$$R_{b,t} = \left(1 - \frac{1}{\alpha\tau_b}\right)^n R_{b,t-n} + \frac{\phi_b}{44/12}Emi^C_{t-1}\sum_{i=0}^{n-1}\left(1 - \frac{1}{\alpha\tau_b}\right)^{n-1-i} \tag{C-4}$$

$$R_{b,t} = \left(1 - \frac{1}{\alpha_{t-n}\tau_b}\right)^n R_{b,t-n} + \alpha_{t-n}\tau_b\frac{\phi_b}{44/12}Emi^C_{t-n}\left[1 - \left(1 - \frac{1}{\alpha_{t-n}\tau_b}\right)^n\right] \tag{C-5}$$

The EBM model, which also has lags, is solved in annual time steps as described in section 4.3.

C.3 Resource depletion

The resource depletion module needs a few modifications when the time step is greater than 1 year—in particular, inter-period resource flows need to be taken into account as is the case for capital/investment stock/flow consistency.

Equation (G-32) needs to reflect the price trends over multiple periods (though the *PTrend* variable is assumed to reflect an annual rate of change between the relevant time periods):

$$PRatio_{r,a,t} = \left[\frac{PX_{r,a,t}/PGDP_{r,t}}{PX_{r,a,t-n}/PGDP_{r,t-n}}\right]^{1/n} \Big/ PTrend_{r,a,t} \tag{C-6}$$

Equation (G-36), a stock equation, needs to take into account the cumulative resource extraction flows between solution periods. These latter are calculated assuming constant growth in extraction between periods. The cumulative extraction between $t-n$ and t is given by:

$$CumExtr_t = \sum_{i=0}^{n-1} Extr_{t-n+i}$$

If we assume constant growth between periods, we have:

$$Extr_{t-n+i} = Extr_{t-n}(1+g)^i$$

where

$$g = \left(\frac{Extr_t}{Extr_{t-n}}\right)^{1/n} - 1$$

and thus

$$CumExtr_t = Extr_{t-n}\sum_{i=0}^{n-1}(1+g)^i = Extr_{t-n}\frac{(1+g)^n - 1}{g}$$

Note that if the growth rate is zero, the growth factor is replaced with the time step, i.e. n. With substitution, equation (C-7) provides the final expression for cumulative extraction.

$$CumExtr_{r,a,t} = \frac{Extr_t - Extr_{t-n}}{\left(\dfrac{Extr_t}{Extr_{t-n}}\right)^{1/n} - 1} \tag{C-7}$$

Equation (G-36), the level of proven reserves, is replaced with equation (C-8). It takes into account cumulative extraction, as described above, and also cumulative new discoveries.

$$Res_{r,a,t} = Res_{r,a,t-n} - CumExtr_{r,a,t} + \left(1 - (1 - \xi_{r,a,t})^n\right) YTD_{r,a,t-n} \tag{C-8}$$

The multi-step expression for the reserve profile equation, equation (G-37), is provided in equation (C-9). The one-year time step equation is nearly identical to the capital stock equation from above from which we can deduce the final closed form.

$$Res^p_{r,a,t} = \left(1 - \rho^x_{r,a,t}\right)^n Res^p_{r,a,t-n} + \xi_{r,a,t} YTD_{r,a,t-n} \frac{\left(1 - \rho^x_{r,a,t}\right)^n - (1 - \xi_{r,a,t})^n}{\xi_{r,a,t} - \rho^x_{r,a,t}} \tag{C-9}$$

The motion equation for unproven reserves, equation (G-40), is replaced with equation (C-10).

$$YTD_{r,a,t} = (1 - \xi_{r,a,t-1})^n YTD_{r,a,t-n} \tag{C-10}$$

The potential supply equation, equation (C-11), is nearly identical except that the resource gap is divided by the time step.

$$XF^{Pot}_{r,a,t} = \rho^x_t Res^p_{r,a,t} + ResGap_{r,a,t}/n \tag{C-11}$$

C.4   Knowledge stock

The starting point for the model implementation is the 1-step motion equation for the knowledge stock[2]:

$$KN_t = (1 - \delta) KN_{t-1} + \sum_{k=0,i=t}^{N,t-N} \gamma_k RD_i = (1 - \delta) KN_{t-1} + G' RDL_{t,t}$$

where the last term is the summation expression in vector form (the double-indexed subscript will become clearer below). $G' RDL$ represents the gamma-weighted sum of the lags of R&D expenditures. From this, we can use induction and convert the one-step expression to an n-step expression, equation (C-12). The first term on the right-hand side is intuitive. The second term adds all of the relevant cumulative weighted-lags and includes the appropriate depreciation for the intermediate years. Say for example we are evaluating knowledge stock in year 2025 with respect to 2020. The sum will be over all years between 2021 and 2025 inclusive. Equation (C-13) calculates the cumulative weighted lags for each year between $t-n+1$ and t, e.g. between 2021 and 2025 for the example above. It does require, nonetheless, the level of R&D expenditures for the intermediate years. Since these have differing weights over time, we need to keep track of these explicitly. In the GAMS code, most equations are indexed over t, which has variable step sizes. The R&D variable is calculated for each year, assuming constant growth in the intermediate years. Equation (C-14) determines the multi-step growth rate. Equation (C-15) defines R&D expenditures for each year, irrespective of the model's time step.

$$KN_t = (1 - \delta)^n KN_{t-n} + \sum_{i=t-n+1}^{t} (1 - \delta)^{t-i} RDL_{i,t} \tag{C-12}$$

$$RDL_{i,t} = \sum_{k=0,j=i-k}^{N,i-N} \gamma_k RD_j \qquad \text{for } t - n < i \le t \tag{C-13}$$

$$RD_t = (1 + gr_t)^n RD_{t-n} \tag{C-14}$$

$$RD_i = (1 + gr_t)^{n-(t-i)} RD_{t-n} \qquad \text{for } t - n < i < t \tag{C-15}$$

---

[2]   In the model the variable RD is given by XFD_{rd}.

In the model implementation, a macro is used to substitute equation (C-13) into equation (C-12). And equations (C-14) and (C-15) can be collapsed to equation (C-16) thereby eliminating the growth rate of R&D expenditures, where the calculations are done for each year between $t - n$ and t.

$$RD_i = RD_{t-n}^{(t-i)/n} RD_t^{(n-(t-i))/n} \qquad \text{for } t - n < i < t \tag{C-16}$$

If R&D is growing at a steady rate of g, we can then write:

$$
\begin{aligned}
KN_t &= (1-\delta) KN_{t-1} + \sum_{k=0}^{N} \gamma_k RD_{t-N} (1+g)^{N-k} \\
&= (1-\delta) KN_{t-1} + RD_t \sum_{k=0}^{N} \frac{\gamma_k}{(1+g)^k}
\end{aligned}
\tag{C-17}
$$

In steady state, the summation term is a constant and we can thus write:

$$KN_t = (1-\delta) KN_{t-1} + \beta RD_t \qquad \text{where } \beta = \sum_{k=0}^{N} \frac{\gamma_k}{(1+g)^k}$$

The formula can also be converted to work with multi-year time steps.

$$
\begin{aligned}
KN_t &= (1-\delta) KN_{t-1} + \beta RD_t \\
&= (1-\delta) [(1-\delta) KN_{t-2} + \beta RD_{t-1}] + \beta RD_t \\
&= (1-\delta)^2 KN_{t-2} + (1-\delta) \beta RD_{t-1} + \beta RD_t \\
&= (1-\delta)^3 KN_{t-3} + (1-\delta)^2 \beta RD_{t-2} + (1-\delta) \beta RD_{t-1} + \beta RD_t \\
&\;\;\vdots \\
&= (1-\delta)^n KN_{t-n} + \beta \sum_{i=0}^{n-1} (1-\delta)^i RD_{t-i}
\end{aligned}
$$

Assuming a constant growth rate for R&D expenditures, the last expression becomes:

$$KN_t = (1-\delta)^n KN_{t-n} + \beta \sum_{i=0}^{n-1} (1-\delta)^i RD_t (1+g)^{-i}$$

where

$$RD_t = RD_{t-i} (1+g)^i$$

After a bit of algebra, it is straightforward to show that the summation expression simplifies to:

$$\sum_{i=0}^{n-1} \frac{(1-\delta)^i}{(1+g)^i} = \frac{1}{(1+g)^{n-1}} \frac{(1+g)^n - (1-\delta)^n}{g + \delta}$$

The final expression can thus be written as:

$$
\begin{aligned}
KN_t &= (1-\delta)^n KN_{t-n} + \beta \frac{RD_t}{(1+g)^{n-1}} \frac{(1+g)^n - (1-\delta)^n}{g + \delta} \\
&= (1-\delta)^n KN_{t-n} + \beta (1+g) RD_{t-n} \frac{(1+g)^n - (1-\delta)^n}{g + \delta}
\end{aligned}
$$

These formulas rely on $n > N$, or that that the growth rate is uniform back through period $t - N$.

From the last formula, we can show that in steady-state the following relation must hold for the ratio of knowledge stocks relative to R&D expenditures:

$$\frac{KN}{RD} = \beta \frac{1+g}{g + \delta}$$

Implementation

Initializing the stock of knowledge and the pre-base year R&D expenditures is an issue. The strategy followed so far is based on the following steps:

- Assume a given base year level of R&D. Calculate the knowledge stock (KN) expenditure consistent with some assumption about the steady-state growth rate around the base year and the rate of knowledge depreciation. From the expression above, we have:

$$KN_{t_0} = \beta RD_{t_0} \frac{1+g}{g+\delta} = RD_{t_0} \frac{1+g}{g+\delta} \sum_{k=0}^{N} \frac{\gamma_k}{(1+g)^k}$$

  Alternatively, one could make an assumption about the base year knowledge stock and invert the expression to calculate the R&D expenditures consistent with the steady-state assumption.

- Back-cast the R&D expenditures to the earliest period, that should be at least N years back or more, for the largest N:

$$RD_t = RD_{t+1} / (1+g) \qquad \text{for } t_f \leq t < t_0 - 1$$

  where t_f is the first year defined, e.g. 1960, and t_0 is the base year for the simulations, e.g. 2014.

- Back-cast the cumulative lag structure, RDL using the following formula:

$$RDL_{t,t_0} \sum_{k=0,i=t-k}^{N,t-N} \gamma_k RD_i \qquad \text{for } t_f + N \leq t \leq t_0$$

- Back-cast the stock of knowledge. This step is not strictly necessary.

$$KN_t = \frac{KN_{t+1} - RDL_{t+1,t_0}}{1-\delta} \qquad \text{for } t_f + N - 1 \leq t < t_0$$
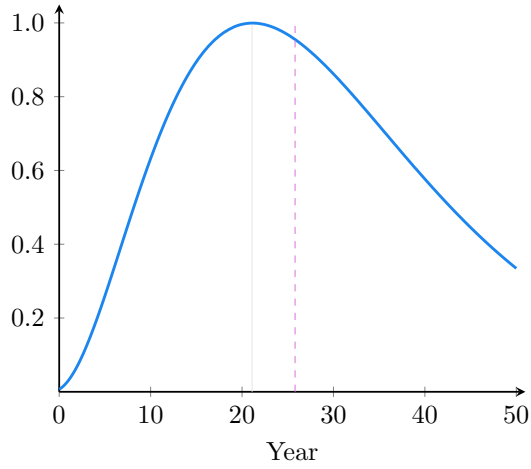
(a) $\delta = 0.7$, $\lambda = 0.9$, $N = 50$, **max=21.1, mean=25.8**
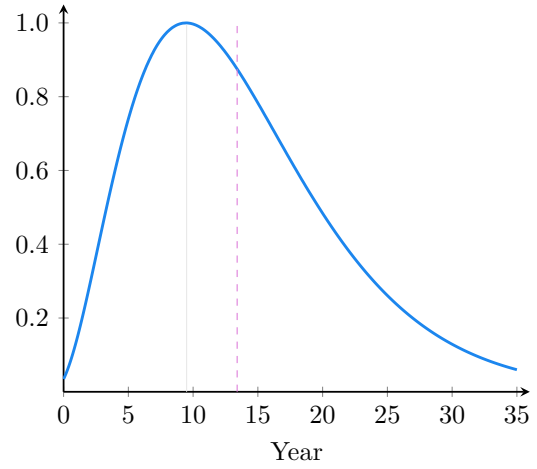
(b) $\delta = 0.7$, $\lambda = 0.8$, $N = 35$, **max=9.5, mean=13.4**
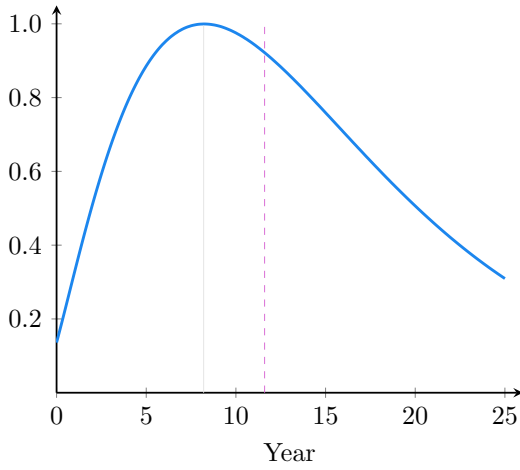
(c) $\delta = 0.6$, $\lambda = 0.85$, $N = 25$, **max=8.2, mean=11.6**
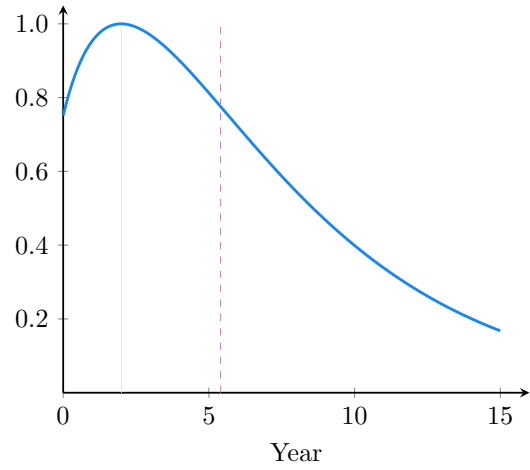
(d) $\delta = 0.4$, $\lambda = 0.8$, $N = 15$, **max=2.0, mean=5.4**
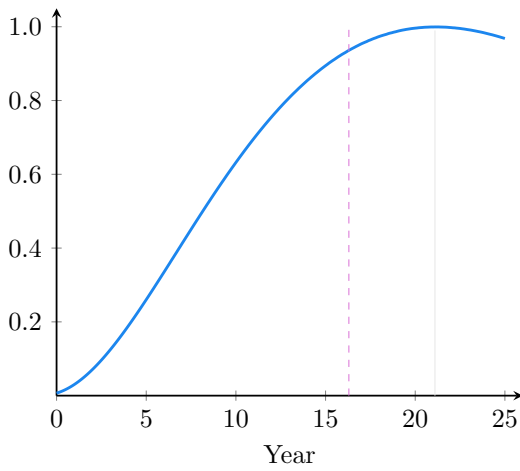
(e) $\delta = 0.7$, $\lambda = 0.9$, $N = 25$, **max=21.1, mean=16.3**
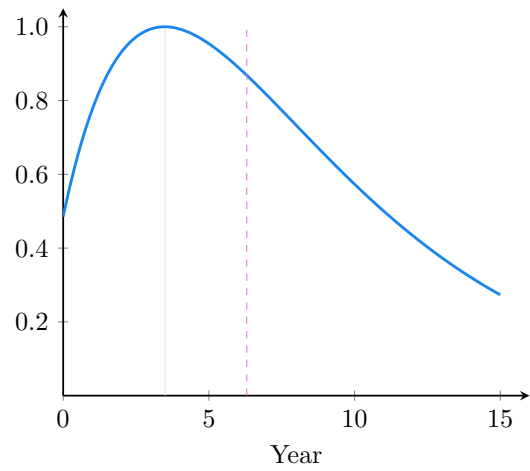
(f) $\delta = 0.5$, $\lambda = 0.8$, $N = 15$, **max=3.5, mean=6.3**

Figure 20: **Gamma distribution examples**

Appendix D

Analytical SAM and GTAP correspondence

This section describes the accounting framework used by the the model. Two different accounting frameworks will be described. The first describes the analytical SAM underlying the model, i.e. how the SAM can be constructed using the variables of the model. The second describes the links between the model variables and the initial database as provided by GTAP.[1] The SAM is not a standard SAM and does not represent the full functionality of the model nor the underlying database. For example, demand is specified at the Armington level and the sales tax are collapsed to a single row dimension.

D.1 The analytical SAM

The analytical SAM is the accounting framework that is derived from the variables of the model. In the absence of a shock, the analytical SAM should reproduce the input SAM. Table D.1 re-produces the analytical SAM. Below are a few explanatory notes:

1. The regional index (r) is dropped for most of the expressions except when needed, for example in the bilateral trade flows.

2. The SAM reflects the differentiation between activities and commodities. The intersection of the activity rows with the commodity columns represents the transformation of domestic production into commodities. In the standard version of the model, the matrix is diagonal and there is a one-to-one mapping between activities and commodities.

3. The trade rows under the commodity columns represents the bilateral imports for region r. These are priced at border, i.e. CIF, prices. The commodity rows under the trade columns represent bilateral exports from region r and are valued at border, i.e. FOB, prices. At the global level, the difference between the two matrices represents the aggregate value of the trade and transport margins.

4. Outbound remittances are calculated post-tax. Inbound remittances accrue to households.

5. In the standard GTAP database, most direct taxes are levied on factor remuneration. There is no balancing between private and public savings. Direct household taxation is calibrated as a balancing item to clear the government accounts under the assumption that the deficit is zero. This can readily be modified with supplemental data. Direct household taxation can be negative, reflecting that net transfers between the government and households is positive.

6. The `BOP` row under the trade columns represents aggregate imports from each region (at border prices). Similarly, the trade rows under the `BOP` column represents the aggregate exports of region r towards each region. The `BOP` accounts reflect the full balance of payments. For each region, the sum of exports (including the export of trade and transport services) less the sum of imports, adjusted by remittances and net capital income, equals the net flow of foreign savings. At the global level, the foreign savings cell should be zero.

---

[1]   This refers to the new standard database, not the 'classic' GTAP database.

Table D.1: **Analytical SAM**

	$ACT(a)$	$COMM(i)$	$LAB(l)$
Activities(a)		$P_{a,i}X_{a,i}$	
Commodities(i)	$\gamma_{i,a}^{eda}PAT_iXA_{i,a}$		
Labor(l)	$PF_{l,a}XF_{l,a}$		
Capital(cap)	$PF_{cap,a}XF_{cap,a}$		
Land(lnd)	$PF_{lnd,a}XF_{lnd,a}$		
Nat. Res.(nrs)	$PF_{nrs,a}XF_{nrs,a}$		
Nat. Res.(wat)	$PF_{wat,a}XF_{nrs,a}$		
Indirect tax	$\sum_i \tau_{i,a}^a\gamma_{i,a}^{eda}PAT_iXA_{i,a}$		
Production tax		$\sum_a \tau_{a,i}^pP_{a,i}X_{a,i}$	
Factor taxes	$\sum_f \tau_{f,a}^vPF_{l,a}XF_{f,a}$		
Import tax		$\sum_s \tau_{s,i,r}^mPWM_{s,i,r}XW_{s,i,r}^d$	
Export tax		$\sum_d \tau_{r,i,d}^ePE_{r,i,d}XW_{r,i,d}^s$	
Direct tax			$\sum_a \kappa_{l,a}^fPF_{l,a}XF_{l,a}$
Household			$\sum_a (1-\kappa_{l,a}^f)PF_{l,a}XF_{l,a}-\sum_d Remit_{d,l,r}$
Government			
Investment			
Depreciation			
Trade margins			
Trade(s)		$PWM_{s,i,r}XW_{s,i,r}^d$	
Balance of payments			$\sum_d Remit_{d,l,r}$

Table D.1 Analytical SAM, ctd.

	CAP	LND	NRS	WAT
Activities(a)				
Commodities(i)				
Labor(l)				
Capital(cap)				
Land(lnd)				
Nat. Res.(nrs)				
Nat. Res.(wat)				
Indirect tax				
Production tax				
Factor taxes				
Import tax				
Export tax				
Direct tax	$\sum_a \kappa^f_{cap,a} PF_{cap,a} XF_{cap,a}$	$\sum_a \kappa^f_{lnd,a} PF_{lnd,a} XF_{lnd,a}$	$\sum_a \kappa^f_{nrs,a} PF_{nrs,a} XF_{nrs,a}$	$\sum_a \kappa^f_{wat,a} PF_{wat,a} XF_{wat,a}$
Household	$\sum_a (1 - \kappa^f_{cap,a}) PF_{cap,a} XF_{cap,a} - YQTF$	$\sum_a (1 - \kappa^f_{lnd,a}) PF_{lnd,a} XF_{lnd,a}$	$\sum_a (1 - \kappa^f_{nrs,a}) PF_{nrs,a} XF_{nrs,a}$	$\sum_a (1 - \kappa^f_{wat,a}) PF_{wat,a} XF_{wat,a}$
Government				
Investment				
Depreciation				
Trade margins				
Trade(s)				
Balance of payments	$YQTF_r$			

Table D.1 Analytical SAM, ctd.

	ITX	PTX	VTX	MTX	ETX	DTX
Activities(a)						
Commodities(i)						
Labor(l)						
Capital(cap)						
Land(lnd)						
Nat. Res.(nrs)						
Nat. Res.(wat)						
Indirect tax						
Production tax						
Factor taxes						
Import tax						
Export tax						
Direct tax						
Household						
Government	$YGOV_{itx}$	$YGOV_{ptx}$	$YGOV_{vtx}$	$YGOV_{mtx}$	$YGOV_{etx}$	$YGOV_{dtx}$
Investment						
Depreciation						
Trade margins						
Trade(s)						
Balance of payments						

Table D.1 Analytical SAM, ctd.

	HHD	GOV	INV	DEPR
Activities(a)				
Commodities(i)	$\gamma_{i,h}^{eda} PAT_i^a XA_{i,h}$	$\gamma_{i,gov}^{eda} PAT_i^a XA_{i,gov}$	$\gamma_{i,inv}^{eda} PA_i^a XA_{i,inv}$	
Labor(l)				
Capital(cap)				
Land(lnd)				
Nat. Res.(nrs)				
Nat. Res.(wat)				
Indirect tax	$\sum_i \tau_{i,h}^a \gamma_{i,h}^{eda} PAT_i XA_{i,h}$	$\sum_i \tau_{i,gov}^a \gamma_{i,gov}^{eda} PAT_i XA_{i,gov}$	$\sum_i \tau_{i,inv}^a \gamma_{i,inv}^{eda} PAT_i XA_{i,inv}$	
Production tax				
Factor taxes				
Import tax				
Export tax				
Direct tax	$\kappa^h YH$			
Household				
Government				
Investment	S^h	S^g		$DeprY$
Depreciation	$DeprY$			
Trade margins				
Trade(s)				
Balance of payments		$ODAOut$		

Table D.1 Analytical SAM, ctd.

	$TMG(fd)$	$TRD(d)$	BOP
$Activities(a)$			
$Commodities(i)$	$PDT_i XTT_i$	$\sum_d PWE_{r,i,d} XW_{r,i,d}$	
$Labor(l)$			
$Capital(cap)$			
$Land(lnd)$			
$Nat.\ Res.(nrs)$			
$Nat.\ Res.(wat)$			
$Indirect\ tax$			
$Production\ tax$			
$Factor\ taxes$			
$Import\ tax$			
$Export\ tax$			
$Direct\ tax$			
$Household$			$\sum_s Remit_{r,l,s} + YQHT_r$
$Government$			$ODAOut$
$Investment$			$PW^{sav} S_r^f$
$Depreciation$			
$Trade\ margins$			$\sum_i PDT_{r,i} XTT_{r,i}$
$Trade(s)$			$\sum_d \sum_i PWE_{r,i,d} XW_{r,i,d}$
$Balance\ of\ payments$		$\sum_s \sum_i PWM_{s,i,r} XW_{s,i,r}$	

D.2 Correspondence to GTAP database

Table D.2 provides the correspondence between the variables in the GAMS model and the standard GTAP database that is used as the starting basis for the GTAP model. Note that The GTAP database reflects an Armington structure that is agent-specific (at the top nest). The model allows for either a national or agent-specific decomposition of domestic absorption across sources. The correspondence herein reflects national sourcing. Hence, the domestic/import split by agent in the GTAP database is aggregated to provide demand at the Armington level.

Table D.2: **Correspondence between GAMS model variables and GTAP database**

GTAP	GAMS	Description
VDFB+VIFB	$PAT_{r,i}XA_{r,i,a}$	Firms' (Armington) purchases at basic prices
VDFP+VIFP	$PA_{r,i,a}XA_{r,i,a}$	Firms' (Armington) purchases at purchaser prices
VDPB+VIPB	$PAT_{r,i}XA_{r,i,h}$	Household (Armington) purchases at basic prices
VDPP+VIPP	$PA_{r,i,h}XA_{r,i,h}$	Household (Armington) purchases at purchaser prices
VDGB+VIGB	$PAT_{r,i}XA_{r,i,gov}$	Government (Armington) purchases at basic prices
VDGP+VIGP	$PA_{r,i,gov}XA_{r,i,gov}$	Government (Armington) purchases at purchasers prices
VDIB+VIIB	$PAT_{r,i}XA_{r,i,inv}$	Investment (Armington) purchases at basic prices
VDIP+VIIP	$PA_{r,i,inv}XA_{r,i,inv}$	Investment (Armington) purchases at purchaser prices
EVFB(f)	$PF_{r,f,a}XF_{r,f,a}$	Firms' purchases of factors at basic prices
EVFP(f)	$PF_{r,f,a}^{p}XF_{r,f,a}$	Firms' purchases of factors at purchaser prices
VOS	$PX_{r,a}XP_{r,a}$	Value of production at producer cost
MAKB	$P_{r,a,i}X_{r,a,i}$	Value of make matrix at basic prices
MAKS	$PP_{r,a,i}X_{r,a,i}$	Value of make matrix at suppliers' prices
EVOS(f)	$(1-\kappa_{r,f,a}^{f})PF_{r,f,a}XF_{r,f,a}$	After tax factor remuneration
VKB	K_{r}^{s}	Initial capital stock
VDEP	$DeprY$	Value of depreciation
VIMB	$PDM_{s,i,r}XW_{s,i,r}^{d}$	Value of bilateral imports tariff inclusive
VCIF	$PWM_{s,i,r}XW_{s,i,r}^{d}$	Value of bilateral imports at border prices
VFOB	$PWE_{r,i,d}XW_{r,i,d}^{s}$	Value of bilateral exports at border prices
VXSB	$PE_{r,i,d}XW_{r,i,d}^{s}$	Value of bilateral exports at producer prices
VST	$PDT_{r,m}XTT_{r,m}$	Domestic exports of trade & transport services
VTWR	$PTMG_{m}XMG_{m,r,i,d}^{m}$	Value of margins from r to d for good i using mode m
POP	Pop_{r}	Population—millions

Appendix E

Dimensions of the GTAP database, release 11

Table E.1: **Regional dimension of the GTAP database (V11)**

1	AUS	**Australia**
		Australia (aus), Christmas Island (cxr), Cocos (Keeling) Islands (cck), Heard and McDonald Islands (hmd), Norfolk Island (nfk)
2	NZL	**New Zealand**
3	XOC	**Rest of Oceania**
		American Samoa (asm), Cook Islands (cok), Fiji (fji), French Polynesia (pyf), Guam (gum), Kiribati (kir), Marshall Islands (mhl), Federated States of Micronesia (fsm), Nauru (nau), New Caledonia (ncl), Niue (niu), Northern Mariana Islands (mnp), Palau (plw), Papua New Guinea (png), Pitcairn (pcn), Samoa (wsm), Solomon Islands (slb), Tokelau (tkl), Tonga (ton), Tuvalu (tuv), United States Minor Outlying Islands (umi), Vanuatu (vut), Wallis and Futura Islands (wlf)
4	CHN	**China**
5	HKG	**Hong Kong SAR (hkg)**
6	JPN	**Japan**
7	KOR	**Republic of Korea**
8	MNG	**Mongolia**
9	TWN	**Taiwan (Province of China)**
10	XEA	**Rest of East Asia**
		Macao SAR (mac), North Korea (prk)
12	KHM	**Cambodia**
13	IDN	**Indonesia**
14	LAO	**Lao, PDR**
15	MYS	**Malaysia**
16	PHL	**Philippines**
17	SGP	**Singapore**
18	THA	**Thailand**
19	VNM	**Vietnam**
20	XSE	**Rest of Southeast Asia**
		Myanmar (mmr), Timor-Leste (tmp)

Table E.1: **Regional dimension of the GTAP database (cont.)**

21	AFG	**Afghanistan**
22	BGD	**Bangladesh**
23	IND	**India**
24	LKA	**Sri Lanka**
25	NPL	**Nepal**
26	PAK	**Pakistan**
27	XSA	**Rest of South Asia**
		Bhutan (btn), Maldives (mdv)
28	CAN	**Canada**
29	USA	**United States**
30	MEX	**Mexico**
31	XNA	**Rest of North America**
		Bermuda (bmu), Greenland (grl), Saint Pierre & Miquelon (spm)
32	ARG	**Argentina**
33	BOL	**Bolivia**
34	BRA	**Brazil**
35	CHL	**Chile**
36	COL	**Colombia**
37	ECU	**Ecuador**
38	PRY	**Paraguay**
39	PER	**Peru**
40	URY	**Uruguay**
41	VEN	**Venezuela, Republica Bolivariana de**
42	XSM	**Rest of South America**
		Falkland Islands (flk), French Guiana (guf), Guyana (guy), South Georgia and the South Sandwich Islands (sgs), Suriname (sur)
43	CRI	**Costa Rica**
44	GTM	**Guatemala**
45	HND	**Honduras**
46	NIC	**Nicaragua**
47	PAN	**Panama**
48	SLV	**El Salvador**
49	XCA	**Rest of Central America**
		Belize (blz)
50	DOM	**Dominican Republic**
51	JAM	**Jamaica**
52	HTI	**Haiti**
53	PRI	**Puerto Rico**
54	TTO	**Trinidad & Tobago**
55	XCB	**Caribbean**
		Anguilla (aia), Antigua and Barbuda (atg), Aruba (abw), Bahamas (bhs), Barbados (brb), Bonaire, Sint Eustatius and Saba (bes), British Virgin Islands (vgb), Cayman Islands (cym), Cuba (cub), Curaçao (cuw), Dominica (dma), Grenada (grd), Montserrat (msr), Saint Barthélemy (blm), Saint Kitts and Nevis (kna), Saint Lucia (lca), Saint Martin (French Part) (maf), Saint Vincent and the Grenadines (vct), Sint Maarten (Dutch part) (sxm), Turks and Caicos Islands (tca), United States Virgin Islands (vir)

Table E.1: **Regional dimension of the GTAP database (cont.)**

56	AUT	**Austria**
57	BEL	**Belgium**
58	BGR	**Bulgaria**
59	HRV	**Croatia**
60	CYP	**Cyprus**
61	CZE	**Czech Republic**
62	DNK	**Denmark**
63	EST	**Estonia**
64	FIN	**Finland**
		Åland Islands (ala), Finland (fin)
65	FRA	**France**
		France (fra), Guadeloupe (glp), Martinique (mtq), Réunion (reu)
66	DEU	**Germany**
67	GRC	**Greece**
68	HUN	**Hungary**
69	IRL	**Ireland**
70	ITA	**Italy**
71	LVA	**Latvia**
72	LTU	**Lithuania**
73	LUX	**Luxembourg**
74	MLT	**Malta**
75	NLD	**Netherlands**
76	POL	**Poland**
77	PRT	**Portugal**
78	ROU	**Romania**
79	SVK	**Slovakia**
80	SVN	**Slovenia**
81	ESP	**Spain**
82	SWE	**Sweden**
83	GBR	**United Kingdom**
84	NOR	**Norway**
		Norway (nor), Svalbard and Jan Mayen Islands (sjm)
85	CHE	**Switzerland**
86	XEF	**Rest of European Free Trade Area (EFTA)**
		Iceland (isl), Liechtenstein (lie)
87	ALB	**Albania**
88	SRB	**Serbia**
89	BLR	**Belarus**
90	RUS	**Russian Federation**
91	UKR	**Ukraine**
92	XEE	**Rest of Eastern Europe**
		Moldova (mda)
93	XER	**Rest of Europe**
		Andorra (and), Bosnia and Herzegovina (bih), Faroe Islands (fro), Gibraltar (gib), Guernsey (ggy), Holy See (vat), Isle of Man (imn), Jersey (jey), Kosovo (uvk), Monaco (mco), Montenegro (mne), North Macedonia (mkd), San Marino (smr), Sark (srk)

Table E.1: **Regional dimension of the GTAP database (cont.)**

94	KAZ	**Kazakhstan**
95	KGZ	**Kyrgyz Republic**
96	TJK	**Tajikistan**
97	UZB	**Uzbekistan**
98	XSU	**Rest of Former Soviet Union**
		Turkmenistan (tkm)
99	ARM	**Armenia**
100	AZE	**Azerbaijan**
101	GEO	**Georgia**
102	BHR	**Bahrain**
103	IRN	**Iran**
104	IRQ	**Iraq**
105	ISR	**Israel**
106	JOR	**Jordan**
107	KWT	**Kuwait**
108	LBN	**Lebanon**
109	OMN	**Oman**
110	PSE	**Palestine**
111	QAT	**Qatar**
112	SAU	**Saudi Arabia**
113	SYR	**Syrian Arab Republic**
114	TUR	**Türkiye**
115	ARE	**United Arab Emirates**
116	XWS	**Rest of Western Asia**
		Republic of Yemen (yem)
117	DZA	**Algeria**
118	EGY	**Egypt**
119	MAR	**Morocco**
120	TUN	**Tunisia**
121	XNF	**Rest of North Africa**
		Libya (lby)

Table E.1: **Regional dimension of the GTAP database (cont.)**

122	BEN	**Benin**
123	BFA	**Burkina Faso**
124	CMR	**Cameroon**
125	CIV	**Côte d'Ivoire**
126	GHA	**Ghana**
127	GIN	**Guinea**
128	MLI	**Mali**
129	NER	**Niger**
130	NGA	**Nigeria**
131	SEN	**Senegal**
132	TGO	**Togo**
133	XWF	**Rest of Western Africa**
		Cape Verde (cpv), Gambia, The (gmb), Guinea-Bissau (gnb), Liberia (lbr), Mauritania (mrt), Saint Helena (shn), Sierra Leone (sle)
134	CAF	**Central African Republic**
135	TCD	**Chad**
136	COG	**Congo**
137	COD	**Democratic Republic of the Congo**
138	GNQ	**Equatorial Guinea**
139	GAB	**Gabon**
140	XAC	**South-Central Africa**
		Angola (ago), São Tomé & Príncipe (stp)
141	COM	**Comoros**
142	ETH	**Ethiopia**
143	KEN	**Kenya**
144	MDG	**Madagascar**
145	MWI	**Malawi**
146	MUS	**Mauritius**
147	MOZ	**Mozambique**
148	RWA	**Rwanda**
149	SDN	**Sudan**
150	TZA	**Tanzania**
151	UGA	**Uganda**
152	ZMB	**Zambia**
153	ZWE	**Zimbabwe**
154	XEC	**Rest of Eastern Africa**
		Burundi (bdi), Djibouti (dji), Eritrea (eri), Mayotte (myt), Seychelles Islands (syc), Somalia (som), South Sudan (ssd)
155	BWA	**Botswana**
156	SWZ	**Eswatini**
157	NAM	**Namibia**
158	ZAF	**South Africa**
159	XSC	**Rest of South African Customs Union**
		Lesotho (lso)
160	XTW	**Rest of the World**
		Antarctica (ata), Bouvet Island (bvt), British Indian Ocean Territory (iot), French Southern Territories (atf)

Table E.2 provides the standard set of commodities in the GTAP Data Base. In the standard database, these are the same as the set of activities. The standard commodity subsets are:

- MARG: OTP, WTP, ATP
- ERG : COA, OIL, GAS, P_C, ELY, GDT
- FUEL: COA, OIL, GAS, P_C, GDT

Table E.2: **Commodity dimension of the GTAP database (V11)**

1	PDR	**Paddy rice**
2	WHT	**Wheat**
3	GRO	**Cereal grains nec**
4	V_F	**Vegetables, fruit, nuts**
5	OSD	**Oil seeds**
6	C_B	**Sugar cane, sugar beet**
7	PFB	**Plant-based fibers**
8	OCR	**Crops nec**
9	CTL	**Bovine cattle, sheep and goats, horses**
10	OAP	**Animal products nec**
11	RMK	**Raw milk**
12	WOL	**Wool, silk-worm cocoons**
13	FRS	**Forestry**
14	FSH	**Fishing**
15	COA	**Coal**
16	OIL	**Oil**
17	GAS	**Gas**
18	OXT	**Other Extraction (formerly omn Minerals nec)**
19	CMT	**Bovine meat products**
20	OMT	**Meat products nec**
21	VOL	**Vegetable oils and fats**
22	MIL	**Dairy products**
23	PCR	**Processed rice**
24	SGR	**Sugar**
25	OFD	**Food products nec**
26	B_T	**Beverages and tobacco products**
27	TEX	**Textiles**
28	WAP	**Wearing apparel**
29	LEA	**Leather products**
30	LUM	**Wood products**
31	PPP	**Paper products, publishing**
32	P_C	**Petroleum, coal products**
33	CHM	**Chemical products**
34	BPH	**Basic pharmaceutical products**
35	RPP	**Rubber and plastic products**

Table E.2: **Commodity dimension of the GTAP database (cont.)**

36	NMM	**Mineral products nec**
37	I_S	**Ferrous metals**
38	NFM	**Metals nec**
39	FMP	**Metal products**
40	ELE	**Computer, electronic and optical products**
41	EEQ	**Electrical equipment**
42	OME	**Machinery and equipment nec**
43	MVH	**Motor vehicles and parts**
44	OTN	**Transport equipment nec**
45	OMF	**Manufactures nec**
46	ELY	**Electricity**
47	GDT	**Gas manufacture, distribution**
48	WTR	**Water**
49	CNS	**Construction**
50	TRD	**Trade**
51	AFS	**Accommodation, Food and service activities**
52	OTP	**Transport nec**
53	WTP	**Water transport**
54	ATP	**Air transport**
55	WHS	**Warehousing and support activities**
56	CMN	**Communication**
57	OFI	**Financial services nec**
58	INS	**Insurance (formerly isr)**
59	RSA	**Real estate activities**
60	OBS	**Business services nec**
61	ROS	**Recreational and other services**
62	OSG	**Public Administration and defense**
63	EDU	**Education**
64	HHT	**Human health and social work activities**
65	DWE	**Dwellings**

The power-version of the GTAP database splits the standard electricity sector ('ELY') into 12 electricity-based activities—11 of which are different power technologies with differentiated cost structures and 1 activity for transmission and distribution. The database assumes full diagonality of the power structure 'make' matrix. A typical model implementation is likely to keep the differentiated cost structures but collapse demand to a single electricity commodity.[1]

Table E.3: **Additional dimensions of the power database (V11)**

1	TND	**Electricity transmission and distribution**
2	NUCLEARBL	**Nuclear power**
3	COALBL	**Coal power baseload**
4	GASBL	**Gas power baseload**
5	WINDBL	**Wind power**
6	HYDROBL	**Hydro power baseload**
7	OILBL	**Oil power baseload**
8	OTHERBL	**Other baseload**
9	GASP	**Gas power peakload**
10	HYDROP	**Hydro power peakload**
11	OILP	**Oil power peakload**
12	SOLARP	**Solar power**

The standard version of GTAP has 8 endowments or factors of production—of which 5 are labor types. The first three labor types in Table E.4 are typically associated with *unskilled* labor and the remaining two would therefore be designated *skilled* labor.[2] It should be noted that in the default configuration, land payments are only attributed in the agricultural sectors—both crops and livestock—but not forestry. Natural resource payments are available for forestry (frs), fisheries (fsh), coal mining (coa), oil and gas extraction (oil and gas) and other mining extraction (oxt).

Table E.4: **GTAP endowments (V11)**

1	TECH_ASPROS	**Technical and professional workers**
2	CLERKS	**Clerical workers**
3	SERVICE_SHOP	**Service shop**
4	OFF_MGR_PROS	**Management**
5	AG_OTHLOWSK	**Agriculture and other low-skill workers**
6	CAPITAL	**Capital**
7	LAND	**Land**
8	NATLRES	**Natural resources**

Table E.5 provides the definitions for the greenhouse (GHG) and non-greenhouse gases. The set EM incorporates all air emissions. The set EMN incorporates all air emissions with the exception of CO_2. The set GHG includes the four greenhouse gases (CO_2, CH_4, N_2O and FGAS). The set NCO2 represents the greenhouse gases except CO_2. The set NGHG includes all non-greenhouse gases.

The land-use version of the GTAP database, also known as the AEZ database, divides land-use into 18 categories that are a cross of six growing length periods, from less than 60 days to over 300 days, and three broad types of climate—tropical, temperate and boreal.[3]

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[1]   The 12 power activities are added to the ERG subset.
[2]   Walmsley and Carrico (2016).
[3]   See Lee et al. (2009) and Baldos (2017).

Table E.5: **Air emissions**

1	CO2	**Carbon dioxide**
2	CH4	**Methane**
3	N2O	**Nitrous oxide**
4	FGAS	**Fluorinated gases**
4	C2F6	**Hexafluoroethane** (PFC-116, C_2F_6)
5	C3F8	**Octafluoropropane** (PFC-218, C_3F_8)
6	C4F10	**Perfluorobutane** (PFC-31-10, C_4F_{10})
7	C5F12	**Perfluoropentane** (PFC-41-12, C_5F_{12})
8	C6F14	**Perfluorohexane** (PFC-51-14, C_6F_{14})
9	cC4F8	**Octafluorocyclobutane** (PFC-318, C_4F_8)
10	CF4	**Carbon tetrafluoride** (PFC-14, CF_4)
11	HCFC141b	**1,1-Dichloro-1-fluoroethane** ($C_2H_3Cl_2F$)
12	HCFC142b	**1-Chloro-1,1-difluoroethane** ($C_2H_3ClF_2$)
13	HFC125	**Pentafluoroethane** (C_2HF_5)
14	HFC134	**1,1,2,2-Tetrafluoroethane** ($C_2H_2F_4$)
15	HFC134a	**1,1,1,2-Tetrafluoroethane** ($C_2H_2F_4$)
16	HFC143	**1,1,2-Trifluoroethane** ($C_2H_3F_3$)
17	HFC143a	**1,1,1-Trifluoroethane** ($C_2H_3F_3$)
18	HFC152a	**1,1-Difluoroethane** ($C_2H_4F_2$)
19	HFC227ea	**1,1,1,2,3,3,3-Heptafluoropropane** (C_3HF_7)
20	HFC23	**Trifluoromethane** (CHF_3)
21	HFC236fa	**1,1,1,3,3,3-Hexafluoropropane** ($C_3H_2F_6$)
22	HFC245fa	**1,1,1,3,3-Pentafluoropropane** ($C_3H_3F_5$)
23	HFC32	**Difluoromethane** (CH_2F_2)
24	HFC365mfc	**1,1,1,3,3-Pentafluorobutane** ($C_4H_5F_5$)
25	HFC41	**Fluoromethane** (CH_3F)
26	HFC4310mee	**1,1,1,2,2,3,4,5,5,5-Decafluoropentane** ($C_5H_2F_{10}$)
27	NF3	**Nitrogen trifluoride** (NF_3)
28	SF6	**Sulfur hexafluoride** (SF_6)
29	BC	**Black carbon**
30	CO	**Carbon monoxide**
31	NH3	**Ammonia**
32	NMVOC	**Non-methane volatile organic compounds**
33	NOX	**Nitrogen oxides**
34	OC	**Organic carbon**
35	PM10	**Particulate matter 10**
36	PM2_5	**Particulate matter 2.5**
37	SO2	**Sulfur dioxide**

Table E.6: **GTAP land-use database (V11)**

1	AEZ1	**Tropical and arid LGP000_060**
2	AEZ2	**Tropical and dry semi-arid LGP060_119**
3	AEZ3	**Tropical and moist semi-arid LGP120_179**
4	AEZ4	**Tropical and sub-humid LGP180_239**
5	AEZ5	**Tropical and humid LGP240_299**
6	AEZ6	**Tropical and humid; year round growing season LGP300PLUS**
7	AEZ7	**Temperate and arid LGP000_060**
8	AEZ8	**Temperate and dry semi-arid LGP060_119**
9	AEZ9	**Temperate and moist semi-arid LGP120_179**
10	AEZ10	**Temperate and sub-humid LGP180_239**
11	AEZ11	**Temperate and humid LGP240_299**
12	AEZ12	**Temperate and humid; year round growing season LGP300PLUS**
13	AEZ13	**Boreal and arid LGP000_060**
14	AEZ14	**Boreal and dry semi-arid LGP060_119**
15	AEZ15	**Boreal and moist semi-arid LGP120_179**
16	AEZ16	**Boreal and sub-humid LGP180_239**
17	AEZ17	**Boreal and humid LGP240_299**
18	AEZ18	**Boreal and humid; year round growing season LGP300PLUS**

Appendix F

User Guide

F.1 Data preparation facility

F.1.1 Introduction

[NEW: 31-Mar-2025] The data preparation facility has been revamped. The two main changes is that the workflow has been converted from a Windows command file to a GAMS file, and that the secondary aggregation, which used to be incorporated in the data initialization phase of the model code, now occurs as part of the workflow. The secondary aggregation is needed for cases when the resulting data aggregation is no longer 'diagonal'. The filter program, which removes small numbers and is described further below, requires a diagonal database.[1] Thus the user first prepares an aggregation mapping file which aggregates the GTAP database to a diagonal aggregated database. A subsequent mapping file, applied after the filter routine, aggregates the intermediate and diagonal database to the desired final database. N.B. The secondary aggregation is required even if the final database is diagonal. This is due to the fact that the ENVISAGE model requires that the commodity labels differ from the activity labels (as they are both subsets of a master set of SAM labels). The filter program explicitly requires that the two sets of labels are identical (or aliased).

The data facility is composed of a number of modules.

1. The first is the *primary* aggregation facility. The aggregation facility is designed to work with any GTAP-conformable database, including the database produced by the aggregation facility. It reads a user-prepared map file that contains the aggregation mappings. The aggregation facility aggregates all relevant GTAP data files. The *primary* aggregation must lead to a diagonal database for the next step, which is the filtering process.

2. The second module is called the 'filter' module. First developed by Tom Rutherford for his 'GTAPinGAMS' project[2] and integrated into Wolfgang Britz's CGEBox framework[3], filtering is designed to remove small numeric values from the database that have minimal impact on data balances and model results. The 'filter' program insures that the resulting database is nonetheless balanced. It currently only works with a diagonal GTAP database. Subsequent to the filtering routine, another procedure is invoked to adjust the emission and energy volume datasets to match the adjusted values in the filtered database.

3. The third module is the *secondary* aggregation facility. This enables the user to create a non-diagonal database (for example to collapse all the electricity commodities into a single stream). This step is required even if the user chooses not to change the aggregation as the ENVISAGE models requires the commodity and activity labels to be differentiated. It uses the same code as for the *primary* aggregation, but the user needs to provide an additional mapping file. The main difference between the *primary* aggregation and the *secondary* aggregation is that the latter saves both mapping sets as part of the resultant database, which can be used for documentation purposes.

4. The fourth module is called the 'AlterTax' module. It is described in Malcolm (1998) and is a procedure to modify an initial GTAP database while minimizing changes to the structure of the original database. Though designed to change tax rates, it can be used to modify other features of a database. This module relies on the standard GTAP model itself, which is provided as part of the data preparation facility.

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[1] We are working to convert the filter program to handle non-diagonal databases.
[2] Lanz and Rutherford (2016)
[3] http://www.ilr.uni-bonn.de/em/rsrch/cgebox/cgebox_GUI.pdf

5. The fifth module is optional and is used to aggregate the myriad elasticities for the ENVISAGE model. These are originally from the OECD's 'circular' economy study—with additional mappings to conform to the dimensionality of the chosen GTAP datbase. This step is required to run the ENVISAGE model, but not for other purposes.

6. The sixth module is optional and is used to aggregate the projection database. This step is required to run dynamic simulations. The projections include the SSPs, various UN population projections, the World Bank GIDD projections and various WEO projections from the IMF.

The outputs of each model are contained in different containers. The output from the *primary* aggregation facility is found in the 'Agg1' folder. The output of the filtering procedure is found in the 'Flt' folder. The output from the *secondary* aggregation facility is found in the 'Agg1' folder. And the output of AlterTax is found in the 'Alt' folder. The end results, assuming all has worked appropriately, will be found in the 'Fnl' folder.

Each data project is associated with a code name, for example '10x10'. This code name will be used for all output files and the aggregation facility will prepare a folder in the 'Data' directory that is named using the code name. All output folders will be contained in that directory.

F.1.2 Preliminaries

There are no proscribed ways to setup the directory structures for the various components of Env11 so the setup below is just a suggestion. All of the components rely on the user providing full path names so it should be flexible enough to handle most user preferences. Figure 21 depicts a fairly standard layout. The root directory is *Env11* and it contains three obligatory folders: *Data*, *SatAcct* and *Model*. The first contains the routines for aggregating and preparing the data for the model. It also contains some sub-folders that will be further described below. The *SatAcct* contains miscellaneous data that is used by the aggregation facility and eventually the model. Use of most of the satellite data is optional and is user-specified. The *Model* folder contains the core code for the ENVISAGE Model. The remaining folders are user folders and will contain the files for single projects. Single projects can be identified with a specific aggregation, different dynamic assumptions and/or a different focus.
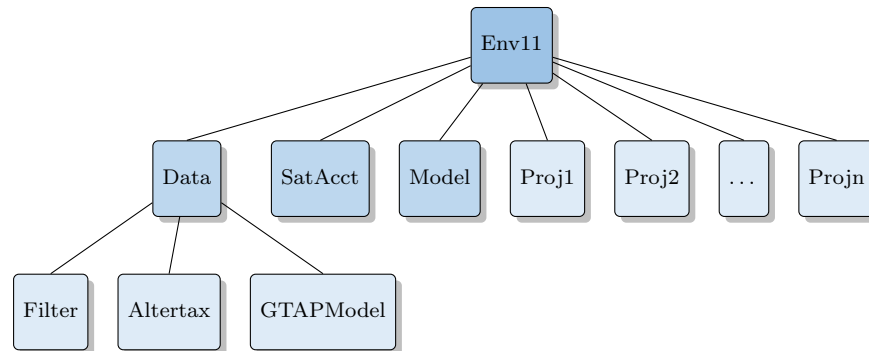
Figure 21: **Possible directory structure**

F.1.3 Workflow

The workflow for the entire aggregation facility is contained in the GAMS file `MakeData.gms`. In principle, the only thing the user needs to modify in the file is the `BaseName` parameter, which is at the top of the file. The `BaseName` typically reflects the name of a specific aggregation and/or project. Names of all the relevant input files will assume that the `BaseName` is part of the filename, and all output files will also contain `BaseName` as part of the output filename—including output folders.

There is a set of flags at the beginning of the file, which allows users to select which parts of the aggregation facility to skip. One could in principle skip the filtering process and `Altertax`, but all other modules should be run.

The only direct input to the `MakeData` file is a user prepared *options* file which must be contained in a sub-folder with the name `BaseName` and the options file must have the name `BaseNameOpt.gms`. If, for example, `BaseName` is equal to 'G20', the option file must be called `G20Opt.gms` and be located in the sub-folder 'G20'.

Table F.1 provides a list of files distributed with the data aggregation facility. In addition, the distribution will have a few sample 'MAP' files that users are likely to use as a starting point for new or modified aggregations. It is assumed that users will have access and appropriate licenses for the GTAP database files (see below for list of GTAP

files). Most files need no user intervention. The exceptions would be the two base 'sets' definition files that would need to be updated with new database releases.[4]

Listing F.1 is an example of the user options in the 'Opt' file. The first four relate to the relevant GTAP version. This example is using version `V11_1POW` of the GTAP Data Base, located in the folder `V:\GTAP\V11_1POW`, with a base name of `GSDF` and a reference year of 2017. Somewhat fortuitously, the updated SSP projections use the same reference year for prices, i.e., the `PPPYear`. All satellite account data is in the folder `satDir`, including the SSP file (`SSPFile`) and the World Bank's projections of education and gender (`GIDDFILE`).

Listing F.1: **Global options for the workflow**

```
1   $setGlobal ver       V11_1POW
2   $setGlobal GTAPDir   v:\GTAP11\%ver%
3   $setGlobal DataBase  GSDF
4   $setGlobal RefYear   2017
5   $setGlobal PPPYear   2017
6   $setGlobal satDir    ..\SatAcct
7   $setGlobal SSPFile   SSPScenOCT2024.gdx
8   $setGlobal GIDDFile  giddProj.gdx
```

The first steps to run this code are: (1) create a sub-folder in the 'Data' folder with the desired `BaseName` as the sub-folder name; (2) create a file in the sub-folder using the `BaseName` appended with 'Opt.gms' and enter the appropriate options as in Listing F.1, and (3) change the `BaseName` option in the file `MakeData.gms`. There are a few additional steps—described next—that are required before running the aggregation facility.

F.1.4 Primary aggregation

The first step in the aggregation facility is to run the primary aggregation of the GTAP Data Base. This is a required step. The output of this aggregation will be a 'diagonal' database, with the same geometry as the standard GTAP Data Base, but aggregated. The reason for 'diagonality' is that the filtering routine, described next, requires diagonality.

Both the primary and secondary aggregation use the same aggregation code, which is provided in the GAMS file `AggGTAP.gms`. It is mostly a stand-alone code, though uses the file `CheckMap.gms` to check the user-based mappings.

The only user input into the primary aggregation is a user-prepared map file, which contains the aggregation mappings for commodities (and activities), regions and factors of production. The mapping file must be contained in the sub-folder called `BaseName` and with the name `BaseNameMap1.gms`, e.g., `G20Map1.gms`. The input files are typically a version of the GTAP database. The output files will be saved in the sub-folder `Agg1` in the `BaseName` sub-folder.

The following describes each section of the primary mapping file:

1. The user must indicate whether this is the primary of secondary aggregation. Both aggregations use the same GTAP code. The main difference between the two is that the secondary aggregation will save the various mapping sets to the resulting GDX file with the aggregate database. These maps can be useful for downstream processes and for documentation purposes.

2. Definition of commodities, indexed by i.

3. Users must declare the margin commodities, but the code will define this set based on the data.

4. Users must declare the activities set, but it will be identical to the commodities set.

5. Definition of regions, indexed by r.

6. Definition of the factors of production (or endowments), indexed by fp.

7. Subsets of the factors, l for labor, cap for capital, lnd for land, and nrs for natural resources. N.B. Users are free to use any labels for the factors of production, for example, `lnd(fp) / terre "Facteur de production 'la terre' /".`

8. Default CET transformation elasticities for allocation factors of production. The user enters this when declaring the parameter `etrae0`. Perfect mobility is assumed when the elasticity is given by `INF`. At this stage, the process uses the GTAP convention of entering CET elasticities as a negative number. It also retains the convention

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[4] The 'sets' definition files are now configured for the new GTAP database configuration that accompanies Version 7 of the GTAP model. These 'sets' definition files have been given the suffix 'F'. The aggregation facility is not backward compatible and only works with the new database format.

that natural resources have very low mobility.[5] The data defined in the parameter `etrae0` is transferred to the parameter `etrae1`, which is indexed by r.

9. Subsets of the factors of production. Sector-specific factors are assumed for all factors with a transformation elasticity of less than 0.01. Mobile factors are assumed when the elasticity is `INF`. Partially mobile factors are not sector-specific or mobile.

10. The 'make' elasticities are entered next. The transformation elasticity defaults to -5 and the CES elasticity to `INF`. With a diagonal matrix, these values are irrelevant. The user can also override these values in the ENVISAGE parameter file.

11. The non-household expenditure elasticities are entered next with default values of 1 for government expenditures, 0 for investment expenditures and 1 for the allocation of international trade and transport services across regions.

12. The aggregation facility allows for combining the returns to land and natural resources with capital returns.[6] In the example in the listing, the natural resource payment is merged in the aggregate `pfd` (processed food), which includes the fisheries sector from the GTAP Data Base.

13. The commodity mapping is provided next. Each GTAP commodity is mapped once to an aggregate commodity. All mappings are verified for consistency.

14. The activity mapping is provided next. In the primary aggregation, code is provided that explicitly assumes a diagonal output database, i.e., a 1-to-1 correspondence between aggregate activities and aggregate commodities. In the secondary aggregation, users have to provide the aggregation mapping explicitly as the activity labels must differ from commodity labels, even in the case of a diagonal output database.

15. The regional mapping is next. Each GTAP region must be assigned to one, and only one, aggregate region.

16. The factor mapping is next, with each GTAP endowment being mapped to one aggregate factor.

17. The last item is for the AEZ version of the database. At the moment, the AEZ version of the ENVISAGE model is under review.

Listing F.2: **Mapping file for primary aggregation**

```
1   $onempty

3   $setGlobal aggStep aggPrimary

5   sets
6      i   "Commodities"   /
7            crp       "Crops"
8            lvs       "Livestock and dairy"
9            frs       "Forestry"
10           coa       "Coal extraction"
11           oil       "Oil extraction"
12           gas       "Gas extraction"
13           oxt       "Other pimary sectors"
14           pfd       "Processed food"
15           xlm       "Other light manufacturing"
16           ppp       "Paper products & publishing"
17           chp       "Chemicals, rubber and plastics"
18           nmm       "Non-metallic minerals"
19           i_s       "Ferrous metals"
20           nfm       "Other metals"
21           xmn       "Other manufacturing"
22           p_c       "Refined oil"
23           etd       "Electricity transmission and distribution"
24           els       "Solar power"
25           elw       "Wind power"
26           elr       "Other renewable power"
```

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[5] The ENVISAGE model assumes that natural resources are sector-specific and have sector-specific supply elasticities.

[6] One would want to do this in the case the sectors where these flows appear are aggregated into a large sector, e.g., other mining with manufacturing. Unless the elasticity of supply is very high, one could be implausibly constraining the supply of the aggregate sector.

```
27          elh       "Hydro power"
28          elc       "Coal-fired power"
29          elg       "Gas power"
30          elo       "Oil power"
31          eln       "Nuclear power"
32          cns       "Construction"
33          atp       "Air transport"
34          wtp       "Water transport"
35          otp       "Other transport"
36          srv       "Services"
37        /

39     m(i)   "Margin commodities"

41     a   "Activities"    /
42          set.i
43        /

45     r   "Regions" /
46          ARG       "Argentina"
47          AUS       "Australia"
48          BRA       "Brazil"
49          CAN       "Canada"
50          CHN       "China"
51          DEU       "Germany"
52          FRA       "France"
53          ITA       "Italy"
54          IND       "India"
55          IDN       "Indonesia"
56          JPN       "Japan"
57          MEX       "Mexico"
58          RUS       "Russian Federation"
59          SAU       "Saudi Arabia"
60          ZAF       "South Africa"
61          KOR       "South Korea"
62          TUR       "Türkiye"
63          GBR       "United Kingdom"
64          USA       "United States"
65          XEA       "Rest of East Asia and Pacific"
66          XSA       "Rest of South Asia"
67          XEC       "Rest of Europe and Central Asia"
68          XMN       "Rest of Middle East and North Africa"
69          XSS       "Rest of Sub-Saharan Africa"
70          XLC       "Rest of Latin America and Caribbean"
71          XEU       "Rest of the European Union"
72          XHY       "Rest of high-income countries"
73        /

75     fp   "Factors of production"   /
76          NSK       "Unskilled labor"
77          SKL       "Skilled labor"
78          CAP       "Capital"
79          LND       "Land"
80          NRS       "Natural resources"
81        /

83     l(fp)   "Labor factors" /
84          NSK       "Unskilled labor"
85          SKL       "Skilled labor"
86        /
87     cap(fp) "Capital" /
88          CAP       "Capital"
89        /
90     lnd(fp) "Land endowment" /
91          LND       "Land"
92        /
93     nrs(fp) "Natural resources" /
94          NRS       "Natural resources"
```

```
 95         /
 96      ;

 98      Parameter
 99         etrae1(fp,r)  "CET transformation elasticities for factor allocation"
100      ;

102      *  Use the GTAP convention that CET elastities are entered as negative numbers

104      parameter etrae0(fp) "CET transformation elasticities for factor allocation" /
105         NSK       inf
106         SKL       inf
107         CAP       inf
108         LND      -1
109         NRS      -0.001
110      / ;

112      etrae1(fp,r) = etrae0(fp) ;

114      set
115         fpf(fp)       "Sector specific factors"
116         fps(fp)       "Sluggish factors"
117         fpm(fp)       "Mobile factors"
118      ;

120      fpf(fp)$(abs(etrae0(fp)) lt 0.01) = yes ;
121      fpm(fp)$(etrae0(fp) eq inf)       = yes ;
122      fps(fp)$(not fpf(fp) and not fpm(fp)) = yes ;

124      *  NEW -- MAKE ELASTICITIES

126      Parameter
127         etraq1(a,r)        "MAKE CET Elasticity"
128         esubq1(i,r)        "MAKE CES Elasticity"
129      ;
130      etraq1(a,r) = -5 ;
131      esubq1(i,r) = inf ;

133      *  NEW -- EXPENDITURE ELASTICITIES

135      Parameter
136         esubg1(r)          "Government expenditure CES elasticity"
137         esubi1(r)          "Investment expenditure CES elasticity"
138         esubs1(i)          "Transport margins CES elasticity"
139      ;

141      esubg1(r) = 1 ;
142      esubi1(r) = 0 ;
143      esubs1(i) = 1 ;

145      set mapt(a) "Merge land and capital payments in the following sectors" /

147      / ;

149      set mapn(a) "Merge natl. res. and capital payments in the following sectors" /
150         pfd
151      / ;

153      *  Commodity mappings: from GTAP (comm) to modeled commodities (i)
154      set mapi(i,comm) /
155         crp.PDR
156         crp.WHT
157         crp.GRO
158         crp.V_F
159         crp.OSD
160         crp.C_B
161         crp.PFB
162         crp.OCR
```

153

```
163    lvs.CTL
164    lvs.OAP
165    lvs.RMK
166    lvs.WOL
167    frs.FRS
168    pfd.FSH
169    coa.COA
170    oil.OIL
171    gas.GAS
172    oxt.OXT
173    pfd.CMT
174    pfd.OMT
175    pfd.VOL
176    pfd.MIL
177    pfd.PCR
178    pfd.SGR
179    pfd.OFD
180    pfd.B_T
181    xlm.TEX
182    xlm.WAP
183    xlm.LEA
184    xlm.LUM
185    ppp.PPP
186    p_c.P_C
187    chp.CHM
188    chp.BPH
189    chp.RPP
190    nmm.NMM
191    i_s.I_S
192    nfm.NFM
193    xmn.FMP
194    xmn.ELE
195    xmn.EEQ
196    xmn.OME
197    xmn.MVH
198    xmn.OTN
199    xmn.OMF
200    etd.TnD
201    eln.NuclearBL
202    elc.CoalBL
203    elg.GasBL
204    elw.WindBL
205    elh.HydroBL
206    elo.OilBL
207    elr.OtherBL
208    elg.GasP
209    elh.HydroP
210    elo.OilP
211    els.SolarP
212    gas.GDT
213    srv.WTR
214    cns.CNS
215    srv.TRD
216    srv.AFS
217    otp.OTP
218    wtp.WTP
219    atp.ATP
220    srv.WHS
221    srv.CMN
222    srv.OFI
223    srv.INS
224    srv.RSA
225    srv.OBS
226    srv.ROS
227    srv.OSG
228    srv.EDU
229    srv.HHT
230    srv.DWE
```

```
231  / ;

233  *  Primary aggregation assumes diagonality: i.e., 1-to-1 mapping of activities and commodities
234  set mapa(a,acts) ;
235  loop(sameas(a,i),
236     loop((acts,comm)$(mapi(i,comm) and sameas(acts,comm)),
237        mapa(a,acts) = yes ;
238     ) ;
239  ) ;

241  *  Regional mapping: from GTAP (reg) to modeled regions (r)
242  set mapr(r,reg) /
243     AUS.AUS
244     XHY.NZL
245     XEA.XOC
246     CHN.CHN
247     XHY.HKG
248     JPN.JPN
249     KOR.KOR
250     XEA.MNG
251     XHY.TWN
252     XEA.XEA
253     XEA.BRN
254     XEA.KHM
255     IDN.IDN
256     XEA.LAO
257     XEA.MYS
258     XEA.PHL
259     XHY.SGP
260     XEA.THA
261     XEA.VNM
262     XEA.XSE
263     XSA.AFG
264     XSA.BGD
265     IND.IND
266     XSA.NPL
267     XSA.PAK
268     XSA.LKA
269     XSA.XSA
270     CAN.CAN
271     USA.USA
272     MEX.MEX
273     XHY.XNA
274     ARG.ARG
275     XLC.BOL
276     BRA.BRA
277     XLC.CHL
278     XLC.COL
279     XLC.ECU
280     XLC.PRY
281     XLC.PER
282     XLC.URY
283     XLC.VEN
284     XLC.XSM
285     XLC.CRI
286     XLC.GTM
287     XLC.HND
288     XLC.NIC
289     XLC.PAN
290     XLC.SLV
291     XLC.XCA
292     XLC.DOM
293     XLC.JAM
294     XLC.HTI
295     XLC.PRI
296     XLC.TTO
297     XLC.XCB
298     XEU.AUT
```

```
299    XEU.BEL
300    XEU.CYP
301    XEU.CZE
302    XEU.DNK
303    XEU.EST
304    XEU.FIN
305    FRA.FRA
306    DEU.DEU
307    XEU.GRC
308    XEU.HUN
309    XEU.IRL
310    ITA.ITA
311    XEU.LVA
312    XEU.LTU
313    XEU.LUX
314    XEU.MLT
315    XEU.NLD
316    XEU.POL
317    XEU.PRT
318    XEU.SVK
319    XEU.SVN
320    XEU.ESP
321    XEU.SWE
322    GBR.GBR
323    XHY.CHE
324    XHY.NOR
325    XHY.XEF
326    XEC.ALB
327    XEC.SRB
328    XEU.BGR
329    XEC.BLR
330    XEU.HRV
331    XEU.ROU
332    RUS.RUS
333    XEC.UKR
334    XHY.XEE
335    XHY.XER
336    XEC.KAZ
337    XEC.KGZ
338    XEC.TJK
339    XEC.UZB
340    XEC.XSU
341    XEC.ARM
342    XEC.AZE
343    XEC.GEO
344    XMN.BHR
345    XMN.IRN
346    XMN.IRQ
347    XMN.ISR
348    XMN.JOR
349    XMN.KWT
350    XMN.LBN
351    XMN.OMN
352    XMN.PSE
353    XMN.QAT
354    SAU.SAU
355    XMN.SYR
356    TUR.TUR
357    XMN.ARE
358    XMN.XWS
359    XMN.DZA
360    XMN.EGY
361    XMN.MAR
362    XMN.TUN
363    XMN.XNF
364    XSS.BEN
365    XSS.BFA
366    XSS.CMR
```

```
367    XSS.CIV
368    XSS.GHA
369    XSS.GIN
370    XSS.MLI
371    XSS.NER
372    XSS.NGA
373    XSS.SEN
374    XSS.TGO
375    XSS.XWF
376    XSS.AGO
377    XSS.CAF
378    XSS.TCD
379    XSS.COG
380    XSS.COD
381    XSS.GNQ
382    XSS.GAB
383    XSS.STP
384    XSS.BDI
385    XSS.COM
386    XSS.ETH
387    XSS.KEN
388    XSS.MDG
389    XSS.MWI
390    XSS.MUS
391    XSS.MOZ
392    XSS.RWA
393    XSS.SDN
394    XSS.TZA
395    XSS.UGA
396    XSS.ZMB
397    XSS.ZWE
398    XSS.XEC
399    XSS.BWA
400    XSS.SWZ
401    XSS.NAM
402    ZAF.ZAF
403    XSS.XSC
404    XSS.XTW
405  / ;

407  *  Factor mapping: from GTAP (endw) to modeled factors (fp)
408  set mapf(fp, endw) /
409    NSK.ag_othlowsk
410    NSK.service_shop
411    NSK.clerks
412    SKL.tech_aspros
413    SKL.off_mgr_pros
414    CAP.Capital
415    LND.Land
416    NRS.NatlRes
417  / ;

419  *  Only used if the LU database exists

421  set AEZ  / AEZ1*AEZ18 / ;
422  set AEZ1 / AEZ1*AEZ18 / ;
423  set mapLU(aez1,aez) ; mapLU(aez1,aez)$sameas(aez,aez1) = yes ;
```

F.1.5 Filtering

The *filter* procedure is used to remove very small transactions from the aggregated (and diagonal) database. It is based on the procedure developed by Tom Rutherford (Lanz and Rutherford (2016)), integrated into CGEBox (Britz and van der Mensbrugghe (2018)) and further modified for use with this aggregation facility. The user can set tolerance levels, as well as regions and/or sectors to exempt from the filtering procedure.

The calling procedure is in the main folder and named `filter.gms`, which will in turn run the filtering program

that is contained in the `filter` sub-folder. The user must prepare a file with filter options. This file must be in the project sub-folder named `BaseName` and it must be named `BaseNameFlt.gms`, for example `G20Flt.gms`. Listing F.3 provides an example of the user options. Users are referred to the original documentation for a description of most of the options. The one that is most used are the exempted sectors, which typically will include the new electricity activities as in many countries these are at relatively low levels and we do not wish to drop them. The input files for the filtering procedure are in the sub-folder `Agg1` and the output files will be in the sub-folder `Flt`.

The `filter` folder contains the remaining GAMS code files, which includes `filter.gms`, `itrlog.gms`, `title.gms`, and `remTinyValues.gms`.

Beyond the listing file, the user should take a brief look at the two log files that are part of the output. The file `BaseNameFlt.prn` in the `BaseName` sub-folder contains the key statistics emerging from the filtering routine. The file `BaseNameLog.txt` contains information that is sent to the standard GAMS `log` file.

Listing F.3: **User option file for the filter program**

```
1   $setGlobal excCombined   0
2
3   $setglobal excSecs "els, elw, elr"
4   $setglobal excRegs
5
6   scalars
7       ifKeepIntermediateConsumption / 1 /
8       ifKeepPrivateconsumption      / 1 /
9       ifKeepGovernmentconsumption   / 1 /
10      ifKeepInvestments             / 1 /
11      ifGDPKeep                     / 1 /
12      ifKeepFactorincomeplusbop     / 1 /
13      ifAdjDepr                     / 1 /
14      abstol                        / 1e-10 /
15      relTol                        / 0.005 /
16      relTolRed                     / 1e-6  /
17      nsteps                        / 5 /
18      minNumTransactions            / 50000 /
19  ;
20
21  file log / %baseName%\%baseName%Log.txt / ;
22  put log ;
```

After the filtering process, the energy and emissions databases are adjusted to be consistent with the post-filtered data. In essence, the pre-filtered coefficients are calculated and applied to the post-filtered values to provide a new set of energy and emission volumes. The adjustment is done with the `AdustAuxAccounts.gms` GAMS code.

F.1.6   Secondary aggregation

The secondary aggregation takes the results from the filtering process and provides the final aggregated database. It is a necessary step irrespective of whether the final database is diagonal as the ENVISAGE model requires different labels for commodities and activities.

The format of the mapping file is nearly identical to the format of the mapping file for the primary aggregation—described above with the following exceptions:

1. The `aggStep` parameter at the beginning of the user file (`BaseNameMap2.gms`) must have the value `aggSecondary`.

2. Activities must be explicitly defined and the labels must differ from the commodity labels.

3. Typically the regional and factor aggregations are not re-defined and simple code can be used to define the mappings.

There are some output differences with the secondary aggregation. Whereas the primary aggregation only saves the original sets and mappings, the secondary aggregation will save both the original sets and mappings, as well as the intermediate sets and mappings. This is potentially useful downstream and for documentation purposes. The only other difference between the primary and secondary aggregation is the calculation of energy combustion. In the case of the primary aggregation, this is calculated using the original database. In the case of the secondary aggregation, it will be based on an aggregation of the energy combustion calculated from the primary aggregation.

Listing F.4: **Map file example for the secondary aggregation**

```
1   $onempty

3   $setGlobal aggStep aggSecondary

5   sets
6       i   "Commodities"   /
7           crp-c   "Crops"
8           lvs-c   "Livestock and dairy"
9           frs-c   "Forestry"
10          coa-c   "Coal extraction"
11          oil-c   "Oil extraction"
12          gas-c   "Gas extraction"
13          oxt-c   "Other pimary sectors"
14          pfd-c   "Processed food"
15          xlm-c   "Other light manufacturing"
16          ppp-c   "Paper products & publishing"
17          chp-c   "Chemicals, rubber and plastics"
18          nmm-c   "Non-metallic minerals"
19          i_s-c   "Ferrous metals"
20          nfm-c   "Other metals"
21          xmn-c   "Other manufacturing"
22          p_c-c   "Refined oil"
23          ely-c   "Electricity"
24          cns-c   "Construction"
25          atp-c   "Air transport"
26          wtp-c   "Water transport"
27          otp-c   "Other transport"
28          srv-c   "Services"
29      /

31      m(i)   "Margin commodities"

33      a   "Activities"   /
34          crp-a   "Crops"
35          lvs-a   "Livestock and dairy"
36          frs-a   "Forestry"
37          coa-a   "Coal extraction"
38          oil-a   "Oil extraction"
39          gas-a   "Gas extraction"
40          oxt-a   "Other pimary sectors"
41          pfd-a   "Processed food"
42          xlm-a   "Other light manufacturing"
43          ppp-a   "Paper products & publishing"
44          chp-a   "Chemicals, rubber and plastics"
45          nmm-a   "Non-metallic minerals"
46          i_s-a   "Ferrous metals"
47          nfm-a   "Other metals"
48          xmn-a   "Other manufacturing"
49          p_c-a   "Refined oil"
50          etd-a   "Electricity transmission and distribution"
51          els-a   "Solar power"
52          elw-a   "Wind power"
53          elr-a   "Other renewable power"
54          elh-a   "Hydro power"
55          elc-a   "Coal-fired power"
56          elg-a   "Gas power"
57          elo-a   "Oil power"
58          eln-a   "Nuclear power"
59          cns-a   "Construction"
60          atp-a   "Air transport"
61          wtp-a   "Water transport"
62          otp-a   "Other transport"
63          srv-a   "Services"
64      /

66      r   "Regions" /
67          ARG     "Argentina"
```

159

```
 68          AUS       "Australia"
 69          BRA       "Brazil"
 70          CAN       "Canada"
 71          CHN       "China"
 72          DEU       "Germany"
 73          FRA       "France"
 74          ITA       "Italy"
 75          IND       "India"
 76          IDN       "Indonesia"
 77          JPN       "Japan"
 78          MEX       "Mexico"
 79          RUS       "Russian Federation"
 80          SAU       "Saudi Arabia"
 81          ZAF       "South Africa"
 82          KOR       "South Korea"
 83          TUR       "Türkiye"
 84          GBR       "United Kingdom"
 85          USA       "United States"
 86          XEA       "Rest of East Asia and Pacific"
 87          XSA       "Rest of South Asia"
 88          XEC       "Rest of Europe and Central Asia"
 89          XMN       "Rest of Middle East and North Africa"
 90          XSS       "Rest of Sub-Saharan Africa"
 91          XLC       "Rest of Latin America and Caribbean"
 92          XEU       "Rest of the European Union"
 93          XHY       "Rest of high-income countries"
 94        /

 96    fp  "Factors of production"  /
 97          NSK       "Unskilled labor"
 98          SKL       "Skilled labor"
 99          CAP       "Capital"
100          LND       "Land"
101          NRS       "Natural resources"
102        /

104    l(fp)   "Labor factors" /
105          NSK       "Unskilled labor"
106          SKL       "Skilled labor"
107        /
108    cap(fp) "Capital" /
109          CAP       "Capital"
110        /
111    lnd(fp) "Land endowment" /
112          LND       "Land"
113        /
114    nrs(fp) "Natural resources" /
115          NRS       "Natural resources"
116        /
117  ;

119  Parameter
120     etrae1(fp,r) "CET transformation elasticities for factor allocation"
121  ;

123  *  Use the GTAP convention that CET elastities are entered as negative numbers

125  parameter etrae0(fp) "CET transformation elasticities for factor allocation" /
126     NSK       inf
127     SKL       inf
128     CAP       inf
129     LND      -1
130     NRS      -0.001
131  / ;

133  etrae1(fp,r) = etrae0(fp) ;

135  set
```

```
136     fpf(fp)      "Sector specific factors"
137     fps(fp)      "Sluggish factors"
138     fpm(fp)      "Mobile factors"
139  ;

141  fpf(fp)$(abs(etrae0(fp)) lt 0.01) = yes ;
142  fpm(fp)$(etrae0(fp) eq inf)       = yes ;
143  fps(fp)$(not fpf(fp) and not fpm(fp)) = yes ;

145  *  NEW -- MAKE ELASTICITIES

147  Parameter
148     etraq1(a,r)       "MAKE CET Elasticity"
149     esubq1(i,r)       "MAKE CES Elasticity"
150  ;
151  etraq1(a,r) = -5 ;
152  esubq1(i,r) = inf ;

154  *  NEW -- EXPENDITURE ELASTICITIES

156  Parameter
157     esubg1(r)         "Government expenditure CES elasticity"
158     esubi1(r)         "Investment expenditure CES elasticity"
159     esubs1(i)         "Transport margins CES elasticity"
160  ;

162  esubg1(r) = 1 ;
163  esubi1(r) = 0 ;
164  esubs1(i) = 1 ;

166  set mapt(a) "Merge land and capital payments in the following sectors" /

168  / ;

170  set mapn(a) "Merge natl. res. and capital payments in the following sectors" /

172  / ;

174  *  Commodity mapping
175  set mapi(i,comm) /
176     crp-c.crp
177     lvs-c.lvs
178     frs-c.frs
179     coa-c.coa
180     oil-c.oil
181     gas-c.gas
182     oxt-c.oxt
183     pfd-c.pfd
184     xlm-c.xlm
185     ppp-c.ppp
186     chp-c.chp
187     nmm-c.nmm
188     i_s-c.i_s
189     nfm-c.nfm
190     xmn-c.xmn
191     p_c-c.p_c
192     ely-c.etd
193     ely-c.els
194     ely-c.elw
195     ely-c.elr
196     ely-c.elh
197     ely-c.elc
198     ely-c.elg
199     ely-c.elo
200     ely-c.eln
201     cns-c.cns
202     atp-c.atp
203     wtp-c.wtp
```

161

```
204     otp-c.otp
205     srv-c.srv
206 / ;

208 set mapa(a,acts) /
209     crp-a.crp
210     lvs-a.lvs
211     frs-a.frs
212     coa-a.coa
213     oil-a.oil
214     gas-a.gas
215     oxt-a.oxt
216     pfd-a.pfd
217     xlm-a.xlm
218     ppp-a.ppp
219     chp-a.chp
220     nmm-a.nmm
221     i_s-a.i_s
222     nfm-a.nfm
223     xmn-a.xmn
224     p_c-a.p_c
225     etd-a.etd
226     els-a.els
227     elw-a.elw
228     elr-a.elr
229     elh-a.elh
230     elc-a.elc
231     elg-a.elg
232     elo-a.elo
233     eln-a.eln
234     cns-a.cns
235     atp-a.atp
236     wtp-a.wtp
237     otp-a.otp
238     srv-a.srv
239 / ;

241 set mapr(r,reg) ; mapr(r,reg)$sameas(r,reg) = yes ;

243 set mapf(fp,endw) ; mapf(fp,endw)$sameas(fp,endw) = yes ;

245 *  Only used if the LU database exists

247 set AEZ  / AEZ1*AEZ18 / ;
248 set AEZ1 / AEZ1*AEZ18 / ;
249 set mapLU(aez1,aez) ; mapLU(aez1,aez)$sameas(aez1,aez) = yes ;
```

F.1.7   Altering the aggregated database

The fourth step in the data processing facility is the ability to make adjustments to the aggregated database. This optional step is known as the *Altertax* procedure described in Malcolm (1998). It is typically used as a pre-processing step to adjust tax rates to a new reference point, for example to modify tariff rates. The *Altertax* procedure uses a GAMS version of the standard GTAP model. All the standard elasticities are over-ridden and set to (near) 1 to minimize deviations in the initial budget shares. The input database for the *Altertax* procedure is in the sub-folder `Agg2` and the output will be in the folder `Alt`. Similar to the filtering procedure, a post-process procedure is implemented to adjust the energy and emissions volumes using the `AdjustAuxAccounts.gms` code.

The calling code is in the main data folder and called `AlterTax.gms`. In principle, the user does not need to modify this file. However, the model code does require a user file to define some key sets used by the model. The user sets need to be defined in a file with the name `BaseNameSets.gms`, which needs to be located in the `BaseName` sub-folder. The sets that need to be defined by the user are the following:

1. `rres`. The model requires a residual region, which is a singleton set. Typically, it will be the largest economy.

2. `rmuv(r)`. The model defines a global price index known as the Manufacturing Unit Value (MUV), which is an index of export prices of manufactured goods for key industrial economies. This index can be used to deflate

real international values, such as net capital flows. Typically, this index is defined over the major industrial economies.

3. `imuv(i)`. This set is also linked to the MUV index and defines the industries included in the calculation of the MUV index.

Listing F.5: **User defined sets for the AlterTax procedure**

```
1   *  Additional sets that are aggregation dependent

3   singleton Sets
4      rres(r)      "Residual region"    / usa /
5   ;

7   Sets
8      rmuv(r)      "MUV regions"        / AUS, CAN, CHN, DEU, FRA, ITA, JPN, KOR, GBR, USA, XHY /
9      imuv(i)      "MUV commodities"    / pfd-c, xlm-c, ppp-c, chp-c, nmm-c, i_s-c, nfm-c, xmn-c /
10  ;
```

The rest of the code for the GTAP model is provided in the sub-folder **GTAPModel**. The core model code is in the file **Model.gms**. The database is read in the file **getData.gms** and the model parameters and variables are initialized and calibrated in the file **cal.gms**. There are some updating statements at the beginning of each solution period that are contained in the file **iterloop.gms**. This file also calls **mvar.gms** to initialize model variables between periods. The model is solved from code in the file **solve.gms**. The output database is created with the file **saveData.gms**.[7] The file **AlterTaxPrm.gms** contains the parameter overrides for the *AlterTax* procedure.

The key user input is the *AlterTax* shock file, which should be in the project sub-folder and called **BaseNameAlt.gms**.[8] This file is included in the core **AlterTax.gms** file for the simulation year with the label 'Shock'. In the standard version of **AlterTax.gms**, time is defined over three periods: 'Base', 'Check' and 'Shock'. The 'Base' period contains the initialized model solution, essentially the base data. The 'Check' period solves the model with no shocks. In essence, this should simply re-produce the base solution. If looking at the GAMS residual check[9], all residuals should be close to zero. It is a good validation test for the model initialization and calibration. The 'Shock' period solves the model with a user specified shock. If the shock is using a time-based variable or parameter, the appropriate time should be specified as either 'Shock', or 'tsim'. A very simple example is:

```
imptax.fx(s,i,"IDN",tsim)$ASEAN(s) = 0.5 * imptax.l(s,i,"IDN",t0) ;
```

which cuts import tariffs by 50% for all imports from ASEAN regions into Indonesia. A more complicated example comes from a potential initialization of marginal abatement cost curves (MACC) for abating non-combustion greenhouse gas emissions. The MACCs are initialized with a negligible carbon price.[10] The code below introduces the shock.

Listing F.6: **Using AlterTax to initialize carbon prices**

```
1       ytax0(r,"ct")     = 1 ;

3       execute_load "%inDir%/%BaseName%NCO2.gdx", gwp, emi_iop, emi_endw, emi_qo ;

5       ProcEmi0(r,em,a)  = gwp(em,r,"AR4")*cscale
6                      *  (sum(i, emi_iop(em, i, a, r))
7                      +   sum(fp, emi_endw(em, fp, a, r))
8                      +   emi_qo(em, a, r)) ;

10      pcarb0(r) = 0.25 ;
```

---

[7] The code in **postsim.gms** is not currently being used, but it can be included at the end of the main file (i.e., **AlterTax.gms**) to produce a CSV file with the model's core indicators, including the SAM.

[8] This file has to be created, even if there is no shock implemented. At a minimum, it should have a single empty line.

[9] Set the GAMS options **limrow** to a positive value.

[10] This is necessary because the MACCs are based on a CES technology in which the initial price must differ from zero.

```
12      ctaxFlag(r,a)$sum(em, pcarb0(r)*(inscale/cscale)*procEmi0(r,em,a) and xp0(r,a)) = yes ;
13      ctax.l(r,a,tsim)$ctaxFlag(r,a)  = 0.0003 ;
14      ctax.lo(r,a,tsim)$ctaxFlag(r,a) = -inf ;
15      ctax.up(r,a,tsim)$ctaxFlag(r,a) = +inf ;
```

First, it calculates the level of process emissions for each sector.[11] It reads emissions from the NCO2 file and sums over all possible sources and converts to CO_2-equivalent using the IPCC's global warming potential. We set the initial carbon price to \$0.25. This version of the GTAP model is not 'emissions' aware. We have added an equation that calculates an output tax (ctax) that would yield the same revenue as the carbon price. This introduces a wedge, in the database, between the unit cost of production and the value of sales, which, when introduced in the ENVISAGE model is exactly equal to the revenues generated by the initial process emissions when priced at \$0.25 per ton of CO2e.

F.1.8 Preparing data for the Envisage model

This section is used to prepare sets and aggregate parameters for use by the ENVISAGE model. The driving code is contained in the GAMS file AggEnvElast.gms, which is in the main Data folder.

The core input files are:

1. The original GTAP Data Base, where the user provides a specific version number.

2. The disaggregated ENVISAGE elasticities that are provided in the satellite data folder. The disaggregated ENVISAGE elasticities are originally sourced from the OECD circular economy project. A new version is created with each new GTAP version, where the user provides a set mapping between the circular economy database and the targeted GTAP version.[12]

3. The aggregated database, which is in folder Fnl.

The user prepares a 'sets' file, which will be used by both the aggregation facility and the ENVISAGE model, i.e., the user should prepare the 'sets' file in the project folder itself, not the project sub-folder in the Data folder. Using the default folder configuration, the sets folder will be in ../BaseName/ and called BaseNameSets.gms.[13] The following describe the main elements of the 'Sets' file:

1. The unskilled (ul) and skilled (sl) labor sets are there for backward compatibility, but are no longer used.

2. The labor nesting in the production structure has two main labor bundles, which appear in different levels of the production nest—with *LAB1* typically towards the top in a traditional capital+energy vs. labor substitution and *LAB2* more closely directly linked with capital. Intuitively, capital requires skilled labor for operations and the skill+capital bundle is a substitute for unskilled labor. The model also includes an *intermediate* demand bundle labeled *LABB*. This is to allow for more subtle distinctions across labor types, for example gender and/or native- vs. foreign-born workers. The user specifies the intermediate labor bundles using a new set *wb*. Each granular labor category is mapped to one, and only one labor bundle indexed by *wb*. And each of the *LABB* bundles is mapped to either *LAB1* or *LAB2*. Examples are provided below after Listing F.7.

3. Similar to the *AlterTax* procedure and the GTAP model used therein, the user needs to define a residual region (rres) as well as the relevant set definitions for the MUV index (rmuv and imuv).

4. Versions of the model may use definitions for education (iedu) and/or health services (ihht).

5. The post-simulation process will prepare pre-defined sector and regional aggregations of most model indicators. The user defines these aggregate sets for commodities (ia), activities (aga) and regions (ra). Though not required, in each case it is useful to include the individual labels for each set, which can be accomplished by including the GAMS code set.xxx, replacing xxx with the relevant set, e.g., set.comm for commodities, set.acts for activities and set.reg for the regions. The user also needs to provide the relevant mappings, which can be hard-coded or defined using GAMS code. The example file in the listing provides examples of both. The set mappings are mapia, mapaga and mapra respectively for commodities, activities and regions.

6. The post-simulation processing also uses aggregations for labor (lagg) using the mapping maplagg.

7. The baseline scenario may use the nsk and skl sets of labor to map to education projections.

---

[11]   The non-normalized level of carbon price revenues must be set to 1 since its default value is 0.

[12]   Not currently publicly available, but the codes are typically provided in the folder GTAP11/Elast.

[13]   The previous aggregation facility had two user defined sets, one in the data folder and one in the project folder. This involved significant duplication, so this has been rationalized and there is now a single user input file.

8. The model has different production nestings for crops, livestock and all other sectors. These are defined respectively by the sets `acr`, `alv` and `ax`.

9. The post-simulation processing may use a standard decomposition of activities into agriculture (`agr`), manufacturing (`man`) and services (`srv`).

10. The user can define the energy activities (`aenergy`) and the fossil fuel activities (`affl`).

11. The user can define the water-related activities (`aw`). N.B. The current version of the model needs to be reviewed in order to model water-use adequately.

12. The model allows for labor market segmentation, which can differ across labor types. The model uses the concepts of zones (z), of which there are three types: rural (`rur`), urban (`urb`) and non-segmented (`nsg`) or economy-wide. The mapping set `mapz` allows the user to define the labor market segmentation. The model will use the labor 'migration' elasticity to define the implementation of segmentation. If the elasticity is set to `INF`, the model assumes perfect mobility across segments.

13. The crop production structure identifies agricultural chemicals as a special input (`frt`) and the livestock production structure identifies feed (`feed`). The water services input (`iw`) is not currently in use.

14. The energy commodities are identified and bundled. The set `e` is the full set of energy commodities, of which `eylc` is the electricity commodity. There is also a subset for the fuel commodities (`fuel`).

15. The ENVISAGE model allows for a separate classification of consumed commodities and uses a transition matrix (or consumer make) matrix approach to convert consumed commodities (k) to marketed commodities (i). In the ideal approach, the transition matrix would be relatively dense and among other things, clearly identify the energy mix for the different consumption bundles (transportation, heating and air conditioning, cooking, etc.). In the absence of such detailed data, the approach to defining the set k focuses on two objectives—at least identify an aggregate energy bundle that has the usual substitution possibilities across energy carriers, and a judicious aggregation of other sectors in order to keep the top-level utility function relatively small and perhaps lined up with existing econometric estimates of consumer income and price elasticities. The subsequent consumer demand nesting will assume a CES structure for the sub-nests. The user thus needs to define k and the mapping from i to k (`mapk`)—which at the moment is an n-to-1 mapping, i.e., the transition matrix has only 0's and 1's. The user can also include definitions for various CPI indices, used for post-simulation processing.

16. The next sub-section identifies the power activities and is related to the power version of the GTAP Data Base. The set `elya` represents all of the power activities, `etd` represents the transmission and distribution activity, and primary power activities are identified with `primElya`. The power activities are bundled in the production make matrix. The user defines so-called power bundles (`pb`). Each power activity, apart from transmission and distribution is then mapped to one of the power bundles.

17. The next section refers to the allocation of land, which uses a nested CET structure. The user defines land bundles (`lb`) and a top bundle (`lb1`). Then land in each activity is mapped to one of the bundles. Examples are provided below.

18. The water sets and bundles are currently ignored.

19. At the bottom of each energy bundle is a set of four bundles designated by `COA`, `OIL`, `GAS` and `ELY`. The user needs to provide a mapping (`mape`) between the primary energy carriers and the four bundles.

20. Non-energy intermediate demand is mapped to either the `ND1` bundle or the `ND2` bundle. For example, in the livestock sector, the `ND2` bundle would have the feed commodities.

21. The next section initializes the emissions cap and trade system. The user-defined set `ra`, should have all combinations of regions that might be part of a cap and trade system. The set `rq` defines the potential carbon clubs, initially set to the empty set. The set `aets` should have all potential combinations of activities/agents that would participate in an ETS.

22. The remaining sets are only defined for the model and not used by the aggregation facility. The first is a mapping of labor types to potential education levels. This is used for the education-based projections.

23. The final item is the sort order for the labels of the SAM. This is useful for loading the SAM-based pivot tables in Excel as the labels will be automatically sorted by Excel.

Listing F.7: **Example 'Sets' file for the Envisage model**

```
1  $onempty
2  */ ---------------------------------------------------------------------------------
3  *
```

```
4   *      Sets needed for the ENVISAGE model, i.e., not defined for the GTAP Model
5   *      Only sets that are aggregation dependent.
6   *      Non-aggregation dependent sets are defined in getData.gms
7   *
8   */ --------------------------------------------------------------------------------

10  *   Subsets ul(l) and sl(l) included for backward compatibility
11  *   !!!! This should come from somewhere else--not part of GTAP

13  set ul(l) "Unskilled labor" /
14     set.lab
15  / ;

17  set sl(l) "Skilled labor" ; sl(l)$(not ul(l)) = yes ;

19  *   !!!! This should come from somewhere else--not part of GTAP
20  *   !!!! Solved with flexible nesting

22  set wb "Intermediate labor bundle(s)" /
23     Single       "Single intermediate labor bundle"
24  / ;

26  set maplab1(wb) "Mapping of intermediate labor demand bundle(s) to LAB1" /
27     Single       "Single intermediate labor bundle"
28  / ;

30  set mapl(wb,l) "Mapping of labor categories to intermedate demand bundle(s)" /
31     Single   .nsk
32     Single   .skl
33  / ;

35  set lr(l) "Reference labor for skill premium" /
36     skl          "Skilled labor"
37  / ;

39  * >>>> MUST INSERT RESIDUAL REGION (ONLY ONE)

41  singleton set rres(r) "Residual region" /
42     USA          "United States"
43  / ;

45  * >>>> MUST INSERT MUV REGIONS (ONE OR MORE)

47  set rmuv(r) "MUV regions" / AUS, CAN, CHN, DEU, FRA, ITA, JPN, KOR, GBR, USA, XHY / ;

49  * >>>> MUST INSERT MUV COMMODITIES (ONE OR MORE)

51  set imuv(i) "MUV commodities" / pfd-c, xlm-c, ppp-c, chp-c, nmm-c, i_s-c, nfm-c, xmn-c / ;

53  set iedu(i) "Education sector(s)" /
54     SRV-c
55  / ;

57  set ihht(i) "Health sector(s)" /
58     SRV-c
59  / ;

61  set ia "Aggregate commodities for model output" /
62     set.comm
63     tagr-c       "Agriculture"
64     tman-c       "Manufacturing"
65     tnrg-c       "Energy"
66     tsrv-c       "Services"
67     tprm-c       "Primary commodities"
68     tind-c       "Industrial commodities"
69     tsvc-c       "Services"
70     ttot-c       "Total"
71  / ;
```

```
73   set mapia(ia,i)  "Mapping for aggregate commodities" /
74      tagr-c   .(crp-c, lvs-c, frs-c)
75      tman-c   .(oxt-c, pfd-c, xlm-c, ppp-c, chp-c, nmm-c, i_s-c, nfm-c, xmn-c)
76      tnrg-c   .(coa-c, oil-c, gas-c, p_c-c, ely-c)
77      tsrv-c   .(cns-c, atp-c, wtp-c, otp-c, srv-c)
78      tprm-c   .(crp-c, lvs-c, frs-c, oxt-c, coa-c, oil-c, gas-c)
79      tind-c   .(pfd-c, xlm-c, ppp-c, chp-c, nmm-c, i_s-c, nfm-c, p_c-c, xmn-c)
80      tsvc-c   .(ely-c, cns-c, atp-c, wtp-c, otp-c, srv-c)
81   / ;
82   mapia(ia,i)$sameas(ia,i) = yes ;
83   mapia("ttot-c",i) = yes ;

85   set iaa(ia)  "Aggregate commodities only" ;
86   loop((i,ia)$(not sameas(i,ia)), iaa(ia) = yes ; ) ;

88   set aga  "Aggregate activities for model output" /
89      set.acts
90      tagr-a       "Agriculture"
91      tman-a       "Manufacturing"
92      tsrv-a       "Services"
93      tnrg-a       "Other"
94      tprm-a       "Primary activities"
95      tind-a       "Industrial activities"
96      tsvc-a       "Services"
97      ttot-a       "Total"
98   / ;

100  set mapaga(aga,a)  "Mapping for aggregate activities" /
101     tagr-a   .(crp-a, lvs-a, frs-a)
102     tman-a   .(oxt-a, pfd-a, xlm-a, ppp-a, chp-a, nmm-a, i_s-a, nfm-a, xmn-a)
103     tnrg-a   .(coa-a, oil-a, gas-a, p_c-a,
104                etd-a, els-a, elw-a, elr-a, elh-a, elc-a, elg-a, elo-a, eln-a)
105     tsrv-a   .(cns-a, atp-a, wtp-a, otp-a, srv-a)
106     tprm-a   .(crp-a, lvs-a, frs-a, oxt-a, coa-a, oil-a, gas-a)
107     tind-a   .(pfd-a, xlm-a, ppp-a, chp-a, nmm-a, i_s-a, nfm-a, p_c-a, xmn-a)
108     tsvc-a   .(etd-a, els-a, elw-a, elr-a, elh-a, elc-a, elg-a, elo-a, eln-a
109                cns-a, atp-a, wtp-a, otp-a, srv-a)
110  / ;
111  mapaga(aga,a)$sameas(aga,a) = yes ;
112  mapaga("ttot-a",a) = yes ;

114  set agaa(aga)  "Aggregate activities only" ;
115  loop((a,aga)$(not sameas(a,aga)), agaa(aga) = yes ; ) ;

117  set ra  "Aggregate regions for emission regimes and model output" /
118     set.reg
119     hic          "High-income countries"
120     g20          "G20 countries"
121     row          "Non G20 countries"
122     lmy          "Developing countries"
123     e27          "European Union-27"
124     eap          "Developing East Asia"
125     sas          "South Asia"
126     eca          "Eastern Europe & Central Asia"
127     mna          "Middle East & North Africa"
128     ssa          "Sub-Saharan Africa"
129     lac          "Latin America & Caribbean"
130     wld          "World total"
131  / ;

133  set mapra(ra,r)  "Mapping for aggregate regions" /
134     e27.(deu, fra, ita, xeu)
135     hic.(aus, can, deu, fra, ita, xeu, gbr, jpn, sau, kor, usa, xhy)
136     g20.(arg, aus, bra, can, chn, deu, fra, ita, ind, idn, jpn, mex, rus, sau, zaf, kor, tur, gbr,
            usa)
137     eap.(chn, idn, xea)
138     sas.(ind, xsa)
```

```
139     eca.(rus, tur, xec)
140     mna.(sau, xmn)
141     ssa.(zaf, xss)
142     lac.(arg, bra, mex, xlc)
143  / ;
144  mapra(ra,r)$sameas(ra,r) = yes ;
145  mapra("lmy",r)$(not mapra("hic",r)) = yes ;
146  mapra("row",r)$(not mapra("g20",r)) = yes ;
147  mapra("wld",r) = yes ;

149  set rra(ra) "Aggregate regions only" ;
150  loop((r,ra)$(not sameas(r,ra)), rra(ra) = yes ; ) ;

152  set lagg "Aggregate labor for output" /
153     set.lab
154     tot         "Total labor"
155  / ;

157  set maplagg(lagg,l) "Mapping for aggregate labor" ;
158  maplagg(lagg,l)$sameas(lagg,l) = yes ;
159  maplagg("tot",l) = yes ;

161  set nsk(l) "Unskilled types for labor growth assumptions" /
162     nsk         "Unskilled labor"
163  / ;

165  set skl(l)  "Skill types for labor growth assumptions" /
166     skl         "Skilled labor"
167  / ;

169  set acr(a) "Crop activities" /
170     crp-a
171  / ;

173  set alv(a) "Livestock activities" /
174     lvs-a
175  / ;

177  set ax(a) "All other activities" /
178     frs-a    "Forestry"
179     coa-a    "Coal extraction"
180     oil-a    "Oil extraction"
181     gas-a    "Gas extraction"
182     oxt-a    "Other pimary sectors"
183     pfd-a    "Processed food"
184     xlm-a    "Other light manufacturing"
185     ppp-a    "Paper products & publishing"
186     chp-a    "Chemicals, rubber and plastics"
187     nmm-a    "Non-metallic minerals"
188     i_s-a    "Ferrous metals"
189     nfm-a    "Other metals"
190     xmn-a    "Other manufacturing"
191     p_c-a    "Refined oil"
192     etd-a    "Electricity transmission and distribution"
193     els-a    "Solar power"
194     elw-a    "Wind power"
195     elr-a    "Other renewable power"
196     elh-a    "Hydro power"
197     elc-a    "Coal-fired power"
198     elg-a    "Gas power"
199     elo-a    "Oil power"
200     eln-a    "Nuclear power"
201     cns-a    "Construction"
202     atp-a    "Air transport"
203     wtp-a    "Water transport"
204     otp-a    "Other transport"
205     srv-a    "Services"
206  / ;
```

```
208  set agr(a) "Agricultural activities" /
209      crp-a, lvs-a, frs-a
210  / ;

212  set man(a) "Manufacturing activities" /
213      coa-a    "Coal extraction"
214      oil-a    "Oil extraction"
215      gas-a    "Gas extraction"
216      oxt-a    "Other pimary sectors"
217      pfd-a    "Processed food"
218      xlm-a    "Other light manufacturing"
219      ppp-a    "Paper products & publishing"
220      chp-a    "Chemicals, rubber and plastics"
221      nmm-a    "Non-metallic minerals"
222      i_s-a    "Ferrous metals"
223      nfm-a    "Other metals"
224      xmn-a    "Other manufacturing"
225      p_c-a    "Refined oil"
226  / ;

228  set srv(a) "Service activities" /
229      etd-a    "Electricity transmission and distribution"
230      els-a    "Solar power"
231      elw-a    "Wind power"
232      elr-a    "Other renewable power"
233      elh-a    "Hydro power"
234      elc-a    "Coal-fired power"
235      elg-a    "Gas power"
236      elo-a    "Oil power"
237      eln-a    "Nuclear power"
238      cns-a    "Construction"
239      atp-a    "Air transport"
240      wtp-a    "Water transport"
241      otp-a    "Other transport"
242      srv-a    "Services"
243  / ;

245  set aenergy(a) "Energy activities" /
246      coa-a    "Coal extraction"
247      oil-a    "Oil extraction"
248      gas-a    "Gas extraction"
249      p_c-a    "Refined oil"
250      etd-a    "Electricity transmission and distribution"
251      els-a    "Solar power"
252      elw-a    "Wind power"
253      elr-a    "Other renewable power"
254      elh-a    "Hydro power"
255      elc-a    "Coal-fired power"
256      elg-a    "Gas power"
257      elo-a    "Oil power"
258      eln-a    "Nuclear power"
259  / ;

261  set affl(a) "Fossil fuel activities" /
262      coa-a    "Coal extraction"
263      oil-a    "Oil extraction"
264      gas-a    "Gas extraction"
265  / ;

267  set aw(a) "Water services activities" /
268  / ;

270  set z "Labor market zones" /
271      rur          "Agricultural sectors"
272      urb          "Non-agricultural sectors"
273      nsg          "Non-segmented labor markets"
274  / ;
```

```
276  singleton set rur(z) "Rural zone" /
277    rur           "Agricultural sectors"
278  / ;

280  singleton set urb(z) "Urban zone" /
281    urb           "Non-agricultural sectors"
282  / ;

284  singleton set nsg(z) "Both zones" /
285    nsg           "Non-segmented labor markets"
286  / ;

288  set mapz(z,a) "Mapping of activities to zones" /
289    rur.(crp-a, lvs-a, frs-a)
290  / ;

292  mapz("urb", a)$(not mapz("rur",a)) = yes ;
293  mapz("nsg", a) = yes ;

295  set frt(i) "Fertilizer commodities" /
296    chp-c
297  / ;

299  set feed(i) "Feed commodities" /
300    crp-c, pfd-c
301  / ;

303  set iw(i) "Water services commodities" /
304  / ;

306  set e(i) "Energy commodities" /
307    COA-c        "Coal"
308    OIL-c        "Oil"
309    GAS-c        "Gas"
310    P_C-c        "Petroleum and coal products"
311    ELY-c        "Electricity"
312  / ;

314  set elyc(i) "Electricity commodities" /
315    ELY-c        "Electricity"
316  / ;

318  set fuel(e) "Fuel commodities" /
319    COA-c        "COA"
320    OIL-c        "OIL"
321    GAS-c        "GAS"
322    P_C-c        "P_C"
323  / ;

325  set k "Household commodities" /
326    agr-k        "Primary agriculture"
327    fud-k        "Other food"
328    lmn-k        "Light manufacturing"
329    dur-k        "Durable goods"
330    trp-k        "Transport services"
331    cns-k        "Construction"
332    srv-k        "Other services"
333    nrg-k        "Energy"
334  / ;

336  set fud(k) "Household food commodities" /
337    agr-k        "Primary agriculture"
338    fud-k        "Other food"
339  / ;

341  set mapk(k,i) "Mapping from i to k" /
342    agr-k . (crp-c, lvs-c, frs-c)
```

```
343     fud-k . (pfd-c)
344     lmn-k . (oxt-c, xlm-c, ppp-c, chp-c)
345     dur-k . (nmm-c, i_s-c, nfm-c, xmn-c)
346     trp-k . (atp-c, wtp-c, otp-c)
347     cns-k . cns-c
348     srv-k . srv-c
349     nrg-k . (coa-c, oil-c, gas-c, p_c-c, ely-c)
350  / ;

352  set CPINDX "Categories of CPI indices" /
353     fud           "Food price index"
354     tot           "Total price index"
355  / ;

357  set mapCPI(cpindx,i) "Mapping from i to CPINDX" /
358     fud     .(crp-c, lvs-c, pfd-c)
359  / ;
360  mapCPI("tot", i) = yes ;

362  set elya(a) "Power activities" /
363     etd-a    "Electricity transmission and distribution"
364     els-a    "Solar power"
365     elw-a    "Wind power"
366     elr-a    "Other renewable power"
367     elh-a    "Hydro power"
368     elc-a    "Coal-fired power"
369     elg-a    "Gas power"
370     elo-a    "Oil power"
371     eln-a    "Nuclear power"
372  / ;

374  set etd(a) "Electricity transmission and distribution activities" /
375     ETD-a         "Electricity transmission"
376  / ;

378  set primElya(a) "Primary power activities" /
379     els-a    "Solar power"
380     elw-a    "Wind power"
381     elr-a    "Other renewable power"
382     elh-a    "Hydro power"
383     eln-a    "Nuclear power"

385  / ;

387  set pb "Power bundles in power aggregation" /
388     GasP          "Gas bundle"
389     OilP          "Oil bundle"
390     coap          "Coal bundle"
391     nucp          "Nuclear"
392     hydp          "Hydro"
393     renp          "Renewables"
394  / ;

396  set mappow(pb,elya) "Mapping of power activities to power bundles" /
397     GasP     .elg-a
398     OilP     .elo-a
399     coap     .elc-a
400     nucp     .eln-a
401     hydp     .elh-a
402     renp     .els-a
403     renp     .elw-a
404     renp     .elr-a
405  / ;

407  set lb "Land bundles" /
408     AGR           "Agriculture"
409  / ;
```

```
411  set lb1(lb) "First land bundle" /
412     AGR          "Agriculture"
413  / ;

415  set maplb(lb,a) "Mapping of activities to land bundles" /
416     AGR      .(crp-a, lvs-a)
417  / ;

419  set wbnd "Aggregate water markets" /
420     N_A          "N_A"
421  / ;

423  set wbnd1(wbnd) "Top level water markets" /
424  / ;

426  set wbnd2(wbnd) "Second level water markets" /
427  / ;

429  set wbndex(wbnd) "Second level water markets" /
430  / ;

432  set mapw1(wbnd,wbnd) "Mapping of first level water bundles" /
433  / ;

435  set mapw2(wbnd,a) "Mapping of second level water bundle" /
436  / ;

438  set wbnda(wbnd) "Water bundles mapped one-to-one to activities" /
439  / ;

441  set wbndi(wbnd) "Water bundles mapped to aggregate output" /
442  / ;

444  set mape(NRG,e) "Mapping of energy commodities to energy bundles" /
445     COA        .COA-c
446     OIL        .OIL-c
447     OIL        .P_C-c
448     GAS        .GAS-c
449     ELY        .ELY-c
450  / ;

452  set mapnd1(i,a) "Mapping of commodities to ND1 bundle" ;
453  set mapnd2(i,a) "Mapping of commodities to ND2 bundle" / / ;

455  mapnd1(i,a)$(not mapnd2(i,a) and not e(i)) = yes ;

457  set rq(ra) "Regions submitted to an emissions cap" ;
458  rq(ra) = no ;

460  set aets "Agent specific ETS coalitions" /
461     All          "ALL agents"
462  / ;

464  $iftheni.modSets %ifModel% == 1
465  set mapETS(aets,aa) "Mapping of agents to ETS coalitions" ;
466  mapETS("All", aa) = yes ;

468  set educMap(r,l,ed) "Mapping of skills to education levels" ;
469  educMap(r,"nsk","elev0")$mapra("lmy",r) = yes ;
470  educMap(r,"skl","elev1")$mapra("lmy",r) = yes ;
471  educMap(r,"skl","elev2")$mapra("lmy",r) = yes ;

473  educMap(r,"nsk","elev0")$mapra("hic",r) = yes ;
474  educMap(r,"nsk","elev1")$mapra("hic",r) = yes ;
475  educMap(r,"skl","elev2")$mapra("hic",r) = yes ;

477  set sortOrder / sort1*sort106 / ;
```

```
479  *  !!!! We might be able to replace this with code

481  set mapOrder(sortOrder,is) /
482      sort1.crp-a
483      sort2.lvs-a
484      sort3.frs-a
485      sort4.coa-a
486      sort5.oil-a
487      sort6.gas-a
488      sort7.oxt-a
489      sort8.pfd-a
490      sort9.xlm-a
491      sort10.ppp-a
492      sort11.chp-a
493      sort12.nmm-a
494      sort13.i_s-a
495      sort14.nfm-a
496      sort15.xmn-a
497      sort16.p_c-a
498      sort17.etd-a
499      sort18.els-a
500      sort19.elw-a
501      sort20.elr-a
502      sort21.elh-a
503      sort22.elc-a
504      sort23.elg-a
505      sort24.eln-a
506      sort25.cns-a
507      sort26.atp-a
508      sort27.wtp-a
509      sort28.otp-a
510      sort29.srv-a
511      sort30.crp-c
512      sort31.lvs-c
513      sort32.frs-c
514      sort33.coa-c
515      sort34.oil-c
516      sort35.gas-c
517      sort36.oxt-c
518      sort37.pfd-c
519      sort38.xlm-c
520      sort39.ppp-c
521      sort40.chp-c
522      sort41.nmm-c
523      sort42.i_s-c
524      sort43.nfm-c
525      sort44.xmn-c
526      sort45.p_c-c
527      sort46.ely-c
528      sort47.cns-c
529      sort48.atp-c
530      sort49.wtp-c
531      sort50.otp-c
532      sort51.srv-c
533      sort52.NSK
534      sort53.SKL
535      sort54.CAP
536      sort55.LND
537      sort56.NRS
538      sort57.itax
539      sort58.ptax
540      sort59.mtax
541      sort60.etax
542      sort61.vtax
543      sort62.ltax
544      sort63.ktax
545      sort64.rtax
546      sort65.vsub
```

173

```
547    sort66.wtax
548    sort67.dtax
549    sort68.ctax
550    sort69.ntmY
551    sort70.regY
552    sort71.hhd
553    sort72.gov
554    sort73.r_d
555    sort74.inv
556    sort75.deprY
557    sort76.tmg
558    sort77.TRD
559    sort78.ARG
560    sort79.AUS
561    sort80.BRA
562    sort81.CAN
563    sort82.CHN
564    sort83.DEU
565    sort84.FRA
566    sort85.ITA
567    sort86.IND
568    sort87.IDN
569    sort88.JPN
570    sort89.MEX
571    sort90.RUS
572    sort91.SAU
573    sort92.ZAF
574    sort93.KOR
575    sort94.TUR
576    sort95.GBR
577    sort96.USA
578    sort97.XEA
579    sort98.XSA
580    sort99.XEC
581    sort100.XMN
582    sort101.XSS
583    sort102.XLC
584    sort103.XEU
585    sort104.XHY
586    sort105.BoP
587    sort106.tot
588 / ;
589 $endif.ModSets
```

Listing F.8: **Simplest labor bundle**

```
1    set wb "Intermediate labor bundle(s)" /
2       Single        "Single intermediate labor bundle"
3    /

5    set maplab1(wb) "Mapping of intermediate labor demand bundle(s) to LAB1" /
6       Single        "Single intermediate labor bundle"
7    /

9    set maplab(wb,l) "Mapping of labor types to intermedate demand bundle(s)" /
10      Single   .Labour
11   /
```

Listing F.9: **Two labor categories in one bundle**

```
1    set wb "Intermediate labor bundle(s)" /
2       Single        "Single intermediate labor bundle"
3    /

5    set maplab1(wb) "Mapping of intermediate labor demand bundle(s) to LAB1" /
```

```
6       Single         "Single intermediate labor bundle"
7     /
```

```
9     set maplab(wb,l) "Mapping of labor types to intermedate demand bundle(s)" /
10      Single   .NSK
11      Single   .SKL
12    /
```

Listing F.10: **Two labor categories in separate bundles**

```
1     set wb "Intermediate labor bundle(s)" /
2       Unskilled       "Unskilled labor bundle"
3       Skilled         "Skilled labor bundle"
4     /
```

```
6     set maplab1(wb) "Mapping of intermediate labor demand bundle(s) to LAB1" /
7       Unskilled       "Map unskilled to LAB1"
8     /
```

```
10    set maplab(wb,l) "Mapping of labor types to intermedate demand bundle(s)" /
11      Unskilled   .NSK
12      Skilled     .SKL
13    /
```

Listing F.11: **Two labor skills by gender**

```
1     set wb "Intermediate labor bundle(s)" /
2       Unskilled       "Unskilled labor bundle"
3       Skilled         "Skilled labor bundle"
4     /
```

```
6     set maplab1(wb) "Mapping of intermediate labor demand bundle(s) to LAB1" /
7       Unskilled       "Map unskilled to LAB1"
8     /
```

```
10    set maplab(wb,l) "Mapping of labor types to intermedate demand bundle(s)" /
11      Unskilled   .(NSK-MAL, NSK-FEM)
12      Skilled     .(SKL-MAL, SKL-FEM)
13    /
```

File name	Description
Core code	
AggGTAP.gms	Main aggregation program
aggSAM.gms	Code to output aggregate SAM in a CSV file
aggNRG.gms	Code to output aggregate energy and emissions in a CSV file
SSPSets.gms	Set definitions for reading SSP data file
saveMap.gms	Code to save aggregation mappings to be loaded into either Word/Excel or LaTeX
makeset.gms	Code to write out core set definitions for models, both GTAP and ENVISAGE
Additional code for **Envisage** *Model*	
makesetEnv.gms	Code to write additional sets for ENVISAGE
AggEnvElast.gms	Code to aggregate ENVISAGE elasticities
aggra.gms	Batinclude code for aggregating elasticities
aggrav.gms	Batinclude code for aggregating elasticities
aggrave.gms	Batinclude code for aggregating elasticities
makeAggSets.gms	Code to write final sets with appropriate suffixes
ConvertLabel.gms	Code to convert labels in aggregated parameter data file
Base data set definitions	
GTAPSets9_2F.gms	Set definitions for standard GTAP database V9.2 (141 × 57)
GTAPSets9_2PF.gms	Set definitions for power+water GTAP database V9.0
GTAPSets10AF.gms	Set definitions for standard GTAP database V10A (141 × 65)
GTAPSets10APOWF.gms	Set definitions for power GTAP database V10A
Miscellaneous	
makedata.cmd	Windows command file for facilitating all data processing
Macro.gms	A basic macro model used to calculate potential investment paths for dynamic scenarios
nipa.gms	Optional GAMS code that will output (in CSV format) the national income and product accounts for the base regions/countries in the GTAP database

The aggregation facility assumes that all of the GTAP-based data files have the same base name and are located in the same folder.[14] Note that some of the files downloaded from GAMS may not follow this convention, in which case they need to be renamed. The aggregation facility is designed to work with the GTAP-provided GDX containers, i.e., there is no need to convert the HAR files.

Table F.2 provides a list of all possible data files that can be used with the data aggregation facility. Some are required, for obvious reasons. Others are optional and will depend on user-based options. Note that the power+water version of the GTAP database includes three extra headers compared to the standard GTAP database. It has total power generated by electricity in Gwhr, water withdrawals in irrigated crops and aggregate water withdrawals from other sectors of the economy.

---

[14] Referred to as '%GTAPBASE%' and '%gtpDir%, respectively, in the GAMS code.

Table F.2: **Base Data Files**

File name	Description
Required GTAP files	
%GTAPBASE%DAT.gdx	File containing base SAM data
%GTAPBASE%PAR.gdx	File containing base parameter values for GTAP model
%GTAPBASE%VOLE.gdx	File containing base energy volume data
%GTAPBASE%EMISS.gdx	File containing base CO_2 emissions data
Optional data files	
%GTAPBASE%NCO2.gdx	File containing base non-CO_2 emissions data
%GTAPBASE%BOP.gdx	File containing remittances and cross-border profit flows. The former is sourced from the 'GMIG' database and the latter from the 'GDYN' database.
%GTAPBASE%ELAST.gdx	Data file containing key land and natural resource supply parameters. This file is not part of the standard GTAP suite and is described in more detail below [TBD]. If the file does not exist, these parameters will be set to default values.
%GTAPBASE%WAGES.gdx	File containing wage/employment splits for labor remuneration. This file is part of the 'GMIG' database. Its use is described below [TBD].
%GTAPBASE%MRIO.gdx	File containing the split of import demand by end-user. This data is used for the Multi-Regional Input-Output version of the model, also referred to as the Global Value Chain (GVC) version. Note that the MRIO database has only three end-users: 'INT' for all intermediate demand; 'CONS' for private and public expenditures; and 'CGDS' for investment expenditures. The initialization module of the CGE models assumes proportionality across end-users within each of the broad categories. Use of this module requires setting the `ifMRIO` flag (see below).[15]
Other data files	
%SSPFILE%	Data file containing projections for population and GDP for 230 countries for all 5 Shared Socio-Economic Pathways (SSPs)—further described below [TBD]. In addition, the file contains the UN's population projections for the 2010, 2012, 2015 and 2017 revisions.
%GIDDLAB%	Data file containing an alternative wage/employment split. The World Bank's Global Income Distribution Dynamics (GIDD) Model provides an alternative set for employment levels. Note that the GIDD data is only provided for two types of labor—unskilled and skilled.
%GIDDPROJ%	Data file containing population projections (including education levels) from the GIDD.
Parameter files	
%EnvElast%	File containing disaggregated elasticities for use with the Envisage Model. Two files are distributed. One for the standard GTAP database and the other for the power+water version.

The SSP file is a combination of data from different sources and time periods. The following describes the data

in the file.

1. *PopHist* This matrix contains population from 1950 through 2100, normally sourced from the last available UN population projection.[16]. The projections are for the Medium variant. It is provided for each of 230 countries[17] for an aggregation of four cohorts: aged between 0 and 14 inclusive (`PLT15`), aged between 15 and 64 inclusive (`P1564`) typically associated with the working age population, aged 65 and above (`P65UP`), and all ages (`PTOTL`).

2. *Pop* This matrix contains the population projections from 2007 through 2100. It is defined for all 5 SSPs and also includes 5 UN projections: Revision 2010, 2012, 2015, 2017 and 2019. The 2007 through 2010 period is from the UN. The 2010-2100 SSP data is from the SSP database, using the growth rates—the 2010 level is based on UN numbers. The SSP database has been annualized using cubic splines and gap filled for missing countries.[18] The population projections have the same 4 cohorts as defined above.

3. *PopScen* This is essentially the same data as in the previous item with additional information for each cohort: gender (`MAL`, `FEM`, `BOTH`), and educational attainment (`ENONE`, `EPRIM`, `ESECN`, `ETERT`, `ETOTL`). It includes the UN projections as well, but only for `ETOTL`. All data is annualized and gap filled.

4. *GDPScen* This matrix contains GDP projections from 2007 through 2100. The standard data (`GDP` and `GDPPC`) use 2007 prices and market exchange rates (MER). Growth from 2007 through 2010 is based on observed rates. There are two versions of the 2010 through 2100 growth rates. The first (V9_1) uses the original SSP database for the entire period (annualized and gap filled for missing countries). It only uses the growth rates as the reference level is 2010 (albeit in 2007 prices and MER). An alternative version (V9_2) still has the 2010 reference year, but uses the Fall 2019 World Economic Outlook (WEO) forecast from the International Monetary Fund (IMF) to replace the 2010-2020 SSP projections with the observed trends (albeit with a projection for 2019 and 2020—excluding the Covid-19 pandemic). The SSP projections include projections from two models: OECD and IIASA.[19] There could be sharp discontinuities in the growth rates at the 2020 transition point. The matrix also contains the data evaluated at 2005 prices and purchasing power parity exchange rates (`GDPPPP05` and `GDPPCPPP05`).

5. *GHGHist* This matrix contains historical data for GHG emissions. It should be used with caution as it is new and not thoroughly vetted. Most of the data is sourced from the World Bank's WDI database. It includes emissions of carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and a composite of fluorinated gases (`FGAS`). It also includes energy use per capita (`NRGUSE`) in tons of oil equivalent per capita.[20] All greenhouse gases are measured in megatons of CO_2-equivalent. WDI is missing data for key countries—mostly due to the transition economies in Europe. For the Annex I countries, the data is sourced from the UNFCCC.

Optional backstop fuel files

[This is ongoing work]

Each backstop is associated with the cost structure of an initial energy source, for example coal-power with CCS is associated with coal-power thermal generation. For the moment, it is assumed that all the increase in cost is embedded in the capital cost. Thus if the total cost of the backstop is twice the 'conventional' cost, all of the increase is embedded in the cost of capital. If χ is the cost multiplier, we have the following formula for the cost shares all the non-capital inputs:

$$X_{i,b} = \frac{X_{i,a}}{VOS_a}$$

where, b is the backstop, a is the associated cost activity, X is the value of the input and VOS is the total cost. The capital share formula is:

$$X_{k,b} = \chi_b - \left(1 - \frac{X_{k,a}}{VOS_a}\right)$$

where k is the capital input.

---

[16] Currently the 2019 revision (https://population.un.org/wpp/)

[17] The country definition for this file continues to hone to the 2010 UN definitions. With Revision 2019, the UN has 235 countries. We map 5 to the original definition. Thus Saint Barthélemy and Saint Martin (French Part) are mapped to Guadeloupe (GLP), Curaçao and Sint Maarten (Dutch part) are mapped to the Netherlands Antilles (ANT) and South Sudan is mapped to Sudan (SDN).

[18] Database construction details are available from the author.

[19] The PIK SSP forecasts were not included because they are available only at a 32-region aggregate level unlike the OECD and IIASA projections. We are also considering adding the CEPII SSP projections.

[20] The data also contains HFC, PFC, SF6, LUCFNET and GHGT.

Filter module files

The filter module files are scattered in two folders—see Table F.3. The *Data* folder contains the calling `filter` routine and two user prepared files. The core files for the Filter module are all in the *Filter* sub-folder.

Table F.3: **Files distributed with the Filter module**

File name	Description
Files in core 'Data' directory	
Filter.gms	GAMS file invoking core filter code
*basename*Flt.gms	User prepared file with filter options
*basename*Map.gms	User prepared file with key mappings—same file used for Aggregation facility
Additional code for Filter module in the 'Filter' directory	
filter.gms	GAMS code containing core filter algorithm
remTinyValues.gms	Code to remove tiny value flows
itrlog.gms	Code that creates diagnostics for the Filter module
title.gms	Code mostly intended for the CGEBox GUI

Altertax module files

The Altertax files are scattered in three folders, see Table F.4. The main *Data* folder contains `Altertax.gms` and the user options file `[basename]Alt.gms`. The sub-folder *GTAPModel* contains the core code for the GAMS-based GTAP model. The sub-folder *Altertax* contains a single file with the parameter definitions needed to run the GTAP model. Note that many of the parameters will be over-ridden by Altertax to make all CES functions (and the utility function) Cobb-Douglas, further described below.

Table F.4: **Files distributed with the Altertax module**

File name	Description
Files in core 'Data' directory	
Altertax.gms	GAMS file invoking core GTAP model code
*basename*Alt.gms	User prepared file with the Altertax shock
Additional code for Altertax module in the 'GTAPModel' directory	
model.gms	GAMS code containing core GTAP model specification
getData.gms	GAMS code to read core database(s)
cal.gms	GAMS code containing variable and parameter initialization and calibration
iterloop.gms	Code containing instructions to prepare a new solve statement
solve.gms	Code invoking a GAMS solver
postsim.gms	Code saving model results in a CSV-formatted file
emiCSV.gms	Code to save emission results in a CSV-formatted file
saveData.gms	Code saving model results in GDX-formatted files with the same structure as the input GTAP-based data files
Additional code for Altertax module in the 'AlterTax' directory	
AlterTaxPrm.gms	GAMS code containing default elasticities for GTAP model.

Data sub-folders

Each aggregation project, with its code, will be associated with a data folder that has the same name as the project. So if the code for the project is '10x10', for example, the data from the data preparation modules will be contained in the folder '10x10'. The output from each module is contained in its own sub-folder of which there are four named **Agg**, **Flt**, **Alt** and **Fnl**. The last folder contains the final results from running the three modules. If a module is skipped, all the data files from the previous module should be copied over to the next folder. Users can create these folders, or use the automated Windows command file called `makedata.cmd` that is further described below. The `makeData` command file also automates the moving of files across folders. Figure 22 provides a visual guide to the directory structure of the output data.
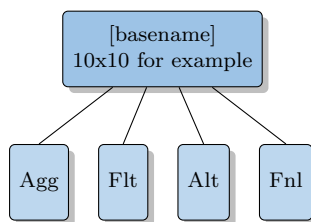
Figure 22: **Directory structure of output data**

F.1.9 Aggregation

From the user perspective aggregation involves the preparation of the map file that will contain the aggregation mappings as well as miscellaneous other information required for the data facility. The mapping file has two components. The first part relates to a generic aggregation of the GTAP database and its components. It hones closely to the GTAPAgg and FlexAgg facilities available for GEMPACK users. The second component is specific to aggregation for the ENVISAGE Model.

Global options

The first part of a map file contains global options for the aggregation. Listing F.12 provides an example of the preamble statements for a mapping file.

Listing F.12: **Global options for the MAP file**

```
 1   $setGlobal DIAG          ON
 2   $setGlobal DYN           ON
 3   $setGlobal MACRO         ON
 4   $setGlobal ifPower       ON
 5   $setGlobal ifWater       OFF
 6   $setGlobal NCO2          ON
 7   $setGlobal ELAST         OFF
 8   $setGlobal LAB           ON
 9   $setGlobal BoP           ON
10   $setGlobal SAVEMAP       TXT
11   $setGlobal LU_MODULE     OFF
12   $setGlobal RD_MODULE     OFF
13   $setGlobal DEPL_MODULE   OFF
14   $setGlobal MRIO_MODULE   OFF
15   $setGlobal ifGENDER      OFF
16   $setGlobal ifBKSTOP      OFF
17   $setGlobal ifMultH       OFF
18   $setGlobal ifDAMAGE      ON

20   *  Only used to override GTAP parameters for Env model

22   $setGlobal OVRRIDEGTAPARM 1
23   $setGlobal OVRRIDEGTAPINC 1

25   *  Select a labor option
26   *  Valid options are:
27   *     noLab  -- ignore employment volumes (all wages are set to 1)
```

```
28   *       agLab  -- calculate ag and non-ag employment (wages uniform within zones)
29   *       allLab -- assume employment data is correct for each sector (wages differ for each sector)
30   *       giddLab -- Use the GIDD labor data

32   $macro IFLABOR noLab

34   $include "GTAPSets10APOWF.gms"

36   $setGlobal gtpDir      "V:/GTAP10/V10APOW"
37   $setGlobal GTAPBASE    "GSDF"
38   $setGlobal SSPFile     "../SatAcct/sspScenV9.gdx"
39   $setGLobal DEPLFile    "../SatAcct/RystadGTAP2014.gdx"
40   $setGlobal giddLab     "../SatAcct/giddLab.gdx"
41   $setGlobal giddProj    "../SatAcct/giddProj.gdx"
42   $setGlobal EnvElast    "../SatAcct/EnvLinkElast10APOW.gdx"
43   $onempty
```

The first set of options turn on or off various features of the aggregation. These are described in further detail in Table .

Table F.5: **Global options for the MAP file**

Option name	*Description*
DIAG	Assumes input database if diagonal. (Alternatives are not yet in use.)
DYN	If this option is set to ON, the aggregation facility will read the SSP and GIDD projection files and create an aggregated scenario file.
MACRO	If this option is set to ON, the aggregation facility use the SSP projections and the aggregated database to run a simple macro model, the purpose of which is to calculate a plausible target path for investment from the reference year to 2100. This further described elsewhere [TBD].
ifPower	If this option is set to ON, the aggregation facility will retrieve the Gwhr information from the GTAP (power) database and aggregate.
ifWater	If this option is set to ON, the aggregation facility will retrieve the water volume information from the GTAP (water) database and aggregate.
NCO2	If this option is set to ON, the aggregation facility will retrieve the non-CO_2 information from the non-CO_2 database and aggregate.
ELAST	If this option is set to ON, the aggregation facility will retrieve the land and natural resource parameters from the relevant 'ELAST' file. It should be set to OFF for an ENVISAGE aggregation as the same parameters are contained in the parameter file.
LAB	If this option is set to ON, the aggregation facility will retrieve the employment data from one of the available databases—either GTAP or the GIDD—and aggregate. The exact type of aggregation will depend on the IFLABOR option described below.
BOP	If this option is set to ON, the aggregation facility will retrieve the additional balance of payments data from the 'BOP' database and aggregate.

Option name	Description
SAVEMAP	This option takes two values: 'TXT' and 'LATEX'. The 'TXT' option will save the aggregation mappings in a file called '[baseBame]Sets.txt'. It is a comma delimited text file (CSV) that contains the aggregation mappings from the original GTAP database to the model-level aggregation. It can be loaded into Excel and the `Data|Convert to Columns` command can be used to convert the data into a table with three columns, using the 'comma' delimiter option. It is then easy to copy the table into Word. The 'LATEX' option creates a file with the name '[baseName]Sets.tex' and creates three relatively generic tables that can be included in a LaTeX file. The user may have to edit the file to convert special characters—such as underscores ('_') and ampersands ('&').
LU_MODULE	If this option is set to ON, the aggregation facility will make use of the GTAP AEZ land-use database. [This is under development.]
RD_MODULE	If this option is set to ON, the aggregation facility will include a new final demand agent labeled `R_D`. N.B. This requires that the label is part of the main *'Sets'* file, e.g., `GTAPSets10AF`.
DEPL_MODULE	This turns on the resource depletion module for coal, oil and gas.
MRIO_MODULE	This turns on the MRIO module. Requires the MRIO database.
ifGENDER	This turns on the gender-aware database for labor. It requires the GIDD employment database.
ifBKSTOP	Creates backstop technologies. Under development.
ifMultH	Used for multi-household databases. Under development.
ifDamage	Turns on the climate damage and adaptation module. Under development.
OVRRIDEGTAPARM	Currently ignored and to be reviewed.
OVRRIDEGTAPINC	If this option is set to 1, the income elasticities will be taken from the input base parameter file. If the option is set to 0, the GTAP income elasticities prevail and override the income elasticities from the parameter file.
IFLABOR	This option takes four possible values. The first value is 'noLab'. In this case the employment volumes are ignored and in effect all wages are set to 1. The second value is 'agLab'. In this case the volume data is used to aggregate agricultural and non-agricultural employment. Within each of these two broad sectors wages are assumed to be uniform across sectors and in effect the implicit inter-sectoral wage differences are ignored. The purpose of this is to minimize potential welfare implications by assuming heterogeneous inter-sectoral wages (apart from the ag/non-ag distinction). The third option is 'allLab' in which case the employment volume data is fully utilized and explicit inter-sectoral wages are assumed to hold. The fourth option is 'giddLab' in which case the GIDD employment volumes are used to determine the wage/employment split. Note that the GIDD option implies a skill/non-skilled labor aggregation.

Following the global options, the next line is an include file for the base data's set definitions. The distribution comes with two prepared set definition files—one for the standard GTAP database and the other for the power database.[21] The following set of lines provide information for locating the input databases. The first is the location of all of the GTAP data files (DAT, PAR, VOLE, EMISS, NCO2, BOP, ELAST and WAGES). The second line provides the base name for the GTAP files, for example GSDF or GSDFP—the latter stands for the power database. The remaining four lines refer to the path name of the other input databases—the SSP projection file, the natural resource depletion data (if the depletion module is being used), the two GIDD databases and the file of disaggregated ENVISAGE elasticities.

---

[21] These are currently available for Version 10A of the GTAP Data Base. Please contact the Center for prior versions.

Standard mappings

The second part of the mapping file contains the standard mappings—this is aimed for the GTAP model and hones very closely to GTAPAGG, the GEMPACK-based aggregation facility.[22] Users define the aggregated sets for activities/commodities, regions and factors of production—with their respective mappings to the full GTAP database. Users have some flexibility in how to define these sets and mappings and the distribution comes with two examples.

Listing F.13 provides an example that is virtually identical to the 10x10 mapping example from GTAP. A first set i is defined that contains the 10 aggregate sectors. Note that since the current database is diagonal, as is the output from the aggregation facility, the set for activities (a) is aliased with the set for sectors (i). Similarly, the user defines set r for regions and fp for the factors of production. These basic sets are subsequently decomposed into a number of subsets—most of which are fairly obvious. Some do have implications for the model:

- The user has flexibility in defining the labor bundles. The model has two fixed labor bundles: *LAB1* and *LAB2*. Subordinate labor bundles are user-defined and then two mappings are required. The first maps the subordinate labor bundles to either *LAB1* or *LAB2*. The second mapping maps the specific labor types to one (and only one) of the subordinate bundles. Say for example there are two skills: `UnSkLab` and `SkLab`—and both are mapped to a single bundle, i.e. are directly substitutable. One subordinate bundle is needed, call is `SINGLE` and it is mapped to *LAB1* (using the mapping set `mapLAB1`). The other mapping maps both skill types to `SINGLE`. An alternative is to map `UnSkLab` to *LAB1* and `SkLab` to *LAB2*. In this case, two subordinate bundles are required, say `NSK` and `SKL` and `UnSkLab` is mapped to the bundle `NSK` and `SkLab` is mapped to `SKL`. Finally, the `NSK` subordinate bundle needs to be mapped to *LAB1* using the mapping set `mapLAB1`. (Any subordinate bundle not mapped to *LAB1* will automatically be mapped to *LAB2*.) This implementation of the labor bundles appears redundant as the subordinate mappings, in either case, are not adding to the model specification. It can help when labor has additional characteristics—such as gender, origin of the worker, or the level of formality. Take the gender example. One could have the subordinate bundles be skilled and unskilled labor, both mapped to *LAB1*, and then have a subsequent mapping that maps the skill/gender qualifications to the relevant subordinate bundles. One would map `UnSkLab_M` and `UnSkLab_F` to `NSK` and `SkLab_M` and `SkLab_F` to `SKL`. It is the user's responsibility to make sure that the substitution elasticities are appropriately set for the respective bundles—in the case of the gender-based bundles, one needs a substitution elasticity between skilled and unskilled workers and two subsidiary elasticities: (1) across gender for unskilled workers; and (2) across gender for skilled workers.

- The subset *lr* determines the labor types used to calculate the skill premium. This became necessary with the new GTAP database that has 5 labor types.

- The *wat* subset should be empty in the absence of the water factor—only available with the power+water database.

- The *mapz* mapping defines the ag vs. non-ag activities. This is only used to determine ag vs. non-ag wages in the case the 'IFLABOR' option is set to *agLab*.

- The set *mapt* will instruct the aggregation facility to merge land and capital payments into capital payments only—for the selected activities. For example, it is possible to remove the land payment in other livestock (the `oap` sector).

- The set *mapn* has the same purpose, but for the natural resource factor. For example, one may wish to delete the natural resource factor payment if the `omn` activity is merged with manufacturing; as depending on the supply elasticity of the natural resource, this may overly restrict output expansion (or contraction).

Post-simulation analysis of the results allow for aggregation of activities, commodities, regions and labor. These are at the full discretion of the user. The aggregate sets are given respectively by *aga*, *ia*, *ra* and *lagg*. The aggregation facility will automatically append the suffixes '-a' and '-c' for the individual activities and commodities, respectively. So it would be usual practice to define the aggregation set items with the '-a' and '-c' suffixes for compatibility. For purposes of post-simulation, these sets will also include the individual set items. For example, the aggregation facility will write out the final set *ra* as the union of the set *r* and *ra*, but the user only needs to provide the definitions for the aggregate items as the aggregation facility will take care of merging the two sets. The definition of these aggregations (with their respective mappings) simplifies the output code. For example, the code to aggregate GDP across regions takes the form:

```
2    loop(ra, put ra.tl, (sum(r$mapra(ra,r), gdp(r)) / ; ) ;
```

---

[22]   The current mappings rely on V10A of the GTAP Data Base.

where the mapping *mapra* defines the aggregation mapping for each region *ra*, including the individually modeled regions.

The aggregation facility requires the factor mobility parameters for the GTAP model. This is given in the parameter `etrae1`. In the sample map file, `etrae1` is initilized with `etrae0` that has no regional index and thus the key factor mobility elasticities in all regions are identical, but this is not a requirement.

The new GTAP standard model has an explicit make matrix. Multi-product activities are governed by a CET function with the transformation elasticity provided by `etraq1` with a default value of 5. The formation of supply from production of multiple activities is governed by a CES function with a substitution provided by `esubq1` with a default elasticity of ∞. Similarly, the new GTAP standard model has new flexibility regarding the CES expenditure elasticities for government, investment and the formation of transportation margins. These are represented respectively in the parameters `esubg1`, `esubi1` and `esubs1`. These are initialized to their default values. At the moment it is best to assume that the input database is diagonal. This is guaranteed by adding code to make the commodity mapping identical to the activity mapping (see lines 266-271).

The aggregation mappings are relatively self-explanatory. The exception may be the mapping *mapl* that will match the user-input labor types to the GIDD labor labels that are fixed.

Listing F.13: **Standard mappings**

```
1   sets
2
3   i   "Commodities"   /
4       GrainsCrops     "Grains and Crops"
5       MeatLstk        "Livestock and Meat Products"
6       Extraction      "Mining and Extraction"
7       ProcFood        "Processed Food"
8       TextWapp        "Textiles and Clothing"
9       LightMnfc       "Light Manufacturing"
10      HeavyMnfc       "Heavy Manufacturing"
11      Util_Cons       "Utilities and Construction"
12      TransComm       "Transport and Communication"
13      OthServices     "Other Services"
14      /
15
16  r   "Regions" /
17      Oceania         "Australia, New Zealand"
18      EastAsia        "East Asia"
19      SEAsia          "Southeast Asia"
20      SouthAsia       "South Asia"
21      NAmerica        "North America"
22      LatinAmer       "Latin America"
23      EU_28           "European Union 28"
24      MENA            "Middle East and North Africa"
25      SSA             "Sub-Saharan Africa"
26      RestofWorld     "Rest of World"
27      /
28
29  fp  "Factors of production"  /
30      UnSkLab         "Unskilled labor"
31      SkLab           "Skilled labor"
32      Capital         "Capital"
33 $iftheni.ifLU "%LU%" == "ON"
34      AEZ1            "LGP000_060"
35      AEZ2            "LGP060_119"
36      AEZ3            "LGP120_179"
37      AEZ4            "LGP180_239"
38      AEZ5            "LGP240_299"
39      AEZ6            "LGP300PLUS"
40 $else.ifLU
41      Land            "Land"
42 $endif.ifLU
43      NatRes          "Natural resource"
44      /
45
46  l(fp)   "Labor factors" /
47      UnSkLab         "Unskilled labor"
48      SkLab           "Skilled labor"
```

```
49            /
50      lr(l)  "Reference labor for skill premium" /
51          SkLab            "Skilled labor"
52          /
53      wb "Intermediate labor bundle(s)" /
54          Single           "Single intermediate labor bundle"
55      /
56      maplab1(wb) "Mapping of intermediate labor demand bundle(s) to LAB1" /
57          Single
58      /
59      mapl(wb,l) "Mapping of labor categories to intermedate demand bundle(s)" /
60          Single.(UnSkLab, SkLab)
61      /
62      cap(fp) "Capital" /
63          Capital          "Capital"
64          /
65      lnd(fp) "Land endowment" /
66  $iftheni.ifLU "%LU%" == "ON"
67          AEZ1             "LGP000_060"
68          AEZ2             "LGP060_119"
69          AEZ3             "LGP120_179"
70          AEZ4             "LGP180_239"
71          AEZ5             "LGP240_299"
72          AEZ6             "LGP300PLUS"
73  $else.ifLU
74          Land             "Land"
75  $endif.ifLU
76          /
77      nrs(fp) "Natural resource" /
78          NatRes           "Natural resource"
79          /
80      wat(fp) "Water resource" /
81          /

83      ra "Aggregate regions for emission regimes and model output" /
84          hic    "High-income countries"
85          lmy    "Developing countries"
86          wld    "World Total"
87          /
88      ia "Aggregate commodities for model output" /
89          tagr-c      "Agriculture"
90          tman-c      "Manufacturing"
91          tsrv-c      "Services"
92          toth-c      "Other"
93          ttot-c      "Total"
94          /
95      aga  "Aggregate activities for model output" /
96          tagr-a      "Agriculture"
97          tman-a      "Manufacturing"
98          tsrv-a      "Services"
99          toth-a      "Other"
100         ttot-a      "Total"
101         /
102     lagg "Aggregate labor for model output" /
103         tot         "Total labor"
104         /
105 ;

107 *  !!!! Explicit assumption about diagonality

109 alias(i,a) ;

111 alias(m,i) ;

113 *  User defined parameters (i.e. not aggregated by aggregation facility)
114 *  NEW -- New region specific

116 Parameter
```

```
117     etrae1(fp,r)  "CET transformation elasticities for factor allocation"
118   ;

120   parameter etrae0(fp) "CET transformation elasticities for factor allocation" /
121       UnSkLab   inf
122       SkLab     inf
123       Capital   inf
124   $iftheni.ifLU "%LU%" == "ON"
125         AEZ1    1.0
126         AEZ2    1.0
127         AEZ3    1.0
128         AEZ4    1.0
129         AEZ5    1.0
130         AEZ6    1.0
131   $else.ifLU
132         Land    1.0
133   $endif.ifLU
134       NatRes   0.001
135   / ;

137   etrae1(fp,r) = etrae0(fp) ;

139   *  NEW -- MAKE ELASTICITIES

141   Parameter
142       etraq1(a,r)        "MAKE CET Elasticity"
143       esubq1(i,r)        "MAKE CES Elasticity"
144   ;
145   etraq1(a,r) = 5 ;
146   esubq1(i,r) = inf ;

148   *  NEW -- EXPENDITURE ELASTICITIES

150   Parameter
151       esubg1(r)          "Government expenditure CES elasticity"
152       esubi1(r)          "Investment expenditure CES elasticity"
153       esubs1(m)          "Transport margins CES elasticity"
154   ;

156   esubg1(r) = 1 ;
157   esubi1(r) = 0 ;
158   esubs1(m) = 1 ;

160   *  This zonal mapping is for labor/volume splits between agriculture and other

162   set mapz(z,a)  "Mapping of activities to zones" /
163       rur.(GrainsCrops,MeatLstk)
164   / ;

166   mapz("urb",a) = not mapz("rur",a) ;
167   mapz("nsg",a) = yes ;

169   * >>>> MUST INSERT RESIDUAL REGION (ONLY ONE)

171   set rres(r) "Residual region" /

173       NAmerica

175   / ;

177   * >>>> MUST INSERT MUV REGIONS (ONE OR MORE)

179   set rmuv(r) "MUV regions" /

181       Oceania
182       EastAsia
183       NAmerica
184       EU_28
```

186

```
186    / ;

188    set mapt(a) "Merge land and capital payments in the following sectors" /

190    / ;

192    set mapn(a) "Merge natl. res. and capital payments in the following sectors" /

194    / ;

196    *  MAPPINGS TO GTAP

198    set mapa(acts,a) /
199        pdr . GrainsCrops
200        wht . GrainsCrops
201        gro . GrainsCrops
202        v_f . GrainsCrops
203        osd . GrainsCrops
204        c_b . GrainsCrops
205        pfb . GrainsCrops
206        ocr . GrainsCrops
207        ctl . MeatLstk
208        oap . MeatLstk
209        rmk . MeatLstk
210        wol . MeatLstk
211        frs . Extraction
212        fsh . Extraction
213        coa . Extraction
214        oil . Extraction
215        gas . Extraction
216        oxt . Extraction
217        cmt . MeatLstk
218        omt . MeatLstk
219        vol . ProcFood
220        mil . ProcFood
221        pcr . GrainsCrops
222        sgr . ProcFood
223        ofd . ProcFood
224        b_t . ProcFood
225        tex . TextWapp
226        wap . TextWapp
227        lea . LightMnfc
228        lum . LightMnfc
229        ppp . LightMnfc
230        p_c . HeavyMnfc
231        chm . HeavyMnfc
232        bph . HeavyMnfc
233        rpp . HeavyMnfc
234        nmm . HeavyMnfc
235        i_s . HeavyMnfc
236        nfm . HeavyMnfc
237        fmp . LightMnfc
238        mvh . LightMnfc
239        otn . LightMnfc
240        ele . HeavyMnfc
241        eeq . HeavyMnfc
242        ome . HeavyMnfc
243        omf . LightMnfc
244        ely . Util_Cons
245        gdt . Util_Cons
246        wtr . Util_Cons
247        cns . Util_Cons
248        trd . TransComm
249        afs . TransComm
250        otp . TransComm
251        wtp . TransComm
252        atp . TransComm
```

187

```
253    whs . TransComm
254    cmn . TransComm
255    ofi . OthServices
256    ins . OthServices
257    rsa . OthServices
258    obs . OthServices
259    ros . OthServices
260    osg . OthServices
261    edu . OthServices
262    hht . OthServices
263    dwe . OthServices
264  / ;

266  $iftheni "%DIAG%" == ON
267      set mapi(comm,i) ;
268      loop((acts,comm)$sameas(acts,comm),
269          mapi(comm,i) = mapa(acts,i) ;
270      ) ;
271  $endif

273  set mapr(reg,r) /
274      aus . Oceania
275      nzl . Oceania
276      xoc . Oceania
277      chn . EastAsia
278      hkg . EastAsia
279      jpn . EastAsia
280      kor . EastAsia
281      mng . EastAsia
282      twn . EastAsia
283      xea . EastAsia
284      brn . EastAsia
285      khm . SEAsia
286      idn . SEAsia
287      lao . SEAsia
288      mys . SEAsia
289      phl . SEAsia
290      sgp . SEAsia
291      tha . SEAsia
292      vnm . SEAsia
293      xse . SEAsia
294      bgd . SouthAsia
295      ind . SouthAsia
296      npl . SouthAsia
297      pak . SouthAsia
298      lka . SouthAsia
299      xsa . SouthAsia
300      can . NAmerica
301      usa . NAmerica
302      mex . NAmerica
303      xna . NAmerica
304      arg . LatinAmer
305      bol . LatinAmer
306      bra . LatinAmer
307      chl . LatinAmer
308      col . LatinAmer
309      ecu . LatinAmer
310      pry . LatinAmer
311      per . LatinAmer
312      ury . LatinAmer
313      ven . LatinAmer
314      xsm . LatinAmer
315      cri . LatinAmer
316      gtm . LatinAmer
317      hnd . LatinAmer
318      nic . LatinAmer
319      pan . LatinAmer
320      slv . LatinAmer
```

188

```
321    xca . LatinAmer
322    dom . LatinAmer
323    jam . LatinAmer
324    pri . LatinAmer
325    tto . LatinAmer
326    xcb . LatinAmer
327    aut . EU_28
328    bel . EU_28
329    cyp . EU_28
330    cze . EU_28
331    dnk . EU_28
332    est . EU_28
333    fin . EU_28
334    fra . EU_28
335    deu . EU_28
336    grc . EU_28
337    hun . EU_28
338    irl . EU_28
339    ita . EU_28
340    lva . EU_28
341    ltu . EU_28
342    lux . EU_28
343    mlt . EU_28
344    nld . EU_28
345    pol . EU_28
346    prt . EU_28
347    svk . EU_28
348    svn . EU_28
349    esp . EU_28
350    swe . EU_28
351    gbr . EU_28
352    che . RestofWorld
353    nor . RestofWorld
354    xef . RestofWorld
355    alb . RestofWorld
356    bgr . EU_28
357    blr . RestofWorld
358    hrv . EU_28
359    rou . EU_28
360    rus . RestofWorld
361    ukr . RestofWorld
362    xee . RestofWorld
363    xer . RestofWorld
364    kaz . RestofWorld
365    kgz . RestofWorld
366    tjk . RestofWorld
367    xsu . RestofWorld
368    arm . RestofWorld
369    aze . RestofWorld
370    geo . RestofWorld
371    bhr . MENA
372    irn . MENA
373    isr . MENA
374    jor . MENA
375    kwt . MENA
376    omn . MENA
377    qat . MENA
378    sau . MENA
379    tur . MENA
380    are . MENA
381    xws . MENA
382    egy . MENA
383    mar . MENA
384    tun . MENA
385    xnf . MENA
386    ben . SSA
387    bfa . SSA
388    cmr . SSA
```

```
389     civ . SSA
390     gha . SSA
391     gin . SSA
392     nga . SSA
393     sen . SSA
394     tgo . SSA
395     xwf . SSA
396     xcf . SSA
397     xac . SSA
398     eth . SSA
399     ken . SSA
400     mdg . SSA
401     mwi . SSA
402     mus . SSA
403     moz . SSA
404     rwa . SSA
405     tza . SSA
406     uga . SSA
407     zmb . SSA
408     zwe . SSA
409     xec . SSA
410     bwa . SSA
411     nam . SSA
412     zaf . SSA
413     xsc . SSA
414     xtw . RestofWorld
415 / ;

417 set mapf(endw, fp) /
418     ag_othlowsk   . UnSkLab
419     service_shop . UnSkLab
420     clerks        . UnSkLab
421     tech_aspros  . SkLab
422     off_mgr_pros . SkLab
423     Capital       . Capital
424 $iftheni.ifLU "%LU%" == "ON"
425     Land          . (AEZ1*AEZ6)
426 $else.ifLU
427     Land          . Land
428 $endif.ifLU
429     NatlRes       . NatRes
430 / ;

432 set mapaez(fpa0, fp) /
433 $iftheni.ifLU "%LU%" == "ON"
434     AEZ1    . AEZ1
435     AEZ2    . AEZ2
436     AEZ3    . AEZ3
437     AEZ4    . AEZ4
438     AEZ5    . AEZ5
439     AEZ6    . AEZ6
440     AEZ7    . AEZ1
441     AEZ8    . AEZ2
442     AEZ9    . AEZ3
443     AEZ10   . AEZ4
444     AEZ11   . AEZ5
445     AEZ12   . AEZ6
446     AEZ13   . AEZ1
447     AEZ14   . AEZ2
448     AEZ15   . AEZ3
449     AEZ16   . AEZ4
450     AEZ17   . AEZ5
451     AEZ18   . AEZ6

453     UnSkLab  . UnSkLab
454     SkLab    . SkLab

456     Capital . Capital
```

```
457        NatlRes . NatRes
458  $else.ifLU
459        (AEZ1*AEZ18). Land
460  $endif.ifLU
461  / ;


464  set maplGIDD(lg, l) "Mapping to GIDD labor database" /
465     nsk.UnSkLab
466     skl.SkLab
467  / ;
```

Sets and mappings for model aggregation

As noted above, the model aggregation can differ from the data aggregation. The data aggregation is built around the concept of a diagonal make matrix—this ensures that the output from any of the three modules always has the same 'geometry'. The model aggregation is intended to allow for a non-diagonal make matrix—this can vastly simplify a model, for example in incorporating the recent GTAP power database. This section allows the user to input the sets and mappings for the model aggregation. Listing F.14 has an example from the '10x10' map file.

Most of the entries in the listing file should be fairly self-explanatory. The user defines two sets, `actf` and `commf`, that will represent the model aggregation. They can replicate the set i from the first section, in which case the make matrix will be the standard diagonal. In the example from the listing file, the two agricultural activities from the data aggregation are merged into one activity called `Agriculture`. Thus the single activity `Agriculture` will produce two commodities: `GrainsCrops` and `MeatLstk`. This is reflected in the two mapping sets `mapaf` and `mapif`. Note that the mappings are based on the data aggregation and not the original GTAP database.[23]

The model needs an `imuvf` subset based on the final model aggregation. This is followed by the definition of the aggregations of commodities, activities and regions, i.e. the set mappings `mapia`, `mapaga` and `mapra`. Note that these mappings are ignored by the model and typically are only used for post-processing of results.

The latest version of the model includes a climate damages and adaption module. This is still under development. The module may use special sector(s) for education and health services. These are provided as subsets `iedu` and `ihht` (see lines 73–81).

The mapping file also includes a user-determined sort order for regions, activities and commodities. GAMS does not preserve the order of set labels. The sort orders can be used upon output to guarantee that output will be sorted according to user preferences. The sort order can also be used to sort labels for Excel (for example in the creation of Pivot tables.).

Listing F.14: **Sets for model aggregation**

```
1  * --------------------------------------------------------------------------------------------
2  *
3  *    Section dealing with model aggregations (to handle non-diagonal make matrix)
4  *
5  * --------------------------------------------------------------------------------------------

7  *  Model aggregation(s)

9  set actf "Model activities" /
10       Agriculture    "Agriculture"
11       Extraction     "Mining and Extraction"
12       ProcFood       "Processed Food"
13       TextWapp       "Textiles and Clothing"
14       LightMnfc      "Light Manufacturing"
15       HeavyMnfc      "Heavy Manufacturing"
16       Util_Cons      "Utilities and Construction"
17       TransComm      "Transport and Communication"
18       OthServices    "Other Services"
19  / ;

21  set commf "Model commodities" /
22       GrainsCrops    "Grains and Crops"
```

---

[23]   The aggregation facility will automatically append the '-a' and '-c' suffixes to activities and commodities, respectively, except for the user-defined aggregations.

```
23      MeatLstk         "Livestock and Meat Products"
24      Extraction       "Mining and Extraction"
25      ProcFood         "Processed Food"
26      TextWapp         "Textiles and Clothing"
27      LightMnfc        "Light Manufacturing"
28      HeavyMnfc        "Heavy Manufacturing"
29      Util_Cons        "Utilities and Construction"
30      TransComm        "Transport and Communication"
31      OthServices      "Other Services"
32  / ;

34  set mapaf(i, actf) "Mapping from original to modeled activities" /
35      GrainsCrops  .Agriculture
36      MeatLstk     .Agriculture
37      Extraction   .Extraction
38      ProcFood     .ProcFood
39      TextWapp     .TextWapp
40      LightMnfc    .LightMnfc
41      HeavyMnfc    .HeavyMnfc
42      Util_Cons    .Util_Cons
43      TransComm    .TransComm
44      OthServices  .OthServices
45  / ;

47  set mapif(i, commf) "Mapping from original to modeled commodities" /
48      GrainsCrops  .GrainsCrops
49      MeatLstk     .MeatLstk
50      Extraction   .Extraction
51      ProcFood     .ProcFood
52      TextWapp     .TextWapp
53      LightMnfc    .LightMnfc
54      HeavyMnfc    .HeavyMnfc
55      Util_Cons    .Util_Cons
56      TransComm    .TransComm
57      OthServices  .OthServices
58  / ;

60  * >>>> MUST INSERT MUV COMMODITIES (ONE OR MORE)
61  *      !!!! Be careful of compatibility with modeled imuv
62  *           This one is intended for AlterTax

64  set imuvf(commf) "MUV commodities" /

66     ProcFood
67     TextWapp
68     LightMnfc
69     HeavyMnfc

71  / ;

73  *  For adaptation module

75  set iedu(commf) "Education sector(s)" /
76     OthServices
77  / ;

79  set ihht(commf) "Health sector(s)" /
80     OthServices
81  / ;

83  *  >>>> Aggregation of modeled sectors and regions

85  set mapia(ia,commf)"mapping of individual comm to aggregate comm" /
86     tagr-c.GrainsCrops
87     tagr-c.MeatLstk
88     tman-c.ProcFood
89     tman-c.TextWapp
90     tman-c.LightMnfc
```

192

```
91      tman-c.HeavyMnfc
92      toth-c.Extraction
93      tsrv-c.Util_Cons
94      tsrv-c.TransComm
95      tsrv-c.OthServices
96    / ;
97    mapia("ttot-c",commf) = yes ;

99    set mapaga(aga,actf)"mapping of individual comm to aggregate comm" /
100     tagr-a.Agriculture
101     tman-a.ProcFood
102     tman-a.TextWapp
103     tman-a.LightMnfc
104     tman-a.HeavyMnfc
105     toth-a.Extraction
106     tsrv-a.Util_Cons
107     tsrv-a.TransComm
108     tsrv-a.OthServices
109   / ;
110   mapaga("ttot-a",actf) = yes ;

112   set mapra(ra,r) "Mapping of model regions to aggregate regions" /
113     hic.(Oceania, NAmerica, EU_28)
114   / ;
115   mapra("lmy", r)$(not mapra("hic",r)) = yes ;
116   mapra("wld", r) = yes ;

118   set maplagg(lagg,l) "Mapping of model labor to aggregate labor" ;
119   maplagg("Tot",l) = yes ;

121   set sortOrder / sort1*sort500 / ;
122   set mapRegSort(sortOrder,r) /

124     sort1 .  Oceania
125     sort2 .  EastAsia
126     sort3 .  SEAsia
127     sort4 .  SouthAsia
128     sort5 .  NAmerica
129     sort6 .  LatinAmer
130     sort7 .  EU_28
131     sort8 .  MENA
132     sort9 .  SSA
133     sort10.  RestofWorld

135   / ;

137   set mapActSort(sortOrder,actf) /

139     sort1   .Agriculture
140     sort2   .Extraction
141     sort3   .ProcFood
142     sort4   .TextWapp
143     sort5   .LightMnfc
144     sort6   .HeavyMnfc
145     sort7   .Util_Cons
146     sort8   .TransComm
147     sort9   .OthServices

149   / ;

151   set mapCommSort(sortOrder,commf) /
152     sort1   .GrainsCrops
153     sort2   .MeatLstk
154     sort3   .Extraction
155     sort4   .ProcFood
156     sort5   .TextWapp
157     sort6   .LightMnfc
158     sort7   .HeavyMnfc
```

```
159      sort8   .Util_Cons
160      sort9   .TransComm
161      sort10  .OthServices
162  / ;
```

Sets and mappings for the Envisage Model

The final part of the map file is specific to the ENVISAGE Model. Many are critical components of ENVISAGE as they often determine model specification. For example, the first two subsets, `acr` and `alv` determine which activities use the crop and livestock production structure, respectively. All other activities will be assigned to the subset `ax` that is the default production structure. Listing F.15 provides an example for the 10x10 map file, though we will provide additional snippets below for a different aggregation tuned to the power and water modules of the model.

The subsets `agr` and `man` are not formally part of the model. These subsets are currently being used to determine activity-specific productivity shifters in dynamic scenarios.[24] They could potentially be used as well in post-simulation processing. The subsets `aenergy`, `affl` and `aw` are currently not being used.

Listing F.15: **Sets for the Envisage Model aggregation**

```
1  * -----------------------------------------------------------------------------------------
2  *
3  *       Envisage section
4  *
5  * -----------------------------------------------------------------------------------------

7  * >>>> Activity related sets and subsets

9  set acr(actf)   "Crop activities" /
10 / ;

12 set alv(actf)   "Livestock activities" /
13 / ;

15 set agr(actf)   "Agricultural activities" /
16       Agriculture      "Agriculture"
17 / ;

19 set man(actf)   "Manufacturing activities" /
20       Extraction       "Mining and Extraction"
21       ProcFood         "Processed Food"
22       TextWapp         "Textiles and Clothing"
23       LightMnfc        "Light Manufacturing"
24       HeavyMnfc        "Heavy Manufacturing"
25 / ;

27 set aenergy(actf) "Energy activities" /
28       Extraction       "Mining and Extraction"
29       Util_Cons        "Utilities and Construction"
30 / ;

32 set affl(actf)  "Fossil fuel activities" /
33       Extraction       "Mining and Extraction"
34 / ;

36 set aw(actf)    "Water services activities" /
37 / ;

39 set elya(actf)  "Power activities" /
40 / ;

42 set etd(actf)   "Electricity transmission and distribution activities" /
43 / ;

45 set primElya(actf) "Primary power activities" /
```

---
[24]   See the model file `initScen.gms`

194

```gams
 46    / ;

 48    set pb    "Power bundles" /
 49       othp      "Other power"
 50    / ;

 52    set mappow(pb,elya) "Mapping of power activities to power bundles" /
 53    / ;

 55    *  >>>> Commodity sets and subsets

 57    set frt(commf) "Fertilizer commodities" /
 58    / ;

 60    set feed(commf) "Feed commodities" /
 61    / ;

 63    set iw(commf) "Water services commodities" /
 64    / ;

 66    set e(commf) "Energy commodities" /
 67    $ontext
 68         Extraction     "Mining and Extraction"
 69         Util_Cons      "Utilities and Construction"
 70    $offtext
 71    / ;

 73    set elyc(commf) "Electricity commodities" /
 74         Util_Cons      "Utilities and Construction"
 75    / ;

 77    set f(commf) "Fuel commodities" /
 78    *     Extraction     "Mining and Extraction"
 79    / ;

 81    *  This zonal mapping is for labor market segmentation in final model

 83    set mapzf(z,actf)  "Mapping of activities to zones" /
 84       rur.Agriculture
 85    / ;

 87    mapzf("urb",actf) = not mapzf("rur",actf) ;
 88    mapzf("nsg",actf) = yes ;

 90    * >>>> Household commodity section

 92    set k "Household commodities" /
 93         GrainsCrops    "Grains and Crops"
 94         MeatLstk       "Livestock and Meat Products"
 95         ProcFood       "Processed Food"
 96         TextWapp       "Textiles and Clothing"
 97         LightMnfc      "Light Manufacturing"
 98         HeavyMnfc      "Heavy Manufacturing"
 99         TransComm      "Transport and Communication"
100         OthServices    "Other Services"
101         Energy         "Energy"
102    / ;

104    set fud(k) "Household food commodities" /
105         GrainsCrops    "Grains and Crops"
106         MeatLstk       "Livestock and Meat Products"
107         ProcFood       "Processed Food"
108    / ;

110    set mapk(commf,k) "Mapping from i to k" /
111         GrainsCrops  .GrainsCrops
112         MeatLstk     .MeatLstk
113         Extraction   .Energy
```

```
114        ProcFood      .ProcFood
115        TextWapp      .TextWapp
116        LightMnfc     .LightMnfc
117        HeavyMnfc     .HeavyMnfc
118        Util_Cons     .Energy
119        TransComm     .TransComm
120        OthServices   .OthServices
121    / ;

123    set cpindx  "CPI indices to be derived by the model" /
124       fud       "Food price index"
125       nfd       "Non-food price index"
126       tot       "Total price index"
127    / ;

129    set mapCPI(cpindx,commf)   "Mapping from commf to CPI index" /
130        fud.GrainsCrops
131        fud.MeatLstk
132        nfd.Extraction
133        fud.ProcFood
134        nfd.TextWapp
135        nfd.LightMnfc
136        nfd.HeavyMnfc
137        nfd.Util_Cons
138        nfd.TransComm
139        nfd.OthServices

141    / ;
142    mapCPI("tot",commf) = yes ;

144    set lb "Land bundles" /
145       agr        "Agriculture"
146    / ;

148    set lb1(lb) "First land bundle" /
149       agr        "Livestock"
150    / ;

152    set maplb(lb,actf) "Mapping of activities to land bundles" /
153       agr        .Agriculture
154    / ;

156    *  !!!! TO BE REVIEWED

158    set lb0    "Default land bundles" / lb1*lb1 / ;
159    set maplb0(lb, lb0) "Mapping of land bundles to original" /
160       agr.lb1
161    / ;

163    set wbnd "Aggregate water markets" /
164       N_A        "N_A"
165    / ;

167    set wbnd1(wbnd) "Top level water markets" /
168    / ;

170    set wbnd2(wbnd) "Second level water markets" /
171    / ;

173    set wbndEx(wbnd) "Exogenous water markets" /
174    / ;

176    set mapw1(wbnd,wbnd) "Mapping of first level water bundles" /
177    / ;

179    set mapw2(wbnd,actf) "Mapping of second level water bundle" /
180    / ;
```

196

```
182  set wbnda(wbnd) "Water bundles mapped one-to-one to activities" /
183  / ;

185  set wbndi(wbnd) "Water bundles mapped to aggregate output" /
186  / ;

188  set NRG "Energy bundles used in model" /
189     coa        "Coal"
190     oil        "Oil"
191     gas        "Gas"
192     ely        "Electricity"
193  / ;

195  set coa(NRG) "Coal bundle used in model" /
196     coa        "Coal"
197  / ;

199  set oil(NRG) "Oil bundle used in model" /
200     oil        "Oil"
201  / ;

203  set gas(NRG) "Gas bundle used in model" /
204     gas        "Gas"
205  / ;

207  set ely(NRG) "Electricity bundle used in model" /
208     ely        "Electricity"
209  / ;

211  set mape(NRG,e) "Mapping of energy commodities to energy bundles" /
212  *  oil        .Extraction
213  *  ely        .Util_Cons
214  / ;

216  *  >>>> Sets required for 'growing' labor by skill

218  set skl(l)   "Skill types for labor growth assumptions" /
219     SkLab
220  / ;

222  set elev / elev0*elev3 / ;

224  set educMap(r,l,elev) "Mapping of skills to education levels" ;

226  *  Use GIDD definitions (i.e. "elev3" has no meaning)

228  educMap(r,"UnSkLab","elev0")$mapra("lmy",r) = yes ;
229  educMap(r,"SkLab","elev1")$mapra("lmy",r)   = yes ;
230  educMap(r,"SkLab","elev2")$mapra("lmy",r)   = yes ;

232  educMap(r,"UnSkLab","elev0")$mapra("hic",r) = yes ;
233  educMap(r,"UnSkLab","elev1")$mapra("hic",r) = yes ;
234  educMap(r,"SkLab","elev2")$mapra("hic",r)   = yes ;

236  $offempty
```

The subsets `elya` and `etd` are used by the power module. The first contains all of the (aggregate) power activities and the second contains the electricity transmission and distribution activity (normally a single activity). If these sets are empty, the distribution of power will be modeled using the standard 'make' specification, i.e. a single CES nest for aggregation. An additional subset, `primElya`, is used post-simulation to calculate primary energy demand. The latter is composed of all combusted fossil fuels and primary electricity production, which excludes thermal power plants to avoid double counting.

The power module allows for a multiple nested CES structure for power aggregation. The example below, Listing F.16, shows the power nesting for a different map file. There are eight power activities (coal, oil, gas, nuclear, hydro, wind, solar, and other renewable), plus the transmission and distribution activity. The eight activities will be mapped to five power bundles (coal, gas, oil, nuclear and renewables), see set `pb`. The power aggregation is based on

197

three nests, see figure 8—that is fully defined with the subsets n the power module of the map file. The subset `elyc` is designed to contain the single electricity commodity that is the output of aggregating electricity output across all electricity activities.

Listing F.16: **A power bundle example**

```
1   set elya(actf) "Power activities" /
2         clp
3         olp
4         gsp
5         nuc
6         hyd
7         wnd
8         sol
9         xel
10        etd
11  / ;

13  set etd(actf)  "Electricity transmission and distribution activities" /
14        etd
15  / ;

17  set primElya(elya) "Primary energy power activities" /
18        nuc
19        hyd
20        wnd
21        sol
22        xel
23        etd
24  / ;

26  set pb   "Power bundles" /
27        coap     "Coal power"
28        gasp     "Gas power"
29        oilp     "Oil power"
30        nucp     "Nuclear power"
31        othp     "Other power"
32  / ;

34  set mappow(pb,elya) "Mapping of power activities to power bundles" /
35        coap.clp
36        gasp.gsp
37        oilp.olp
38        nucp.nuc
39        othp.(hyd, wnd, sol, xel)
40  / ;

42  set elyc(commf) "Electricity commodities" /
43        ely
44  / ;
```

The subsets `frt` and `feed` are used respectively by the `acr` and `lvs` activities to define the *ND2* bundle.[25] The former contains the fertilizer commodities and the latter the feed commodities. The subset `iw` defines the commodities that can be bundled with the water factor in the water module. It could contain, for example, the water services commodity (`wtr`) from the GTAP database.

The subset `e` is a critical component of the energy module, see figures 5 and 10. The subset is also linked to the `NRG` set, the subsets `coa`, `oil`, `gas` and `ely` and the mapping set `mape`. Listing F.17 shows a potential configuration using the standard energy commodities in the GTAP database.[26] In the current configuration the set `NRG` and the name of the subsets `coa`, `oil`, `gas` and `ely` cannot be modified by the user. The subsets, however, can be empty. The subset f is not used by the model itself [TBC]. It is used at times in post-simulation to evaluate CO_2 emissions from the combustion of fuels. In the original GTAP database, this corresponds to the commodities `coa`, `oil`, `gas`, `p_c`, and `gdt`.

---

[25]   From the point of view of the code, these are used to determine the mapping sets `mapi1` and `mapi2`.

[26]   The ordering in the map file is somewhat arbitrary and mostly up to user discretion, unless there are specific dependencies.

```
1  set e(commf) "Energy commodities" /
2        coa
3        oil
4        gas
5        p_c
7        ely
8  / ;

10  set NRG "Energy bundles used in model" /
11     coa          "Coal"
12     oil          "Oil"
13     gas          "Gas"
14     ely          "Electricity"
15  / ;

17  set coa(NRG) "Coal bundle used in model" /
18     coa          "Coal"
19  / ;

21  set oil(NRG) "Oil bundle used in model" /
22     oil          "Oil"
23  / ;

25  set gas(NRG) "Gas bundle used in model" /
26     gas          "Gas"
27  / ;

29  set ely(NRG) "Electricity bundle used in model" /
30     ely          "Electricity"
31  / ;

33  set mape(NRG,e) "Mapping of energy commodities to energy bundles" /
34     COA.(coa)
35     OIL.(oil, p_c)
36     GAS.(gas)
37     ELY.(ely)
38  / ;
```

The mapping set `mapzf` is critical for the rural to urban migration module (if implemented). It would normally correspond to the `mapz` mapping set from the earlier part of the mapping file—but corresponding to the model aggregation, not the data aggregation. If the migration elasticity is set to infinity, the model will assume perfect labor mobility and the `mapzf` mapping set will be ignored.

The top level demand nest for consumer demand is based on a different set of goods than the commodities defined by the set i. This allows for a 'make' or 'transition' matrix approach to consumer demand specification, see figure 9. The household commodities are specified over the set k. It is a simple mapping from the supply commodities (i) to consumed commodities (k). If there is a one-to-one mapping, the consumer make matrix is fully diagonal. At the moment, one of the key purposes of the make matrix is to allow for a consumer energy bundle, with a structure similar to the energy bundle in production, see figure 10. It could however be used to construct other non-diagonal elements. For example crop commodities could be combined into a single consumer crop bundle and the livestock commodities could be combined into a single consumer livestock bundle. The subsequent CES nests would then allocate the top level demand to the relevant components. The subset `fud(k)` can be used by the ENVISAGE model to focus on food demand. Listing F.18 provides an example of a consumer mapping for energy only.[27] N.B. The CPI index, which can be defined over one or more bundles, is defined with respect to the core commodity supply, not with respect to the consumer commodities (i.e., it is defined over i, not over k).

```
1  set k "Household commodities" /
2        crp       "Cereals"
```

_____

[27] Like for the other activities and commodities, the aggregation facility will automatically append the '-k' suffix to consumer labels.

```
 3        osd      "Oil seeds"
 4        xcr      "Other crops"
 5        lvs      "Meat and wool"
 6        rmk      "Raw milk"
 7        frs      "Forestry"
 8        fsh      "Fisheries"
 9        omn      "Other mining"
10        met      "Meat products"
11        vol      "Vegetable oils"
12        mil      "Dairy products"
13        ofd      "Other foods products"
14        b_t      "Beverages and tobacco products"
15        tex      "Textiles"
16        wap      "Wearing apparel"
17        lea      "Leather products"
18        wdp      "Lumber, paper and paper products"
19        crp      "Chemicals, rubber and plastics"
20        mmn      "Basic metals and minerals"
21        ele      "Electronic equipment"
22        mvh      "Motor vehicles"
23        xma      "Other industry & manufacturing"
24        cns      "Construction"
25        ttp      "Trade and transport"
26        bsv      "Financial and business services"
27        dwe      "Dwellings"
28        xsv      "Other services"
29        nrg      "Energy bundle"
30  / ;

32  set fud(k) "Household food commodities" /
33        crp      "Cereals"
34        osd      "Oil seeds"
35        xcr      "Other crops"
36        lvs      "Meat and wool"
37        rmk      "Raw milk"
38        fsh      "Fisheries"
39        met      "Meat products"
40        vol      "Vegetable oils"
41        mil      "Dairy products"
42        ofd      "Other foods products"
43        b_t      "Beverages and tobacco products"
44  / ;

46  set mapk(commf,k) "Mapping from i to k" /
47        cer.cer
48        osd.osd
49        xcr.xcr
50        lvs.lvs
51        rmk.rmk
52        frs.frs
53        fsh.fsh
54        coa.nrg
55        oil.nrg
56        gas.nrg
57        omn.omn
58        met.met
59        vol.vol
60        mil.mil
61        ofd.ofd
62        b_t.b_t
63        tex.tex
64        wap.wap
65        lea.lea
66        wdp.wdp
67        p_c.nrg
68        crp.crp
69        mmn.mmn
70        ele.ele
```

```
71        mvh.mvh
72        xma.xma
73        ely.nrg
74        cns.cns
75        ttp.ttp
76        bsv.bsv
77        dwe.dwe
78        xsv.xsv
79  / ;
```

The land module depends on user-determined land bundles, see figure 13. The model allows for a fair amount of user flexibility. There are three nests, but one of the nests allows for a variable number of land bundles. The mapping of the agricultural activities to the land bundles is also determined by the user. Listing F.19 shows an example of a land bundle that corresponds to that used in the MAGNET model.[28]

The MAGNET specification has only three bundles (L, FCP and COP). We have arranged these into five bundles. The top is split between the hrt bundle and all of the rest (that corresponds to the FCP bundle). We assume that the top evel CET elasticity in MAGNET, σ^1 holds for the top level bundle in ENVISAGEbut also for the bundle composed of activities in the hrt bundle. This has the same impact as mapping directly the hrt activities to the top level bundle as in MAGNET. The *XNLB* bundle will then be composed of the lvs and cer bundles. MAGNET's CET elasticity, σ^2 is then applied to the *XNLB* and lvs bundles, which has the same impact as mapping directly the lvs activities to the *XNLB* bundle as in MAGNET. Finally, MAGNET's σ^3 elasticity is applied to the cer bundle. The set lb0 is used to map the elasticities in the parameter file to the model elasticities—the set is not part of the the the model. [This mapping is still under review and users can override these in the model parameter file].

Listing F.19: **An example of land configuration**

```
1   set lb "Land bundles" /
2     hrt        "Horticulture and other crops"
3     lvs        "Livestock and sugar"
4     cer        "Cereals and oil seeds"
5   / ;

7   set lb1(lb) "First land bundle" /
8     hrt        "Horticulture and other crops"
9   / ;

11  set maplb(lb,actf) "Mapping of activities to land bundles" /
12    hrt.v_f
13    htr.ocr
14    lvs.lvs
15    lvs.sug
16    cer.wht
17    cer.gro
18    cer.osd
19  / ;

21  set lb0    "Default land bundles" / lb1*lb3 / ;
22  set maplb0(lb, lb0) "Mapping of land bundles to original" /
23    hrt.lb1
24    lvs.lb2
25    cer.lb3
26  / ;
```

The water module has three basic bundle types:

1. Water bundles that have activities mapped to them—for example irrigation water in crop activities

2. Water bundles that are linked to aggregate output indices—for example industrial water use

3. Exogenous bundles not linked directly to the economy—for example ground water recharge or water for environmental services.

These fundamental water bundles are structured in a nested CET nest for which users have significant control. In the example in Listing F.20 there is one bundle of the first type (crops), three bundles of the second type (livestock,

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[28]   edepot.wur.nl/310764, page 74).

201

municipal and industrial) and two bundles of the third type (environmental services and ground water recharge). There are two intermediate bundles that form the top level CET nest—agriculture and non-agriculture, see figure 15. The set `wbnd` defines all of the possible bundles. The set `wbnd1` defines the top level bundles—in principle there could be more than two. The set `wbnd2` defines the second level bundles. The set `wbndEx` defines the exogenous bundles. These will be subtracted from total water supply. The mapping set `mapw1` maps fundamental bundles to the top-level intermediate water bundles. The mapping set `mapw2` maps all activities to one of the fundamental bundles. The subset `wbnda` indicates which water bundles are mapped directly to activity-based water demand. With the current water database, only irrigated crops use water directly. The subset `wbndi` indicates which water bundles are mapped to output indices. The elasticities for the water module need to be provided in the user-built parameter file—see below.

Listing F.20: **An example of water configuration**

```
1   set wbnd "Aggregate water markets" /
2      agr      "Agriculture"
3      nag      "Non-agriculture"
4      env      "Environmental services"
5      grd      "Ground water recharge"
6      crp      "Crops"
7      lvs      "Livestock"
8      ind      "Industrial use"
9      mun      "Municipal use"
10  / ;

12  set wbnd1(wbnd) "Top level water markets" /
13     agr      "Agriculture"
14     nag      "Non-agriculture"
15  / ;

17  set wbnd2(wbnd) "Second level water markets" /
18     crp      "Crops"
19     lvs      "Livestock"
20     ind      "Industrial use"
21     mun      "Municipal use"
22  / ;

24  set wbndEx(wbnd) "Exogenous water markets" /
25     env      "Environmental services"
26     grd      "Ground water recharge"
27  / ;

29  set mapw1(wbnd,wbnd) "Mapping of first level water bundles" /
30     agr.(crp,lvs)
31     nag.(mun,ind)
32  / ;

34  set mapw2(wbnd,actf) "Mapping of second level water bundle" ;
35  mapw2("crp",acr) = yes ;
36  mapw2("lvs",alv) = yes ;
37  mapw2("ind",man) = yes ;
38  mapw2("mun",actf)$(not (acr(actf) or alv(actf) or man(actf))) = yes ;

40  set wbnda(wbnd) "Water bundles mapped one-to-one to activities" /
41     crp      "Crops"
42  / ;

44  set wbndi(wbnd) "Water bundles mapped to aggregate output" /
45     lvs      "Livestock"
46     ind      "Industrial use"
47     mun      "Municipal use"
48  / ;
```

The remaining section of the map file relates to the use of the IIASA- or GIDD-based education projections to shape the growth of skilled and unskilled labor. This is not fundamental to the model, but is part of defining the dynamic scenario.

One subset defines skilled labor, `skl`. This is different from the `ul` and `sl` subsets that are used to allocate labor

between the *LAB1* and *LAB2* bundles. The purpose here is only to drive the relative growth of unskilled and skilled labor. The mapping set, `educMap`, determines the mapping between labor skills and the education projections. There are two different education projections. The IIASA education projections have four classifications: none, primary, secondary and tertiary (labeled `ENONE`, `EPRIM`, `ESECN` and `ETERT` in the SSP database, but labeled `ELEV0*ELEV3` in the scenario file.) The GIDD database has three classifications: 0-6 years of education, 6-9 years of education and 9 or more years (labeled `EDUC0_6`, `EDUC6_9`, and `EDUC9UP` in the GIDD database, but labeled `ELEV0*ELEV2` in the scenario file.) If using the IIASA database, all four education levels should be mapped to the skills. If using the GIDD database, only three of the levels are active. The mappings can vary across regions. For example, in high-income countries, skilled workers might be only tertiary and above. In developing countries, skilled workers may also include workers with a secondary education.

Running the aggregation

The aggregation is coded in GAMS and can be run in the GAMS IDE or from a Windows command console. For the latter, the command line is:

```
gams aggGTAP --basename=[basename] --ifAlt=[OFF|ON] --model=[GTAP|ENV]
        --ifCSV=[0|1] --ifAggTrade=[0|1]
```

The aggregation routine is named `AggGTAP.gms`. The user needs to provide the base name of the project, for example `10x10`. The aggregation facility will prepare additional output if the user wishes to use AlterTax. In that case, the `ifAlt` setting should be set to `ON`, otherwise it should be set to `OFF`. Aggregation is handled somewhat differently for the GTAP and ENVISAGE models. The model option should take the value `GTAP` for the GTAP model, and `ENV` for the ENVISAGE model. The `ifCSV` option takes either the value 0 or 1. If it is set to 1, the aggregation facility will output the SAM, energy and emissions data in a CSV formatted file that can be uploaded into Excel. The `ifAggTrade` option takes either the value 0 or 1. If it is set to 1, the output SAM will collapse bilateral trade to a single trade account, otherwise the SAM will have the full bilateral trade (on both the import and export side).

If the aggregation is intended for the ENVISAGE Model, the *aggGTAP* routine will invoke the *convertLabel* program that will convert the labels in the parameter file to the labels needed for the model.

F.1.10   Filtering

The filtering algorithm was developed by Tom Rutherford to remove small value flows from the aggregated database (Lanz and Rutherford (2016)). The distributed version has been modified by Wolfgang Britz to handle some additional features. The user needs to prepare a very short include file with the algorithms's options (`[basename]Flt.gms`). Listing F.21 is an example of the filter options file.

The filter options file

The consecutively applied filtering and rebalancing approach is an extension of the method and code developed by Tom Rutherford.[29] It deletes components of the SAM depending on their shares on specific totals, according to the 'Relative tolerance' (`relTol`):

- Domestic and imported intermediate demand of a commodity are dropped relative to its total output
- Private/government/investment domestic respectively import demand of a commodity are dropped relative to total Private/government/investment domestic respectively import demand
- Trade flows of a product are dropped if both shares on total exports of that product and its exporter and on imports of that product and its importer are below the relative threshold
- Production is dropped if net production of a commodity, i.e. after intermediate use of that commodity in its own production is deducted, is below the relative threshold with regard to total net production

Listing F.21: **An example of a filter file**

```
1  scalars
2  ifKeepIntermediateConsumption / 1 /
3  ifKeepPrivateconsumption      / 1 /
```

---

[29]   The following explanations are adapted from Britz's documentation. See http://www.ilr.uni-bonn.de/em/rsrch/cgebox/cgebox_GUI.pdf.

```
 4  ifKeepGovernmentconsumption   / 1 /
 5  ifKeepInvestments             / 1 /
 6  ifGDPKeep                     / 1 /
 7  ifKeepFactorincomeplusbop     / 1 /
 8  ifAdjDepr                     / 1 /
 9  abstol                        / 1e-10 /
10  relTol                        / 0.005 /
11  relTolRed                     / 1e-6  /
12  nsteps                        / 5 /
13  minNumTransactions            / 50000 /
14  ;

16  file log / %baseName%flt.log / ;
17  put log ;

19  $ontext
20  $setglobal excRegs
21  $setglobal excSecs
22  $offtext

24  $setglobal excSecs "sol, wnd, xel"
25  $setGlobal excCombined  1
```

The absolute tolerance level, `absTol`, deletes any trade flow value that is below the tolerance level in absolute terms. With a value of 1.E-10, that preliminary deletion step is skipped. It is generally not recommended to use absolute deletion thresholds above 1.E-6 in combination with rebalancing as the subsequent relative thresholds will anyhow apply more refined rules.

The filtering process imposes restrictions which should maintain the regional SAMs balanced. Additional constraints ensure that production activities require added value and intermediate inputs, if not already otherwise found in the data base. As filtering systematically removes elements from the SAM and the trade matrices, the process implies without further corrections shrinking the economies. During rebalancing, the algorithm can therefore add penalties for deviations from the following aggregate transactions:

- Intermediate consumption
- Private consumption
- Public consumption
- Investment
- Factor income plus BOP
- GDP

The imposition of these penalties is driven by the relevant flags in the filter options file. By adding these penalties terms, the non-deleted entries (and thus most important transactions) tend to be scaled upwards. It is generally recommended to use these penalties terms. The code will also scale all non-deleted trade flows to approximately maintain the total volume of international trade and related international transport margins.

The absolute and relative thresholds are stepwise enforced. For the first few steps, exponential increases are used, starting with minus half the number of steps. For six steps, to give an example, the first thresholds applied will the 1.E-3 of the final one, next 1.E-2 and finally 10%. The remaining steps will use equal linear increases between 10% and the desired final ones. Once the final thresholds are active, filtering is still applied several times until no small values are found any longer. The code should ensure that the resulting transactions are still fully consistent with each other, i.e. both the resulting trade matrices and the SAMs are balanced. The changes imposed by filtering and subsequent balancing are stored in the log file. Inspecting how the stepwise enforcement of the thresholds impacts on the number of non-zero items can inform on an appropriate level for tolerances to be used.

The SAMs used during filtering are—as in the GTAP database—defined in Million dollars. An absolute threshold of 1.E-6 will hence delete any economic transactions worth a single dollar or less. In SAMs with high regional and sectoral detail, even such tiny transactions might make up to 10% of the non-zero entries. Increasing the threshold to $1000 might remove 1/4 or more of all non-empty transactions. Similar results are found from using relative tolerances of 0.001%.

Thanks to balancing, also rather dis-aggregated versions of the model with large number of sectors and regions can be used. The biggest impact of the filtering is typically on transactions related to bilateral trade flows. Here, often 50% or more of the flows account for only 1% of the total value of these transactions. Thus, tiny changes in the relative tolerance can have a considerable impact on the number of deleted transaction, and one might need to

experiment with settings in the range around 1.E-1 to 1.E-4 to find a compromise between sparsity and the implied changes on structure of the economy. For very large data sets (e.g. a 1:1 version) filtering thresholds above 1% might be needed to yield reasonable model sizes. The user can additionally define a minimum number of transactions to be kept, which reduces the need to experiment with different thresholds as the filtering process will stop once less than the desired number of transactions is reached. Tests with the model have shown that the model in full resolution of the GTAP 8.1 data base without filtering, i.e. 57 sectors and 134 regions, can be solved in partial trade liberalization scenarios, solution has failed with other shocks on models with more than 400,000 transactions, especially if the global bank mechanism active. A close look at the filtering statistics is recommended, to avoid sharp impacts on the structure of the economy. A more detailed discussion on the relation between model dis-aggregation, filtering, solution behavior and simulated welfare impacts is provided in Britz and van der Mensbrugghe (2016).

Special treatment for specific regions and sectors When building a data base for a project, it might be desirable to apply less aggressive filtering thresholds for specific regions and/or sectors in the focus of the application. The algorithm therefore allows defining lists of regions/sectors with accompanying specific thresholds. The codes for regions/sectors needs to be inputted in the global defines called `excRegs` and `excSecs`. 'Reduced thresholds only in combination' will apply the different threshold only to the intersection of the inputted regions and sectors (`excCombined`), otherwise, all regions and sectors inputted will be receive different thresholds. Take an example where you enter for regions `xoc` and for sectors `pdr`. If 'Reduced thresholds only in combination' is NOT switched on, all transactions of the region `xoc` and all transactions for the sector `pdr` will be treated differently. If the 'Reduced thresholds only in combination' is active, only the transaction relating both to `pdr` and the region `xoc` are exemptions. However, filtering for the remaining sectors/regions has still an impact on these exemptions. For example, if production of a sector in a region is dropped, the related export flows need to be dropped as well, affecting potentially transactions in regions and sectors where tighter thresholds are used. Tests have however indicated that very few transactions are lost in regions/sector where stringent thresholds are applied as long as the overall filtering thresholds are not too aggressive.

Diagnostics The filter listing file (`filter.lst`) and the log file (`[basename]flt.log`) provide a number of useful diagnostics regarding the filtering process. Users can also load the resulting 'CSV' file that contains the SAM and energy/emission values both pre- and post-filtering.

Running the filter program

The filter routine is coded in GAMS and can be run in the GAMS IDE or from a Windows command console. For the latter, the command line is:

```
gams filter --basename=[basename] --ifCSV=[0|1] --ifAggTrade=[0|1]
```

The filter routine is named `filter.gms`. The user needs to provide the base name of the project, for example `10x10`. The `ifCSV` option takes either the value 0 or 1. If it is set to 1, the aggregation facility will output the SAM, energy and emissions data in a CSV formatted file that can be uploaded into Excel. The `ifAggTrade` option takes either the value 0 or 1. If it is set to 1, the output SAM will collapse bilateral trade to a single trade account, otherwise the SAM will have the full bilateral trade (on both the import and export side).

F.1.11   Altering a database

The third module in the data preparation routine is called Altertax, first developed by Malcolm (1998) for the GTAP model. The Altertax module is optional. It allows the user to make changes to the structure of the database that minimizes the distortions from the original database. It is typically used to change tax rates, for example import tariffs. The distributed version of Altertax relies on a version of the GTAP model written in GAMS.[30] The main thrust of Altertax is to convert most of the model elasticities to 1 thereby all CES functions are essentially converted to Cobb-Douglas functional forms. The latter have the property of conserving value shares. In addition, the Altertax version of the GTAP model assumes a Cobb-Douglas utility function for private consumption. A fixed current account is also imposed for the balance of payment closure. The overrides are provided in the `AlterTax.gms` file.

---

[30]   The GAMS-based GTAP model has been developed by GTAP staff and documentation is available upon request. It differs from the Rutherford *GTAPinGAMS* model (Lanz and Rutherford (2016)) as it is intended to exactly replicate the specification of the standard GTAP model in GEMPACK (Corong et al. (2017) and Hertel (1997)).

Option file

If running Altertax, the user needs to prepare an option file that specifies the nature of the alteration—it is to be named `[basename]Alt.gms`. The file can be empty, in which case Altertax would simply re-balance the initital database—though this will already have been done in the filter routine if it was used. Listing F.22 shows a very simple change: a rise of 40% on the import tariff of motor vehicles in Argentina. One could of course overlay the entire import tariff schedule for Argentina in this file—including any bilateral dimensions. Note that the activity and commodity labels will need the appropriate suffix.

Listing F.22: **An example of an Altertax shock**

```
2   imptx.fx(s,"mvh-c","arg",tsim) = 1.4*imptx.l(s,"mvh-c","arg",tsim) ;
```

A more complicated, though artificial example is provided in Listing F.23. This example shows how to phase in the shock to ease numerical convergence. The shock is a 50% cut in all tariffs. The user provides the number of iterations to phase in the shocks, say for example 4. The iteration count is first checked to see if it is 1, in which case there is no iteration phase and the shock is imposed in one shot. If the iteration count is greater than 1, the shock will be phased in. The `solver` needs to be invoked for the first $n-1$ iterations, the nth iteration will be solved in the standard (and final) invocation of the solver.

Listing F.23: **An example of an Altertax file**

```
1    *  -------------------------------------------------------------------------------------------------
2    *   Altertax shock
3    *
4    *   If new policy needs to be phased in, the basic implementation is
5    *
6    *      p(it) = pfinal*(it/n) + pinitial*(1-it/n)
7    *
8    *   See below for an example
9    *
10   *   Set the number of iterations on the command line, for example --niter=4

12   *   Cut initial tariffs by 50%
13   *  -------------------------------------------------------------------------------------------------

15   if(niter(tsim) eq 1,

17   *   No phase in of cuts

19       imptx.fx(r,i,rp,tsim) = 0.5*imptx.l(r,i,rp,"base") ;

21   else

23   *   Phase in the shock

25       for(iter=1 to niter(tsim),
26           imptx.fx(r,i,rp,tsim) = 0.5*imptx.l(r,i,rp,"base") * (iter/niter(tsim))
27                                   + imptx.l(r,i,rp,"Base") * (1 - iter/niter(tsim)) ;
28           if(iter < niter(tsim),
29               $$batinclude "solve.gms" gtap
30           ) ;
31       ) ;
32   ) ;
```

Running Altertax

The Altertax routine is coded in GAMS and can be run in the GAMS IDE or from a Windows command console. For the latter, the command line is:

```
gams AlterTax --BaseName=[basename] --niter=1 --ifCSV=[0|1] -idir=GTAPModel
```

The filter routine is named `Altertax.gms`. The user needs to provide the base name of the project, for example `10x10`. The user needs to provide the number of iterations for a phase-in of the shock. For most modest shocks, a value of 1 should be sufficient. Note that it is up to the user to make sure the Altertax option file contains the necessary code for the phase in of the shock. The `ifCSV` option takes either the value 0 or 1. If it is set to 1, the aggregation facility will output the SAM, energy and emissions data in a CSV formatted file that can be uploaded into Excel. The `iDir` option provides a pointer to the folder containing the GTAP model code. Under default configurations the model code is a sub-folder in the *Data* directory with the name *GTAPModel*.

If the `ifCSV` option is set to 1, the SAM and energy/emissions data will be output in a CSV-formatted file for input into Excel. The CSV file will have a 'time' dimension that takes three values. 'Year' equal to 1 corresponds to the base year initialization of model variables and parameters and should correspond to the input database. 'Year' 2 corresponds to a first simulation with no shock. It should re-produce the base data as well, i.e. the results in 'Year' 1 and 2 should be identical to within a very small tolerance level. 'Year' equal to 3 will correspond to the post-shock structure of the database and comparison with either 'Year' 1 or 2 will highlight deviations from the initial database.

Dealing with process emissions

Process emissions are integrated into the production structure, see for example equation (P-4). However, there is no price on process emissions in the base data and thus no way to calibrate the base year parameters. We can use 'Altertax' to introduce a small tax on process emissions. This is done in such a way as to raise the unit cost of production by a small amount—the amount that would be equivalent to the revenues generated by the process emissions using the low tax rate.

For this purpose, the GTAP Model in GAMS, that is the core of Altertax has been modified to include an additional production tax, which is specific to each activity.[31] The additional production tax, labeled `ctax`, is tacked onto to the unit cost of production, `PX`, and is a new price wedge that affects the price of output as it enters the 'make' module. The level of `ctax` is endogenous. It is set such that the value of the revenues associated with `ctax` is equal to the value of the revenues that would be generated by the process emissions given the user-specified carbon tax. The model therefore includes the following equation:

$$\tau_{r,a}^c PX_{r,a} XP_{r,a} = \sum_{em} PCARB_r\, prEmi_{r,em,a}$$

where τ^c is labeled `ctax`. All information on the right-hand side is given—the user provides $PCARB$[32] and the data provides the base year process emissions for each activity. The other changes in the code reflect necessary changes by the introduction of `ctax`. The resulting database looks like a standard GTAP database, but in the absence of process emissions, it would not be consistent.

Listing F.24 provides the code snippet with the shocks that are introduced into the 'Alt' shock file. The first part extracts the process emissions from the 'NCO2' database and calculates the aggregate process emissions for each GHG and activity. N.B. The user selects the conversion factor, i.e., the global warming potential. It should be consistent between the data preparation phase, i.e., Altertax, and the model simulations. The second part of the code initializes the 'carbon' tax. In the standard code, `ctax` is zero. This code endogenizes `ctax` and sets the desired price for the tax on process emissions.

Listing F.24: **Initial pricing of process emissions in Altertax**

```
1   *  Introduce a 'small' tax on process emissions
2   *  Assume we are using 'AR4' and set a price of 25 cents per tCO2eq

4   execute_load "%inDir%/%BaseName%NCO2.gdx", gwp, emi_iop, emi_endw, emi_qo ;

6   ProcEmi0(r,em,a)  = gwp(em,r,"AR4")*cscale
7                     *  (sum((i0,a0)$(mapa0(a0,a)), emi_iop(em, i0, a0, r))
8                     +   sum((fp,a0)$mapa0(a0,a), emi_endw(em, fp, a0, r))
9                     +   sum(a0$mapa0(a0,a), emi_qo(em, a0, r))) ;

11  pcarb0(r) = 0.25 ;
12  ytax0(r,"ct")     = 1 ;
13  ctaxFlag(r,a)$sum(em, pcarb0(r)*(inscale/cscale)*procEmi0(r,em,a)) = yes ;
14  ctax.l(r,a,tsim)$ctaxFlag(r,a) = 0.0003 ;
15  ctax.lo(r,a,tsim)$ctaxFlag(r,a) = -inf ;
16  ctax.up(r,a,tsim)$ctaxFlag(r,a) = +inf ;
```

---

[31] The standard production tax, labeled `prdtx` in the model code, is introduced as part of the 'make' module.
[32] The code reflects the scaling options for emissions and the value data.

After transferring the data files from the data folder to the simulation folder, the user needs to initialize the parameter `pcarb0` for the simulations—it should have the same value as that used for Altertax. This can be set in the 'Prm' file, for example:

```
pcarb0(r,ghg,a) = 0.25 ;
```

The 'check' simulation should be balanced, the calibration of the GHG bundle should yield small shares, and 'small' revenues should be generated by the initially low carbon price.

F.1.12   Integrated command file

The distribution is delivered with a Windows command file, `makeData.cmd`, that can be used to run all of the modules in sequence. To run the command, type the following in a Windows console:

```
makeData [baseName] [-ifFilter] [-ifAlt] [-ifEnv]
```

This will invoke the `makeData` command file. There are three options. To invoke the filter module, enter the `-ifFilter` option on the command line. To invoke the Altertax module, enter the `-ifAlt` option on the command line. To prepare an aggregation for the ENVISAGE Model, enter the `-ifEnv` option on the command line. N.B. We have made every attempt to make the `makeData` command file as robust as possible—but coding a Windows command file is not for the faint-hearted. We would be happy to have any feedback on its robustness and usefulness.

The `makeData` command file automates the copying and moving of files. In a first step, it will create a folder with the name of the aggregation code, `[basename]` if it does not exist. It will also create the four sub-folders: `Agg`, `Flt`, `Alt` and `Fnl`. The aggregation routine creates 10 GDX files (and optionally the CSV file). The filter routine only modifies the five data files: `[basename]Dat.gdx`, `[basename]Vole.gdx`, `[basename]Emiss.gdx`, `[basename]NCO2.gdx` and `[basename]Wages.gdx`. The `makeData` command file copies the other (non-modified) files from the `Agg` folder to the `Flt` folder. Similarly for Altertax, the command file will copy the non-modified files from the `Flt` folder to the `Alt` folder. If the respective routines are not invoked, the command file will copy ALL files from one folder to the next. In a final step, all the GDX files in the `Alt` folder will be copied to the `Fnl` folder from where the user can copy the final data and parameter files to the working folder for subsequent simulations.

F.2   Model Simulations

F.2.1   Introduction

We introduce in this section how to run ENVISAGE model simulations. There are three fundamental types of simulations:

- Comparative static. Comparative static involves introducing a shock to the benchmark database with no dynamic elements—such as factor accumulation, technology and preference changes, etc. (of course a comparative static shock could include a shock to one of these elements). The key difference between the comparative static and dynamic version of the model is the specification of the capital account as described in the model description. Capital markets close with a CET transformation function in the comparative static version, which also has no vintage capital. In the dynamic version, installed capital is assumed to be only partially mobile across activities and the model explicitly incorporates vintages.

- Recursive dynamic with baseline calibration. This version of the model is dynamic and the model is used to determine some model parameters subject to some targets. For example, GDP growth may be exogenous and labor productivity is calibrated to achieve a given growth target, and/or the investment to GDP ratio is targeted and the household savings rate is adjusted to meet the target, etc.

- Recursive dynamic shock scenario with pre-calibrated dynamic parameters. This type of dynamic scenario will use the results from the baseline calibration scenario for some of the underlying dynamic trends, for example labor productivity. In the absence of a shock, this type of simulation should re-produce the baseline scenario.

There is no single best way to run model simulations—in either comparative static or dynamic mode as GAMS provides great flexibility. We introduce herein a unified system that can work for all three types of simulations. This system is useful because it highlights the differences across the simulation types. It does have some potential drawbacks that will be highlighted below.

F.2.2 Preliminaries

The model code is composed of a series of GAMS file, this helps with modularizing the code. Typically, all of the core model code will be available in a single folder and the user will set the `idir` option in GAMS to the location of the core model code, say for example in the *Model* directory.

Table F.6 is a list of the files that constitute the core model code. The model's declarations and equation specification are contained in the `model.gms` file.

Table F.6: **Distributed model files**

File name	Description
cal.gms	Calibration of model parameters
closure.gms	Default closure assumptions
compScen.gms	'Scenario' file for comparative static simulatons
getData.gms	File that reads the GDX-based input files.
init.gms	Initialization of model variables
initScen.gms	Scenario file for dynamic simulations
InitVar.gms	Inter-period initialization of model variables
initVint.gms	Second period initialization of vintage volume variables
iterloop.gms	Inter-period code
miscDat.gms	Currently this file contains only an energy conversion table. It is used post-simulation.
model.gms	Core model specification
postsim.gms	Post-simulation statements—mostly creation of output CSV files
recal.gms	Inter-period core code to update vintage technology parameters and potentially other dynamically calibrated parameters such as the Armington preference parameters
recalnnn.gms	Code that updates technology parameters such as `and1`
recalnnt.gms	Code that updates technology parameters such as `aland`
recalnrg.gms	Code that updates technology parameters such as `aNRG`
recalvat.gms	Code that updates technology parameters such as `axp`
recalvnn.gms	Code that updates technology parameters such as `ava`
recalvnt.gms	Code that updates technology parameters such as `ak`
recalxanrg.gms	Code that updates technology parameters such as `aeio` with no energy nesting
recalxanrgn.gms	Code that updates technology parameters such as `aeio` with energy nesting
sam.gms	Code that writes out the simulation SAMs in CSV format
SaveParm.gms	Code that writes out key parameters
scale.gms	Code that scales model variables/equations. Currently not used.
solve.gms	Code that invokes the solver.

Figure 23 provides a schematic view of a simulation. In the current setup, the user prepares a project specific file with the name `[basename]Opt.gms`. This file contains the common options and statements across all simulations in the project. It is designed to handle both comparative static and dynamic simulations. Many of the global options can be over-ridden in subsequent simulation files (for example `runSim.gms`), however, it is normally not possible to override set definitions such as time. The first step in a simulation file, such as `runSim.gms` is to read the options file. The options file will also load a number of the core GAMS code and some user files. The 'Sets' file, which is user-based, is generated by the aggregation facility and normally requires no further input. The parameter file, described below does need user input for each project. The core code that is read includes the model specification (`model.gms`), reading of GDX-based input data files (`getData.gms`)[33], initialization of model variables (`init.gms`), model calibration (`cal.gms`) and the default closure rules (`closure.gms`). After defining the default closure the model is ready to be solved.

---

[33] [1-Dec-2018] Reading of the base data files was separated from the model initialization routine. This allows the user to make direct changes to the database before variable initialization and parameter calibration.

The model is solved over time—even in the case of comparative static simulations—albeit skipping the first period, which will contain the initialized and calibrated solution using the base data. A number of statements are executed at the beginning of each period that initializes variables, may recalibrate technology parameter (to be described), updates time-based shocks in the case of dynamic scenarios and introduces shocks for both comparative static and dynamic simulations. Once all of the period updating is finished, the solver is invoked. The simulation file will loop over all time periods—as long as the model converges. After looping over all periods, model results will be saved—optionally in a CSV file and always in GDX format.

In a typical application, the user will create a project directory with the base name of the project. The simulation files will be contained in that directory. The distribution comes with several files that provide examples of how to run both comparative static and dynamic simulations. Beyond the aggregated data files, the user needs to prepare a file with the model parameters. This file can be fairly generic if based on an aggregation of one of the existing parameter files. However, there a number of options that users need to define to complete the model specification—such as the labor market closure.

The user-based parameter file

While most of the inputs to the model are prepared by the aggregation facility, it is up to the user to prepare the file with project-specific model parameters. The file has the name `[basename]Prm.gms`. There is a generic parameter file that simply initializes all model parameters to those generated by the aggregation facility. These can be overridden—after the ones from the aggregation facility are loaded, but before the initialization of the model parameters. The read-in parameters will have a suffix of '0'. The model parameters have no suffix.

The aggregated parameters reflect values used by the OECD's Environment Directorate Env-Linkages Model (Chateau et al. (2014)). The `Envisage` Model has some new features not fully reflected in the original Env-Linkages Model, such as the power and water module. The file does include some initial levels for the power elasticities, but [for the moment] does not include the elasticities for the water module. These must be entered by the user if the water module is active. Another critical set of elasticities is the natural resource supply elasticities. [NEW] It is the user's responsibility to enter these elasticities (for the original model aggregation). The user must enter a pair of elasticities for each natural resource and for all regions. The parameter to be initialized is `etanrfx0` that takes three indices: region, activity and a special one that takes the values of 'lo' and 'hi'. This latter is represented by the set `lh`. These refer to the supply elasticity to apply as a function of market conditions. The 'lo' supply elasticity is to be used for a market under contraction. The 'up' supply elasticity is to be used for an expanding market. Intuition suggests that the 'lo' elasticity will be greater than the 'hi' elasticity, i.e. it is easier to contract supply than to expand it. These should be carefully scrutinized and potentially adjusted during baseline simulation runs.

[NEW] The user must enter the investment allocation elasticities to be used with the flexible capital flow specification. The initial elasticities are to be initialized with the parameter `epsRor0`. The default value for the GTAP model is 10.

[NEW] There are a number of parameters to be entered by the user for the USAGE-inspired capital account closure. The user must enter the bounds on the growth of capital in any given year. These are entered in the parameters 'grKMin0' and 'grKMax0'. Normal trend growth is entered in the parameter 'grKTrend0'. The curvature of the logistic function at the equilibrium point of the capital supply schedule is entered in the parameter 'chigrK0'. The user must also enter the so-called normal rate of return, from which the deviations are evaluated. This is entered in the parameter 'RoRn0'.

The comparative static version of the model uses a weighted average of the vintage-dependent elasticities. Users are free to change the weights, where the default values are 0.8 for *Old* and 0.2 for *New*.

There are a number of other key assumptions that are included in the parameter file. The first relates to the labor market assumptions. The user must initialize the matrix of options in the parameter `labHyp`. It has a regional and skill index and 11 attributes described below.

1. `omegam`. This determines labor market segmentation. A finite value will implement labor market segmentation—typically defined over agricultural versus non-agricultural activities. Perfect labor mobility is implemented if the value is infinity (`INF`). If it is finite, this will be the value of the labor mobility elasticity in equation (G-1), ω^m, the elasticity of rural to urban migration with respect to the expected urban wage premium.

2. `migr0`. This determines the initial level of migration as a percent of the rural labor force. For example, if it is set to 1.0, the initial level of rural to urban migration will be set to 1% of the rural labor force. It is ignored if the migration elasticity is infinity.

3. `uezRur0`, `uezUrb0`. These parameters provide the initial unemployment rate (in percent) in respectively the rural and urban labor markets. In the case of full labor mobility, only the urban unemployment rate is active and the rural rate is ignored.

Figure 23: **Simulation flow chart**
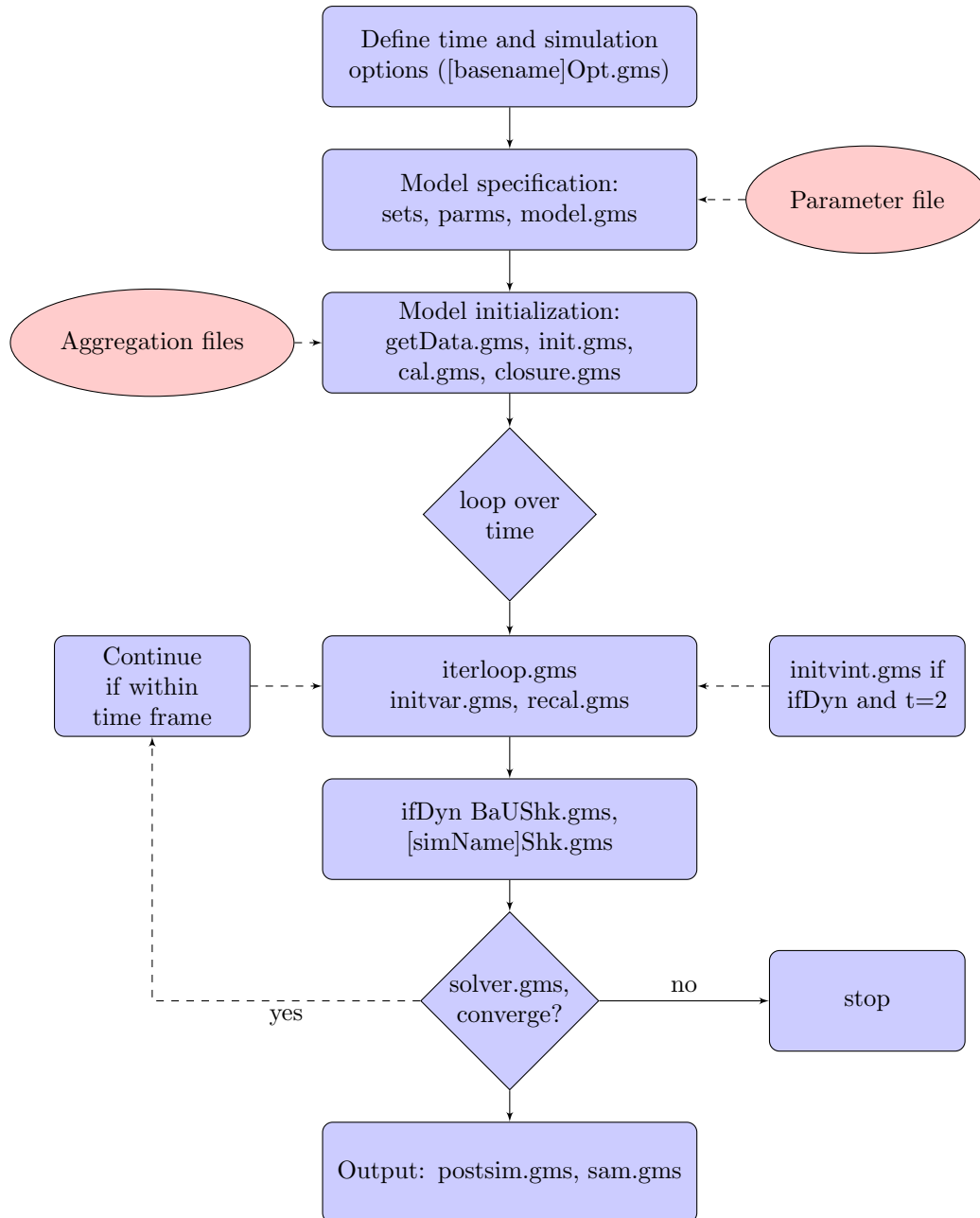
4. `uezminzRur0`, `uezminzUrb0`. These parameters provide the lower bound for the unemployment rate (in percent) in respectively the rural and urban labor markets. The rural rate is ignored in the case of full labor mobility. An error will be issued if the initial unemployment rate is less than the minimum unemployment rate.

5. `resWageRur0`, `resWageUrb0`. These parameters determine the initial level of the reservation wage with respect to the prevailing wage. A value of 1 indicates that the reservation wage is binding and the the unemployment rate is greater than (or equal to) the minimum unemployment rate. A value less than one indicates the distance between the initial equilibrium wage and the reservation wage. The rural reservation wage is ignored in the case of full labor mobility. A value of `NA` indicates full employment (at all times). An error will be issued if the value is greater than 1.

6. `omegarwg omegarwue omegarwp`. The reservation wage is a function of three indicators—the growth of per capita GDP, the unemployment rate and the CPI. The elasticity is positive for income growth and the CPI. The reservation wage is negatively related to the unemployment rate, i.e. an increase in unemployment would tend to dampen the reservation wage. To avoid problems with division by zero, the relationship between the reservation wage and unemployment is converted to a relationship between the reservation wage and the employment rate, i.e. $(1 - UE)$. Thus if the elasticity of the reservation wage is ω^{ue}, the elasticity of the reservation wage with respect to the employment rate is $-\omega^{ue}(1 - UE)/UE$. The user is expected to enter the elasticity with respect to the employment rate and it should be a positive number.

There are three additional parameters that allow for control of the dynamics of the capital market. The first, `cap_out_Ratio0`, allows for overriding the initial capital to GDP ratio. The initial capital stock is an estimate derived from the Penn World Tables.[34] For some countries and/or regional aggregations the estimates can lead to significant jumping off problems in the initial years. A value of `NA` will ignore the override. The parameter is entered as a multiplicative adjustment of the initial capital stock. The default value is 1.0. The second is the depreciation rate, `deprT`, which for some rapidly growing countries is too low. The default GTAP value is 4%. The third is an investment target, `invTarget0`. Users can input this for particular years, e.g. 2030 and 2050. Code in the 'Opt' file will translate these targets into linear changes for the investment target, for example between 2011 and 2030 and then between 2030 and 2050, in order to smooth the path between target years.

The final section allows the user to implement 'twists' to the Armington preference parameters. There are three sets of twist parameters and one twist-related regional mapping set. The top-level Armington twists require two different parameters depending on the value of `ArmFlag`. In the case of national sourcing, i.e. when `ArmFlag = 0`, the relevant twist parameter is `twt1`, which is region-, commodity and time-specific. With agent-based sourcing, i.e. when `ArmFlag = 1`, the relevant twist parameter is `tw1`, that is in addition agent-specific. A positive value, for example 0.02, would lead to a change in the ratio of imports to domestic demand of 2 percent—assuming constant prices. The parameter `tw2` is applied to the second level nesting. The r index refers to the importing country. The twist can be applied to one single region, or a group of regions. The user designates the targeted exporters in the regional mapping set `rtwtgt`. The first regional index in the mapping set refers to the targeted exporters and the second regional index refers to the importer. For example, to increase the import shares of China, Japan and Korea in the U.S. using the twist, the relevant mapping is `(chn,jpn,kor).usa`. Note that in this implementation, the same twist is applied uniformly across the targeted exporters.

F.2.3 Common options

The first set of global options are entered as definitions through GAMS' `$setGlobal` statement. The following enumerates the options:

1. **wDir**. By default this is set to the active directory.

2. **SSPMOD**. This option defines the choice of the economic model used for the SSP GDP projections. There are currently two choices: `OECD` and `IIASA`.[35]

3. **SSPSCEN**. This option defines which SSP to use for the GDP projections (and will be combined with the model choice). There are five valid options: `SSP1-SSP5`.

4. **LABSCEN**. This option influences the relative growth of skilled versus unskilled labor. If it is set to one of the SSPs, the education profiles from the SSPs—with the user mappings of skilled workers to education levels (`educMap`) together with the `skLabgrwgt` parameter (see below)—will determine the growth rate of skilled workers. The growth of unskilled workers will be determined residually so that the overall growth or workers

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[34] http://www.rug.nl/ggdc/productivity/pwt/

[35] The PIK projections have not been processed since they reflect a fixed aggregation of 32 regions, whereas the OECD and IIASA projections were done for most countries.

matches the growth of the working age population. If this parameter is not equal to one of the SSPs, both skilled and unskilled workers will grow at the same rate as the working age population, i.e. no differentiation of growth by skill.

5. **POPSCEN**. This option defines which population projection to use. There are ten valid options. The IIASA SSP population projections are labeled `SSP1-SSP5`. The scenario file also includes four UN population projections: `UNMED2010`, `UNMED2012`, `UNMED2015` and `UNMED2017`. Finally, the World Bank's GIDD projections are available with the label `GIDD`. The latter is intended to duplicate UNMED2015. It is only available through 2050.

6. **OVERLAYPOP**. This option allows to replace GTAP's base year population level with that from the scenario database. In any case, only the growth rates from the population projections are used from the initial base year level.[36]

7. **TASS**. This option determines the specification for the aggregate land supply curve. Valid options are: `KELAS` for iso-elastic supply, `LOGIST` for logistic supply curve, `HYPERB` for hyperbola supply curve, and `INFTY` for infinitely elastic (i.e. horizontal) supply curve.

8. **WASS**. This option determines the specification for the aggregate water supply curve. It has the same valid options as the aggregate land supply curve.

9. **utility**. This option determines the specification of the household utility function. Five options are available: `CD` for the Cobb-Douglas utility function, `LES` for the linear expenditure system, `ELES` for the extended linear expenditure system, `AIDADS` for the AIDADS demand system, and `CDE` for the Constant-difference-in-elasticity demand system. The `CD`, `LES` and `AIDADS` are new options. The LES is calibrated to the Frisch parameter, which is currently parameterized in the calibration routine (`cal.gms`), but it would be preferable to have this entered as user input. AIDADS is coded, but there is no calibration. It has been tested assuming the LES as one of the special cases of AIDADS. The Cobb-Douglas simply uses the base year budget shares to calibrate the marginal budget shares and the subsistence minima are set to zero.

10. **NRITER**. This option controls the number of iterations when running the model convergence in single country mode. As this has not been tested with the latest version of the model, it is best to set this option to 0.

11. **savfFlag**. This option controls the capital account closure. Three options are currently available. A value of `capFix` uses the fixed capital account closure rule. A value of `capFlexGTAP` uses the flexible capital account closure rule of the GTAP model. This closure allocates global savings in order to equalize expected returns across regions. A value of `capRFix` fixes the ratio of savings relative to nominal GDP. A value of `capFlexUSAGE` uses the flexible capital account closure rule inspired by the USAGE model. This closure allocates global savings as a function in deviations of the regional rate of return from the 'normal' rate of return.

12. **intRate**. The user sets this option to the desired value for the global real interest rate. The default value is 5% (i.e. 0.05). Note that this parameter is currently only used by the USAGE capital account closure.

13. **costCurve**. Cost curves are used to implement downward trends of new technologies (e.g. renewable electricity). These can be implemented using either a hyperbolic (`HYPERB`) or with a logistic function (`LOGIST`).

14. **NTM_MODULE**. This flag takes the value of `ON` to implement the non-tariff measure module, or `OFF` otherwise. It expects to read a GDX-based satellite account with the *basename* followed by NTM, e.g., `ANX1NTM.gdx`.

15. **MRIO_MODULE**. This flag takes the value of `ON` to implement the MRIO module, or `OFF` otherwise. It expects to read a GDX-based satellite account with the *basename* followed by MRIO, e.g., `ANX1MRIO.gdx`.

16. **RD_MODULE**. This flag takes the value of `ON` to implement the R & D module, or `OFF` otherwise. It expects to read a GDX-based satellite account with the *basename* followed by R_D, e.g., `ANX1R_D.gdx`. If the file does not exist, it will assume that the cost structure is identical to the cost structure for government expenditures and scale the costs to some percentage of GDP, which will be specified in the `KnowledgeData0` parameter of the user parameter file. The R & D expenses will be extracted from government expenditures.

17. **DEPL_MODULE**. This flag takes the value of `ON` to implement the resource depletion module for coal oil and gas resources, or `OFF` otherwise. It expects to read a GDX-based satellite account with the *basename* followed by DEPL, e.g., `ANX1DEPL.gdx`.

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[36] There is currently an inconsistency in the population levels in the SSP database and GTAP. In the former, population is in levels and in the latter it is millions. The inputs from the SSP database are currently scaled by a million to make them consistent with the GTAP units. The population scale factor can be used to improve the population scale on an as-need basis.

18. **CLIM_MODULE**. This flag takes the value of `ON` to implement the climate module. The module is initialized from a GAMS file, which is calibrated for a specific reference year, for example, 2014. (For the moment, there is no version available for GTAP V11 with a 2017 reference year.)

19. **DAMAGE_MODULE**. This flag takes the value of `ON` to implement the climate damage and adaptation module, or `OFF` otherwise. This is under development.

20. **FBS_MODULE**. This flag takes the value of `ON` to implement the nutrition module, or `OFF` otherwise. It expects to read a GDX-based satellite account with the *basename* followed by `FBS`, e.g., `ANX1FBS.gdx`.

21. **ifBKSTP**. This flag takes the value of `ON` to implement backstops, or `OFF` otherwise. This is under development.

22. **ifDEPR0**. This flag takes the value of `ON` to run the model on a gross investment basis, or `OFF` to run the model on a net investment basis. When set to `ON`, the incoming `VDEP` array is set to 0.

23. **ifVINT**. This flag takes the value of 1 to run dynamic simulations under the vintage capital assumption, or 0 to run dynamic simulations with a single vintage.

The remaining global options are entered as scalars. The first set provide the context for the model: comparative static and recursive dynamic with or without dynamic calibration.

1. **ifDyn**. This option takes two values. A value of 0 indicates a comparative static model. A value of 1 indicates a dynamic model. In the standard package, this will be set automatically when the simulation is invoked and the user sets the `simType` option. Valid values are `CompStat` and `RcvDyn`.

2. **ifCal**. This option takes two values and is only meaningful for dynamic scenarios. A value of 0 runs a dynamic scenario with pre-calibrated trends (generated by a baseline). A value of 1 runs a dynamic calibration scenario where specific dynamic trends are calibrated to exogenous indicators such as GDP. This value will be set in dynamic variables by setting the `ifCal` option when invoking the simulation.

3. **ifVint**. This options takes the value 0 or 1. It is set automatically when the simulation is invoked by the user. A value of 1 implements vintage capital, which is valid only for dynamic simulations. It should be set to 0 for comparative static simulations.

The remaining global options are typically invariant across simulations. The following list enumerates the options.

1. **inScale**. Scale factor for input data. The input SAM is typically in millions. A scale of 10^{-6} has been found to be a useful scaling factor.

2. **outScale**. Scale factor for output data. This is typically just the inverse of `inScale`.

3. **popScale**. Scale factor for population. Often set to 10^{-6}, though need to check on the population level inconsistencies.

4. **lScale**. Scale factor for labor volumes. [Need to verify scale in 'wages' database.]

5. **eScale**. Scale factor for energy. The energy volumes are in million tons of oil equivalent (MTOE). A scale factor of 0.001 is typically used.

6. **watScale**. Scale factor for water, typically 10^{-12} is used. [Check units of volume database.]

7. **cScale**. Scale factor for emissions, typically 0.001 is used. [Check units for emissions.]

8. **ifCEQ**. Convert emissions to CEq. Input emissions are in CO_2eq. If this flag is set to 1, the units will be converted to Ceq. The climate module [tbd] is based on emissions in Ceq.

9. **ArmFlag**. Set to 1 for agent-based Armington. If the flag is set to 0, the top level Armington sourcing is done at the aggregate level. [This is a new option.]

10. **MRIO**. Set to 1 for agent-based second-level Armington, i.e. to use the MRIO version of the model. [This is a new option.]

11. **ifNRG**. Set to 1 to use energy volumes. This will provide a volume/price split for energy commodities.

12. **ifNRGNest**. Set to 1 for energy nesting. A value of 0 will have only a single nested energy nest.

13. **ifMCP**. Set to 1 for MCP. A value of 0 will use NLP. The objective function is Walras' Law.

14. **ifLandCET**. Set to 1 to use CET for land allocation. A value of 0 will use the additive CET specification.

15. **ifSUB**. Set to 1 to substitute out equations. A value of 0 will have the full model specification without substitution. This will considerably increase the size of the model.

16. **IFPOWER**. Set to 1 for power module, which requires the power database, else set to 0.

17. **IFWATER**. Set to 1 for water module, which requires the water database, else set to 0

18. **ifAggTrade**. Set to 1 to aggregate trade in SAM. A value of 0 will have the full bilateral trade matrices output as part of the SAM.

19. **skLabgrwgt**. Set to between 0 and 1. This determines the growth assumptions for skilled and unskilled labor. The growth of skilled labor is driven by the growth of the appropriate education categories. The growth of unskilled labor will then be calculated by residual since the growth of overall labor will be determined by the growth of the working age population (15-64). One can use this parameter to modulate this calculation. If the weight is set to 1, then the growth rate of skilled labor will match exactly the growth of the corresponding education categories, ignoring any resulting impact on the growth of unskilled labor. At the other extreme, if the weight is set to 0, the growth of both skilled and unskilled will be identical and equal to the growth rate of total labor, i.e. the education profiles are fully ignored. Users can choose an intermediate value between 0 and 1.

20. **EXR**. This is an exchange rate, which defaults to 0. It can be useful to present results in a different currency, e.g., euro, yen, rands, etc., when working on a study for a particular country or region.

F.2.4 Comparative static simulations

In the current setup, there is very little difference between running comparative static and simulations dynamic simulations. Both the `runsim.gms` and `[basename]Opt.gms` files are coded to run both types of simulations. One of the key differences between the two is the time framework. The standard comparative static simulation has three 'time' periods labeled `base`, `check` and `shock`. The user is free to modify these in the 'Opt' file. One of the primary purposes of the comparative static version of the model is to test model initialization, calibration and homogeneity. It is also extremely useful to test new model specifications or parameterizations. With a new model version—for example a new aggregation, new specification or parameterization, it is always recommended practice to do a full diagnostic check.

A standard diagnostic check is to run the comparative static model with a single homogeneity shock in the `shock` time period. The `base` year is never run, it is meant to be able to replicate the initial database. The `check` simulation replicates `base` if the initialization and calibration are working correctly. For this reason, it is important to look at the largest residual in the listing file (or console). Open the listing file and do a search for `LOOPS`. This should take you to the diagnostics for the first simulation, i.e. the `check` simulation. Model diagnostics will appear by scrolling down—for example model size. The key diagnostic is `INITIAL POINT STATISTICS`. The maximum of `F` should be 0 or near zero. The largest error should reflect more or less the precision of the input SAM. One may also want to observe the initial Jacobian to see the range of the minimum and maximum elements. The range should be relatively narrow. A good maximum should be no larger than `1.0e003`. A larger number is an indication of a scaling issue.

A second check of the model is to look at the values for the `LHS` in the list code, which stands for the left-hand side. This is controlled by the `LIMROW` option. It may default to either 0 or 3. In the case of 0, no equations will be listed. In the case of 3, up to 3 in any block will be listed. Setting `LIMROW` to a higher number will increase the number of equations printed in a block, for example: `Options limrow=100 ;`. The equations listing shows the value of the residual of an equation, i.e. it collects all endogenous variables on the left-hand side and evaluates the expression and it collects all additive exogenous variables and parameters on the right-hand side and evaluates the right-hand side expression. It then displays the value of the left-hand side that should match the displayed right-hand side. In most cases, the right-hand side will evaluate to 0, but it does not have to.[37] If there is a serious initialization/calibration problem, it may be necessary to increase `limrow` and visually go through the equation listings. For a quick inspection, do a search for `LHS` to find the first equation in the listing file. Then search for `****`. The four asterisks indicate an 'infeasible' solution. Note that in most cases, the infeasibility is likely to be a very small number—and typically reflects the accounting precision of the incoming SAM. To avoid nuisance infeasibility, one can increase the tolerance level by using the `tolinfrep` attribute of a model. In the default settings, the infeasibility tolerance has been set to `1.0e-005`, and with a well-balanced SAM, this will generally avoid any listed infeasibilities.

The homogeneity test requires a shock file. The shock file should have the same name as the simulation. Thus if the simulation is assigned the name `COMP`, the user should create the a file named `COMPShk.gms`. Listing F.25 illustrates one way of implementing the homogeneity shock. It tests the value of the time period, and then increases the value of the exogenous numéraire by 50%.

Listing F.25: **Homogeneity shock**

```
2   if(sameas(tsim,"shock"),
```

---

37   For example the labor equilibrium condition will have the exogenous supply of labor on the right-hand side.

```
3     pnum.fx(tsim) = 1.5*pnum.l(tsim) ;
4   ) ;
```

It is useful to do a quick check of the diagnostics by loading the output CSV file into Excel—and most convenient in Excel's pivot tables. The first thing to look at is the resulting SAM's. All accounts should be perfectly balanced for each individual region and for each time period. The second diagnostic is to compare the `base` values with the `check` values. They should be identical to within vary narrow margins. A third check is to compare the `check` values with the `shock` values. All volume variables should be identical and all value variables and prices should have increased by the same percentage amount as the numéraire, for example 50%.

These are minimal diagnostics, but the user may want to do others—particularly if there is a change in specification. These would include shocks to taxes, for example tariffs, shock to factor stocks, productivity shocks, etc.

The distribution comes with an option file for the '10x10' aggregation, as well as the generic `runsim.gms` file. The latter is set to handle many different situations but users may find it useful and/or necessary to make a copy of it and insert modifications. The distribution also comes with a command file that is needed to run the file in a Windows console. To run the homogeneity test, the command line could be the following:

```
runsim Homog Comp CompStat 0
```

The first argument will be the simulation name. In comparative static simulations the second argument is ignored, but must be present. In dynamic simulations it is the name of the baseline file. The third argument must be `CompStat` for comparative static simulations. The fourth argument is ignored for comparative static simulations, it is used for dynamic simulations. It is best to set it to zero.

Users will need to modify the `runsim` command file for their installations. An example is depicted below.

```
gams runSim --simName=%1 --BauName=%2 --simType=%3 --ifCal=%4 --baseName=10x10
    --odir=v:\Output\EnvLink\10x10 -idir=..\model5n
    -scrdir=v:\Output\EnvLink\10x10 -ps=9999 -pw=150
```

The `runSim` command file expects at least the four arguments as described above. It is setup for a specific project. In this case it is setup for a project named '10x10'. The user needs to specify a folder for the output directory. Use '.' to specify the current directory. It is also required to specify the model directory. In the example above, the model directory is at the same level as the simulation directory and named `model5n`. The remaining parameters are optional and the user is free to add others.[38]

[NEW 01-Dec-2018] The code in the 'Opt' file has been modified somewhat to improve the handling of starting point for the model simulation. The old code allowed to start a simulation by reading in a previous baseline using the `execute_loadpoint` option of GAMS. This is handy if you are running shock simulations starting in a future year relative to the reference year. The new code allows the user to specify any simulation as a starting point. The user can identify the starting simulation on the command line with the `startName` option. The Windows command file `runsimR.cmd` shows an example of how to use an existing comparative static simulation as a starting point. If there is no additional shock, the model should solve immediately.

F.2.5   Dynamic simulations

Dynamic simulations are in principle only somewhat more complicated than comparative static simulations—though in practice raise considerably more problems. There are two key differences. The first is the time dimension. This is under the discretion of the user. For GTAP V9, the starting year should be set at 2011. The terminal year and the intermediate years are at full discretion. The model is setup to handle year step sizes of more than 1 [though this probably needs additional testing since it hasn't been used in a while]. One potential problem with multi-year time steps could be convergence, though this has proven to be less of a problem further out when the model has reached some sort of steady state. The second key difference is the implementation of the vintage capital formulation. This in principle should be transparent to the user.

---

[38]   Simulation output can be voluminous depending on the aggregation and the time span of a simulation. With increasing use of backups and the cloud, it is sometimes useful to store output on non-critical storage in order to minimize bandwidth problems and exceeding allowed storage capacity.

In principle there is nothing to change in the 'Opt' file nor in the `runsim.gms` file. There are default statements that are implemented in the file `iterloop.gms` that generate the necessary closures for dynamic simulations—either the baseline, or pre-calibrated scenarios. Users have some control in shaping the baseline by modifying or adding statements in the `BaUShk.gms` file. In the distributed file, there are two adjustments to the standard baseline. The first can phase out net capital flows between two years. The second targets the investment to GDP ratio for some given future year—both the target and the year are provided by the user. It endogenizes the savings rate in order to achieve the desired investment ratio.

Other key dynamic assumptions are embedded in the `initScen.gms` file that is in the *Model* directory. In particular the default assumptions on labor productivity wedges (`glAddShft` and `glMltShft`), autonomous yield growth (`yexo`), autonomous energy efficiency improvement (`aeei` and `aeec`) and improvements in trade margins (`tteff`) are included in this file.[We may revisit this looking ahead as it is not satisfactory to have these assumptions included in the core model code.]

The mechanics of the baseline are fairly straightforward—shaping the baseline is not always straightforward. One of the key issues to arise is the growth of the capital stock. Base year conditions may lead to extreme jumping off conditions. For example over-investment may lead to sharply declining rates of return, and under-investment the reverse. The initial growth of the capital stock will depend on base year investment (and savings) rates, net capital flows, the initial stock of capital and the depreciation rate. On balance, one would like to have as a starting point for the baseline relatively steady returns to capital, unless there is a good reason to expect rates of return to rise or drop. It may involve adjusting the initial capital stock and rates of depreciation. It is not possible to change the initial savings ate including the capital account, but these can be adjusted over time to yield a desired path for capital accumulation and rates of return. This is one of the purposes for using a long-run target for the investment to GDP.

To run the baseline simulation, the command line could be the following:

```
runsim BaU BaU RcvDyn 1
```

The first argument and second arguments are the name of the baseline. In the case of running the baseline, the second argument is redundant, but necessary. The third argument is to invoke a dynamic-type scenario. The fourth argument invokes the code for dynamic calibration. On occasion, the model will fail to converge to the terminal year and typically this is due to factor prices going to some extreme—notably zero for capital in the case of investment exceeding demand for capital. One may have to restrict the simulation to an intermediate year and assess intermediate results to understand the nature of the problem. This can be done by restricting the time loop, for example `loop(tsim$(years(tsim) le 2020)`.

Once the user has a satisfactory baseline, dynamic shock simulations can be implemented. They are associated with a specific shock file with a base name linked to the simulation name, `[simName]Shk.gms`, for example `[SSP2ccShk.gms]` to run the SSP2 baseline with climate change impacts. To run a pre-calibrated dynamic scenario, the command line could be the following:

```
runsim SSP2CC BaU RcvDyn 0
```

The first argument is the name of the simulation, and `runSim.gms` will try and open and implement a shock file with a filename that incorporates the simulation name. The second is the name of the baseline file. At a minimum this is necessary because the shock simulation needs to extract from the baseline calibrated information from the baseline—notably the labor productivity parameter and real government expenditures. Both are assumed to be exogenous in the shock simulations.[39]

[NEW 01-Dec-2018] The code in the 'Opt' file has been modified to allow for more flexibility in terms of starting from an existing simulation. The old code only allowed for starting from the baseline simulation. The new code allows for starting from any simulation (including the baseline). An existing simulation is not always the best starting point for solving for a given year, particularly for a future year. The code allows the user to provide a year until which the simulation starts from an existing simulation or the solution from a previous period. To test the baseline simulation, for example, one could re-run the baseline with no shock but using the calibrated variables from the baseline, for example:

---

[39]   Users can extract other information from the baseline on an as-need basis.

```
runsimrd noShk BaU RcvDyn 0 BaU 2030
```

where the command file `runsimrd.cmd` is used to invoke GAMS. This says to run a calibrated recursive dynamic scenario using the calibrated variables from the baseline scenario called BaU. In addition, use as the starting point the BaU scenario through the year 2030. If the dynamics is set up correctly, the model should solve within 1 iteration in each period. In effect, the *noShock* scenario is similar in concept to the *Check* scenario for comparative statics. The rule of thumb is that you can use the baseline till the first year of a shock. In most cases, initializing with the previous period's result is better once the shock has been implemented as the shock tends to lead to sharp deviations from the baseline. For example, if a shock starts in the year 2020, use the baseline values through 2020 as a starting point, then for subsequent years, the model with initialize values with the solution from the previous period—not the baseline simulation. The Windows command file `runAll.cmd` contains the batch commands to run the baseline and the no-shock scenario in sequence. Note that it will make a copy of the 'runSim.gms' file for each of the simulations and thus the list files will have separate names.

F.3   Post-simulation processing

There is some rudimentary post-simulation processing that is still under development. It involves two steps. The first step is a GAMS file that reads in a solution and produces a number of model-based indicators and stores them in different CSV cubes. The CSV cubes can hold results from one or more simulations. These can be loaded into Excel files, preferably into pivot tables, and/or alternatively they can be loaded into statistical or graphical packages such as R for automated generation of figures.

F.3.1   User inputs

The user typically prepares two files—a Windows command file, which facilitates automating the processing of the GDX file(s), and a user-based option file that is specific to the simulations. The user-based option file is assumed to have the name 'baseNameTab.gms', where 'baseNameTab' is replaced with the base name for the set of simulations, for example '10x10'. Listing F.26 provides an example of a 'Tab' file. There are a number of global options that first need to be initialized.

1. **xclDir**. This sets the directory for holding the Excel files. N.B. For the moment it is best to use the full path name as testing with relative paths has created errors. For example do not use `".\Doc\"` or `"Doc\"`. This seems to be an issue with the use of the VB scripts under Windows.

2. **inDir**. This sets the directory for the CSV files. Note that this will be initialized from the command line parameter '%oDir%'.

3. **wDir**. This sets the working directory. It normally defaults to the directory that was used to invoke the 'makCSV' program.

4. **simTgt**. The following set of options initialize the fields for the pivot tables. `simTgt` should be set to the code for a valid simulation or scenario, such as `BaU`.

5. **regTgt**. `regTgt` should be set to the code for a valid region in the model. Refer to the 'Sets' file to see the valid regions for a specific aggregation.

6. **timeTgt**. `timeTgt` should be set to a valid reporting year. Typically this will be the base, or reference, year.

7. **actTgt**. `actTgt` should be set to a valid activity, which can also be referenced in the 'Sets' file.

The user next provides the reporting years. The set `tr` is a subset of the full simulation time framework, which can include all solution years. The time framework also differs between comparative static and dynamic scenarios. The next option also permits sub-setting the number of activities reported. It defaults to all activities and activity aggregations. The user also needs to specify a conversion factor that is used to convert output from 'primary' electricity activities. 'Primary' electric activities are those that do not rely on energy inputs such as coal or gas. The latter have a conversion loss typically assumed to be a factor of 3, i.e. it takes 3 MTOE of coal or gas to produce 1 MTOE of electricity.[40] This factor is only valid for the power-enabled version of the model.

The final set of options is for the cubes (i.e. tables) that will be extracted from the GDX files. The set `tables` contains the full list of tables that are available in the `makTab.gms` file. The user determines which of the tables to produce by defining the table subset `ifTab`.

---

[40]   See https://unstats.un.org/unsd/envaccounting/londongroup/meeting13/LG13_12a.pdf.

Listing F.26: 'Tab' file example for creating CSV cubes

```
 2   * ------------------------------------------------------------------------------
 3   *
 4   *     Options for creating CSV cubes
 5   *
 6   *     Users typically select years to output and CSV cubes to create
 7   *
 8   * ------------------------------------------------------------------------------

10   *  Set folder for Excel files

12   $setGlobal xclDir  "v:\env10\10x10\doc\"

14   *  Options for CreatePivot file

16   $setGlobal indir      %oDir%
17   $setGlobal wdir       %system.fp%
18   $setGlobal modDir     "..\Model"

20   *  AgMIP cube?
21   $setGlobal AgMIP No

23   $setGlobal DEPLFlag   ON
24   $setGlobal simTgt     BaU
25   $setGlobal regTgt     EastAsia
26   $setGlobal timeTgt    2011
27   $setGlobal actTgt     Agriculture-a

29   *  Select report years

31   $iftheni "%simType%" == "compStat"

33      set tr(t)   "Reporting years" / base, check, shock / ;
34      set trb(t) "Bilateral trade years" / check, shock / ;
35      set trm(t) "MRIO years" / check, shock / ;

37   $elseifi "%simType%" == "RcvDyn"

39      set tr(t)   "Reporting years" / 2014, 2017, 2020, 2025, 2030 / ;
40      set trb(t) "Bilateral trade years" / 2014, 2030 / ;
41      set trm(t) "MRIO years" / 2014, 2030 / ;

43   $endif

45   *  Select reporting activities (a subset of aga -- activities + aggregate activities

47   set aggaga(aga) "Activities to report" ;

49   *  Report all

51   aggaga(aga) = yes ;

53   *  Select MRIO reporting commodities

55   set mrioc(r,i) "Commodities to report" ;
56   mrioc(r,i) = no ;

58   scalar elyPrmNrgConv "Primary electric conversion factor" / 3 / ;

60   set cpiLab / CPIFUD, CPINFD, CPITOT / ;
61   set mapCPILab(CPINDX,CPILAB) / TOT.cpiFUD, TOT.CPINFD, TOT.CPITOT / ;

63   set fpagg / nsk, skl, cap, nrs, lnd / ;
64   set mapfp(fpagg,fp) /
65      nsk.nsk
66      skl.skl
```

```
67      cap.cap
68      nrs.nrs
69      lnd.lnd
70   / ;

72   *  Pivot tables to create

74   *  List of tables

76   set tables /
77      gdppop       "Macro data"
78      factp        "Factor prices"
79      kappah       "Household direct tax rate"
80      rgovshr      "Government expenditures"
81      savinv       "Savings investment balance"
82      xp           "Output by activity"
83      va           "Value added by activity and factor"
84      inv          "Investment"
85      emi          "Emissions"
86      cost         "Production costs"
87      ydecomp      "Growth decomposition"
88      trade        "Trade by sector"
89      fdem         "Final demand"
90      bilat        "Bilateral trade"
91      lab          "Labor demand"
92      pow          "Power module"
93      sam          "SAM module"
94      MRIO         "MRIO table"
95      tot          "Terms of trade module"
96      nrg          "Energy module"
97      depl         "Depletion variables"
98      climate      "Climate module"
99      shock        "For future use"
100     demand       "Demand"
101     fbs          "Nutrition module"
102     AgMIP        "Cube for AgMIP submission"
103  / ;

105  *  Selected tables

107  set ifTab(tables) /
108     gdppop       "Macro data"
109     emi          "Emissions"
110     sam          "SAM module"
111  *  MRIO         "MRIO table"
112  *  climate      "Climate module"
113  *  factp        "Factor prices"
114  *  kappah       "Household direct tax rate"
115  *  rgovshr      "Government expenditures"
116  *  savinv       "Savings investment balance"
117  *  xp           "Output by activity"
118  *  va           "Value added by activity and factor"
119  *  inv          "Investment"
120  *  cost         "Production costs"
121  *  ydecomp      "Growth decomposition"
122  *  tot          "Terms of trade"
123  *  trade        "Trade by sector"
124  *  fdem         "Final demand"
125  *  bilat        "Bilateral trade"
126  *  lab          "Labor demand"
127     pow          "Power sector variables"
128     nrg          "Energy module"
129  *  depl         "Depletion variables"
130  *  shock        "For future use"
131     fbs          "Nutrition module"
132  *  AgMIP        "Cube for AgMIP submission"
133  / ;
```

F.3.2 Extracting the indicators

The user can invoke the `makCSV` GAMS program by entering the following command (or equivalent in the GAMS IDE):

```
gams makCSV --simname=BaU --BaUName=BaU --simType=RcvDyn --ifCal=1
    --ifAppend=0 --BaseName=10x10 --odir=z:\Output\Env10\10x10 -idir=..\Model
```

Many of the command line options are identical to the `runsim` command. The user specifies the name of the simulation (`simname`), the name of the baseline simulation, potentially the same (`BaUname`)[41], the simulation type (`simType`), and the dynamic calibration flag (`ifCal`). One of the key command line arguments is `ifAppend`. If `ifAppend` is set to 0, all CSV cubes will be created anew—erasing any existing CSV cubes. If `ifAppend` is set to 1, the list of indicators being extracted will be appended to the end of an existing CSV file—thus enabling the concatenation of indicators from multiple scenarios in a single cube. If running from a Windows command file—the first invocation of `makCSV` should set the append parameter to 0, and all subsequent invocations should set the append parameter to 1. The output directory should be where the GDX files are located and will also be the location of the output CSV files.

Once the CSV cubes have been created, a new command file will also have been created the contains the commands to convert the CSV cubes into Excel files, or, if the Excel files already exist, will 'refresh' the pivot tables contained in the Excel file. This command file will only work in a Windows-based terminal window as the commands rely on VB scripts. To invoke the command, simply type 'baseNamePivot', for example '10x10Pivot'. The command file will have been created by the 'makCSV' program. It will only invoke the appropriate commands for the requested tables. Sample 'runtab.cmd' files will automatically invoke the command.

The whole system relies on four GAMS files and one 'VBS' file that are typically located in the 'Model' directory. The five files are:

1. **makCSV**. `makCSV.gms` is the master GAMS file that reads the options, invokes the indicator extraction file (makTab) and creates the 'baseNamePivotPivot.cmd' file.

2. **makTab**. `makTAB.gms` is the core GAMS file that will extract the data from GDX files, create indicators, and save them to the CSV files.

3. **setupPivot**. `setupPivot.gms` is invoked by the `makCSV.gms` to create the command lines for the 'baseNamePivot.cmd' file. It is a small file invoked using 'batinclude' with arguments that simplifies the creation of the command file.

4. **CreatePivot**. `createPivot.gms` writes the VB script that creates the Excel files with the built-in pivot tables. It is only invoked for Excel files that are being created for the first time. Otherwise, the 'refresh' VB script is invoked.

5. **refresh**. `refresh.vbs` is a VB script that refreshes a pivot table in an Excel file (without opening Excel).

F.3.3 Standard indicators

The GAMS file 'makCSV.gms' is the file that contains code to create indicators from simulation results. It extracts selected model results from a simulation (stored as a GDX container), creates an indicator (potentially without transformation) and in most cases will also aggregate across user-defined regions and sectors. Classes of indicators are stored together in CSV cubes. There are several advantages to having indicators in separate cubes. First, it keeps the sizes of the CSV files to a reasonable size; making them easier and quicker to load. Second, the geometry of the cube can be tailored to specific indicators. For example, macroeconomic indicators typically do not need sector or other qualifiers. Table F.7 describes the current list of CSV cubes.

The following sections describe each of the indicators. The left-hand side should be thought of as being indexed by aggregate region (R), time and simulation. Normally each modeled region is mapped to its own aggregate region, but could also be mapped to a true aggregate region that would be composed of 1 or more modeled regions. The right-hand side is normally composed of a model indicator. There is often a one-to-one mapping, i.e. the aggregation is just a simple sum of a model indicator.

---

[41]   The baseline is on occasion used to calculate indicators relative to the baseline.

The GDPPop cube

The formulas below calculate the indicators contained in the `GDPPop` cube. The formulas need little explanation. GDP at 2005 PPP exchange rates uses the base year PPP exchange rate derived from the SSP scenario file. The first two population indicators, `P1564` and `PopT`, refer to the exogenous information that is used as input for a model simulation. The indicator `Pop` refers to the population that emerges from the model. This could differ from `PopT` depending on the initial level of the population—though the growth rates should be identical.

$$RGDPMP_R = \sum_{r \in R} RGDPMP_r$$

$$RGDPMPPPP_R = \sum_{r \in R} RGDPMP_r PPP = \sum_{r \in R} RGDPMP_r \frac{GDPScen_{Mod,SSP,GDPPPP05,r,t0}}{GDPScen_{Mod,SSP,GDP,r,t0}}$$

$$GDPMP_R = \sum_{r \in R} GDPMP_r$$

$$PGDPMP_R = \sum_{r \in R} GDPMP_r \Big/ \sum_{r \in R} RGDPMP_r$$

$$P1564_R = \sum_{r \in R} PopT_{r,P1564}$$

$$PopT_R = \sum_{r \in R} PopT_{r,PTOTL}$$

$$Pop_R = \sum_{r \in R} Pop_r$$

$$RGDPPC_R = \sum_{r \in R} RGDPMP_r \Big/ \sum_{r \in R} Pop_r$$

The GDP cube also contains some indicators of welfare. The indicators EV, EVG and EVI represent the equivalent variation for households, government and investment expenditures, respectively. The household welfare indicator is derived from the indirect utility function for the user-chosen utility function. Government and investment expenditure functions have a simple formula for equivalent variation based on the CES preference function. Note that the model allows for multiple households, though the standard GTAP database only includes a single representative household. The remaining three indicators measure the volume of private, public and investment expenditures.

$$EV_R = \sum_{r \in R} \sum_h EV_{r,h}$$

$$EVG_R = \sum_{r \in R} YFD_{r,gov} \frac{YFD_{r,gov,t0}}{YFD_{r,gov}}$$

$$EVI_R = \sum_{r \in R} YFD_{r,inv} \frac{YFD_{r,inv,t0}}{YFD_{r,inv}}$$

$$EVT_R = EV_R + EVG_R + EVI_R$$

$$XFD_R = \sum_{r \in R} \sum_h XFD_{r,h}$$

$$XFDG_R = \sum_{r \in R} XFD_{r,gov}$$

$$XFDI_R = \sum_{r \in R} XFD_{r,inv}$$

The FactP cube

The `FactP` cube provides aggregate (or economy-wide) factor prices. The labels for capital, land and water return are hard-coded. The labels for the aggregate wages will be the same as the labels for the labor types (e.g. `nsk` and `skl`).

$$AW_{R,l} = \sum_{r \in R} \sum_{a} PF_{r,l,a} XF_{r,l,a} \bigg/ \sum_{r \in R} \sum_{a} XF_{r,l,a}$$

$$TRENT_R = \sum_{r \in R} \sum_{a} PF_{r,cap,a} XF_{r,cap,a} \bigg/ \sum_{r \in R} \sum_{a} XF_{r,cap,a}$$

$$PTLand_R = \sum_{r \in R} PTLand_r TLand_r \bigg/ \sum_{r \in R} TLand_r$$

$$PTNRS_R = \sum_{r \in R} \sum_{a} PF_{r,nrs,a} XF_{r,nrs,a} \bigg/ \sum_{r \in R} \sum_{a} XF_{r,nrs,a}$$

$$PTH2O_R = \sum_{r \in R} \sum_{a} PF_{r,wat,a} XF_{r,wat,a} \bigg/ \sum_{r \in R} \sum_{a} XF_{r,wat,a}$$

The Kappah cube

This cube contains only the average rate of transfer between households and government.[42]

$$Kappah_R = \sum_{r \in R} \kappa_r^h YH_r \bigg/ \sum_{r \in R} YH_r$$

The RGovShr cube

This cube contains only the average ratio of real government expenditure to real GDP.[43]

$$RGovShr_R = 100 \cdot \sum_{r \in R} XFD_{r,gov} \bigg/ \sum_{r \in R} RGDPMP_r$$

---

[42]   The main purpose of this is to assess the impacts of the standard closure rule that has the government deficit fixed and net household transfers adjust to meet the fiscal target.

[43]   In the standard baseline, this ratio is fixed and drives the growth of real government expenditures. In shock simulations, the ratio is endogenous and real government expenditures are fixed to baseline levels.

Table F.7: **CSV cubes**

File name	Description
GDPPop	Contains GDP and population including: real GDP at market price (`rgdpmp`), nominal GDP at market price (`gdpmp`), GDP at market price deflator (`pgdpmp`), working age population (`P1564`), input population (`PopT`), simulated population (`Pop`), and real GDP per capital (`rgdppc`).
Factp	Contains aggregate factor prices including: wages identified by skill, capital (`trent`), land (`ptland`), natural resources (`ptnrs`) and water, if available (`pth2o`).
kappah	Contains net direct transfers to housholds as a share of income (`kappah`).
RGovShr	Contains real government expenditures as a share of real GDP (`rgovshr`).
savinv	Contains various indicators relating to savings and investment including: nominal GDP at market price (`gdpmp`), real GDP at market price (`rgdpmp`), real investment as a share of real GDP (`rinvshr`), aggregate real investment (`rinv`), aggregate nominal investment (`inv`), aggregate nominal household saving (`savh`), household savings rate (`aps`), nominal government saving (`savg`), nominal foreign saving (`savf`) and nominal depreciation (`deprY`).
Output	Contains various indicators relating to output: unweighted output (`xp`), output weighted by base year prices (`xpw`), nominal output (`xpd`), output price using un-weighted volumes (`px`), output price using base year price-weighted volumes (`pxn`), GAPS unweighted volume (`xpg_GAPS`), and Envisage output using GAPS base year volume (`xpg_Env`).
VA	Contains various indicators relating to value added by activity and factor: nominal value added (`va_d`), value added weighted by base year prices (`va_n`), price of value added using base-year price-weighted volumes (`pva_n`), unweighted volumes (`va`), and price of value added using unweighted volumes (`pva`).
INV	Contains various indicators relating to investment by sector: volume of sectoral investment (`inv_sec`), nominal investment by sector (`invd_sec`), sectoral share of aggregate nominal investment (`invd_shr`), sectoral share of nominal value added (`va_shr`), and ratio of sectoral investment share and value added share (`invRatio`).
EMI	Contains various indicators relating to greenhouse gas emissions: total emissions (`emi`), emissions linked to intermediate and final demand (`emi_io`), emissions linked to factor use (`emi_fp`), and emissions linked to output (`emi_xp`).
Cost	Contains various indicators relating to decomposing production cost: change in production cost (`delpx`), change in output (`delxp`), contribution of change in cost arising from change in input price (`pcshr`), contribution of change in cost arising from change in technology mix (`xcshr`), and cost share of input (`cshr`).
YDecomp	Contains various indicators relating to decomposing GDP growth: nominal GDP at factor cost (`gdpfc`), real GDP at factor cost (`rgdpfc`), real GDP at factor cost deflator (`pgdpfc`), contribution of GDP growth from factor growth (`qdel`), and contribution of GDP growth from factor productivity (`ldel`).
Trade	Contains various indicators relating to trade: nominal exports (`exp_d`), real exports (`exp`), export tax rate (`etax`), nominal imports at FOB prices (`imp_fob_d`), real imports at FOB prices (`imp_fob`), nominal imports at CIF prices (`imp_cif_d`), real imports at CIF prices (`imp_cif`) import tariff (`mtax`) and domestic absorption (`Absorb`).
Lab	Contains various indicators relating to labor markets: labor supply (`ls`), wage (`twage`), real wage (`trwage`), rural to urban migration (`migr`), labor supply by zone (`lsz`), average wage by zone (`awage`), equilibrium wage by zone (`ewage`) average real wage (`arwage`) and average equibrium wage (`aewage`).
Power	Contains various indicators relating to power: output (`XP`), and average cost (`PX`).

The SavInv cube

The `SavInv` cube contains the indicators below. Note that investment should equal aggregate savings (household, government and foreign) plus depreciation for any regional aggregation. Moreover, the sum of foreign saving at the world level should equal 0.

$$RGDPMP_R = \sum_{r \in R} RGDPMP_r$$

$$GDPMP_R = \sum_{r \in R} GDPMP_r$$

$$RInvShr_R = 100 \cdot \sum_{r \in R} XFD_{r,inv} \bigg/ \sum_{r \in R} RGDPMP_r$$

$$RInv_R = \sum_{r \in R} XFD_{r,inv}$$

$$Inv_R = \sum_{r \in R} YFD_{r,inv}$$

$$SavH_R = \sum_{r \in R} S_r^h$$

$$SavG_R = \sum_{r \in R} S_r^g$$

$$SavF_R = \sum_{r \in R} PW^{sav} S_r^f$$

$$DeprY_R = \sum_{r \in R} DeprY_r$$

$$APS_R = 100 \cdot \sum_{r \in R} S_r^h \bigg/ \sum_{r \in R} YD_r$$

The Output cube

The output cube contains various indicators related to output. Three reflect different measures of the level of output—nominal (`XPD`), unweighted real (`XP`) and weighted real (`XPW`). For most purposes the appropriate index is the weighted real—particularly when aggregating across activities using different units. Potential exceptions are when aggregating energy indicators as these may be evaluated in the same units, for example MTOE or GWhr. Similarly there are two price indices: unweighted (`PX`) and weighted (`PXN`). The cube also contains the total factor productivity parameter (for installed or *Old* capital).

$$XP_{R,A} = \sum_{r \in R} \sum_{a \in A} XP_{r,a}$$

$$XPW_{R,A} = \sum_{r \in R} \sum_{a \in A} PX_{r,a,0} XP_{r,a}$$

$$XPD_{R,A} = \sum_{r \in R} \sum_{a \in A} PX_{r,a} XP_{r,a}$$

$$PX_{R,A} = \sum_{r \in R} \sum_{a \in A} PX_{r,a} XP_{r,a} \bigg/ \sum_{r \in R} \sum_{a \in A} XP_{r,a}$$

$$PXN_{R,A} = \sum_{r \in R} \sum_{a \in A} PX_{r,a} XP_{r,a,0} \bigg/ \sum_{r \in R} \sum_{a \in A} PX_{r,a,0} XP_{r,a,0}$$

$$lambdaxpOld_{R,A} = \sum_{r \in R} \sum_{a \in A} \lambda_{r,a,Old}^{xp} PXv_{r,a,Old,0} XPv_{r,a,Old} \bigg/ \sum_{r \in R} PXv_{r,a,Old,0} XPv_{r,a,Old}$$

[Optional] There are two additional volume indicators linked to an external estimate of base year volumes.[44] The first indicator is the external indicator itself—in its own units and with its own growth trends (e.g. from an FAO or IEA projection). The second is the growth trend using the Envisage results but re-based to that of the external data. This allows for easier comparison of trends between institutions. In the case of the comparison with the FAO GAPS projections, the FAO projections are labeled `xpg_GAPS` and the Envisage projections are labeled `xpg_Env`. Note that the base years for GAPS and Envisage do not line up (2012 versus 2011), nor do the time steps line up.

The VA cube

The value added cube contains various indicators related to value added. In addition to the various endowments that are part of the model definition, the indicators also include aggregations for labor (`tlab`), all non-labor factors (`tcap`) and all factors (`tot`). In most cases, the VA_N indicator is the most appropriate for assessing aggregate value added. The VA indicator can be used in some cases to evaluate labor in person years depending on the calibration choices of the user.

$$VA\_D_{R,A,V} = \sum_{r \in R} \sum_{a \in A} \sum_{f \in V} PF_{r,f,a} XF_{r,f,a}$$

$$VA\_N_{R,A,V} = \sum_{r \in R} \sum_{a \in A} \sum_{f \in V} PF_{r,f,a,0} XF_{r,f,a}$$

$$PVA\_N_{R,A,V} = \sum_{r \in R} \sum_{a \in A} \sum_{f \in V} PF_{r,f,a} XF_{r,f,a} \Big/ \sum_{r \in R} \sum_{a \in A} \sum_{f \in V} PF_{r,f,a,0} XF_{r,f,a}$$

$$VA_{R,A,V} = \sum_{r \in R} \sum_{a \in A} \sum_{f \in V} XF_{r,f,a}$$

$$PVA_{R,A,V} = \sum_{r \in R} \sum_{a \in A} \sum_{f \in V} PF_{r,f,a} XF_{r,f,a} \Big/ \sum_{r \in R} \sum_{a \in A} \sum_{f \in V} XF_{r,f,a}$$

The INV cube

The investment cube contains investment by sector—both in volume and value terms. The formulas take into account multi-year step sizes. The `InvRatio` indicator measures an activity's share of investment relative to its share of value added (in nominal terms).

$$INV\_SEC_{R,A} = \sum_{r \in R} \sum_{a \in A} \frac{\delta_{r,t-n}}{1-(1-\delta_{r,t-n})^n} \left[\sum_v K_{r,a,v,t}^v - (1-\delta_{r,t-n})^n \sum_v K_{r,a,v,t-n}^v \right] \frac{1}{\chi_r^k}$$

$$INVD\_SEC_{R,A} = \sum_{r \in R} \sum_{a \in A} \frac{\delta_{r,t-n}}{1-(1-\delta_{r,t-n})^n} \left[\sum_v K_{r,a,v,t}^v - (1-\delta_{r,t-n})^n \sum_v K_{r,a,v,t-n}^v \right] \frac{PFD_{r,inv,t-n}}{\chi_r^k}$$

$$INVD\_SHR_{R,A} = \frac{\sum_{r \in R} \sum_{a \in A} \frac{\delta_{r,t-n}}{1-(1-\delta_{r,t-n})^n} \left[\sum_v K_{r,a,v,t}^v - (1-\delta_{r,t-n})^n \sum_v K_{r,a,v,t-n}^v \right] \frac{PFD_{r,inv,t-n}}{\chi_r^k}}{\sum_{r \in R} \sum_a \frac{\delta_{r,t-n}}{1-(1-\delta_{r,t-n})^n} \left[\sum_v K_{r,a,v,t}^v - (1-\delta_{r,t-n})^n \sum_v K_{r,a,v,t-n}^v \right] \frac{PFD_{r,inv,t-n}}{\chi_r^k}}$$

$$VA\_SHR_{R,A} = \frac{\sum_{r \in R} \sum_{a \in A} \sum_f PF_{r,f,a} XF_{r,f,a}}{\sum_{r \in R} \sum_a \sum_f PF_{r,f,a} XF_{r,f,a}}$$

$$InvRatio_{R,A} = \frac{INVD\_SHR_{R,A}}{VA\_SHR_{R,A}}$$

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[44] This is particularly useful where the external estimate of base year volumes is in recognizable units—such as tons, joules, etc.

The Emissions cube

The emissions cube contains 5 indicators: total emissions, emissions linked to consumption of intermediate and final demand, emissions linked to factor-use and emissions linked to output. Emission indicators are output for all types of emissions. Physical emissions are all denominated in unit of gigatons.[45] Greenhouse gases are also output in units of CEq and CO2eq. If the model uses units of CEq, CO2eq is calculated by multiplying the emission level by a factor of 44/12. In the contrary case, emissions are converted to CEq using the factor 12/44.

$$Emi_{R,Tot,em} = \sum_{r \in R} EmiTot_{r,em}$$

$$Emi\_io_{R,A,em} = \sum_{r \in R} \sum_{a \in A} \sum_{i} Emi_{r,em,i,a}$$

$$Emi\_io_{R,fd,em} = \sum_{r \in R} \sum_{i} Emi_{r,em,i,fd}$$

$$Emi\_fp_{R,A,em} = \sum_{r \in R} \sum_{a \in A} \sum_{fp} Emi_{r,em,fp,a}$$

$$Emi\_xp_{R,A,em} = \sum_{r \in R} \sum_{a \in A} Emi_{r,em,Tot,a}$$

The Trade cube

The first set of equations relate to exports. These are aggregated over regions and commodities, where the index I is an aggregate commodity, which may be an individually modeled commodity. The first two measure aggregate exports by commodity I at current FOB prices and at constant FOB prices. The third formula measures the export tax.

$$EXP\_D_{R,I} = \sum_{s \in R} \sum_{i \in I} \sum_{d} PWE_{s,i,d} XW_{s,i,d}$$

$$EXP_{R,I} = \sum_{s \in R} \sum_{i \in I} \sum_{d} PWE_{s,i,d,0} XW_{s,i,d}$$

$$ETAX_{R,I} = 100 \cdot \sum_{s \in R} \sum_{i \in I} \sum_{d} PWE_{s,i,d} XW_{s,i,d} \Big/ \sum_{s \in R} \sum_{i \in I} \sum_{d} PE_{s,i,d} XW_{s,i,d} - 100$$

The next set of five equations relate to imports. The first two equations evaluate current and constant imports at FOB prices, i.e. these indicators ignore the trade and transport margins. The next two equations use CIF import prices. The final equation evaluates the import tariff in percentage terms. Note that the variable XW represents the supply of exports and is adjusted by the iceberg parameter to convert to demand for imports.

$$IMP\_FOB\_D_{R,I} = \sum_{d \in R} \sum_{i \in I} \sum_{s} PWE_{s,i,d} \lambda^w_{s,i,d} XW_{s,i,d}$$

$$IMP\_FOB_{R,I} = \sum_{d \in R} \sum_{i \in I} \sum_{s} PWE_{s,i,d,0} \lambda^w_{s,i,d} XW_{s,i,d}$$

$$IMP\_CIF\_D_{R,I} = \sum_{d \in R} \sum_{i \in I} \sum_{s} PWM_{s,i,d} \lambda^w_{s,i,d} XW_{s,i,d}$$

$$IMP\_CIF_{R,I} = \sum_{d \in R} \sum_{i \in I} \sum_{s} PWM_{s,i,d,0} \lambda^w_{s,i,d} XW_{s,i,d}$$

$$MTAX_{R,I} = 100 \cdot \sum_{d \in R} \sum_{i \in I} \sum_{s} PDM_{s,i,d} \lambda^w_{s,i,d} XW_{s,i,d} \Big/ \sum_{s \in R} \sum_{i \in I} \sum_{s} PWM_{s,i,d} \lambda^w_{s,i,d} XW_{s,i,d} - 100$$

Domestic absorption is defined as the sum of demand by source, i.e. domestic and imported. Its evaluation depends on the Armington trade specification. The first formula evaluates domestic absorption in the case of the agent-specific Armington specification. The second formula measures domestic absorption in the case of the national-agent Armington specification.

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[45]   Base non-greenhouse gas emissions are in units of gigagrams. They are divided by 1000 to convert to gigatons.

$$Absorb_{R,I} = \sum_{d \in R} \sum_{i \in I} \sum_{aa} \gamma_{r,i,aa}^{edd} PDT_{r,i} XD_{r,i,aa} + \gamma_{r,i,aa}^{edm} PMT_{r,i} XM_{r,i,aa}$$

$$Absorb_{R,I} = \sum_{d \in R} \sum_{i \in I} \sum_{aa} \gamma_{r,i,aa}^{eda} PAT_{r,i} XA_{r,i,aa}$$

F.3.4 Calculating the indicators

Calculating the indicators involves running the `makCSV` file for each simulation. The command line arguments are similar to those used to run the simulations themselves. One key argument is the `ifAppend` argument. Typically it is set to '0' for the first simulation and '1' for subsequent simulations. This forces the results to be concatenated in the CSV cubes. The sequence is easiest to run by using an existing command file that contains the full suite of commands to create the CSV cubes. In addition, the command file contains the code to create or refresh the Excel files and the pivot tables in the Excel file. If the Excel file already exists, the command file will request a refresh of the pivot table(s). The advantage of this is that it will preserve any worksheets, charts, tables that a user may have created and saved using a previous CSV cube. The `refresh.vbs` script is invoked to refresh an existing Excel file. If the Excel file does not exist, a new one will be created with a single pivot table.

F.3.5 Creating the Excel files

The user can request a new Excel file by modifying the code in `CreatePivot.gms`. The basic purpose of the GAMS program is to create a Visual Basic script (vbs) that will automatically create a pivot table in an Excel file based on the relevant CSV cube. Each cube has its own geometry and there is a set of built-in geometries in the GAMS file—for example `VarByCost` or `VarByActivity`. When using `CreatePivot.gms`, a specific geometry must be chosen for each CSV cube that corresponds to the geometry of the cube. Users are free to add new geometries, using the existing ones as a guideline. Some of the geometries allow for labels to be sorted—see for example `VarByActivity`. The pivot tables that are created all have two dimensions—a row and a column. The rows will have the position number 1, columns position number 2, and the values position number 5. All other dimensions normally default to position number 3. Users can modify the pivot table geometry once the file has been created (and this will be preserved when the files are refreshed). It can be convenient to have some of the labels sorted automatically—though VB script can sometimes be fussy about this. Regions are sorted in all cases.[46] Some of the geometries allow for sorting by either activity or commodity.

F.3.6 Installing a 'CSV' driver for Excel

Creating Pivot tables in Excel requires the installation of the appropriate data base driver. Most Excel implementations include drivers for Excel and Access files, but not for CSV files. The so-called ODBC driver can be installed from a package available from Microsoft at https://www.microsoft.com/en-us/download/details.aspx?id=54920.[47] Depending on the computer, it is possible to download either the 32-bit or 64-bit driver, with, respectively, file names of `accessdatabaseengine.exe` and `accessdatabaseengine_X64.exe`. Once installed, the user must make the driver 'active'. To do this, the user enters `ODBC Data Sources` in the search box of the task bar. This will open a dialog box that indicates the existing ODBC drivers. If the `MS Access Text Driver` is already available, nothing further needs to be done. If the `MS Access Text Driver` is not yet available, installation requires clicking on the `Add...` button. This should bring up a list of drivers including `Microsoft Access Text Driver (*.txt, *.csv)`. Finish entering the required information—`Data Source Name` and `Description`, for example *MS Access Text Driver* and **.txt and *.csv files*, and press the `OK` button when done. N.B. Some installations may require Administrator privileges to be able to install an ODBC driver.

F.3.7 Creating graphs in R

A number of R scripts are available to create and save figures automatically. Most of these will generate a single graph for each region (both model regions and the agggregate regions) as well as for each simulation. The scripts are designed to save all the graphs for a simulation in a simulation folder. The scripts have the following outline:

- The scripts are called from a command line and the only argument is the name of CSV cube, for example `GDPPop`, without the `.csv` extension.

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[46] Only the model core regions are sorted, it would be nice to have sorting for the aggregate regions as well.
[47] Last accessed on 20-Jan-2022

- Each script will read a script common to all that sets some global parameters, such as input and output directories. This common script is called `PlotOpt.R`. It normally only needs to be reviewed for a new model configuration.

- Each script also relies on two user-based files that contain the list of regions (`regions.csv`) and the time periods (years.csv). These files are aggregation specific.

- The script relies on two libraries—`sqldf` and `ggplot2`. The first allows for reading CSV files as if they are cubes and users can then use the standard SQL commands to select and filter the incoming data.[48] The graphs are created using the `ggplot2` package.

- The function `read.csv.sql` is used to read in the CSV cube. This allows to skip some fields altogether and/or to select specific rows of the cube. For example, the `plotRGDPoc.R` script reads only two variables in the cube—`Pop` and `rgdpmp`.[49]

- The body of the script loops over all possible simulations and regions and creates a figure for each combination based on the indicators read in from the CSV cube.

- The figures are stored by simulation. If the parent folder is called `PROJNAME\DOC` then the figures will be stored in `PROJNAME\DOC\SIM1`, `PROJNAME\DOC\SIM2`, ..., `PROJNAME\DOC\SIMn`. There could be other ways to organize the folder structure depending on the specific needs of the user.

Running of the scripts can be automated. See for example the file `plotAll.cmd`.

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[48] The alternative is to use the standard `read.csv` function. The downside to this is that it will read the entire cube, even if only a small slice is desired.

[49] Unfortunately, `read.csv.sql` does not strip double quotes from the incoming cube so the R script has been designed to handle the string labels with double quotes.

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