Prizes and Incentives in Elimination Tournaments

By SHERWIN ROSEN*

Contestants who succeed in attaining high ranks in elimination career ladders rest on their laurels in attempting to climb higher, unless top-ranking prizes are given a disproportionate weight in the purse. A large first-place prize gives survivors something to shoot for, independent of past performances and accomplishments.

Several recent papers have clarified the problem of incentives when competitors are paid on the basis of rank or relative performance (Edward Lazear and myself, 1981; Jerry Green and Nancy Stokey; 1983; Barry Nalebuff and Joseph Stiglitz, 1984; Bengt Holmstrom, 1982; James Malcomson, 1984; Lorne Carmichael, 1983; Mary O'Keefe, W. Kip Viscusi, and Richard Zeckhauser, 1984). The main focus so far has been to examine the economic efficiency of these schemes. However, a much longer tradition in statistics views relative comparisons as an experimental design for ranking and selecting contestants. These two views are joined in this work.

I investigate the incentive properties of prizes in sequential elimination events, where rewards are increasing in survival. The inherent logic of these experiments is to determine the best contestants and promote survival of the fittest; and to maintain the "quality of play" as the game proceeds through its stages. Athletic tournaments immediately come to mind, but much broader interest in this class of problems arises from its potential application to career games, where the tournament analogy is supported (James Rosenbaum, 1984). Many organizations have a triangular structure (for example, Martin Beckmann, 1978) and most top level managers come up through the ranks (Kevin J. Murphy, 1984). A career trajectory is, in part, the outcome of competition among peers to attain higher ranking and more remunerative positions over the life cycle. The structure of rewards influences the nature and quality of competition at each stage of the game.

What needs to be explained is the marked concentration of rewards in the top ranks. For example, the top four ranks receive 50 percent or more of the total purse in tennis tournaments. Concentration is less extreme in the executive labor market, but nonetheless, earnings rise more than proportionate to rank in most firms. I show below that an elimination design requires an extra reward for the overall winner to maintain performance incentives throughout the game.

The economics of this result derives from the survival aspects of the game. A competitor's performance incentives at any stage are set by an option value. The loser's prize is guaranteed at that stage, but winning gives the option to continue on to all successive rungs in the ladder. There are fewer steps remaining to be attained as the game proceeds, and the option value plays out. It expires in the final match because advancement opportunities vanish. At that point, the difference in prize money between winning and losing must incorporate the equivalent of the survival option that maintained incentives at earlier stages. The extra weight of rewards at the top is due to the no-tomorrow aspects of the final stage of the game. It extends the horizon of players surviving to those stages, and makes the game appear of

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infinite length to a contestant, *as if* there are always many steps left to attain, no matter how far one has climbed in the past. This result obviously bears a family resemblance to the role of a "pension" in a finitely repeated principal and agent problem (Gary Becker and George Stigler, 1984; Lazear, 1981).

Section I describes the game, and Section II sets forth the nature of contestants' strategies. Sections III and IV analyze the problem when the inherent talents of competitors are known, while Section V analyzes the case where talents are unknown.

I. Design of the Game

For analytical tractability and simplicity, the ideas are best revealed by a paired-comparison structure, as in a tennis-ladder. The tournament begins with 2^N players and proceeds sequentially through N stages. Each stage is a set of pairwise matches. Winners survive to the next round, where another pairing is drawn, and losers are eliminated from subsequent play. Half are cut from further consideration at each stage. Thus, in a career game, those eligible for promotion to some rank have attained the rank immediately below it. Those who are passed over at any stage are out of the running for further promotions. The top prize W_1 is awarded to the winner of the final match, who has won N matches overall. The loser of the final match achieves second place overall and is awarded prize W_2 for having won $N-1$ matches. Losers of the semifinals are both awarded W_3, etc.

Define s as the number of stages remaining to be played. Then all players eliminated in a match where s stages remain are awarded prize W_{s+1}. Define the interrank spread $\Delta W_s = W_s - W_{s+1}$ as the marginal reward for advancing one place in the final ranking. These increments determine incentives to advance through the stages. Prizes are increasing in survival: $\Delta W_s > 0$ for all s.

I am concerned in this work with studying how prizes affect performance and selection, and with finding some characterizations of the relative reward structure required to maintain incentives as the game proceeds. This is a piece of a larger problem of the "optimal" prize structure, the study of which requires specifying how incentives affect the social value of the game. These complex matters are not well understood. So rather than tying results to an arbitrary input-output technology, a common feature of the larger problem obviously requires that players work at least as hard, if not harder, in the later stages of the game as in the early stages. We don't want contestants to lay down near the end. For example, in a hierarchical organization, the decisions made at the top are more important than those made further down the pyramid (see my 1982 paper): shirking and lack of talent have more serious consequences at the top of the organization than at the bottom.

Rank-order schemes are encountered when individual output and input are difficult to measure on a cardinal scale, an inherent feature of managerial and many other types of talent; or when common background noise contaminates precise individual assessments of value-added. Competition is inherently head-to-head in most athletic games, and cardinality in any sense other than probability of winning has little meaning. Ordinality is inherent because the point scores used to calibrate performance contain many arbitrary elements, as in a classroom test. Many of these same considerations apply to selection of managerial talents though competition is not strictly paired comparisons.

Given the rules, these issues may be finessed for studying the connection between prizes, incentives, and selection by specifying how players' actions affect the probability of winning. Let i index a player and let j index an opponent in some match. Consider a game in which there are m types of players. Index the ability type of the i player by I and the ability type of the opponent j player by J. Both I and J take on m possible values, $1,2,\dots,m$, with $m \le 2^N$. Let x_{si} and x_{sj} denote the intensity of effort expended by players i and j in a match when s stages remain to be played, and let γ_I and γ_J represent their abilities or natural talents for the game. Then $P_s(I,J)$ is the probability that a player of type I wins in a match against a player of type J (possibly the same type) with

$$(1)\quad P_s(I,J)=\frac{\gamma_I h(x_{si})}{\gamma_I h(x_{si})+\gamma_J h(x_{sj})},$$

where $h(x)$ is increasing in x and $h(0)\geq 0$. A player increases the probability of winning the match by exerting greater effort, given the talent and effort of the opponent and own talent. To simplify the problem, the win technology is assumed identical at every stage (s enters only through the x's).

When both players exert the same level of effort, the win probability is $P_s(I,J)=\gamma_I/(\gamma_I+\gamma_J)$, and its inverse is a bookmaker's "morning line" or "true-to-form" actuarially fair payoffs per dollar bet on player-type I. Notice from (1) that common, multiplicative environmental factors do not affect $P_s(I,J)$. Let the common factor multiply $\gamma h(x)$ for both players. Then whether the commonality is match-, stage-, or tournament-specific, it factors out of the probability calculation and has no effect on either incentives or selection. Equation (1) is a logit when $h(x)$ is exponential. Alternatively, think of $\gamma h(x)$ as the arrival rate of a Poisson process. Then (1) can be given a racing game interpretation, as in the recent literature on patent races (Glenn Loury, 1979).

II. Strategies

A player's decision of how much effort to expend in any match depends on weighing the benefit of greater effort (increasing the probability of surviving) against its costs. There are two complications. First, the value of advancing depends on how the player assesses future effort should eligibility be maintained. This forward-looking effect is analyzed by backward recursion. Second, current actions depend on the anticipated behavior of the current and all future possible opponents. The sequential character of the game allows this to be analyzed by adopting Nash noncooperative strategies as the equilibrium concept. Discounting between stages is ignored and risk neutrality is assumed.

Define $V_s(I,J)$ as the value to a player of type I of playing a match against an opponent of type J when s possible stages remain to be played. Assume, for now, that all players' talents are common knowledge. Let $c(x)$ be the cost of effort in any match, assumed identical for all players, $c'(x)>0$, $c''(x)\geq 0$, and $c(0)=0$. The value of the match consists of two components: one is W_{s+1}, the prize earned if the match is lost and the player is eliminated, an event which occurs with probability $1-P_s(I,J)$. The other is the value of achieving a final rank superior to $s+1$ if the match is won. Let $EV_{s-1}(I)$ represent the expected value of eligibility in the next stage. This is a weighted average over J of $V_{s-1}(I,J)$, where the weights are the probabilities that the I player will confront an opponent of type J in the next stage. These probabilities depend on the activities of players in other matches and the rules for drawing opponents at each stage. The probability of continuation is $P_s(I,J)$, and costs $c(x)$ are incurred for either outcome, so the fundamental equation for this problem is

$$(2)\quad V_s(I,J)=\max_{x_{si}}\left[P_s(I,J)EV_{s-1}(I)\right.$$
$$\left.+\left(1-P_s(I,J)\right)W_{s+1}-c(x_{si})\right].$$

The max in (2) is understood on Nash assumptions as conditioned on the given current and expected future efforts of all other players remaining alive at s and on the optimum actions taken by the player in question in subsequent matches.

Substituting (1) into (2) and differentiating with respect to x_{si} yields the first-order condition

$$(3)\quad \gamma_I\gamma_J h_j h_i'/\left(\gamma_I h_i+\gamma_J h_j\right)^2$$
$$\times\left[EV_{s-1}(I)-W_{s+1}\right]-c_i'=0,$$

where $h_i=h(x_{si})$, $h_i'=dh(x_{si})/dx_{si}$, etc. The second-order condition is

$$(4)\quad D=c_i'\left[\left(h_i''/h_i'\right)-2\gamma_I h_i'\right.$$
$$\left./\left(\gamma_I h_i+\gamma_J h_j\right)\right]-c_i''<0.$$

Note that (4) allows $h''>0$ so long as it is bounded. There is also a global condition.

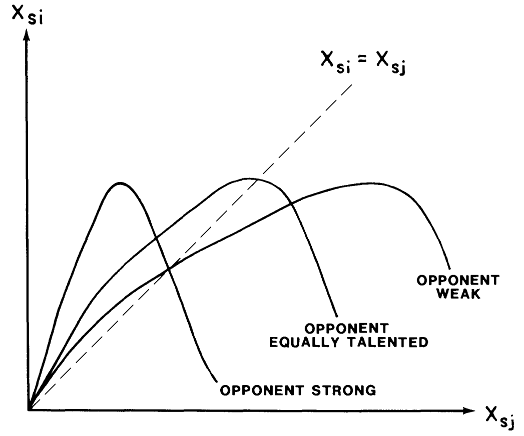

FIGURE 1

Equation (3) indicates that effort in any match is controlled by $EV_{s-1}(I)-W_{s+1}$. This difference between winning and losing must be positive for the player to have an interest in maintaining eligibility into the next stage. Otherwise, it is best to default and exert no effort.

Equation (3) defines the best-response function for player i. Differentiating with respect to the current opponent's effort,

$$(5)\quad \partial x_i/\partial x_j = \frac{(h_j'/h_j)c_i'}{-D(\gamma_I h_i+\gamma_J h_j)} \times(\gamma_I h_i-\gamma_J h_j).$$

Player i's best reply is increasing in x_j when x_j is small enough, but is decreasing when the opponent's effort is sufficiently large. It has a turning point at $\gamma_I h(x_i)=\gamma_J h(x_j)$. The turning point occurs at $x_i=x_j$ for equally talented players ($\gamma_I=\gamma_J$). It turns at some value $x_j>x_i$ when i is playing a weaker opponent ($\gamma_I>\gamma_J$) and it turns at some $x_j<x_i$ when the opponent is the stronger player ($\gamma_I<\gamma_J$). See Figure 1. Analysis is confined to pure-strategy equilibria.[1]

III. Incentive Maintaining Prizes: Equally Talented Contestants

The solution is transparent when all players are equally talented (there is only one type). Then $EV_{s-1}(I)=V_{s-1}$, because each player knows for sure that an opponent of equal skill will be confronted at every stage. From Figure 1, the best-reply function is the same for all players and has a turning point at $x_i=x_j$. Therefore, the equilibrium is symmetric: $x_{si}=x_{sj}=x_s$ for all i and j and $P_s=1/2$ in equilibrium. Each match is a close call in expected value. The common level of effort when s stages remain which satisfies (3) is

$$(6)\quad (V_{s-1}-W_{s+1})(h'(x_s)/h(x_s))/4=c'(x_s).$$

Define the elasticities

$$(7)\quad \eta(x)=xh'(x)/h(x),$$
$$\varepsilon(x)=xc'(x)/c(x),$$
$$\mu(x)=\eta(x)/\varepsilon(x).$$

Then (6) becomes

$$(8)\quad (V_{s-1}-W_{s+1})\mu(x_s)/4=c(x_s).$$

Substituting (8) into (2) and using $P_s=1/2$,

$$(9)\quad V_s=(1/2)(1-\mu(x_s)/2)\times(V_{s-1}-W_{s+1})+W_{s+1} = \beta_s V_{s-1}+(1-\beta_s)W_{s+1},$$

[1]The best reply may jump down to zero at some point because either (4) fails beyond that point or default ($x_i=0$) is a global optimum while (3) is local. Pure strategies characterize equilibrium when these jumps occur (if they do) at sufficiently large x_i. This requires certain bounds on the curvature of the $h(x)$ and $c(x)$ functions and some limits on the degree of heterogeneity (the γ's) among players (for example, a very weak player might just lay down against a very strong one). The rules of the game determine $c(x)$ and $h(x)$ (see O'Keeffe et al.). Rules and initial screening of entrants must be suitably constrained to guarantee pure-strategy equilibria. Nalebuff and Stiglitz analyze random strategies in one-shot games. Stephen Bronars (1985) shows that a weak player might employ a riskier strategy against a stronger opponent, but (1) is not suitably parameterized to consider this.

where

$$(10) \qquad \beta_s = (1/2)(1-\mu(x_s)/2).$$

The recursion (9) holds if (3) is a global maximum, and no player has incentives to default from x_s defined by (6). This requires, from (9), that $V_s - W_{s+1} = \beta_s(V_{s-1} - W_{s+1}) > 0$. Otherwise taking the sure loss is a better choice. Therefore $\beta_s > 0$, or, from (10) and (7), $\eta(x_s)/2\varepsilon(x_s) < 1$, or $\mu(x) < 2$. There is no pure-strategy equilibrium in this game if any player has an incentive to default.

The sense of the no-default condition $\eta(x)/\varepsilon(x) < 2$ is related to the problem of an arms race. If the elasticity of response of effort is large relative to the elasticity of its cost, then players' efforts to win results in a negative sum game in pure strategies. It is not optimal to default if the opponent does, but at the local equilibrium the costs of contesting have been escalated so much that both want to default. In fact, (9) implies that for given prizes, players are better off when there is less scope for actions to affect outcomes: V_s is decreasing in $\mu(x)$, so the rules of the game must be devised to balance two conflicting forces: games which greatly constrain the effect of actions on outcomes are inefficient and unproductive: whereas competition is destructive if these constraints are relaxed too much.[2]

Assuming $0 < \beta_s < 1$, for all s and using $V_0 = W_1$ as a boundary condition, the solution to (9) is

$$(11) \quad V_s = (\beta_1\beta_2\dots\beta_s)\Delta W_1 + (\beta_2\dots\beta_s)\Delta W_2 + \dots + \beta_s \Delta W_s + W_{s+1}.$$

The value of maintaining eligibility at any stage is the sure prize the player has guaranteed by surviving that long, plus the discounted sum of successive interrank rewards that may be achieved in future matches. Herein lies the "option" value of an elimination design. Manipulating (11) yields an expression for $V_{s-1} - W_{s+1}$, which controls performance incentives, from (8):

$$(12) \quad (V_{s-1} - W_{s+1}) = (\beta_1\dots\beta_{s-1})\Delta W_1 + (\beta_2\dots\beta_{s-1})\Delta W_2 + \dots + \Delta W_s.$$

Incentives are determined by the discounted sum of interrank spreads.

What reward structure maintains incentives to perform at a common value throughout all stages of the game? Here $x_s = x^*$ for all s and $\beta_s = \beta$ is a constant for all s, from (10). Then (12) implies

$$(13) \quad (V_{s-1} - W_{s+1}) - \beta(V_{s-2} - W_s) = \Delta W_s \qquad \text{for } s = 2,3,\dots,N.$$

Constant performance requires that $(V_{s-1} - W_{s+1})$ is a constant. Suppose $V_{s-1} - W_{s+1} = k$, where k is determined so that $x_s = x^*$ solves (3). Then from (13),

$$(14) \quad k(1-\beta) = \Delta W = \Delta W_s \qquad \text{for } s = 2,3,\dots,N$$

and since final-round effort depends only on ΔW_1, from (12),

$$(15) \quad k = \Delta W_1 = \Delta W/(1-\beta) > \Delta W.$$

The incentive-maintaining prize structure requires a constant interrank spread from second place down, from (14). However, it requires a larger interrank spread at the top, from (15). Prizes rise *linearly* in increments $\Delta W = k(1-\beta)$ from rank $N+1$ up through rank 2, but the first place prize takes a distinct *jump* out of sync with the general linear pattern below it. The incentive-maintaining prize distribution weighs the top prize more heavily than the rest.[3] See Figure 2.

[2] Contestants have incentives to introduce new techniques and styles of play to create a winning edge. These are sources of technical change in career games. Athletic games use a supreme authority to maintain the integrity of the game. Innovations which escalate the collective costs of competition relative to social value are prohibited.

[3] This analysis determines only relative prizes across ranks, not their absolute level. More structure on technologies and the social value of the game must be introduced to examine the latter (for example, see Lazear's and my article). Here we require that the purse is large enough to support $V_s > 0$ for all s. This implies

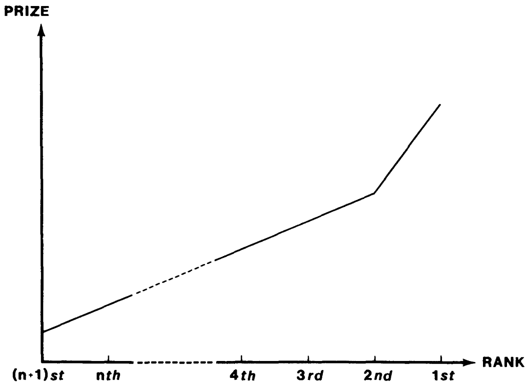

FIGURE 2

The proof supports the economic interpretation of this surprising conclusion. The final-round spread has to replace the earlier option value of achieving possible higher ranks at earlier stages. Substitute (14) and (15) into (12):

$$(16)\quad (V_{s-1}-W_{s+1})$$
$$=\beta^{s-1}\Delta W_1+\Delta W(\beta^{s-2}+\beta^{s-3}+\dots+1)$$
$$=\Delta W\left[(\beta^{s-1}/(1-\beta))+\beta^{s-2}+\beta^{s-3}+\dots+1\right]$$
$$=\Delta W(1+\beta+\beta^2+\beta^3+\dots)\quad \text{for all } s,$$

The extra increment at the top converts the value of the difference between winning and losing at each stage into a perpetuity of constant value at all stages. It effectively extends the horizon of the players and makes them behave *as if they are in a game which continues forever*. This horizon-extending feature of the top prize is one of the reasons why observed rewards are concentrated toward the top ranks. It is clear that concentrating even more of the purse on the top creates incentives for performance to increase as the game proceeds through its stages. For example, if the winner takes all, then every term other than the one in ΔW_1 in (12) vanishes and the difference in value between winning and losing increases as the game proceeds, through the force of discounting: effort is smallest in the first stage and largest in the finals.

The result in (16) and (17) is robust to a number of modifications:

(*i*) *Risk Aversion*. Suppose preferences take the additive form $U(W)-\Sigma_s c(x_s)$, where $c(x)$ is as before and $U(W)$ is increasing, but not necessarily linear in W. Then the entire analysis goes through by replacing W_s with $U(W_s)$ wherever it appears. Incentive maintenance requires a constant difference in the utility of rewards $U(W_{s+1})-U(W_{s+2})$ in all stages prior to the finals, but still requires a jump in the interrank difference in utility of winning the finals. If players are risk averse ($U''(W)<0$), the incentive-maintaining prize structure requires strictly increasing interrank spreads, with an even larger increment between first and second place. The prize structure is everywhere convex in rank order, with greater concentration of the purse on the top prizes than when contestants are risk neutral. The spread has to be increasing to "buy off" survivor's risk aversion and maintain their interest in advancing to higher ranks.

(*ii*) *Symmetric Win-Technologies*. The derivation of (14) and (15) rests only on that property that P_s is 1/2 in equilibrium. Hence Figure 2 is independent of the specific form of (2) and holds for *any* win technology resulting in a symmetric equilibrium. Furthermore, the result extends to more than pairwise comparisons: there might be n-way comparisons at each stage. In the Poisson case, the probability of advancing becomes $h(x_i)/\Sigma^n h(x_k)$. Then $\beta_s=(1/n)(1-(n-1)\mu(x_s)/n)$, but the logic otherwise remains unchanged.

(*iii*) *Stage Effects*. The nature of competition may vary across stages. For example, in a corporate hierarchy the pass-through rate may fall at each successive rank. Similarly, μ_s may be smaller in the later stages because higher-ranking positions are more demanding than lower-ranking ones. In

an upper bound on feasible x^*. Another upper bound on x^* is implied by contestants' outside opportunities, but is ignored.

either case, β_s decreases as the game proceeds, and interrank spreads must be increasing to undo the incentive dilution effects of greater discounting of the future, which otherwise reduces the option value of continuation. These considerations increase the convexity of the rank/reward structure.[4]

IV. Heterogeneous Contestants with Known Talents

In heterogeneous populations, elimination designs promote *survival of the fittest* and progressive elimination of weaker contenders. The conditional mean ability of survivors tends to increase as the game proceeds and differences in survivors' talents are compressed relative to the initial field. This increasing homogeneity among surviving members across stages extends the incentive-maintenance result above to the limit of the last few stages of a long game. For by continuity of the best-reply functions in ability parameters, EV_{s-1} is approximately V_{s-1} among relatively homogeneous survivors in the final stages. The extra final-round incremental prize remains necessary to maintain incentives toward the end.

This section shows that the value of the continuation option is increasing in ability, which is why the design encourages survival of the fittest. However, analysis is complicated by progressive increasing strength-of-field effects. That stronger opponents are likely to be encountered in later stages reduces the value of continuation, while the greater likelihood of being matched against a weaker opponent in the current stage increases it. Therefore, the nature of the game is affected by the rules for drawing opponents, such as seeding. A simulation of a two-stage game illustrates these issues.[5]

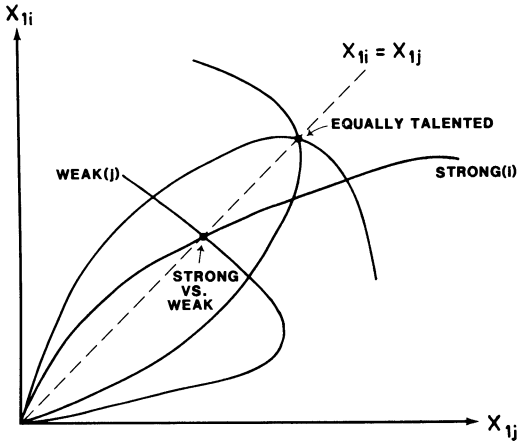

FIGURE 3

To simplify, assume two player types and constant elasticity cost and $h(x)$ functions. Type 1 is stronger than type 2 ($\gamma_1 > \gamma_2$). Since we have to keep track of each player's talent, the definition of β_s must be extended to

$$(17)\quad \beta_s(I,J) = P_s(I,J)[1-\mu P_s(J,I)],$$

where the P's are evaluated at equilibrium. Equation (17) includes (10) when $I = J$ because $P_s(I,I) = 1/2$.

Finals. Since $EV_1 - W_2 = \Delta W_1$ for all γ_I, symmetry of (3) implies $x_{1i} = x_{1j}$ irrespective of players' talents: $P_1(I,J) = \gamma_I/(\gamma_I + \gamma_J)$ in equilibrium. Effort is greater in a final match involving equally talented contestants than in one which matches a stronger against a weaker player (Figure 3). Using the same manipulations as before, we find

$$(18)\quad V_1(I,J) = \beta_1(I,J)\Delta W_1 + W_1.$$

Since $P_1(1,2) > P_1(2,1)$—a stronger player has a winning edge against a weak one in equilibrium, we have $\beta_1(1,2) > \beta > \beta_1(2,1)$.

Semifinals. Let π_1 denote the probability that the winner of the match in question

[4] The analysis of direct effort spillovers across stages is complicated by the fact that there may be asymmetric as well as symmetric pure-strategy equilibria. At symmetric equilibrium it is easy to show that fatigue and "burnout" requires more concentration on the top prize to penalize early-round "coasting." The force of "momentum" or learning requires less concentration at the top to maintain constant quality of play.

[5] Notice that a player is interested in what players in other matches are doing at any given stage because those outcomes determine who likely opponents will be in future stages. Equilibrium at each stage is a simultaneous 2^s player game: the problem does not disassemble pairwise and its complete solution must be simulated.

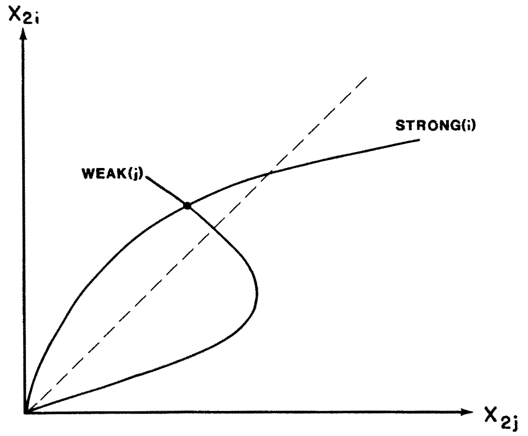

FIGURE 4

will confront a strong player in the finals. This depends on the identities and efforts chosen by players in the other match, but these actions are given to the opponents in this match in the Nash solution. Therefore,

$$(19)\quad EV_1(I)=[\pi_1\beta_1(I,1)+(1-\pi_1)\times\beta_1(I,2)]\Delta W_1+W_2=\tilde{\beta}_1(I)\Delta W_1+W_2,$$

where

$$\tilde{\beta}_1(I)=\pi_1\beta_1(I,1)+(1-\pi_1)\beta_1(I,2)$$

and

$$(20)\quad EV_1(I)-W_3=\tilde{\beta}_1(I)\Delta W_1+\Delta W_2.$$

π_1 is smaller for the strong contestant implies $\tilde{\beta}_1(1)>\tilde{\beta}_1(2)$ because $\beta_1(1,2)>\beta_1(2,1)$.

There are two possible types of matches in the semi's. The equilibrium is symmetric if $I=J$, with $P_2(1,1)=P_2(2,2)=1/2$. If $I\neq J$, the equilibrium is *not* symmetric because the stronger player has a greater value of continuation in (19) and (20). The strong player exerts greater efforts to win in equilibrium and $P_2(1,2)>\gamma_1/(\gamma_1+\gamma_2)=P_1(1,2)$ (see Figure 4). We find

$$V_2(I,J)=\beta_2(I,J)[\tilde{\beta}_1(I)\Delta W_1+\Delta W_2]+W_3.$$

Furthermore,

$$(21)\quad \beta_2(1,2)>\beta_1(1,2)>\beta>\beta_1(2,1)>\beta_2(2,1),$$

which implies $V_2(1,2)>V_2(2,1)$ and $\tilde{\beta}_2(1)>\tilde{\beta}_2(2)$.

These formulas generalize for arbitrary s:

$$(22)\quad V_s(I,J)=\beta_s(I,J)[(\tilde{\beta}_1(I)\tilde{\beta}_2(I)\dots\tilde{\beta}_{s-1}(I))\Delta W_1+(\tilde{\beta}_2(I)\dots\tilde{\beta}_{s-1}(I))\Delta W_2+\dots+\Delta W_s]+W_{s+1}EV_{s-1}-W_{s+1}$$
$$=(\tilde{\beta}_1(I)\tilde{\beta}_2(I)\dots\tilde{\beta}_{s-1}(I))\Delta W_1+(\tilde{\beta}_2(I)\dots\tilde{\beta}_{s-1}(I))\Delta W_2+\tilde{\beta}_{s-1}(I)\Delta W_{s-1}+\Delta W_s$$

$$\tilde{\beta}_s(I)=\pi_s\beta_s(I,1)+(1-\pi_s)\beta_s(I,2),$$

where π_s is the probability a strong opponent will be encountered at s. An easy induction proves $\tilde{\beta}_s(1)>\tilde{\beta}_s(2)$ for all s, so (22) implies that the value of continuation is larger for stronger players at every stage of the game. The second expression in (22) also implies that a strong player works harder in a strong-weak match than a weak player does: the weak are eliminated with probability in excess of $\gamma_1/(\gamma_1+\gamma_2)$ at every stage except the last.

Since the value of the game is larger for stronger players, equilibrium in matches involving unequally talented players is asymmetric (except in the finals) and the definition of incentive maintenance must be extended. The most straightforward extension is a requirement that the same level of effort be maintained in all stages *within* any given type of pairing I against J: it is not feasible for effort to be maintained at a constant value *across* match types due to heterogeneity. Even this question cannot be answered in its entirety without additional structure, because the inequality in (21) cannot be extended in general. However, we have the following analytical result for the

TABLE 1—TWO-STAGE, TWO-TYPES SIMULATION ($\gamma_1 = 2$, $\gamma_2 = 1$)

	Spread: $\Delta W_1/\Delta W_2$				
	1	2	3	5	8
A. Semifinals (s = 2)					
$x_2(1,1)$	120.3	118.1	116.6	115.1	113.9
$x_2(2,2)$	92.6	76.4	60.7	55.5	47.7
$x_2(1,2)$	92.7	82.6	76.1	68.3	62.0
$x_2(2,1)$	81.6	66.7	57.7	47.4	39.6
$P_2(1,2)$	.69	.71	.73	.74	.76
B. Finals (s = 1)					
$x_1(1,1) = x_1(2,2)$	83.3	125.0	150.0	178.5	200.0
$x_1(1,2) = x_1(2,1)$	74.1	111.1	133.4	158.8	177.8
Pr(1,1)	.24/.48	.26/.51	.28/.53	.28/.55	.29/.57
Pr(2,2)	.04/.09	.04/.08	.03/.07	.03/.06	.03/.06
Pr(1,2)	.72/.43	.70/.41	.70/.40	.69/.39	.68/.37
C. Expected Total Effort					
Random	540.7	574.7	598.9	616.0	632.4
Seeds	507.4	537.2	554.4	573.3	586.9

Notes: Simulation for $\gamma_1 = 2$, $\gamma_2 = 1$, $\eta = \varepsilon = \mu = 1.0$. The term $x_s(I, J)$ is equilibrium effort expended by player of ability type I in match against opponent of ability type J when s stages remain; $P_2(1,2)$ is probability strong player wins semifinal round match against weak opponent; $P_1(1,2) = 2/3$ because final-round equilibrium is symmetric. Finals pairing probability Pr(I, J) is equilibrium probability of type I against type J in finals: First number refers to *random* initial draw, second number to strong/weak *seeds* in first round. The last rows give expected effort summed over all players in all matches and stages.

last two stages:

If the prize distribution is linear at the top ($\Delta W_1 = \Delta W_2$), effort by both players in strong-weak matches is larger in the semifinals than in the finals; and effort in matches between similar types is also larger in the semi's than in the finals.

The first part follows from the fact that $\tilde{\beta}_1(1)$ necessarily exceeds $\tilde{\beta}_1(2)$; while the second part follows from Section III (and in fact holds true for all stages when the prize structure is linear everywhere). The best reply for each player in any type of match is larger in the semi's than in the finals when $\Delta W_1 = \Delta W_2$. Consequently, the extra incremental prize at the top remains necessary to extend the horizon and help insure that the final match is the best match.

A small simulation for a two-stage game illustrates these ideas and shows some effects of seeding. To simplify the calculations, I chose $h(x) = c(x) = x$ (so $\varepsilon = \eta = \mu = 1.0$). Further, $\gamma_1 = 2$ and $\gamma_2 = 1$: true-to-form odds in a (1,2) match are 2-to-1 in favor of type 1. The simulation assumes that the game begins with two players of each type. The total purse is fixed at 1000 and $W_3 = 0$. The rank-prize structure is linear when $\Delta W_1/\Delta W_2 = 1$.

The first two rows of Table 1 show what might happen in a random draw which pulls strong-strong and weak-weak in the initial round. This happens half the time and guarantees a strong-weak final match pairing. Column 1 demonstrates that finals effort is smaller than semifinals effort when prizes are linear. Comparing across columns, we see that the final-round increment has to be quite large for strong players to exert more effort in the finals than in the semi's. The results are qualitatively similar for the other initial-round pairing possibility. These mixed matches would be assured by seeing, but occur only half the time with random draws. Notice that effort is smaller in mixed matches than in like matches, and that effort differences across player types are smaller in mixed matches. Strong players work very hard at round 1 to knock each other off and get into the finals when they know that their next opponent will be weak.

The probabilities of various final match-type pairings are shown in panel B. Neither seeding nor random draw guarantees that the best players survive to the finals, but seeding *doubles* the probabilities that they do. Under random draw, the most probable (by far) final match is strong-weak. These probabilities are fairly insensitive to spread because the strong-player win-probability in a mixed first-round match is insensitive to the prize distribution with this parameter configuration. Notice that seeding makes a strong-strong final match the most probable outcome, but it comes at the cost of increasing the probability of a weak-weak final. However, this latter probability is small in either case.

Comparing across columns, we see that semifinals effort decreases and finals effort increases as the spreads grow larger. However, panel C shows that the second effect exceeds the first: expected total effort over all matches and stages increases with the spread. Most remarkably, total effort is greater when the initial draw is random than seeded. Seeding produces less variance in efforts in the first round, a lower mean in that round, and it most likely produces a better match among more talented opponents in the finals. The final interrank spread must be greatly elevated in the seeding game to produce expected total effort comparable to the no-seeding game. This suggests that seeds are observed when not simply total effort expended, but the distribution of the quality of play among players and stages, and guaranteeing the best match at the end, are important for the social productivity of the game. It justifies my reluctance to specify an additive social value function for the purposes of calculating an "optimal" prize structure.

V. Heterogeneous Contestants with Talents Unknown

Suppose we are interested in choosing the best out of T possible contestants. A round-robin design matches each player against every other and chooses the one with the largest overall win percentage. A sequential or knockout design eliminates a contender from further consideration after a certain number of losses. The sequential design promotes survival of the fittest and saves sampling costs by eliminating likely losers early in the game, but provides less precise information than the round-robin. The design choice comes down to comparing sampling costs with the value of more precision or the loss of making errors. H. A. David (1959, 1969) suggests that knockout designs have advantages over round-robins in selecting the best contestant, and Jean Gibbons, Ingram Olkin, and Milton Sobel (1977) prove it using sequential statistical decision theory. These issues are of great practical importance in medical trials. However, it is not possible to apply statistical decision theory alone to selection in human populations because no account is taken of contestants' incentives to optimize against the experimental design.

The main ideas are best illustrated in the case of "symmetric ignorance." Consider a sequential single elimination design, in which there are m types of contestants, all of whom share the same priors on the talents of others and who are equally ignorant about their own and others' talents. The distribution of types is common knowledge, and there is no private information. Estimates of own talents and the strength of the surviving field are updated as the game proceeds. This changing information feeds back into each contestant's strategy at every stage. When contestants have no more information about themselves than their surviving opponents do, it is clear that the interesting equilibrium is symmetric, because all survivors share the same information set—the same winning record, and choose the same strategy.

Let $\alpha_s(I)$ denote the probability that a player is type I when s stages remain to be played, and let $\tilde{\alpha}_s(J)$ denote the player's assessment that the current opponent is type J. Then, from Bayes' rule, the player's assessment of himself when $s-1$ stages remain, conditional on surviving (winning at stage s) is

$$(23)\quad \alpha_{s-1}(I) = \Pr(\text{win at stage } s|I)\,\alpha_s(I)\Big/\Pr(\text{win at stage } s) = \alpha_s(I)\sum_J \tilde{\alpha}_s(J)P_s(I,J)\Big/\sum_I\sum_J \alpha_s(I)\tilde{\alpha}_s(J)P_s(I,J)$$

where $P_s(I, J)$ is the win technology in (1); $\sum_J \tilde{\alpha}_s(J) P_s(I, J)$ is the conditional probability of winning given that one is type I. The denominator is the unconditional probability of winning at stage s. Since the initial prior is common, information is common at all stages, so $\alpha_s(I) = \tilde{\alpha}_s(I)$ in equilibrium. Furthermore, all contestants choose the same effort for given s, and $P_s(I, J) = \gamma_I/(\gamma_I + \gamma_J)$ in equilibrium: survival chances for each type run true to form at each stage. Finally, the unconditional equilibrium survival probability is always $1/2$ in paired comparisons. In equilibrium (23) becomes

$$(24) \quad \alpha_{s-1}(I) = 2\alpha_s(I) \times \sum_J \alpha_s(J)[\gamma_I/(\gamma_I + \gamma_J)].$$

Equation (24) implies survival of the fittest. To illustrate, suppose there are two types, with $\gamma_1 > \gamma_2$. Let α_s be the expected proportion of stronger (type-1) players alive at s. Then (24) is

$$(25) \qquad \alpha_{s-1} - \alpha_s = \alpha_s(1 - \alpha_s)\omega,$$

where $\omega = (\gamma_1 - \gamma_2)/(\gamma_1 + \gamma_2)$ is the difference in form probabilities between types. The solution to (25) looks like a logistic. The weak are eliminated at the largest rate when $\alpha_s = 1/2$, and are eliminated at a slower rate elsewhere. The rate of elimination of the weak also depends on ω. Convergence is very fast when ω is large. For example, if $\alpha_n = 1/2$ and ω is close to unity (its maximum possible value) over 99 percent of expected survivors are strong after only three stages. More stages are required to select the fittest members of the population the smaller the initial values of α and ω.

In choosing a strategy a player must assess own and opponents' talents at each stage. The problem is illustrated for the case of two types, strong (γ_1) and weak (γ_2). We have

$$(26) \quad V_s(\alpha_s, \tilde{\alpha}_s) = \max\{\Pr(\text{win}|\alpha_s, \tilde{\alpha}_s) \times [V_{s-1}(\alpha_{s-1}, \tilde{\alpha}_{s-1}) - W_{s+1}] - c(x_{si})\},$$

where the win probability is conditioned on the information available at the beginning of stage s:

$$(27) \quad \Pr(\text{win}|\alpha_s, \tilde{\alpha}_s) = \alpha_s[\tilde{\alpha}_s P_s(1,1) + (1 - \tilde{\alpha}_s) P_s(1,2)] + (1 - \alpha_s)[\tilde{\alpha}_s P_s(2,1) + (1 - \tilde{\alpha}_s) P_s(2,2)],$$

with $(\alpha_s, \tilde{\alpha}_s)$ updated according to (23). Thus in choosing x_s the player weighs the possibilities of own and opponent's talent pairings by the information currently available. This information depends on past data, exogenous as of stage s. The player's assessment of the future strength of an opponent, $\tilde{\alpha}_{s-1}$, depends on the given efforts of players in other matches. However, the player's assessment of his own talent in the next stage depends on today's actions and outcomes, from (23), and this (the value of information) also enters the calculation for choice of x_s. The Bayesian link between stages s and $s-1$ introduces an interstage linkage in strategies that is not present when talents are known.

The first-order condition for this problem is

$$(28) \quad \frac{\partial \Pr(\text{win}|.)}{\partial x_{si}}[V_{s-1}(\alpha_{s-1}, \tilde{\alpha}_{s-1}) - W_{s+1}] + \Pr(\text{win}|.)[\partial V_{s-1}(\alpha_{s-1}, \tilde{\alpha}_{s-1})/\partial \alpha_{s-1}] \times (\partial \alpha_{s-1}/\partial x_{si}) - c'(x_{si}) = 0.$$

The derivative $\partial \Pr(\text{win}|.)/\partial x_{si}$ is calculated from (27) and $\partial \alpha_{s-1}/\partial x_{si}$ is calculated from (23), both given x_{sj}. An expression for the information term $\partial V_{s-1}/\partial \alpha_{s-1}$ is found by applying the envelope property to (26):

$$(29) \quad \partial V_s(\alpha_s, \tilde{\alpha}_s)/\partial \alpha_s = [V_{s-1}(\alpha_{s-1}, \tilde{\alpha}_{s-1}) - W_{s+1}] \times (\partial \Pr(\text{win}|\alpha_s, \tilde{\alpha}_s)/\partial \alpha_s).$$

The symmetric solution is characterized by (28) evaluated at $\alpha_s = \tilde{\alpha}_s$ and $x_{si} = x_{sj}$ for all s.

Writing $V_s = V_s(\alpha_s, \alpha_s)$ detailed calculations at equilibrium yield

$$(30a)\quad \partial V_s/\partial \alpha_s = (V_{s-1} - W_{s+1})(\omega/2);$$

$$(30b)\quad \partial \Pr(\text{win}|.)/\partial x_{si} = (h'/h) \times\left[(1/4) - \alpha_s(1-\alpha_s)(\omega^2/2)\right];$$

$$(30c)\quad \partial \alpha_{s-1}/\partial x_{si} = -\alpha_s(1-\alpha_s) \times \omega\left[\left(\omega\alpha_s - \frac{\gamma_1}{\gamma_1+\gamma_2}\right)^2 + \frac{\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2}\right].$$

Condition (30a) shows that the value of continuation is increasing in own-assessment of talent, and that its incremental value is increasing in ω, the difference in form probabilities. The value of information is small when contestants are not very different from each other. The marginal effect of effort on winning (in (30b)) is decreasing in population heterogeneity (ω) and in the uncertainty with which players assess themselves at each stage (α). Uncertainty is a force that dampens incentives to perform and is greatest at $\alpha_s = 1/2$. This effect disappears as uncertainty is resolved. Equation (30c) shows that greater effort *reduces* the posterior assessment of strength.[6] Given the equilibrium effort of the opponent, the winning contestant is more probably of greater talent if less effort has been expended. The elimination design places extra value on strength, and private incentives to experiment to discover own strength is another force tending to make players hold back efforts at earlier stages. However, this term also vanishes as uncertainty is resolved.

Substituting (30) in (28) and manipulating into elasticity form, we have

$$(31)\quad c(x_s) = \left[(\mu/4) - A_s\right](V_{s-1} - W_{s+1}) - B_s(V_{s-2} - W_s),$$

where

$$(32)\quad A_s = \mu\alpha_s(1-\alpha_s)(\omega^2/2)$$

$$B_s = \mu(\omega^2/4)\alpha_s(1-\alpha_s) \times\left[\left(\omega\alpha_s - \frac{\gamma_1}{\gamma_1+\gamma_2}\right)^2 + \frac{\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2}\right] \quad \text{for } s \geq 2$$

$$B_1 = 0$$

The boundary condition $B_1 = 0$ holds because information has no value in the finals. Equation (25) is used to calculate A_s and B_s. Substituting into the value function and subtracting W_{s+2} provides a recursion for the increments $V_s - W_{s+2}$ in (31):

$$(33)\quad V_s - W_{s+2} = (\beta - A_s)(V_{s-1} - W_{s+1}) + B_s(V_{s-2} - W_s) + \Delta W_{s+1},$$

with boundary condition $V_0 - W_2 = \Delta W_1$.

Conditions (31) and (33) plus the calculation of A_s and B_s from (32) and (25) represent the complete solution of the symmetric ignorance problem. Notice that this solution converges in the limit to that of equal known talents (Section III) as α_s approaches unity, because A_s and B_s go to zero. Hence the extra increment in the final interrank spread is required for incentive maintenance in a sufficiently long game, irrespective of the initial distribution of talents.[7] By a similar

[6] Updating own-assessment of talent conditional on losing has no value because losers are eliminated. It has value in games with double or more eliminations, but equilibrium is not symmetric. Nor is it symmetric if contestants observe finer information than each player's previous win-loss record.

[7] The remaining case is one of private information, where each player knows his own type, but has prob-

token, the earlier result holds approximately when heterogeneity is small.

In fact, heterogeneity must be quite large for the value of information to have much effect on the incentive maintaining prize structure of Figure 2. For example, consider the case where $\gamma_1 = 2$, $\gamma_2 = 1$ and $\mu = 1$. The strong type wins two-thirds of the time and $\omega = 1/3$. Direct calculation reveals that B_s is of order 10^{-3}, and A_s is of order 10^{-2}. Therefore the second difference effects in (31) and (33) are negligible and Figure 2 is a very close approximation to the incentive maintaining prize structure. When the strong player wins three-fourths of the time, the corresponding orders of magnitude are 10^{-2} for both terms, so the approximation in Figure 2 remains very good: there are only a few minor wiggles.

Significant departures from Figure 2 occur when there are major differences between types, but this is mainly due to the incentive dilution effects of uncertainty. Even when the strong player wins 90 percent of the time, the terms in B_s remain of order 10^{-2} and the second difference (value of information) terms are negligible. But the terms in A_s show more variation with α_s. The term $(\beta - A_s)$ is smallest in those stages where uncertainty is largest. The interrank spread must be increased in those stages for x_s to be maintained, to overcome larger discounting of the future. Thus in a tournament where the proportion of strong players is relatively small in the first round, early-round incentive-maintaining prizes are approximately linear because there is little uncertainty. As the weak players are eliminated and α_s rises toward $1/2$, uncertainty is *increasing* and the interrank spread has to increase to overcome this effect. If the game is long enough for α_s to exceed $1/2$, uncertainty is decreasing and interrank spreads are decreasing for incentive maintenance. They increase again toward the end, due to the horizon effects. If the initial field is equally split ($\alpha_N = 1/2$), resolution-of-uncertainty acts to distribute the prize money more equally across the ranks. If the initial proportion α_N is small and the game is long, incentive-maintaining prizes redistribute from the extremes toward the middle.

Finally, the expected selection recursions in (24) show that the social value of information is independent of x_s in the symmetric equilibrium: all information in selecting strong players for survival is embedded in the elimination design itself, and incentives for contestants to produce private information come to naught. The attempt by all players to gain informational advantages in calculating their private strategies cancel each other out because of the ordered quality of competition. No one obtains an informational edge over that inherent in the design. There is a role for the prize structure to discourage these socially useless actions, and this requires less concentration of the prize money at the top, to reduce the private value of information. However the calculations above suggest that these effects are relatively minor unless differences in talents are large.

VI. Conclusions

The chief result is identifying a unique role for top-ranking prizes to maintain performance incentives in career and other games of survival. Extra weight on top-ranking prizes is required to induce competitors to aspire to higher goals independent of past achievements. There are many rungs in the ladder to aspire to in the early stages of the game, and this plays an important role in maintaining one's enthusiasm for continuing. But after one has climbed a fair distance, there are fewer rungs left to attain. If top prizes are not large enough, those that have succeeded in achieving higher ranks rest on their laurels and slack off in their attempts to climb higher. Elevating the top prizes effectively lengthens the ladder for higher-ranking contestants, and in the limit makes it appear of unbounded length: no matter how far one has climbed, there is

abilistic assessments of opponents' types. Then Bayesian updating applies to opponents only. The analysis of this case is conceptually straightforward, but the equilibria are not symmetric and few analytical results are available. It is omitted for that reason. Still, the result on concentration of the purse on the top applies because survivors at the last few stages are relatively homogeneous.

always the same length to go. In examining the relation between wages and marginal products, the concept of marginal productivity must be extended to take account of the value to the organization of maintaining incentives and selecting the best personnel to the various rungs, not only the contribution at each step. Payments at the top have indirect effects of increasing productivity of competitors further down the ladder.

There is another interesting class of questions in this type of competition. Adam Smith held the opinion that there is natural tendency for competitors to overestimate their survival chances ("overweaning conceit"), while Alfred Marshall held the opposite opinion. Further analysis shows how biased assessments of talent affect survival. There is a clear disadvantage to pessimism and underestimation of own talents. The pessimist doesn't try hard enough because opponents appear relatively stronger, and also because the true value of continuation is underrated. An elimination design is disadvantageous to the timid. They do not survive very long. The effects of overestimation and optimism are more complicated. For strong players and among any contestants in a field of comparable types, optimism has two effects: the optimist has a tendency to slack off due to underestimation of the relative strengths of the competition, but overestimates the own-value of continuation, which induces greater effort. Optimism has no clear-cut effects on altering survival probabilities. However, the second effect vanishes in the finals, and winning chances are reduced. Optimism has positive survival for weak players in a strong field. A weaker player who feels closer to the average field strength than is true, works harder on both counts and is not eliminated as quickly as another weak competitor with more accurate self-assessments.

When contestants' abilities are unknown, private incentives to optimize against the design for personal informational advantage lead to socially useless actions. These in the end do not produce any more information than is already embodied in the game itself and must be discouraged by concentrating less of the purse at the top. There are also private incentives for a contestant to invest in signals aimed at misleading opponents' assessments. It is in the interest of a strong player to make rivals think his strength is greater than it truly is, to induce a rival to put forth less effort. The same is true of a weak player in a weak field. However, it is in the interests of a weak player in a strong field to give out signals that he is even weaker than true, to induce a strong rival to slack off. Weighting the top prizes less heavily reduces these inefficient signaling incentives.

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